EUROPHYSICS LETTERS 1 April 1991

Europhys. Lett., **14** (7), pp. 655-660 (1991)

Nonlinear Dynamical Coupling Observed near the Threshold of Convection in a Two-Layer System.

P. CARDIN and H.-C. NATAF
Département de Géologie, Ecole Normale Superieure
24, rue Lhomond, 75231 Paris Cedex 05, France

(received 14 November 1990; accepted in final form 25 January 1991)

PACS. 47.20 – Hydrodynamic stability and instability.
PACS. 68.10E – Interface elasticity, viscosity, and viscoelasticity.
PACS. 91.45F – Convection currents.

Abstract. – Laboratory experiments on Rayleigh-Bénard convection in a 2-layer system have been conducted. Remarkable nonlinear coupling is observed for Rayleigh numbers not far above the critical one. Two examples are described: the first one displays a steady resonance pattern with two rolls above each roll of the bottom layer, while in the second example, a pseudo-periodic time oscillation is observed, with a single time-dependent horizontal wavelength. In both cases, the coupling is predominantly «thermal», implying that some kind of interface viscosity is present.

Introduction. – Rayleigh-Bénard convection in a 2-layer system is expected to display a very wide variety of dynamical behaviours. Even at the threshold of convection, marginal stability analyses predict several possible types of coupling. The most common one is «mechanical» coupling, where rolls rotate in a gearlike fashion, with downwellings of the upper layer above uprisings of the lower layer [1]. Under some conditions, «thermal» coupling (uprisings above uprisings) can prevail [2-4]. Finally, overstability can occur, with oscillations at the threshold of convection [5-7].

Two-layer convection has also been chosen as a frame for a major study of nonlinear dynamical coupling [8]. Although the problem treated is then fairly idealized, it should make 2-layer convection an attractive candidate for observing nonlinear interactions.

However, only a few experimental observations have been presented so far [3, 6, 9]. In this paper, we report on exploratory experimental results that show nonlinear coupling for Rayleigh numbers only a few times critical. The first example displays a 2-to-1 stationary coupling, and is close to the case treated by Proctor and Jones [8]. In the second example, we observe oscillations between two structures of different wavelengths. In both cases, the coupling is predominantly «thermal». This can only be explained if some kind of interface viscosity is present [3, 4].

Experimental set-up. – The experimental set-up is essentially the same as described in Nataf *et al.* [3]. We briefly recall its main features. The two superposed liquids are enclosed in a lucite frame sandwiched between two horizontal copper plates. The inner dimensions of the frame are (50 (Z: height) × 250 (X) × 125 (Y)) mm^3. The tank is placed on a moving base that allows computer-controlled translations in the three directions of space (X, Y, Z). A strioscopy method is used to observe the temperature field: a laser beam enters the tank at a given X and Z position; it is deflected by the variations of the index of refraction due to the temperature gradients it encounters; the deflection is measured on a position-sensitive photodetector.

Two pairs of liquids are used: silicon oil over glycerol (as in Nataf *et al.* [3]), and silicon «light» over silicon «dense». The latter pair is obtained by letting separate a mixture of equal volumes of methyl phenyl polysiloxane oil (Rhodorsil oil 550) and an ordinary methyl polysiloxane (silicon oil) of similar viscosity (Rhodorsil 47V100). The two transparent oils thus obtained have very similar properties, and a very small interface tension. The properties of the liquids used are listed in table I.

TABLE I. – *Physical properties of the liquids.*

	SI units	Glycerol	Rhodorsil silicon 47 V 500	Rhodorsil silicon 550	Rhodorsil silicon 47 V 100
k thermal conductivity	W m^{-1} K^{-1}	0.294	0.16	0.146	0.16
ρ density (25 °C)	kg m^{-3}	$1.26 \cdot 10^3$	$0.97 \cdot 10^3$	$1.07 \cdot 10^3$	$0.97 \cdot 10^3$
C_p specific heat	J kg^{-1} K^{-1}	$2.62 \cdot 10^3$	$1.46 \cdot 10^3$	$1.50 \cdot 10^3$	$1.46 \cdot 10^3$
κ thermal diffusivity	m^2 s^{-1}	$0.89 \cdot 10^{-7}$	$1.13 \cdot 10^{-7}$	$0.91 \cdot 10^{-7}$	$1.3 \cdot 10^{-7}$
ν kinematic viscosity (25 °C)	m^2 s^{-1}	$7.45 \cdot 10^{-4}$	$4.99 \cdot 10^{-4}$	$1.25 \cdot 10^{-4}$	$1.00 \cdot 10^{-4}$
α thermal expansion	K^{-1}	$4.9 \cdot 10^{-4}$	$9.45 \cdot 10^{-4}$	$7.5 \cdot 10^{-4}$	$9.45 \cdot 10^{-4}$
Σ interfacial tension (25 °C)	N m^{-1}	$25 \cdot 10^{-3}$		$< 5 \cdot 10^{-3}$	
$\partial\Sigma/\partial T$ temp. derivative of Σ	N m^{-1} K^{-1}	$-1.3 \cdot 10^{-4}$		?	
ν_{int} interface viscosity	m^3 s^{-1}	$\geqslant 10^{-5}$		?	

Marginal stability has been computed for these two pairs of liquids by Cardin *et al.* [4], following methods proposed by Rasenat *et al.* [6] and Wahal and Bose [7], among others. For a given pair of liquids, the only varied parameters are the d_0 ratio (the thickness of the upper layer divided by the thickness of the lower layer), and ΔT (the temperature difference between the bottom and top plates). Individual Rayleigh numbers are computed for each layer, according to

$$\mathrm{Ra}_i = \frac{\alpha_i g \beta_i d_i^4}{\kappa_i \nu_i},$$

where «i» is either «t» (top) or «b» (bottom), β is the temperature gradient of the conductive state (with the relations: $\beta_b d_b + \beta_t d_t = \Delta T$, and $k_b \beta_b = k_t \beta_t$), d_i is the thickness of the «i» layer, and g is the acceleration due to gravity. Other symbols are defined in table I. The Rayleigh numbers listed in table II are computed assuming no interface viscosity.

Silicon oil over glycerol: thermal coupling and 2*-to-*1 *resonance.* – This is the pair of liquids used by Nataf *et al.* [3]. In that paper, we reported on observations with $d_0 = 1$ (both layers have the same thickness). «Thermal» coupling was observed in all cases. Classical marginal stability analysis predicts «mechanical» coupling instead (*e.g.* [6]). However, «thermal» coupling is indeed predicted if interface viscosity is present, and large enough.

Interface viscosity enters the balance of tangential stress at the interface through a nondimensional number $N_{vi} = \nu_{int}/\nu_b d_b$, where ν_{int} is the interface viscosity [7]. For values of N_{vi} larger than about 2, «thermal» coupling takes over «mechanical» coupling [4]. In that paper [4], we also describe simple mechanical experiments that demonstrate that some interface viscosity is present in the silicon oil/glycerol system. A rough estimate of its magnitude was performed, yielding a N_{vi} value of the order of or larger than 1. The interface viscosity is probably due to the presence of some yet to be determined surfactant at the interface between the two liquids.

In the present study, we varied the depth ratio d_0. For d_0 values between 1 and 0.61, we observed the same kind of «thermal» coupling. But for d_0 less than 0.61, we obtained a different structure. It is a structure with two wavelengths: two small rolls in the upper layer lie above one large roll of the lower layer. Simple thermal coupling is therefore not possible anymore. Figure 1 shows lines of equal horizontal gradient of temperature for such a case, with $d_0 = 0.61$. The parameters for this experiment are given in table II. The image was built by contouring strioscopic measurements obtained on an XZ grid with a 1 mm step in both X and Z. In the upper layer, four rolls are visible. Their width is approximately equal to the depth of that layer. In the bottom layer, two large square rolls are seen. At the sides, the coupling is thermal, with downwellings above downwellings. The very smooth pattern in the top layer indicates that convection is weak, while the more localized gradients in the bottom layer reveal more vigorous downwellings. This is in agreement with the individual Rayleigh numbers: $Ra_b = 4900$, $Ra_t = 1900$, which suggest a more active bottom layer. It is interesting to note that the uprising in the middle of the lower layer is almost invisible. Only the change in sign of the horizontal gradient indicates that there is a roll boundary there. It

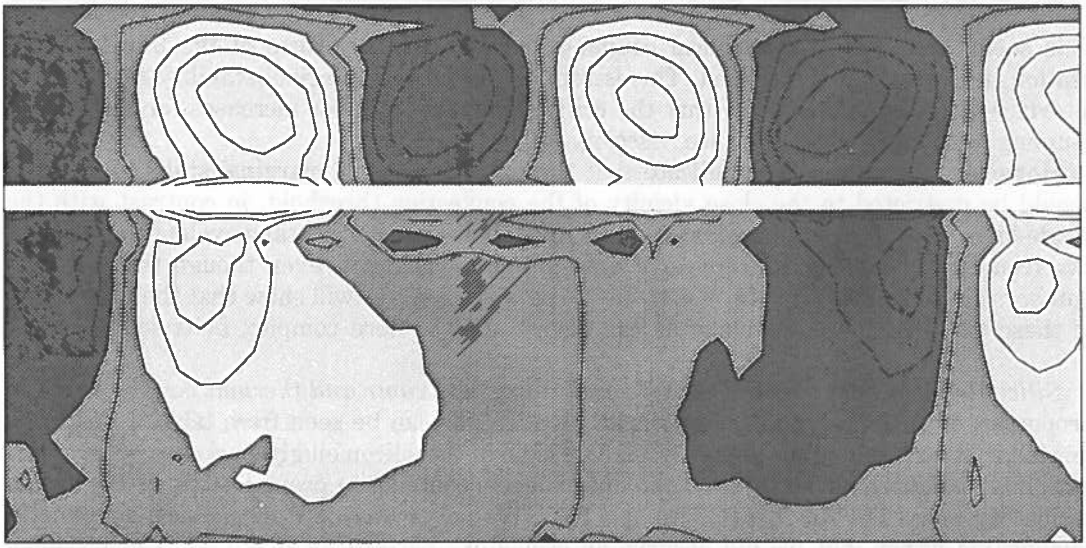

Fig. 1. – Lines of equal horizontal gradient of temperature in a 2-layer convection experiment. The top layer consists of silicon oil, and the bottom one is made of glycerol. Parameters for this experiment are given in table II. Light shading is for «positive» horizontal gradient (*i.e.* temperature increases when going from left to right), while heavy shading is for negative horizontal gradient. The contour interval is 0.02 °C/mm. There are two small rolls in the upper layer above each large roll of the lower layer. Note that the uprising current is hardly visible between the two downwellings in the lower layer.

TABLE II. – *Experimental parameters.*

	47 V 500	silicon «light»
	glycerol	silicon «dense»
T_t top plate temperature (°C)	22.8	17.1
T_b bottom plate temperature (°C)	27.5	21.0
d_t top layer thickness (mm)	19	29
d_b bottom layer thickness (mm)	31	21
ν_t top layer kinematic viscosity (m²/s)	$5.15 \cdot 10^{-4}$	$1.30 \cdot 10^{-4}$
ν_b bottom layer kinematic viscosity (m²/s)	$7 \cdot 10^{-4}$	$2.5 \cdot 10^{-4}$
Ra_t top layer Rayleigh number	1900	18500
Ra_b bottom layer Rayleigh number	4800	3300
Ra_c bottom layer critical Rayleigh number	1080	340
λ_c critical wavelength (mm)	78	66

thus seems that where mechanical coupling should prevail because of the 2-to-1 configuration, it is very much inhibited. This is reminiscent of the marginal stability results with interface viscosity, which show that the critical Rayleigh number increases for mechanical coupling with increasing interface viscosity [4].

However, our results also indicate that the extrapolation of marginal stability analysis should be restricted to the close vicinity of the convection threshold, in contrast with the single-layer case. Indeed, marginal stability only allows one horizontal wavelength, whereas two (coupled) wavelengths are clearly seen in our experiment, even though the Rayleigh numbers are moderate ($\mathrm{Ra}/\mathrm{Ra}_c = 4.4$). In the next section, we will show that the interaction of these two horizontal wavelengths can also lead to a more complex behaviour.

Silicon «light» over silicon «dense»: oscillatory behaviour and thermal coupling. – The properties of these two liquids are almost identical, as can be seen from table I. Interface tension is at least one order of magnitude less than in the silicon oil/glycerol case. «Classical» marginal stability analysis [6] predicts «mechanical» coupling to prevail, with an overstable oscillatory behaviour for $d_0 = 0.95$ to $d_0 = 1.05$. We never observed mechanical coupling in this system either, but we did observe an oscillatory pattern for $d_0 = 1.38$. The Rayleigh numbers for this experiment are given in table II. Using strioscopy, we measured the horizontal gradient of temperature as a function of X, at mid-depth of each layer. We then derived the positions of the uprisings and downwellings in the tank from the zero-crossings of these profiles. Measurements were taken every 3 hours. Figure 2 shows the time-evolution of the role pattern over a time lapse of 60 hours.

A roughly periodic time variation is observed, with a period of approximatively 20 hours ($\simeq 15 d_b^2/\kappa_b$). Clearly, more work would be needed to assert the exact temporal behaviour. Nevertheless, several features of this record are noteworthy. The oscillation is between two patterns with different horizontal wavelengths. The long wavelength ($\lambda_l \simeq 67$ mm) is close to the «natural» wavelength of the upper layer ($2d_t = 58$ mm), while the short one ($\lambda_s \simeq 48$ mm) is close to the natural wavelength of the lower layer ($2d_b = 42$ mm). The long wavelength is also close to the computed critical wavelength ($\lambda_c = 66$ mm). We note that the rolls of the top and bottom layers are superposed at all times, and that the coupling is always «thermal». It is interesting to see that the evolution from the short wavelength to the long one is more or less continuous, while the transition from the long one to the short one occurs through an intermediate stage, where convection seems to loose its structure in the X-direction. Moreover, the current situated in the middle of the tank bifurcates into two currents on its sides, and a new current of opposite sign replaces it in the middle.

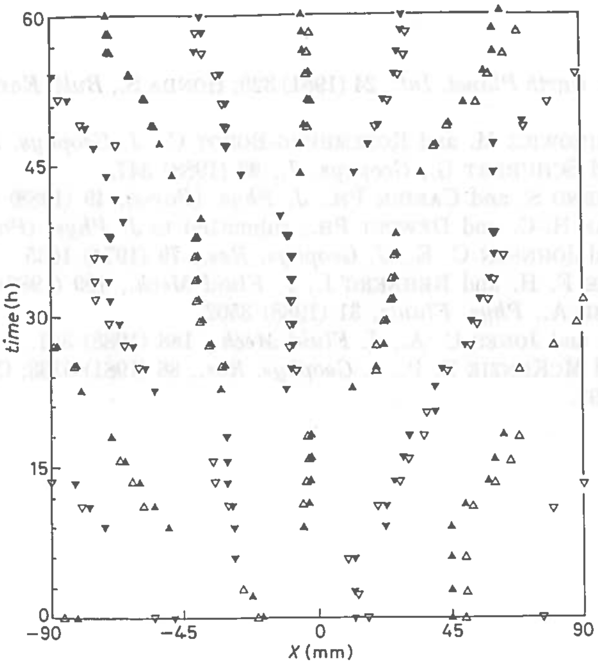

Fig. 2. – Evolution with time of the positions of uprisings (up-triangles) and downwellings (down-triangles) in the upper layer (empty symbols), and in the lower layer (filled symbols). The top fluid is silicon «light», and the bottom one is silicon «dense». Parameters are given in table II. The side walls are at $X = -125$ mm and $+125$ mm, outside of the portion shown. Note the bifurcation of the central current.

The oscillations observed here are quite different from the oscillations predicted by marginal stability analysis. The latter only involves one horizontal wavelength, and the oscillation is from a rather thermally coupled pole to a rather mechanically coupled one [4, 6]. On the contrary, our experiment displays two wavelengths, and coupling is thermal at all times. This clearly implies nonlinear interactions of the convective rolls across the interface, even though the Rayleigh numbers are not large again ($\mathrm{Ra}/\mathrm{Ra}_c \simeq 9.7$).

Conclusion. – We have described two examples of remarkable dynamical behaviours in a 2-layer convective system. These justify the interest that is growing for this type of system. Especially encouraging is the observation that nonlinear coupling occurs even when the «individual» Rayleigh numbers are fairly low. This suggests that an accurate prediction of the observed coupling should be possible using a simple description of convection in each layer [8]. However, care should be taken in modelling the role of the interface between the two fluid layers. Indeed, «thermal» coupling is found to be the dominant type of coupling in our experiments. This is possible only because some kind of interface viscosity is present [4]. Interface viscosity must therefore be taken into account for accurate comparisons to be performed with the experimental results.

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This work was supported by CNRS-INSU-DBT, «Instabilités» contribution no. 235. Contribution no. 130 of ENS-Paris-Géologie.

REFERENCES

[1] BUSSE F. H., *Phys. Earth Planet. Int.*, 24 (1981) 320; HONDA S., *Bull. Earthquake Res. Inst.*, 57 (1982) 273.

[2] CSEREPES L., RABINOWICZ M. and ROSEMBERG-BOROT C., *J. Geophys. Res.*, 43 (1988) 12009; ELLSWORTH K. and SCHUBERT G., *Geophys. J.*, 93 (1988) 347.

[3] NATAF H.-C., MORENO S. and CARDIN PH., *J. Phys. (Paris)*, 49 (1988) 1707.

[4] CARDIN PH., NATAF H.-C. and DEWOST PH., submitted to *J. Phys. (Paris)*.

[5] RICHTER F. M. and JOHNSON C. E., *J. Geophys. Res.*, 79 (1974) 1635.

[6] RASENAT S., BUSSE F. H. and REHBERG I., *J. Fluid Mech.*, 199 (1989) 519.

[7] WAHAL S. and BOSE A., *Phys. Fluids*, 31 (1988) 3502.

[8] PROCTOR M. R. E. and JONES C. A., *J. Fluid Mech.*, 188 (1988) 301.

[9] RICHTER F. M. and MCKENZIE D. P., *J. Geophys. Res.*, 86 (1981) 6133; OLSON P., *J. Geophys. Res.*, 13 (1984) 11293.