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Convection experiments in fluids with highly temperature-dependent viscosity and the thermal evolution of the planets

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Convection experiments in fluids whose viscosity depends on temperature were carried out. Ratios of the top viscosity over the bottom viscosity up to 10^5 have been studied. Horizontally-averaged temperature versus depth profiles are presented, as well as accurate Nusselt number measurements and temperature isogradient cell pattern for Rayleigh numbers (R) in the moderate range. The difference between the actual interior temperature and the mean of the top and bottom temperatures increases with the viscosity ratio. This difference reaches 25% of the overall temperature drop for a viscosity ratio of 10^5. As previously reported, the Nusselt number (Nu) first decreases when the viscosity ratio increases, as compared to the constant viscosity case, provided the Rayleigh number is defined using the viscosity at the mean of the top and bottom temperatures. However, the trend is reversed for viscosity ratios larger than ~5000. This is consistent with an $Nu-R$ relationship based on the critical Rayleigh number.

All planetary thermal evolution models using the parameterized convection approach divide the convective planet into a conductive lid and a convective region of constant viscosity underneath. Due to the lack of knowledge about convection in a temperature-dependent viscosity fluid, assumptions have to be made in order to define the cut-off between the two regions. The most commonly made assumptions are not supported by our experiments. Our experimental results can be used for extrapolating to situations of geophysical interest and for answering basic questions such as: what is the interior temperature of the convective system as a function of the rheological law and heat flux, and, what is the thickness of the lid?

1. Introduction

It has long been recognized that the thermal evolution of the planets is critically regulated by the rheological behavior of their interiors. Indeed, the efficiency of convection in removing heat from a planetary body depends on the viscosity, which in turn is a strong function of its temperature. Thus, the internal temperature of a planet is expected to be regulated by the coupled action of convection and temperature-dependent rheology. Tozer has developed and defended this argument in a number of papers (e.g. Tozer, 1965, 1967, 1972). The point has been accepted and most thermal evolution models from that time on take the regulating effect into account. Often in these models, convection takes place under a rigid lid, i.e. the lithosphere. Tozer's argument is applied to the convective region but, in order to get a complete thermal model, it is then necessary to choose a criterion for defining the lid. This criterion varies widely from author to author and rests upon as-

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sumptions which are still to be tested. Indeed, while a large number of such parameterized models have appeared in the literature, little work has been done to increase our basic knowledge and physical understanding of convection in a temperature-dependent viscosity fluid. After the early numerical results of Foster (1969) and Torrance and Turcotte (1971), interest was renewed in the subject by the experiments of Booker (1976) and their interpretation by Booker and Stengel (1978). The relationship between Nusselt and Rayleigh numbers (Nu, R) derived from their experiments has been widely used, and it is only recently that other aspects of convection in variable viscosity fluids have been investigated. This includes studies of the planform of convection (Richter, 1978; White, 1982) and calculations for the onset of convection for a variety of viscosity laws (Jaupart and Parsons, 1982). The finite-amplitude range has been addressed by several numerical experiments (i.e. Torrance and Turcotte, 1971; Houston and De Bremaecker, 1975; Hsui, 1978). A more systematic study of the physics of convection with temperature-dependent viscosity is now developing (Daly, 1978, 1980a, b; Ivins et al., 1982; Kenyon and Turcotte, 1981; Fowler, 1981; Morris, 1981). The experiments described here are mainly aimed at a better understanding of the physical aspect of convection with temperature-dependent viscosity. At the same time, we summarize approaches used in planetary thermal evolution modeling and show how our experiments can be used to test some of these. In section 2 we present a summary of these models and stress the assumptions they are based on, and formulate simple questions that ultimately we want to answer using our experimental data and related theoretical considerations. In section 3 the experimental set-up is briefly described, emphasizing the thermal structure of the convective state. This includes accurate measurements of the Nusselt number, profiles of the horizontally-averaged temperature versus depth, and interferometric photographs of the cells' temperature structure. Results for viscosity ratios up to 10^5 in the moderate Rayleigh number range are presented in section 4. Our Nusselt number measurements are in fair agreement with previous ones (Booker, 1976; Booker and Stengel, 1978) and the empirical law found by these authors can be extended to higher viscosity ratios. However, we issue a warning against the misuse of this law in geophysical situations. In section 5, we suggest a simple method for extrapolating our results to much higher viscosity ratios. The study of convection in a temperature-dependent viscosity fluid is only one (important) step towards a clearer picture of the thermal state of the planets and some of the limitations and desirable extensions are pointed out.

2. Thermal evolution models for the planets

An abundant geophysical literature has built up in recent decades which uses a parameterized convection approach to construct thermal evolution models. Such models have proved useful in addressing and answering questions such as the internal temperature of the planets (Tozer, 1972), tectonic regimes in the past (McKenzie and Weiss, 1975), balance between heat production and heat flux (Daly, 1978; Daly and Richter, 1978; Schubert et al., 1980), heat sources distribution and the depth of convection (Sharpe and Peltier, 1978; Richter and McKenzie, 1981b; Schubert and Spohn, 1981), to quote only a few. Instead of solving the complete set of equations which govern convection, parameterized convection uses the relationship which relates the Nusselt number to the Rayleigh number independent of the actual details of the convective structure. The Nusselt number describes the efficiency of convection in transporting heat: it is the ratio of the convective heat flux over the heat flux one would get if heat was conducted across the layer, or alternatively it is the vertical temperature difference required to conduct a quantity of heat across the layer divided by the (smaller) temperature difference required by convection to carry the same amount of heat. In the constant viscosity case and with isothermal boundaries, the Rayleigh number is an external parameter defined as

$$R = (\alpha g d^3 \Delta T)/(\kappa \nu) \quad (1)$$

where α is the coefficient of thermal expansion, g is the acceleration due to gravity, d is the depth of the fluid layer, ΔT is the temperature drop across it, κ is the thermal diffusivity and ν is the kinematic viscosity. The Rayleigh number describes the propensity of the fluid to convect: the larger the number, the more vigorous is the convection. Similar Rayleigh numbers can be defined for different conditions (internal heating, constant heat flux, etc.) by using other externally-given parameters (heat production, heat flux, etc.). The relationship

$$Nu = a(R/R_c)^{\beta} \tag{2}$$

where β is between 1/4 and 1/3, and R_c is the critical Rayleigh number, gives good results for quite a variety of conditions (see Rossby, 1969; Turcotte and Oxburgh, 1967; Sharpe and Peltier, 1978; Schubert, 1979; Richter and McKenzie, 1981a). The existence of such a simple relationship is basically due to the way convection 'works': convective motions are organized in cells whose typical dimensions are of the same order as the depth of the convective layer; heat is conducted through a boundary layer at the bottom, advected in plumes around an isothermal core to the upper boundary layer which conducts it to the top surface. However, when there is a very large viscosity variation across the layer, one expects a thick rigid lid to develop on top of the convective region where Nu–R relationship applies. The problem is then to determine the thickness of the 'lid' or the temperature drop across it. In other words, one has to define where the cut-off between a more or less rigid and conducting lid and the convective regime takes place. This cut-off presumably depends on the viscosity law and on the vigor of convection (i.e. the Rayleigh number).

The central issue, be it in the laboratory or in nature, is simply stated. Suppose we know the rheology, the heat flux at the surface and other 'external' parameters such as conductivity, depth. What then is the internal temperature, what is the temperature at the base of the layer, and what is the thickness of the 'lithosphere'? At present, the physics of temperature-dependent viscosity convection is not known well enough to give an answer to these questions of geophysical interest. Reflecting the incomplete knowledge in this domain, different answers have been given to the cut-off question. The answers, which are necessary for a complete thermal model, rest on 'reasonable' assumptions that vary from author to author. In his original approach, Tozer (1972) takes the viscosity at the base of the lid to calculate the Rayleigh number and the parameterized Nusselt number for the convective region beneath the lid, and then assumes that the lid thickness is the one which minimizes the bottom temperature (i.e. maximizes the overall Nusselt number). Sometimes a constant lid thickness is assumed: for lunar models, Turcotte et al. (1972) choose 300 km whereas Turcotte et al. (1979) take 200 km. However, most authors favor the constant viscosity cut-off assumption: a particular value of the viscosity is used to define the base of the lid. Quite a variety of values can be found in the literature. McKenzie and Weiss (1975) choose 10^{21} poise in their model for the upper mantle of the Earth; Sharpe and Peltier (1978) choose 10^{25} poise under continents in a whole mantle model; Schubert et al. (1980) take 10^{27} poise in their lunar model; 10^{25} poise is chosen by Turcotte et al. (1979) for Venus and Earth models. In all these models, the Rayleigh number used in the parameterization is defined with the viscosity in the interior of the convective region. Note that when the surface rigid lid actually participates in convection (the oceanic plates on Earth), then the argument developed above of a conductive lid breaks down as the convective boundary layer then extends to the surface. Although the convective behavior is even more complicated and poorly understood in that case, boundary layer arguments suggest that the simple $Nu \alpha R^{1/3}$ relationship might hold for the system as a whole (Olson and Corcos, 1980). Thermal models of this latter type for the Earth often have no 'lid', and have a free surface instead (Sharpe and Peltier, 1978; Turcotte et al., 1979; Davies, 1980; Schubert et al., 1980). Clearly, experiments with rigid boundaries are not designed to deal with this problem. On the other hand, such experiments can help us in getting a better understanding of convection in a temperature-dependent viscosity fluid and testing some of the assumptions used in the thermal models with a lid.

3. Experimental set-up

In our experiments, attention was directed towards the thermal structure of the convective fluid. Three techniques were used:

(1) a differential interferometry technique which gives an instantaneous global picture of the vertical or horizontal gradient of temperature in the fluid seen in a vertical cross-section, thereby obtaining quantitative information on the structure of cells and the within-cell convection;

(2) a platinum wire resistance technique by which horizontally-averaged temperature can be measured as a function of depth;

(3) a heat flux measuring method which enables us to obtain accurate Nusselt number measurements.

The convective fluid is contained in a box whose horizontal boundaries are thick copper plates kept at uniform temperature, and the vertical ones are 1 cm-thick insulating lucite walls. L-100 polybutene or Tate and Lyle's Golden Syrup were used as working fluids depending on the degree of viscosity variation desired. Figure 1 shows the kinematic viscosity of these two fluids as a function of temperature. Temperatures are set from -20 to $+80°C$. Viscosity ratios (viscosity at the top over viscosity at the bottom) from 10 to 10^3 are obtained with the polybutene and from 10^3 to 10^5 with the Golden Syrup.

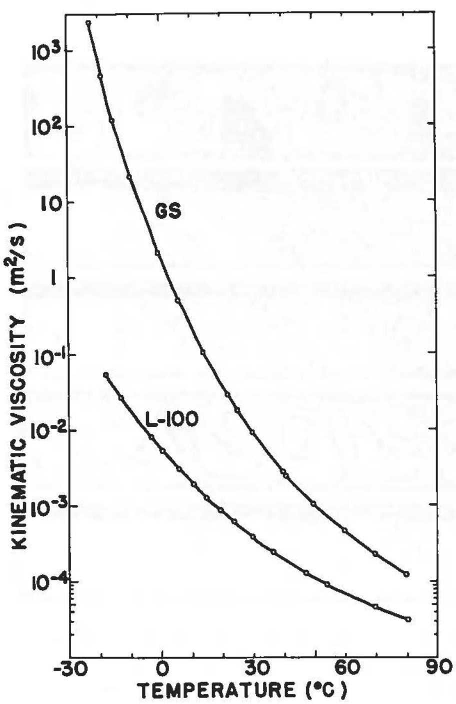

Fig. 1. Kinematic viscosity of L-100 polybutene and Golden Syrup (GS) as a function of temperature. Viscosity ratios up to 2×10^5 were achieved in our experiments.

The experimental set-up for differential interferometry is very similar to the one described in Nataf et al. (1981). For this set-up, the internal dimensions of the frame which contains the fluid are: 1 cm × 2 cm × 20 cm. The width is only twice the depth so that a reasonable number of isogradient fringes is obtained. However, all the temperature profiles and Nusselt number measurements are performed in a larger, otherwise similar, tank in order to decrease the influence of the lateral boundaries. The horizontal dimensions are then 10 cm × 20 cm and the depth is either 1 or 2.4 cm.

The horizontally-averaged temperature was obtained by measuring the electrical resistance of several platinum wires which run in a horizontal plane within the fluid. The accuracy was $\sim 0.1°C$. Temperature versus depth profiles were obtained by changing the depth of the measuring plane from the top plate to the bottom one. The depth of the wires is controlled by two verniers with an accuracy of ~ 0.1 mm. The Nusselt number measurements were done by monitoring the electrical dissipation in resistance wires within the bottom copper plate whose temperature was measured by thermocouples. In order to minimize heat losses other than to the fluid, the bottom plate was placed in a lucite box which was itself contained in an aluminum guard heater whose temperature was set as nearly as possible equal to the temperature of the copper plate in contact with the fluid. The heat loss correction was then less than 2%.

Further details of the experimental set-up and procedure are given in Richter et al. (1982).

4. Results

In the following, unless otherwise specified, the viscosity at the mean of the top and bottom temperatures is used to define the Rayleigh number (as in Torrance and Turcotte, 1971; Booker, 1976). The interferometric photographs show quite a variety of patterns depending on the viscosity ratio and the Rayleigh number. Figure 2 gives a few examples. One striking feature is the very small wavelength which characterizes experiments with high viscosity ratio, especially when the Rayleigh number is large. This is due to the concentration of the motion towards the bottom where the viscosity is lowest. However, one should keep in mind that for these runs the width of the tank is only twice its depth. The motion in the highly viscous top part might thus be made even more difficult. The 'eyes pattern' in the horizontal isogradient photographs correspond to uprising currents when they are close to the bottom and to downwelling currents when near the top (see Nataf et al., 1981). A marked asymmetry between the uprising and downwelling currents is shown by the isogradient photographs. The horizontal gradient is larger in the downwelling currents than in the uprising ones.

The existence of a lid (the very viscous region at the top) where the heat transfer is mainly by conduction is best seen on the horizontally-averaged temperature versus depth profiles. Figure 3 shows some of these profiles. Most remarkable is the increase of the interior temperature when the viscosity ratio is increased. In a temperature-dependent viscosity fluid, convection 'chooses' an interior temperature much higher than the mean of the top and bottom temperatures (which is the interior temperature for a constant viscosity fluid). We define the 'offset' ϵ as the difference between the actual interior temperature T_{int} and the mean of the top and bottom temperatures $\bar{T}$, normalized by the total temperature difference ΔT. For a given viscosity ratio, the offset is independent of the Rayleigh number (see Fig. 3) provided the Rayleigh number is large enough. Actually, the offset, as we define it, is zero for any viscosity ratio at the convection threshold. However, for a given viscosity ratio, it reaches an equilibrium value for a Rayleigh number of ~ 6000. For a Rayleigh number of ~ 30000, another transition

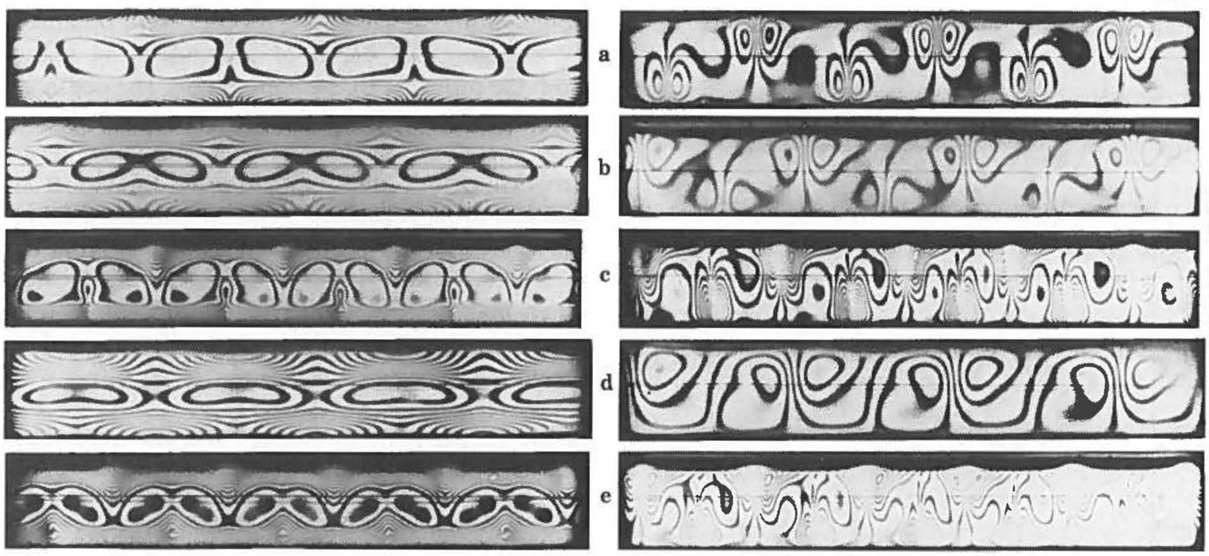

Fig. 2. Vertical (left) and horizontal (right) isogradients of temperature. Convection cells are seen as in a vertical cross-section. The difference in gradient between two adjacent black fringes is 13°C cm^{-1}. Plate temperatures are (in °C) as follows. Top: (a) 20.0; (b) 9.6; (c) 0.0; (d) −16.7; (e) −14.7. Base: (a) 60.1; (b) 49.8; (c) 80.0; (d) 46.9; (e) 14.8. Viscosity ratios: (a) 12; (b) 18; (c) 180; (d) 360; (e) 980. Rayleigh numbers: (a) 20000; (b) 9800; (c) 39500; (d) 5000; (e) 22000. Note the asymmetry between uprising and downwelling currents, the decrease of the wavelength for high viscosity ratios, and the shift of the center of the fringe patterns towards the bottom.

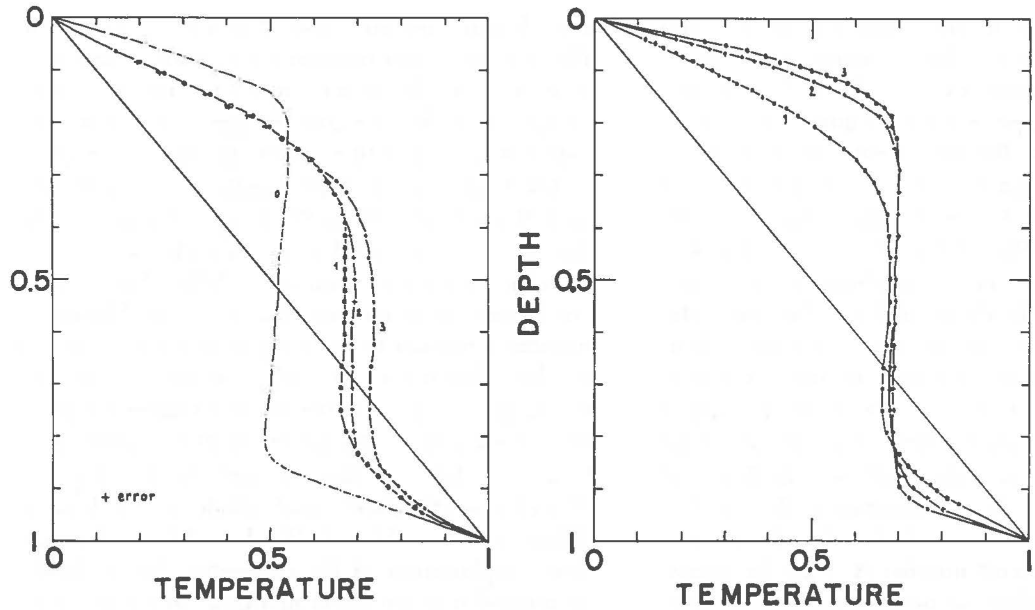

Fig. 3. Horizontally-averaged temperature versus depth profiles. On the left, experimentally measured profiles are shown for increasing viscosity ratios. The profile labeled 0 is for silicon oil with a viscosity ratio of 2 and a Rayleigh number of 1.2×10^5. The three other profiles have the same Rayleigh number 1.2×10^4 and increasing viscosity ratios: (1) 460; (2) 4150; (3) 49000. Note the increase of the interior temperature with increasing viscosity ratio. On the right, all profiles correspond to the same viscosity ratio of 180. The Rayleigh numbers are: (1) 2.35×10^4 (2) 2.05×10^5 (3) 5.90×10^5. They all give almost the same value of the interior temperature.

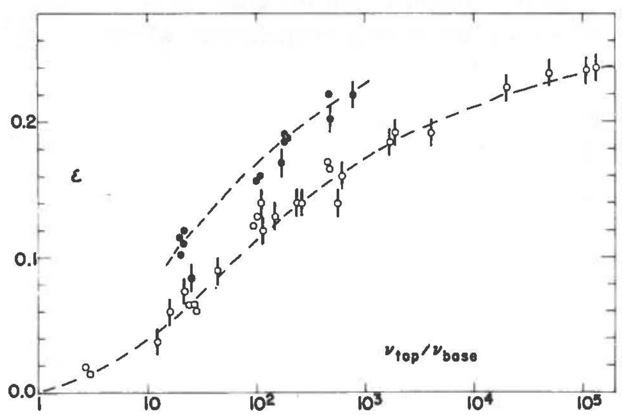

Fig. 4. Offset of the interior temperature versus viscosity ratio. The offset ϵ is defined as the difference between the interior temperature and the mean of the top and bottom temperatures, normalized with the total temperature drop across the layer. Full dots are for Rayleigh numbers larger than 30000. Circles are for Rayleigh numbers less than 30000. The two sets of data roughly define two curves (dashed lines).

seems to occur which brings the offset to a higher equilibrium value. As shown in Fig. 4, the offset values thus roughly define two curves when plotted against the viscosity ratio. Numerical experiments using the mean field approach (see Daly, 1980b) also show that the offset reaches an equilibrium value when the Rayleigh number is increased. In these numerical experiments, the sub-transition is not seen however, and the offset values fall on the upper curve of the laboratory results (S.F. Daly, personal communication, 1981). It is therefore likely that this transition corresponds to a change in the pattern of convection, presumably the appearance of the spoke regime (D.B. White, personal communication, 1981).

The Nusselt number describes the efficiency of convective heat transport. It has been noted that the Nusselt number decreases when the viscosity ratio is increased and the Rayleigh number kept constant (Torrance and Turcotte, 1971; Booker, 1976). This is when the Rayleigh number is defined using the viscosity at the mean of the top and bottom temperatures. For a constant viscosity fluid, the relationship $Nu_0 = 0.184\ R^{0.281}$ describes very well the experimental results when R is larger than 4000 (Rossby, 1969). Booker (1976) reports Nusselt number measurements for L-8 polybutene in the viscosity ratio range 1 to 300 and Rayleigh numbers between 1.5×10^4 and 4×10^5. Using Rossby's (1969) relationship as a reference, Booker (1976) shows that the Nu/Nu_0 ratio decreases when the viscosity ratio increases (Nu is the measured Nusselt number for the considered viscosity ratio and Nu_0 is the Nusselt number given by Rossby's relationship at the same Rayleigh number). Using the same data set, Booker and Stengel (1978) find that the decrease of the Nu/Nu_0 ratio can be entirely accounted for by the increase of the critical Rayleigh number R_c with the viscosity ratio. Indeed these authors claim that the relationship $Nu = 1.49\ (R/R_c)^{0.281}$, where R_c is the critical Rayleigh number for the considered viscosity ratio, fits all their data. We performed accurate measurements of the Nusselt number for viscosity ratio up to 2×10^5. Figure 5 shows the Nu/Nu_0 values we find together with Booker's results and Booker and Stengel's empirical relationship. For a given viscosity ratio, R_c is actually very sensitive to the chosen viscosity law. The curve shown on Fig. 5 is for L-100 polybutene at a mean temperature of 25°C. However, curves for the R_c values computed by Booker and Stengel (1978) for their experiments fall within 1% of the chosen curve.

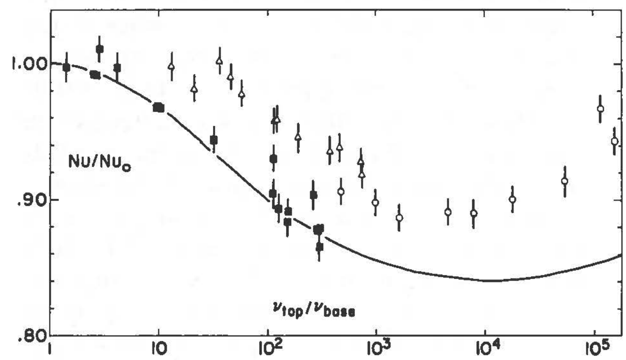

Fig. 5. Nu/Nu_0 versus viscosity ratio. The Nusselt number is normalized using Nu_0 the Nusselt number for a constant viscosity fluid at the same Rayleigh number. Full squares are Booker's (1976) data. The curve is the empirical relationship of Booker and Stengel (1978): $Nu(p_\nu)/Nu_0 = (1707.7/R_c(p_\nu))^{0.281}$ calculated from the critical Rayleigh numbers R_c we computed for L-100 polybutene at a mean temperature of 25°C. Triangles are our results for L-100 polybutene and circles are for Golden Syrup.

Qualitatively, our experiments are in agreement with Booker and Stengel's idea. In particular, the Nu/Nu_0 ratio is found to increase after the viscosity ratio reaches a value of ~ 5000. This is precisely what we would expect as the critical Rayleigh number is shown to decrease after such a value is reached (Richter et al., 1982). However, the two investigations do not give the exact same results in the viscosity ratio domain where they overlap. This is due in part to uncertainties in the material properties of the fluids used which can produce an almost uniform offset in Nu/Nu_0. This is the most likely explanation of the difference but it should be noticed that our experiments are at lower values of the Rayleigh number (from 10^4 to 3×10^4).

At this point, we return to the planetary evolution models and see how well the assumptions they use work in terms of our experiments. Booker and Stengel's (1978) paper, as Schubert's (1979) review and Davies' (1980) paper among others, implies that the relationship in eq. 2 can be applied in thermal evolution models. However, Booker and Stengel's relationship applies when the Rayleigh number is defined with the viscosity at the mean of the top and bottom temperatures, whereas most

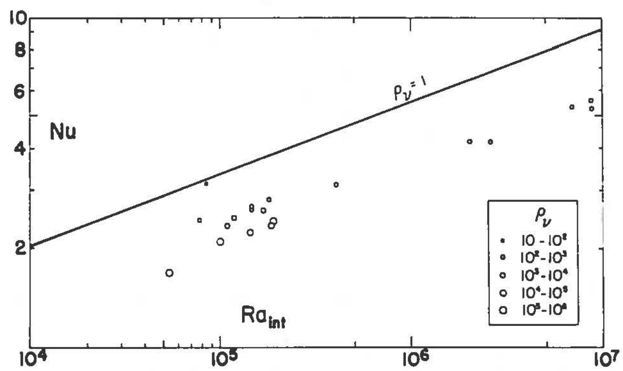

Fig. 6. Nusselt number versus the Rayleigh number used in most thermal evolution parameterized models: the 'interior' Rayleigh number (see text). Our data points fall increasingly below the constant viscosity curve when the viscosity ratio increases.

thermal models use a Rayleigh number based on the viscosity in the interior or the viscosity corresponding to the average temperature of the layer. As pointed out by Daly (1978, 1980a), these two quantities are very different due to the 'offset' of the interior temperature we described above. In Fig. 6, we plot our Nusselt number measurements against the Rayleigh number usually used in thermal evolution models, that is

$$R_{\text{int}} = \frac{\alpha g d^4 q}{k \kappa \nu_{\text{int}}} \tag{3}$$

where q is the heat flux and ν_{int} is the viscosity corresponding to the interior temperature. The latter is readily deduced from our horizontally-averaged temperature profiles. When the viscosity ratio increases, the Nusselt number values fall increasingly down below the constant viscosity curve, and errors as large as a factor of 2 will be made by using the constant viscosity relationship.

The viscosity cut-off assumption can also be tested in our experiments once we define a 'lid'. It is tempting to choose the transition temperature as that above which an approximately symmetrical temperature profile is achieved. The horizontally-averaged temperature profile is then divided into two portions: a lid at the top between dimensionless temperatures 0 (top) and 2ϵ, and a convective region from 2ϵ to 1 (bottom). The offset ϵ and the thickness δ of the lid thus defined can be measured from the temperature profiles. These show a constant temperature gradient in the 'lid' so that the assumption of a conductive lid seems reasonable. However, if we define a reduced Rayleigh number and a reduced Nusselt number for the convective sublayer using the ϵ, δ and ν_{int} values measured, the reduced Nusselt number is up to 15% higher than the one predicted by the constant viscosity Nu–R relationship for the reduced Rayleigh number. This suggests that the convective velocity in the lid, although very small, contributes to the heat transport by a non-negligible amount. It is likely that the use of a strain-averaged viscosity in the sense of Parmentier et al. (1976) would give a result closer to the constant viscosity case (S.F. Daly, personal communication, 1981; see also Jaupart and Parsons, 1982). This cannot be done for our experiments, however, as we did not measure velocities.

With our definition of the lid, it is simple to show that the assumption of a cut-off viscosity at its base does not hold for our experiments. Imagine the following experiment: for a fixed upper temperature (taken to be zero) and for a given fluid, the temperature drop ΔT is increased across the fluid. The temperature at the base of the lid is $2\epsilon\,\Delta T$. The viscosity ratio ρ_ν increases with ΔT so that ϵ increases with ΔT as well. Thus, the temperature at the base of the lid increases yielding a decrease of the viscosity at the base of the lid. The only way to keep the latter constant would be to have the offset decreasing when the viscosity ratio increases. This is clearly not the case in our experiments.

5. Discussion

The demonstration of the inadequacy of some widely used assumptions when applied to our experiments need not imply that these are equally inadequate when applied to the planets. In many respects our experiments are in a transitional parameter range which is not directly relevant to the discussion of planetary evolution. However, the problem of large viscosity variations has to be properly addressed and experiments of the kind shown here can hopefully guide us in this task. Indeed a simple argument, namely that when the top part of the convective layer is really behaving rigidly one can add some more rigid material on the top without changing the circulation underneath, shows that results in the transitional viscosity ratio range can be extrapolated to very large viscosity ratios (Richter et al., 1982). Ultimately, the understanding of convection in a fluid with temperature-dependent viscosity and its parameterization must be based on careful laboratory and numerical experiments if the results are to be believed.

One obvious drawback of our approach when looking at the Earth is that for the Earth the rigid top layer can play an active role in convection, this being the case at least in the oceanic regions. Clearly the results of our experiments and extrapolations where the lid behaves as a rigid conductive passive top will not apply to all parts of the Earth. Even with free boundaries, convection takes place under a rigid lid when the viscosity ratio is large enough (Torrance and Turcotte, 1971; Daly, 1980a; Jaupart and Parsons, 1982) and thus it appears that the type of 'plate tectonics convection' the Earth exhibits is very peculiar. It might be possible to account for it when zones of weakness are present in the rigid lid (Kopitzke, 1979; Jacoby and Schmeling, 1982). Even so, we are far from having a consistent parameterized scheme to use in that case. The horizontal dimensions of the plates might play a critical role (Daly, 1980a). Furthermore, a third of the surface of the Earth is covered with continental lithosphere which does not participate actively in convection. Therefore, the two kinds of regime (with and without a passive lid) tend to be present on Earth and interactions between the two modes might be very important for the total thermal balance (Sharpe and Peltier, 1978; Rabinowicz et al., 1980; Nataf et al., 1981).

Thermal convection in planets with a rigid outer shell is certainly easier to handle and lies more within the scope of this paper. The results we have presented here and previous authors' work begin to address some of the basic questions in this domain.

Acknowledgments

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