OCEAN NOISE and SIGNAL PROCESSING

CARGESE-APRIL,2013

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MOTIVATION:ONE MAN’S NOISE IS ANOTHER MAN’S SIGNAL

- **WHITE NOISE**: ”UNCORRELATED” sensor to sensor -NON-PROPAGATING
- **ISOTROPIC NOISE**:PROPAGATING IN ALL DIRECTIONS WITH EQUAL AMPLITUDE
- CONFUSED WITH WHITE NOISE BECAUSE WHEN SENSED AT HALF WAVE LENGTH--same a white noise
- **SEA SURFACE NOISE**: PROPAGATING PARTIALLY CORRELATED-HAS DIRECTIONALITY:
- **SHIPPING NOISE:** PROPAGATING WHEN FAR AWAY-APPEARS TO HAVE SAME PROPERTIES AS SURFACE NOISE
- **BIOLOGICAL NOISE**
- **SEISMIC NOISE**
- **THERMAL NOISE**

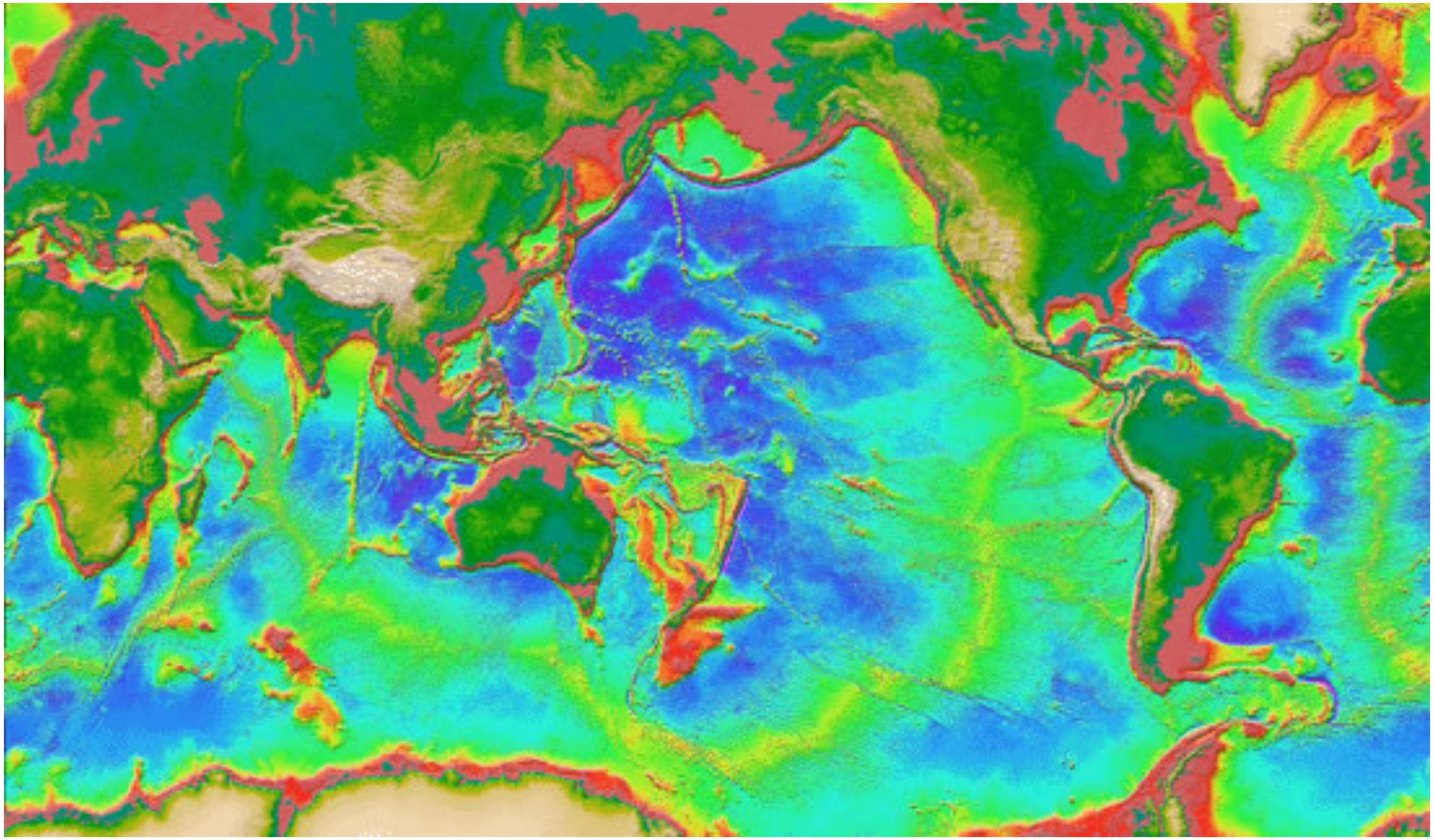
- USUALLY TRY TO DETECT SIGNAL IN NOISE
- USE DIRECTIONALITY/CORRELATION OF NOISE FOR INVERSION
- ACOUSTIC DAYLIGHT,INVERSION
- **THIS TALK WILL END EMPHASIZING UTILITY OF NOISE:**

TREATING NOISE AS THE SIGNAL

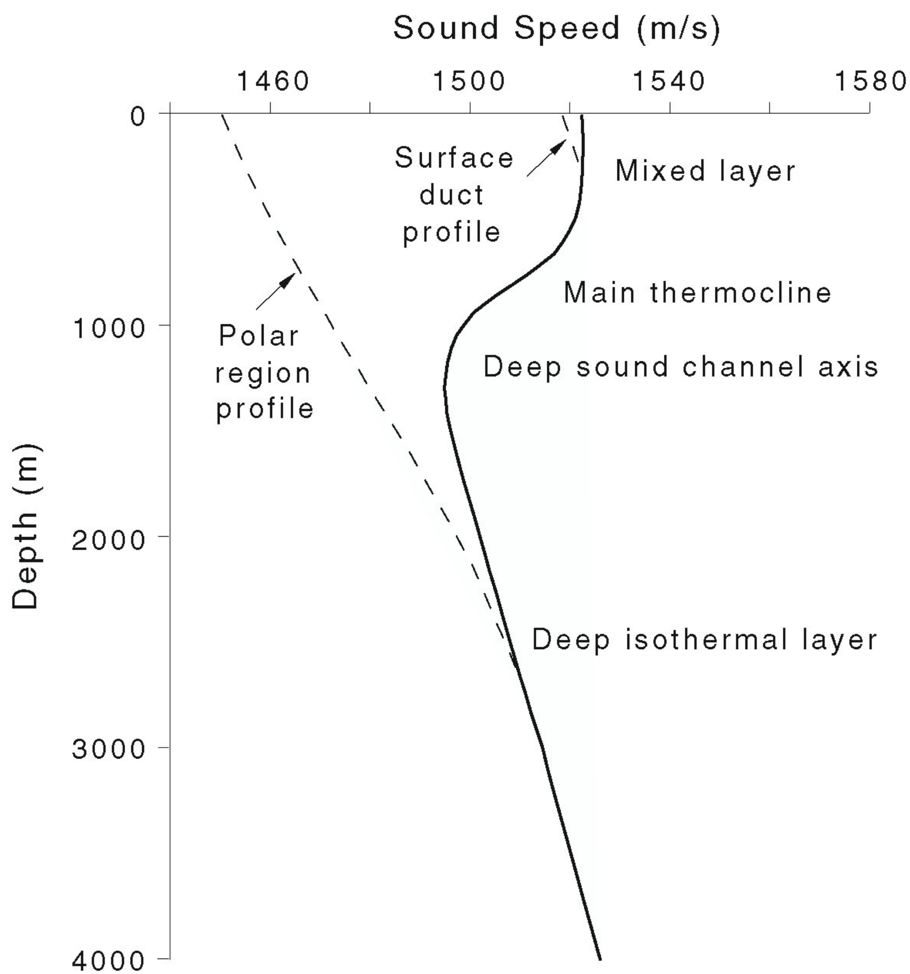
OUTLINE

- **OVERVIEW:OCEAN ACOUSTICS & NOISE**
  - OCEAN ENVIRONMENT
  - PROPAGATION
  - DIFFERENT TYPES OF NOISE
  - SIGNAL PROCESSING: NOISE AS A NUISANCE
- **EXTRACTING COHERENT INFORMATION FROM NOISE**
  - SENSOR-SENSOR CORRELATION
  - BEAMFORMING →BEAM-BEAM CORRELATION
- **EXTRACTING SCATTERING/STRUCTURE PROPERTIES FROM NOISE**

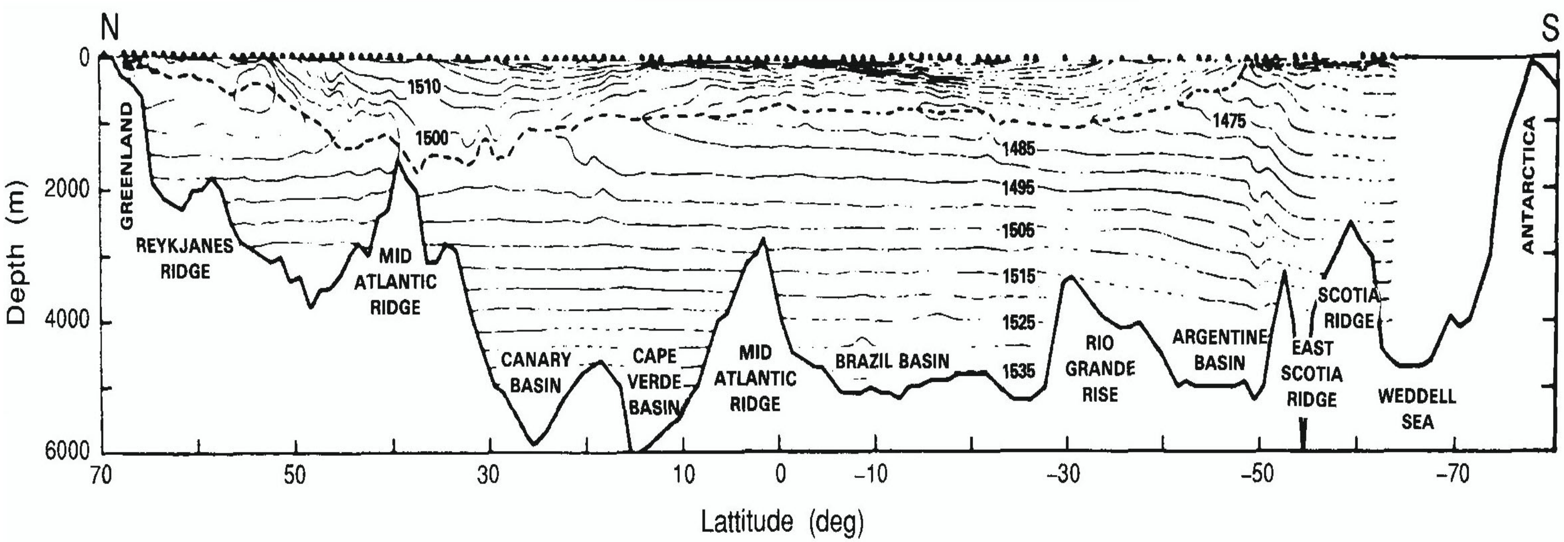
ACOUSTICS IN THE OCEAN



GENERIC SOUND SPEED STRUCTURE



GLOBAL SOUND SPEED STRUCTURE

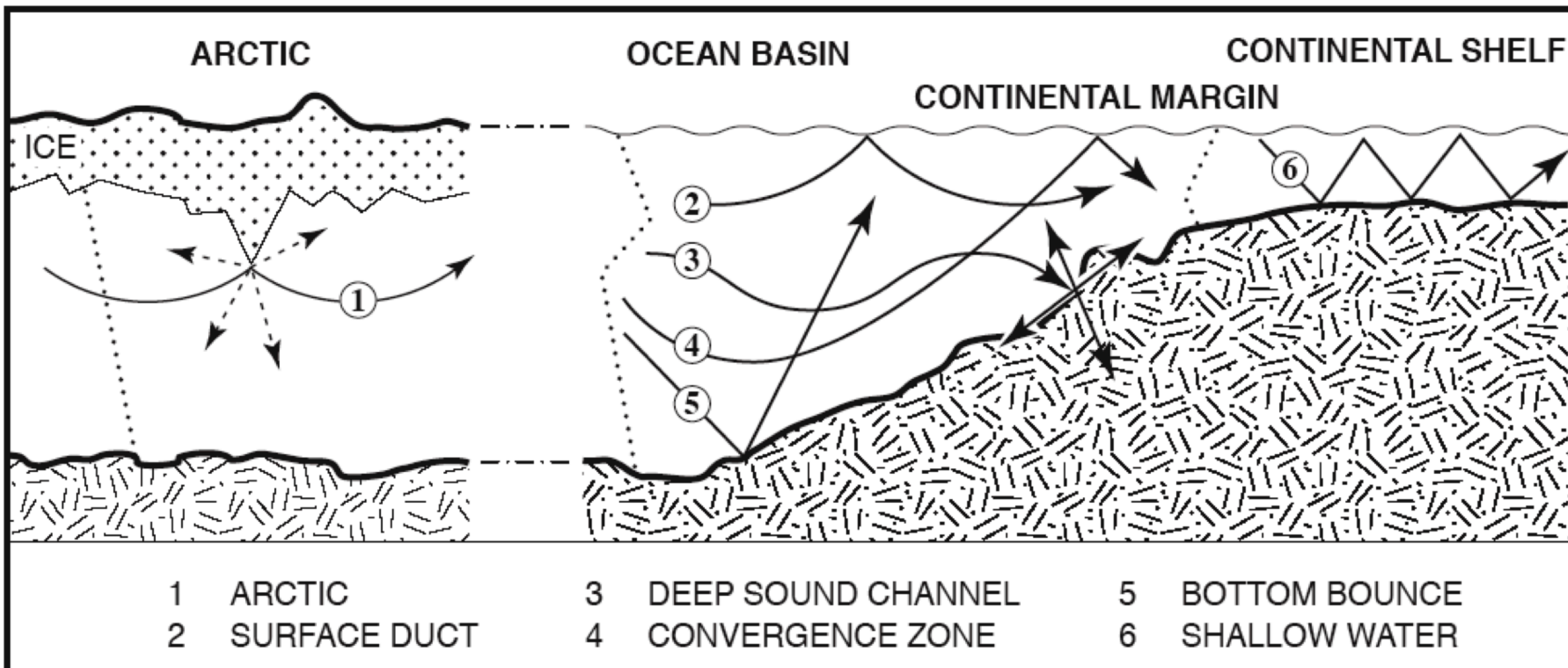


- OCEAN ENVIRONMENT
- PROPAGATION
- DIFFERENT TYPES OF NOISE
- SIGNAL PROCESSING: NOISE AS A NUISANCE

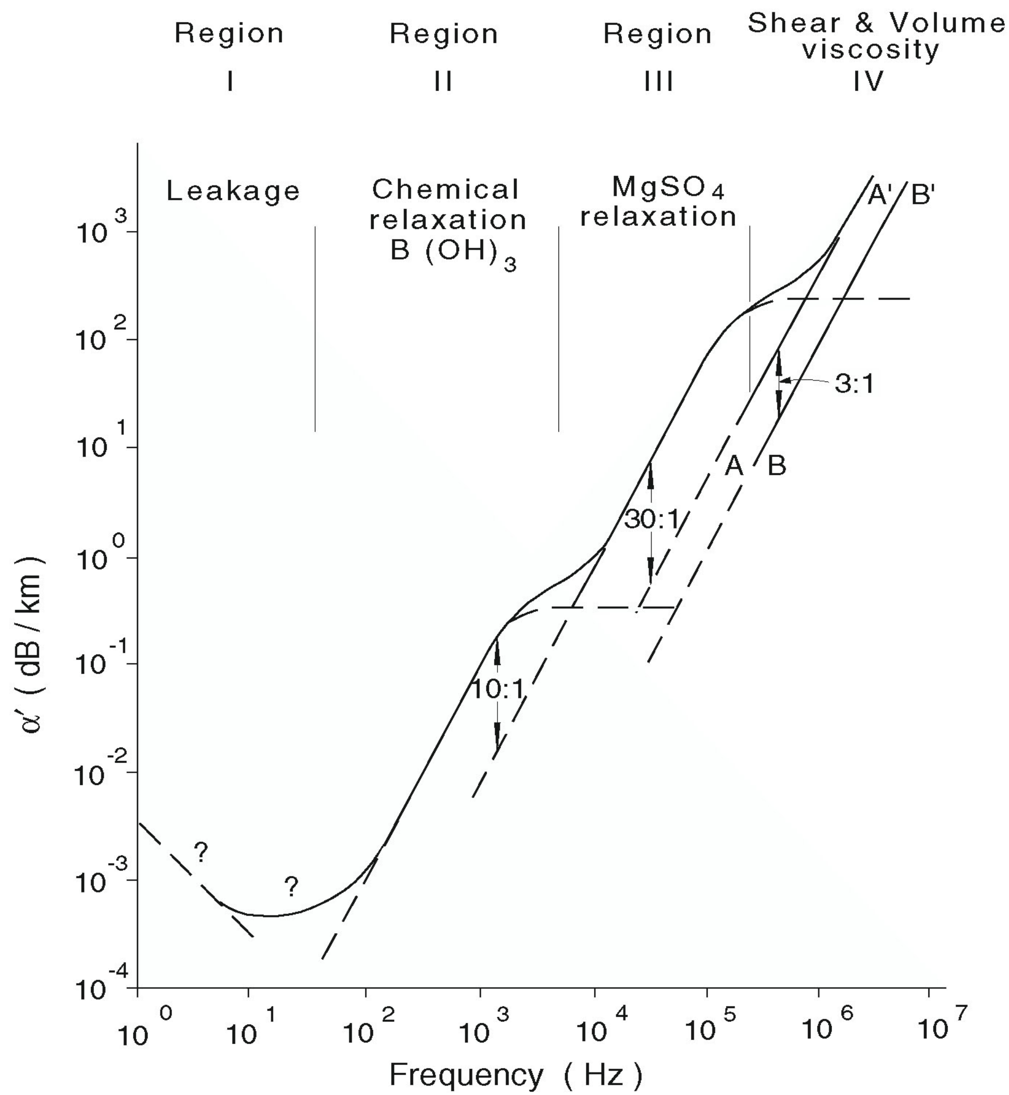
SNELL'S LAW:

SOUND LIKES LOW SPEEDS

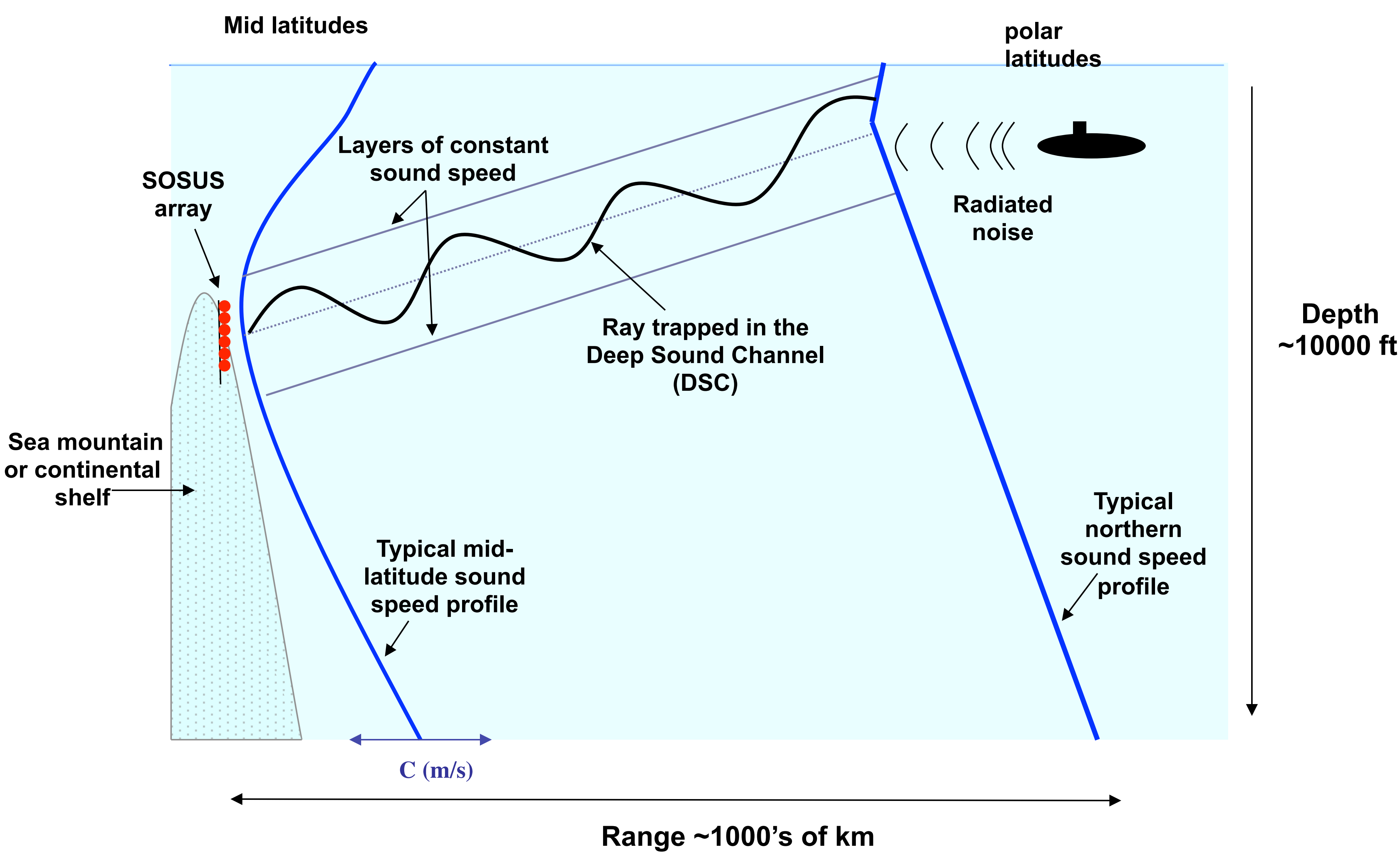
SCHEMATIC OF SOUND PROPAGATION PATHS



ATTENUATION OF SOUND IN SEAWATER (URICK)

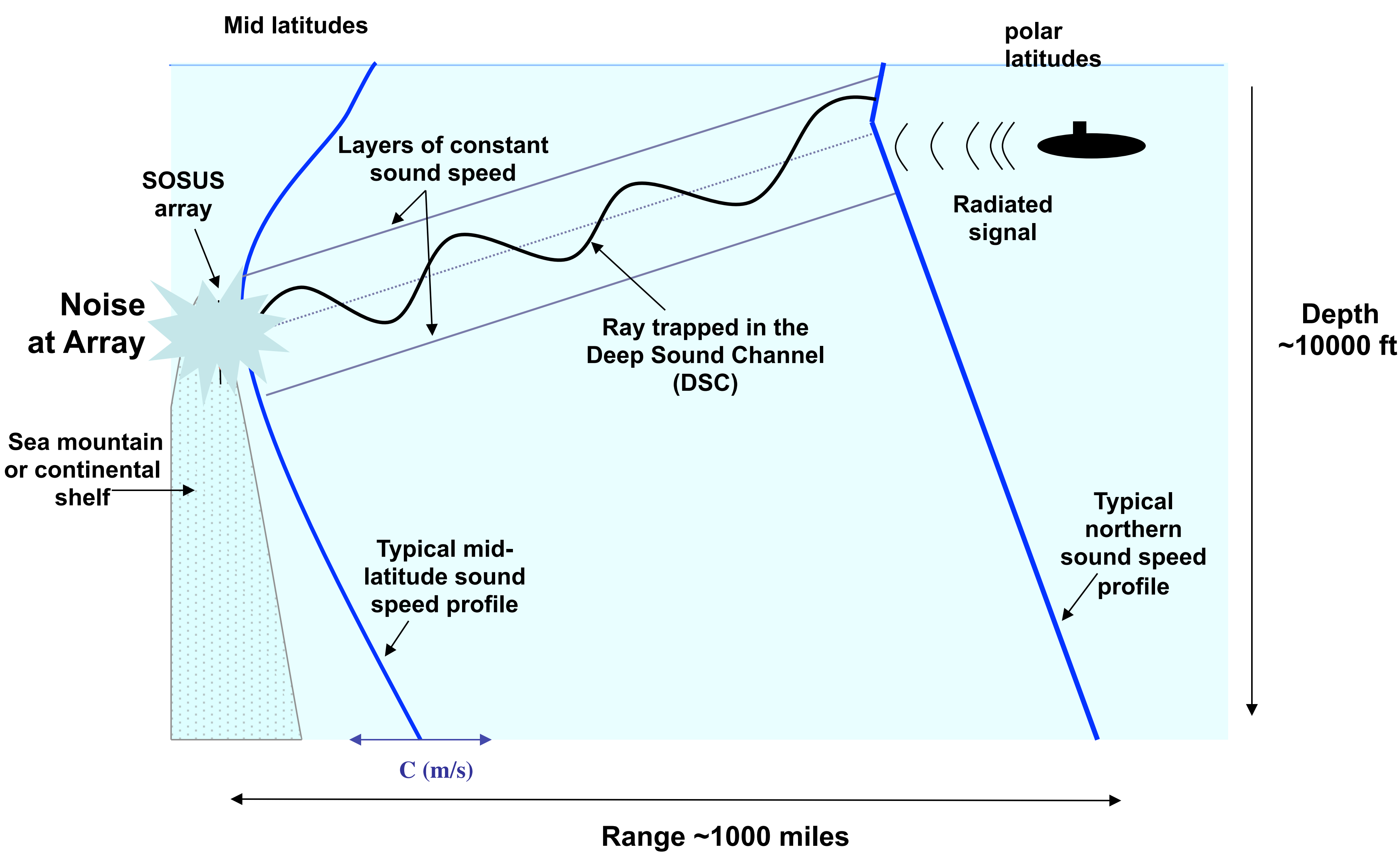


Historical Underwater Acoustics



Box 1

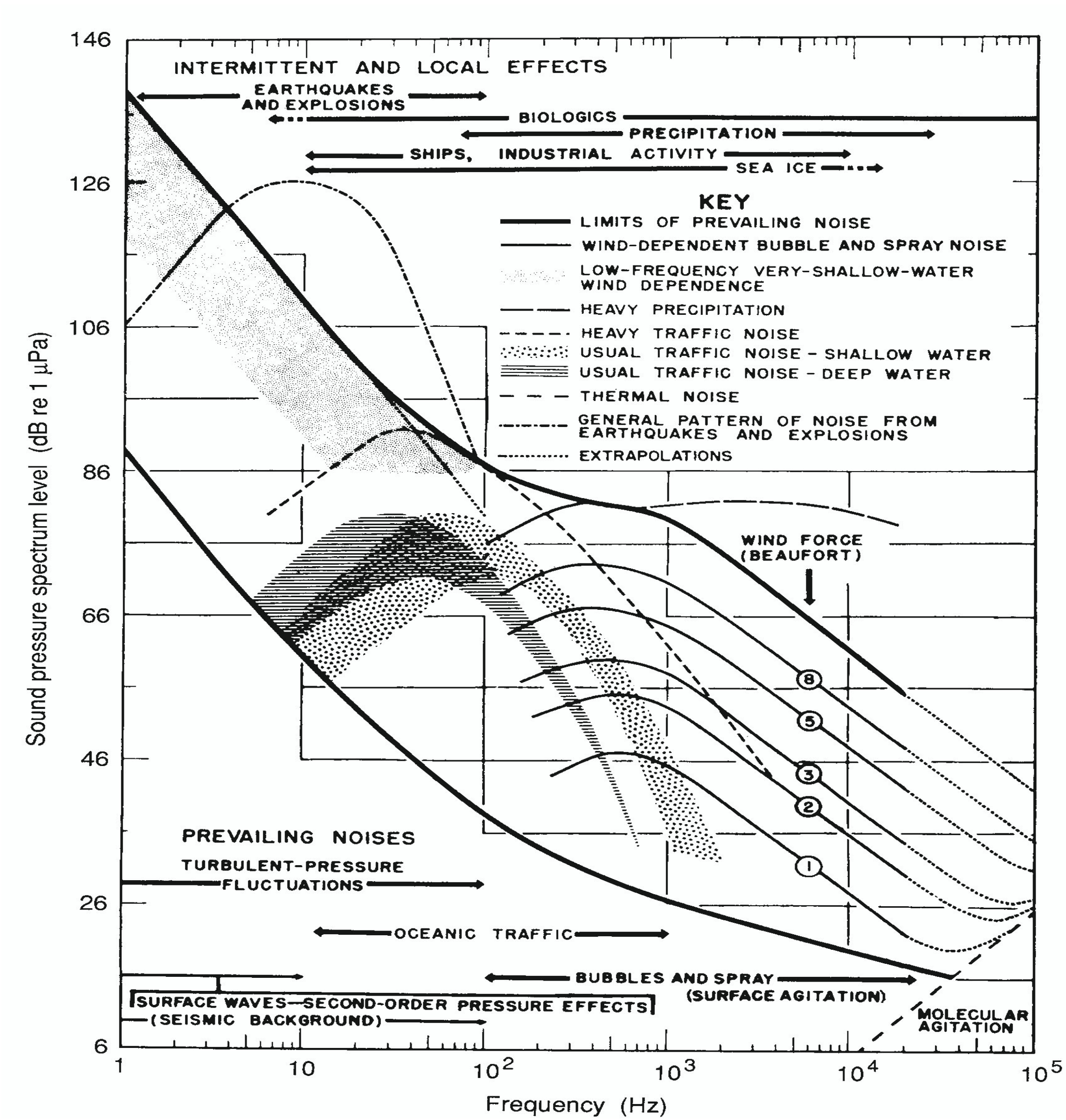
Historical Underwater Acoustics



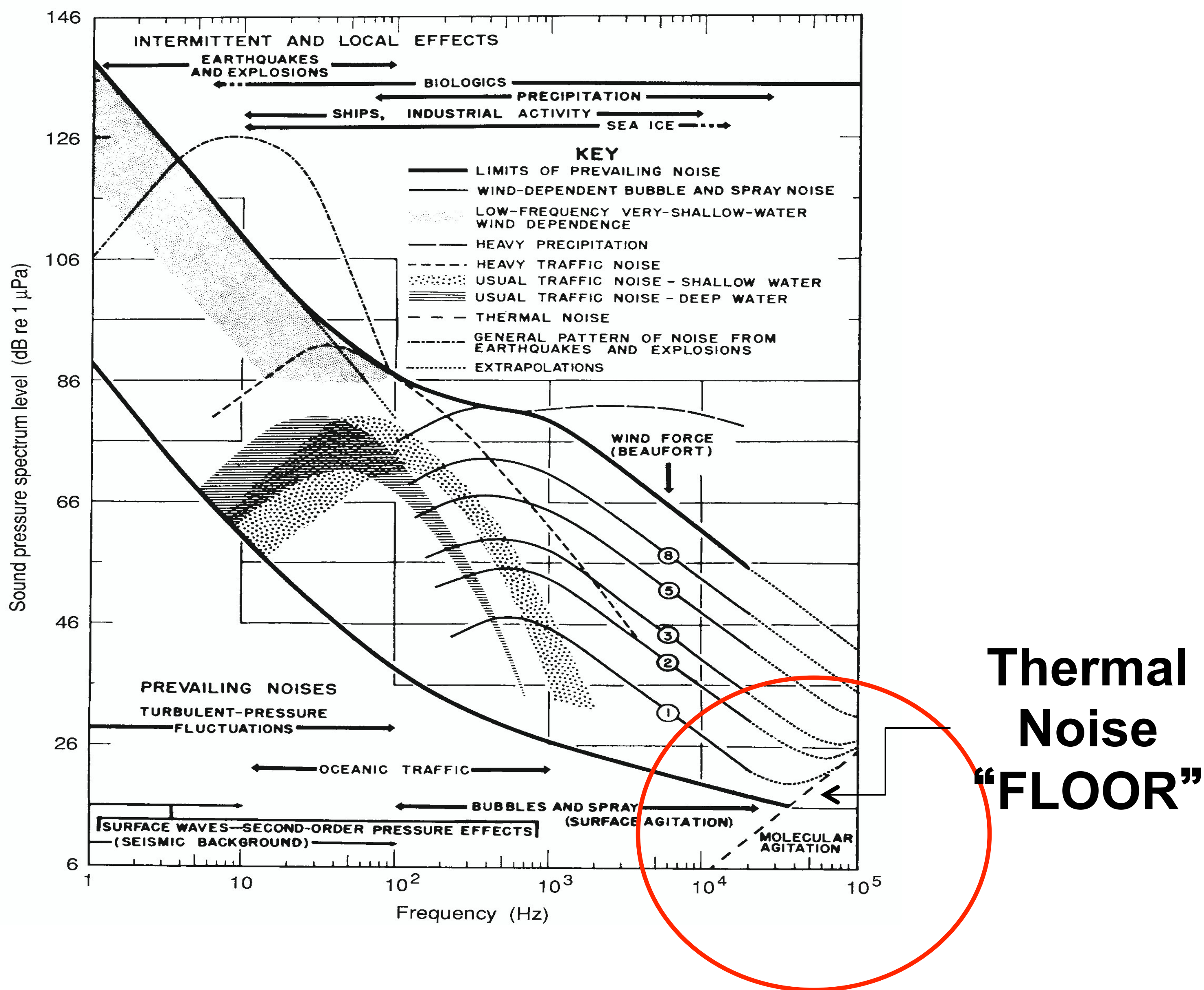
Box 1

- OCEAN ENVIRONMENT
- PROPAGATION
- DIFFERENT TYPES OF NOISE
- SIGNAL PROCESSING: NOISE AS A NUISANCE

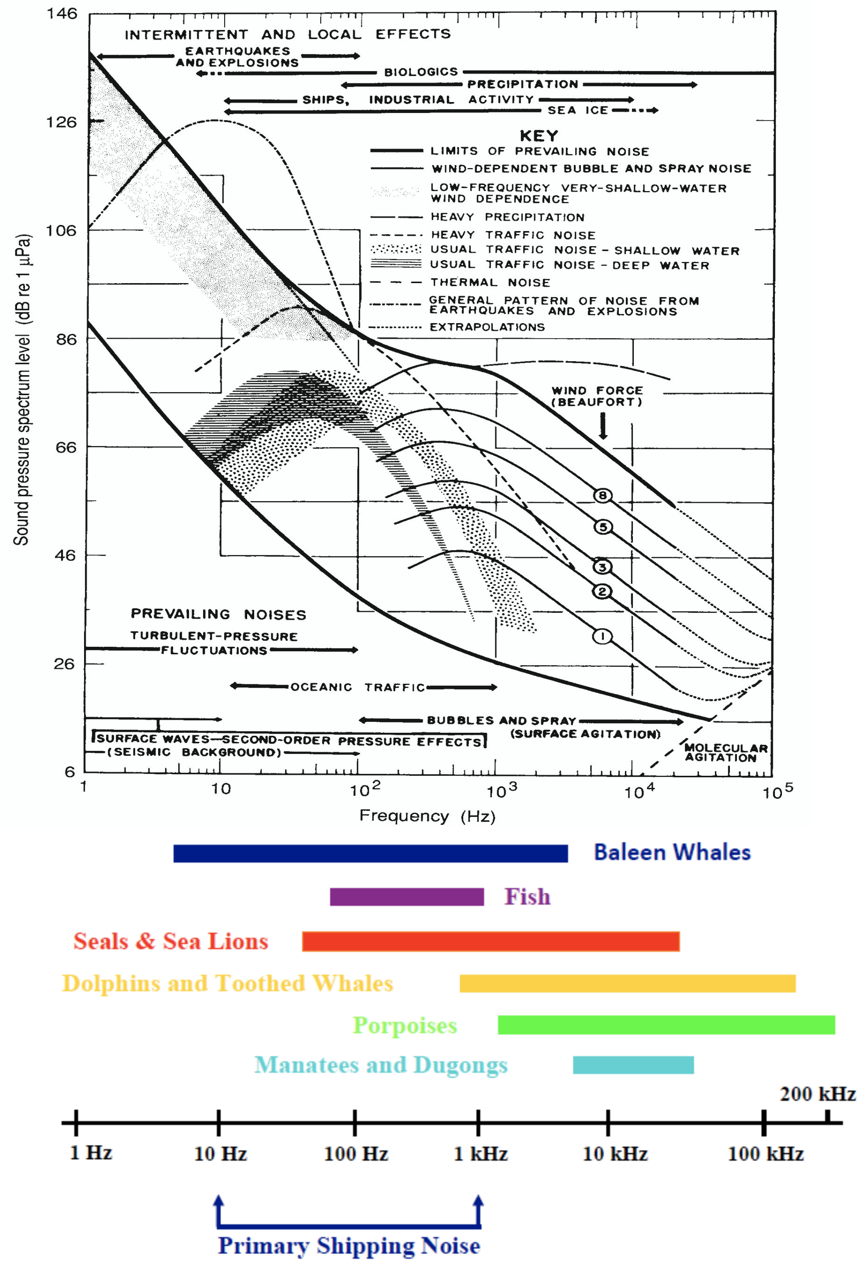
AMBIENT NOISE SPECTRA (WENZ)



AMBIENT NOISE SPECTRA (WENZ)

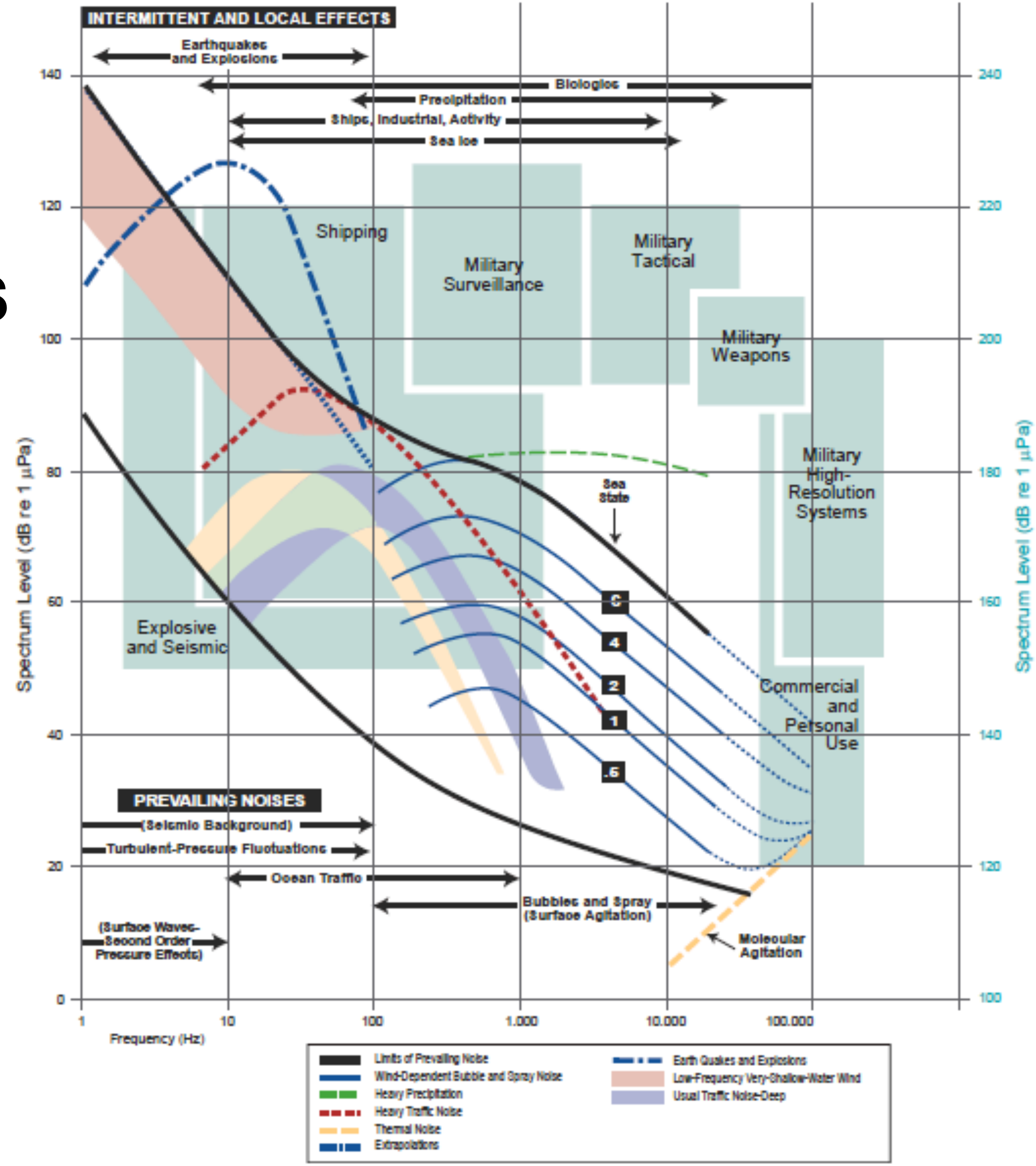


BIOLOGICS



INTERMITTENT AND LOCAL EFFECTS
EARTHQUAKES AND EXPLOSIONS
BIOLOGICS
PRECIPITATION
SHIPS, INDUSTRIAL ACTIVITY
SEA ICE
KEY
LIMITS OF PREVAILING NOISE
WIND-DEPENDENT BUBBLE AND SPRAY NOISE
LOW-FREQUENCY VERY-SHALLOW-WATER WIND DEPENDENCE
HEAVY PRECIPITATION
HEAVY TRAFFIC NOISE
USUAL TRAFFIC NOISE - SHALLOW WATER
USUAL TRAFFIC NOISE - DEEP WATER
THERMAL NOISE
GENERAL PATTERN OF NOISE FROM EARTHQUAKES AND EXPLOSIONS
EXTRAPOLATIONS
WIND FORCE (BEAUFORT)
8
5
3
2
1
PREVAILING NOISES
TURBULENT-PRESSURE FLUCTUATIONS
OCEANIC TRAFFIC
BUBBLES AND SPRAY (SURFACE AGITATION)
SURFACE WAVES—SECOND-ORDER PRESSURE EFFECTS
(SEISMIC BACKGROUND)
MOLECULAR AGITATION
Sound pressure spectrum level (dB re 1 µPa)
146
126
106
86
66
46
26
6
1
10
10^2
10^3
10^4
10^5
Frequency (Hz)
Baleen Whales
Fish
Seals & Sea Lions
Dolphins and Toothed Whales
Porpoises
Manatees and Dugongs
200 kHz
1 Hz
10 Hz
100 Hz
1 kHz
10 kHz
100 kHz
Primary Shipping Noise

NOISE LEVELS AND SOURCE LEVELS



BRADLEY,STERN
NRC 2008

Ships Underway	Broadband Source Level (dB re 1 µPa at 1 m)
Tug and Barge (18 km/hour)	171
Supply Ship (example: Kigoriak)	181
Large Tanker	186
Icebreaking	193
Seismic Survey	**Broadband Source Level (dB re 1 µPa at 1 m)**
Air gun array (32 guns)	259 (peak)
Military Sonars	**Broadband Source Level (dB re 1 µPa at 1 m)**
AN/SQS-53C (U. S. Navy tactical mid-frequency sonar, center frequencies 2.6 and 3.3 kHz)	235
AN/SQS-56 (U. S. Navy tactical mid-frequency sonar, center frequencies 6.8 to 8.2 kHz)	223
SURTASS-LFA (100-500 Hz)	215 dB per projector, with up to 18 projectors in a vertical array operating simultaneously
Ocean Acoustic Studies	**Broadband Source Level (dB re 1 µPa at 1 m)**
Heard Island Feasibility Test (HIFT) (Center frequency 57 Hz	206 dB for a single projector, with up to 5 projectors in a vertical array operating simultaneously
Acoustic Thermometry of Ocean Climate (ATOC)/North Pacific Acoustic Laboratory (NPAL) (Center frequency 75 Hz)	195

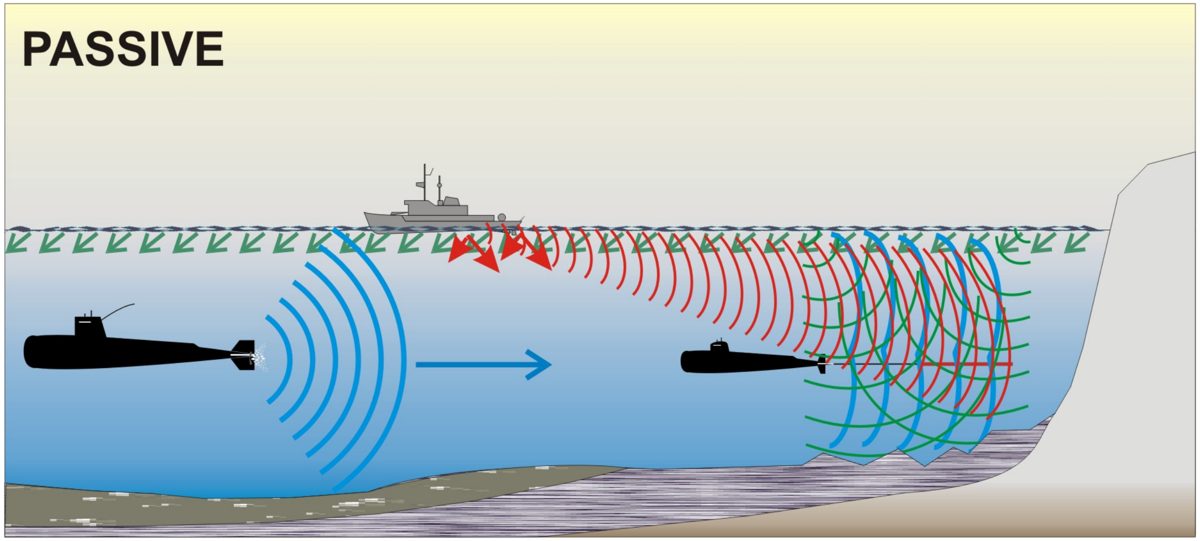
Man Made Sounds

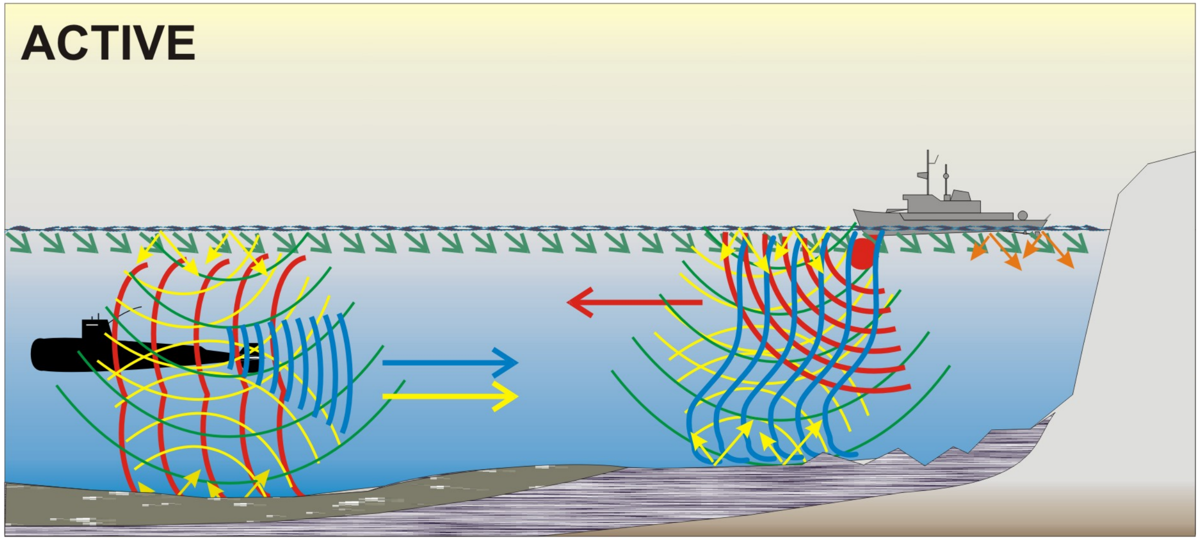
Animal Sounds

Source	Broadband Source Level (dB re 1 µPa at 1 m)
Sperm Whale Clicks	163-223
Beluga Whale Echolocation Click	206-225 (peak-to-peak)
White-beaked Dolphin Echolocation Clicks	194-219 (peak-to-peak)
Spinner Dolphin Pulse Bursts	108-115
Bottlenose Dolphin Whistles	125-173
Fin Whale Moans	155-186
Blue Whate Moans	155-188
Gray Whale Moans	142-185
Bowhead Whale Tonals, Moans and Song	128-189
Humpback Whale Song	144-174
Humpback Whale Fluke and Flipper Slap	183-192
Southern Right Whale Pulsive Call	172-187
Snapping Shrimp	183-189 (peak-to peak)

- OCEAN ENVIRONMENT
- PROPAGATION
- DIFFERENT TYPES OF NOISE
- SIGNAL PROCESSING: From NOISE AS A NUISANCE

TYPICAL SONAR VIEW OF NOISE: NUISANCE





ARRAY GAIN: Signal adds up faster than noise

$$\frac{S^2}{N^2} = \frac{\overline{\left[\sum_{i=1}^{m} s_i(t)\right]^2}}{\overline{\left[\sum_{i=1}^{m} n_i(t)\right]^2}}, \longrightarrow \frac{S^2}{N^2} \equiv \frac{\sum_{i,j=1}^{m} s_{ij}}{\sum_{i,j=1}^{m} n_{ij}}, \longrightarrow \frac{S^2}{N^2} = \frac{s^2}{n^2}\frac{\sum_{i,j=1}^{m} \hat{s}_{ij}}{\sum_{i,j=1}^{m} \hat{n}_{ij}}.$$

$$\mathrm{AG} = 10\log\frac{S^2/N^2}{s^2/n^2} = 10\log\frac{\sum_{i,j=1}^{m} \hat{s}_{ij}}{\sum_{i,j=1}^{m} \hat{n}_{ij}},$$

Incoherent: no correlation between sensors: no X-terms in sum->m terms Vs m^2 for coherent

Isotropic noise $\hat{n}_{ij} = \frac{\sin(kd)}{kd}, \quad i \neq j$ is uncorrelated at $\lambda/2$

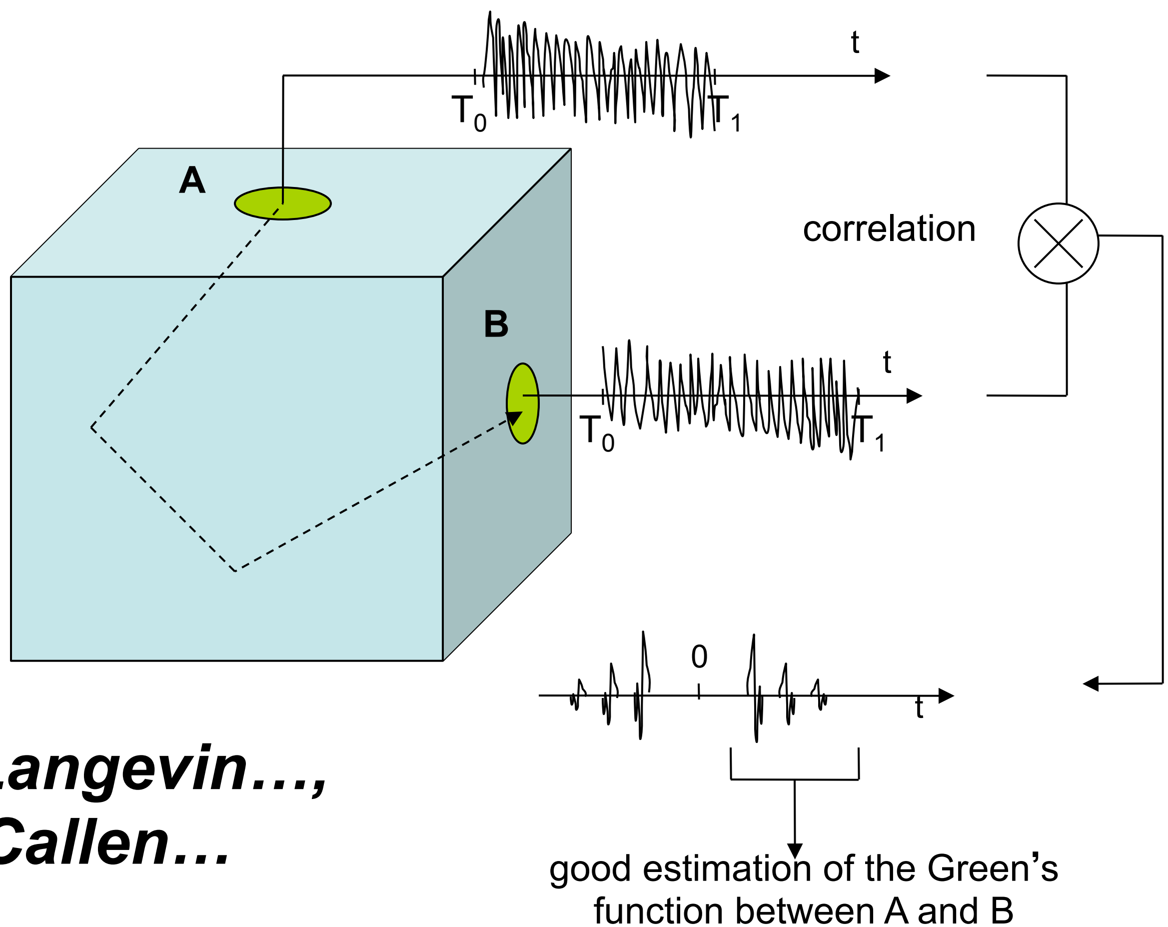
$\longrightarrow$ AG = 10 *log* m

- **EXTRACTING COHERENT INFORMATION FROM NOISE**

Coherent signals from thermal noise

First experimental demonstration in ultrasonics (0.1 – 0.9 MHz)

R.L. Weaver & O.I. Lobkis, Phys. Rev. Lett., 2001



History: Einstein, Langevin…, Johnson,Nyquist, Callen…

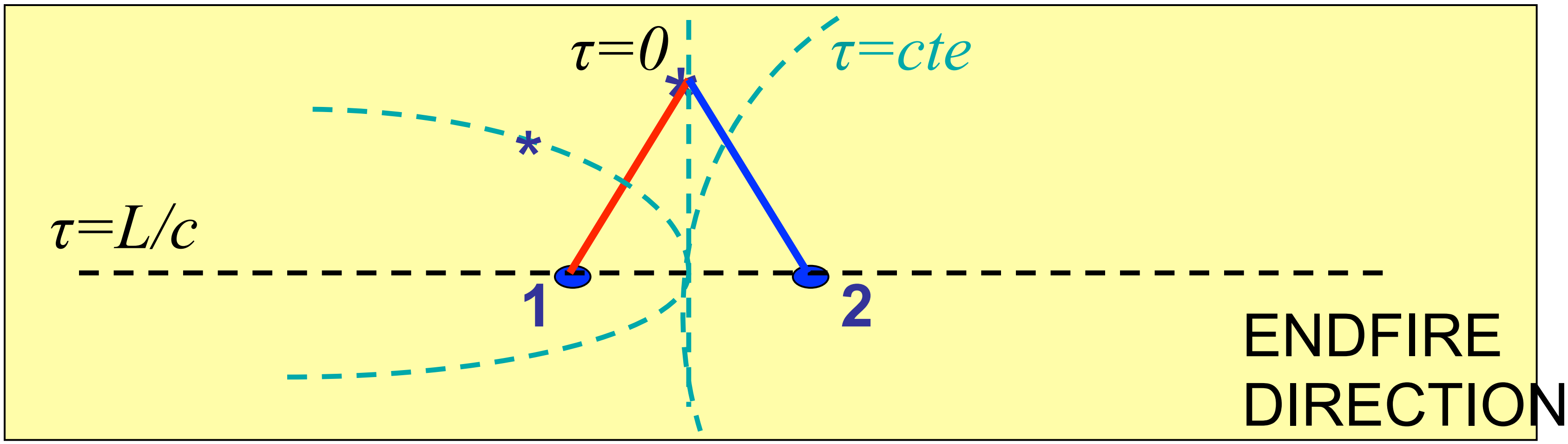
NOISE CORRELATIONS IN OCEAN ACOUSTICS

VIA

- FREE SPACE
- WAVEGUIDE
- BEAM FORMING

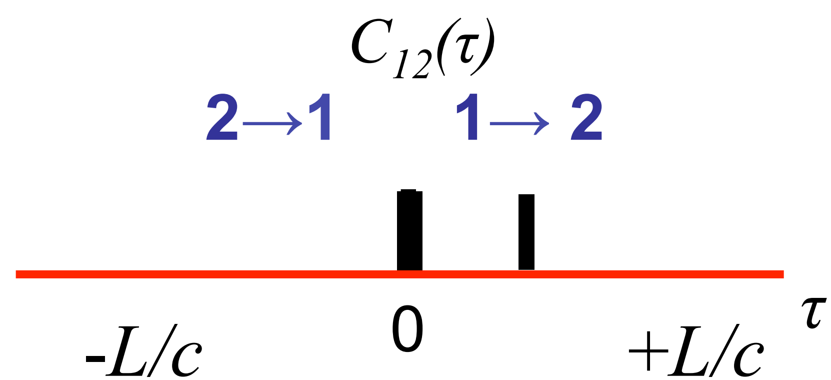
Noise cross-correlation: Free space

$$C_{12}(\tau) = \int_{-\infty}^{\infty} P(\mathbf{r}_1, t)\, P(\mathbf{r}_2, t+\tau)\,\mathrm{d}\,t.$$



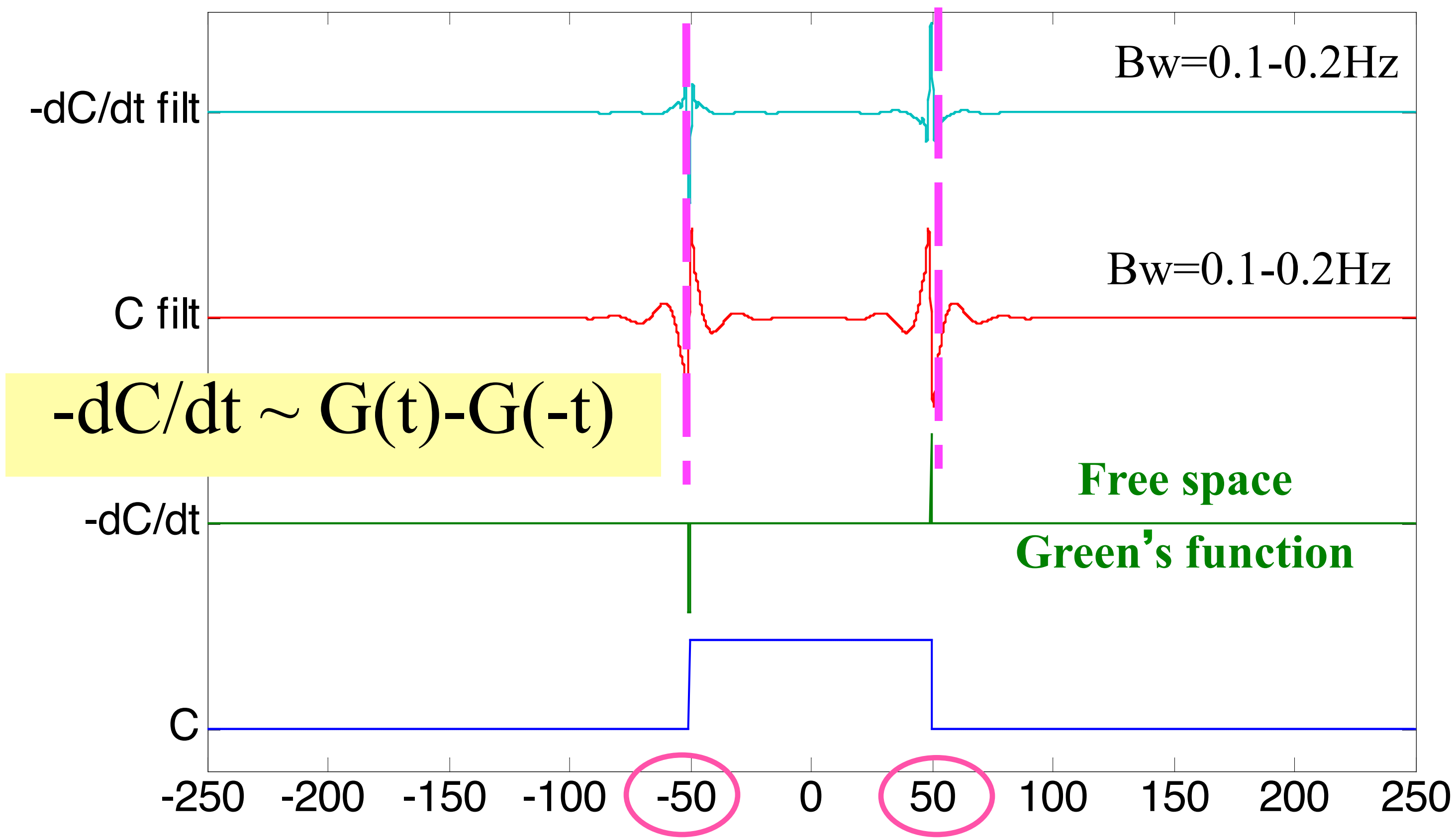
Noise sources yielding constant time-delay τ, lay on same Hyperbola

$$\tau = \frac{|r_2 - r_s| - |r_1 - r_s|}{c} \leq \frac{L}{c}$$



Isotropic distribution of uncorrelated random noise sources

C, dC/dt, band-limited signal

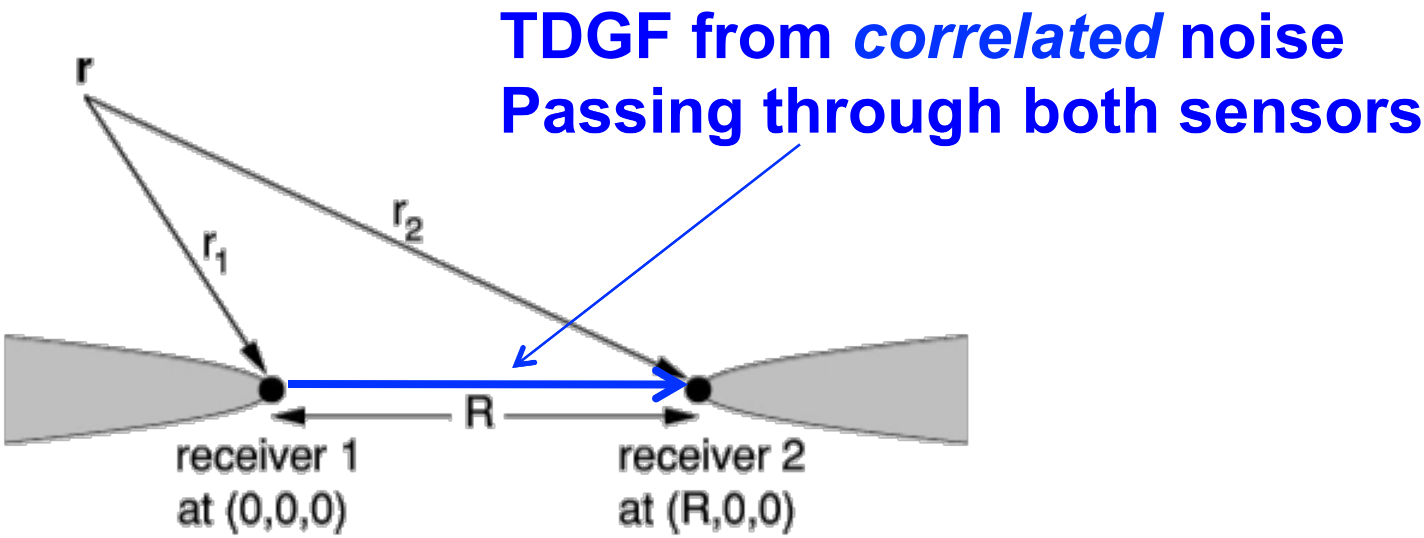


•With cross-correlation process the phase of the source signal is removed,

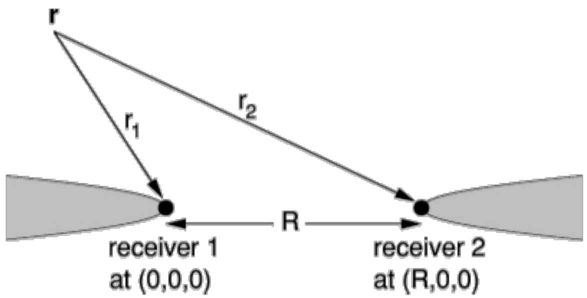
→Arrival time is given by the center of the pulse (envelope maximum)

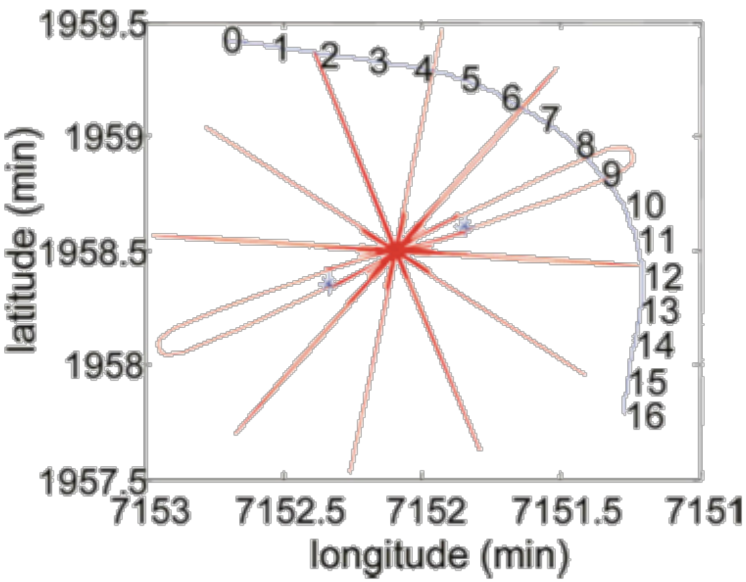
•Isotropic noise distribution → Symmetric Correlation function.

Physics of Correlation Process

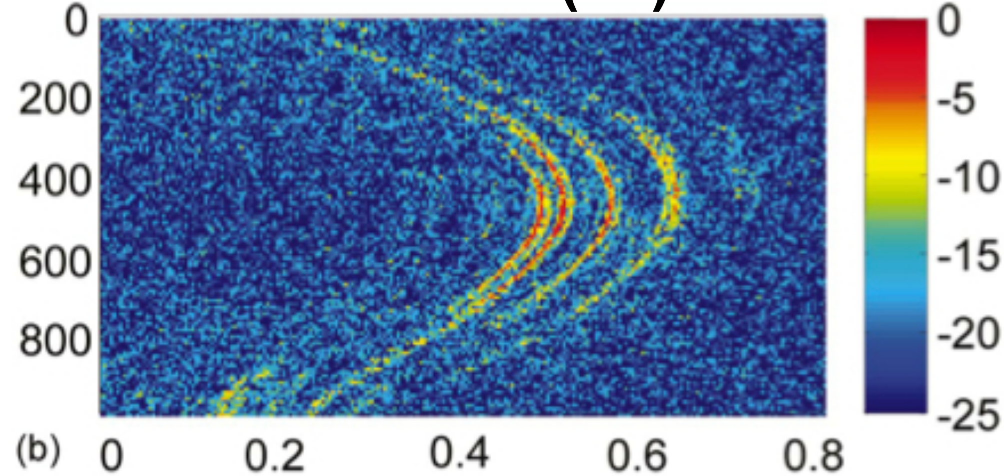


Geometry of correlated noise –endfire demo:

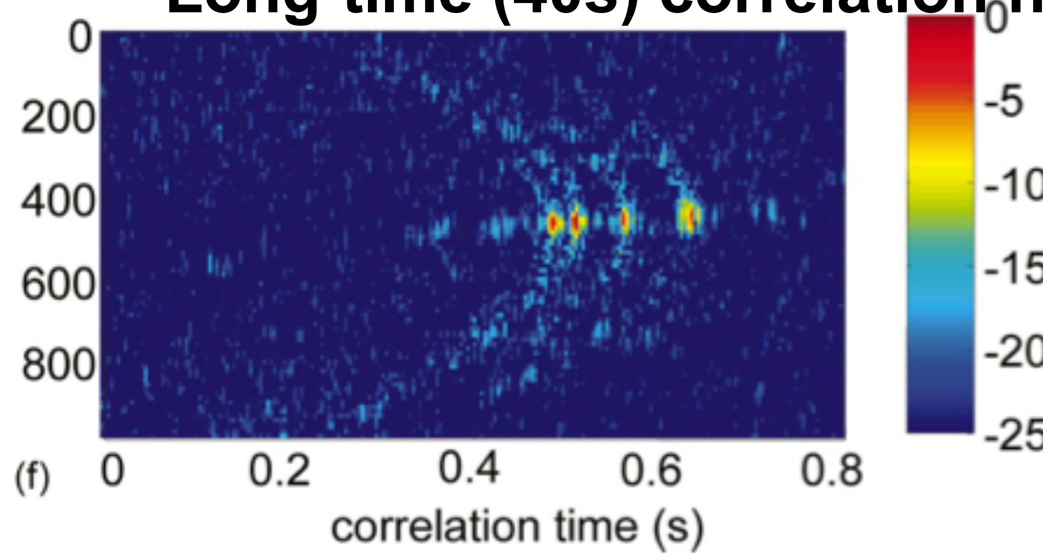




Short-time (5s) correlation



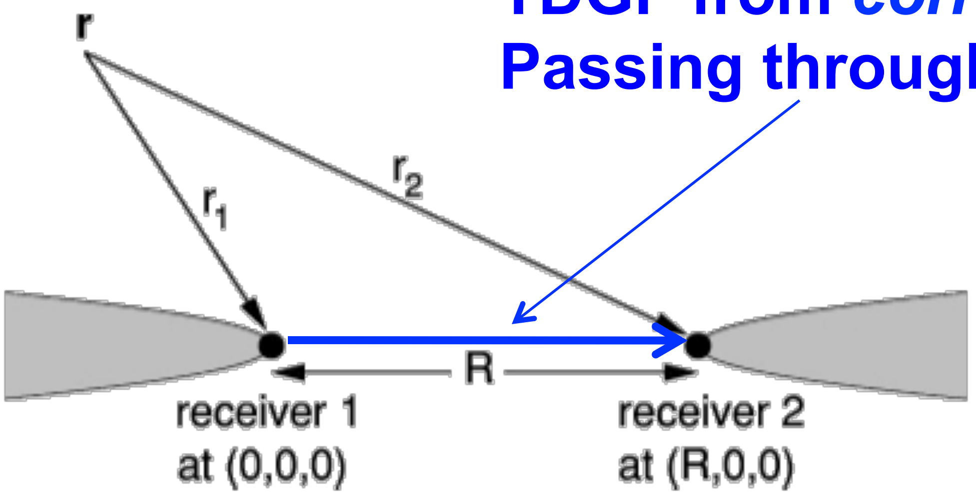
Long-time (40s) correlation he



Roux & Kuperman (JASA, 2004), Sabra et al. (JASA, 2005 & IEEE. J. Ocean. Eng. 2005), Snieder (Phys.Rev 2

Physics of Correlation Process

TDGF from *correlated* noise
Passing through both sensors



Physics of Correlation Process

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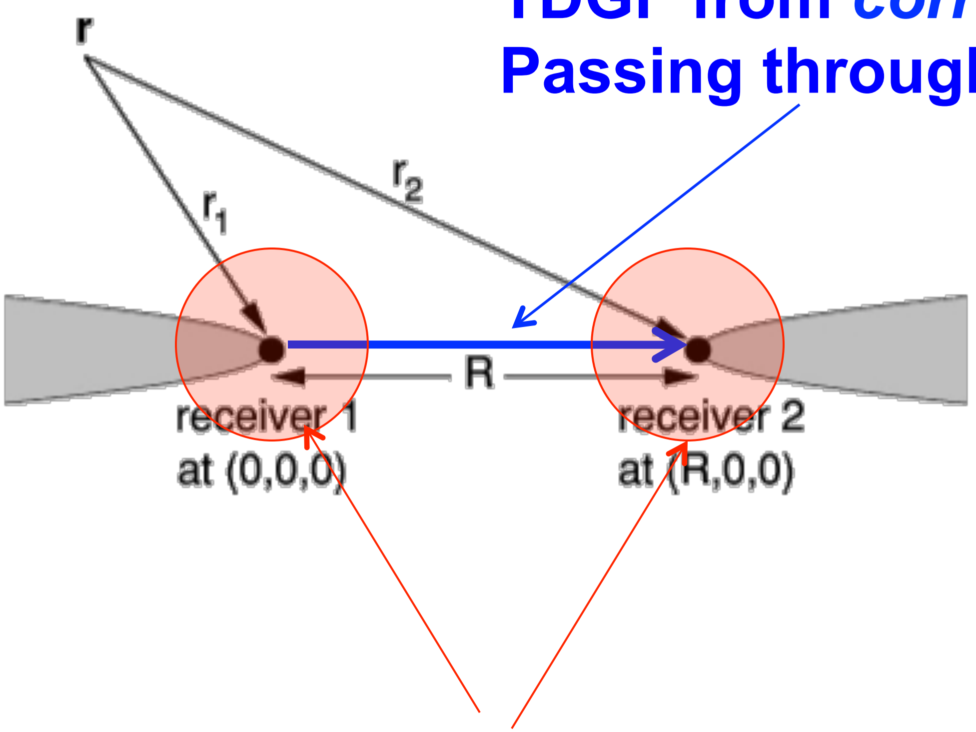


Uncorrelated noise at each sensor

Experimental estimation of covariance function

Experimental goal is to estimate covariance function from realization(s)

$$C_{ab}(\tau) = \frac{1}{\Delta t}\int_0^{\Delta t} dt\; P_a(t)P_b(t+\tau)$$

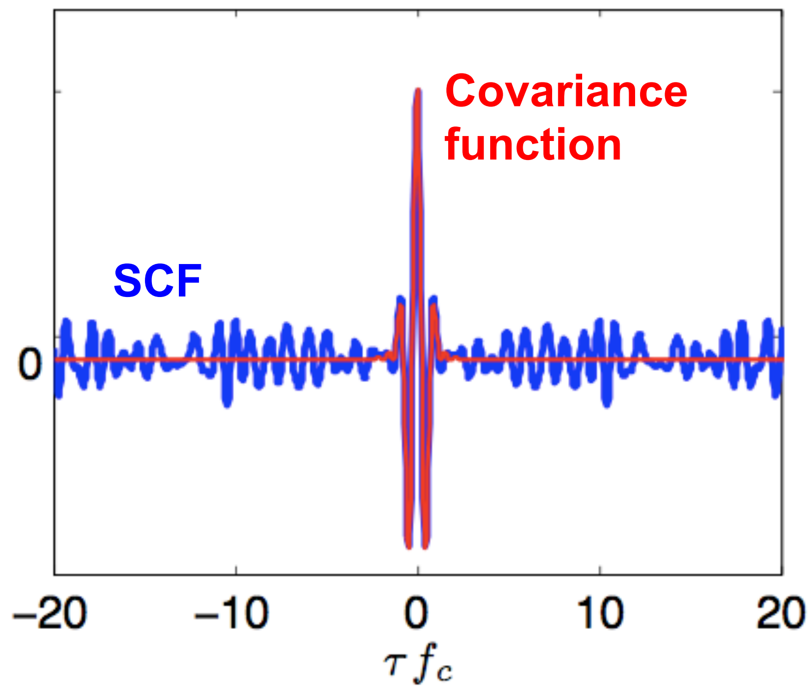
We measure the sample correlation function (SCF)

Ensemble average of SCF realizations converges to covariance function

$$W_{ab}(\tau) = \lim_{M\to\infty}\frac{1}{M}\sum_{m=1}^{M}\{C_{ab}(\tau)\}_m$$

Likewise with the infinite time average of a single realization of the SCF

$$W_{ab}(\tau) = \lim_{\Delta t\to\infty} C_{ab}(\tau)$$

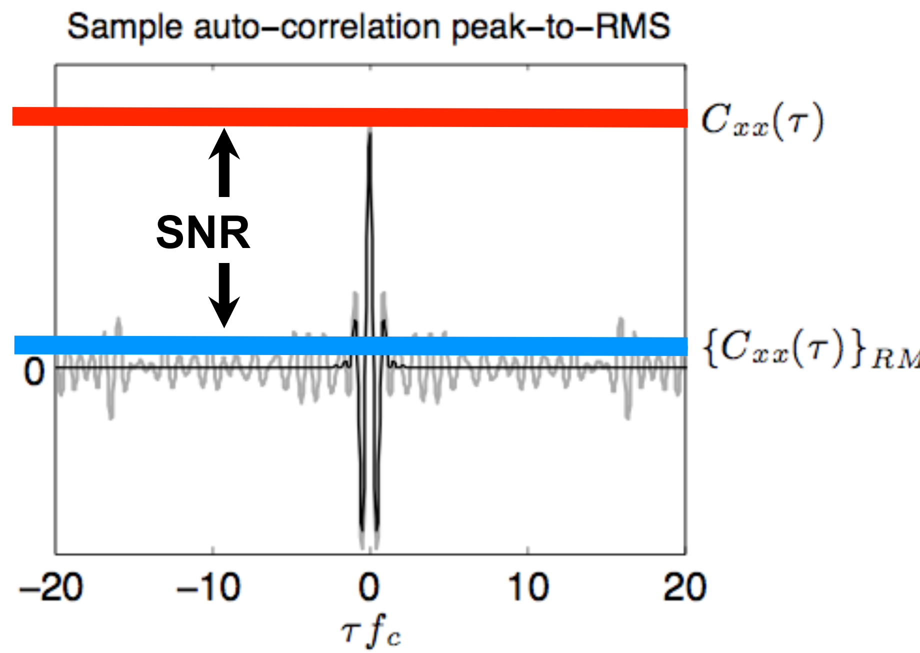


SNR

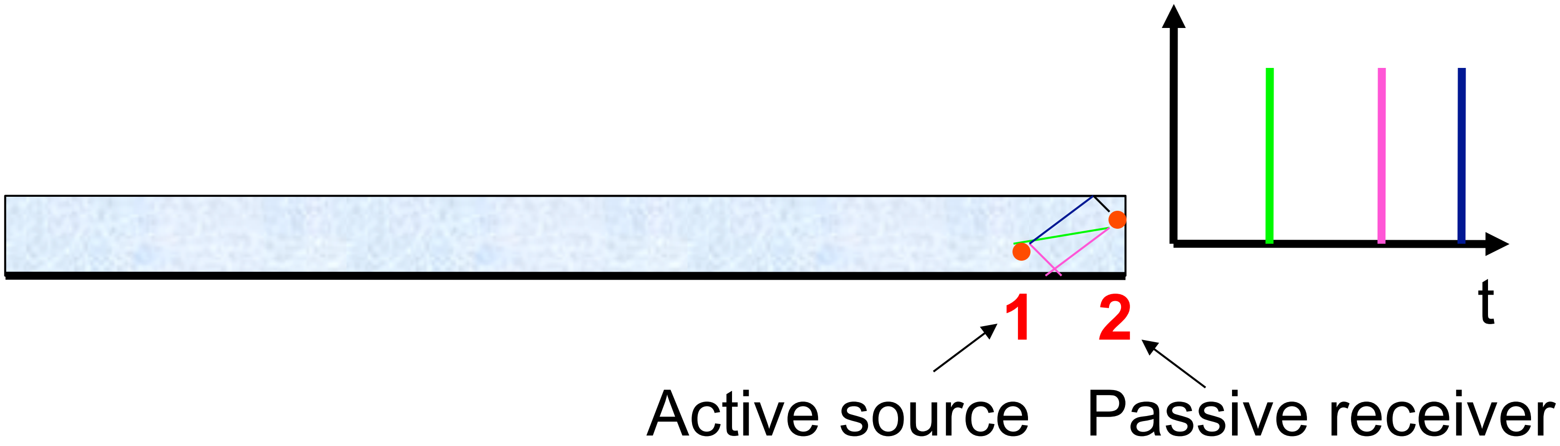
coherent energy
at time τ

$$SNR = \frac{C_{ab}(\tau)}{\sqrt{\frac{1}{\Delta t}\int_0^{\Delta t} d\tau C_{ab}^2(\tau)}}$$

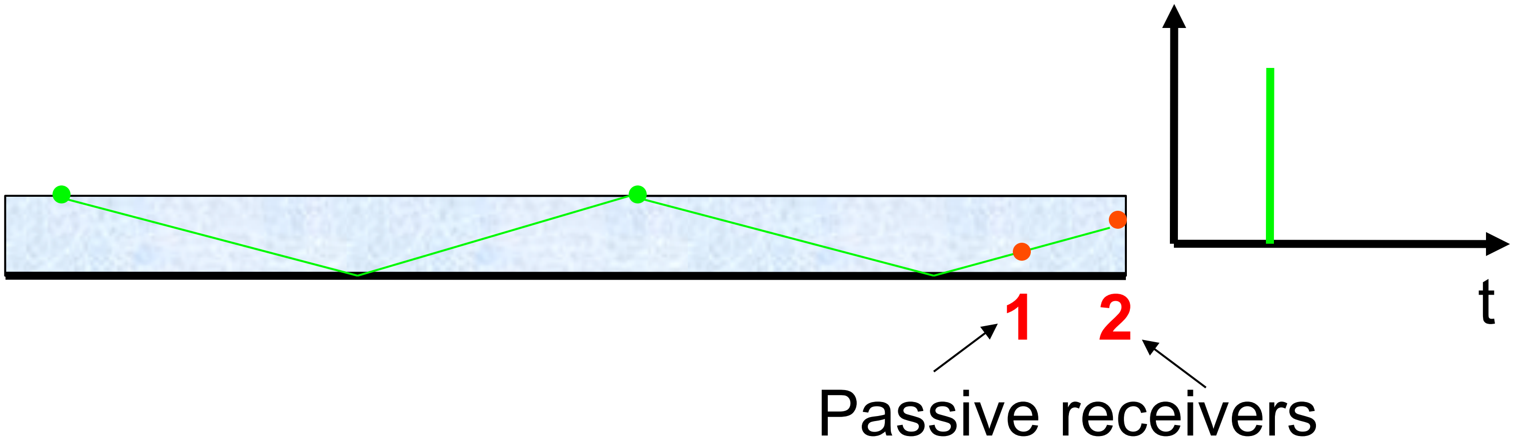
RMS = total energy
incident on sensors



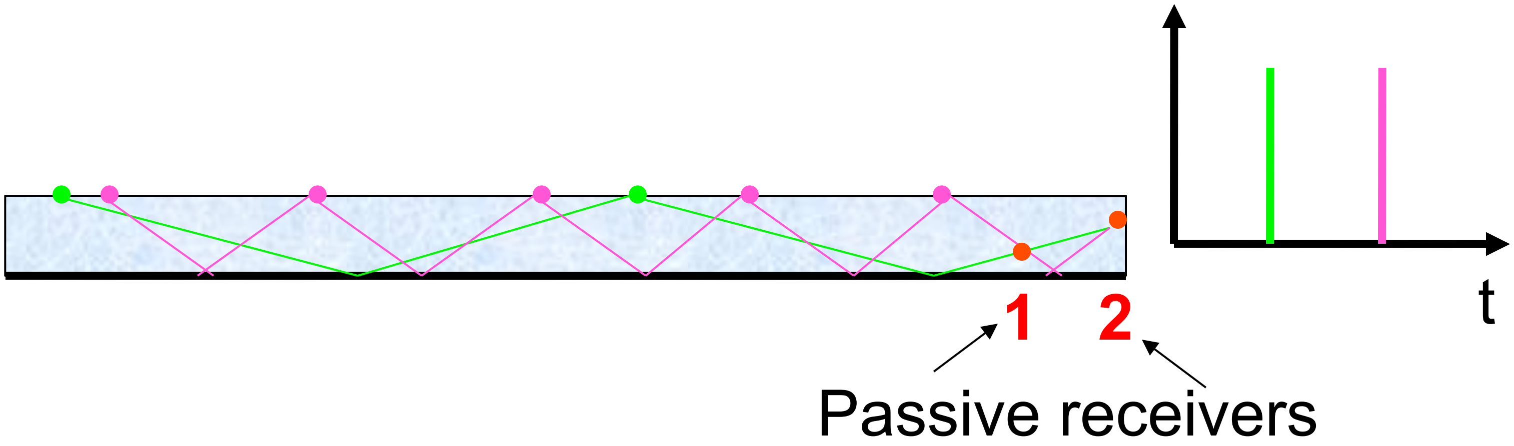
Waveguide:Arrival times



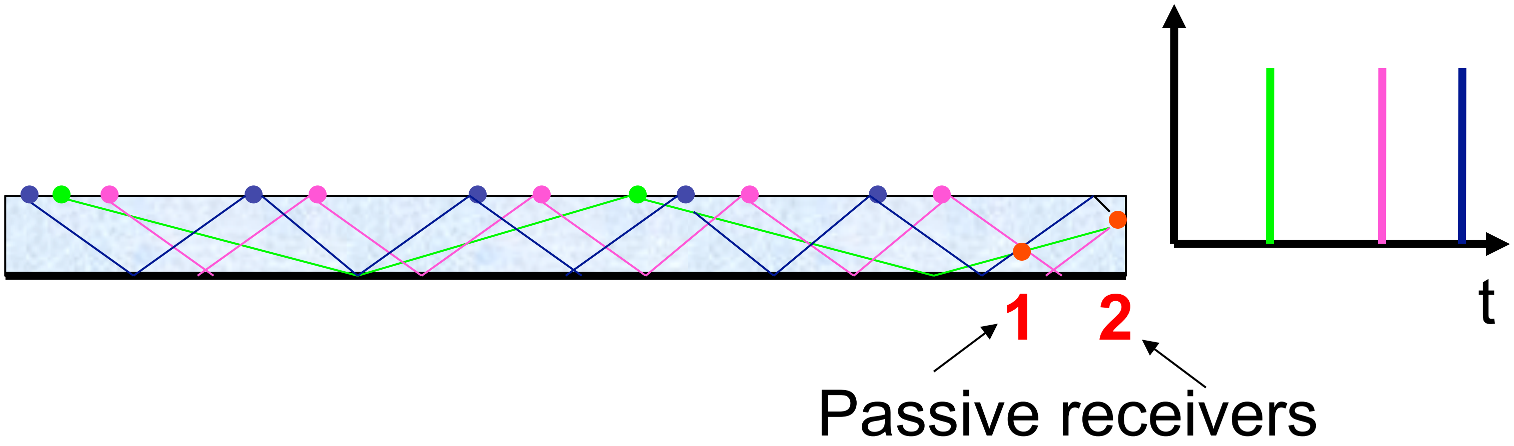
Correlation times with noise sources



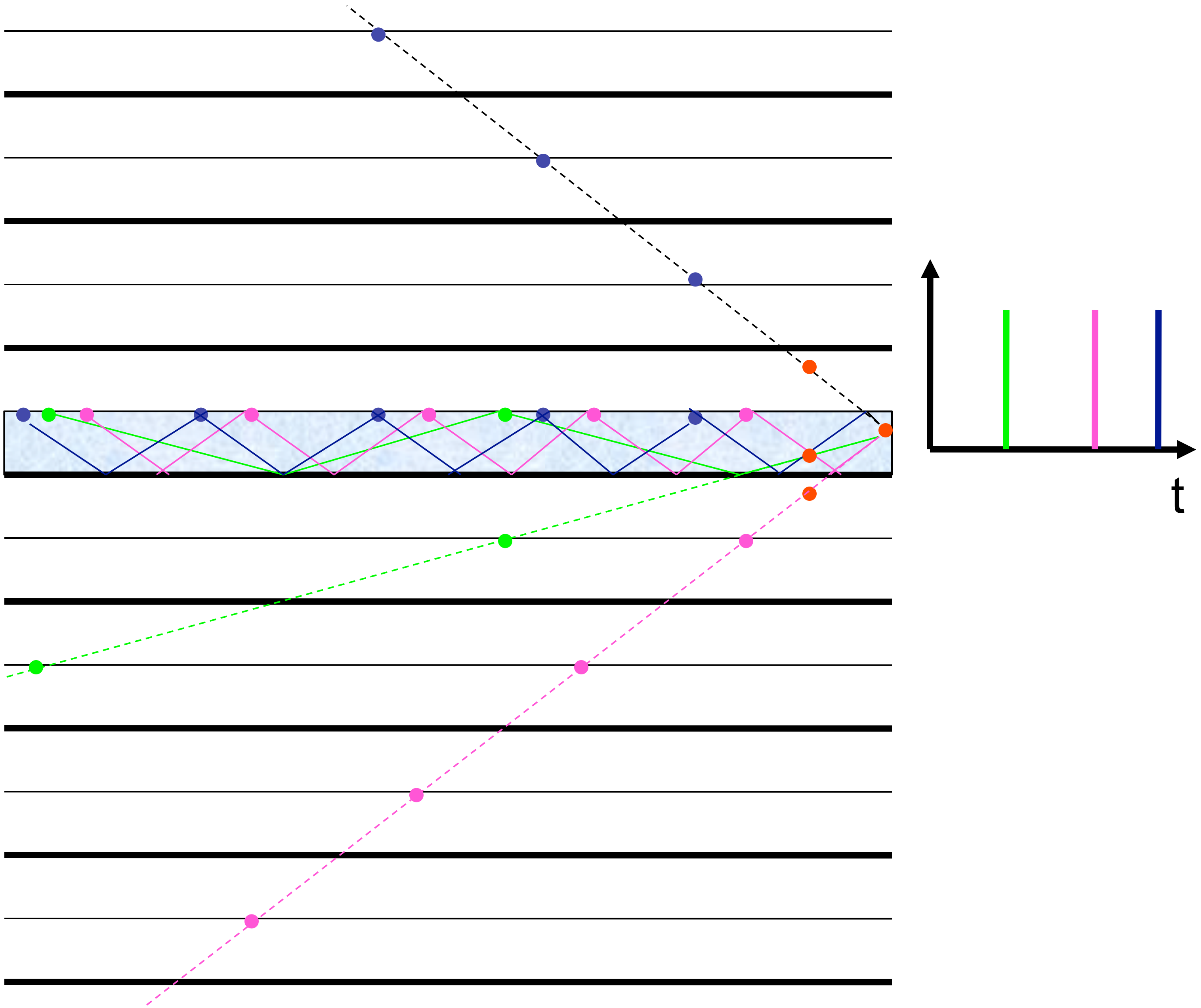
Correlation times with noise sources



Correlation times with noise sources



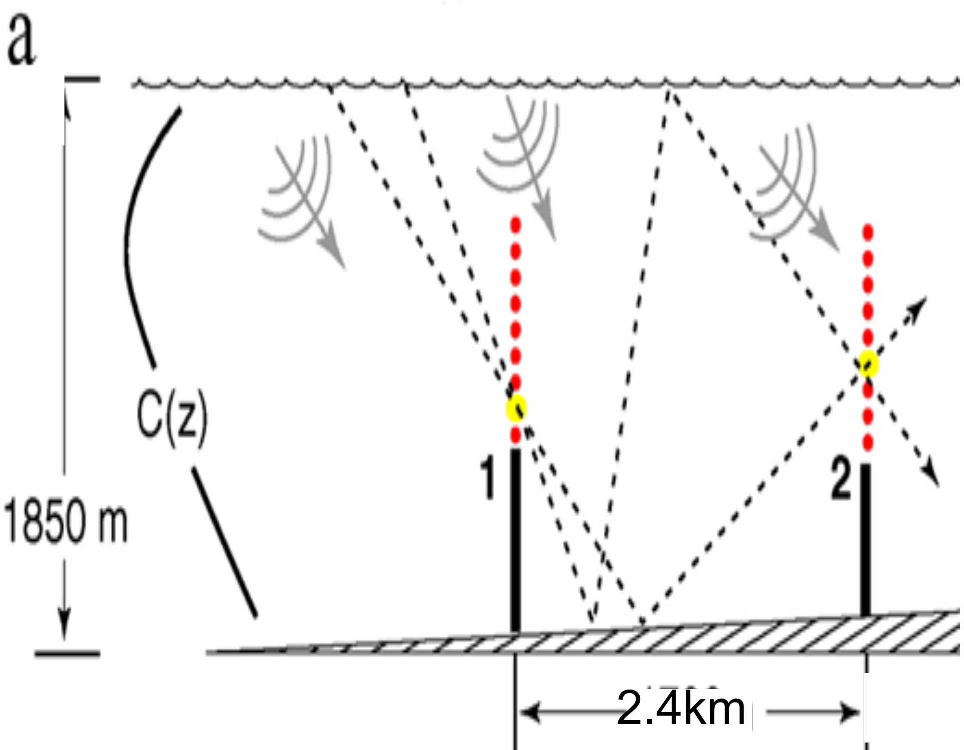
Correlation times-Noise Source Images



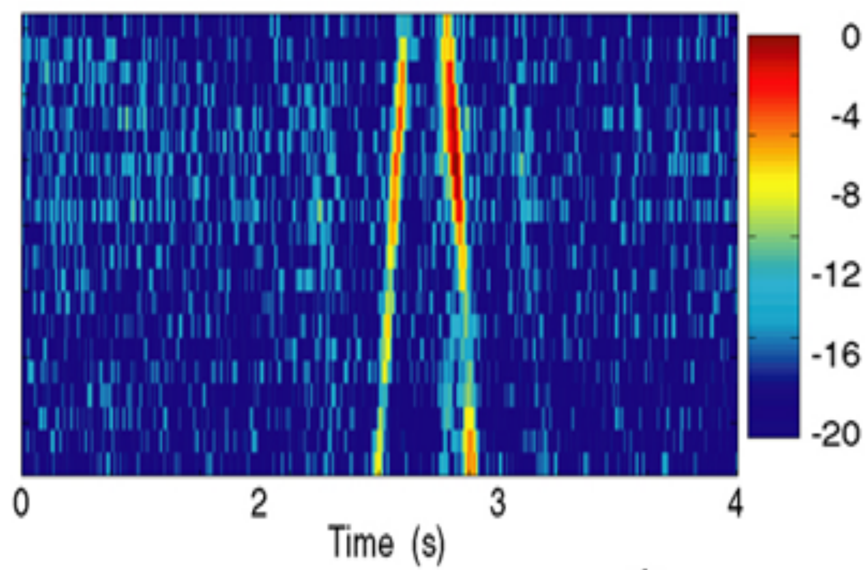
Underwater Acoustics

(non-free space)

NPAL data



Experimental results (70 – 130 Hz)



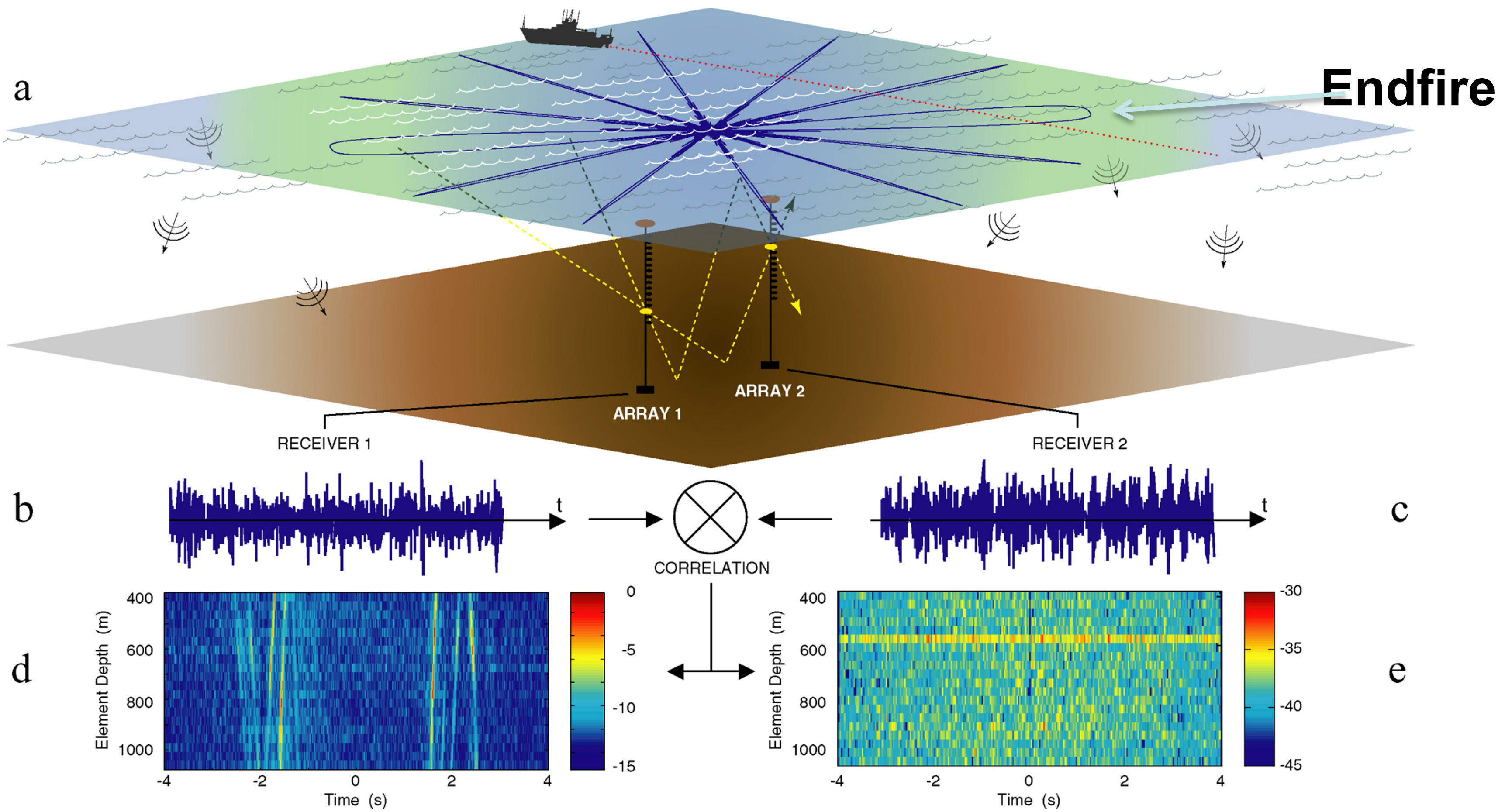
Noise events propagating through receivers 1 and 2 average-up coherently over the long-time in the cross-correlation function.

Coherent wavefronts yield an estimate of the Green's function between 1 and 2.

Roux & Kuperman (JASA, 2004), Sabra et al. (JASA, 2005), (IEEE. J. Ocean. Eng. 2005)

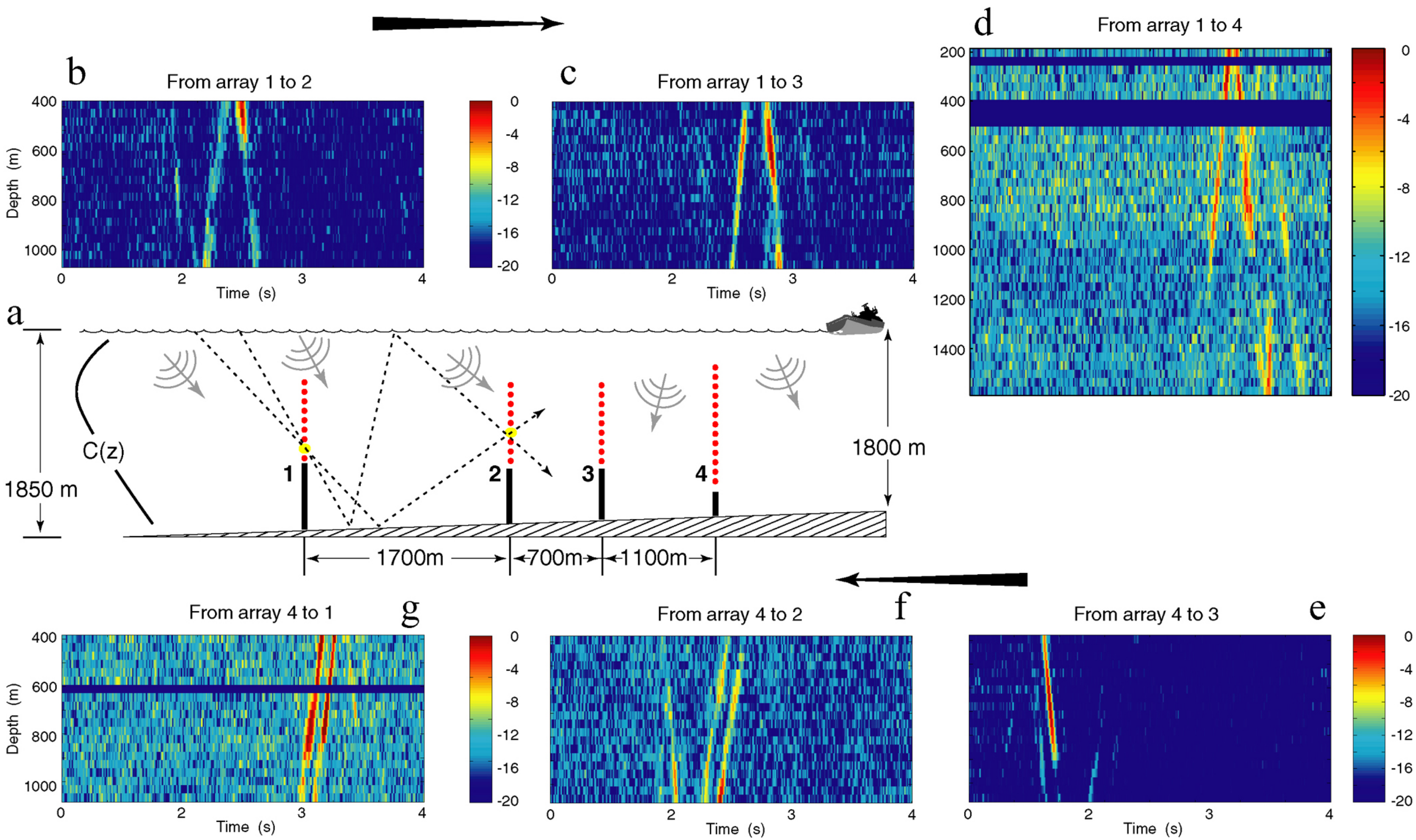
Experimental results (70 – 130 Hz)

NPAL experiment

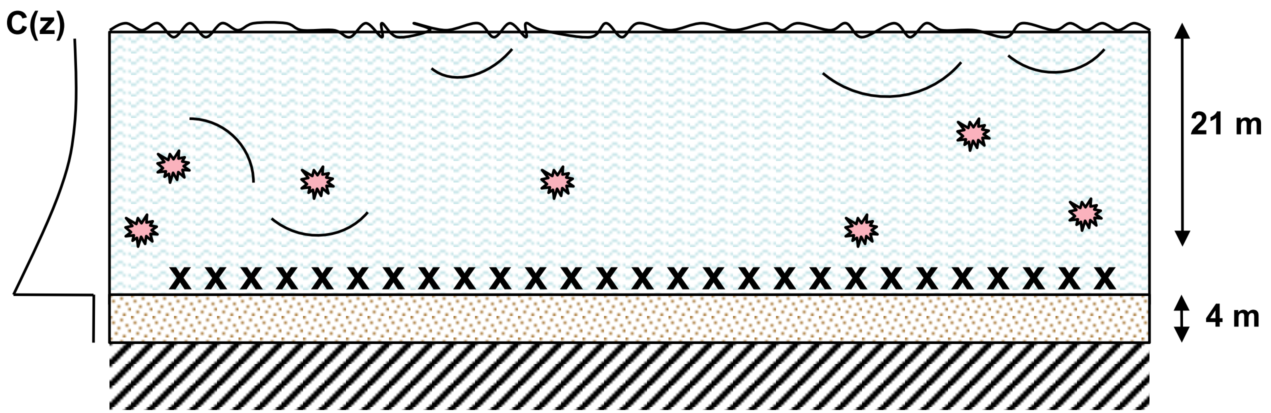


Roux,Kuperman-2004 (using NPAL data)

Experimental results (70 – 130 Hz)



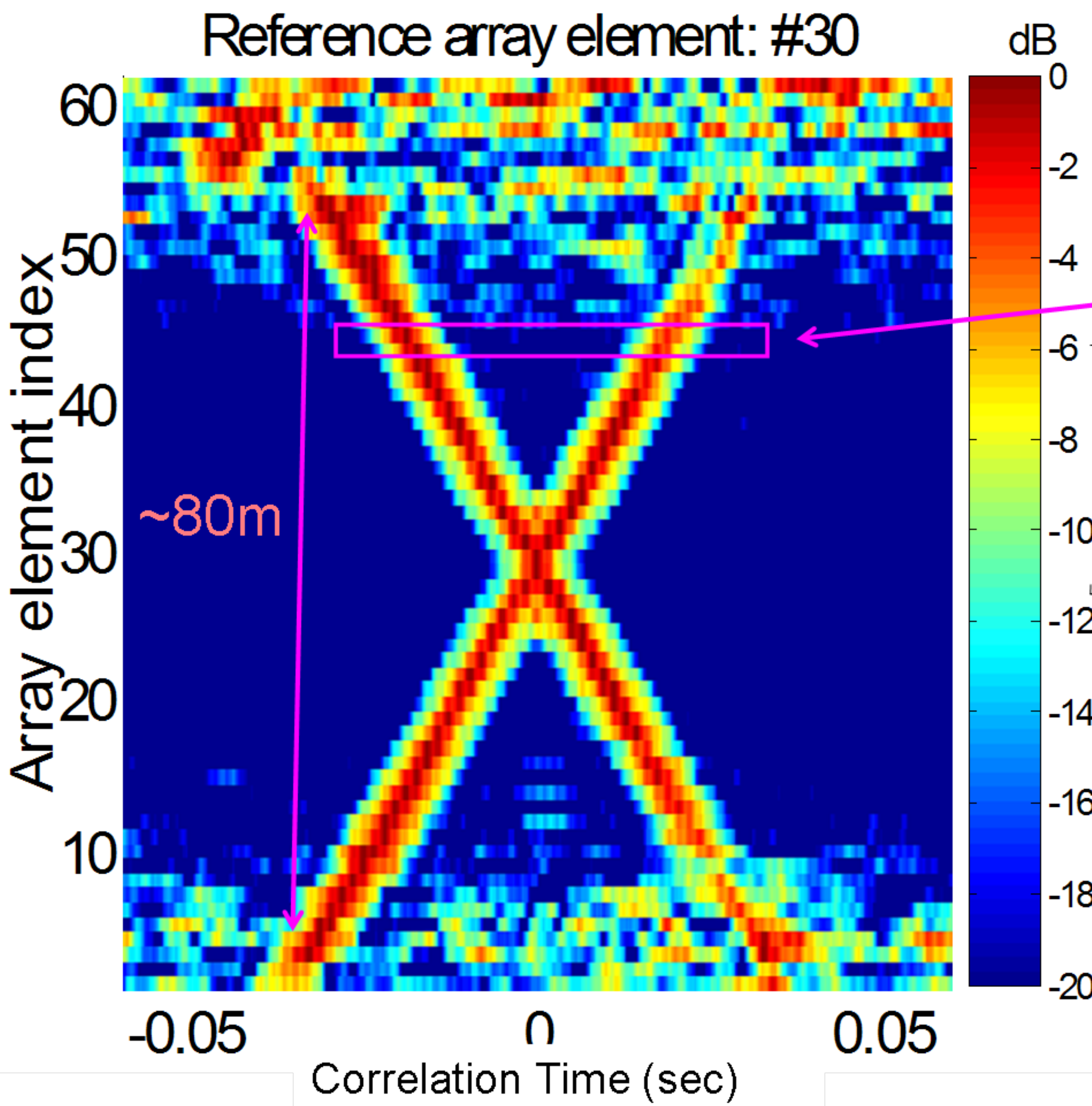
“Croaker” Fish

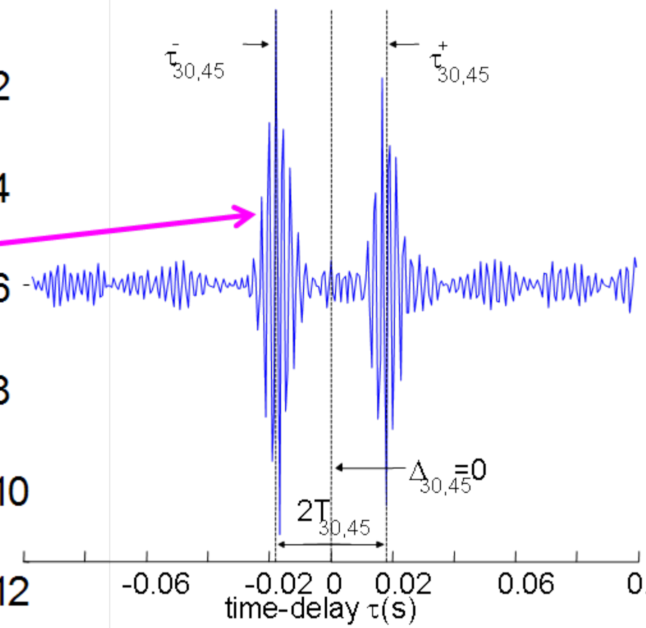


Fish Noise in Very Shallow Water

D’Spain et al

Horizontal Coherent Wavefronts

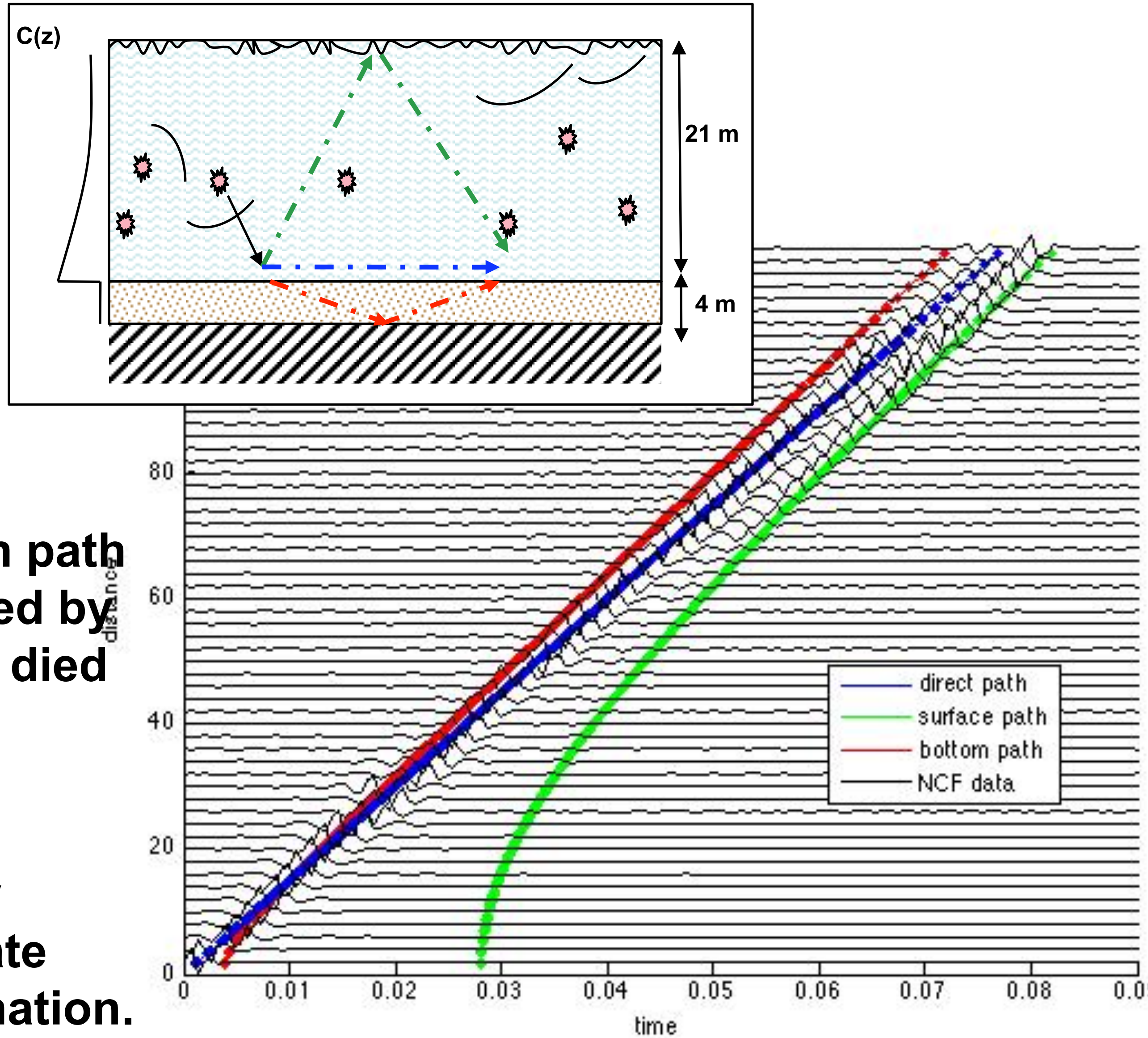




JD 151, 5.26. AM (GMT).

11 min Data.
Bw=[350Hz-700Hz]

Looking for the sediments



Sediment reflection path but it was dominated by the direct path and died out over longer distances

---working on array processing to isolate bottom path/information.

Fried et al 2008

THE PROBLEM

TIME OF ARRIVAL PEAKS
FROM
SENSOR-SENSOR
CORRELATION PROCESSING
TAKES TOO LONG TO BUILD UP

ISSUE: CORRELATION TIME *vs*
OCEAN TIME SCALES

NOISE CORRELATION PROCESSING USING SPATIAL ARRAYS

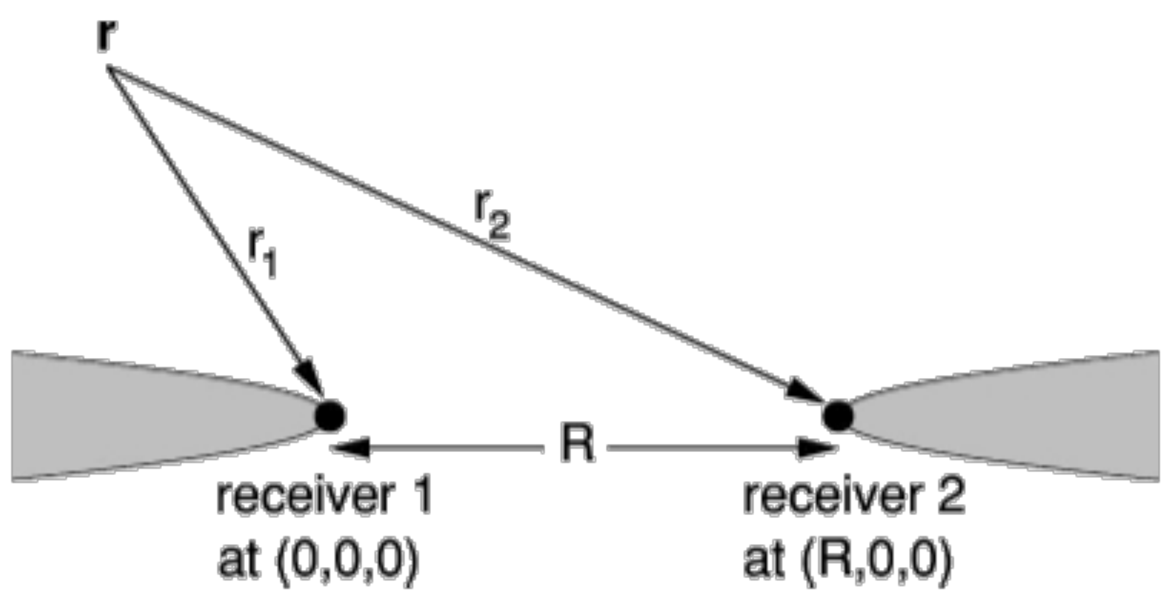
- TAKE ADVANTAGE OF PHYSICS
- RESULT: ENHANCES EXTRACTION OF TIME OF ARRIVAL STRUCTURE OF TDGF
- VERTICAL ARRAYS vs HORIZONTAL ARRAYS?

Geometry of correlated noise

Noise that contributes to the correlation comes from 'endfire' to the sensors

The major contribution is from noise in-line (endfire lobes, (anti-)Fresnel zones, etc).

Stationary phase argument.



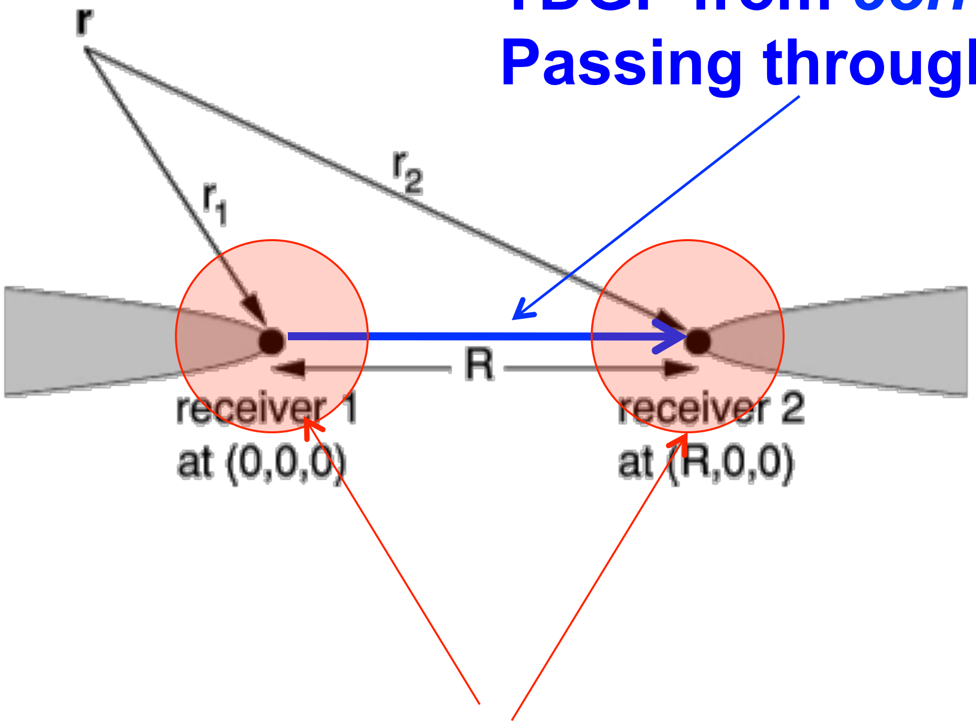
Snieder (Phys.Rev 2004), etc…

Moving on from correlation between sensors to correlation between arrays

- **Our Previous work has shown:**
  - **Time-of-arrival information of noise correlation function along line array agree with physics of noise propagation between sensors correlated**
  - **Can retrieve multiple arrivals – direct, and surface**
  - **Relative strength of surface reflection path points to critical angle at water/bottom interface**
  - **<u>ISSUE: </u>PROCESSING TIME SCALES vs OCEAN TIMES SCALES**
  - **Potential Solutlon: BEAMFORMING**

Physics of Correlation Process

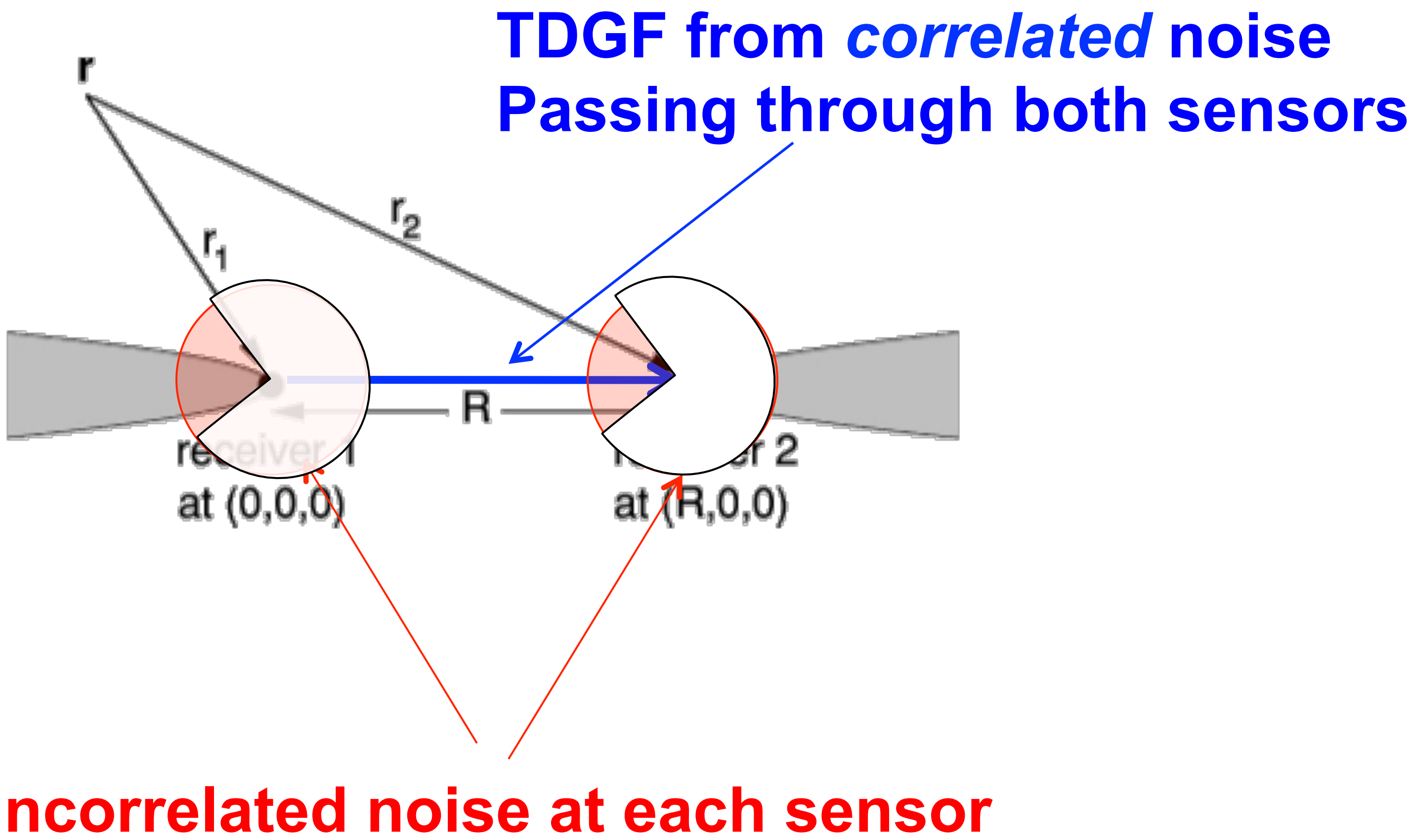
TDGF from *correlated* noise
Passing through both sensors



Uncorrelated noise at each sensor

Even a Vertical Array w/ Azimuthal Symmetry does not discriminate in horizontal

Physics of Correlation Process

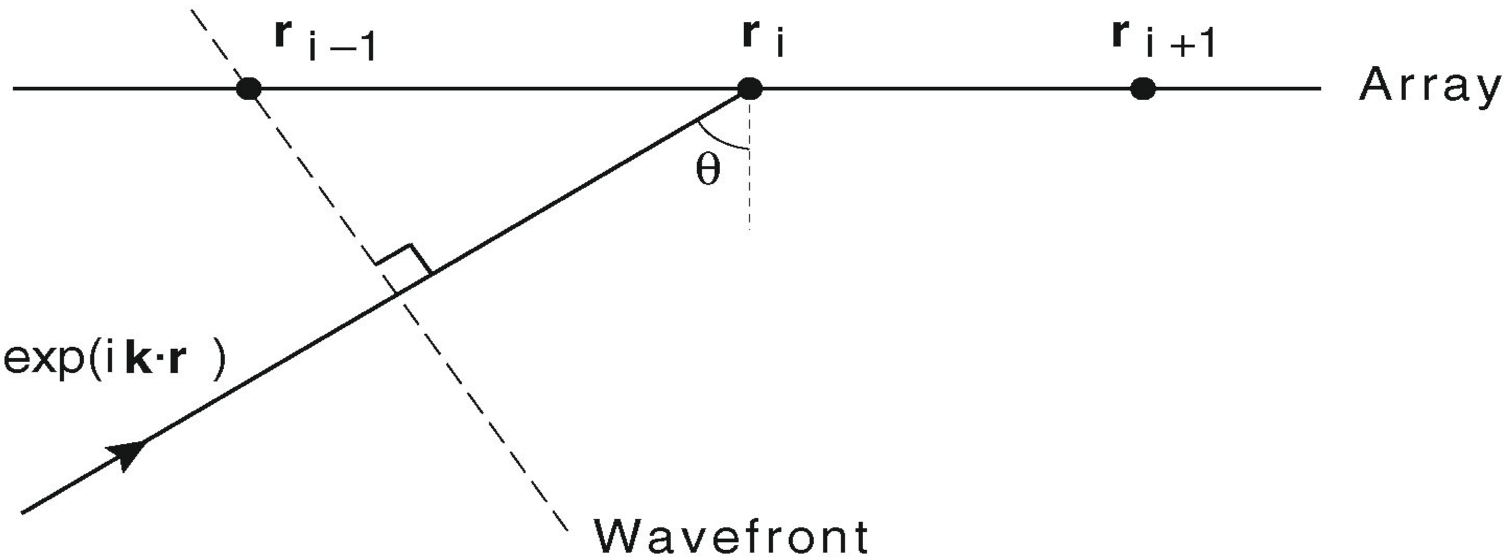


Horizontal Array can reduce incoherent noise

GOAL OF ARRAYS OR ANTENNAS:

1. ADD UP MORE "SIGNAL" THAN "NOISE"
2. LOOK IN A CERTAIN DIRECTION
   - TOWARD A SIGNAL OF INTEREST
   - LOW SIDELOBES
3. ADAPTIVE PROCESSING: USE DATA FOR HIGH RESOLUTION AND MINIMIZE SIDELOBES

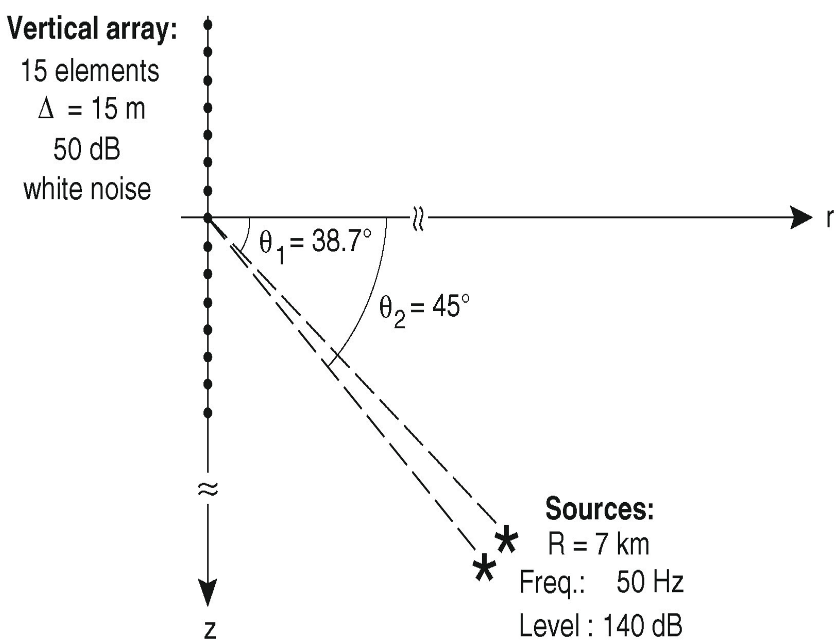
PLANE WAVE BEAMFORMING

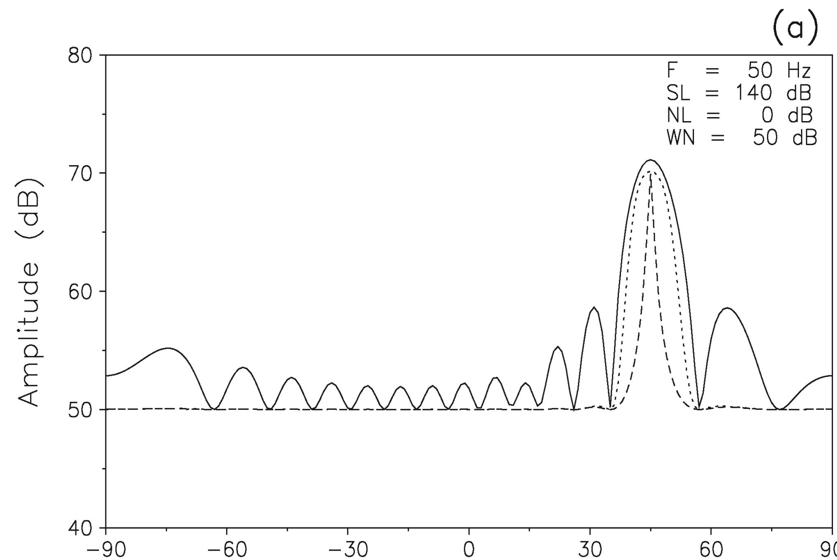


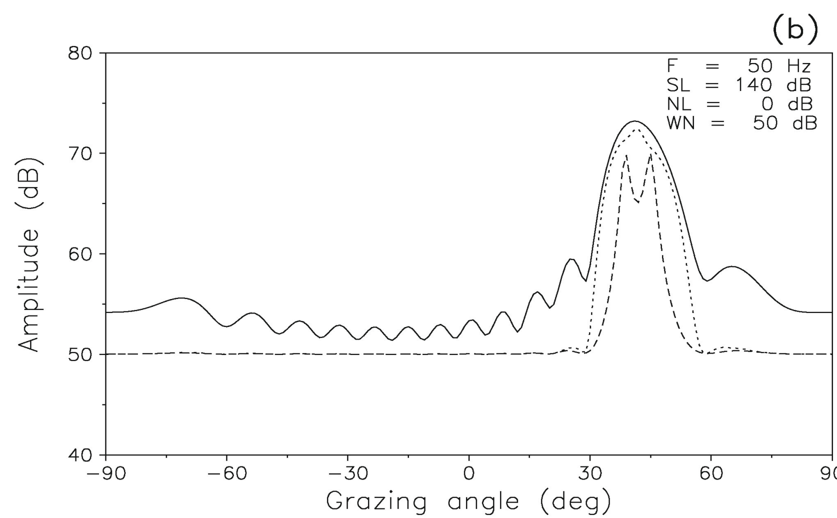
s=Signal vector; d=s + noise
w="replica" vector (usually from a model)

If w=s (or d), then w^+d is maximum
→ At correct angle, each element w_i^* is cc of s_i

LINEAR AND ADAPTIVE BEAMFORMING



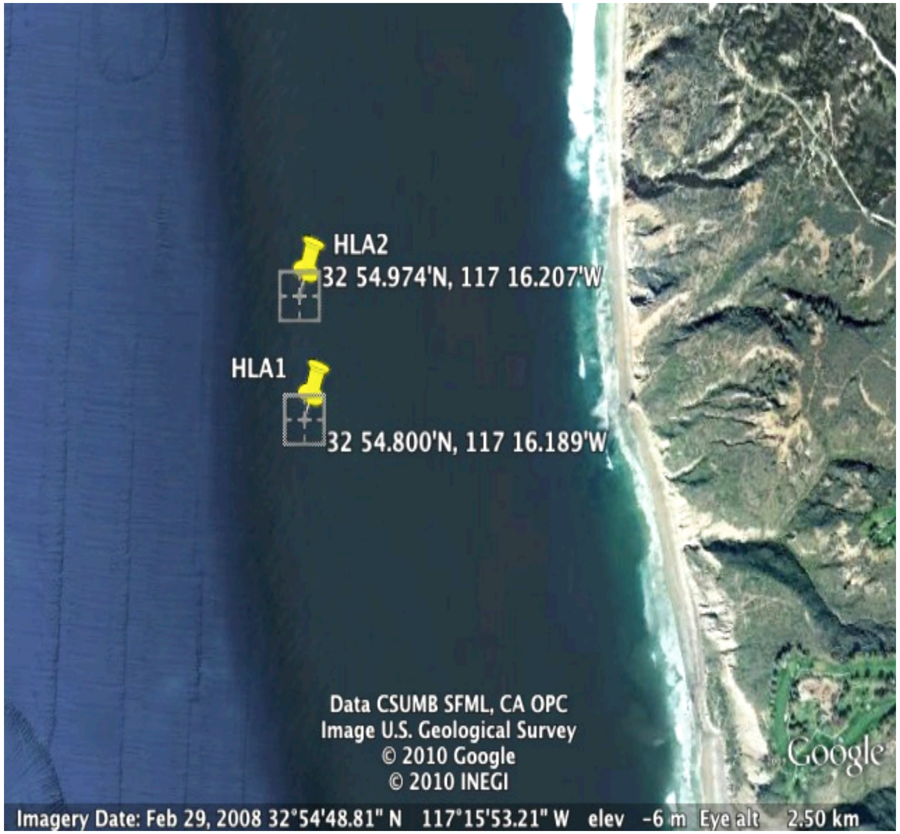


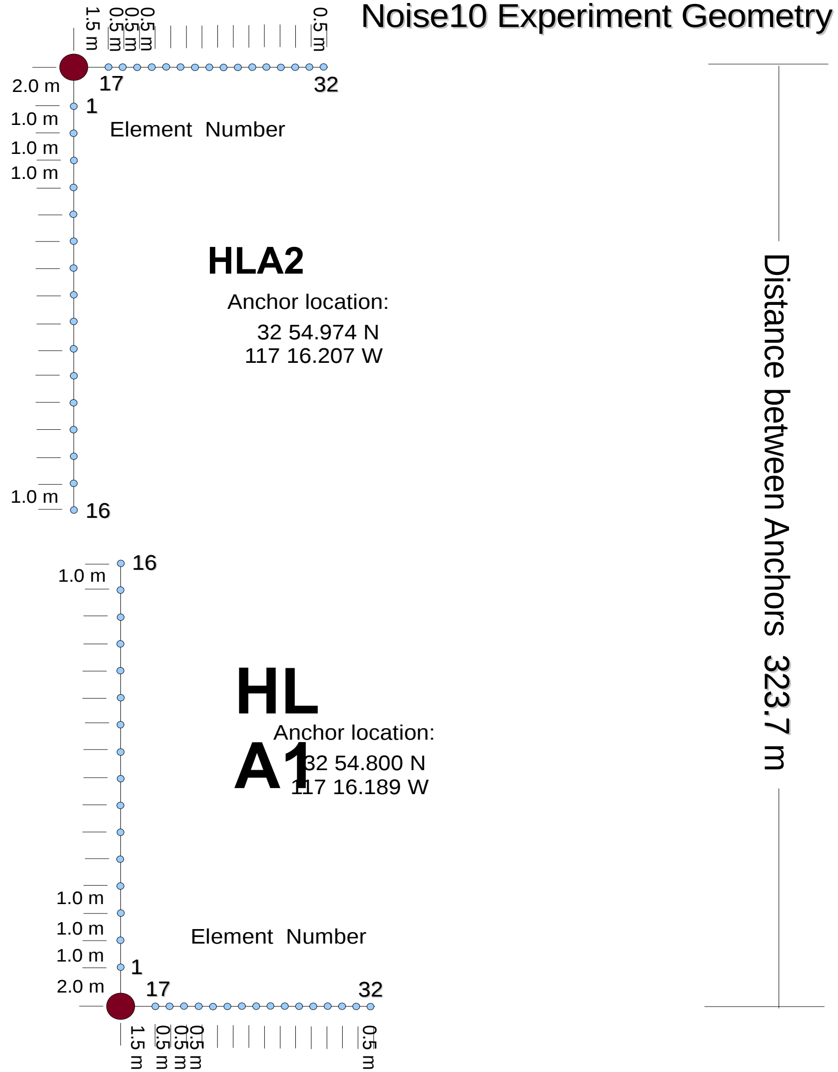


Noise correlation between two Horizontal arrays

- Diffuse surface noise (1-2 kHz), shallow water
- Incorporating beams in correlation – 323 m
- Analysis of SNR to see buildup time and array size, shape and noise directionality effects

Noise10 Experiment

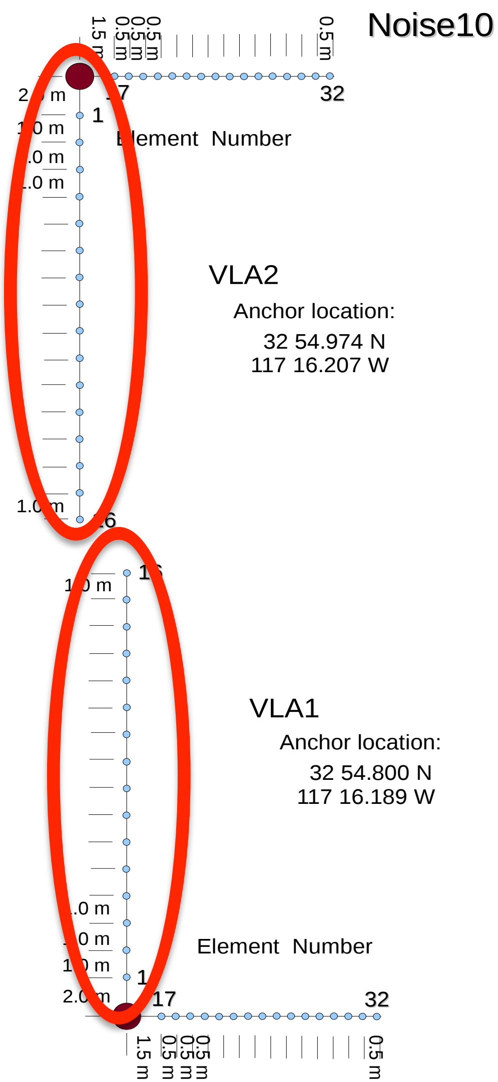




- •Location of arrays mounted on the ocean bottom of 20m depth
- •Beach one side, ocean on the

expect arrival at ~.2

Compare beampatterns



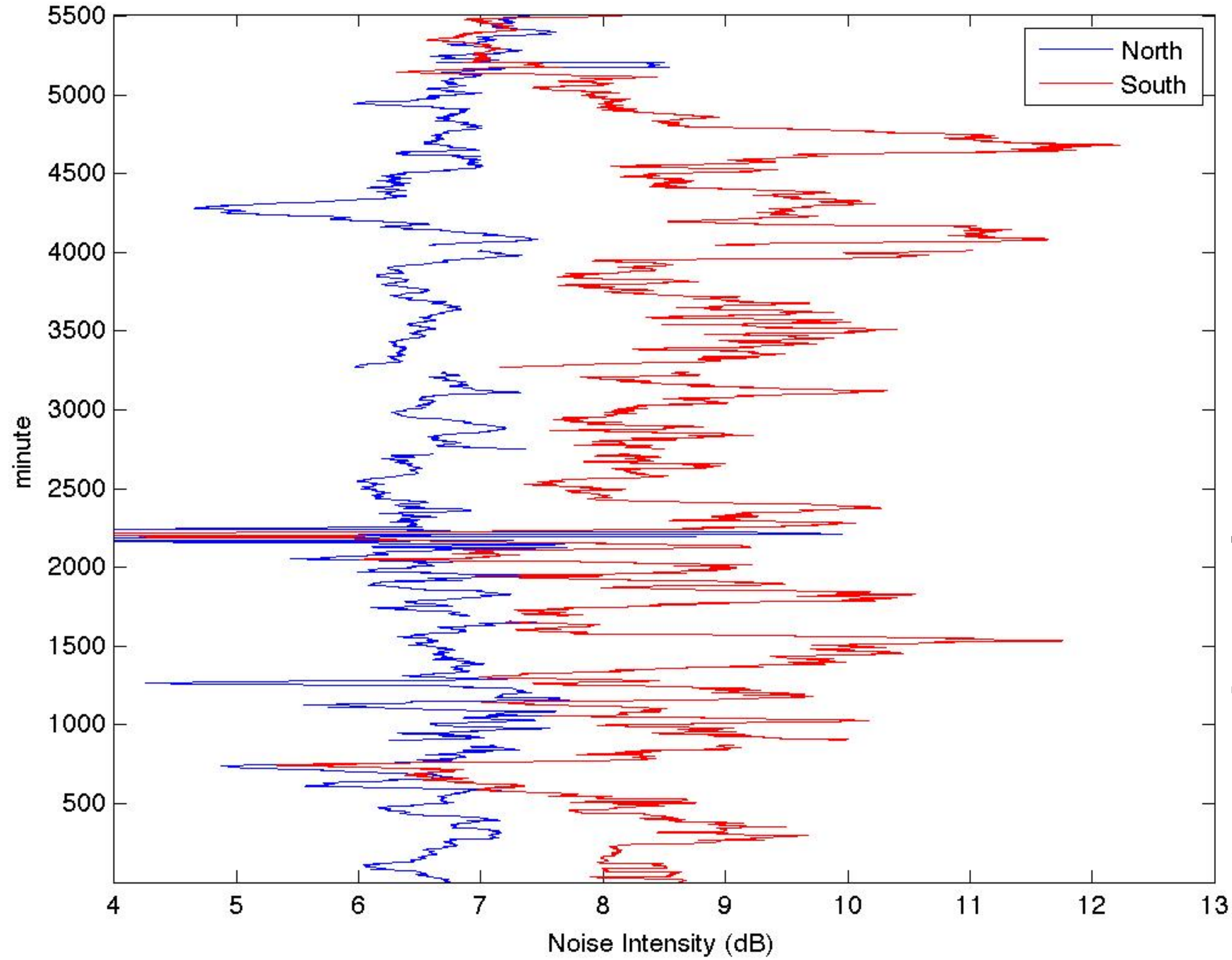
Use elements 1-16 of each array (sensors in line between arrays) to look north and south.

Average over all frequencies 1-2kHz.

Get a very rough measure of noise field – large side lobe effects can't be

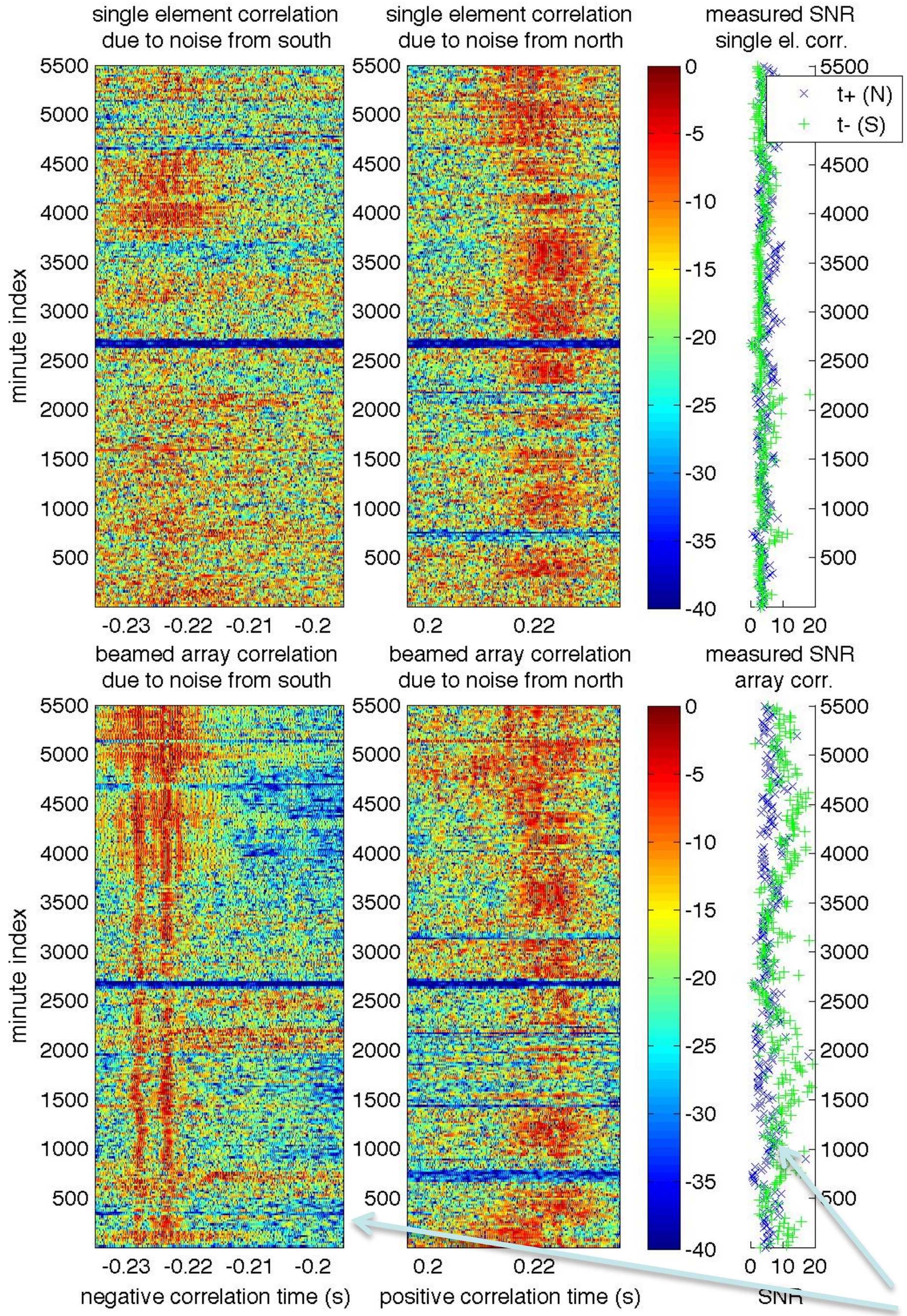
Compare beampatterns

But what about the difference between looking North



~ 30% more energy from South than from North

1-2 kHz Noise

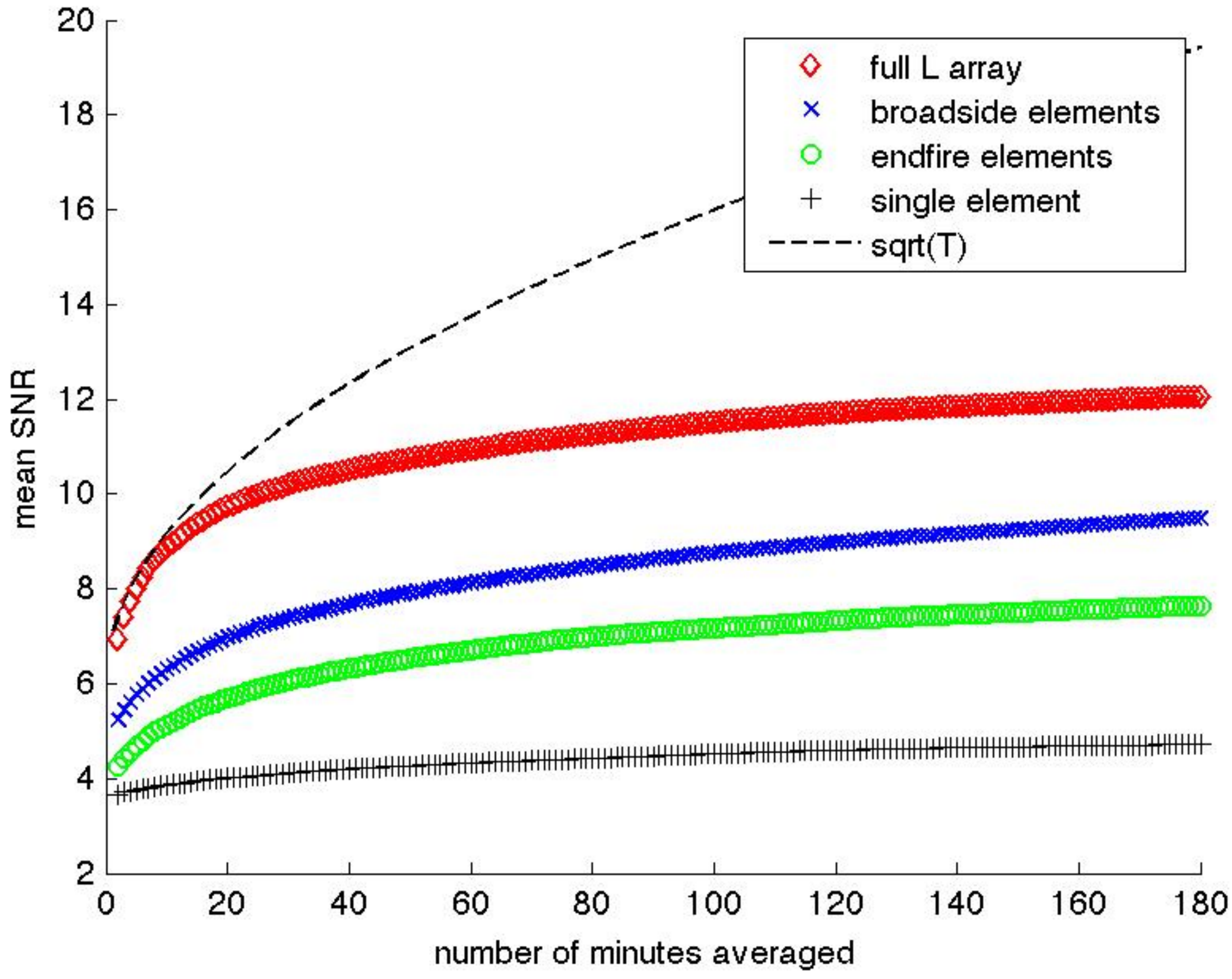


Single Sensor Correlations

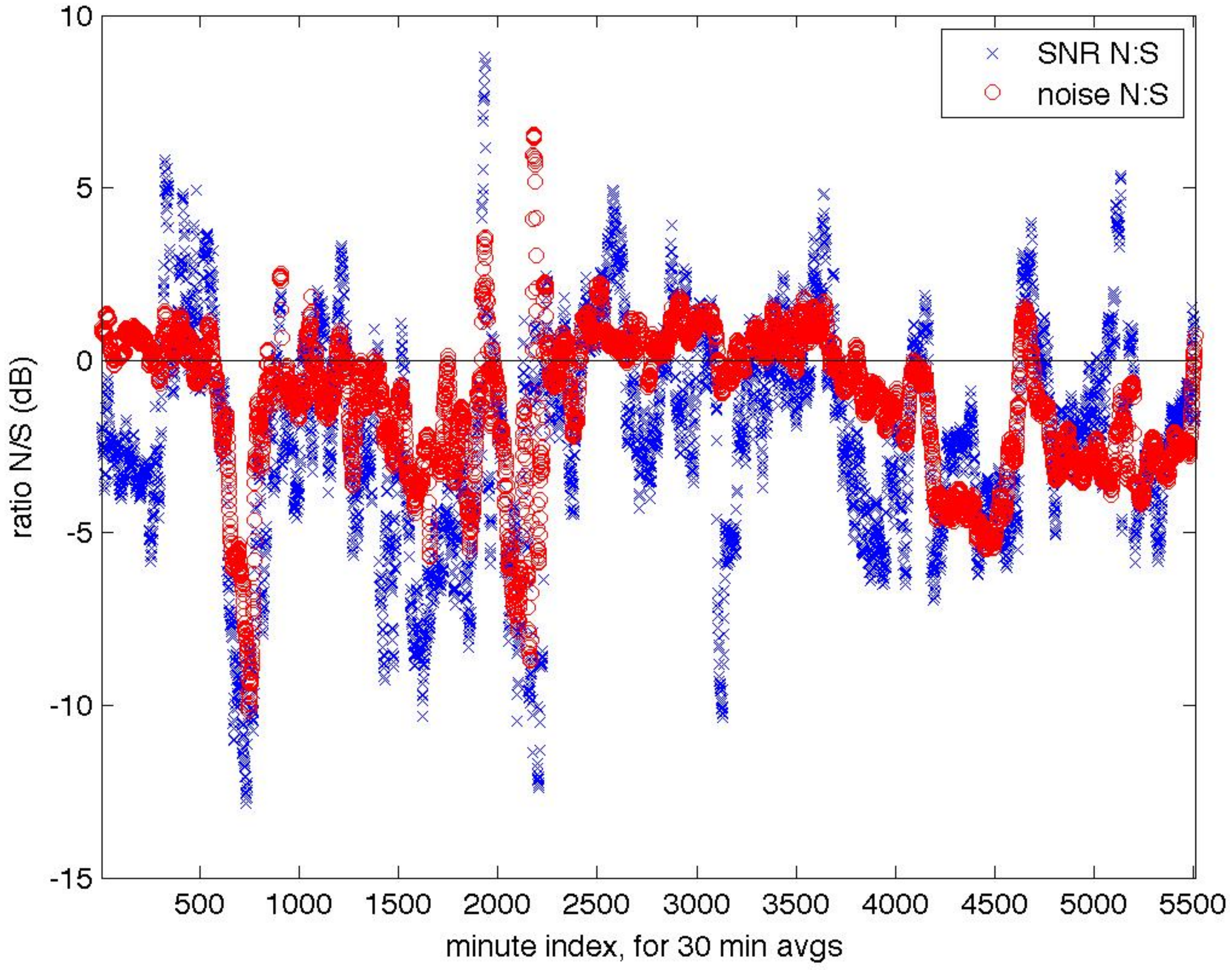
Beam Correlations between L-Shaped Arrrays

Green is noise from south

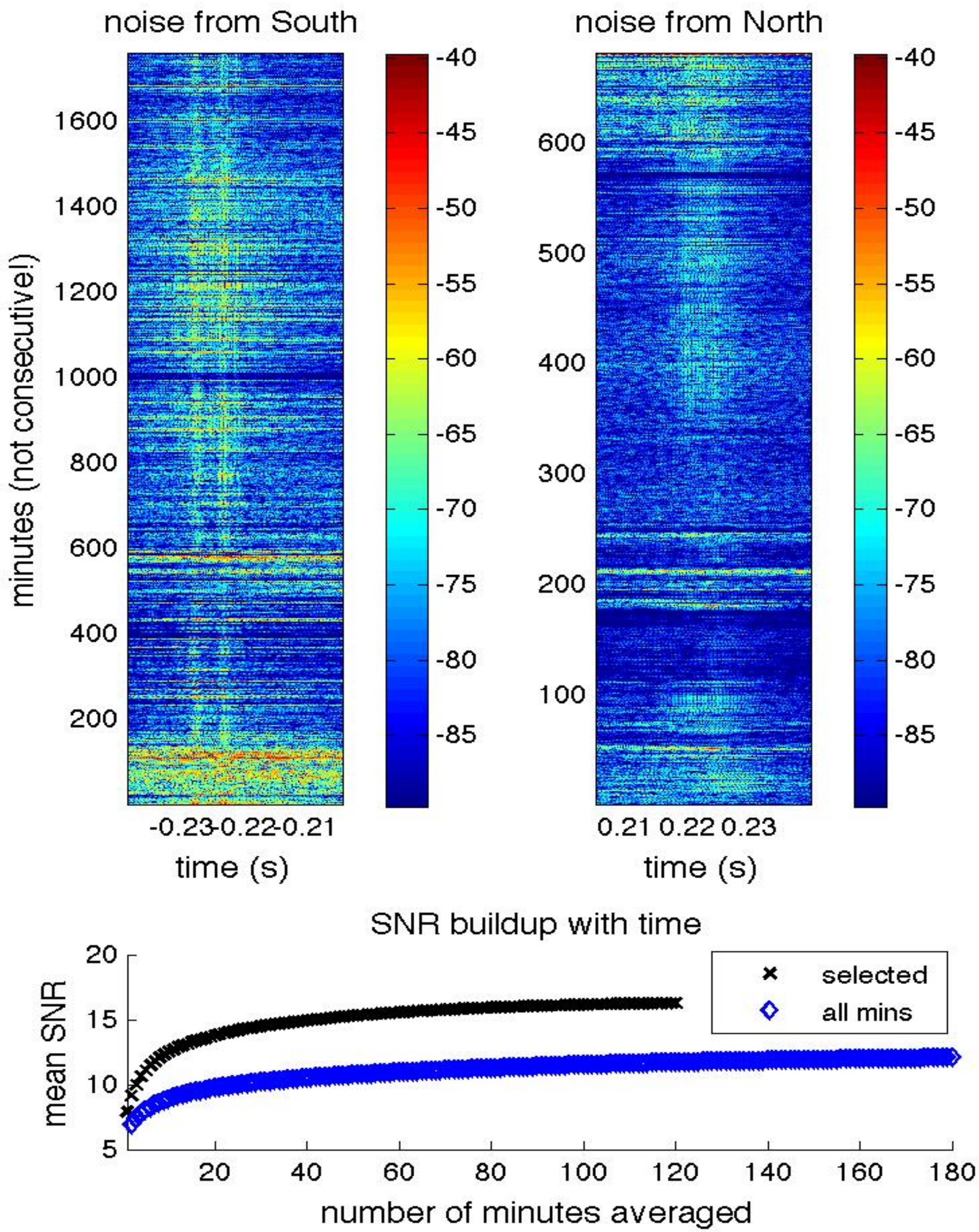
SUMMARY OF ARRAY RESULTS



Correspondence of Noise Directionality with SNR of North/South Correlation Peaks



Selective
Or
“a priori”
Correlation
processing



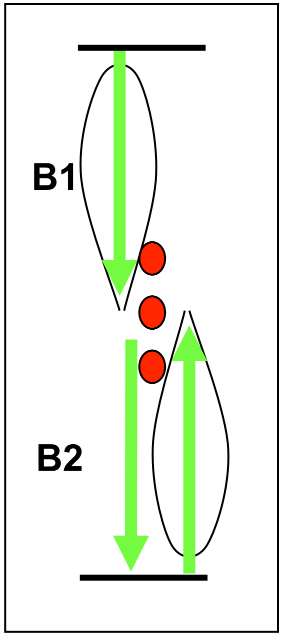
LESSONS LEARNED: Noise correlation between two Horizontal arrays

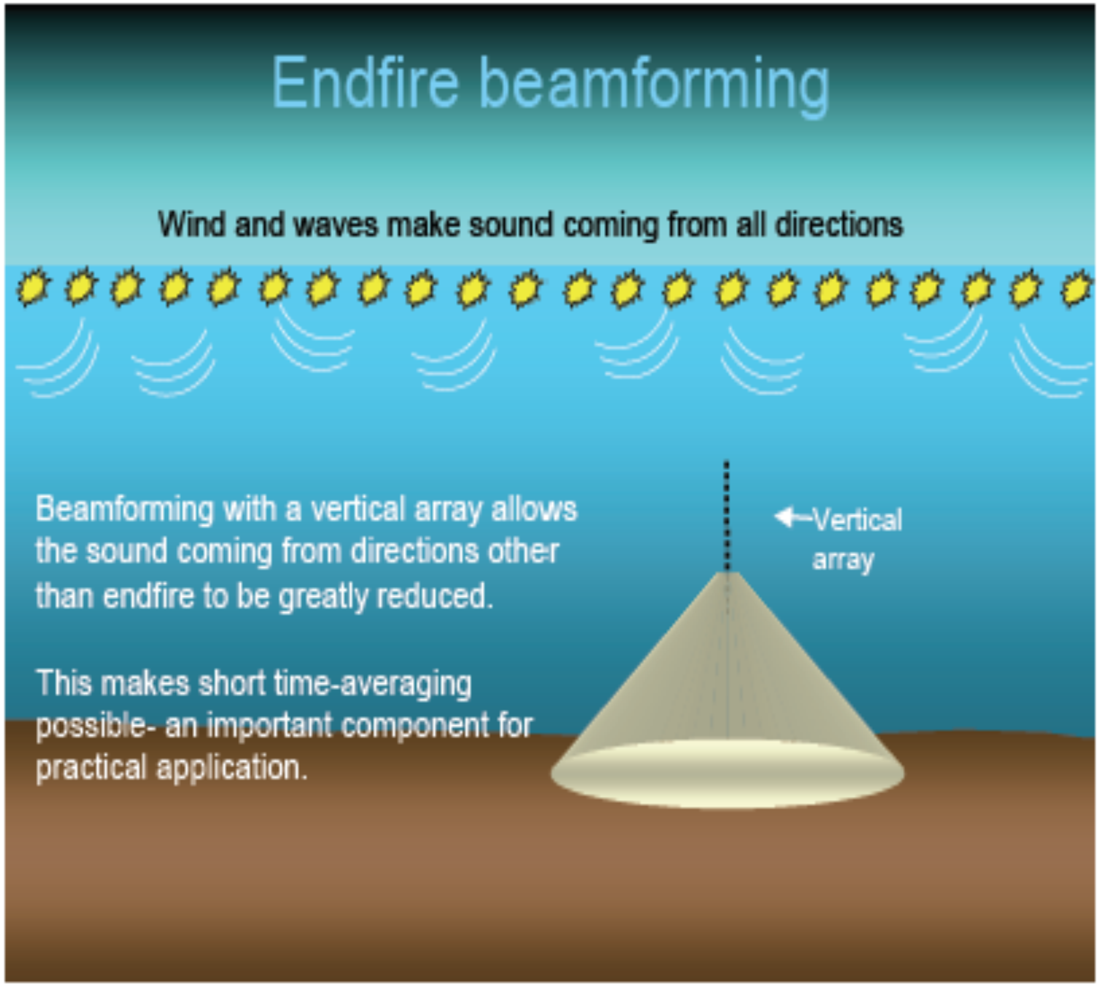
- **Beam-Beam Correlation Processing enhances extraction of arrival times**
- **The processing is based on**
  - **Beams along endfire**
  - **Reduction of uncorrelated noise (vs sensors that receive all noise) since the correlation peaks must ("overcome") emerge from this noise**
- **The emergence time is (still) less than √T because ocean is not stationary over correlation time interval**
- **Selective ('a priori") correlation processing reduces the total build up of uncorrelated noise so that emergence time <u>AND</u> SNR of correlation peaks are enhanced.**

Passive fathometer:

(Horizontal Array used as Vertical Array)

Using ambient noise on a drifting array we can map the bottom properties

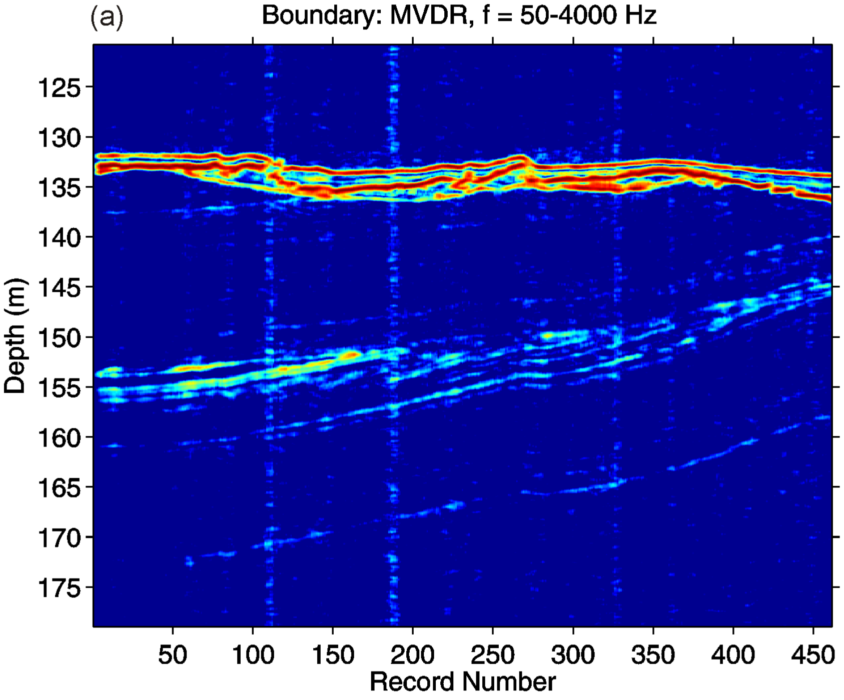




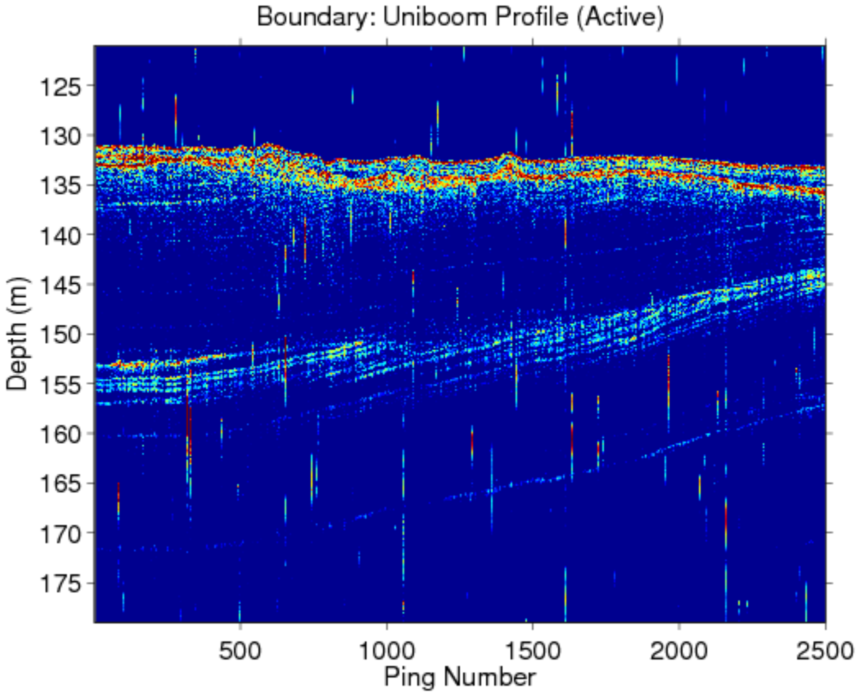
Siderius et al., JASA 2006,
Gerstoft et al., JASA 2008,
Harrison, JASA 2009,
Traer et al., JASA 2009,
Siderius et al., JASA 2010

Passive fathometer (drifting array)

Ambient noise 50-4000 Hz



Boomer



Adaptive processing gives better resolution of reflections

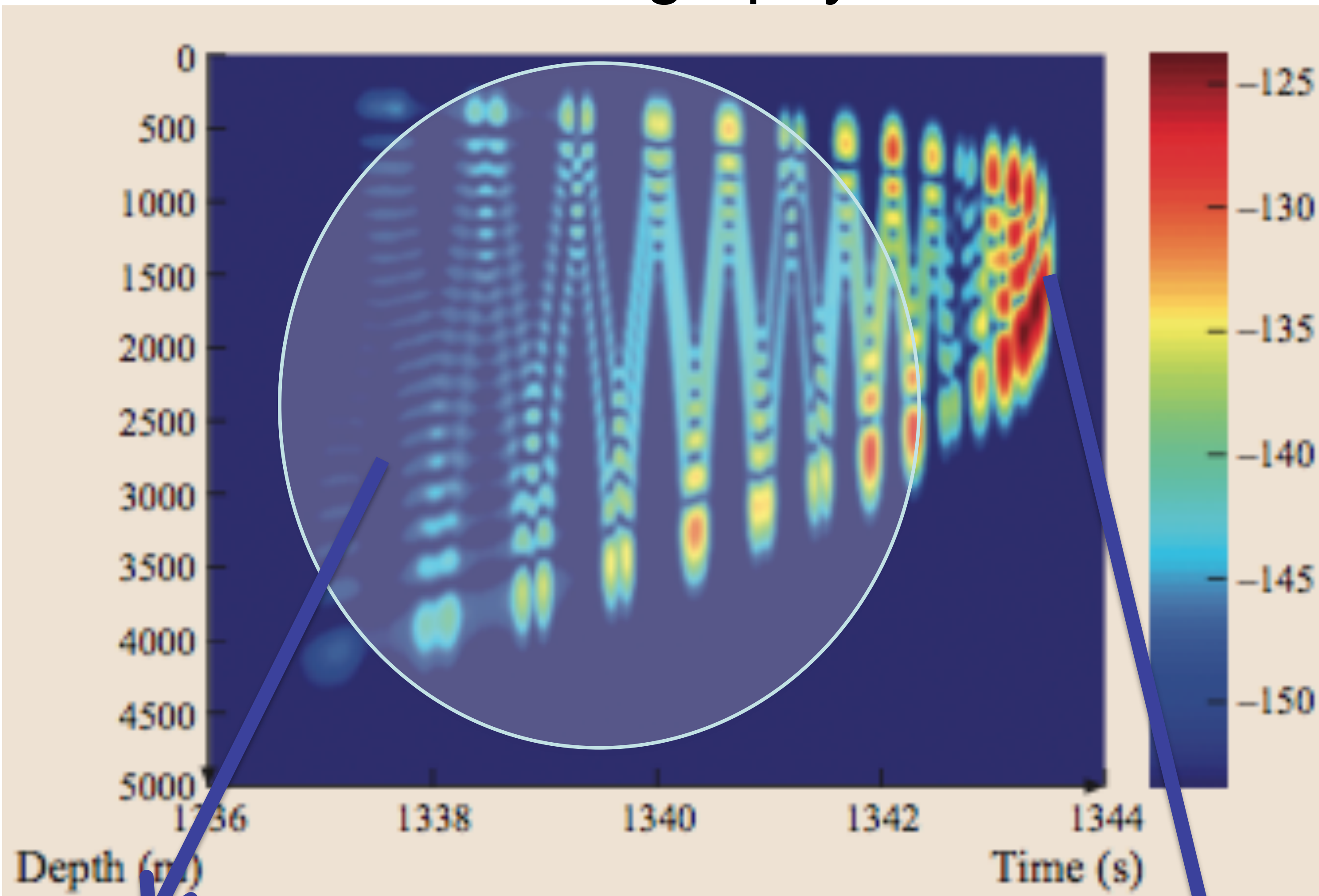
Siderius, Gerstoft et al., 2009   NURC data

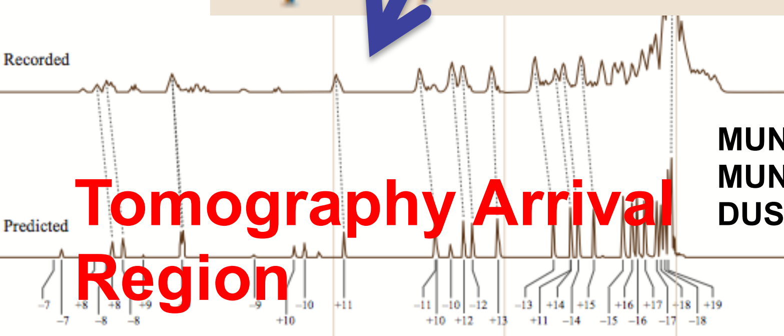
A Conclusion: Noise Correlation Processing Appears Promising for Geophysical Inversion

BUT NOT YET

for “Ocean Tomography” [why?->

Tomography





MUNK,WUNCH, ’82
MUNK,WORCESTER,WUNCH,’95
DUSHAW, ET AL ’09,…

Tomography Arrival Region

Last Arrival Cres

Conclusions: Noise Correlation Processing has become a tool for Geophysical Inversion

BUT NOT YET

for “Ocean Tomography”

BASIC ISSUE TO STILL OVERCOME:

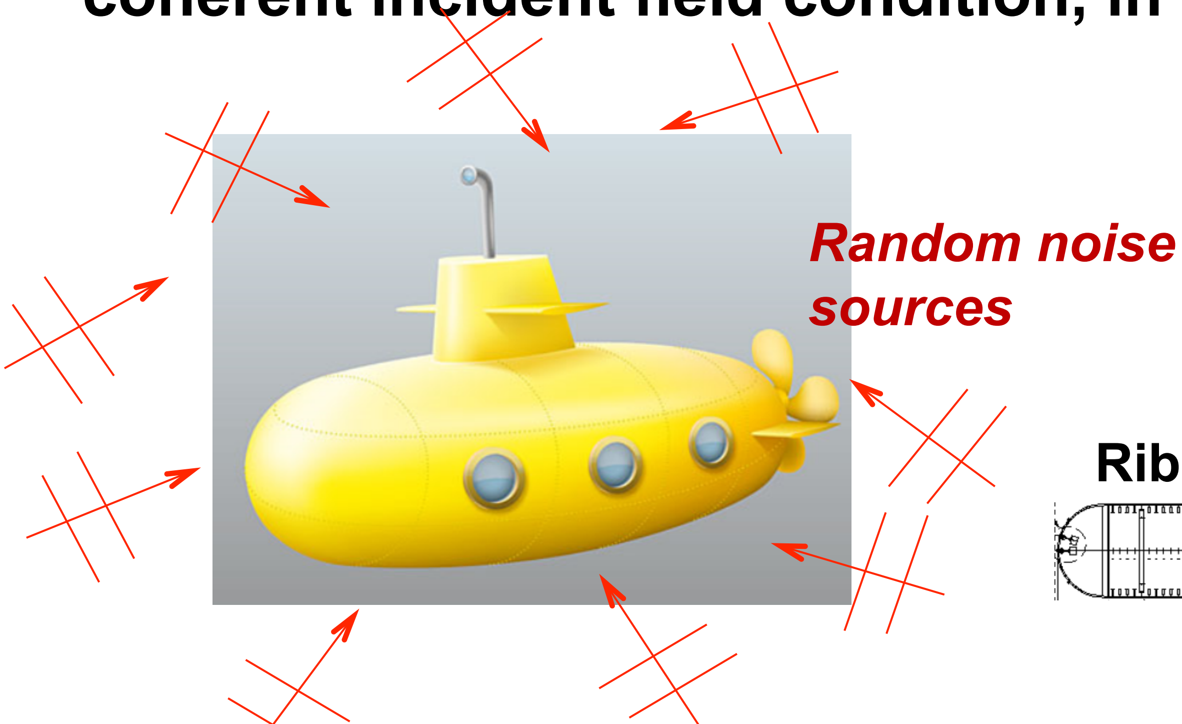
Extracting time of arrival structure (or equivalent) in short enough time interval to within time scale of ocean phenomenon under study

Underwater Acoustics Motivated Problem: Object Scattering

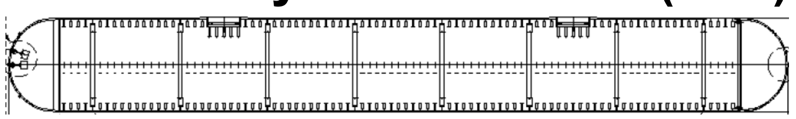
- Examples: Submarines, Mines…
- Issue: Determining Sattering Properties of Object is Difficult
  - Experimentally {Huge Effort/Facility}
  - Computationally {e.g., gazillion degrees of freedom(DOF) finite element calculation}
- BUT: Scatterer is ultimately observed by system with limited DOF
- **SOLUTION: NOISE!!!**

OBJECTIVES

- **Measure the structural Green's function of an elastic object (structural impedance matrix) excited by an external random noise field, by using measurements of surface velocity & pressure.**
- **With this information predict the scattered field for any coherent incident field condition, in any medium.**



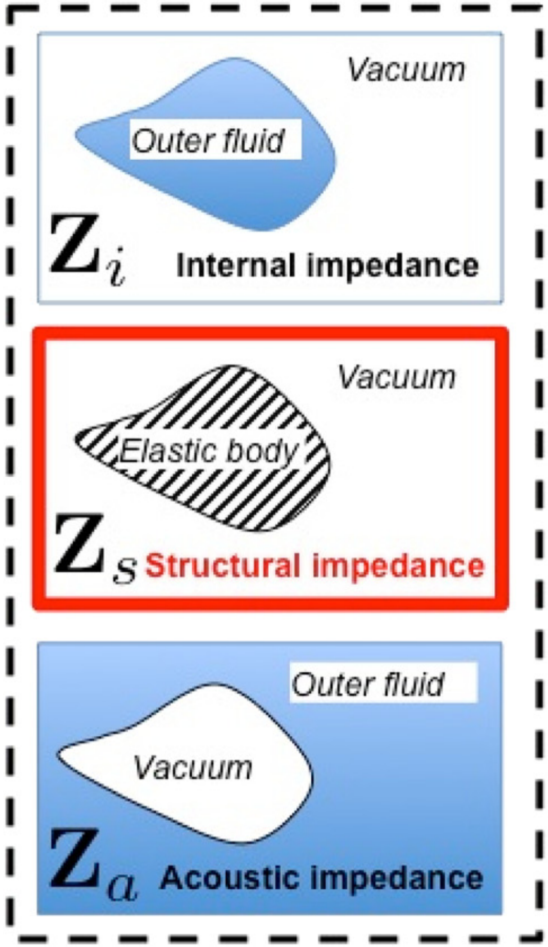
Ribbed cylindrical shell (C50)



Rakotonarivo, Kuperman, Williams (2013), ***Prediction of a body's structural impedance and scattering properties using correlation of random noise*** **, JASA**

INTRODUCTION

We need three *surface* impedances to characterize the scattering from an elastic body given the incident pressure field:



$$\mathbf{p} = \mathbf{p_i} + \mathbf{p_s} \text{ and } \mathbf{v} = \mathbf{v_i} + \mathbf{v_s}$$

$\mathbf{p_i} = -\mathbf{Z_i v_i}$ **Incident Fields**

(i.e. Interior Neumann Green fcn.)

$\mathbf{p} = -\mathbf{Z_s v}$ **Total Fields**

$\mathbf{Z_s}$ **is the Structural Impedance**

$\mathbf{p_s} = \mathbf{Z_a v_s}$ **Scattered Fields**

(i.e. Exterior Neumann Green fcn.)

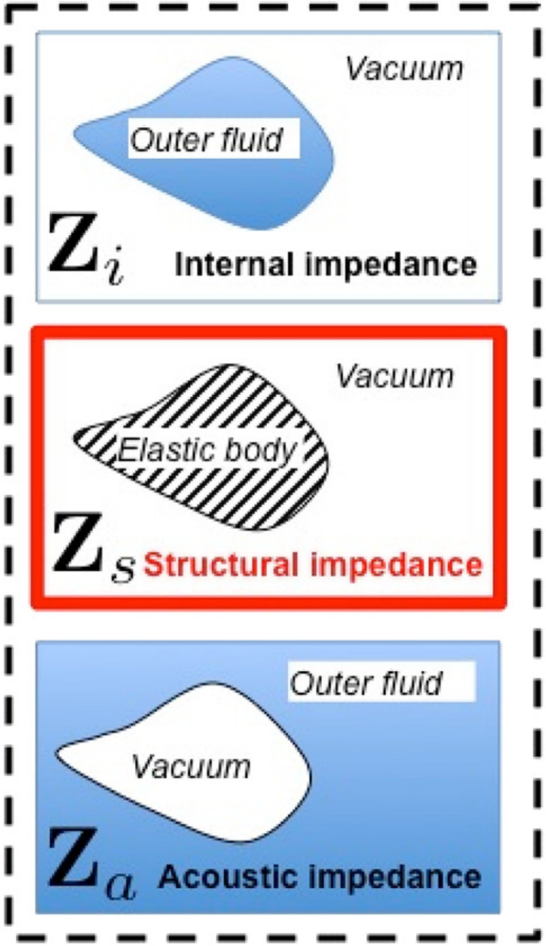
Borgiotti (1990); Gaumond et Yoder (1995); Lucifredi and Schmidt (2004); Bobrovnitskii (2006)

INTRODUCTION

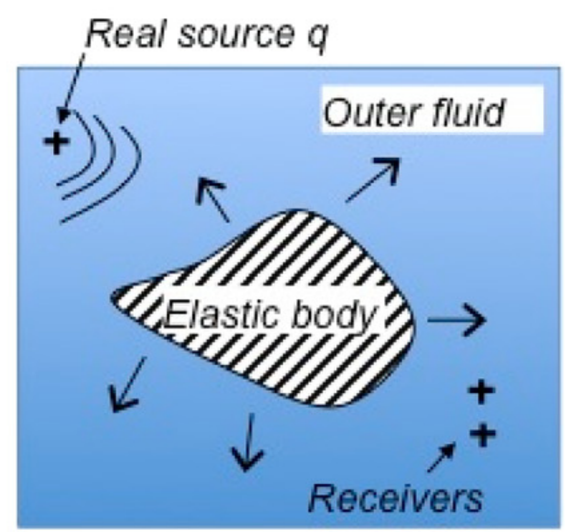
Simple manipulation of the impedances yields[1],

where $\mathbf{p}_s$ is the scattered field on the surface:

$$\mathbf{p}_s = \underbrace{\left(\frac{1}{\mathbf{Z}_a} + \frac{1}{\mathbf{Z}_s}\right)^{-1} \left(\frac{1}{\mathbf{Z}_i} - \frac{1}{\mathbf{Z}_s}\right)}_{Q=\text{Scattering Matrix}} \mathbf{p}_i$$



Z_s contains the physics of the elastic body *when placed in a vacuum*



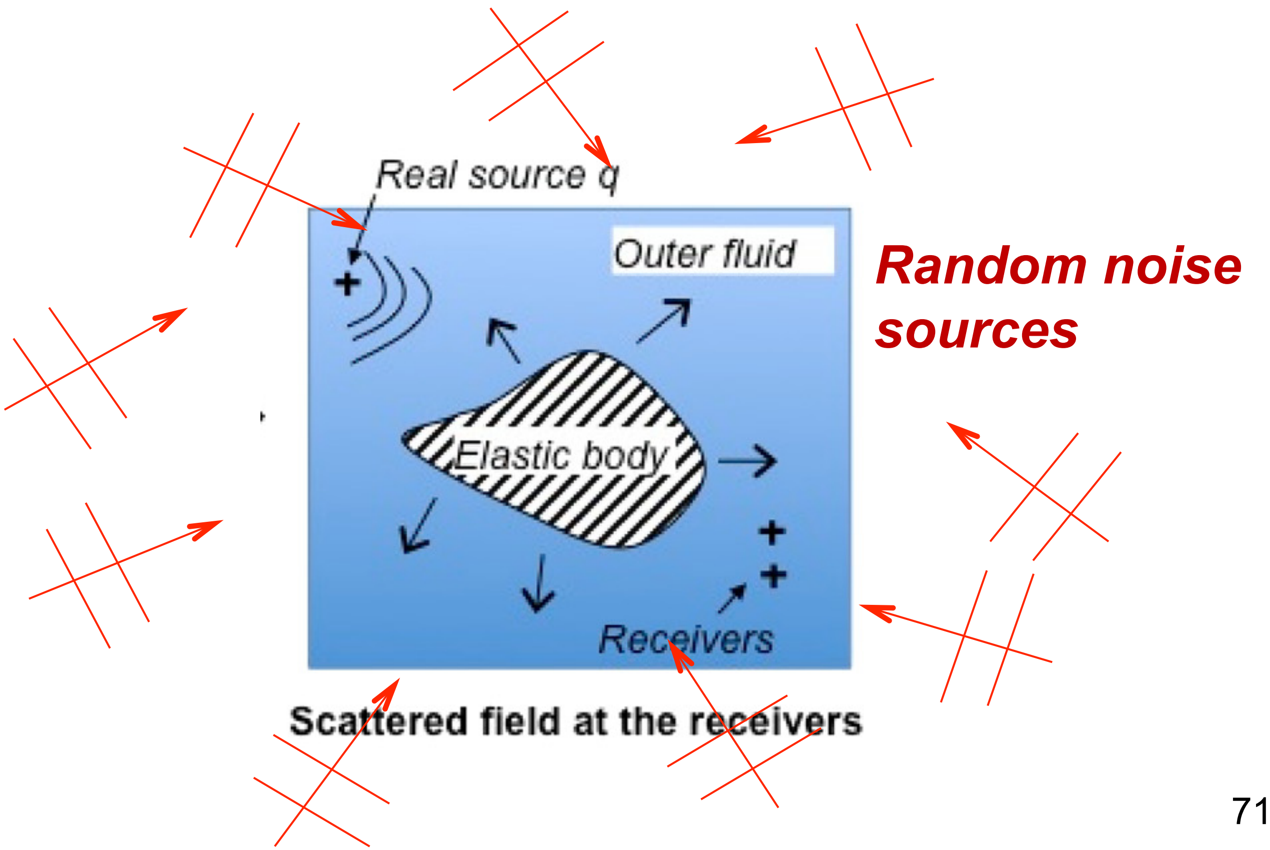
Scattered field at the receivers

[1]Bobrovntiskii (2006), *A new impedance-based approach to analysis and control of sound scattering (JSV)*

Borgiotti (1990); Gaumond et Yoder (1995); Lucifredi and Schmidt (2004); Bobrovnitskii (2006)

FIRST OBJECTIVE

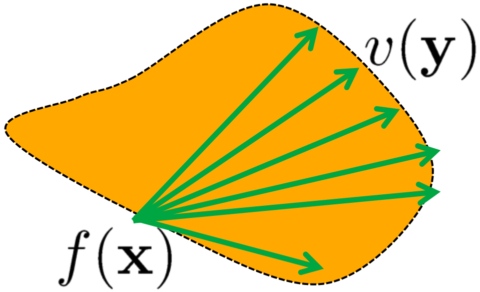
- **Measure Z_S: the elastic object's structural impedance matrix by placing it in a random noise field and measuring surface normal velocity and pressure.**



WHAT is the STRUCTURAL IMPEDANCE?

Structural admittance Green's function definition:

Discretize & Invert *G*: $\mathbf{f} = \mathbf{Z_s v}$



Structural impedance matrix

$$\mathbf{p} = -\mathbf{Z_s}\,\mathbf{v}$$

$$\begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_N \end{pmatrix} = -\underbrace{\begin{pmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{pmatrix}}_{\mathbf{Z_s}} \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{pmatrix}$$

Pressure field <u>at the object surface</u>

Normal velocity <u>at the object surface</u>

Borgiotti (1990); Gaumond et Yoder (1995); Lucifredi and Schmidt (2004); Bobrovnitskii (2006)

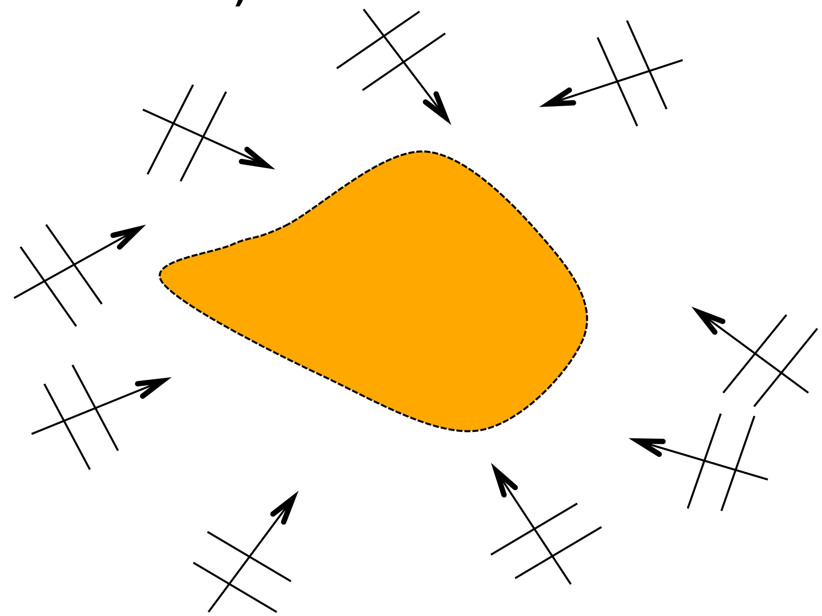
MEASUREMENT of the STRUCTURAL IMPEDANCE

Cross-Correlation method to predict Z_s from the noise

$$\mathbf{p} = -\mathbf{Z_s}\mathbf{v}$$

Multiply both sides by

L realizations (random noise sources)



Outer Products, cross-correlations of all sensor pairs, are averaged over *L* realizations

$$\langle \mathbf{p}\mathbf{p}^H \rangle = -\mathbf{Z_s} \langle \mathbf{v}\mathbf{p}^H \rangle$$

Pressure field <u>at *N* surface nodes</u>

Normal velocity <u>at *N* surface nodes</u>

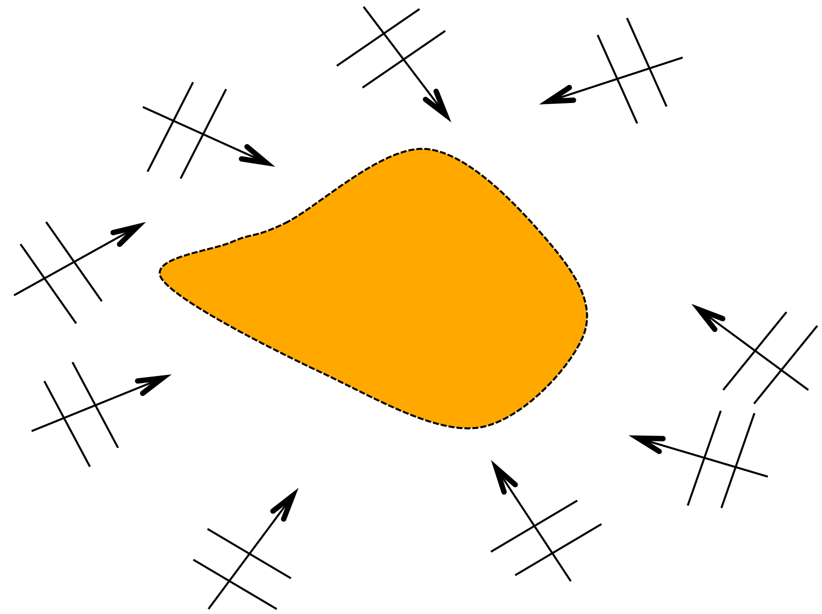
Rakotonarivo, Williams, Kuperman (2013)

MEASUREMENT of the STRUCTURAL IMPEDANCE

After ensemble averaging:

$$\langle \mathbf{p}\mathbf{p}^H \rangle = -\mathbf{Z}_\mathbf{s} \langle \mathbf{v}\mathbf{p}^H \rangle$$

All are N x N matrices



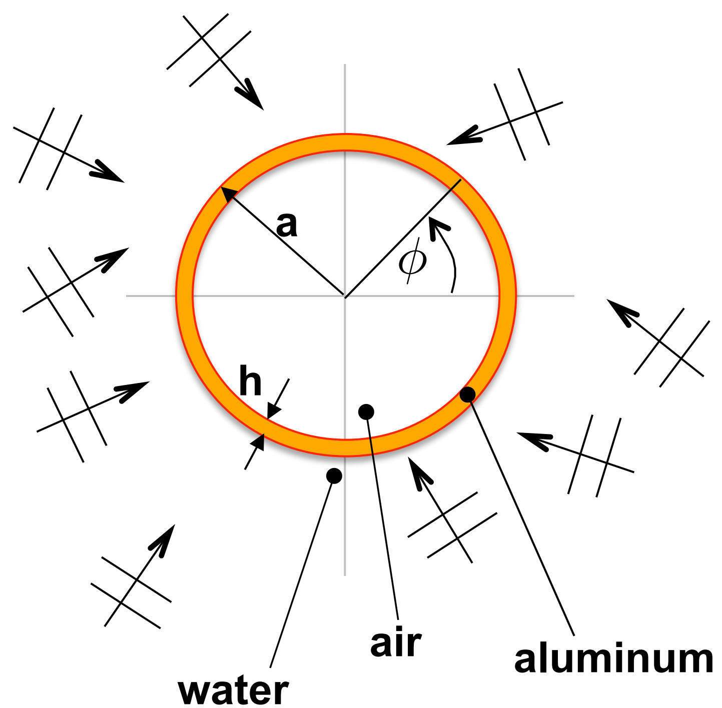
If sufficient number of spatially random realizations, we can invert : $\langle \mathbf{v}\mathbf{p}^H \rangle$

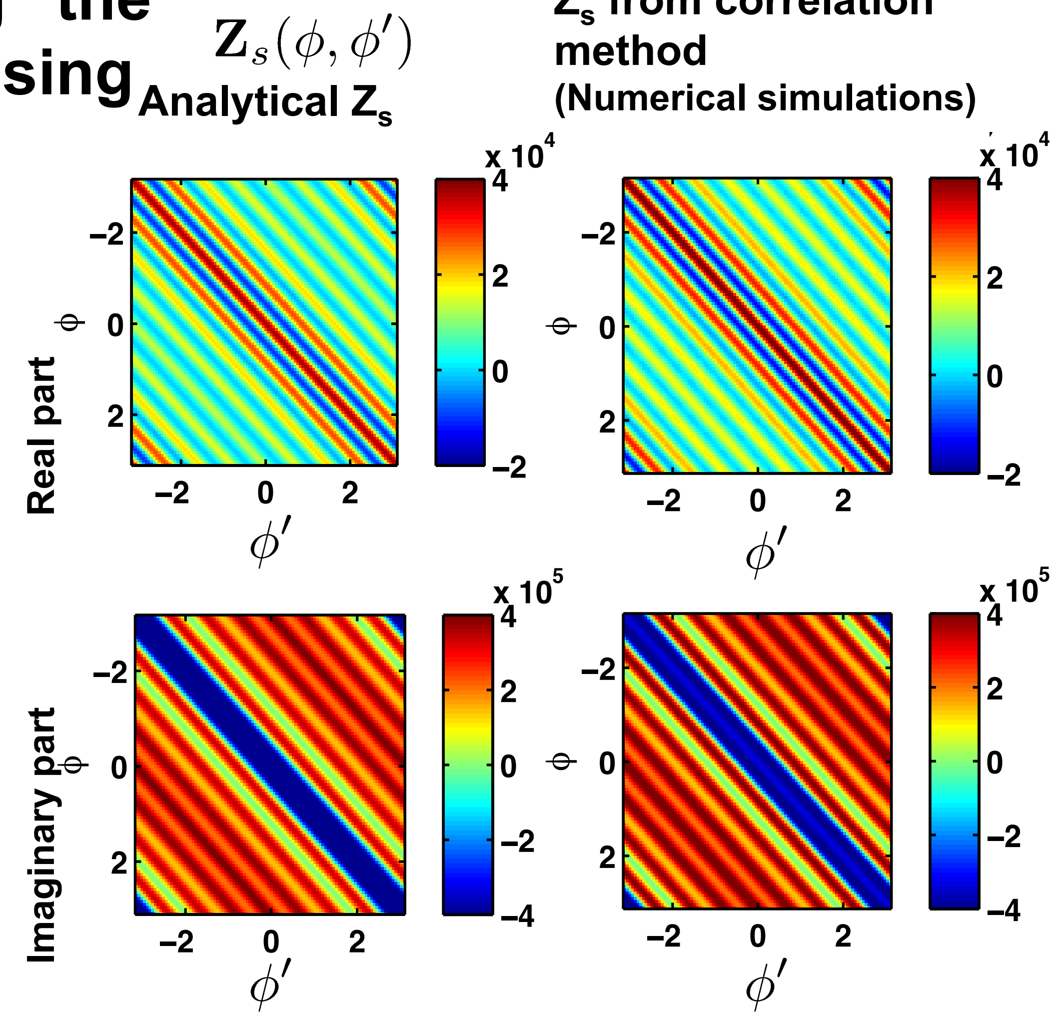
$$\mathbf{Z}_\mathbf{s} = -\langle \mathbf{p}\mathbf{p}^H \rangle \langle \mathbf{v}\mathbf{p}^H \rangle^{-1}$$

Rakotonarivo, Williams, Kuperman (2013)

MEASURED Z_s COMPARED with EXACT RESULT (in vacuo)

- **Success at "measuring" the structural Impedance using random noise**



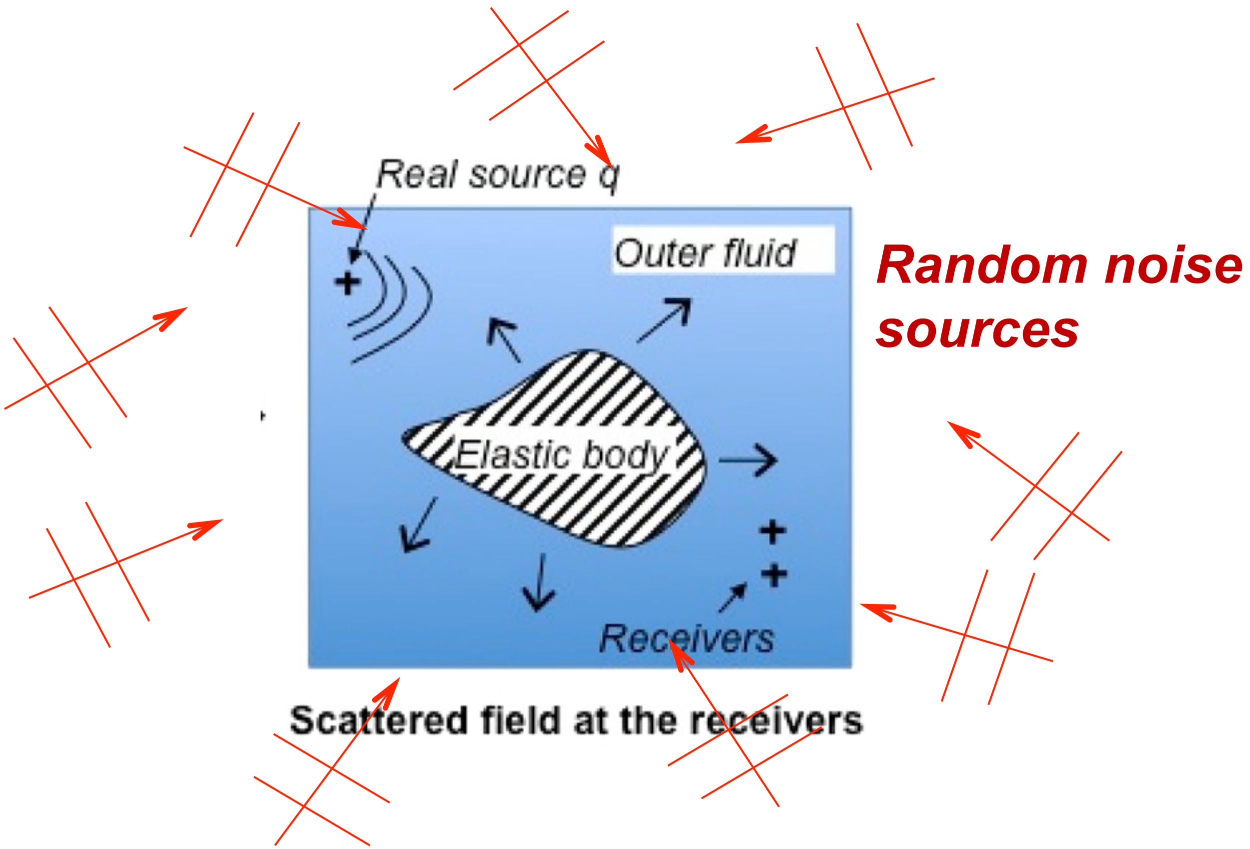


Simulation at 2 kHz, h=1 cm, a= 22.5 cm, damping 0.05

Rakotonarivo, Kuperman, Williams (2013), *Prediction of a body's structural impedance and scattering properties using correlation of random noise* , JASA

SECOND OBJECTIVE

- **Given Z_s predict the scattered field for any coherent incident field condition, in any medium.**



Lucifredi and Schmidt (2006), *Subcritical scattering from buried elastic shells,* JASA

SCATTERED FIELD CALCULATION from MEASUREMENT of $\mathbf{Z}_s$

Again we had:

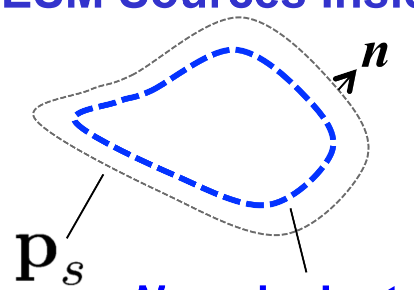
$$\mathbf{p}_s = \underbrace{\left(\frac{1}{\mathbf{Z}_a} + \frac{1}{\mathbf{Z}_s}\right)^{-1} \left(\frac{1}{\mathbf{Z}_i} - \frac{1}{\mathbf{Z}_s}\right)}_{Q=\text{Scattering Matrix}} \mathbf{p}_i$$

$$\mathbf{p}_s = \mathbf{Q}\,\mathbf{p}_i$$

How do we compute $\mathbf{Z}_a$ and $\mathbf{Z}_i$ for a general surface?

CALCULATION of RADIATION IMPEDANCE Z_a for GENERAL SURFACE

- **Use Equivalent Source Method (ESM) to obtain $\mathbf{Z}_a$**



$\mathbf{s}$ = Source strength

$\mathbf{G}$ = Free-space Green's function *for the medium of interest*

$$\mathbf{p}_s = \mathbf{G}\,\mathbf{s}$$

$$\mathbf{v}_s = \mathbf{G}_v\mathbf{s}$$

$$\mathbf{G}_v = \frac{1}{i\rho c k}\frac{\partial \mathbf{G}}{\partial n}$$

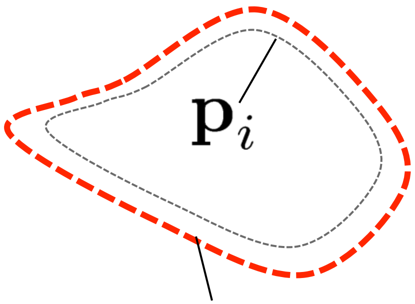
Eliminate $\mathbf{s}$:

$$\mathbf{p}_s = \mathbf{G}\mathbf{G}_v^{-1}\mathbf{v}_s = \mathbf{Z}_a\mathbf{v}_s$$

$$\mathbf{Z}_a = \mathbf{G}\mathbf{G}_v^{-1}$$

CALCULATION of INTERNAL FIELD IMPEDANCE Z_i for GENERAL SURFACE

- Again use ESM to obtain $\mathbf{Z}_i$



$$\mathbf{p}_i = \mathbf{G}_i \mathbf{s}_i$$

$\mathbf{s}_i$ = Source strength

$\mathbf{G}_i$ = Free-space Green's function for the medium of interest

$$\mathbf{v}_i = -\mathbf{G}_{iv} \mathbf{s}_i$$

$$\mathbf{G}_{iv} = \frac{1}{i\omega\rho} \frac{\partial \mathbf{G}_i}{\partial n}$$

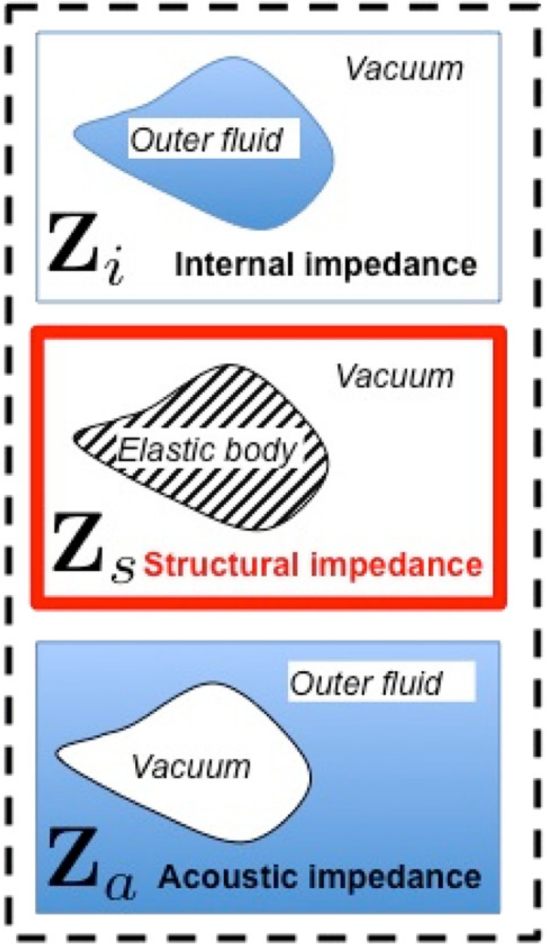
Eliminate $\mathbf{s}_i$:

$$\mathbf{p}_i = -\mathbf{G}_i \mathbf{G}_{iv}^{-1} \mathbf{v}_i = -\mathbf{Z}_i \mathbf{v}_i$$

$$\mathbf{Z}_i = \mathbf{G}_i \mathbf{G}_{iv}^{-1}$$

SUMMARY

- Given Incident field we have Scattered field at surfac



$$\mathbf{p}_s = \mathbf{Q}(\mathbf{Z}_i, \mathbf{Z}_s, \mathbf{Z}_a)\,\mathbf{p}_i$$

$$\mathbf{Z}_i = \mathbf{G}_i \mathbf{G}_{iv}^{-1}$$

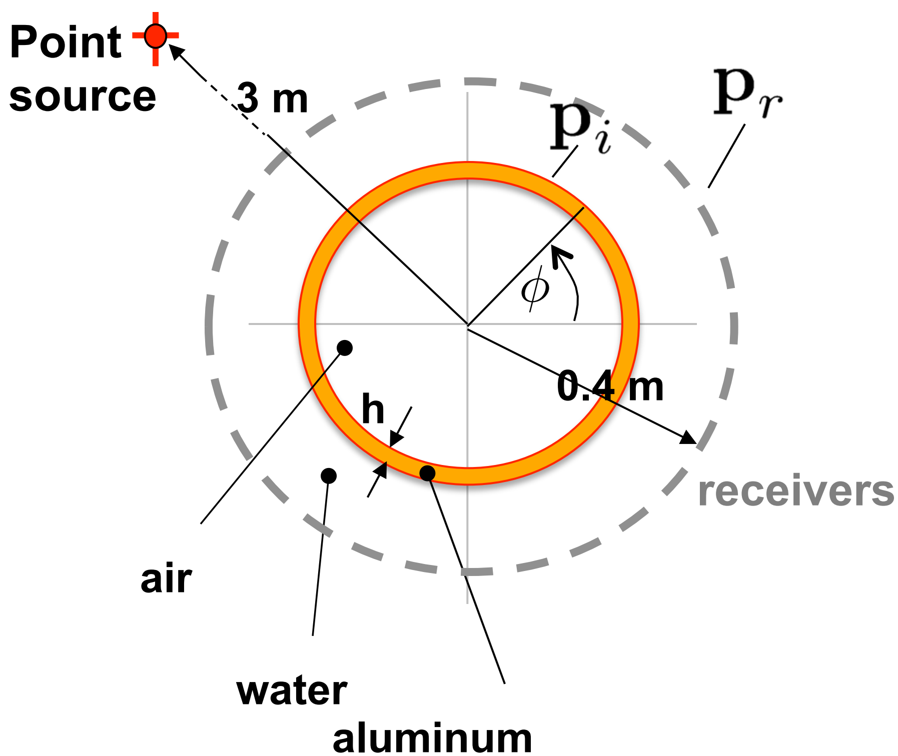
$$\mathbf{Z_s} = -\langle \mathbf{p}\mathbf{p}^H \rangle \langle \mathbf{v}\mathbf{p}^H \rangle^{-1}$$

$$\mathbf{Z}_a = \mathbf{G}\mathbf{G}_v^{-1}$$

SCATTERED FIELD NUMERICAL EXPERIMENT

$$\mathbf{p}_r = \mathbf{G}_r \mathbf{G}^{-1} \underbrace{\mathbf{Q}\, \mathbf{p}_i}_{\mathbf{p}_s}$$

- Back to cylindrical shell example, compute $\mathbf{p}_r$ for a point source at 3 m, 100 receivers $\mathbf{p}_r$ at r = 0.4 m

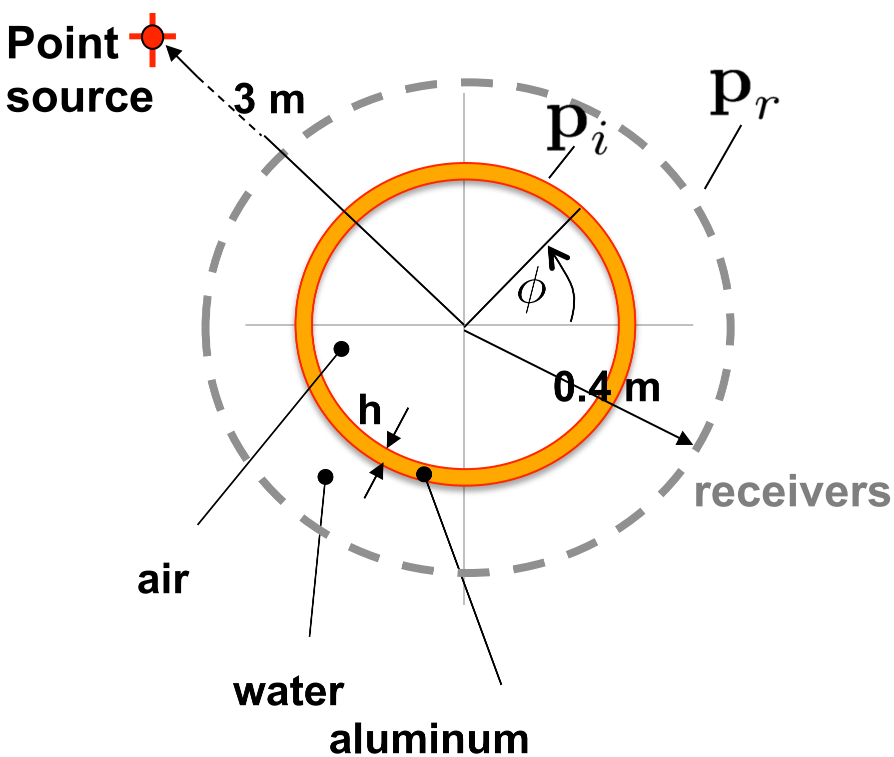


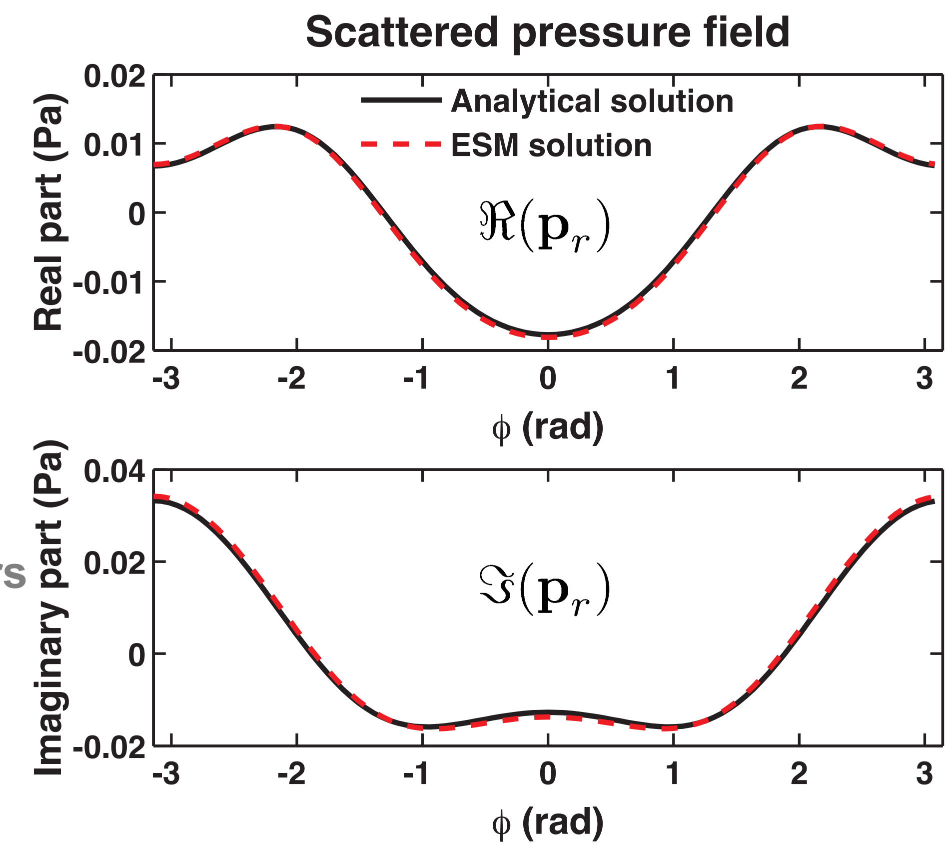
Simulation at 2 kHz, ka=1.87

N= 100 surface nodes/virtual sources

RESULTING SCATTERED FIELD (p_r) at RECEIVERS COMPARED with EXACT SOLUTION

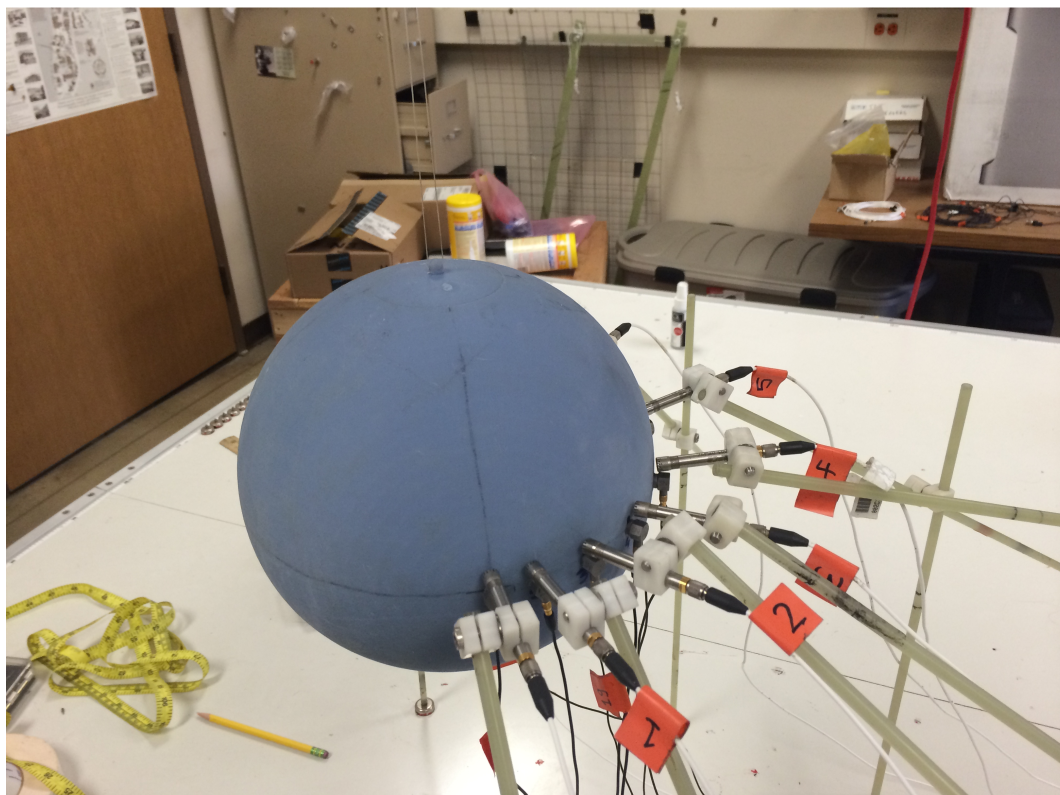
- **Success at predicting the scattered field using the structural Impedance measurement**





CONCLUSIONS AND PERSPECTIVES

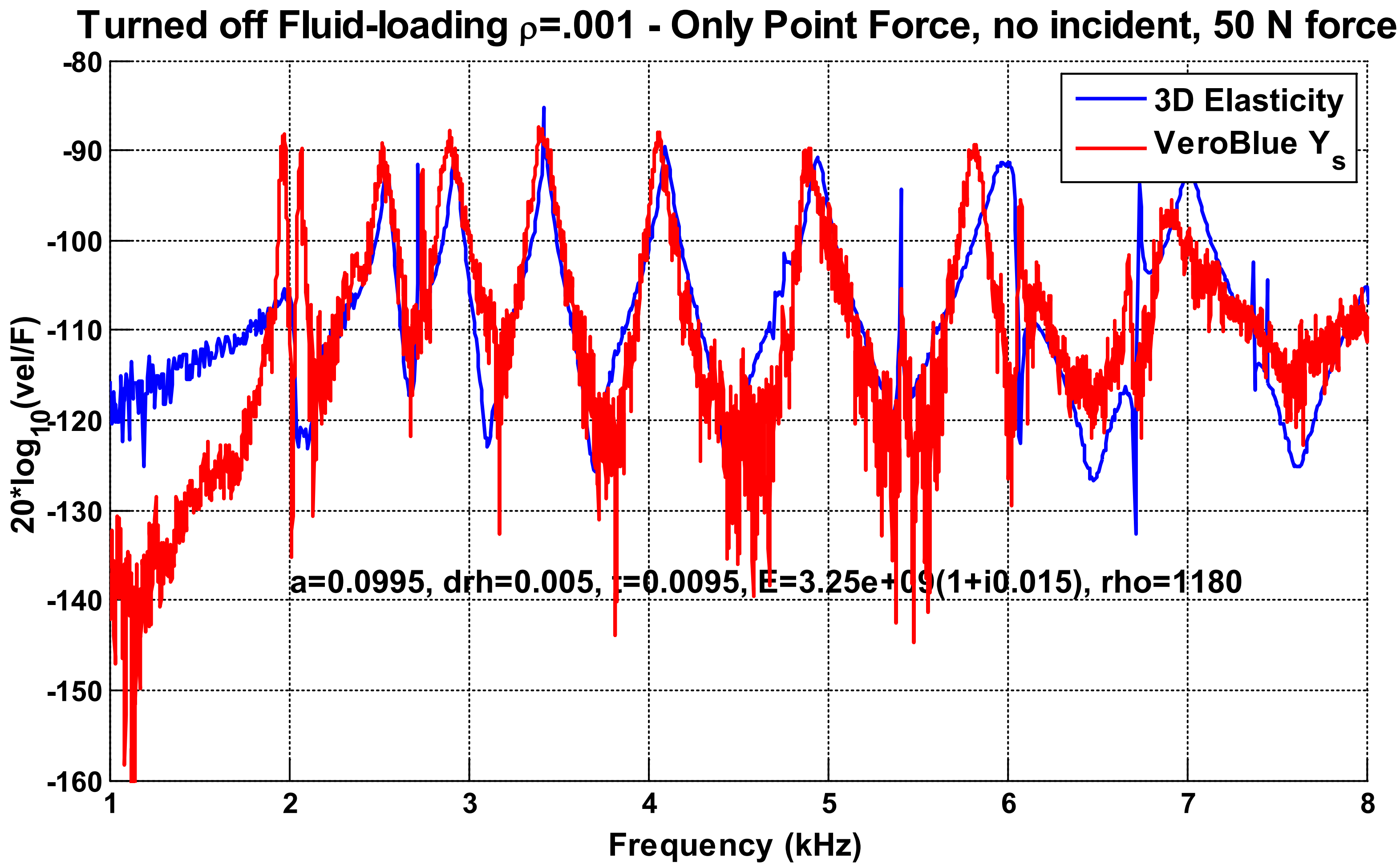
- Correlation method to predict the structural impedance using random noise sources, ESM to yield radiation impedance and internal impedance
- Scattered field prediction for a given incident field
- Coming Next: Experimental investigation – spherical shell



Preliminary Results

(Back to a point Force, not incident field j_n and j_n' set to zero in A_1 and A_2 terms, c=343 but \rho=.001 instead of 1.2).
Really good agreement. Note fluid loading is very small effect.

Driving point admittance: $Y_s = -\langle vv^H \rangle \langle vp^H \rangle^{-1}$



Thank You