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A SIMPLE METHOD FOR INVERTING THE AZIMUTHAL ANISOTROPY OF SURFACE WAVES

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Abstract. We investigate the problem of retrieving anisotropy as a function of depth in the mantle, from the observed azimuthal variations of Love and Rayleigh wave velocities. Following the approach of Smith and Dahlen, this azimuthal dependence is expressed in terms of a Fourier series of the azimuth θ. For the most general case of anisotropy (provided it is small enough), some simple linear combinations of the elastic tensor coefficients are shown to describe the total effect of anisotropy (both polarization anisotropy and azimuthal anisotropy) on the propagation of surface waves. For the terms that do not depend on the azimuth the combinations are related to the elastic coefficients of a transversely isotropic mantle. For the azimuthal terms the relevant combinations are explicited. It is found that the partial derivatives of the azimuthal terms with respect to these combinations are easy to compute for they are proportional to the partial derivatives of a transversely isotropic model in the case of a plane-layered model. In a first approximation the same property holds true for a spherical earth and we calculate from PREM all the partial derivatives needed for performing the inversion of the azimuthal anisotropy of surface waves in the period range 50-300 s. It is observed that very shallow anisotropy can be responsible for substantial azimuthal variations up to the longest periods. With this approach it is also easy to compute the azimuthal variations of surface wave velocities produced by any anisotropic model. When a C_{ij} elastic tensor is chosen for the upper mantle, azimuthal variations up to 2% are obtained for Rayleigh waves. The azimuthal variations of Love wave velocities are very small. The 2θ term of the azimuthal variations of Rayleigh wave velocities is the dominant term. Its fast axis corresponds to the fast axis of P waves.

Introduction

The evidence that much of the upper mantle may be anisotropic is steadily growing. In particular, the interest in azimuthal anisotropy is increasing. Since intrinsic anisotropy requires that crystals be both anisotropic and oriented in preferential directions, observations of seismic anisotropy can provide information about the mineralogy and the deep structure of the mantle. Over the last few years, there has been a large increase in the number of observations implying anisotropy in the earth (for a review, see Crampin et al. [1984]). The early evidence was the discrepancy between Rayleigh and Love wave dispersion [Anderson, 1961, 1966; Aki and Kaminuma, 1963], and the azimuthal dependence of P_n velocities [Hess, 1964; Morris et al., 1969; Raitt et al., 1971]. Azimuthal variations have now been well documented for different areas in the world for body waves [Bibee and Shor, 1976; Bamford, 1977; Hirn, 1977; Talandier and Bouchon, 1979; Fuchs, 1983] as well as for surface waves [Forsyth, 1975; Mitchell and Yu, 1980; Montagner, 1985]. The Rayleigh - Love discrepancy is widespread for fundamental modes [McEvilly, 1964; Forsyth, 1975; Schlue and Knopoff, 1979; Journet and Jobert, 1982; Montagner, 1985] but also for overtones [Leveque and Cara, 1983]. This discrepancy is accounted for when using a transversely isotropic medium with a vertical symmetry axis (PREM of Dziewonski and Anderson [1981]). On a global scale, long-period surface waves have been used to map lateral variations of anisotropy in the upper mantle within this framework of a transversely isotropic earth [Nataf et al., 1984]. Tanimoto and Anderson [1984] have recently determined the azimuthal variation of Love and Rayleigh wave phase velocities on a global scale. They show that the fast directions of Rayleigh waves appear to correlate with flow directions in the mantle, but it is not known to what depth the anisotropy extends.

As we see, observational surface waves studies call for two kinds of anisotropy: On the one hand, variations of phase or group velocities with the azimuth of the path (azimuthal anisotropy); on the other hand inconsistent values for the azimuthally averaged velocities of Love and Rayleigh waves, explained in terms of transverse isotropy (also called polarization anisotropy). From an observational point of view it must be noted that there is a difference between these two forms of anisotropy. Azimuthal anisotropy is observed directly for body waves as well as for surface waves; on the contrary, transverse isotropy is not observed directly but is inferred from the inversion of Rayleigh and Love wave data. To our knowledge, no inversion has been performed that takes into account simultaneously azimuthal anisotropy and Rayleigh - Love discrepancy.

From a theoretical point of view the propagation of surface waves in a completely

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anisotropic plane-layered medium has been studied by Crampin [1970] and applied to a few special cases [Crampin, 1970; Maupin, 1985]. Smith and Dahlen [1973] have studied the effect of a slight elastic anisotropy on the propagation of Love and Rayleigh surface waves. They give the azimuthal dependence of the phase velocities of surface waves, which involves the variation with depth of some combinations of the canonical harmonic components of the elastic tensor, which are hard to handle.

The purpose of the present paper is to provide the basis for an inversion method of the azimuthal variations of surface wave velocities. From the azimuthal variations at different periods our goal is to retrieve the variations with depth of some combinations of the elastic coefficients. Using Smith and Dahlen's approach, we show that the partial derivatives involved can be calculated very simply. The combinations of the elastic coefficients are explicited. The method is then applied to two realistic mantle models in order to assess the relative importance of the different terms involved. It is shown that depending on whether the anisotropy is confined to the lithosphere or to the asthenosphere, the resulting azimuthal variation of the phase velocity of Rayleigh waves presents a different signature.

Relating Local Elastic Coefficients to the Anisotropy of Surface Waves

We follow the approach of Smith and Dahlen [1973]. They calculate the azimuthal dependence of surface wave velocities for the most general slightly anisotropic elastic plane-layered medium with its 21 independent elastic coefficients. Rayleigh's principle is combined with Backus' [1970] harmonic tensor decomposition. To the first order in the anisotropy, the azimuthal dependence of both Love - and Rayleigh - wave phase velocity is found to be of the form

$$C(\omega,\theta) = A_1(\omega) + A_2(\omega)\cos 2\theta + A_3(\omega)\sin 2\theta + A_4(\omega)\cos 4\theta + A_5(\omega)\sin 4\theta$$

where ω is the angular frequency and θ is the azimuth of the wave number vector measured clockwise from the north. The various azimuthal terms $A_i(\omega)$ are depth integral functions that involve some canonical harmonic elastic components $\gamma_\sigma^{lm\phi}(z)$ as derived from Backus [1970]. Indeed, these harmonic components are difficult to handle, and we have preferred to express the different $A_i(\omega)$ using the Cartesian elastic coefficients $C_{ij}(z)$ in the usual simplified index notation (see Fuchs [1983], for example). The details of the calculations for Love waves are given in the appendix, and we present only the results in the text. It is found that the perturbed phase velocity can be expressed as

$$C(k,\theta) = C_0(k) + \delta C(k,\theta)$$

where k is the modulus of the wave number vector, $C_0(k)$ is the phase velocity for the isotropic medium considered as a reference model and $\delta C(k,\theta)$ is the first - order perturbation in phase velocity dispersion. We assume that the anisotropy is slight enough that quasi - Love modes and quasi - Rayleigh modes can be defined [Crampin, 1977]. Love - and Rayleigh - wave dispersion is successively considered as follows.

Love Waves

The first - order perturbation in Love wave phase velocity $\delta C_L(k,\theta)$ can be expressed as

$$\delta C_L(k,\theta) = \frac{1}{2C_{0L}(k)}\left[L_1(k) + L_2(k)\cos 2\theta + L_3(k)\sin 2\theta + L_4(k)\cos 4\theta + L_5(k)\sin 4\theta\right] \quad (1)$$

where

$$L_0(k) = \int_0^\infty \rho . W^2 dz$$

$$\begin{aligned}
L_1(k) &= \frac{1}{L_0}\int_0^\infty (W^2.dN + \frac{W'^2}{k^2}.dL)dz \\
L_2(k) &= \frac{1}{L_0}\int_0^\infty -G_c \cdot \frac{W'^2}{k^2} dz \\
L_3(k) &= \frac{1}{L_0}\int_0^\infty -G_s \cdot \frac{W'^2}{k^2} dz \\
L_4(k) &= \frac{1}{L_0}\int_0^\infty -C_c . W^2 dz \\
L_5(k) &= \frac{1}{L_0}\int_0^\infty -C_s . W^2 dz
\end{aligned} \quad (2)$$

where W(z) is the unperturbed Love displacement eigenfunction and W' its derivative with respect to the depth z.

Rayleigh Waves

The first - order perturbation in Rayleigh wave phase velocity $\delta C_R(k,\theta)$ can be expressed as

$$\delta C_R(k,\theta) = \frac{1}{2C_{0R}(k)}\left[R_1(k) + R_2(k)\cos 2\theta + R_3(k)\sin 2\theta + R_4(k)\cos 4\theta + R_5(k)\sin 4\theta\right] \quad (3)$$

where

$$R_0(k) = \int_0^\infty \rho(U^2 + V^2)dz$$

$$\begin{aligned}
R_1(k) &= \frac{1}{R_0}\int_0^\infty \left[dA.V^2 + dC.\frac{U'^2}{k^2} + dF.\frac{2U'V}{k} + dL(\frac{V'}{k}-U)^2\right]dz \\
R_2(k) &= \frac{1}{R_0}\int_0^\infty \left[B_c.V^2 + H_c.\frac{2U'V}{k} + G_c(\frac{V'}{k}-U)^2\right]dz \\
R_3(k) &= \frac{1}{R_0}\int_0^\infty \left[B_s.V^2 + H_s.\frac{2U'V}{k} + G_s(\frac{V'}{k}-U)^2\right]dz \\
R_4(k) &= \frac{1}{R_0}\int_0^\infty C_c.V^2 dz \\
R_5(k) &= \frac{1}{R_0}\int_0^\infty C_s.V^2 dz
\end{aligned} \quad (4)$$

where U(z) and V(z) are the unperturbed Rayleigh displacement eigenfunctions, U in the horizontal direction and V in the vertical direction of propagation; U' and V' are their derivatives with respect to z. The sign convention for U, V, W is the same as that of Smith and Dahlen [1973].

The 13 depth - functions A, C, F, L, N, B_c, B_s, G_c, G_s, H_c, H_s, C_c, C_s are linear combinations of the elastic coefficients C_{ij} and are now explicitly given as follows

Constant term (independent of azimuth)

$$A = {}^3/_8\,(C_{11}+C_{22}) + {}^1/_4\,C_{12} + {}^1/_2\,C_{66}$$

$$C = C_{33}$$

$$F = {}^1/_2\,(C_{13}+C_{23}) \qquad (5a)$$

$$L = {}^1/_2\,(C_{44}+C_{55})$$

$$N = {}^1/_8\,(C_{11}+C_{22}) - {}^1/_4\,C_{12} + {}^1/_2\,C_{66}$$

2θ - azimuthal term

$$B_c = {}^1/_2\,(C_{11}-C_{22})$$

$$B_s = C_{16}+C_{26}$$

$$G_c = {}^1/_2\,(C_{55}-C_{44})$$

$$G_s = C_{54} \qquad (5b)$$

$$H_c = {}^1/_2\,(C_{13}-C_{23})$$

$$H_s = C_{36}$$

4θ- azimuthal term

$$C_c = {}^1/_8\,(C_{11}+C_{22}) - {}^1/_4\,C_{12} - {}^1/_2\,C_{66}$$

$$C_s = {}^1/_2\,(C_{16}-C_{26}) \qquad (5c)$$

It may be noted that all these expressions are relative to the components of the total elastic tensor and not of the perturbed one. This is possible because only linear combinations of the C_{ij} are involved. If we consider the constant terms L_1 and R_1, five independent combinations of elastic coefficients are involved. They correspond to the case of a transversely isotropic medium with vertical symmetry axis after averaging over all azimuths. The elastic coefficients of this equivalent transversely isotropic medium have been defined, following Love [1927, p. 196] and Takeuchi and Saito [1972], as A, C, F, L, N.

Let us recall that in a really transversely isotropic medium, A, C, L, N can be determined from measurements of the P - and S - wave velocities propagating perpendicular or parallel to the axis of symmetry:

$$A = \rho V_{PH}^2,\quad C = \rho V_{PV}^2,\quad N = \rho V_{SH}^2,\quad L = \rho V_{SV}^2$$

where ρ is the density.

Some of the previous combinations have already been derived in the expressions that describe the azimuthal dependence of body waves (see Crampin et al.[1984], for example) in a weakly anisotropic medium.

$$\rho V_P^2 = A + B_c\cos 2\theta + B_s\sin 2\theta + C_c\cos 4\theta + C_s\sin 4\theta$$

$$\rho V_{SP}^2 = D - C_c\cos 4\theta - C_s\sin 4\theta$$

$$\rho V_{SR}^2 = F + G_c\cos 2\theta + G_s\sin 2\theta$$

We use Crampin's notations for the coefficients B_c, B_s, G_c, G_s, C_c, C_s. However, H_c, H_s do not appear in body wave azimuthal dependence and are typical of surface waves. On the other hand, we do not follow Crampin's notation for theconstant term and prefer the most commonly used notation A, C, F, L, N.

From equations (2) and (4) the partial derivatives of phase velocities with respect to the different parameters p_i can be calculated:

$$(\delta C_1)_\omega = \frac{C}{U}\cdot(\delta C_1)_k = \sum_{i=1}^{n_p}\frac{C}{U}\int_0^\infty\left[\left(\frac{\partial C_1}{\partial p_i}\right)_{k,p_j\neq i}\frac{dp_i}{\Delta h}\right]dz$$

where δC_1 is anyone of the azimuthal phase velocity amplitudes in (1) and (3). The subscripts k, ω, p_j are used to show that the derivatives are taken along k, ω, p = constant. Δh is the normalizing thickness for the partial derivatives. U is the group velocity and n_p the number of parameters.

The partial derivatives with respect to the five elastic coefficients of a transversely isotropic earth A, C, F, L, N are in the literature [Anderson and Dziewonski, 1982]. One important point is that the determination of the partial derivatives for the azimuthal terms does not require additional computations because the kernels that appear in the azimuthal terms are already present in the constant terms L_1 and R_1. In fact, as can be seen from (2) and (4) the partial derivatives of the azimuthal terms with respect to the azimuthal combinations of the elastic coefficients (B_c, C_c, G_c,...) are exactlv equal to the partial derivatives of the constant term with respect to the corresponding transverse isotropy parameters (A, N, L,...). Surface wave propagation (or normal mode solution in a spherical earth) can be calculated completely for transverse isotropy with a vertical symmetry axis [Anderson, 1961; Takeuchi and Saito, 1972]. Our point is that the partial derivatives with respect to the five elastic parameters of a transversely isotropic earth (PREM, [Dziewonski and Anderson, 1981]) obtained from these calculations can be used directly to invert the azimuthal variations of the phase velocities of surface waves. In the next section, we take advantage of this property to present these partial derivatives.

Partial Derivatives for the Azimuthal Anisotropy of Surface Waves

In the period range that we consider (50 - 300 s), the influence of the sphericity of the earth cannot be neglected. We make no attempt here to carry out an analysis of azimuthal anisotropy for normal modes in a spherical earth. Dahlen and Smith [1975] and Woodhouse and Dahlen [1978] have given the general expressions for the influence of anisotropy perturbation on the period of normal modes using Rayleigh's principle. However the integration has not been performed so as to give the variation of the eigenperiod with azimuth. It is not possible to simply transpose the decomposition made for surface waves in a flat - layered medium to normal modes in a spherical Earth. In particular, the concept of azimuth of propagation itself is not a natural one for normal modes. Although the theory presented above is valid for surface waves in a flat mantle, we will calculate the partial derivatives with respect to ρ, A, C, F, L, N in a spherical earth. We think that by applying a flat

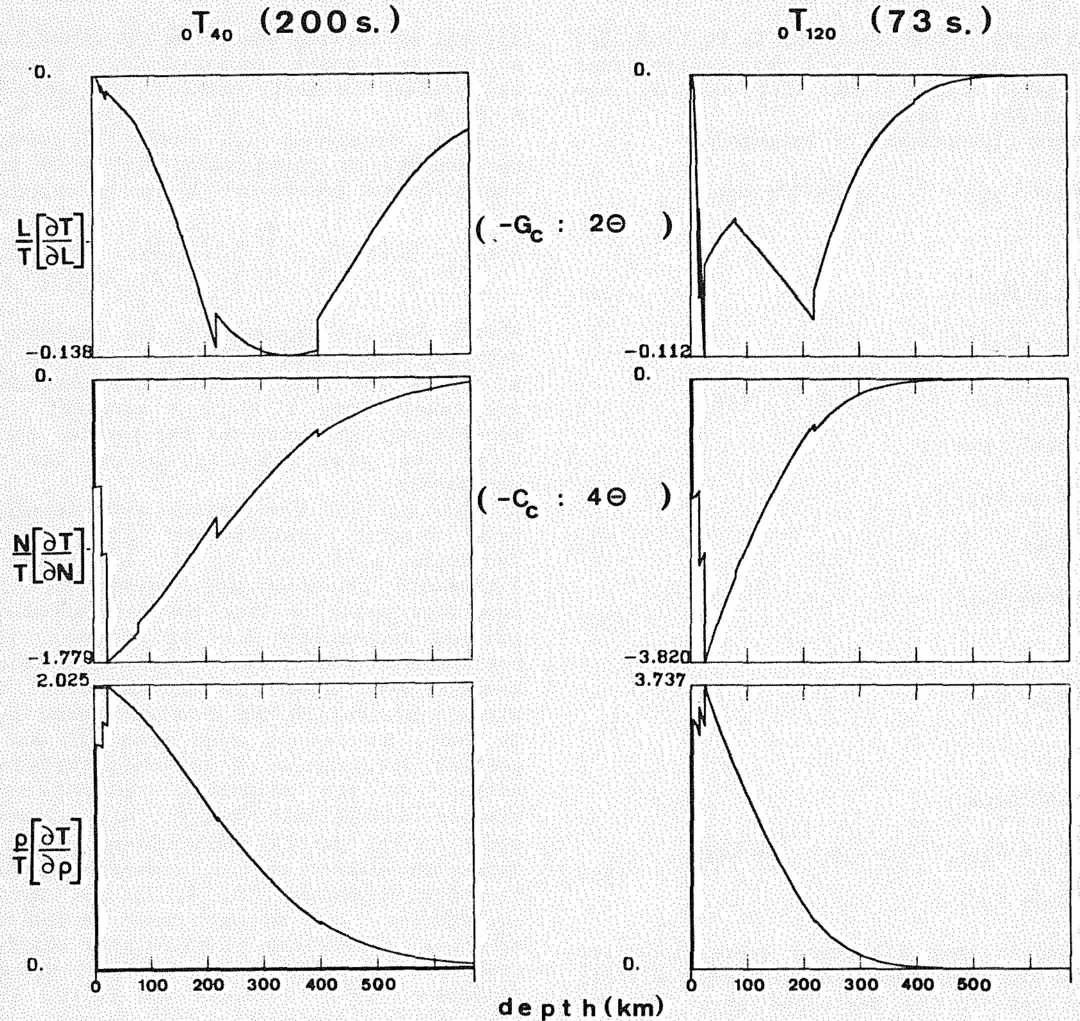

Fig. 1. Partial derivatives for Love waves. The plots are the partial derivatives of the period of the normal modes $_0T_{40}$ (left) and $_0T_{120}$ (right) with respect to the elastic coefficients of a transversely isotropic earth L and N, and the density ρ, as a function of depth in the upper mantle. (The partial derivatives with respect to A, C, F are null for Love waves). The plots are normalized to their maximum amplitudes, which are given for a Δh = 1000-km thick perturbed layer. In parenthese, the combination of elastic coefficients that has the same partial derivative for the azimuthal terms (2θ and 4θ). Note that the amplitude of the L - partial is very small.

layered theory to a sound spherical earth model, we obtain more realistic dispersion properties and eigenfunctions than by applying that theory, in its legitimate frame, to a necessarily oversimplified flat earth model. Sphericity cannot produce azimuthal variations in itself. If the actual problem of azimuthalanisotropy were solved, the sphericity corrections to add to our approximated theory would probably amount to some 10 % of the few percent we are inverting. We use the program "EOS" of Dziewonski to compute the periods and partial derivatives of the normal modes. The earth model we choose, is PREM [Dziewonski and Anderson, 1981]. The computation includes the effects of gravity and anelastic attenuation.

Figure 1 shows the partial derivatives for Love waves. The partial derivative with respect to L is equal to the partial derivative with respect to $-G_c$ in the 2θ - azimuthal term; the partial for N is equal to the partial for $-C_c$ in the 4θ - term. As expected, the amplitude of the N - kernel is much larger, which just means that Love waves are much more sensitive to SH - velocity ($\sqrt{N/\rho}$) than to SV - velocity ($\sqrt{L/\rho}$). Therefore, we expect the 4θ - azimuthal term to be dominant over the 2θ - term for Love waves. However, this prediction depends on the relative strengths of the G_c and C_c coefficients to expect for a realistic anisotropic mantle. We also note that the 4θ - term is mostly sensitive to the shallow C_c - structure.

Figure 2 displays the partial derivatives for Rayleigh waves. There are now three partials that contribute to the 2θ - azimuthal term. One of them is very shallow (the B_c - partial, equal to the A - partial). A second one is sensitive to much deeper structure (the G_c - partial, equal to the L - partial). The third partial (the H_c - partial, equal to the F - partial) has a mixed

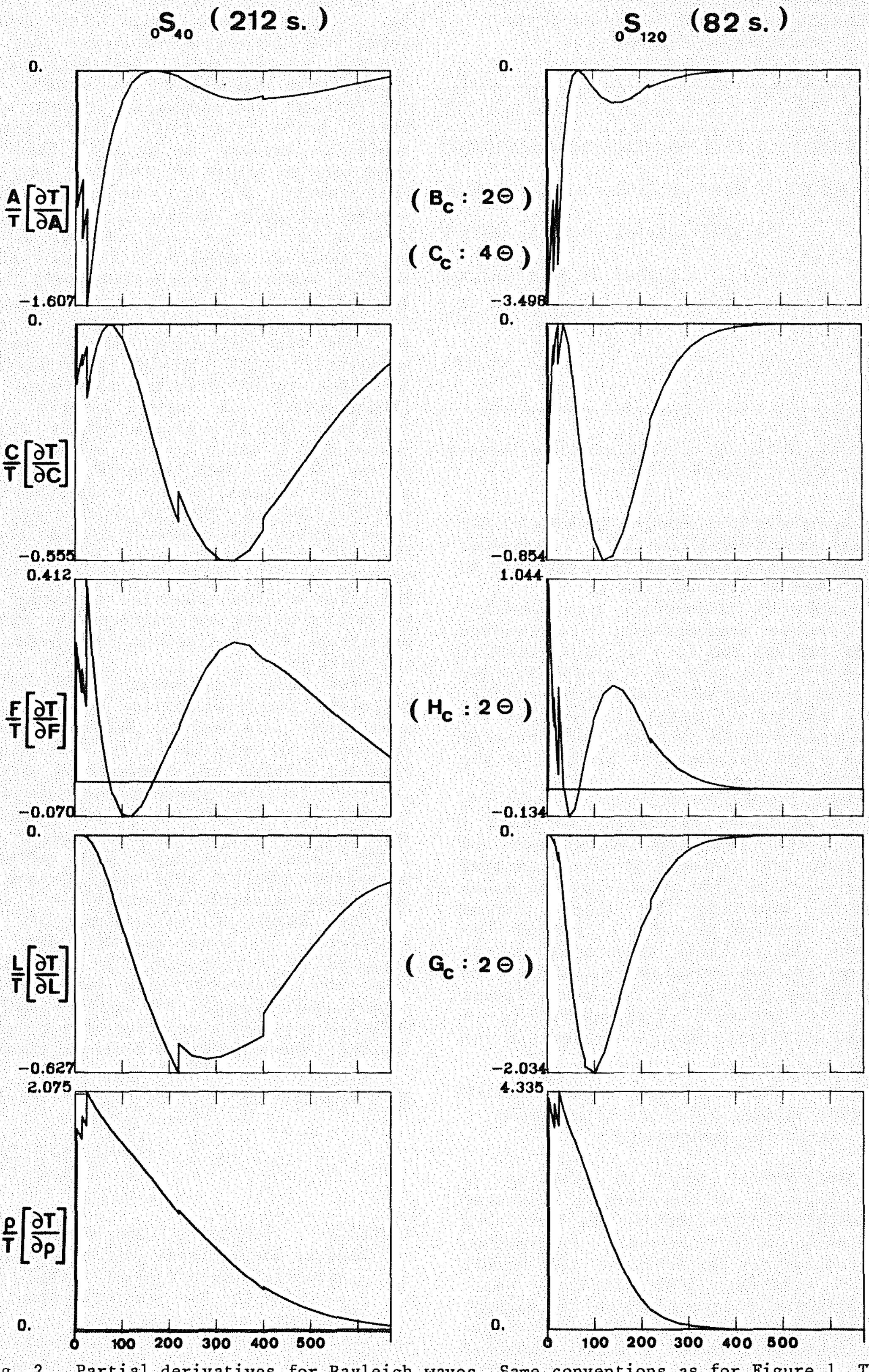

Fig. 2. Partial derivatives for Rayleigh waves. Same conventions as for Figure 1. The partial with respect to N has not been plotted. It is null in a flat earth, but it takes a non zero value in a spherical earth. However, for the periods considered here, its amplitude is small (less than 15% of the amplitude of the L - partial). Note that three partials contribute to the 2θ - azimuthal term.

TABLE 1. Elastic Coefficients C_{ij} chosen for both "Lithospheric" and "Asthenospheric" Anisotropic Regions.

i	j = 1	j = 2	j = 3	j = 4	j = 5	j = 6
1	2.3648	0.7253	0.7228	-0.0008	-0.0196	0.0000
2		2.2081	0.7187	0.0169	0.0164	-0.0040
3			2.2016	0.0182	-0.0024	0.0041
4				0.7494	-0.0058	-0.0103
5					0.7921	0.0128
6						0.7877

In megabars.
From anisotropic Table 2d of Peselnick and Nicolas (1978) for uppermost oceanic mantle.

sensitivity and a somewhat smaller amplitude. The 4θ - azimuthal term depends on the C_c -parameter only. The corresponding partial (A - partial) is very shallow, even at long periods.

Density cannot be responsible for azimuthal anisotropy of course. Nevertheless, it can affect its amplitude through the L_0 - or R_0 - terms in equations (2) and (4). This is a second - order effect that can be ignored at the present time. Note that the plotted ρ - partials are those for the complete transversely isotropic medium; they do not describe the influence of density on the azimuthal terms.

At this stage the main conclusion is that the observation of azimuthal anisotropy for long - period Love - or - Rayleigh waves does not necessarily imply the presence of deep azimuthal anisotropy. Indeed, shallow anisotropy can cause large azimuthal anisotropy for long - period Rayleigh waves through the B_c - partial. To further assess the relative importance of the various terms creating azimuthal anisotropy, we need to determine the values of the relevant combinations of elastic coefficients in a realistic mantle. This is what we present in the next section.

Application to Two Realistic Upper Mantle Structures: Anisotropy in the Lithosphere or in the Asthenosphere

From a geodynamical point of view it is important to know if we can resolve azimuthal anisotropy beneath the lithosphere. In fact, deep anisotropy could bring valuable information concerning convective currents in the mantle [Tanimoto and Anderson, 1984; Nataf et al., 1984]. To check that point, we apply the partial derivatives of the previous section to two simple models: a "lithospheric" model in which azimuthal anisotropy is restricted to the lithosphere: from the base of the crust (at 24 - km depth) to 100 - km depth; an "asthenospheric" model, in which azimuthal anisotropy runs from 140 - km to 400 - km depth, the rest of the mantle (including the lithosphere and the spinel - zone) bearing no azimuthal anisotropy.

To estimate plausible values for the G_c, B_c,... combinations that enter the azimuthal terms, we choose a realistic C_{ij} matrix of elastic coefficients. Peselnick and Nicolas [1978] have calculated the matrix of elastic coefficients for uppermost oceanic mantle on a massif scale from field observations (Antalya ophiolite complex in Turkey), fabric data and experimental values of single - crystal elastic coefficients. The C_{ij} matrix is the weighted average of many samples oriented in the field covering a 15 - km long outcrop. This is still very small as compared to the wavelength of the surface waves we are dealing with (100 - 1000 km). From P_n - studies and models of the formation of oceanic lithosphere it is possible to infer that anisotropy remains homogeneous on horizontal length - scales in excess of 1000 km. We take their results to build both our lithospheric and asthenospheric anisotropic examples. The C_{ij} matrix (their Table 2d) is reproduced in Table 1, with respect to the x, y, z axes. The x - axis is normal to the ridge, the y - axis is parallel to it and the z - axis is vertical. We note that their data produce a 4% P - wave anisotropy, somewhat smaller than actually observed P_n - anisotropies in the oceans [e.g., Morris et al., 1969; Raitt et al., 1971]. Therefore we think that the azimuthal anisotropy we derive should not overestimate the azimuthal anisotropy to be expected for surface waves in the earth.

Given the C_{ij} matrix, it is easy to calculate the combinations G_c, B_c,... that we need for discussing azimuthal anisotropy, according to equations (5). The values of the relevant ratios of combinations are given in Table 2. Strictly speaking, we should take into account the pressure - and temperature - dependence of these ratios. However, they are poorly known in the depth range we consider and we neglect them at this stage in order to keep our models as simple as possible.

The azimuthal variations of the phase velocities of Love and Rayleigh waves are computed by integrating the relevant partials times the associated ratios over the anisotropic region. For example, the $\cos 2\theta$ - anisotropy of Rayleigh wave phase velocity at a given period T_0 for the "asthenospheric" model is obtained as

$$\frac{\Delta C}{C}(T_0)\cos 2\theta = \frac{-C}{U}\left\{\int_{140\text{km}}^{400\text{km}} \left[\left(\frac{B_c}{A}\right)\frac{A}{T}\left(\frac{\partial T}{\partial A}\right)_k + \left(\frac{H_c}{F}\right)\frac{F}{T}\left(\frac{\partial T}{\partial F}\right)_k + \left(\frac{G_c}{L}\right)\frac{L}{T}\left(\frac{\partial T}{\partial L}\right)_k\right]\frac{dz}{\Delta h}\right\}\cos 2\theta$$

TABLE 2. Ratios of the Combinations of the Elastic Coefficients Used to Calculate the Azimuthal Anisotropy of Surface Waves for Our Two Realistic Models.

	$G_{c,s}/L$	$C_{c,s}/N$	$C_{c,s}/A$	$B_{c,s}/A$	$H_{c,s}/F$
cos	0.028	-0.005	-0.002	0.034	0.003
sin	-0.007	0.003	0.001	-0.002	0.006

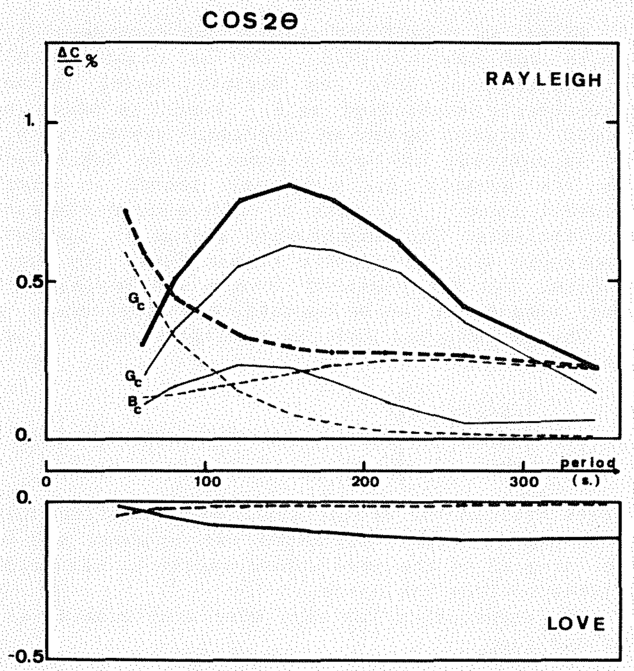

Fig. 3. The cos2θ - azimuthal anisotropy amplitude of Rayleigh waves (top) and Love waves (bottom) as a function of period for the two tested models. The dashed line is for the "lithospheric" model and the continuous line for the "asthenospheric" model. Also drawn (fine lines) are the individual contributions of the G_c and B_c terms to the Rayleigh wave cos2θ - anisotropy for both models. Note the large anisotropy at long-period for the "lithospheric" model.

where C and U are the unperturbed phase and group velocities at the period T_0.

The cos2θ - amplitudes are given in Figure 3 as a function of period. As expected, the 2θ - anisotropy of Love waves is very small (less than 0.2% peak to peak). On the contrary, Rayleigh waves have a significant 2θ - anisotropy for both models. The "asthenospheric" model yields a maximum 1.6% peak to peak anisotropy at 150 s. The "lithospheric" model reaches the same value at 50s, but its anisotropy remains high (0.5% peak to peak) up to the largest periods. As shown in Figure 3, its long - period anisotropy comes from the B_c - contribution. The very shallow sensitivity of the B_c - partial allies with the large value of the B_c/A ratio to give that strong effect. We also note that the H_c - contribution to the 2θ - term is always negligible (not plotted).

As can be seen from Table 2, the sine - terms are much smaller than the cosine - terms, except for H_c and H_s whose contribution is negligible. The θ=0 direction thus corresponds to a maximum velocity when the cosine - amplitude is positive (as for Rayleigh waves) and to a minimum when the cosine - amplitude is negative (as for Love waves). Rayleigh waves are faster along the direction of fast P - waves (which coïncides with the normal to the ridge, the x - axis in Table 1), whereas Love waves are faster 90° from that direction. As noted by Maupin [1985], that property arises from the difference in particle motion between Love and Rayleigh waves.

Figure 4 shows our results for the cos4θ - anisotropy. That term is negligibly small for Rayleigh waves (less than 0.1% peak to peak). For Love waves, the amplitude is also rather small (less than 0.2% peak to peak) and we note that the variation with period is weak.

To assess the generality of the results that we present, we have tried other plausible C_{ij} matrices. In particular, we have considered the case of the hartzburgite sample given by Peselnick and Nicolas [1978]. On the sample scale the coherence is better and the resulting anisotropy larger. The cos2θ - Rayleigh wave anisotropy is still the dominant feature with a maximum 4% peak to peak variation. The cos4θ - anisotropy of Rayleigh waves remains negligible. These two trends seem to be indeed robust features for any realistic upper mantle anisotropy, when the fast axis is horizontal. On the other hand, the amplitude and even the sign of the cos4θ - Love wave anisotropy is quite sensitive to the chosen C_{ij} matrix. For the hartzburgite C_{ij}, its amplitude reaches half the Rayleigh cos2θ - amplitude and it is negative. The cos2θ - Love wave anisotropy remains very small.

Discussion and Conclusions

Figures 3 and 4 indicate that realistic anisotropic models of the mantle produce significant azimuthal variations of the velocities of surface waves. The 2θ - term of Rayleigh wave azimuthal variations is the dominant term and a robust feature. Its fast axis is the fast axis of P - waves. The peak to peak amplitude of the variation reaches 1.6%, a value that compares favorably with observed azimuthal variations in the Pacific [Forsyth, 1975; Mitchell and Yu, 1980; Montagner, 1985] or on a global scale [Tanimoto and Anderson, 1984]. We should stress that the tensor of the elastic coefficients that we adopted for our models is truly not extreme in the sense that the P_n - azimuthal anisotropy it produces is lower than actually observed in most oceans. The 4θ - term

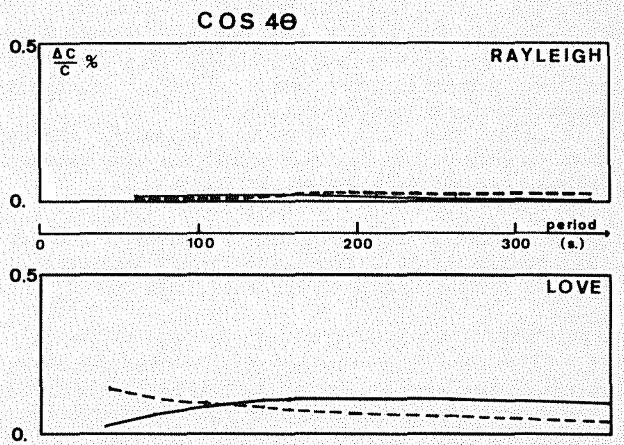

Fig. 4. The cos4θ - azimuthal anisotropy amplitude of Rayleigh waves (top) and Love waves (bottom). Same conventions as in Figure 3.

for Rayleigh waves is found to be negligible. For Love waves, on the contrary, the 4θ - term can be larger than the 2θ - term. The amplitude of both terms is however quite small and somewhat dependent on the details of the C_{ij} matrix. Since the 4θ - variation requires a very complete azimuthal coverage in order to be detected, we think that Love wave azimuthal dependence should be rather difficult to retrieve.

Concerning the variation of the 2θ - term of Rayleigh wave with period, we note that the observation of azimuthal anisotropy at long - period does not necessarily mean that deep anisotropy is present. Indeed, our "lithospheric" model yields peak to peak amplitudes up to 0.5% at 300 s. This is because two terms contribute to the 2θ - azimuthal variations of Rayleigh waves: one is sensitive to very shallow structure (the Bc - combination), and one to deep structure (the Gc - combination). In order to retrieve some information about the variation of anisotropy as a function of depth, it is necessary to invert the observed azimuthal anisotropy variations, using a period range as large as possible.

In this paper we have set the bases for such an inversion. We have shown how partial derivatives for the inversion of the azimuthal variations of surface waves velocities could be derived very simply from the partial derivatives of phase velocity of a transversely isotropic earth. We have presented examples of these partial derivatives calculated from PREM. Using the linear combinations of the elastic coefficients derived in equation (5), it is possible to test any C_{ij} anisotropic matrix. In turn, the inversion of the observed azimuthal variations of Rayleigh and Love wave velocities could bring most valuable information about the petrology of the mantle and about the preferred orientations of crystals at depth. Once related to the deformation field that produce them, these preferred orientations could be used to map convection currents at depth [Tanimoto and Anderson, 1984].

We note that the B_c - combination, whose value at shallow depth significantly contributes to the 2θ - azimuthal variation of Rayleigh wave velocity, is the combination that describes the 2θ - azimuthal variation of P_n - velocity [Backus, 1965]. Since many data are becoming available on the azimuthal variation of P_n - velocities around the world, it could be possible to correct Rayleigh wave data for this shallow contribution and obtain reliable estimates of the deeper anisotropy.

Finally, we note that the present theory needs a more rigourous extension to the problem of normal modes in a spherical earth.

Appendix: Direct Calculation of First - Order Perturbation of Surface Wave Phase Velocity in a Slightly Anisotropic Medium

The calculations which lead to equations (1), (2), (3), (4), (5) of text, are detailed using the same approach as Smith and Dahlen [1973] but without Backus' harmonic tensor decomposition. We express the different azimuthal terms as a function of the elastic coefficients C_{ij} expressed in the commonly used simplified index notation.

First, we recall the assumptions considered by Smith and Dahlen [1973]. Let us consider the propagation of Love and Rayleigh waves in an arbitrarily stratified half - space in which a right - handed Cartesian coordinate system (x, y, z) is defined. The half - space may be described by its density ρ(z) and its fourth - order elastic tensor Γ(z) with 21 independent elastic coefficients. The unperturbed medium is assumed isotropic with an elastic tensor $\Gamma_0(z)$. In that medium, the two cases of Love - and Rayleigh - wave dispersion can be successively considered.

The unperturbed Love wave displacement is of the form

$$\vec{u}(r,t)=\begin{bmatrix}-W(z)\sin\theta\\ W(z)\cos\theta\\ 0\end{bmatrix}\exp\left(i\left[k(x\cos\theta+y\sin\theta)-\omega t\right]\right) \quad (A1)$$

where W(z) is the scalar depth eigenfunction for Love waves, k the horizontal wave number, and θ the azimuth of the wave number vector k, measured clockwise from the north.

The unperturbed Rayleigh wave displacement is of the form

$$\vec{u}(r,t)=\begin{bmatrix}V(z)\cos\theta\\ V(z)\sin\theta\\ iU(z)\end{bmatrix}\exp\left(i\left[k(x\cos\theta+y\sin\theta)-\omega t\right]\right) \quad (A2)$$

where V(z) and U(z) are the scalar depth eigenfunction for Rayleigh waves.

The associated strain tensor $\varepsilon(\vec{r},t)$ is defined by:

$$\varepsilon_{ij}(\vec{r},t) = 1/2\left(u_{i,j} + u_{j,i}\right) \quad (A3)$$

where ,j denotes the differentiation with respect to the j - th coordinate. The medium is perturbed from $\Gamma_0(z)$ to $\Gamma_0(z)+\gamma(z)$, where γ(z) is small compared to $\Gamma_0(z)$. This means that we are in the approximation where we can still consider quasi - Love modes and quasi - Rayleigh modes [Crampin, 1977].

From Rayleigh's principle, the first - order perturbation δC(k) in phase velocity dispersion is [Smith and Dahlen, 1973]

$$\delta C(\vec{k}) = \frac{C}{2\omega^2}\,\frac{\int_0^\infty \gamma_{ijkl}\cdot\varepsilon_{ij}\varepsilon^*_{kl}\,dz}{\int_0^\infty \rho u_k u_k^*\,dz} \quad (A4)$$

where u_i and ε_{ij} are the displacement and strain for the unperturbed half - space and the asterisk denotes complex conjugation.

Now because of the symmetry of the tensors γ(z) and ε, we use the simplified index notation c_{ij} and ε_i for the elements γ_{ijkl} and ε_{ij}, but we must take into account the number n_{ij} of coefficients γ_{ijkl} for each c_{ij}. Therefore equation (A4) can be written as

$$\delta C(\vec{k}) = \frac{C}{2\omega^2}\,\frac{\int_0^\infty n_{ij}c_{ij}\cdot\varepsilon_i\varepsilon_j^*\,.dz}{\int_0^\infty \rho u_k u_k^*dz} \quad (A5)$$

We detail only the calculations for Love waves. Equations (A1) and (A3) give the various components of strain:

TABLE A1. Calculation of the Various $c_{ij}\varepsilon_i\varepsilon_j^*$ With the Simplified Index Notation, for Love Waves

n_{ij}	ij	$c_{ij}\varepsilon_i\varepsilon_j^*$
1	11	$c_{11}\alpha^2\beta^2.k^2W^2$
1	22	$c_{22}\alpha^2\beta^2.k^2W^2$
1	33	0
2	12	$-c_{12}\alpha^2\beta^2.k^2W^2$
2	13	0
2	23	0
2	24	0
4	14	$c_{14}(-i\alpha^2\beta).kWW'/2$
4	15	$c_{15}(i\alpha\beta^2).kWW'/2$
4	16	$c_{16}(-\alpha\beta)(\alpha^2-\beta^2).k^2W^2/2$
4	24	$c_{24}(-i\alpha^2\beta).kWW'/2$
4	25	$c_{25}(i\alpha\beta^2).kWW'/2$
4	26	$c_{26}\alpha\beta(\alpha^2-\beta^2).k^2W^2/2$
4	34	0
4	35	0
4	36	0
4	44	$c_{44}\alpha^2.W'^2/4$
8	45	$c_{45}(-\alpha\beta).W'^2/4$
8	46	$c_{46}(-i\alpha)(\alpha^2-\beta^2).kWW'/2$
4	55	$c_{55}\beta^2.W'^2/4$
8	56	$c_{56}(i\beta)(\alpha^2-\beta^2).kWW'/4$
4	66	$c_{66}(\alpha^2-\beta^2).k^2W^2/4$

$\alpha = \cos\theta$; $\beta = \sin\theta$.

$$\varepsilon_1 = -i\alpha\beta.kW$$
$$\varepsilon_2 = i\alpha\beta.kW$$
$$\varepsilon_3 = 0.$$
$$\varepsilon_4 = 1/2\ \alpha W'$$
$$\varepsilon_5 = -1/2\ \beta W'$$
$$\varepsilon_6 = 1/2\ (\alpha^2-\beta^2).kW$$

where $\alpha = \cos\theta$ and $\beta = \sin\theta$.

In Table A1, the different terms $n_{ij}c_{ij}\varepsilon_i\varepsilon_j^*$ of (A5) are given. We note that when $c_{ij}\varepsilon_i\varepsilon_j^*$ is an imaginary complex, its contribution to $\delta C(k,\theta)$ is null. When all the contributions are summed, the different terms $\alpha^k\beta^l$ present are such that k+l is even. Therefore, these terms can be developped as a Fourier series in θ with only even terms. Finally it is found that:

$$\delta C_L(k,\theta) = \frac{C}{2\omega^2 L_0}\int_0^\infty dz\,\{k^2W^2[\tfrac{1}{8}(c_{11}+c_{22}-2c_{12}+4c_{66})] + W'^2[\tfrac{1}{2}(c_{44}+c_{55})] + \cos 2\theta.W'^2[\tfrac{1}{2}(c_{44}-c_{55})] - \sin 2\theta.W'^2 c_{45} - \cos 4\theta.k^2W^2[\tfrac{1}{8}(c_{11}+c_{22}-2c_{12}-4c_{66})] + \sin 4\theta.k^2W^2[\tfrac{1}{2}(c_{26}-c_{16})]\} \quad (A6)$$

The procedure is exactly the same for Rayleigh waves, starting from the displacement given by (A2).

If we consider a transversely isotropic medium with vertical symmetric axis, we have

$$c_{11} = c_{22} = dA \qquad c_{33} = dC$$
$$c_{12} = d(A - 2N) \qquad c_{13} = c_{23} = dF$$
$$c_{44} = c_{55} = dL \qquad c_{66} = dN$$
$$c_{14} = c_{24} = c_{15} = c_{25} = c_{16} = c_{26} = 0$$

and (A6) reduces to

$$\delta C_L(k,\theta) = \frac{1}{2C_L L_0}\int_0^\infty (W^2.dN + \frac{W'^2}{k^2}.dL)dz \quad (A7)$$

we find again the expressions of Takeuchi and Saito [1972, p. 268] for Love waves. Keeping only the constant term of equation (A6), we can obtain an equivalent transversely isotropic model from equation (A7), by setting

$$dN = 1/8\ (c_{11} + c_{22} - 2c_{12} + 4c_{66})$$

$$dL = 1/2\ (c_{44} + c_{55})$$

then we obtain the equivalent transversely isotropic model; the three other equivalent parameters A, C, F are obtained from Rayleigh waves and are given in the text.

If we call C_{ij} the elastic coefficients of the total elastic tensor, we can set

$$N = 1/8\ (C_{11} + C_{22} - 2C_{12} + 4C_{66})$$

$$L = 1/2\ (C_{44} + C_{55})$$

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