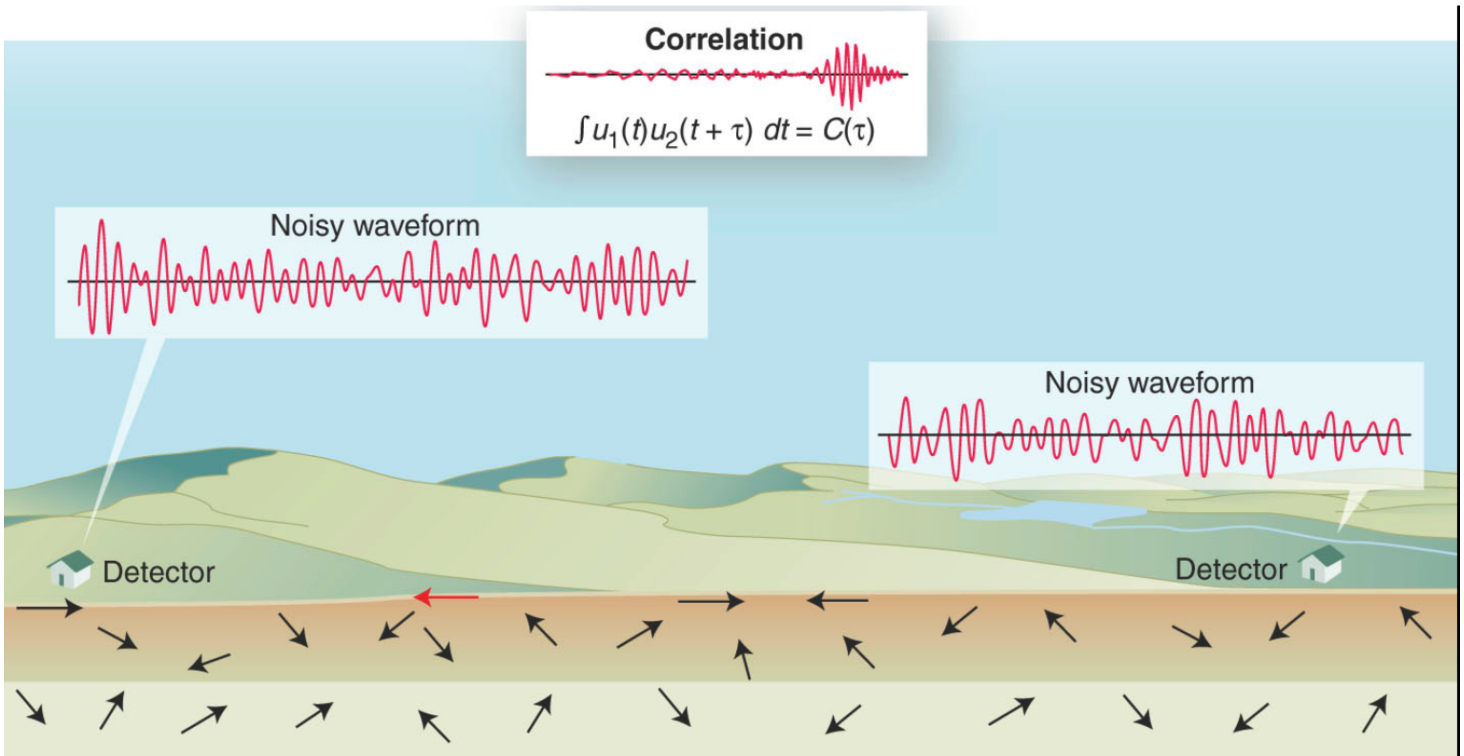
Field-field correlations in ultrasonics and seismology, and the Retrieval of Green's Function from noise

- *RL Weaver*
- *University of Illinois*



With thanks to my collaborators, including:
Xiaodong Song, J Yoritomo, Oleg Lobkis   U Illinois
David Yang   LANL
The team of Ondes et Structures *et al*   Grenoble

Cargese May 2015

This subfield of seismology grew out of a theoretical insight that seismic noise correlations

Noise record ψ at positions x,y

$$C(\vec{x},\vec{y};\tau) = \int \psi(\vec{x},t)\psi(\vec{y},t+\tau)dt$$

"lapse time τ"

Should be equal (*with <u>numerous</u> caveats and clarifications*) to

$$G(\vec{x},\vec{y};\tau)$$

the medium's Green function, representing the response you would have at position $\vec{x}$ given a unit impulse at $\vec{y}$

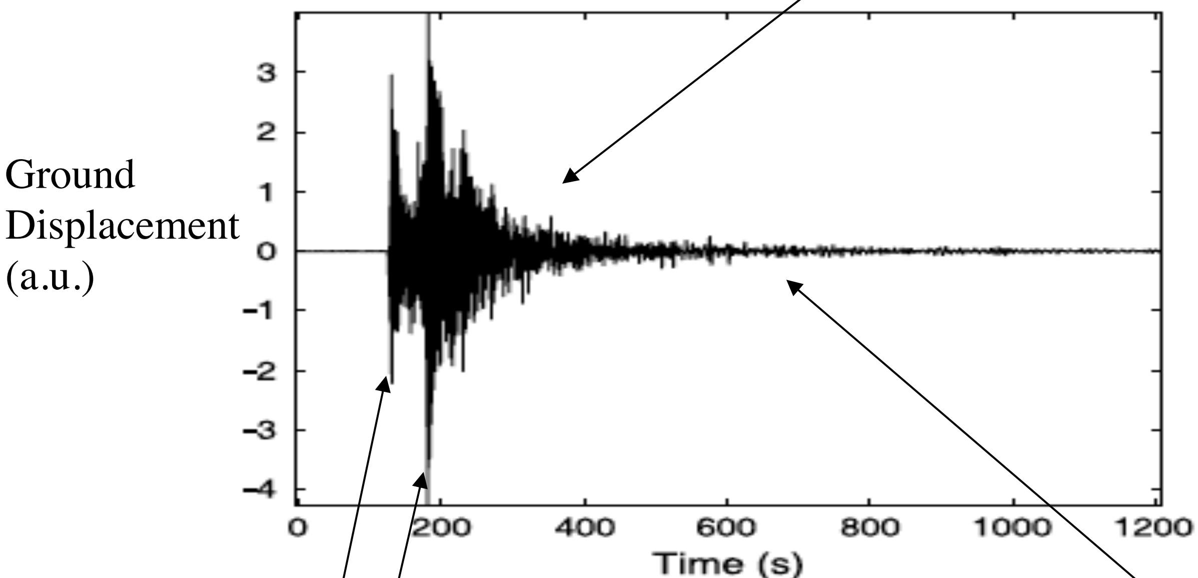
That is, by cross-correlating random noise, we can construct what we'd get if we could do an active experiment by artificially generating seismic waves.

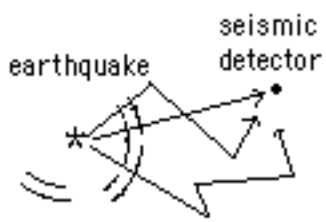
Potentially very convenient!

Major caveat: The noise has to be fully diffuse, "equipartitioned"

One way to have full diffuseness is to have the noise be multiply scattered, e.g. "coda"

For example in earthquakes like this





Typical frequencies

$f \sim$ Hz

Wavelengths

l ~ km

There are (two) main arrivals (P and S or R),

and also lots of random looking multiply scattered "coda"

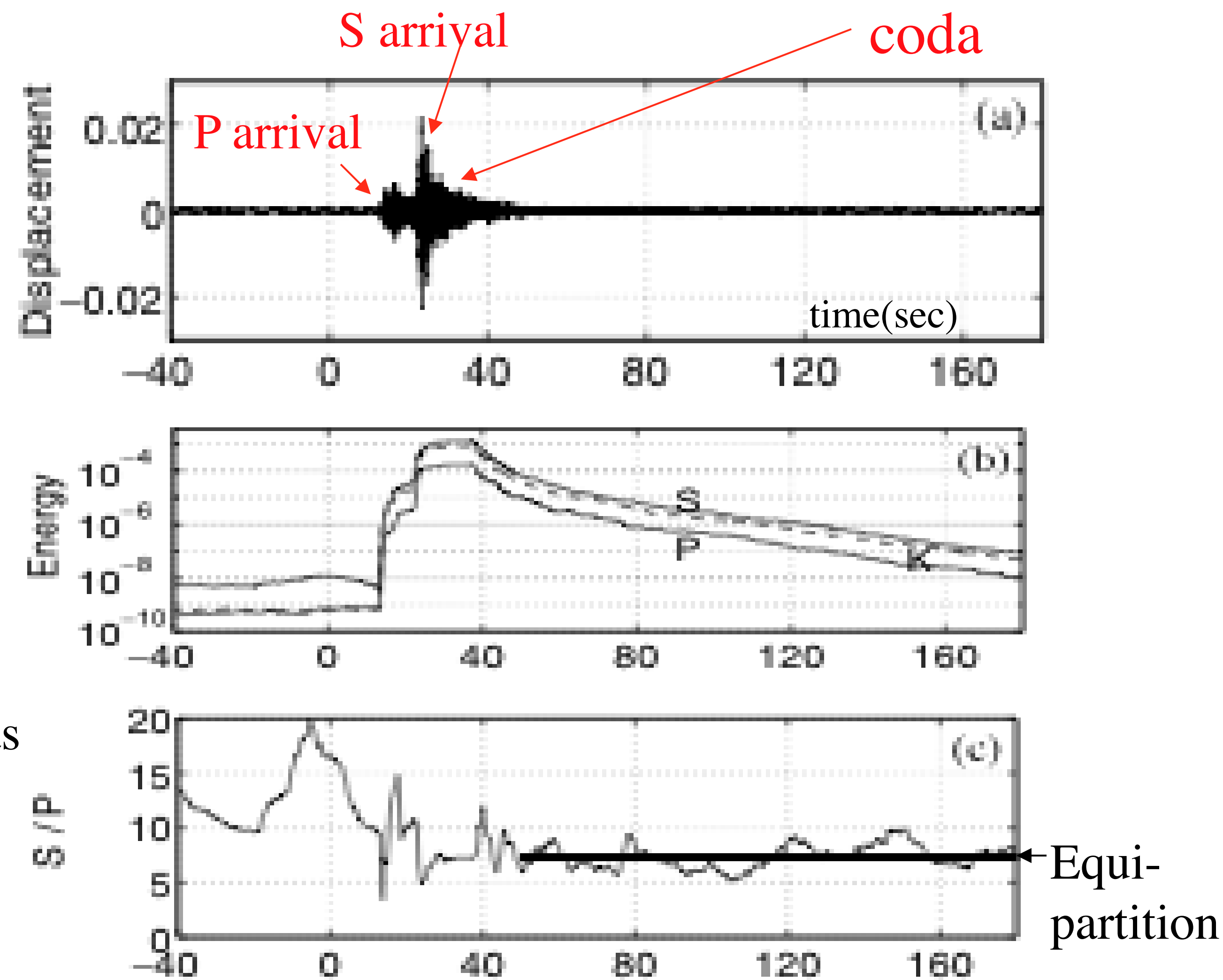
History of the approach . . .

Conversations at a 1999 workshop,
about the seismic coda - which appeared to be equipartitioned

An earthquake record

Ray arrivals are followed by low amplitude noise, or "*coda*"

The coda appears to achieve a steady state ratio of its energy contents
For example, its shear-to-dilational energies: S/P



Phys Rev Lett **86** 3447-50 (2001)

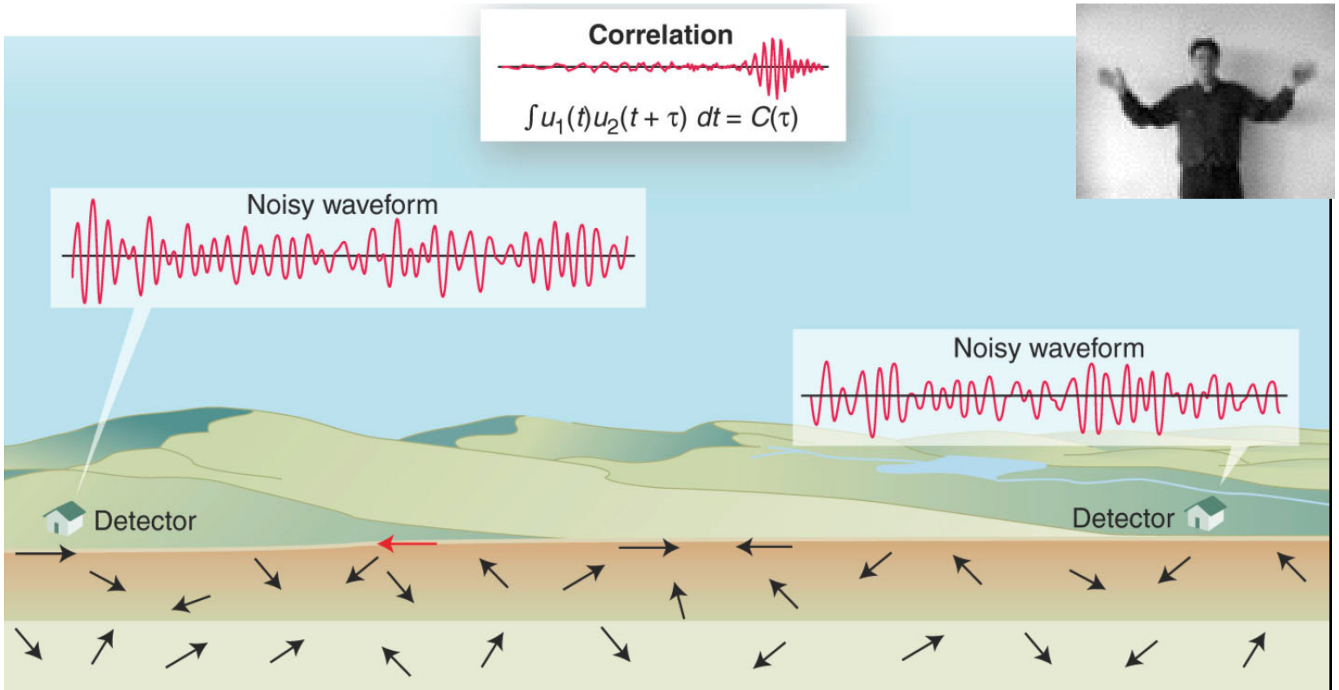
History of the approach continued. . .

I pointed out

"if a wave field (e.g. seismic coda) is multiply scattered to the point of being equipartitioned, the field's correlations should be Green's function,

And we could recover lots of information without using a controlled source"

Hand-waving plausibility argument . . .



that there could well be a signature, an "arrival,"
at the correct travel time

- due to those few rays that happen to be going the right way

  But G exactly?

  And where's the proof?

  And won't other rays from other directions obscure the effect?

Standard Proofs . .

For a conventional diffuse acoustic field (applies to coda?)
can invoke a modal picture (for closed systems)
or a plane wave picture (for homogeneous systems)

For a thermally diffuse field
can refer to a picture in terms of the normal modes
or prove it using the fluctuation-dissipation theorem

The simplest proof involves a common definition of a fully diffuse field, from room acoustics, or from the physics of thermal phonons, in terms of the normal mode expansion for the field in a finite body

$$\psi(\vec{x},t) = \mathrm{Re}\sum_{n=1}^{\infty} a_n\, u_n(\vec{x})\exp(i\omega_n t)$$

For which we assert modal amplitude statistics

$$< a_n a_m^* > = \delta_{nm} \quad 2F(\omega_n)/\omega_n^2 \qquad \text{"equipartition"}$$

n.b: this follows from maximum entropy where F ~ energy per mode ($k_B T$)

It is then straightforward to derive

$$C(\tau) \equiv < \psi(\vec{x},t)\psi(\vec{y},t+\tau) > =$$

$$\mathrm{Re}\sum_{n=1}^{\infty} F(\omega_n)\, u_n(\vec{x})\, u_n(\vec{y})\exp(-i\omega_n \tau)/\omega_n^2$$

$$C(\tau) \equiv \langle \psi(\vec{x},t)\psi(\vec{y},t+\tau) \rangle =$$

$$\sum_{n=1}^{\infty} F(\omega_n) u_n(\vec{x}) u_n(\vec{y}) \cos(\omega_n \tau) / \omega_n^2$$

Compare with the modal representation for G . . .

$$G_{xy}(\tau) = H(\tau) \sum_{n=1}^{\infty} u_n(\vec{x}) u_n(\vec{y}) \sin(\omega_n \tau) / \omega_n$$

(H= *unit step function*)

We may conclude

$\partial C/\partial\tau$ = -{G - $G^{\text{time reversed}}$} convolved with $F(\tau)$

An alternative proof,

based on G's role as a *propagator of initial conditions*

$$\psi(\vec{r},t+\tau) = \int d\vec{a}\ \ \psi(\vec{s}+\vec{a},t)\ \dot{G}(\vec{s}+\vec{a},\vec{r},;\tau)\ +\ \int d\vec{a}\ \ \dot{\psi}(\vec{R}+\vec{a},t)\, G(\vec{s}+\vec{a},\vec{r};\tau)$$

ψ at position r and a later time t + τ

may be constructed in terms of an integral of ψ

over all other positions at an earlier time t.

Now take the expectation $C_{s\rightarrow r}(\tau)$ =

$$C(\tau) = <\psi(\vec{r},t+\tau)\psi(\vec{s},t)>$$

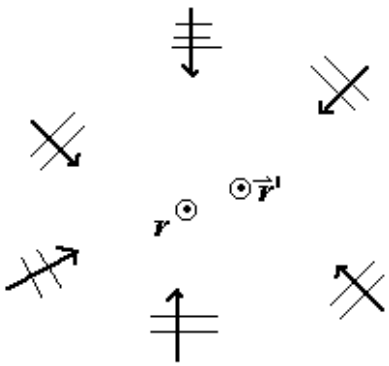
$$= \int d\vec{a}\ \ <\psi(\vec{s}+\vec{a},t)\psi(\vec{s},t)>\ \dot{G}(\vec{s}+\vec{a},\vec{r},;\tau)\ +\ \int d\vec{a}\ \ <\dot{\psi}(\vec{s}+\vec{a},t)\psi(\vec{s},t)> G^{ac}(\vec{s}+\vec{a},\vec{r};\tau)$$

$C_{s\rightarrow r}(\tau)$ is seen to be a *spatial convolution* of the

<u>equal time C(τ=0)</u> with <u>G</u>

<u>A proof applicable to ballistic media</u> (i.e those with no significant scattering)

If the field in the vicinity of the receivers is
an incoherent superposition of plane waves – of equal intensity in all directions



$$\tilde{\psi}(\vec{r},\omega) = \int A(\theta) \exp(-i\omega\hat{\theta}\bullet\vec{r}\,/\,c)\; d\theta$$

(2-d)

with $\quad < A >= 0; \qquad < A(\theta)A^{*}(\theta') >= \delta(\theta-\theta')$

Then the field-field correlation is

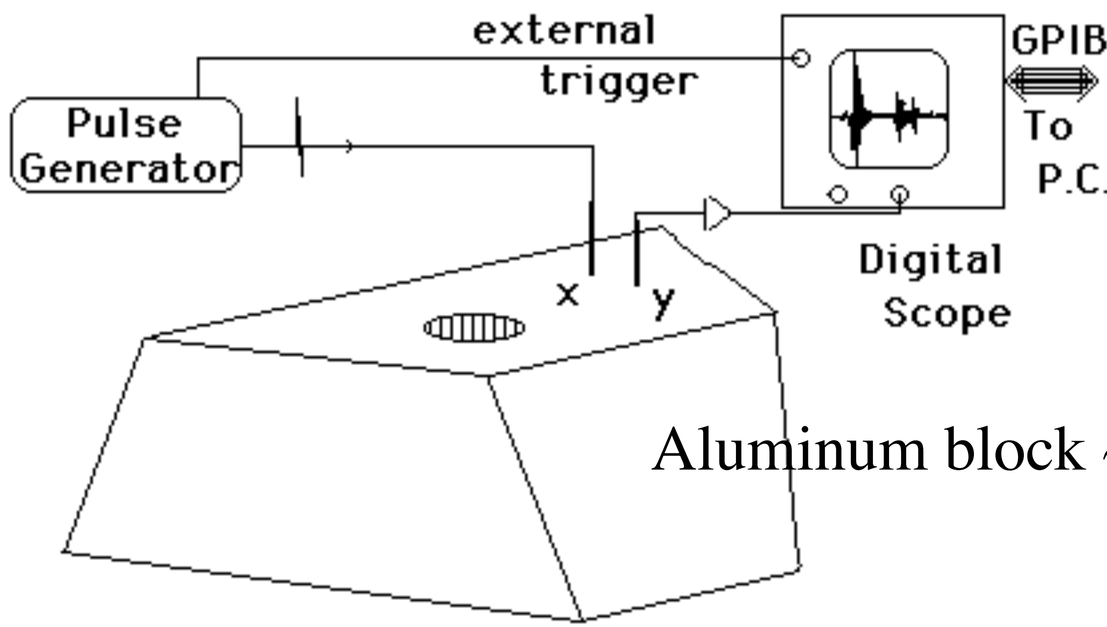
$$< \tilde{\psi}(\vec{r},\omega)\; \tilde{\psi}(\vec{r}\,',\omega)^{*} >= \int \exp(-i\omega\hat{\theta}\bullet(\vec{r}-\vec{r}\,')\,/\,c)\; d\theta =$$

$$2\pi J_{o}(\omega\,|\,\vec{r}-\vec{r}\,'\,|\,/c) \sim \operatorname{Im} G$$

QED

Laboratory Verification ?

An ultrasonic "pitch-catch" measurement

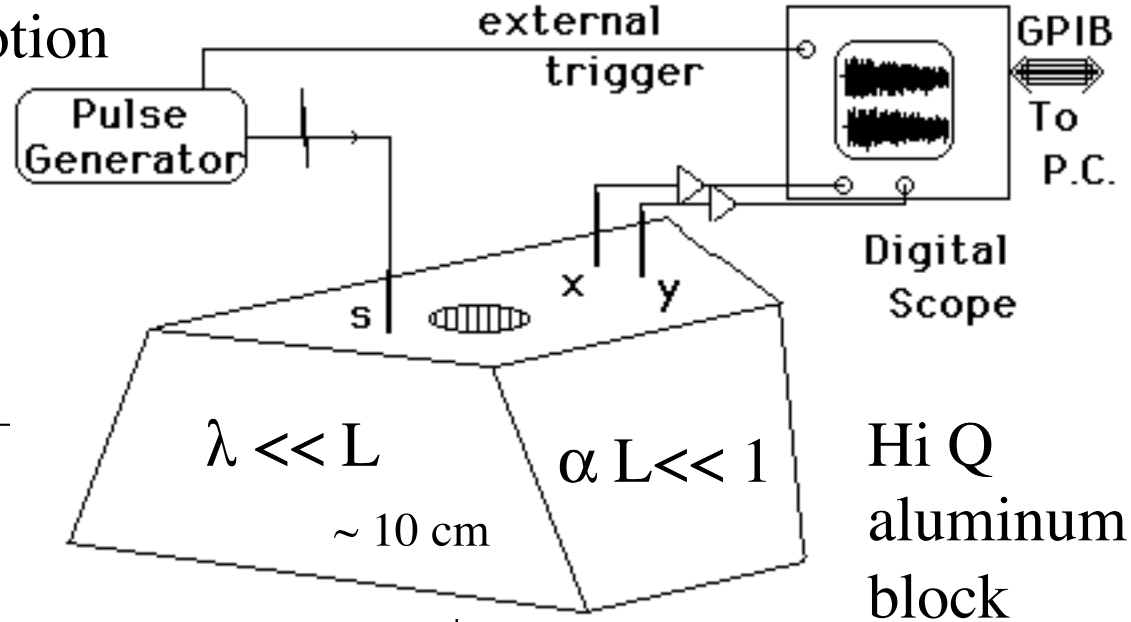


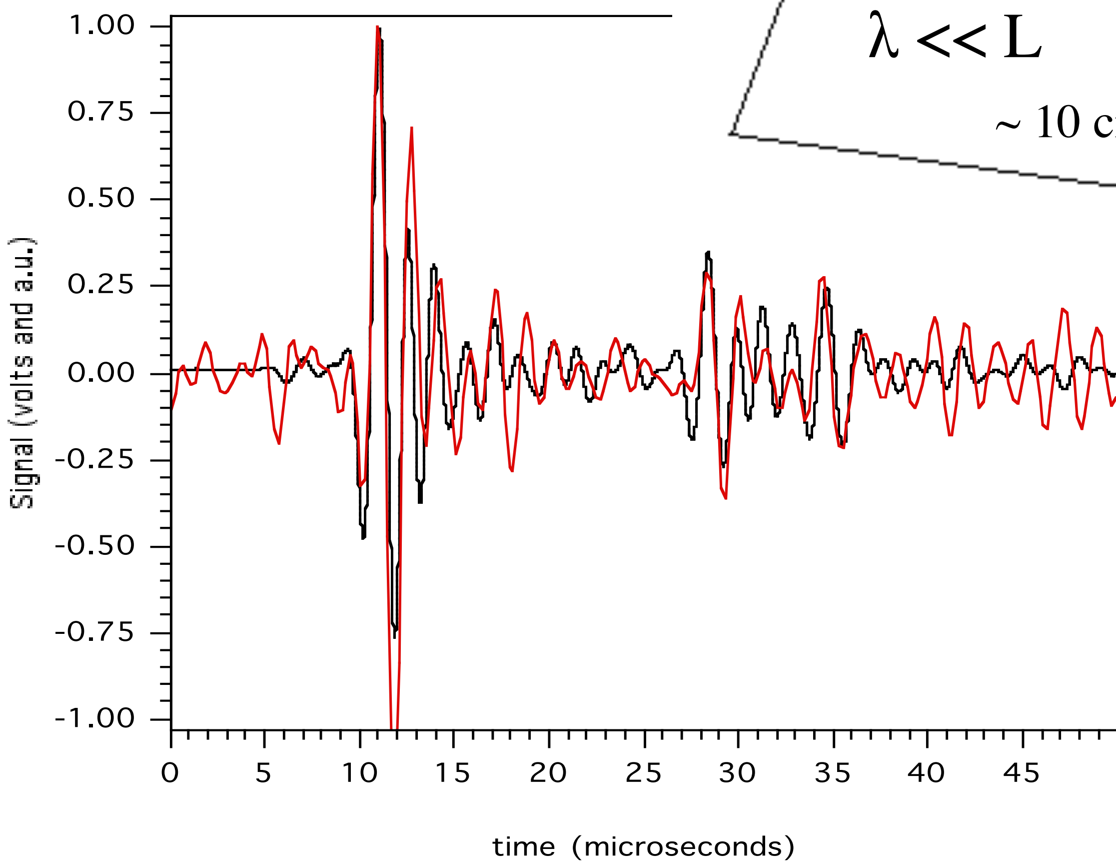
Aluminum block ~ 10 cm wide

An impulse (with frequencies up to MHz) is applied at position x.

The resulting mechanical motion (wavelengths λ~mm)
is detected at position y.

Now apply the source at position "s"
And detect the resulting motion
at x and y.





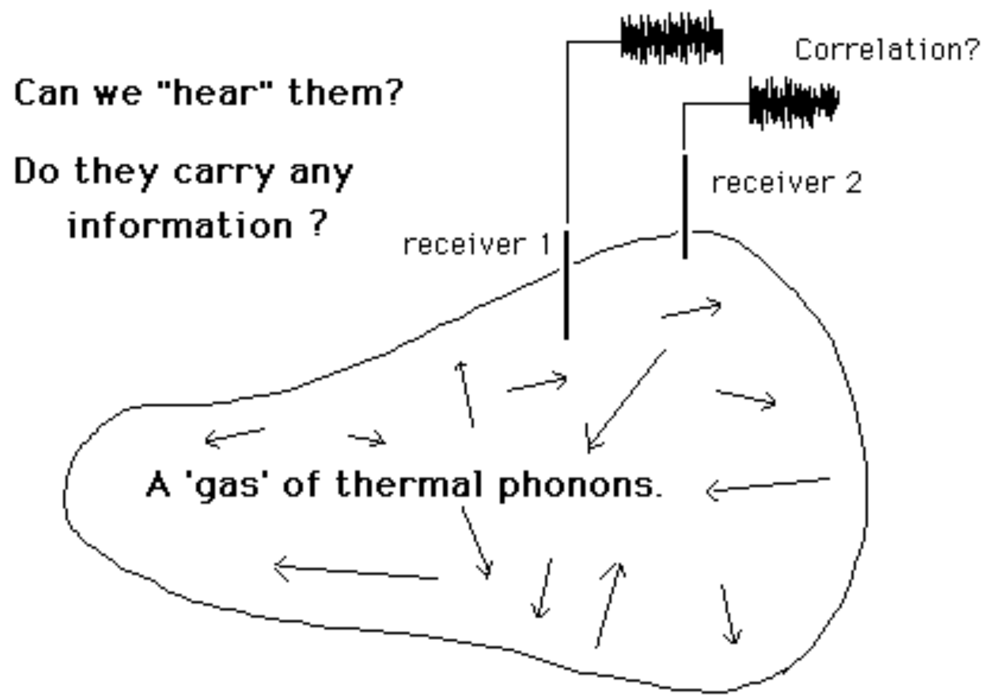
direct pitch/catch signal
x ➔ y

Correlation between signals
s ➔ x & s ➔ y

J Acoust Soc Am. **110**, (2001)

The best diffuse field is that provided by thermal fluctuations of elastic waves

A gas of phonons as it were . . . .



The strength of a thermal ultrasonic field at MHz frequencies

1) Classical Thermal Fluctuation analysis tells us;

Each mode has small energy $kT \approx 4.2 \times 10^{-21}$ joules

For typical solids,

with mode counts below 1 MHz of ~ 300 modes / cm^3

We have energy densities of ~ 10^{-12} Joules / m^3

and rms strain amplitudes of ~ 3×10^{-12}

and rms displacement amplitudes of

~ 10^{-15} meter

2) How difficult is it to detect such weak signals?

We'll see . . . .

3) Why should we do so?

Answer:

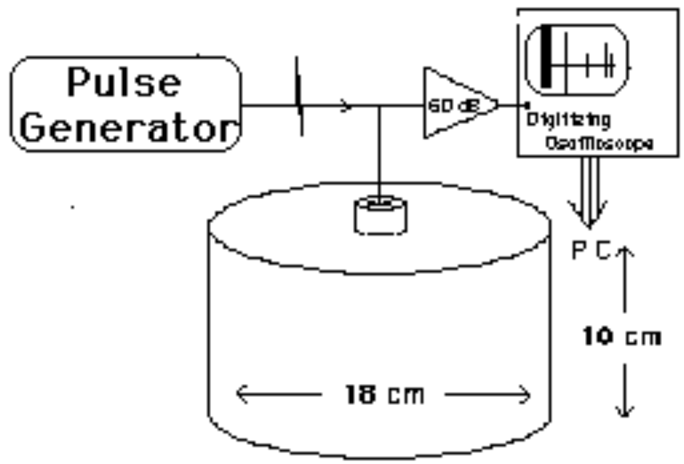
They are perfectly diffuse,

and carry ultrasonic information

Laboratory verification in the thermal case:

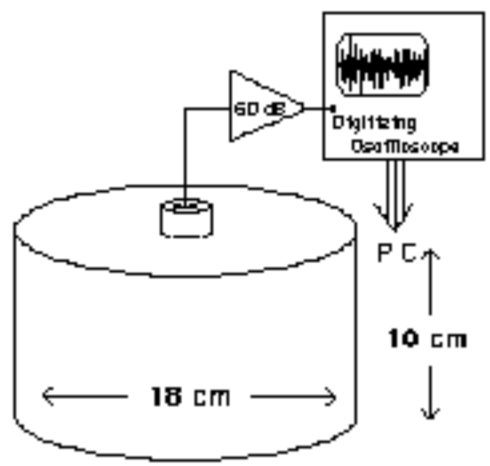
Comparison of a
Direct Pulse-Echo
Signal,

(conventional ultrasonics)

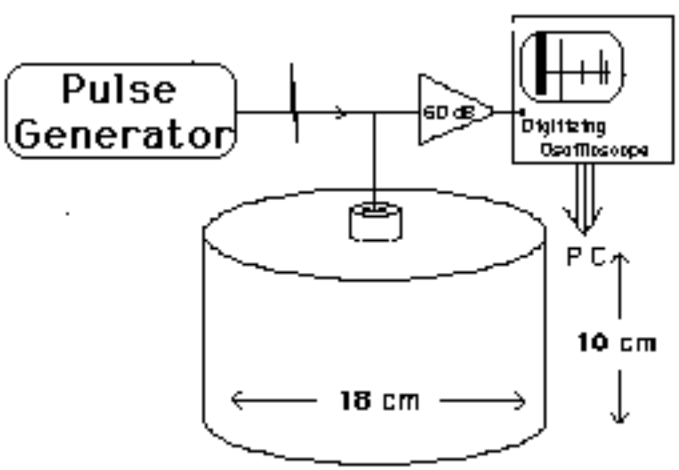


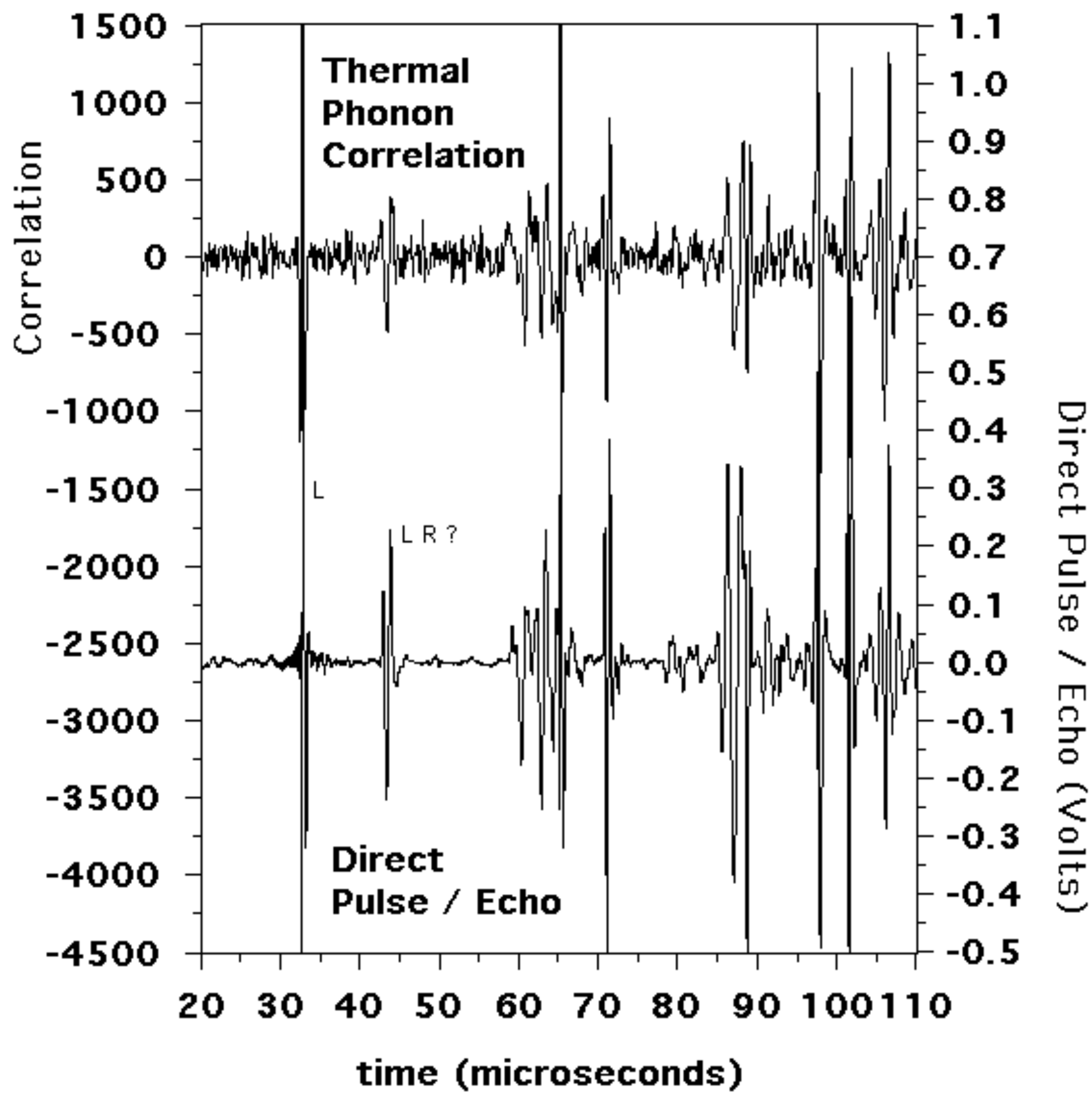
and

Thermal Noise
Correlation

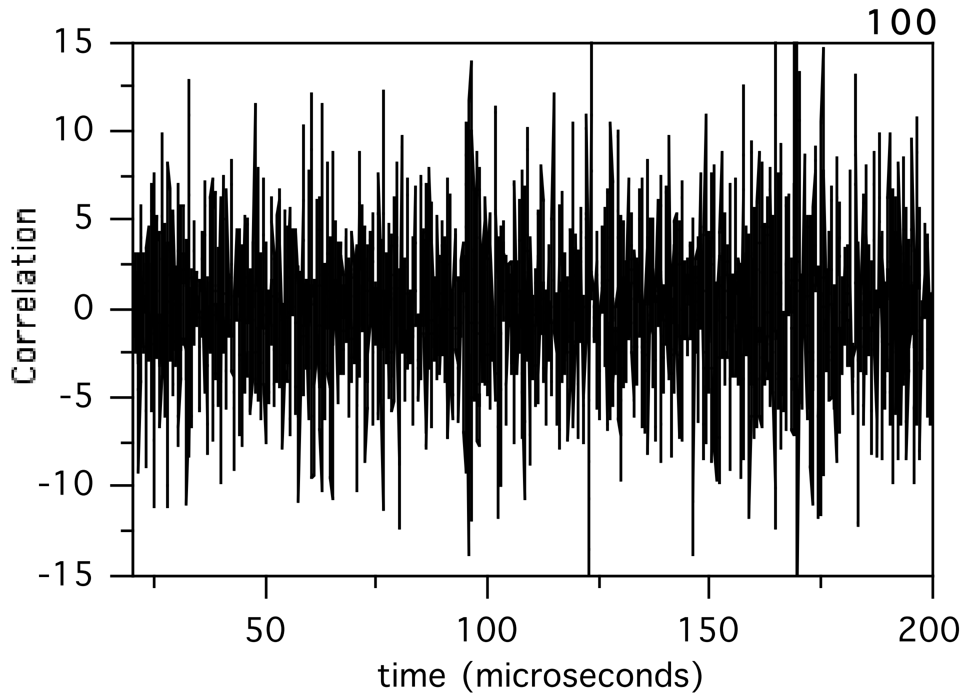


Direct Pulse-Echo Signal



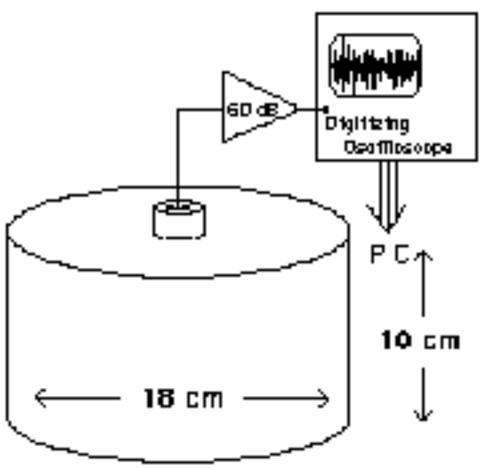


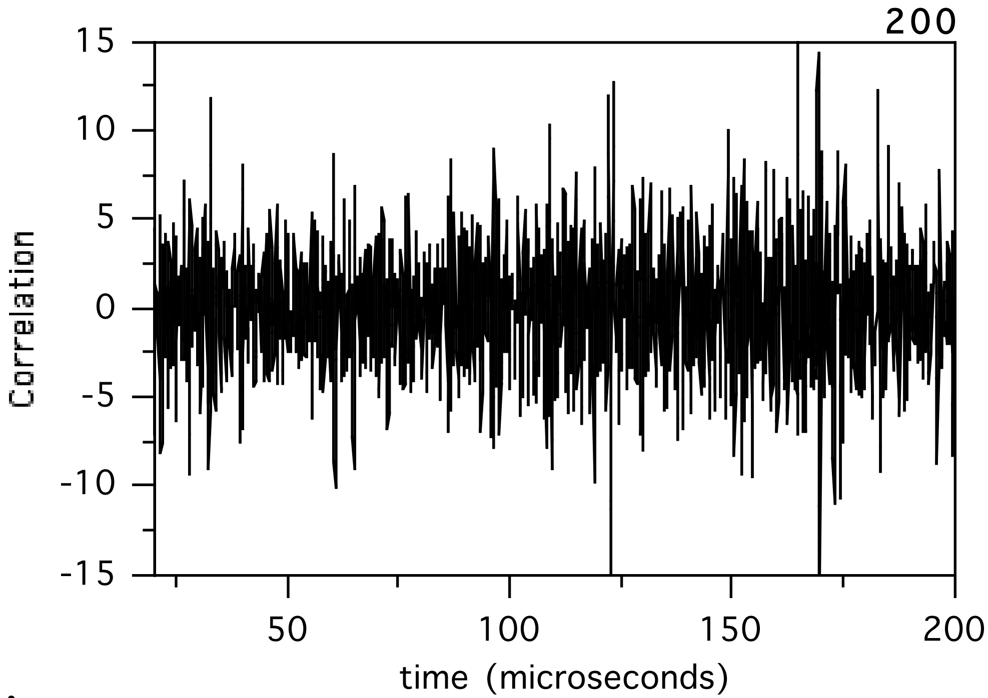
Thermal noise correlations

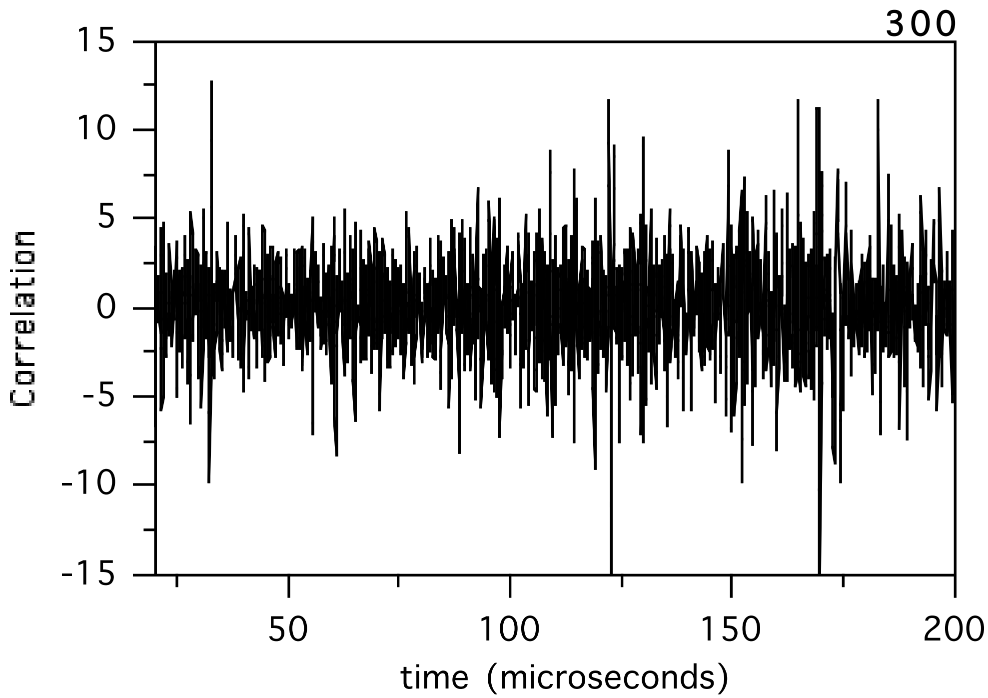


After
Capturing
320 msec
of noise data

(and taking
9 seconds to do so)




200
Correlation
15
10
5
0
-5
-10
-15
50
100
150
200
time (microseconds)


300
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Correlation
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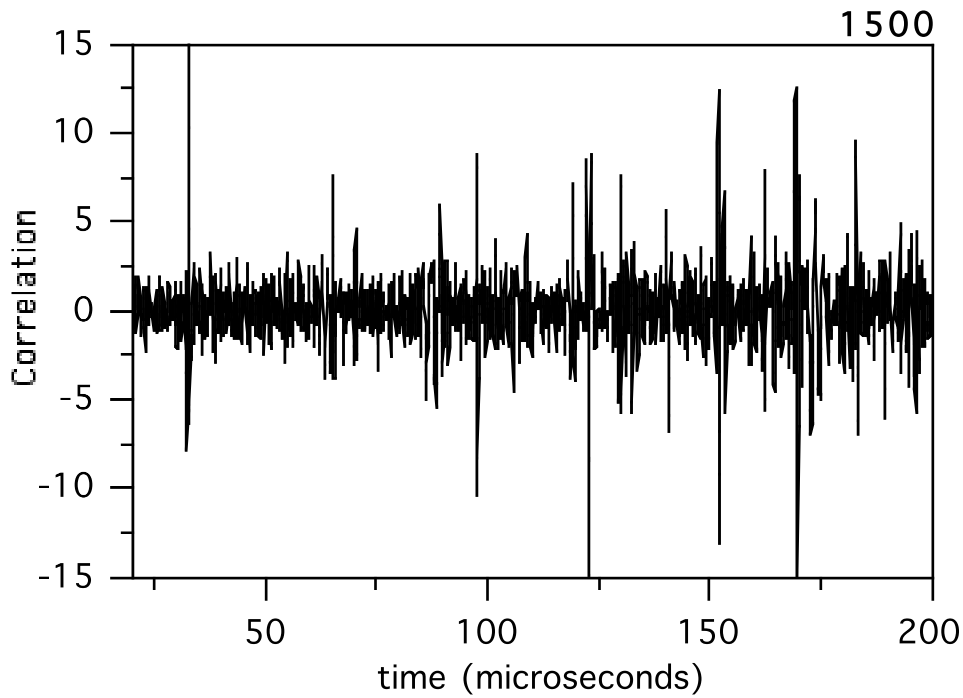
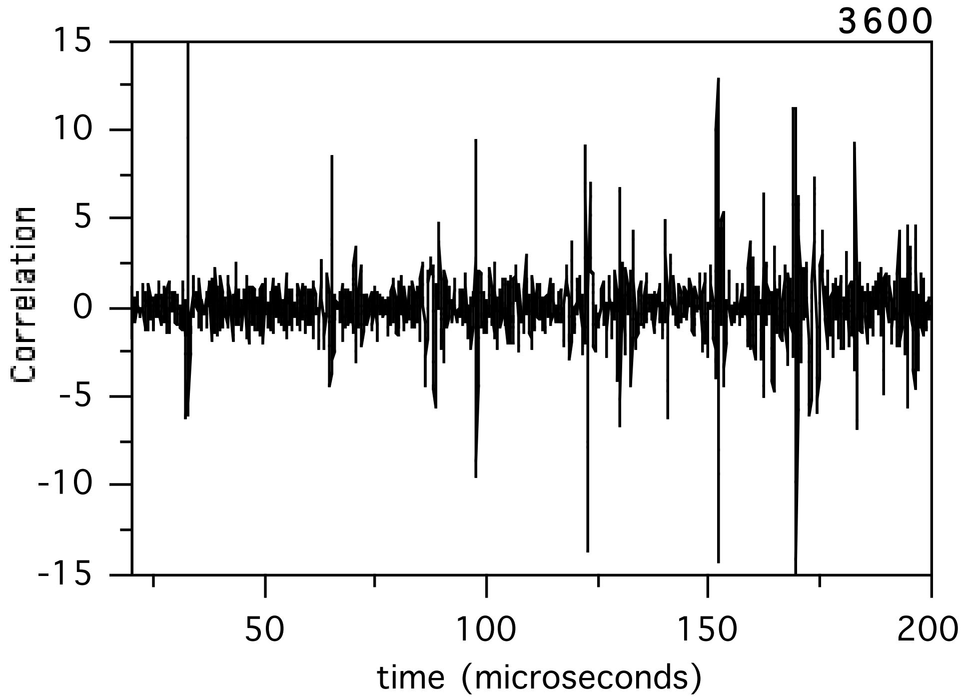
time (microseconds)



1500
15
10
5
0
-5
-10
-15
Correlation
50
100
150
200
time (microseconds)

•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10
5
0
-5
-10
-15
Correlation
50
100
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200
time (microseconds)

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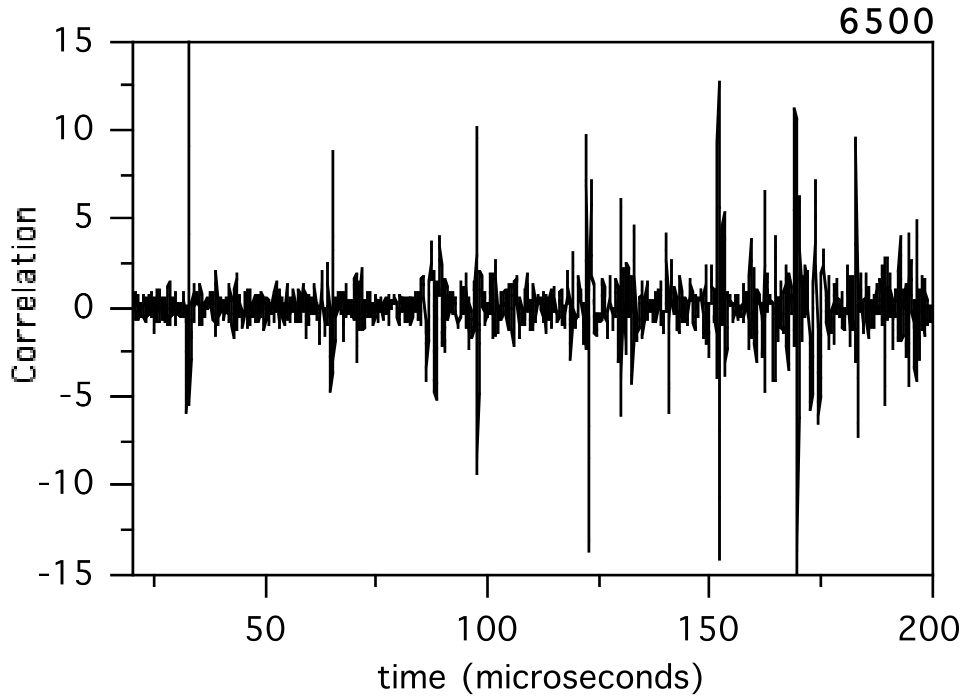

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-10
-15
Correlation
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150
200
time (microseconds)

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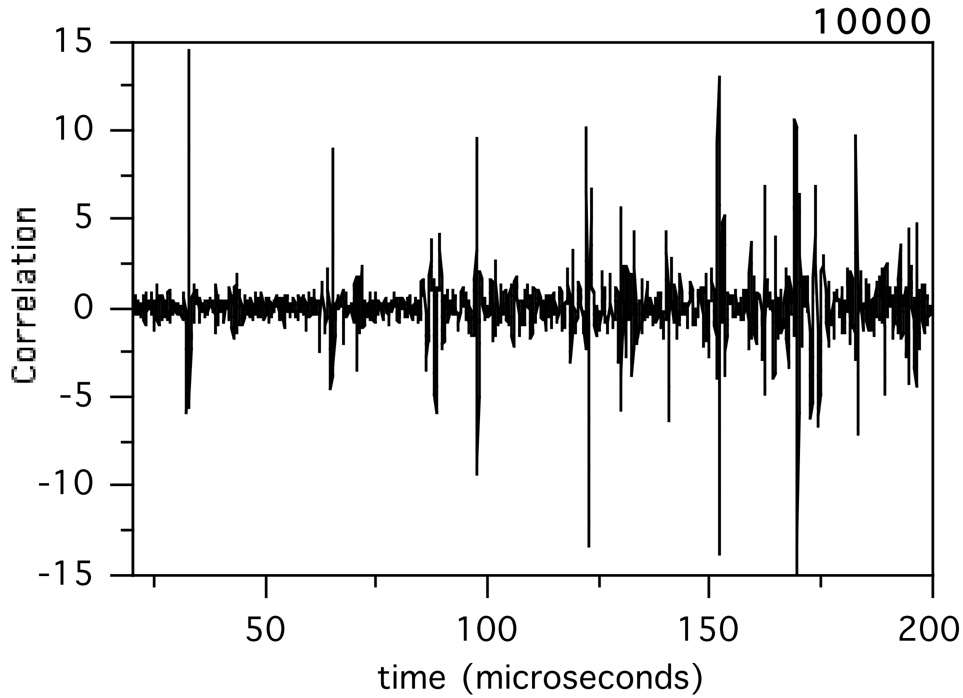
10000
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-5
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-15
Correlation
50
100
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200
time (microseconds)

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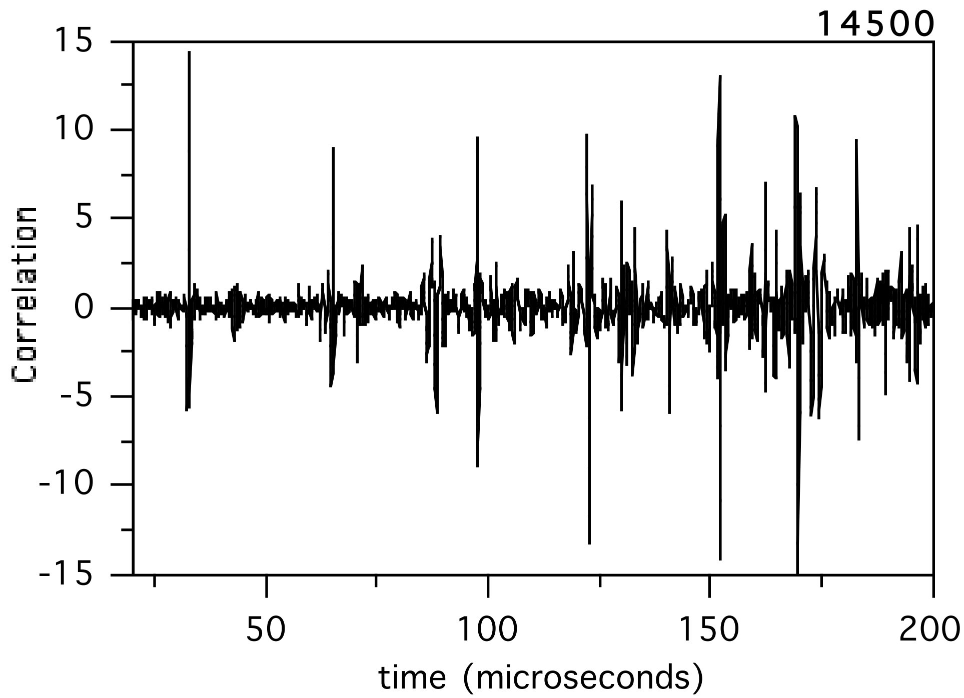

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0
-5
-10
-15
Correlation
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100
150
200
time (microseconds)

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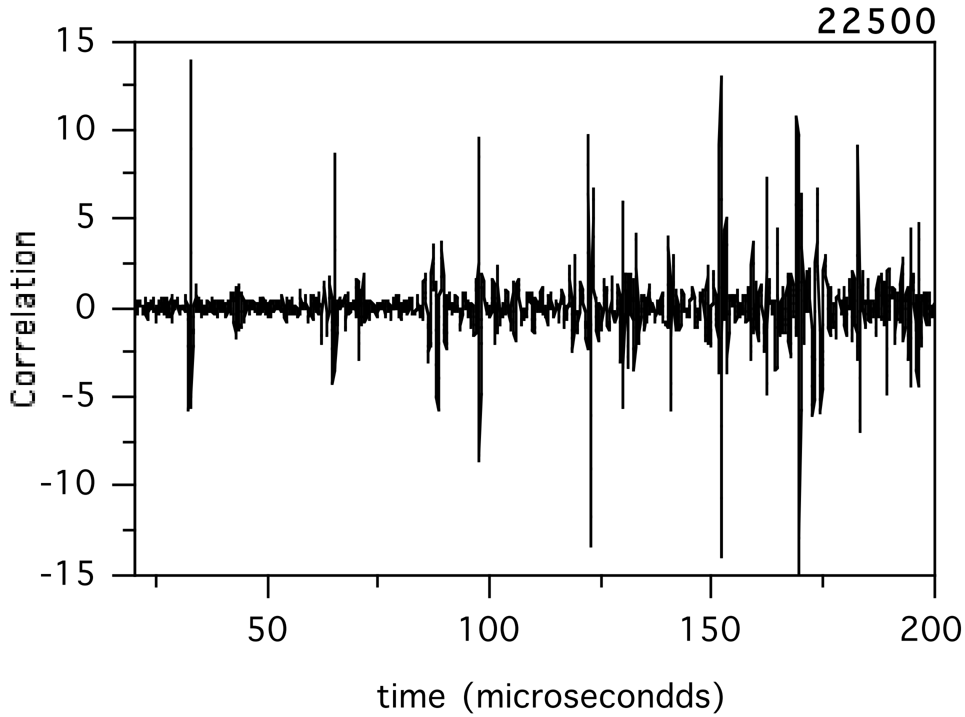
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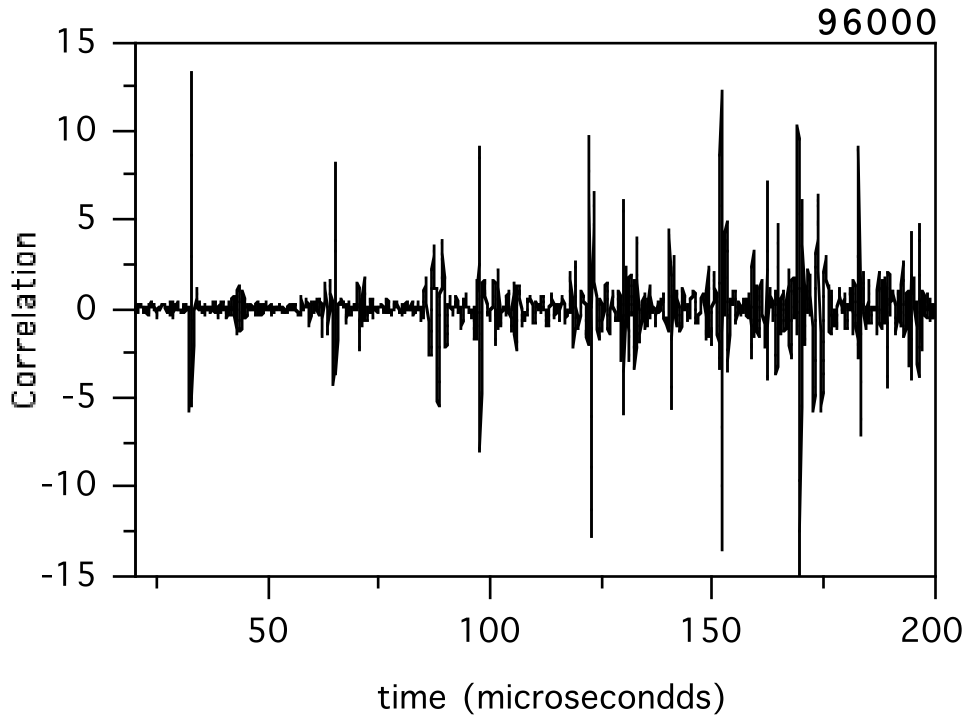
-10
-15
Correlation
50
100
150
200
time (microsecondds)

•


96000
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Correlation
50
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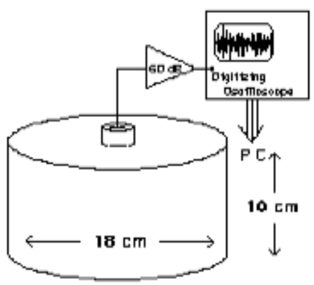
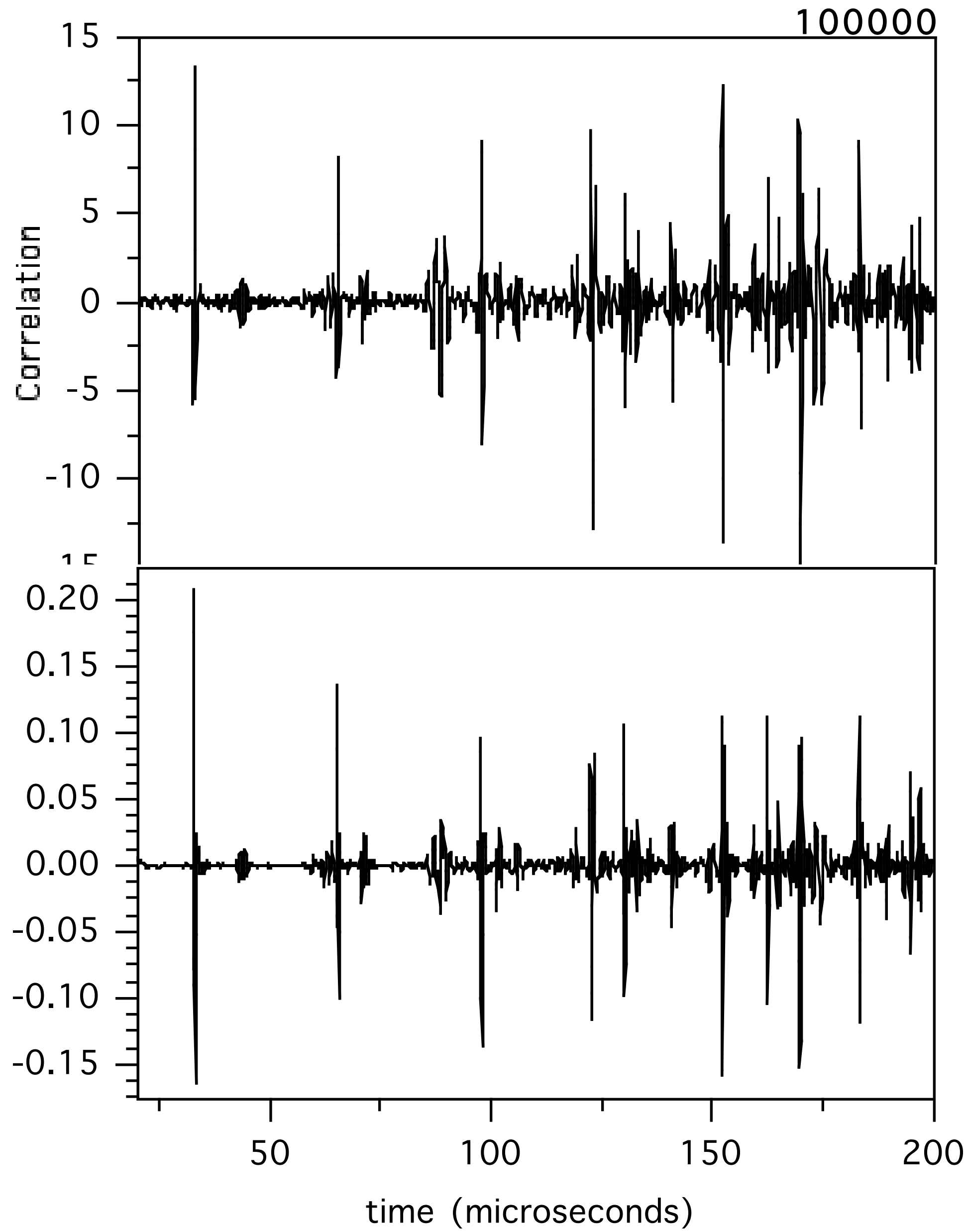
time (microsecondds)

•

Thermal noise correlation

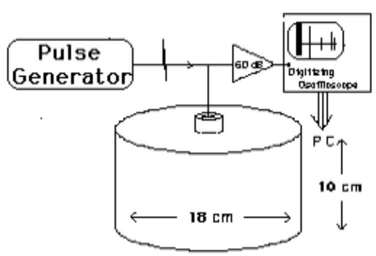




After Capturing 320 seconds Of data

(and taking 2.5 hours to do so)

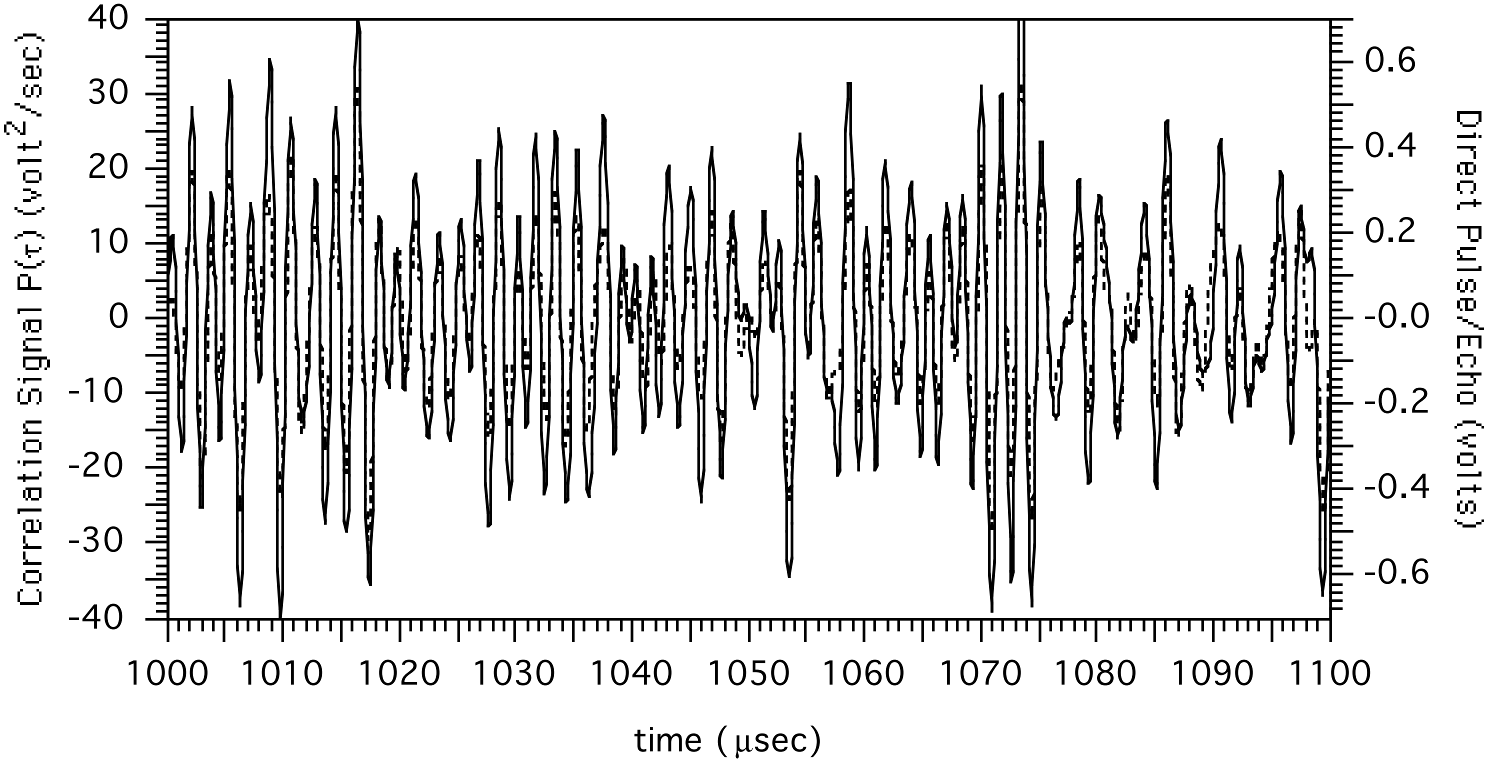
Direct pulse-echo



PRL (2001)

Comparison at later times

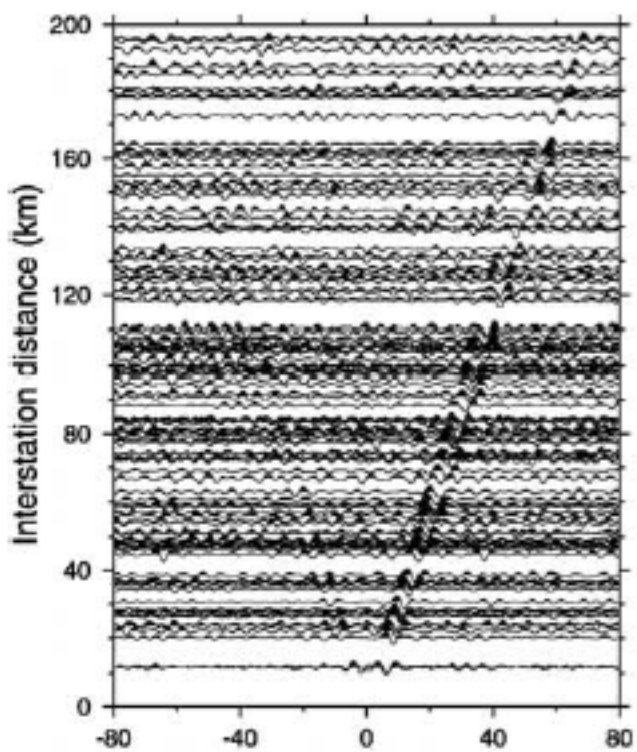
(~ 1 msec, after rays have traveled ~3 meters)



This led Paul et al (Grenoble team) in 2003
to cross correlate the coda from a set of 100 Alaskan earthquakes as measured on an array of many (N) seismic stations

From this they constructed N(N-1)/2 cross correlations $C(\tau)$

and stacked them against
inter-station distance:



Lapse time τ (seconds)

From this we note two things

1) It works; we clearly see the signs of propagation
2) It works *badly*:

    Low Signal/Noise

    $C(\tau) \neq C(-\tau)$

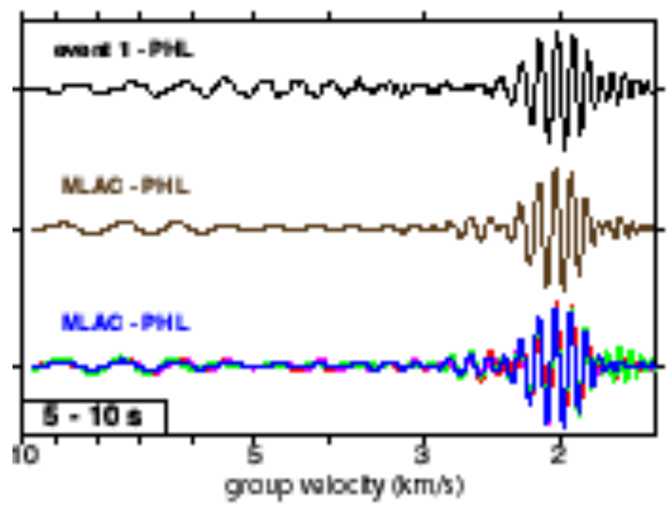
Far more successful, it turns out, were efforts to correlate Ambient Seismic Noise (rather than coda)

Advantage : there is LOTS of it. It is virtually continuous
Disadvantage: it is not equipartitioned, it is not multiply scattered

Nevertheless…Shapiro et al constructed correlations of Ambient noise in So Cal in 2005

Example traces C(τ):

We note
1) It works
2) Improved S/N

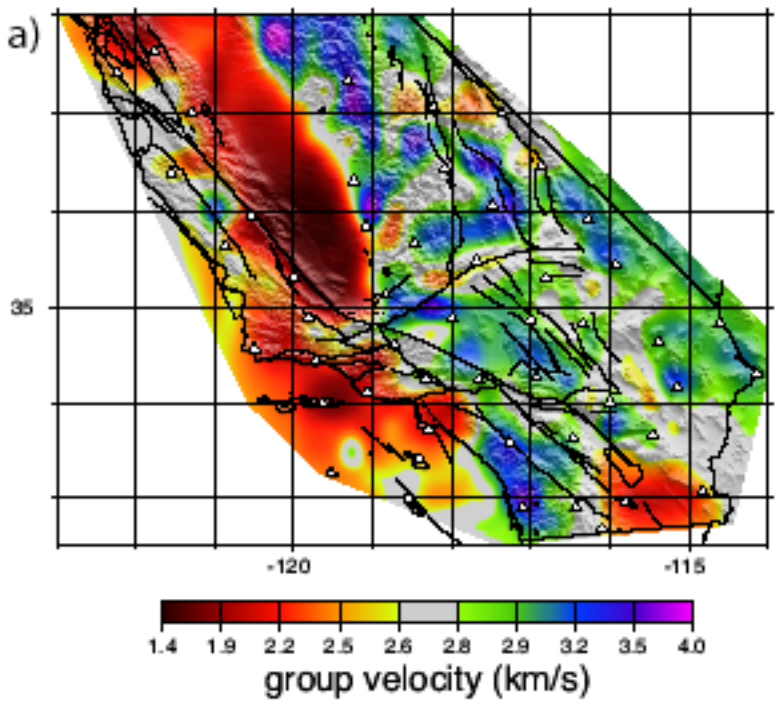


Earthquake

1 month correlation

four 1 month correlations

The work led to the first of several striking maps of
Seismic Velocity,
obtained by tomographically inverting the arrival times in the C(τ)

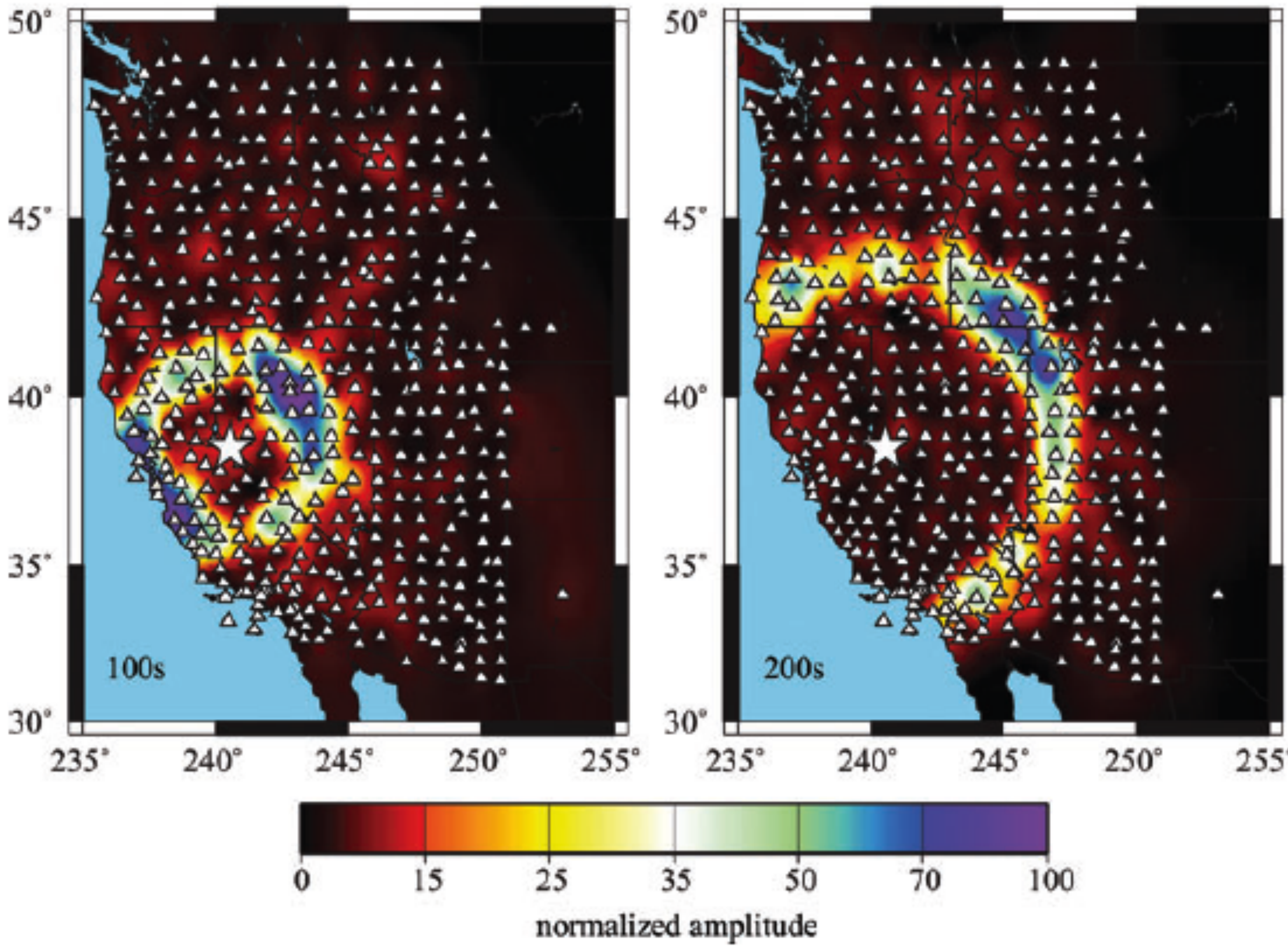


A map of Surface-Wave Velocity in California

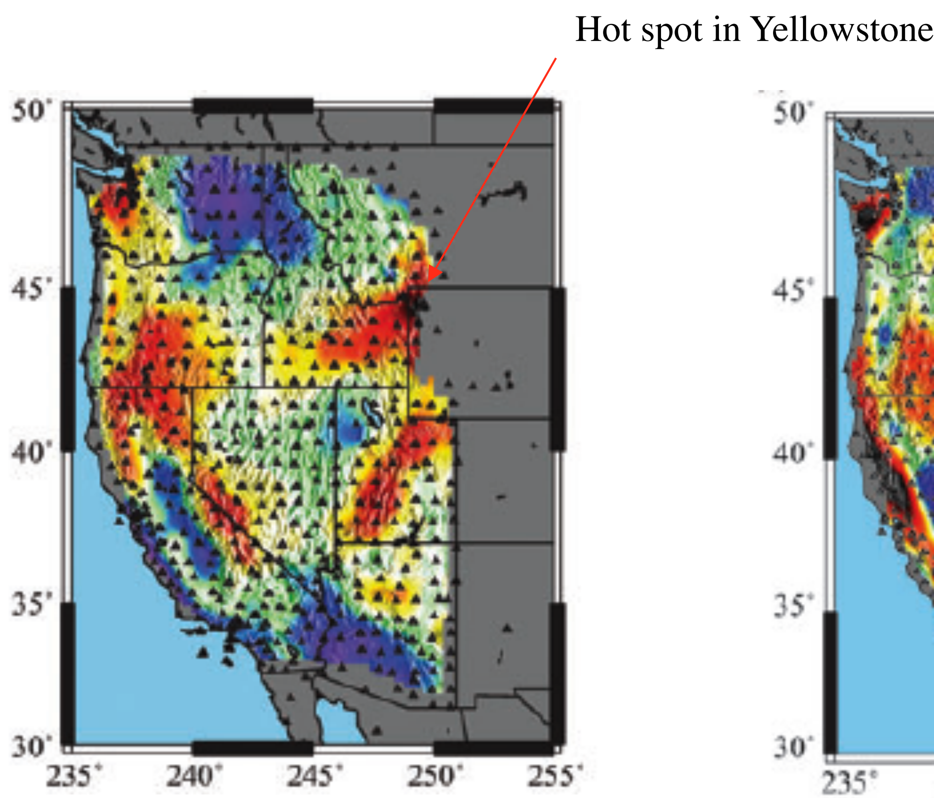
Obtained from correlating seismic noise based on correlations between each pair of a network of 62 So Cal stations
(Shapiro et al, Science 2005)

Frequencies ~0.02 < f < 1 Hz; 3km < λ <150 km

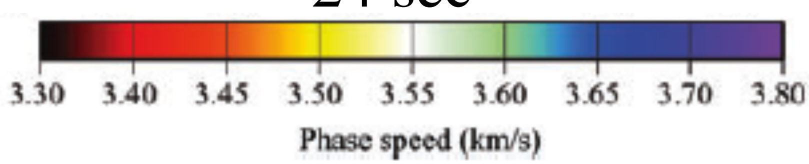
Lin and Ritzwoller and Snieder (2009) Geophys J Int
3 years of data on a larger array

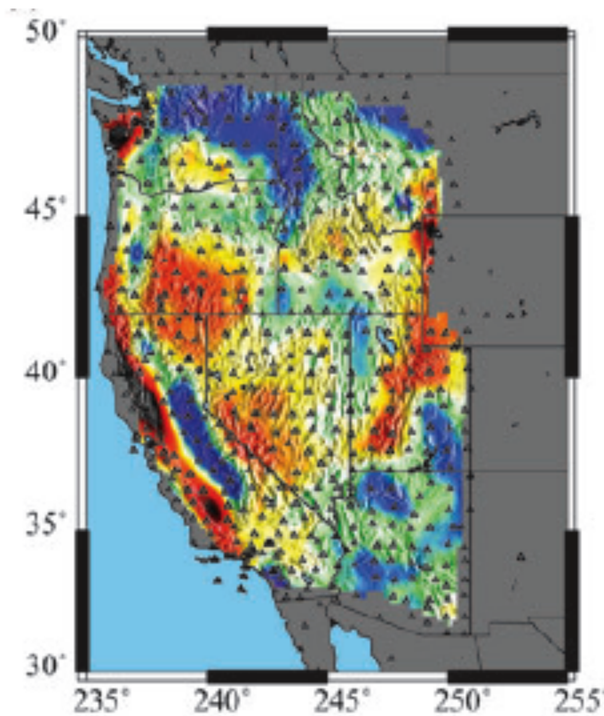


Tomographically generated maps of wave speed

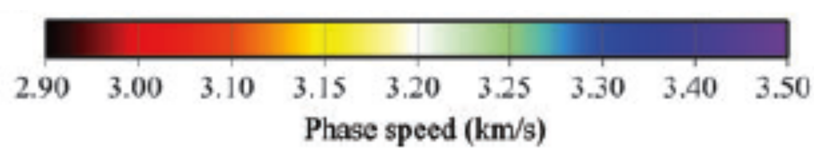


24 sec



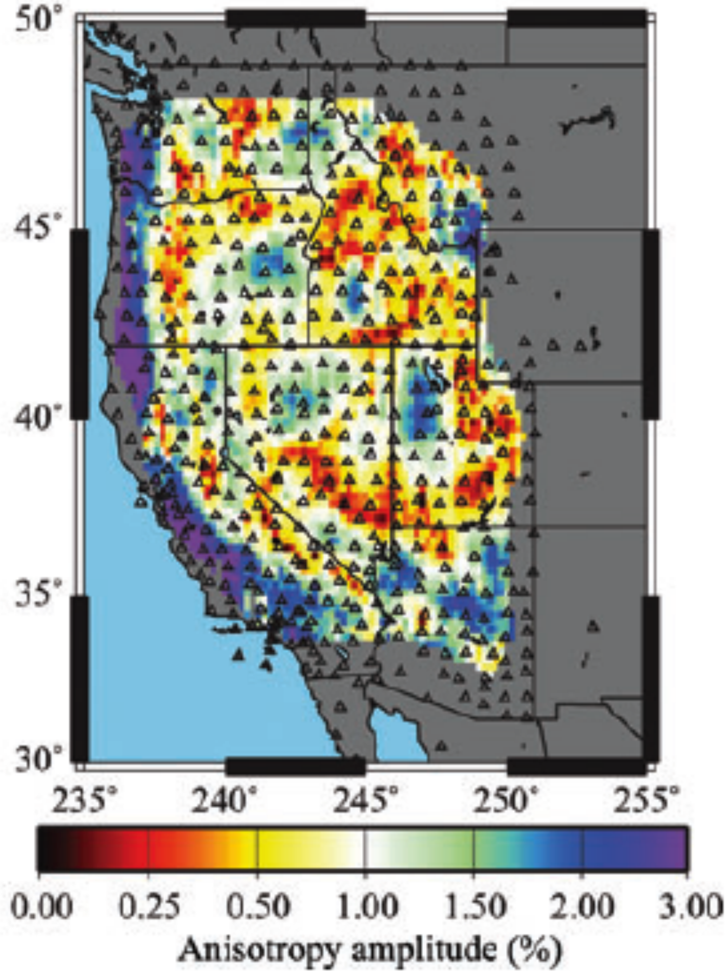


12 sec



Different properties at different frequencies
i.e, different depths

They even resolve ~ 1% anisotropies in wave speed

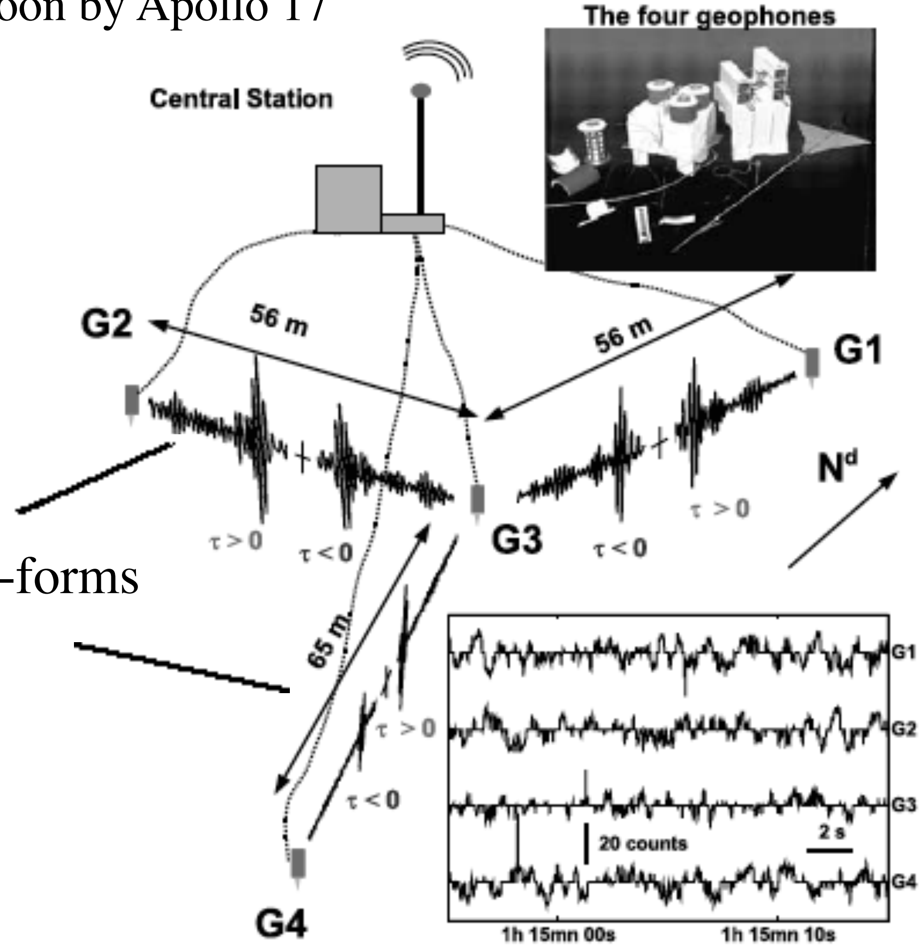


A similar procedure from the moon . . .

E. Larose, A. Khan, Y. Nakamura, M. Campillo : Lunar Subsurface Investigated from Correlation of Seismic Noise, Geophys. Res. Lett. 32 (16), L16201 (2005)

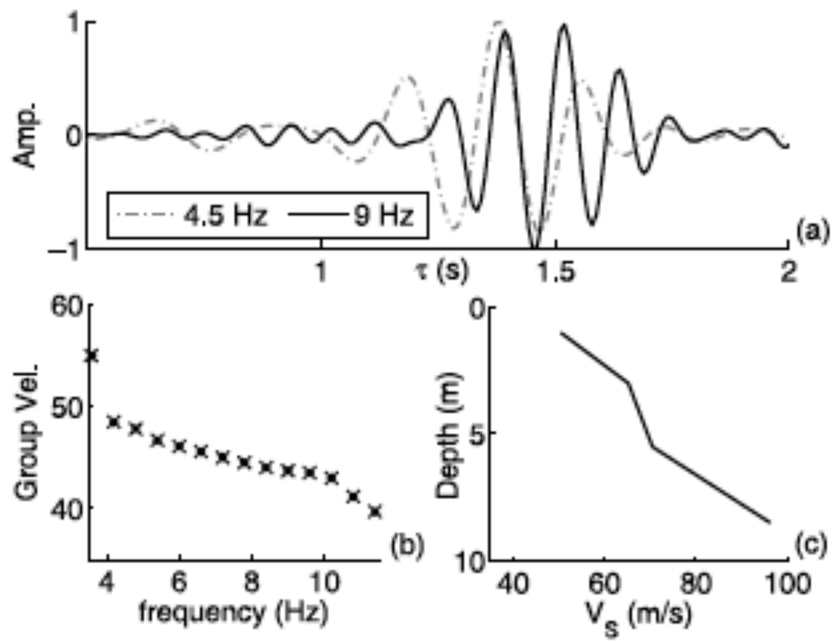
Noise collected over 16 months from four geophones left on the moon by Apollo 17

Correlation wave-forms



Typical noise record, varies in intensity with the time of "day"

Analysis of the lunar correlation waveforms gives subsurface wavespeed profile



But proofs that depend on full diffusivity
and/or finite bodies and closed acoustic systems,
may not be relevant for practice.

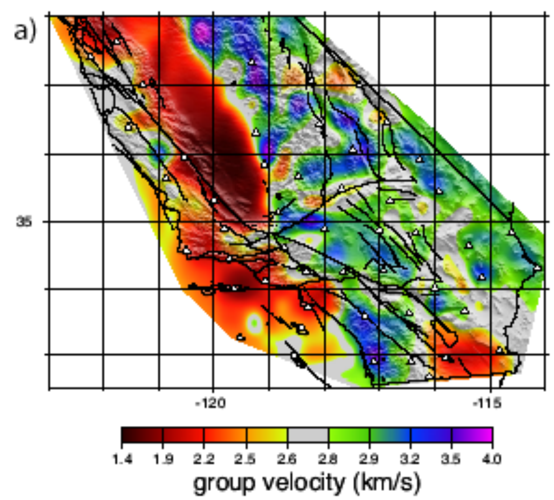
Ambient seismic noise(*), for example, is

NOT fully diffuse

It has preferred propagation directions (sources in ocean storms)

and it is not multiply scattered

Nevertheless, these
maps appear to be correct;
the method appears to work



Why does it work?

*Late *coda* appears
fully diffuse, but
there isn't enough of it.

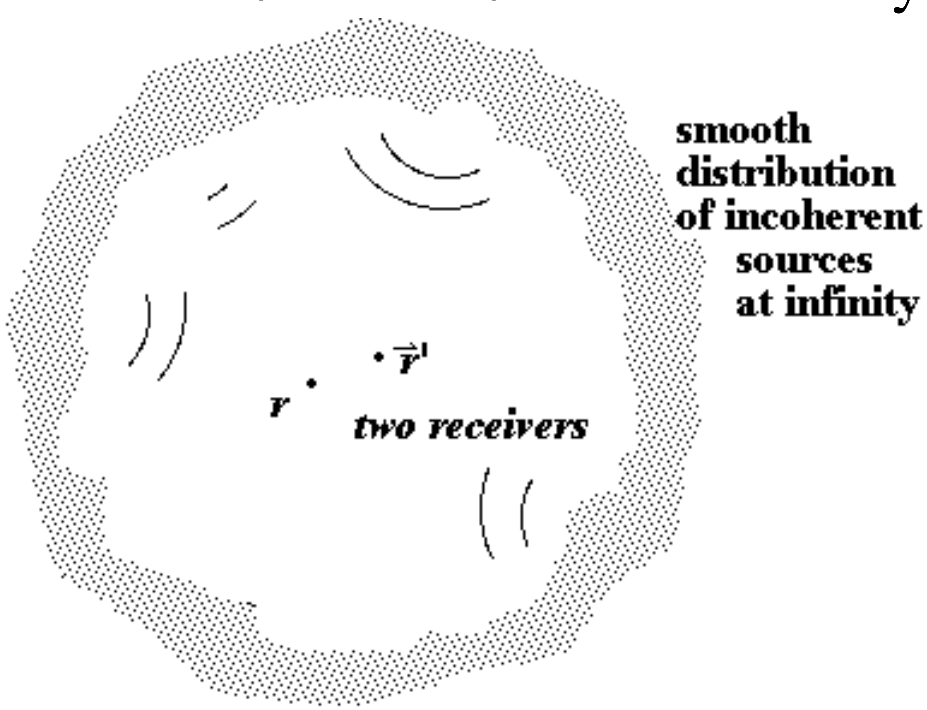
What if an incident field does not have isotropic intensity?
What it it is not equipartitioned?

May we still assert $C(\tau) \sim G(\tau)$?

It transpires that

➔ There is an asymptotic validity to assertion

Consider a homogeneous medium with incoherent sources at infinity

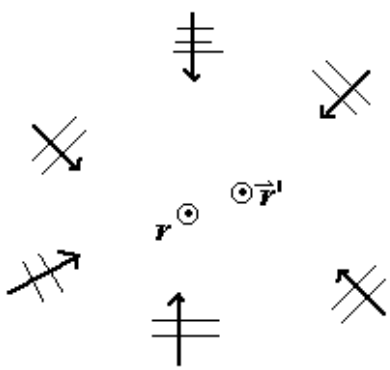


Intensity distribution $B(\theta)$

The field in the vicinity of the origin is a superposition of plane waves

$$\tilde{\psi}(\vec{r},\omega) = \int A(\theta)\exp(-i\omega\hat{\theta}\bullet\vec{r}\,/\,c)\;d\theta \qquad (\text{2-d})$$

with $< A >= 0;$ $\quad < A(\theta)A^{*}(\theta') >= B(\theta)\delta(\theta-\theta')$



i.e, incident plane waves with intensity B(θ) – different in different directions

This implies that the field-field correlation is

$$< \tilde{\psi}(\vec{r},\omega)\;\tilde{\psi}(\vec{r}\,',\omega)^{*} >= \int B(\theta)\exp(-i\omega\hat{\theta}\bullet(\vec{r}-\vec{r}\,')\,/\,c)\;d\theta$$

Exact (assuming no scattering)

If special case B(θ) = constant, this is

$$< \tilde{\psi}(\vec{r},\omega)\, \tilde{\psi}(\vec{r}\,',\omega)^* >= B\int \exp(-i\omega\hat{\theta}\bullet(\vec{r}-\vec{r}\,')/c)\; d\theta$$

$$= 2\pi B\; J_0(\omega\,|\,\vec{r}-\vec{r}\,'\,|/c) \sim \operatorname{Im} G \sim G - G^{TR}$$

and we recover the previous theorem.

If B (θ) ≠ constant,

and if $\omega\,|\,\vec{r}-\vec{r}\,'\,|/c >> 1$, we can evaluate by stationary phase

$$< \tilde{\psi}(\vec{r},\omega)\, \tilde{\psi}(\vec{r}\,',\omega)^* >\sim B(0)\int \exp(-i\omega\,\cos\theta\,|\,\vec{r}-\vec{r}\,'\,|/c)\; d\theta$$

$$\sim\; B(0)\exp(-i\omega\,|\,\vec{r}-\vec{r}\,'\,|/c)/\sqrt{\omega\,|\,\vec{r}-\vec{r}\,'\,|/\pi c}$$

Which looks like the asymptotic form for the Hankel function

Thus the identification is retained in the asymptotic limit, $\omega|\Delta r\,|\,/\,c >> 1$:   C ∼ G…

But…. proportionality depends on intensity B(0) in the "on-strike" direction

If B ≠ constant, then…

$$C = <\psi(\vec{r},t)\psi(\vec{r}\,',t')> = \int B(\theta)\exp(-i\omega\hat{\theta}\bullet(\vec{r}-\vec{r}\,')/c + i\omega(t-t'))\, d\theta\, \tilde{S}(\omega) d\omega$$

We evaluate in the asymptotic limit of large receiver separation

wavelet S(t) related to power spectrum of noise

$$= \frac{-1}{4\pi}\sqrt{\tfrac{2\pi}{\omega x}} \int_0^{+\infty} d\omega\, i \exp(i\omega(t - x/c))\tilde{S}(\omega) \quad \times$$

$$\{B(0)e^{i\pi/4} + B''(0)\tfrac{1}{2\omega x}e^{3i\pi/4} - B(0)\tfrac{i}{8\omega x}e^{5i\pi/4} ..) + c.c.$$

Leading term

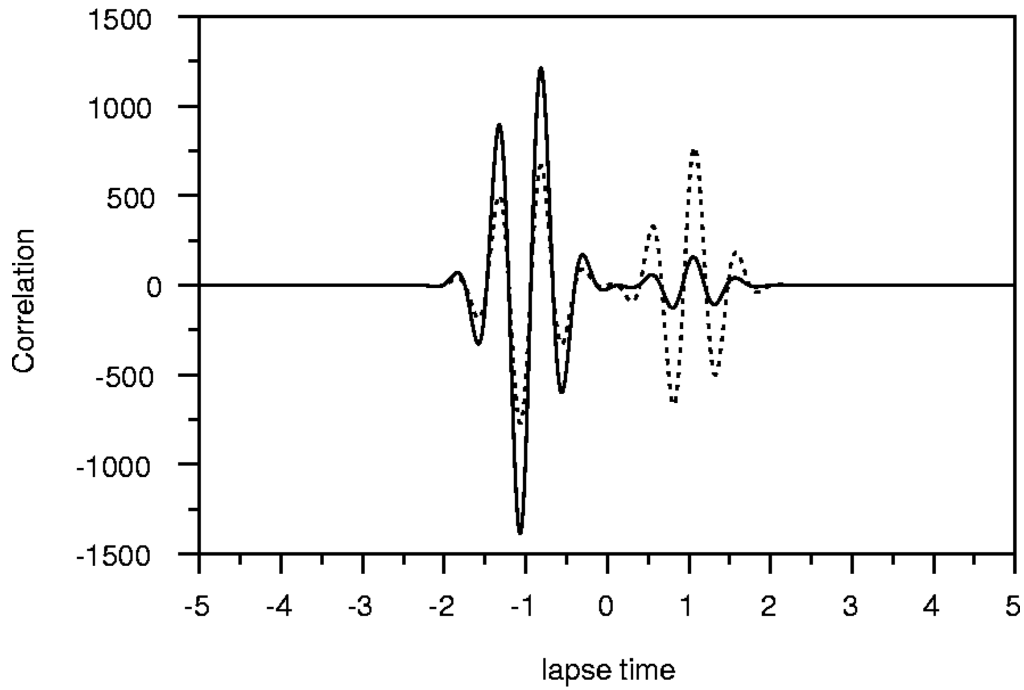
first correction

We see that the apparent arrival time is delayed relative to $|r-r'|/c$ by a fractional amount $[B''(0)/B(0)] / 2k^2|r-r'|^2$

>The effect of non-isotropic B on arrival time is small in practice
>Hence the high quality of the maps of seismic velocity
In-*spite of* ambient seismic noise being not equipartitioned!

Comparison of Correlation waveform (solid line)
and time-symmetrized G (dashed line)

For case of non-trivial noise directionality $B(\theta) = 1 - 0.8 \cos \theta$



Numerical simulations

Froment et al 2009

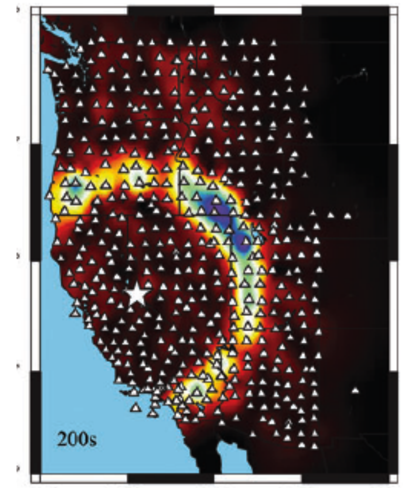
Our rough identification is retained: C shows propagation

But a) precise assertion fails, $G \neq dC/dt$

b) large differences in amplitudes at positive and negative time

c) there are *tiny* shifts of apparent arrival time, as predicted

In sum, the method works well for arrival times, hence the good maps



The method is well suited to seismology because

➔Stations are asymptotically well separated (more than a wavelength)

➔Controlled sources are *highly* inconvenient, (earthquakes and nuclear explosions)

➔Advent of large arrays of long-period seismic stations and world-side access to their digitized time records

Once upon a time seismologists would record seismic time-records, ignore the noise, and examine the earthquakes

Now they often throw out the earthquakes and keep the noise.

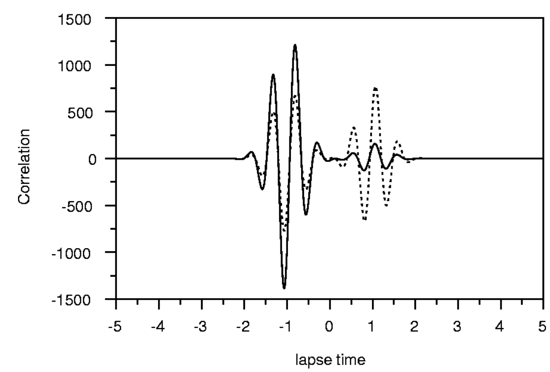
Anisotropy of ambient intensity has little impact on apparent velocities.

BUT, there are other consequences of imperfectly partitioned noise:

➔ non-symmetric Correlations $C(\tau) \neq C(-\tau)$

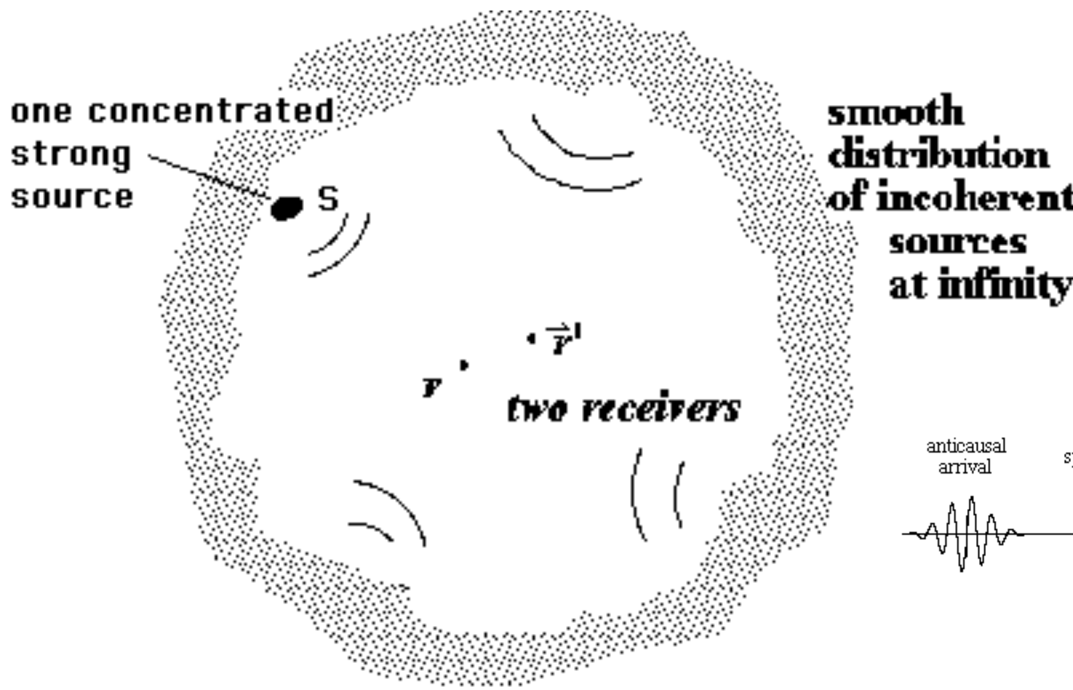
➔ Spurious features in the correlations

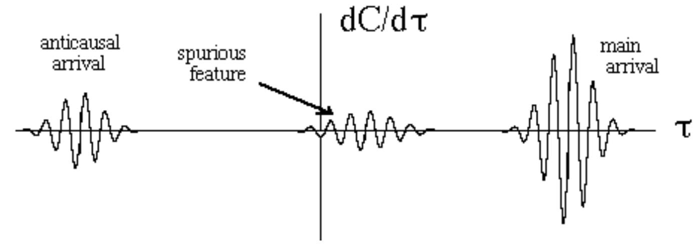
➔ Amplitude information is hard to interpret



One consequence of non-fully-diffuse noise occurs

if there are <u>point</u> sources of small angular size $\delta\theta < 1/k|r-r'|$



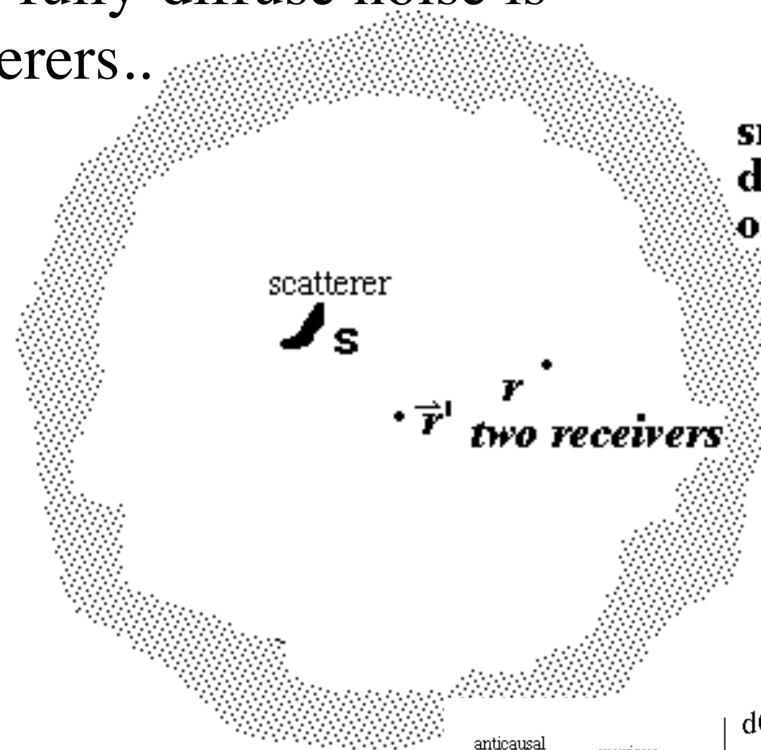


C(τ) will include a spurious arrival

at $\tau = |\mathbf{r}-\mathbf{S}|/c - |\mathbf{r'}-\mathbf{S}|/c < |\mathbf{r}-\mathbf{r'}|/c$

non-causal

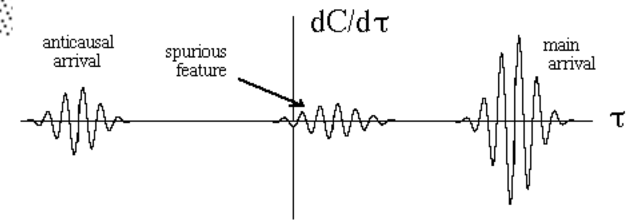
Another consequence of non-fully-diffuse noise is spurious arrivals due to scatterers..



smooth distribution of incoherent sources at infinity

$B(\theta)$

Intensity distribution
$B(\theta) \neq$ constant



Correlations in the presence of a scatterer will show

a direct arrival at $\tau = |\mathbf{r}-\mathbf{r}'|/c$

an indirect arrival at $\tau = |\mathbf{r}-\mathbf{s}|/c + |\mathbf{r}'-\mathbf{s}|/c$ } parts of G

and

a spurious arrival at $\tau = |\mathbf{r}-\mathbf{s}|/c - |\mathbf{r}'-\mathbf{s}|/c$

non causal

Disappears if field is Equipartitioned !!

Yet another caveat is the possible occurrence of
ghost arrivals -

even when the field is multiply scattered

Recall….

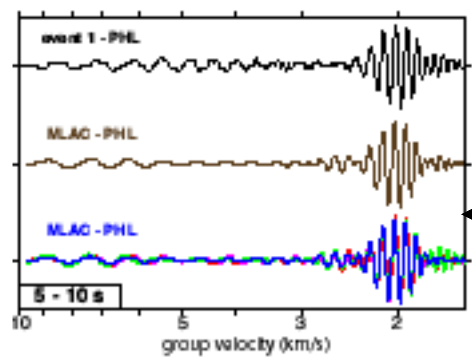
$\partial C/\partial \tau = -\{G - G^{\text{time reversed}}\}$ convolved with $F(\tau)$

So… if $F(\omega)$ has structure
- due possibly to sources in resonant regions, or even just next to a coherent reflector.

…. there can be extra features in C.

Another consequence of poorly partitioned noise can be that
Amplitude Information
is difficult to interpret

Arrival time is evident



Arrival amplitude?
Is this meaningful?

If we really had *G*,
we'd be able to infer attenuation also
we'd be able to infer ground motion amplitude for earthquakes

Amplitude of C ?

Can we retrieve magnitude of G as well as arrival times ?

Theory (now with due care for all proportionalities):

$$C_{sr}(\tau) = \int_T dt\, \psi_s(t)\psi_r(t+\tau) = T < \psi_s(t)\psi_r(t+\tau) >$$

$$= T \frac{< \psi_s(t)^2 >}{\pi n} F(\tau) \otimes \dot{G}_{sr}^{TS}(\tau) \quad \times \quad B_{s \to r}$$

Proportionality includes

Directivity B of noise field at ‘s’ towards ‘r’

Mean square signal $< \psi_s{}^2 >$ at s

modal density n at s

Frequency filter function F (normalized to $F(0) = 1$)

Permits a “deconvolution” $\tilde{G}_{sr} \sim \tilde{C}_{sr} / \tilde{C}_{ss}$?

$$C_{sr}(\tau) = T\,\frac{<\psi_s(t)^2>}{\pi n}\,F(\tau)\otimes\dot{G}_{sr}^{TS}(\tau) \quad \times \quad B_{s\to r}$$

We might use this in a number of ways

1) To predict strong ground motion

2) To analyze amplitudes for attenuation

3) Compare C with its variance, so as to predict S/N

4) For a theory of C3

What is the variance of C (over realizations of noise process) ?

$$\operatorname{var} C = < C^2 > - < C >^2$$

$$= < \int_T \psi(\mathbf{s},t)\psi(\mathbf{r},t+\tau)\,dt \int_T \psi(\mathbf{s},t')\psi(\mathbf{r},t'+\tau)\,dt' > - < \int_T \psi(\mathbf{s},t)\psi(\mathbf{r},t+\tau)\,dt >^2$$

Looks complicated,

BUT – inasmuch as the field ψ is a Gaussian process, the expression breaks up into pair-wise terms

$$\operatorname{var} C \approx \int_{TT} < \psi(\mathbf{r})^2 >< \psi(\mathbf{s})^2 > F^2(t-t')\,dt\,dt'$$

$$= < \psi(\mathbf{r})^2 > < \psi(\mathbf{s})^2 > T \int F^2(\mu)\,d\mu \;\approx\; < \psi(\mathbf{r})^2 > < \psi(\mathbf{s})^2 > T \;\frac{\sqrt{\pi}\Delta}{2\sqrt{2}}$$

Proportional to

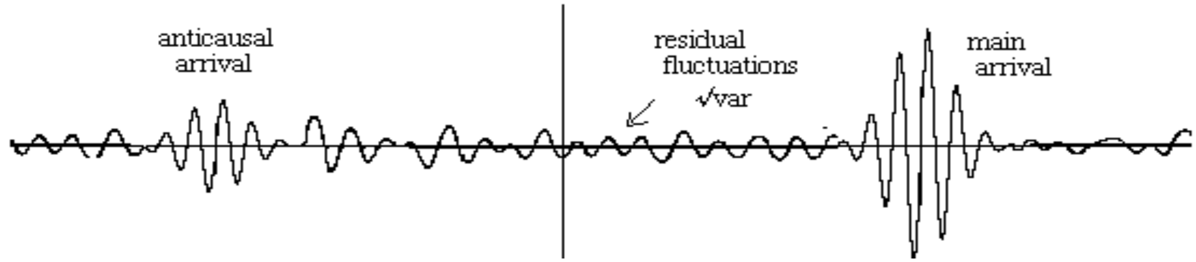
the integration time T,

to the time-domain duration Δ of the filter *F*

and the means square signals at s and at r

And independent of τ

We can compare < C > and √varC:



$$S/N = \frac{C(\tau_{\max})}{\sqrt{\operatorname{var} C}} = B_{s\to r}\sqrt{\frac{<\psi_s^2>}{<\psi_r^2>}}\sqrt{\frac{1}{2\pi\omega R}}\sqrt{\frac{2\sqrt{2}T}{\sqrt{\pi}\Delta}}\exp(-\alpha R)$$

The Signal to Noise ratio (for amplitudes) improves with integration time T like

$$\sqrt{\frac{T}{\Delta}} = \sqrt{\frac{integration\,time}{filter\ duration}} \approx \sqrt{\frac{months}{10's\,seconds}}$$

And degrades with distance like

$$\frac{1}{2\pi}\sqrt{\frac{\lambda}{R}}\exp(-\alpha R)$$

Upshot: S/N in most practice should be pretty good!

Can we retrieve attenuation?

Theory: Ray amplitudes X depend on

attenuation α

site factors s

"on-strike" intensity *B*

$$X_{i\to j} = s_i s_j\, B_i(\hat{n}_{i\to j})\sqrt{2\pi/\omega_o\,|\vec{r}_i-\vec{r}_j|}\,\exp\!\left(-\int_{\vec{r}_i}^{\vec{r}_j}\alpha(\vec{r})\,d\ell\right)$$

Noise intensity B varies in space (via a radiative transfer equation):

$$\hat{n}\cdot\vec{\nabla}B(\vec{r},\hat{n})+2\alpha(\vec{r})B(\vec{r},\hat{n}) = P(\vec{r},\hat{n}) + \oint B(\vec{r},\hat{n}')p(\vec{r},\hat{n},\hat{n}')d\hat{n}'$$

↑ sources (P) ↑ Scattering into direction n from n'

Can we ignore scattering and sources on the continent?

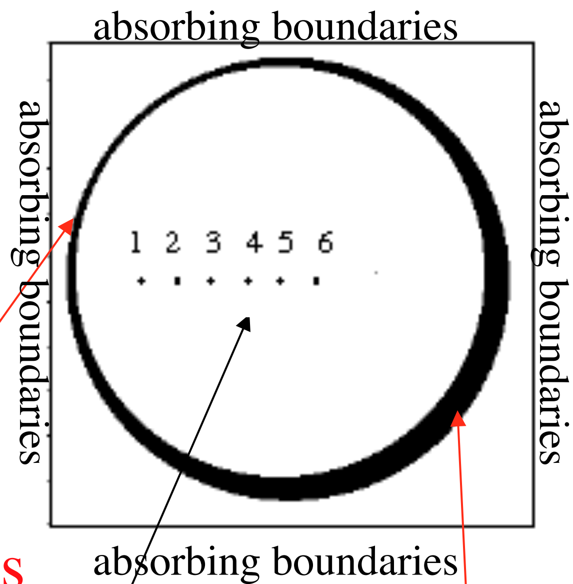
If so, then . . .

$$\hat{n}\cdot\vec{\nabla}B(\vec{r},\hat{n})+2\alpha(\vec{r})B(\vec{r},\hat{n}) = 0$$

and,

$$B(\vec{r},\hat{n}) \sim \exp(-2\alpha\ distance)$$

Some numerical experiments:



Weak sources

A linear array of six stations
separations: 81 km

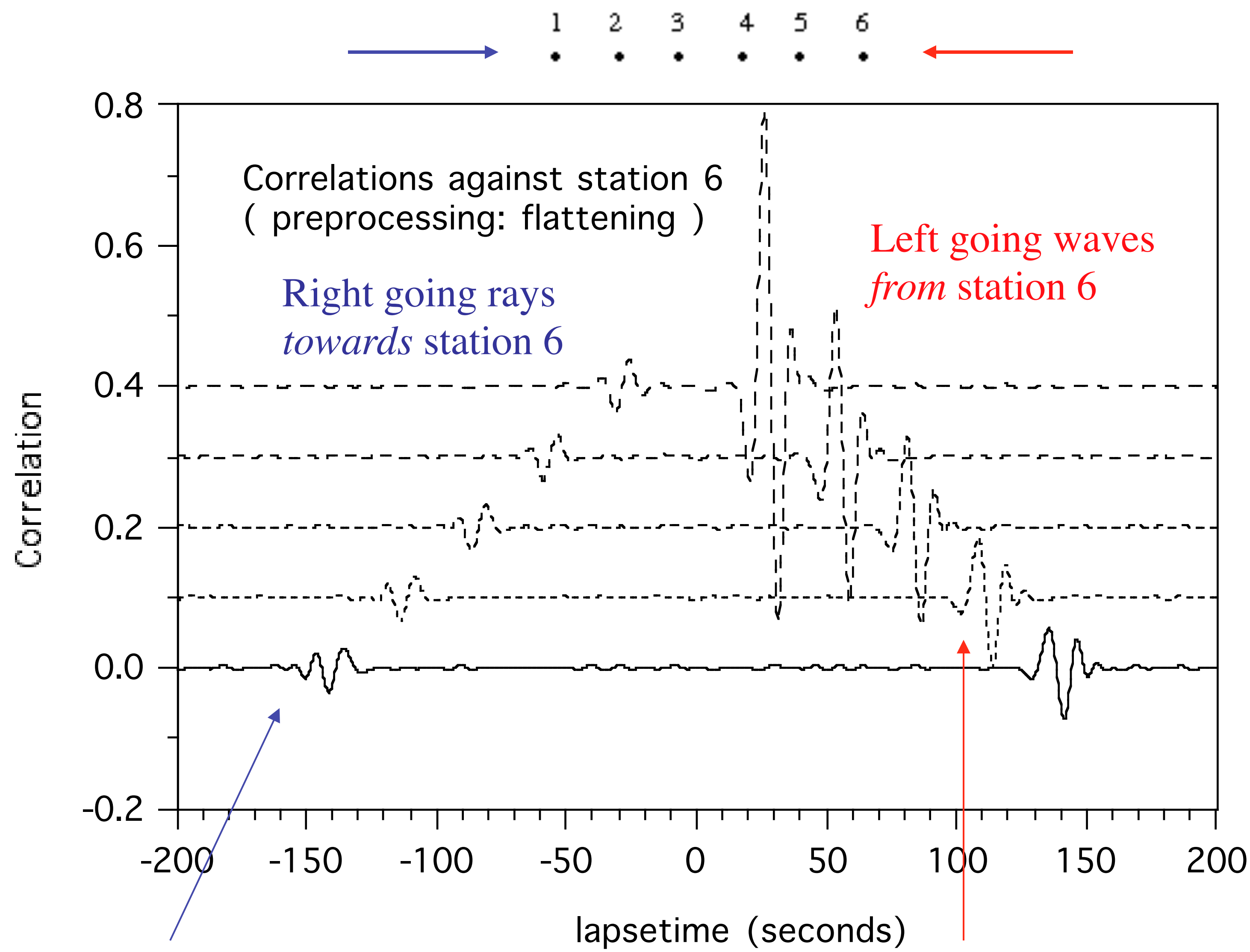
Region of strong sources

A 271 x 271 square mesh
(corresponding to 800 x800 km)

Constant attenuation a

Six receivers

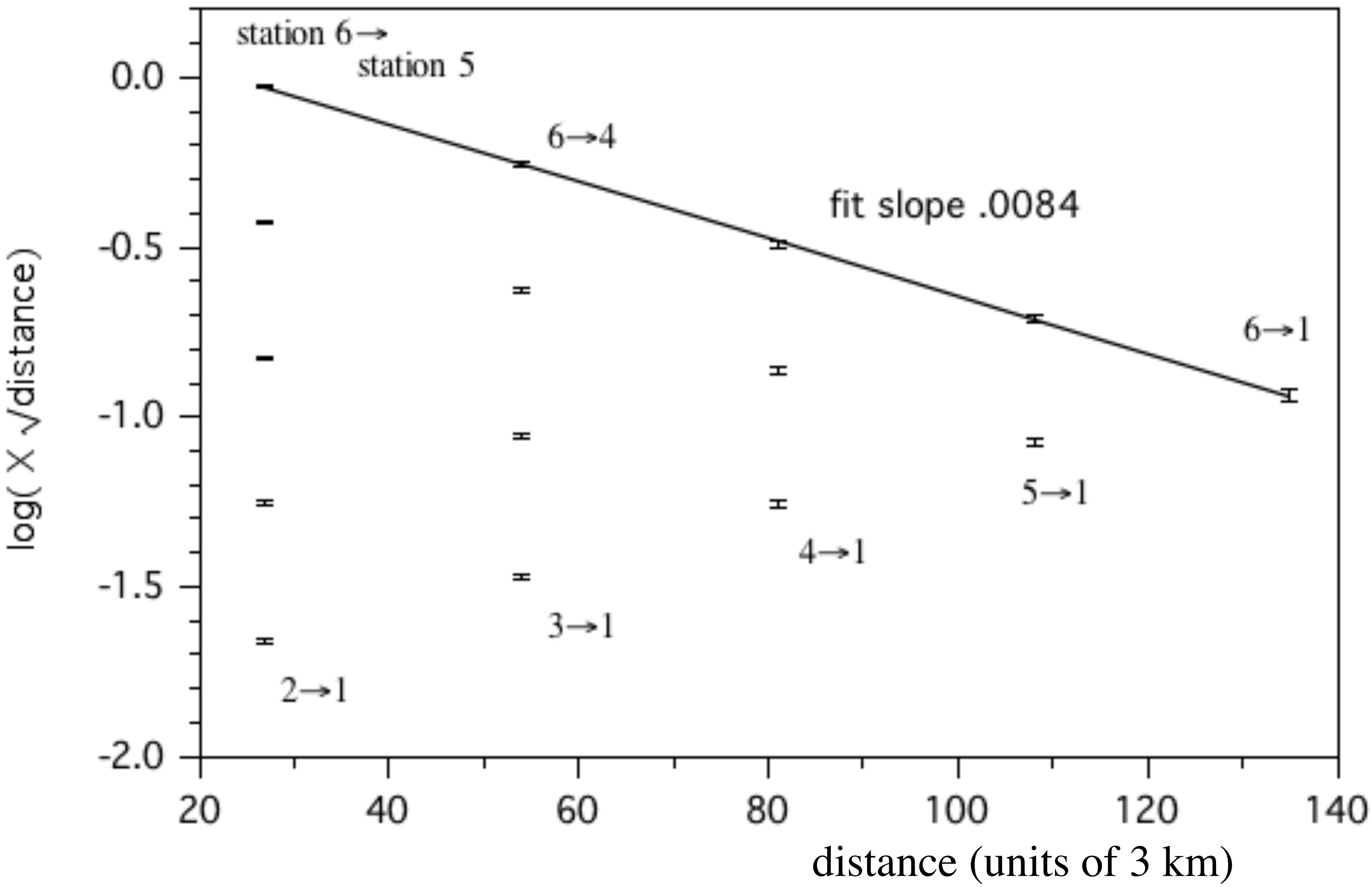
four "months" of time-varying
and direction-varying
band-limited
ten second ambient noise
(without EQs)



Note an apparent absence of attenuation!

Note the diminishment of amplitude due to geometric + anelastic attenuations (at constant *B*)

All 15 left-going amplitudes



Excellent fit to theory

both for coherent decay $\exp(-\alpha \text{ distance})$

and for *B* on strike (i.e vertical offsets) $\exp(-2\alpha \text{ distance})$

So *can we* retrieve attenuation?

In practice it has proved difficult

There are complications from

dispersion

and from (if scattering is present)

spurious features

coda

pulse broadening

secondary sources such that $B \neq \exp(-2\alpha R)$

Perhaps one way to ameliorate the imperfectness of ambient noise is the so-called C^3 method in which we

Correlate the Coda of the (ambient noise) Correlations

Stehly et al 2008

Given many ancillary stations $a = 1,2..N$

and each station's ambient noise $\psi_a(t)$

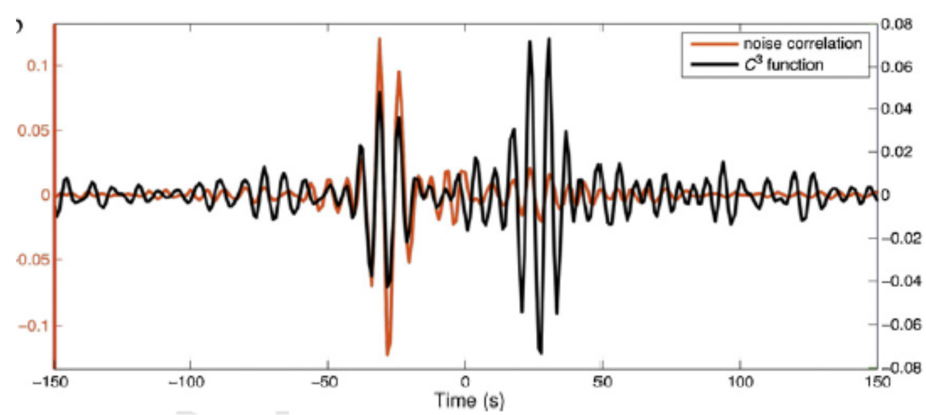
and the noise at two stations of interest $\psi_s(t)$ and $\psi_r(t)$

1) Construct the 2N cross correlations $C_{as}(\tau)$ and $C_{ar}(\tau)$
2) Discard their short τ support i.e. only keep the late τ parts of $C(\tau)$

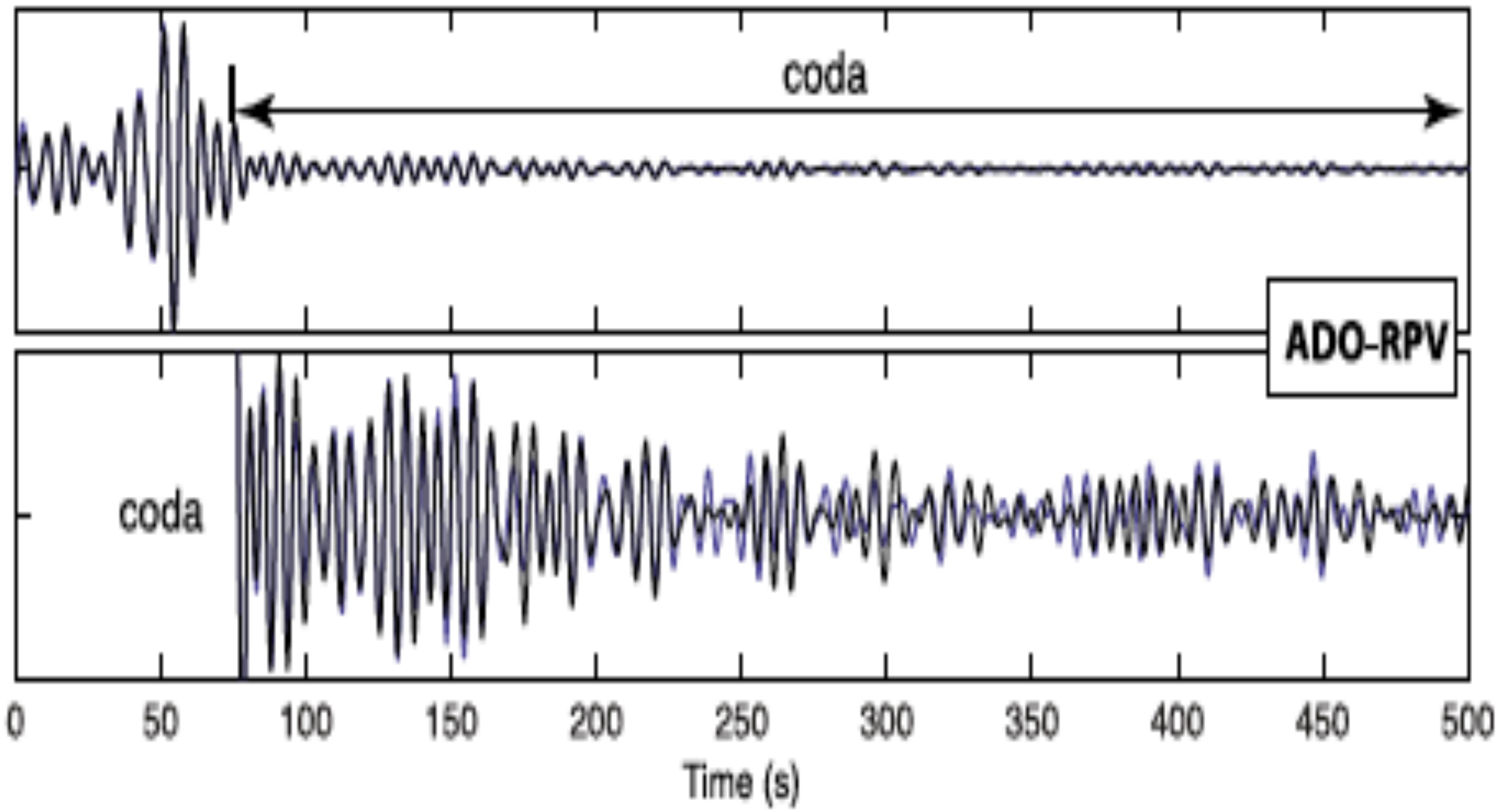
(This is presumably multiply scattered and thus more fully diffuse)

3) Construct $C^3_{sr}(\tau) = \Sigma_a \int C_{as}(t)\, C_{ar}(t+\tau)\, W(t)\, dt$

Froment *et al* 2011 using N=150 European stations:



C3 has been used by Ma and Beroza (GRL 2012)



So Cal, stations separated by ~ 60 seconds, periods ~ 7 seconds
Comparison of codas from two 6-month data sets

Ma and Beroza had

N = 155 stations
useful coda window of τ_{max} ~ 400 seconds
filter duration ~ 10 seconds

The figure of merit is therefore S/N ~

$$B\frac{1}{2\pi}\sqrt{\frac{\lambda}{R}}\sqrt{N\tau_{\max}/\Delta} \sim 10\sqrt{\frac{\lambda}{R}}$$

Suggesting good C3 S/N out to distances of 10's of wavelengths

C3 method has been employed by
Froment et al 2011
Ma and Beroza 2012
Zhang and Yang 2013

My query is..

When do we expect C3 to help us?

Can we quantify conditions under which C3 is worth trying?

I think yes……

When a typical ray of the ambient noise does NOT scatter before it dissipates or escapes.

(i.e. when the ambient noise is very non isotropic)

(especially when $B(\theta) = 0$!)

When $C(\tau)$ is contaminated by spurious features.

When $C(\tau)$ has a long coda with $S/N > 1$ i.e high coda-Q

When there are many ancillary stations.

What IS the strength of the 'empirical coda' ?
answer:

$$<\psi(\mathbf{a},t)\psi(\mathbf{s},t+\tau)>^2 = \left(\frac{<\psi_a^2>^2 \Delta}{8\sqrt{2}\pi^{3/2} n\kappa}\right) \frac{1}{\tau} \exp(-R^2/4\kappa\tau)\exp(-2\sigma\tau)$$

κ = diffusivity = c × meanfreepath /2
independent of Directivity D
$\sigma = \pi f/Q_{coda}$
R = a-s separation

Compare variance of C

$$\operatorname{var} C \approx <\psi(\mathbf{a})^2><\psi(\mathbf{s})^2> \frac{\sqrt{\pi}\Delta}{2\sqrt{2}T}$$

We find that an empirical coda is stable (i.e above its variance) (and therefore worth using) at lapse times τ less than τ_{max} with

$$2\sigma\tau_{max} = \log\left(\frac{T}{\tau_{max}} \frac{1}{4\pi^2 n\kappa}\right) \qquad ; \; \tau_{max} \textit{ typically } < \sim 5\frac{Q_{coda}}{2\pi f}$$

This estimate of $\tau_{\max}$ then permits us to estimate C3's S/N

Recall C's S/N:

$$S/N = B_{s\to r}\frac{1}{2\pi}\sqrt{\frac{\lambda}{R}}\sqrt{\frac{2\sqrt{2}T}{\sqrt{\pi}\Delta}}\exp(-\alpha R)$$

To apply this to C3, we must….

remove B

replace T with $\tau_{\max}$

replace Δ with $\sqrt{2}\Delta$

and insert factor of $\sqrt{N_a}$

C3's S/N

$$S/N = \frac{1}{2\pi}\sqrt{\frac{\lambda}{R}}\sqrt{\frac{2\sqrt{2}\tau_{crit}}{\sqrt{2\pi}\Delta}}\sqrt{N_a}\exp(-\alpha R)$$

Adequate at short R

Ratio

$$\frac{S/N \quad C3}{S/N \quad C} = \frac{\sqrt{N_a}\sqrt{\tau_{crit}/2T}}{B_{s->r}}$$

Usually not an improvement!

Conclusion about C^3:

C3 will usually not improve S/N (unless *B* was very bad)
Though it will give adequate S/N at moderate distances

So why use C3?

It has promise to,

Give adequate S/N, at least at moderate s-r distances,
Depress or Eliminate spurious features
and Control Amplitudes for retrieval of attenuation.
(also – To Permit use of asynchronous records)

In Sum . .

It has been over 12 years now, and Green's function retrieval is still active, still hot,

Applications in

- High resolution seismic velocity maps
- Maps of attenuation too? Of scattering?
- Monitor temporal changes in a medium
- Ocean Acoustics
- Atmospheric Infrasound
- Vibrations of Buildings

We still need better understanding of
the effects of imperfectly diffuse fields
and ways to ameliorate them