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ONDES ET IMAGES

Green's function retrieval in multiple scattering media

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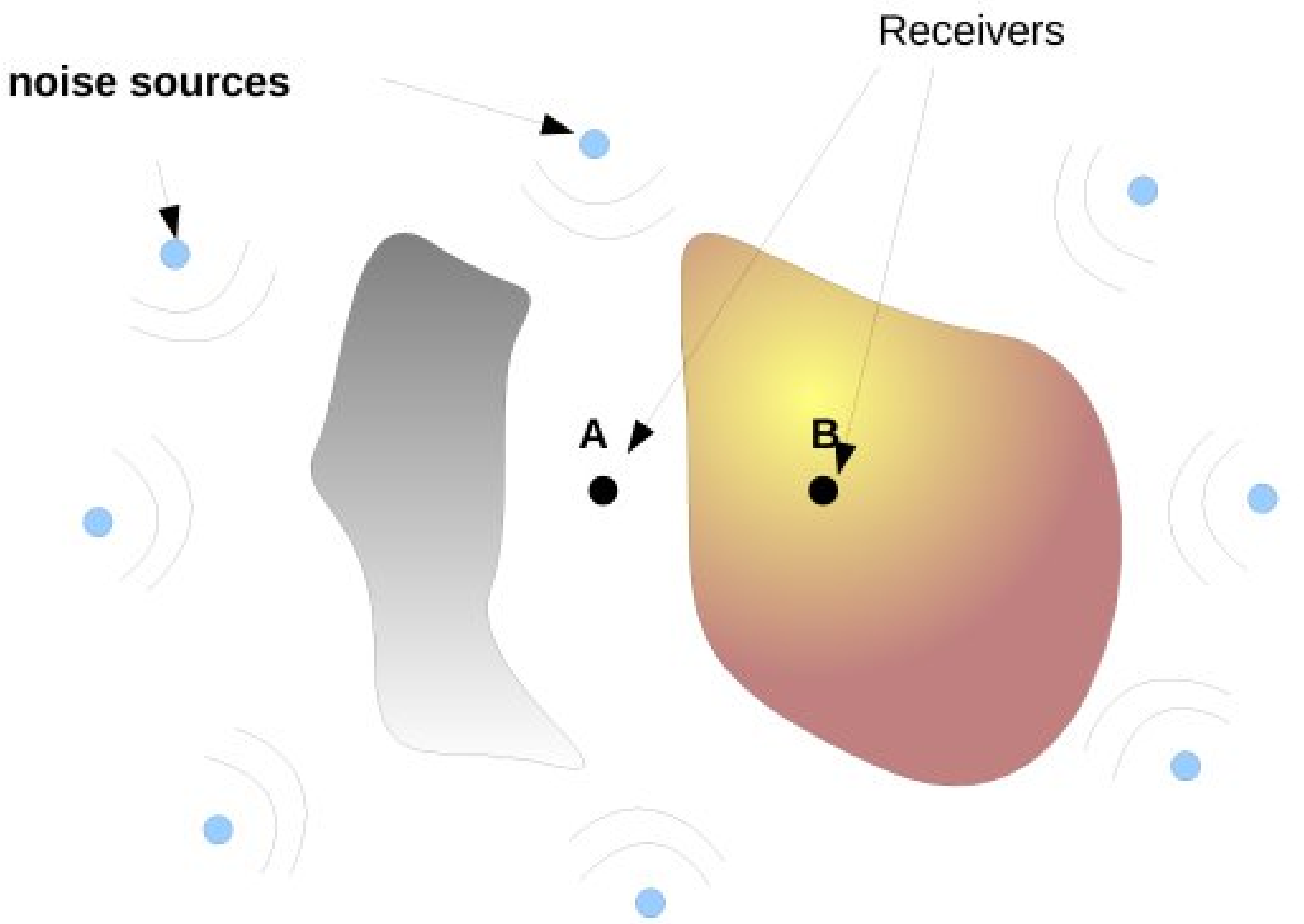
Part. Supported by ANR grant OPTRANS

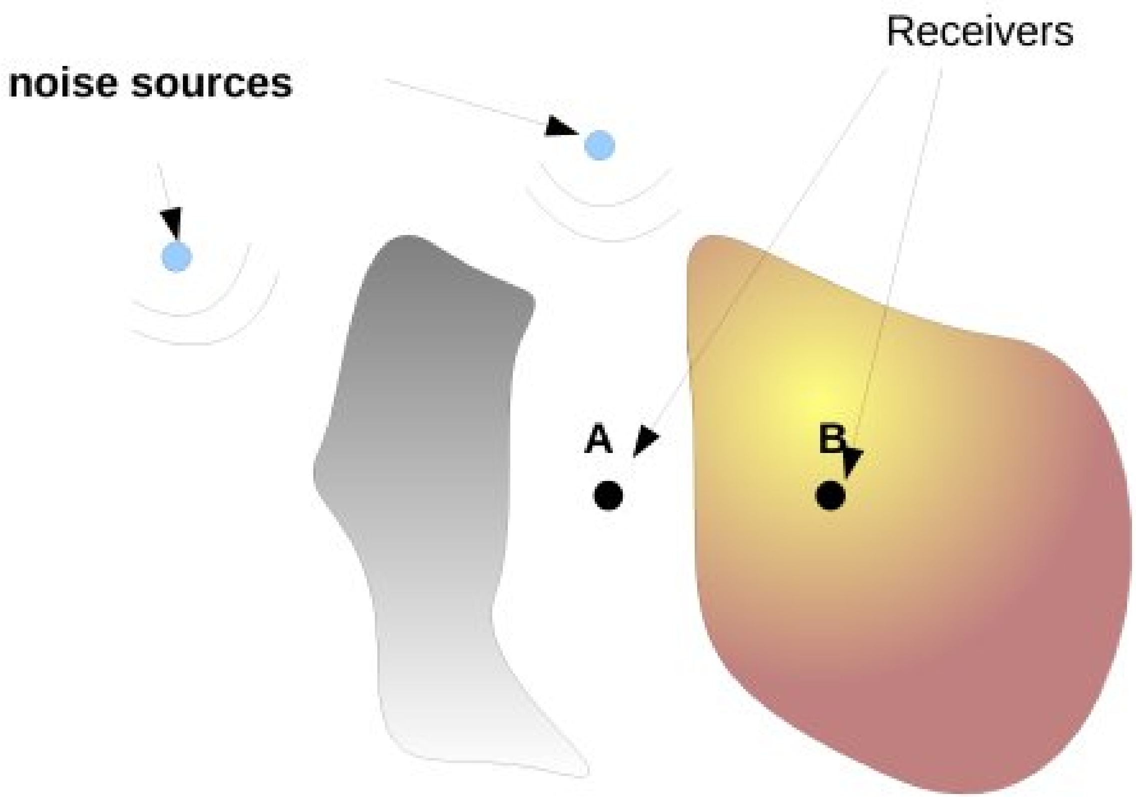
Noise correlation



Isotropic and uniforme noise sources

$$\Rightarrow \quad \frac{\partial C(A,B,t)}{\partial t} \propto G(A,B,-t) - G(A,B,t)$$

Noise correlation

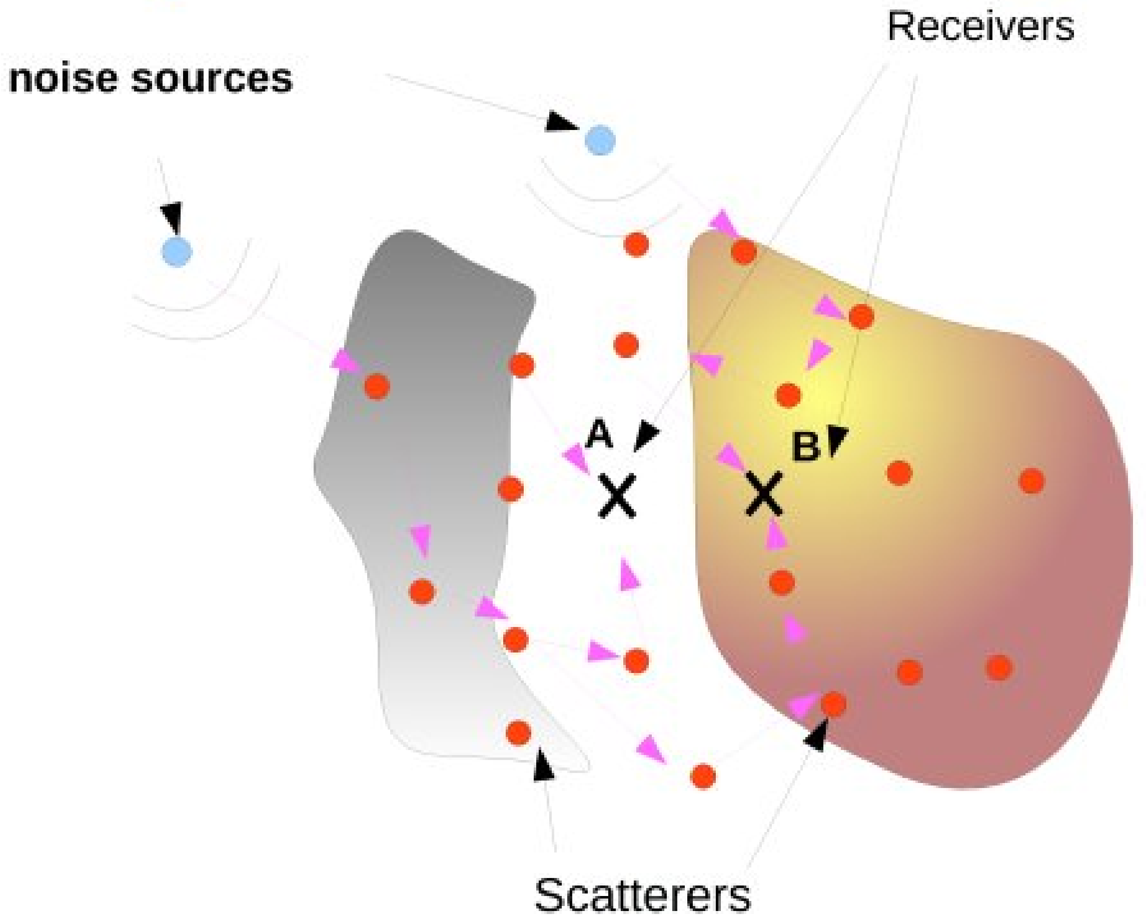


Anisotropic and non-uniforme noise sources

$$\frac{\partial C(A,B,t)}{\partial t} \neq G(A,B,-t) - G(A,B,t)$$

e.g. Roux et al. 2005

Multiple scattering media



Scatterers plays the rôle of secondary sources

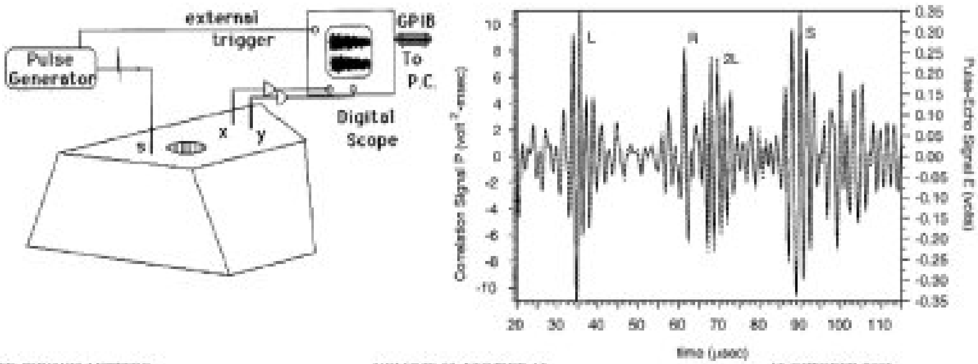
$$\frac{\partial C(A,B,t)}{\partial t} \sim G(A,B,-t) - G(A,B,t)$$

Experimental validation

VOLUME 87, NUMBER 13 PHYSICAL REVIEW LETTERS 24 SEPTEMBER 2001

Ultrasonics without a Source: Thermal Fluctuation Correlations at MHz Frequencies

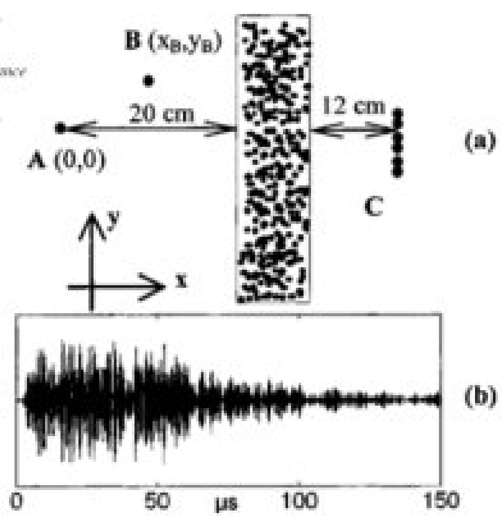
Richard L. Weaver and Oleg I. Lobkis
Theoretical & Applied Mechanics, University of Illinois, Urbana, Illinois 61801
(Received 5 April 2001; published 7 September 2001)



APPLIED PHYSICS LETTERS VOLUME 83, NUMBER 15 13 OCTOBER 2003

How to estimate the Green's function of a heterogeneous medium between two passive sensors? Application to acoustic waves

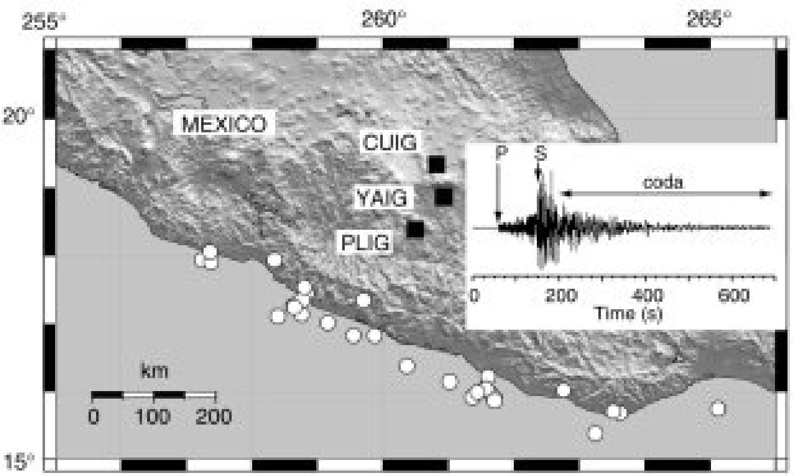
Arnaud Derode[a]
LOA, Université Paris 7, CNRS UMR 7587, ESPCI, Paris, France
Eric Larose and Michel Campillo
LGIT, Université Joseph Fourier, CNRS UMR 5559, Grenoble, France
Mathias Fink
LOA, Université Paris 7, CNRS UMR 7587, ESPCI, Paris, France



SCIENCE VOL 299 24 JANUARY 2003

Long-Range Correlations in the Diffuse Seismic Coda

Michel Campillo* and Anne Paul

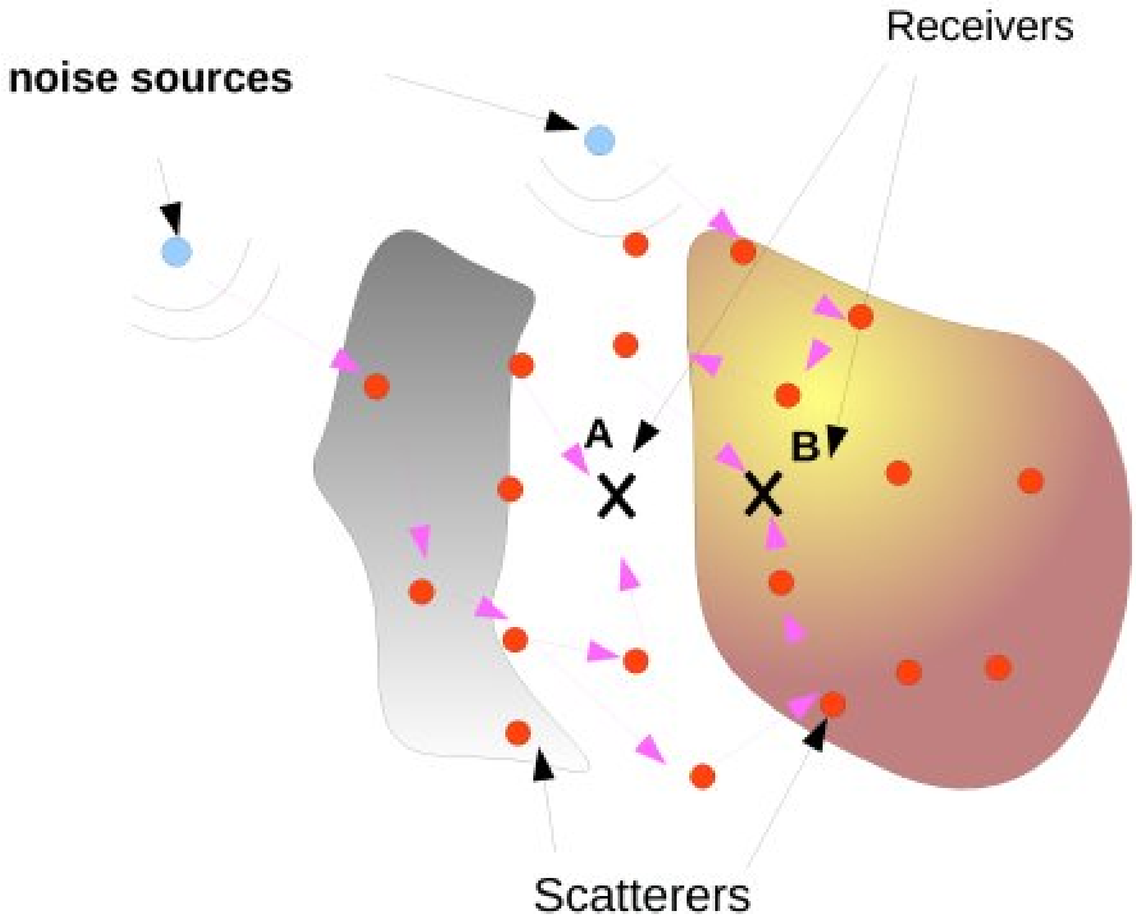


Scattering helps to estimate the Green's function

Analysis

- Integral equations of propagations [Weaver et al.-2004, Wapenaar-2004,...]
- Modal decomposition [Weaver-2001, ...]
- Asymptotic limit of diffusion [Tiggelen & Skipetrov – 2003]
- Averaging on source positions [Larose et al. - 2008,...]
- Diagrammatic approach on a single realization [Margerin & Sato - 2011], [Garnier & Papanicolaou - 2009]

But



$$\frac{\partial C(A,B,t)}{\partial t} \sim G(A,B,t) - G(A,B,-t)$$

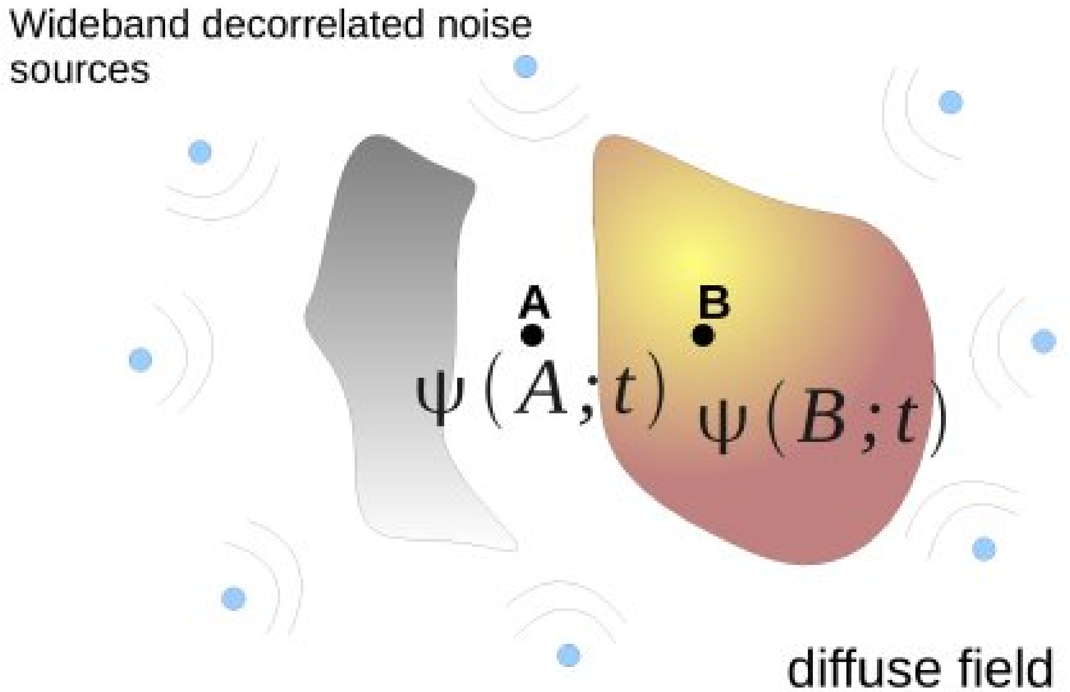
The correlation estimates the Green's function with, without the scatterers or its average?

Outlines

1) Relationship between correlation and time reversal
2) Noise Correlation function in multiple scattering media
    1) A single realization
    2) Mean Value over realizations
    3) Fluctuations around mean value

1 – Time Reversal and Correlation

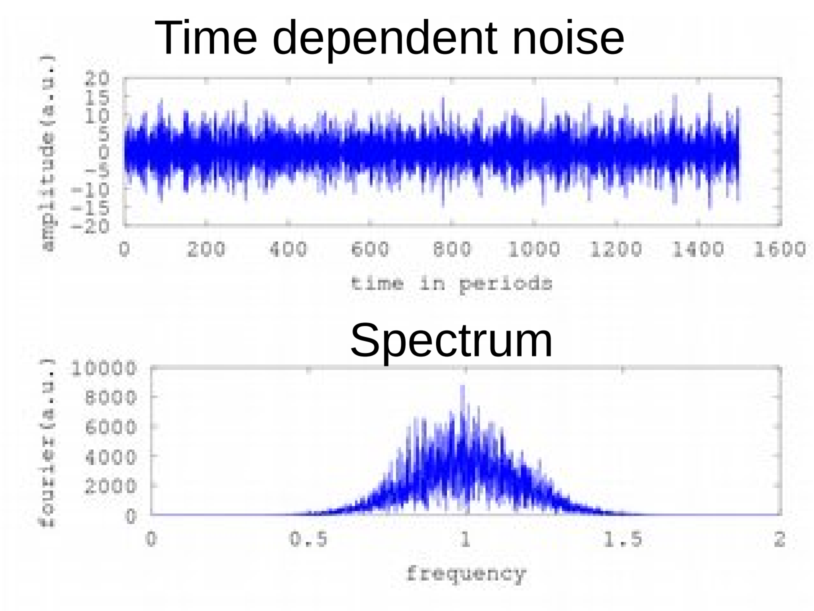
Diffuse noise sources

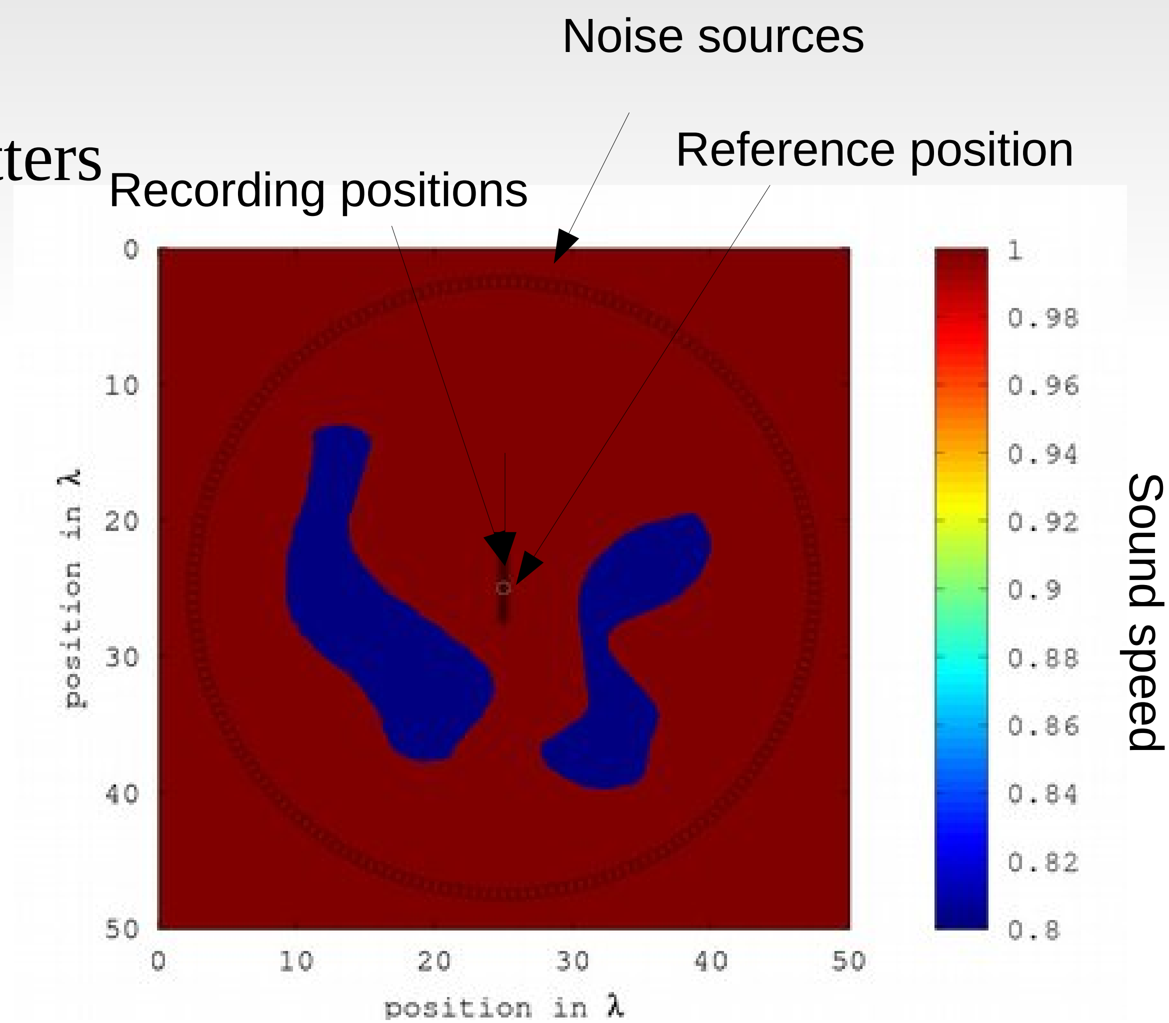


$$C(A,B;t) \equiv \int \psi(A;\tau)\psi(B;\tau-t)\,d\tau$$
$$= \psi(A;t) \otimes \psi(B;-t)$$

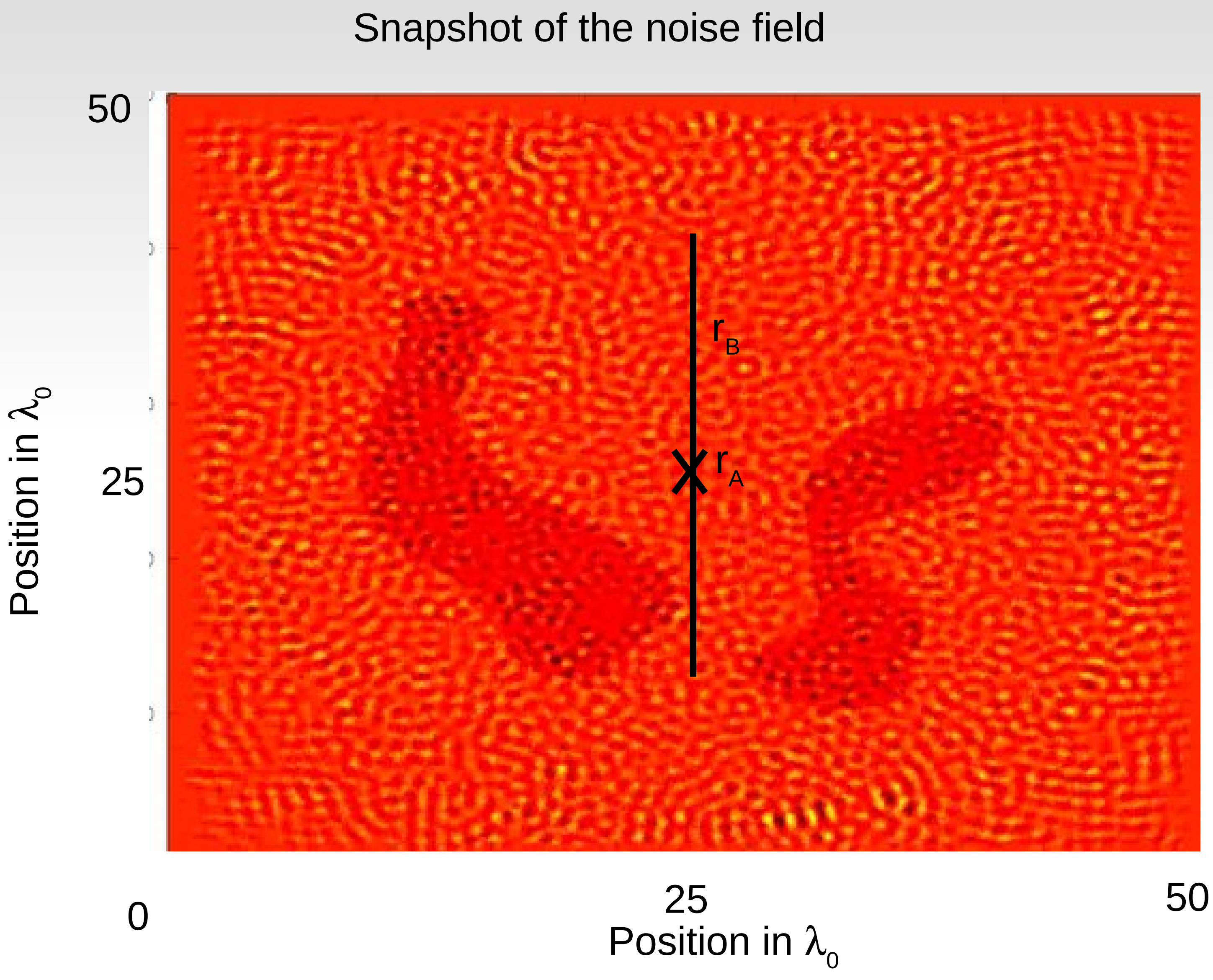
Numerical simulation

- FDTD acoustic (scalar) simulation
- Aborbing boudary conditions
- 50λ x 50λ square
- 200 decorrelated noise emitters
- Wideband source





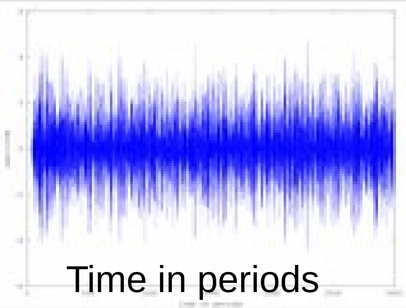
Random noise field



Noise correlation function

ψ(A,t)

Amplitude (a.u.)



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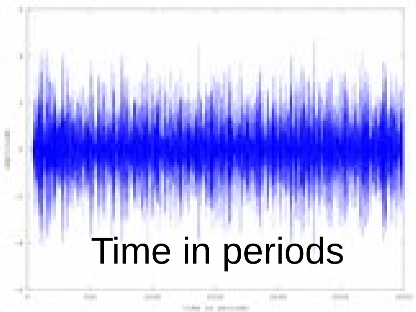
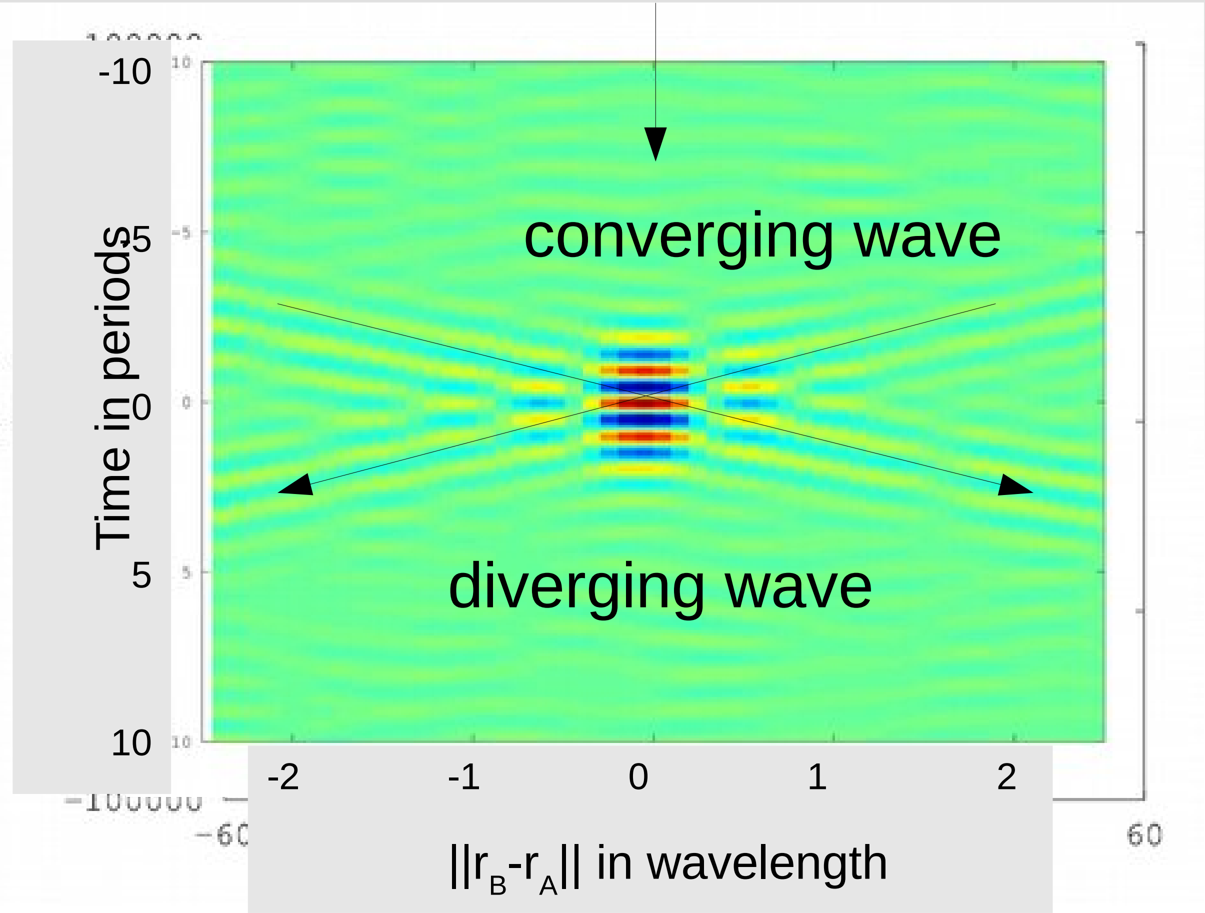


ψ(B,-t)

Noise correlation function NCF



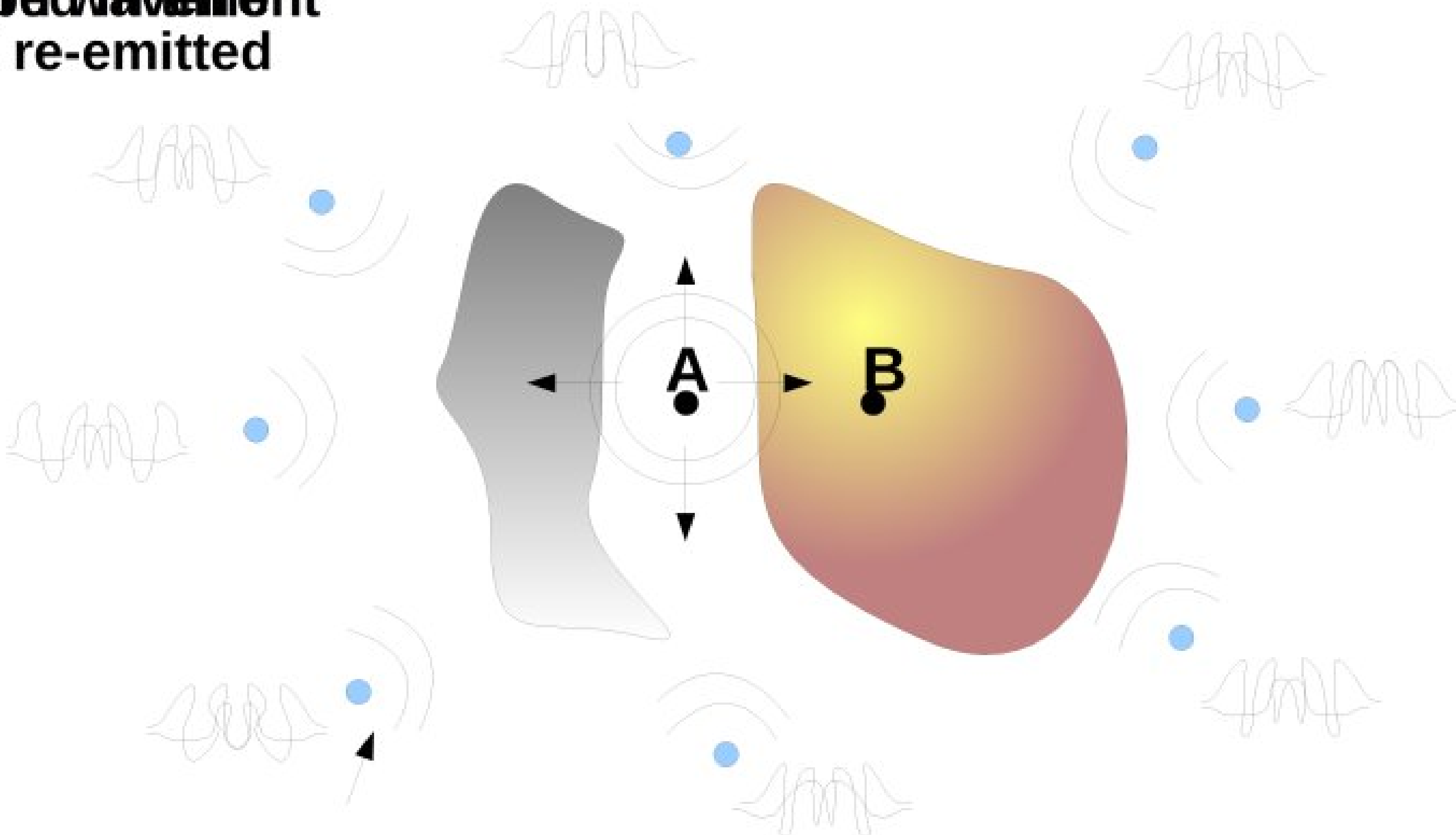
$$\frac{\partial C(A,B,t)}{\partial t} \propto G(A,B,-t) - G(A,B,t)$$

1 - Correlations and time reversal

Time Reversal

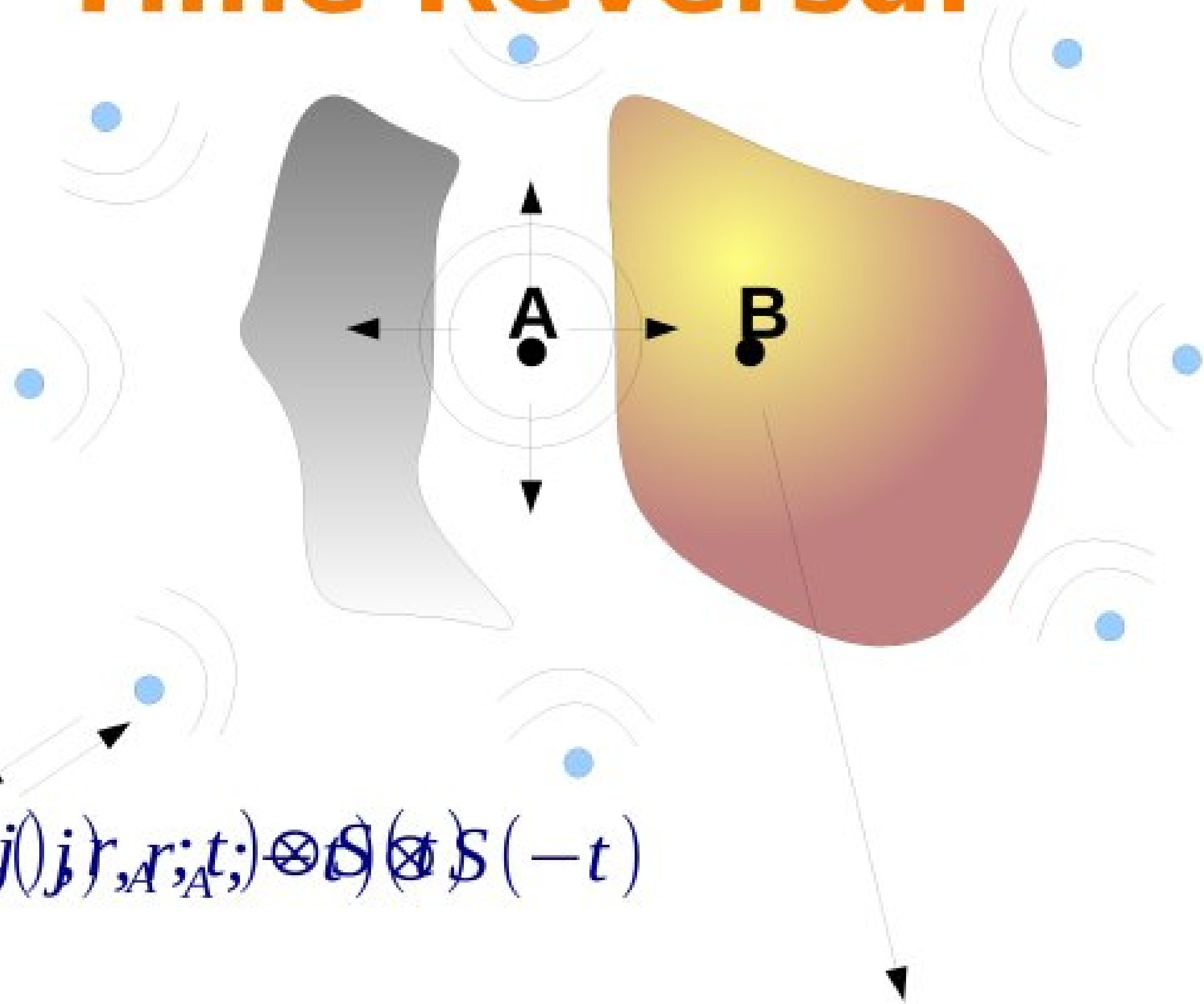
Recordings of the forward wavefront

All signals are flipped in time and re-emitted



Transceivers

Time Reversal



$$\psi(r_l;t) = G(r_S(i), r_A; t) \otimes S(-t)$$

Expression of the Time Reversed field

$$G(r_B, r_S(i); t) \otimes G(r_S(i), r_A; -t) \otimes S(-t)$$

Time reversal vs Correlations

$$\psi_{RT}(B;t)=\sum_i G(r_S(i),r_A;-t)\otimes G(r_B,r_S(i);t)\otimes S(-t)$$

$$C(A,B;t)=\Delta T\sum_i G(r_A,r_S(i);-t)\otimes G(r_B,r_S(i);t)\otimes S(t)$$

But

$S(t)$ is symmetric

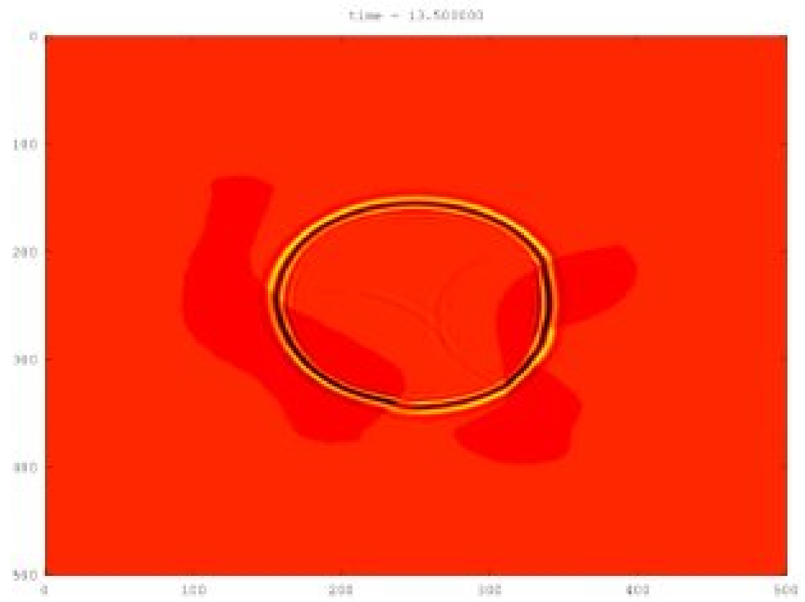
And the medium is reciprocal $\quad G(r_A,r_S(i);t)=G(r_S(i),r_A;t)$

$$\psi_{RT}(B;t)=\Delta T\, C(A,B,t)$$

Derode et al., JASA, APL 2003

→ Time Reversal equivalent to correlation

Green's function

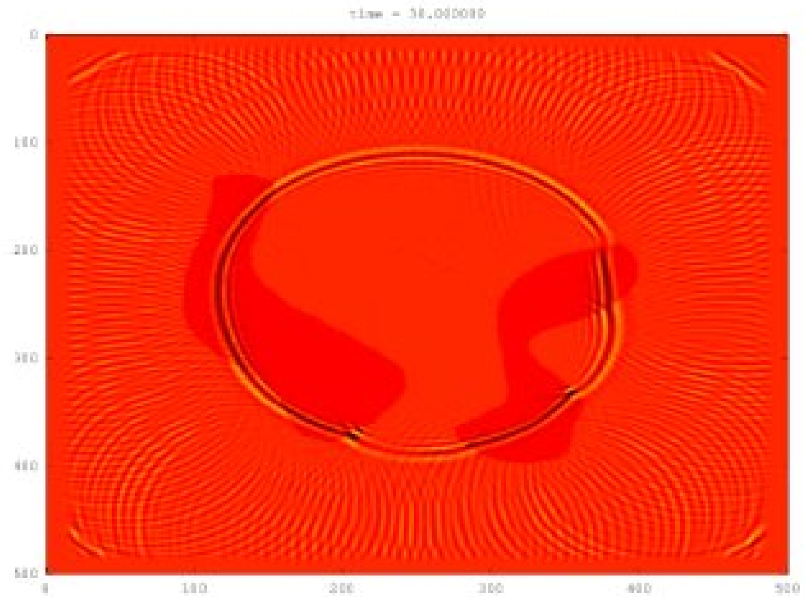


Forward step

Diverging wave

$$G(r_i, B, t) \otimes s(t)$$

Time-Reversed wave



Backward step

Converging wave followed by a diverging one

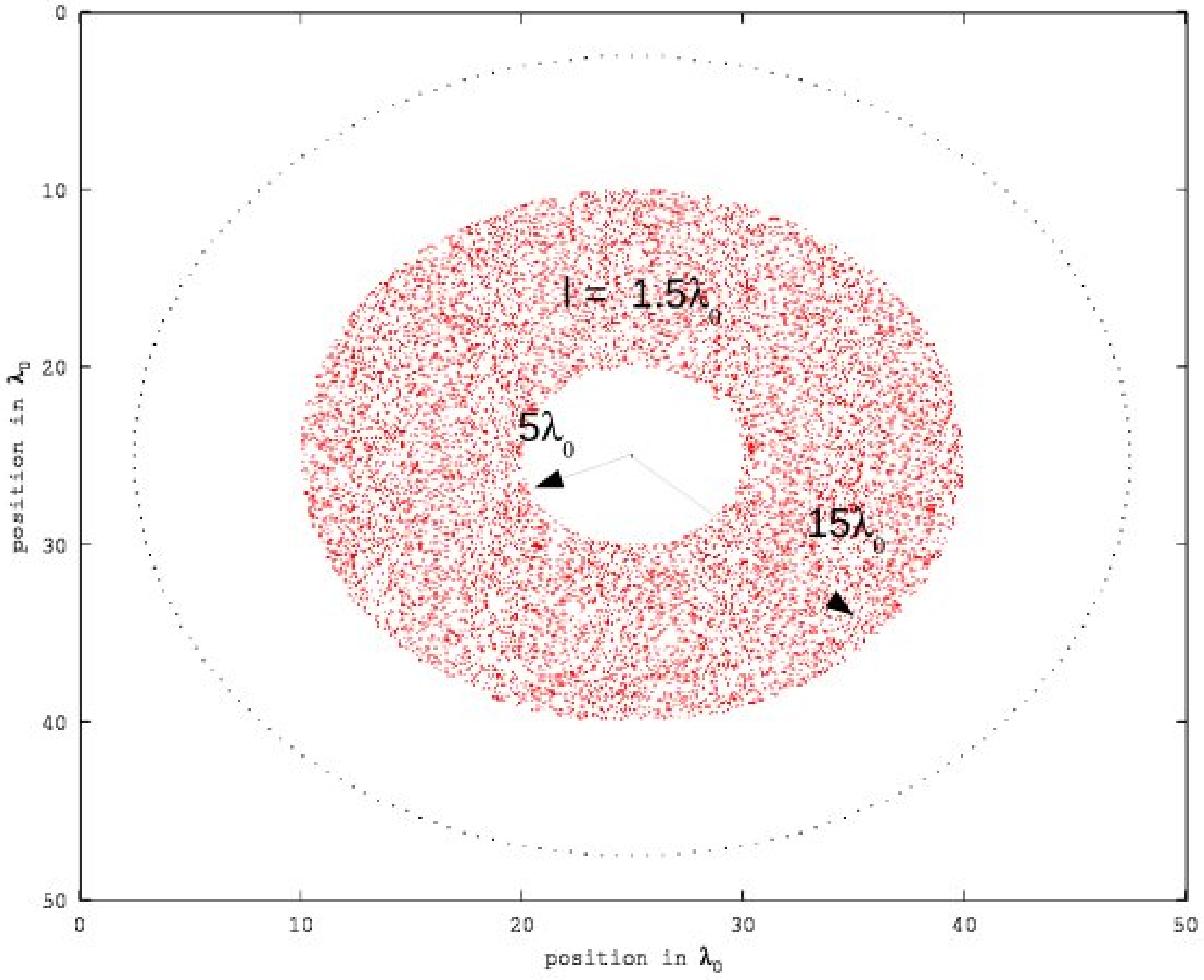
$$C(A,B,t) \propto [G(A,B,-t) - G(A,B,t)] \otimes s(t)$$

~ 2 - 3 mn simulation instead of several hours !!!!

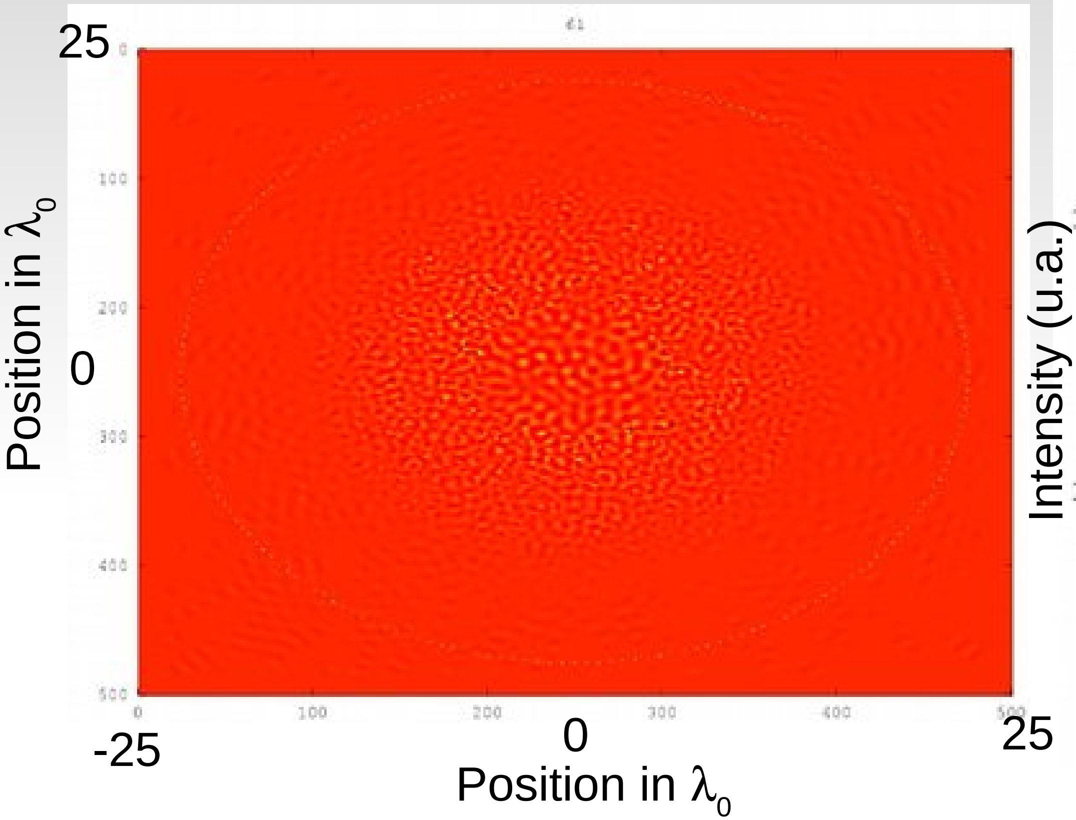
2.1- Correlations in scattering media

A single realization

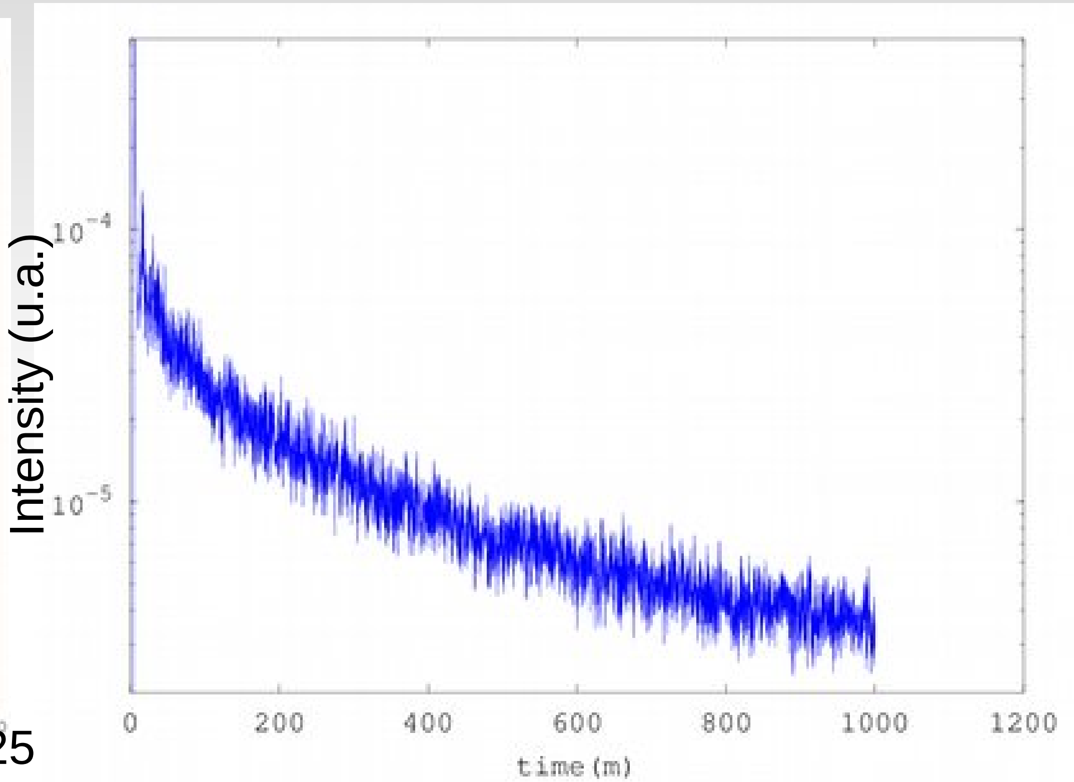
Multiple scattering medium



Green's function



Linear scale

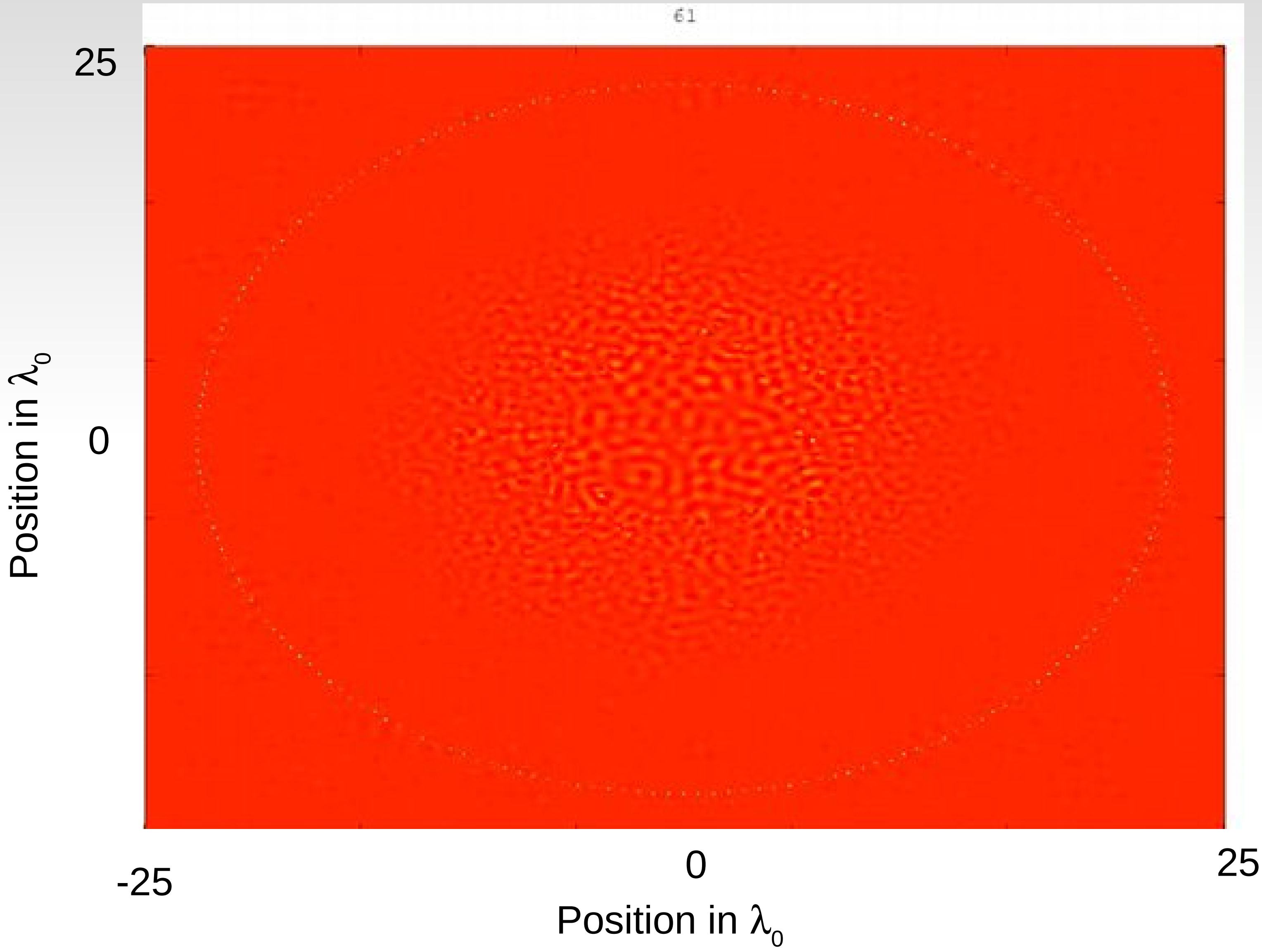


Time of Flight distribution

Noise Correlation Funtion

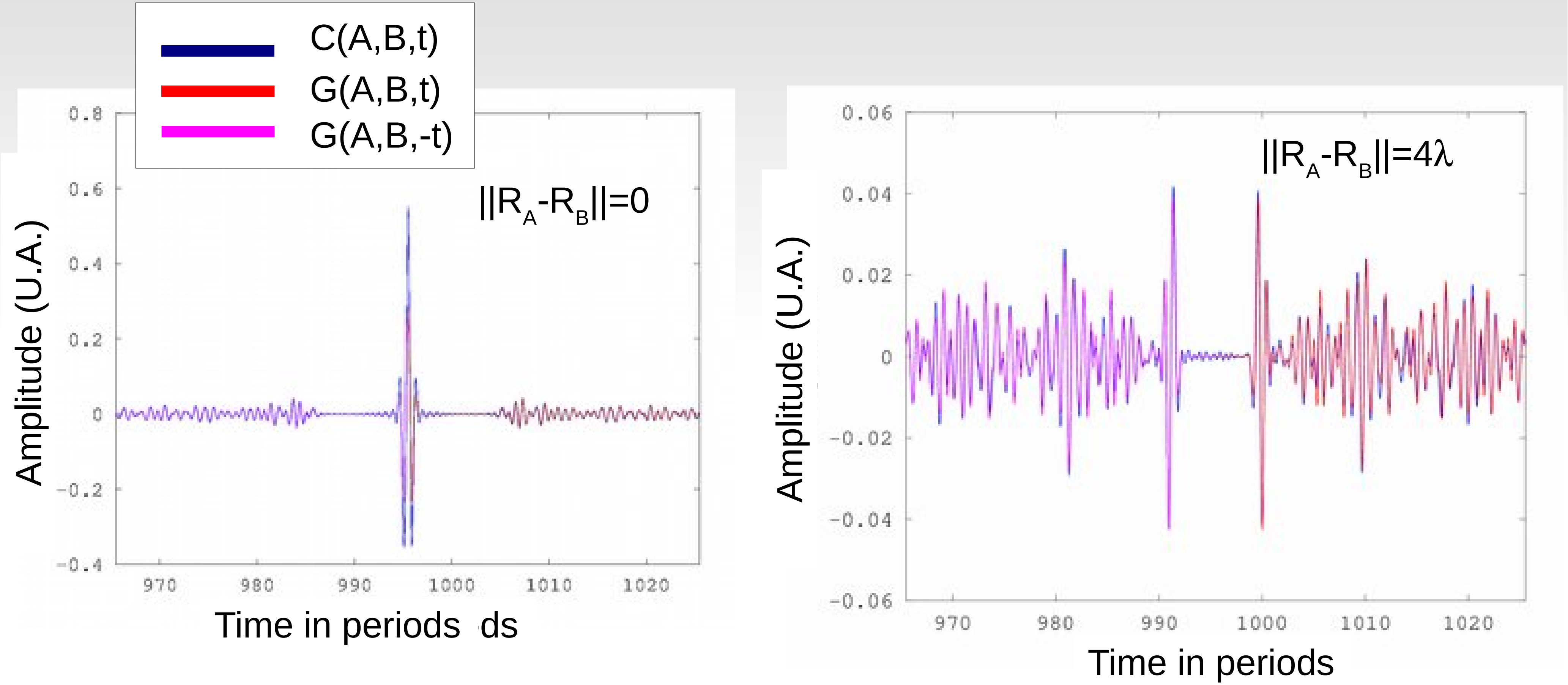
•200 uncorrelated noise sources

$$C(A,B,t)$$



Linear scale

Comparison b/w NCF and Green's function



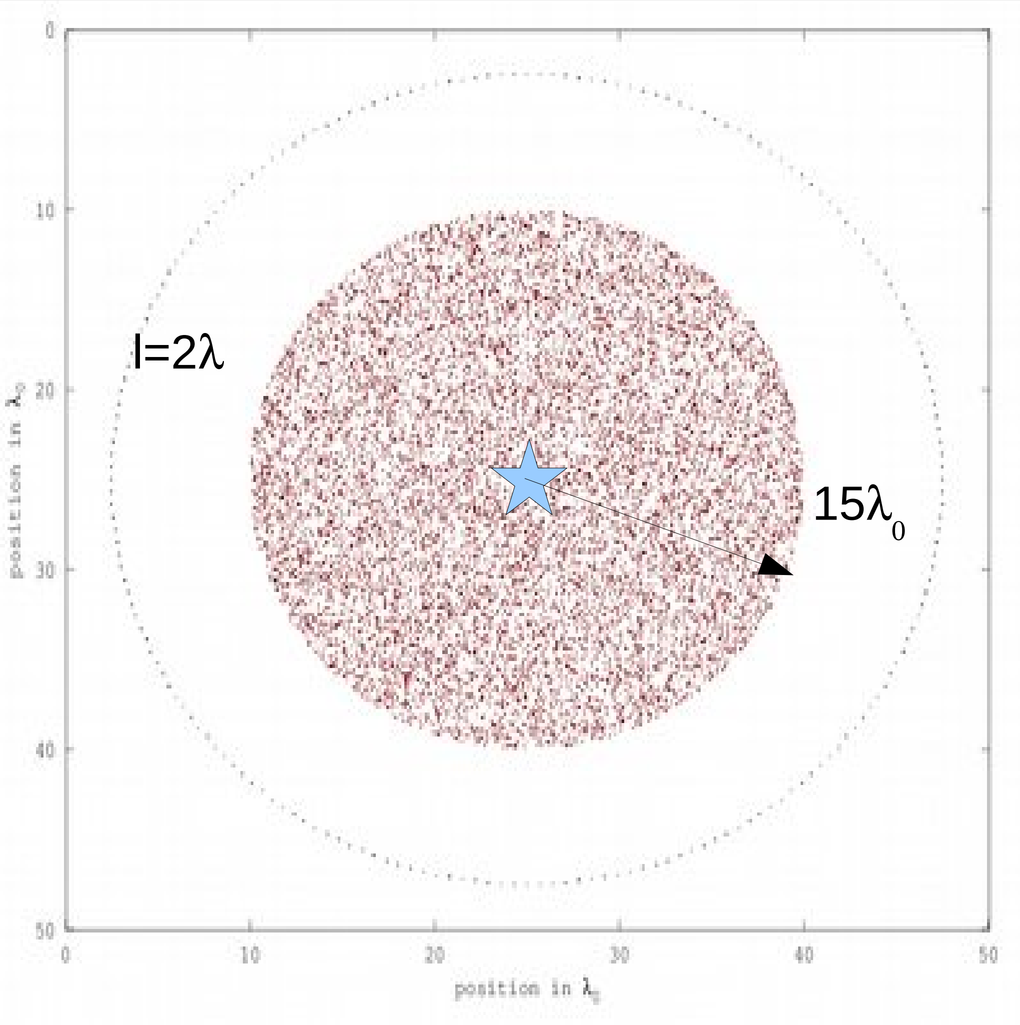
$$\partial_t C(A,B,t) \propto G(A,B,t) - G(A,B,-t)$$

Still valid in random media !

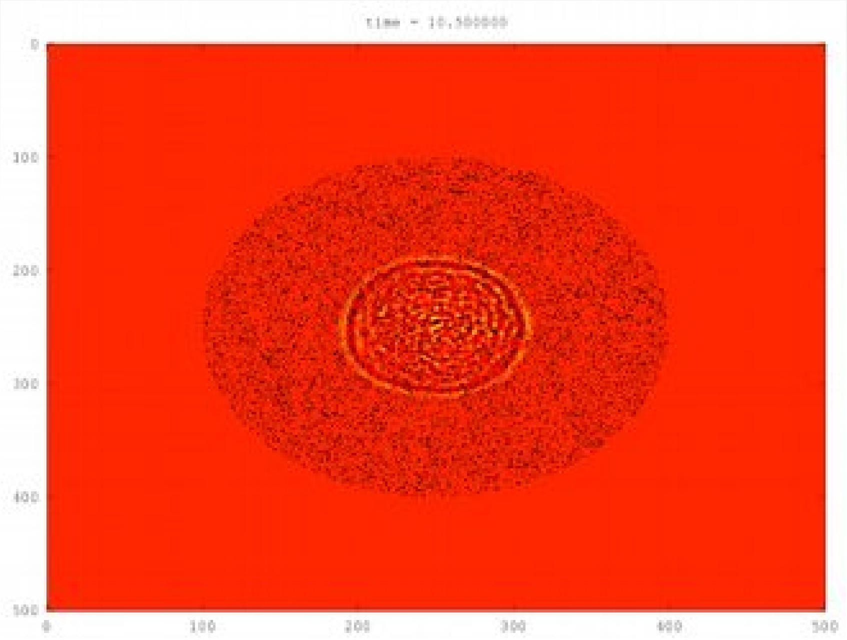
Derode et al., JASA 2003

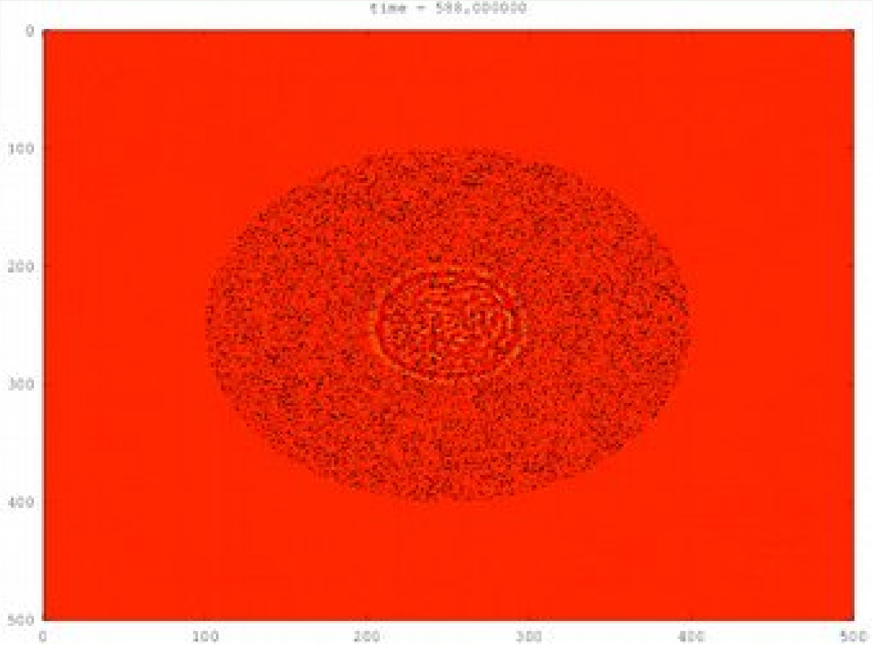
Margerin & Sato, Wave Motion, 2011

Correlation inside random media

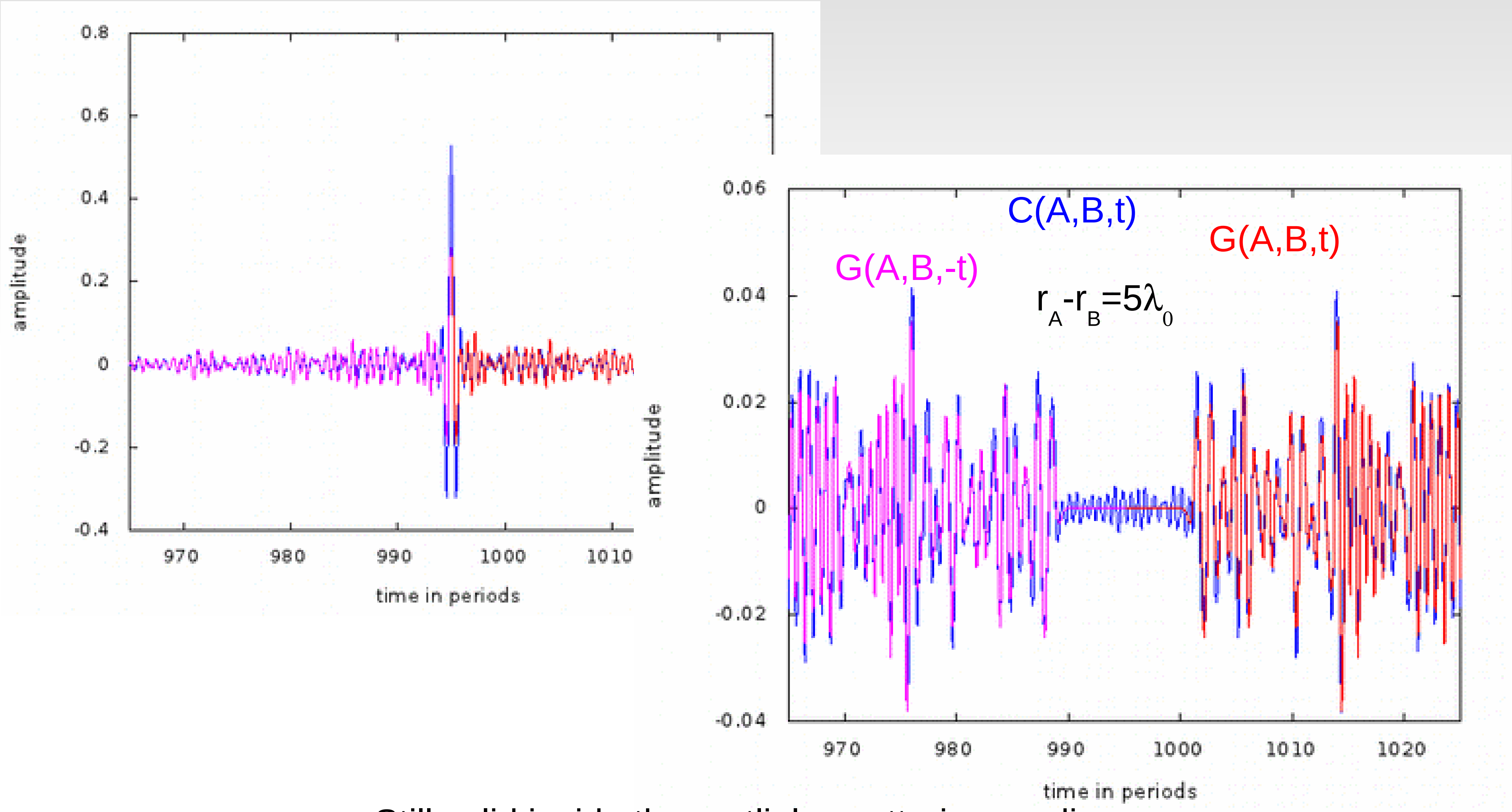


Correlation inside random media



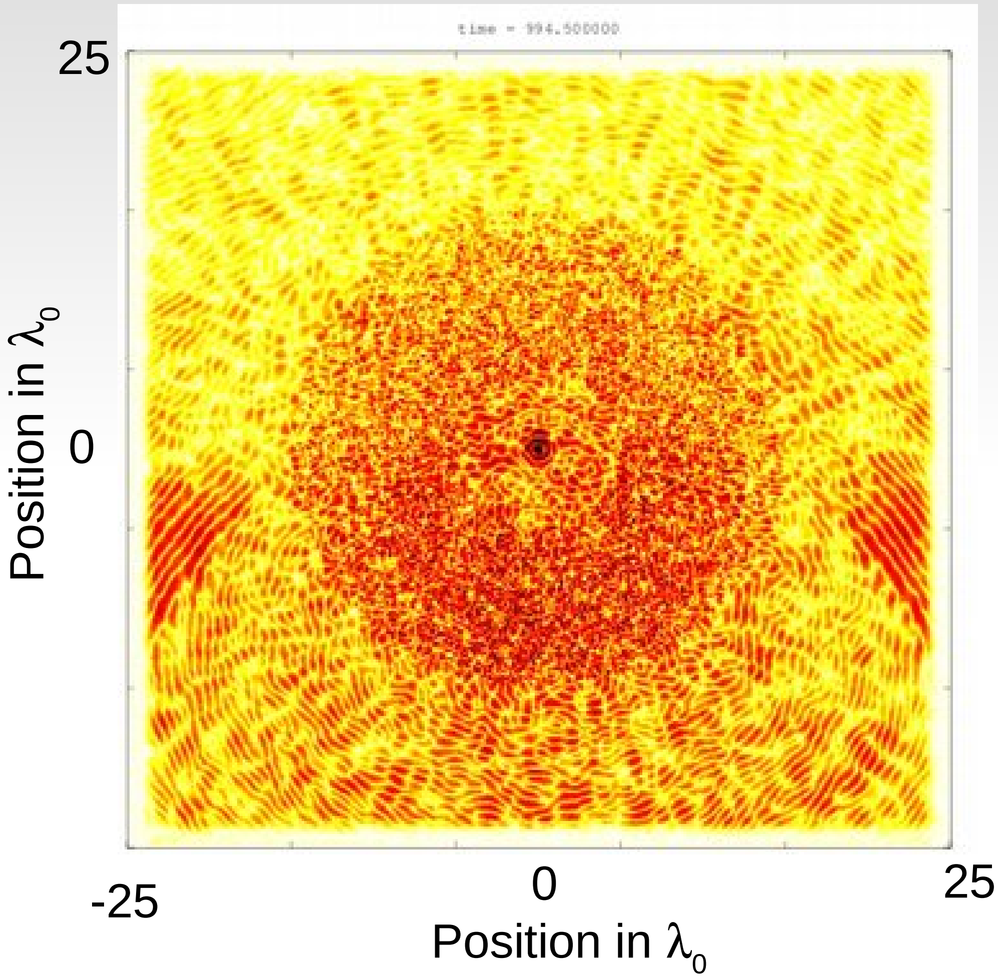


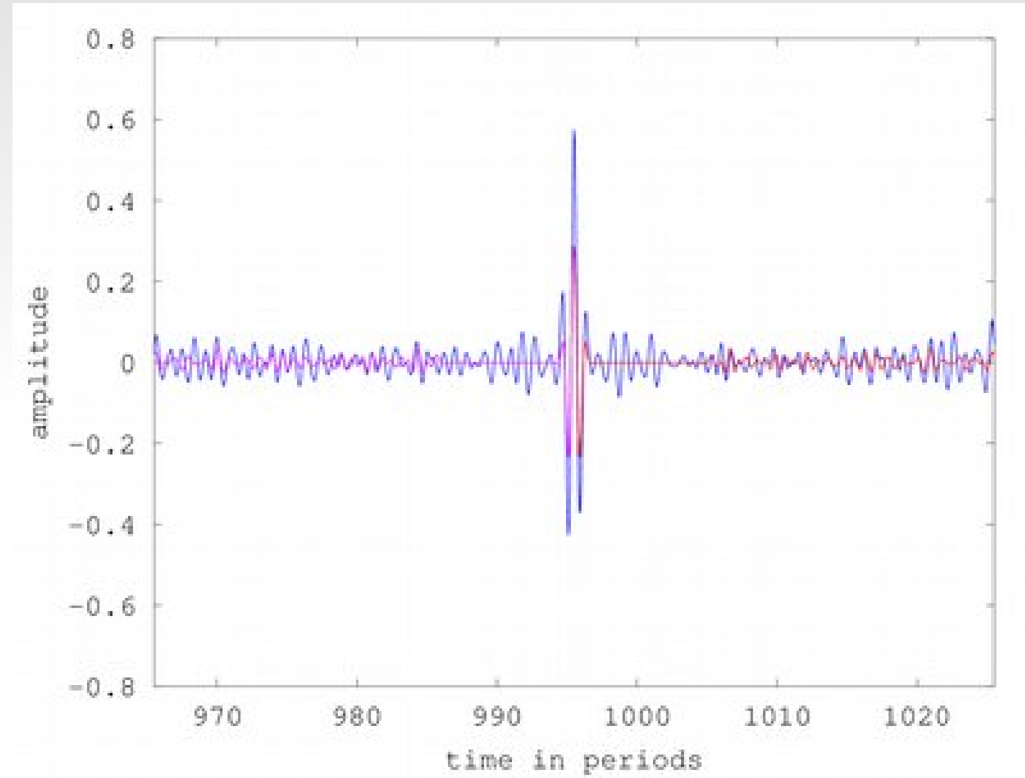
Time signals



Still valid inside the mutliple scattering media

1 noise source on single realization





Estimation of the Green Function on a single realization

2.2 - NCF in multiple scattering media :

Mean value

Average

When the noise sources are equirepartitionned, i.e., uniformaly distributed

$$\partial_t C(A,B,t) \propto G(A,B,t) - G(A,B,-t)$$

Trivial

$$\partial_t \langle C(A,B,t)\rangle \propto \langle G(A,B,t)\rangle - \langle G(A,B,-t)\rangle$$

$$\langle C(A,B,\omega)\rangle \propto \langle \Im G(A,B,\omega)\rangle$$

Average

When the noise sources are **NOT** equirepartitionned, i.e., uniformaly distributed

$$\langle C(A,B,t)\rangle \propto \textbf{?}$$

Ergodicity arguments imply that the scattering plays the role of secondary sources?

Derode, Larose, Campillo, Fink APL 2003, Larose, et al. Geophysics 2006, Bal et al. 2002

Result of multiple scattering theory at long time

$$\lim_{t\to\infty}\int \mathrm{dr}\langle I_{\omega\mathbf{k}}(\mathbf{r},t)\rangle = -\frac{1}{\omega}\mathrm{Im}\,\langle G(\omega,\mathbf{k})\rangle \times \frac{\sum_{\mathbf{k'}} -\frac{1}{\omega}\mathrm{Im}\,\langle G(\omega,\mathbf{k'})\rangle S(\omega,\mathbf{k'})}{\sum_{\mathbf{k'}} -\frac{1}{\omega}\mathrm{Im}\,\langle G(\omega,\mathbf{k'})\rangle}.$$

[Tiggelen-PRL2003]

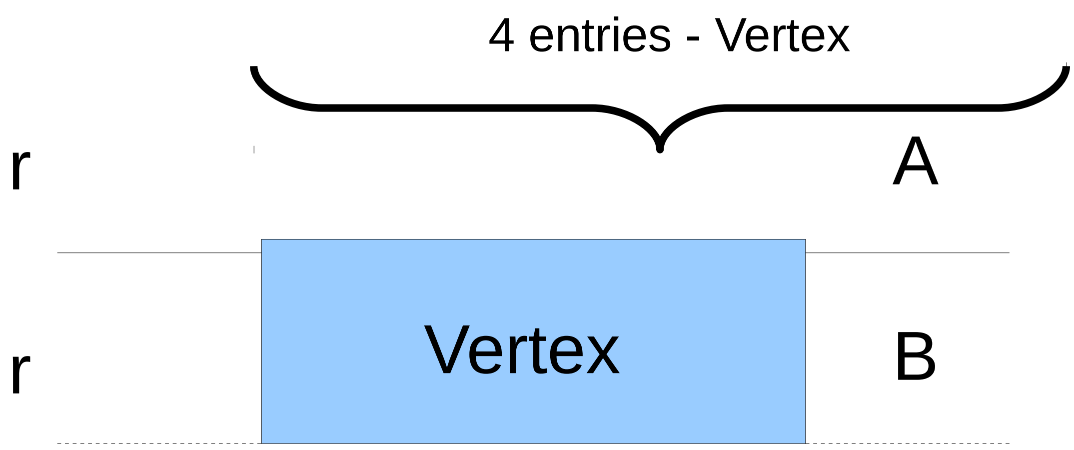
One noise source

$$\langle C(A,B,t)\rangle = \langle G(B,r;-t)\otimes G(A,r;t)\rangle$$

r : noise source
position

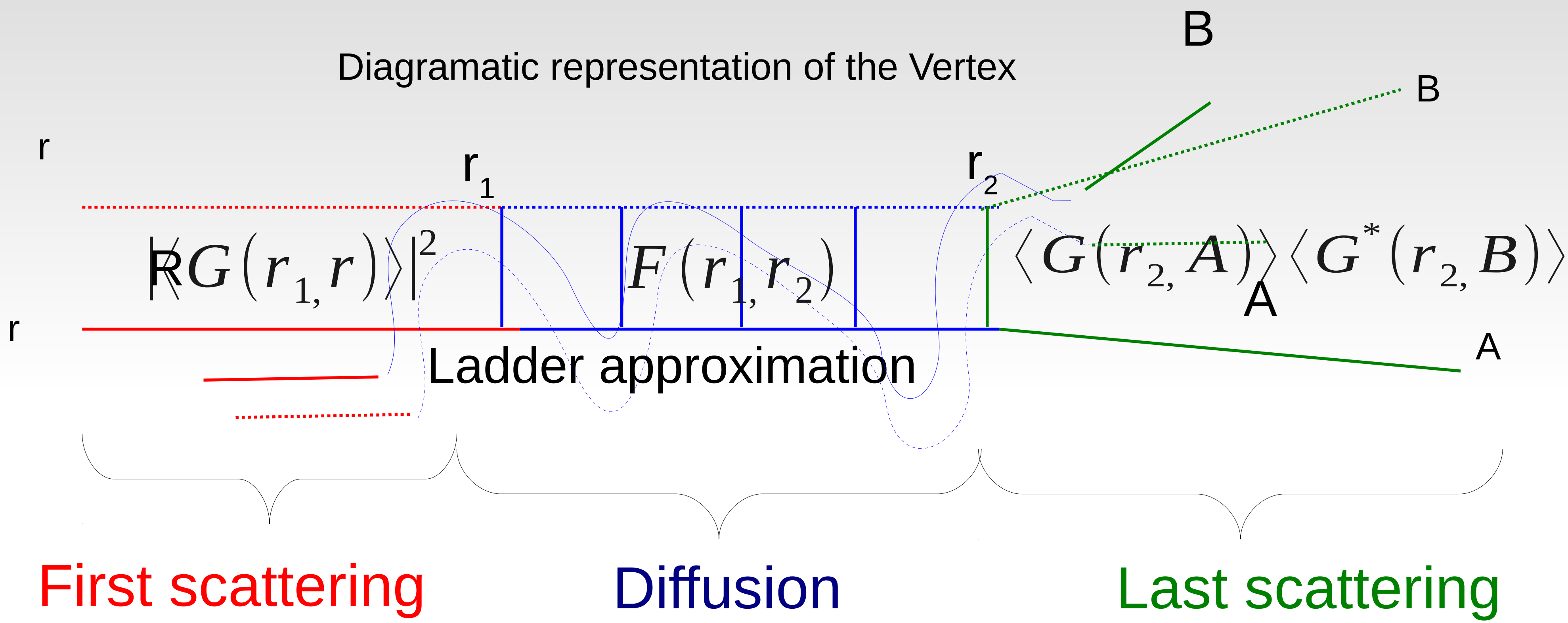
Fourier transform

$$\langle C(A,B,\omega)\rangle = \langle G(B,r;\omega)^{*}G(A,r;\omega)\rangle$$



Julien de Rosny et al. ,PHYSICAL REVIEW E 70, 046601 (2004)

Boltzman approximation



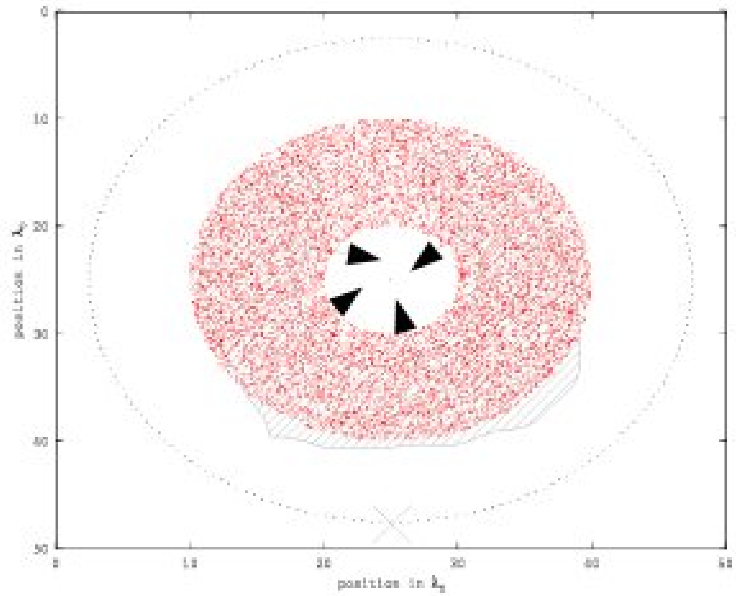
$$C(A,B,\omega)$$

$$\langle C(A,B,\omega)\rangle = \langle G(r,A,\omega)\rangle\langle G^*(r,B,\omega)\rangle +$$

$$\frac{1}{c_0}\int IL(r_2)\langle G(r_2,A,\omega)\rangle\langle G^*(r_2,B,\omega)\rangle d^2r$$

Halo as secondaty sources

$$IL(r_2)=\frac{1}{c_0}\iint_{\delta V}|\langle G(r_1,r,\omega)\rangle|^2 F(r_1,r_{2,}\omega)\,d^2r\,d^2r_1$$



When the halo is uniform around A and B (at less than a *l),* the field is diffuse for <C>

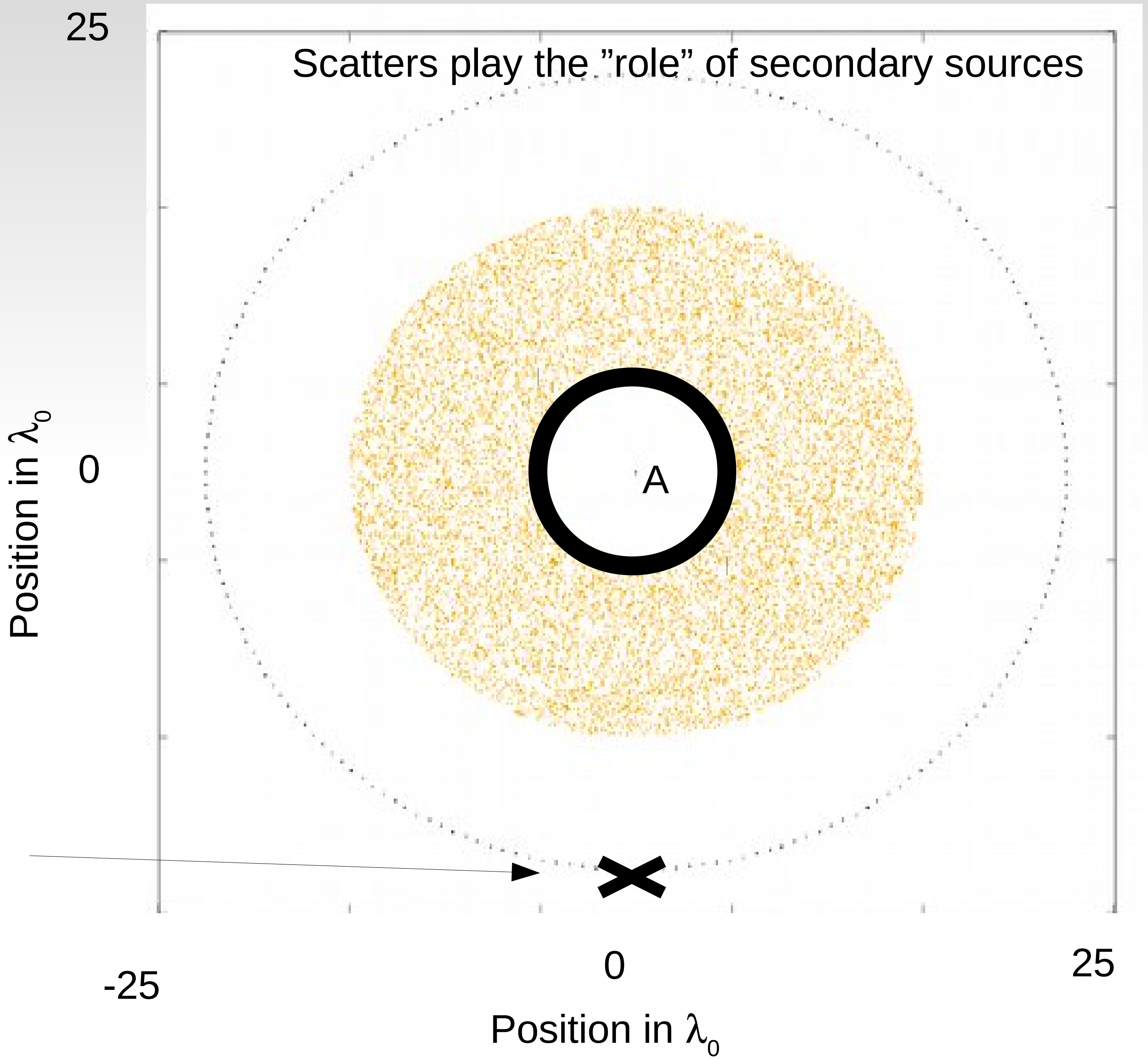
$$\partial_t\langle C(A,B,t)\rangle\approx[\langle G(A,B,-t)\rangle-\langle G(A,B,t)\rangle]$$

Numerical validation

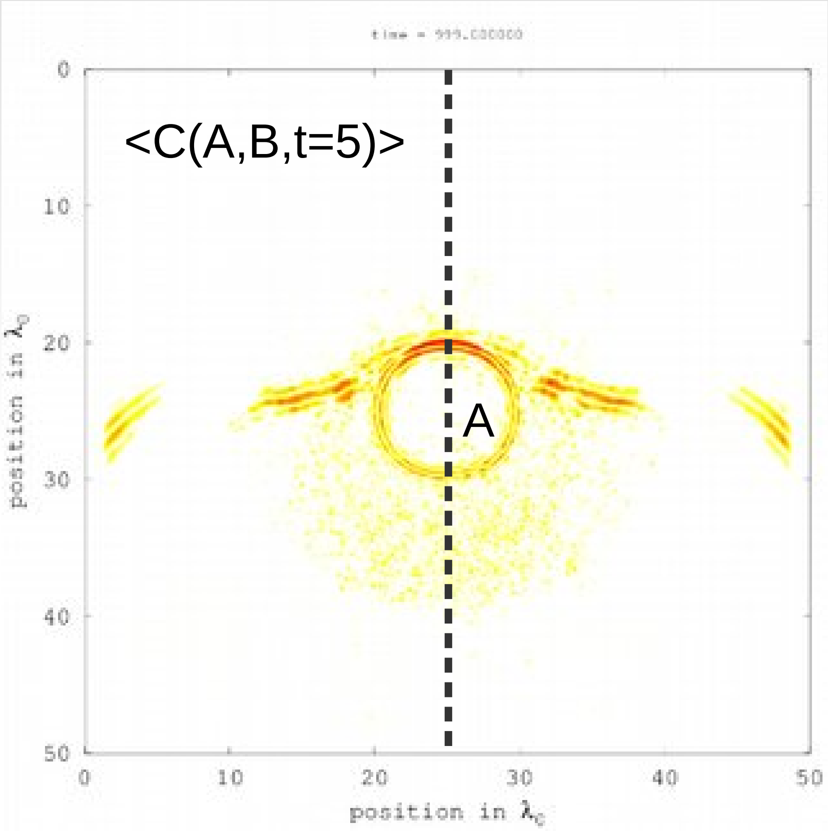
- 1 uncorrelated noise source
- 270 disorder realizations

$$\langle C(A,B,t)\rangle$$

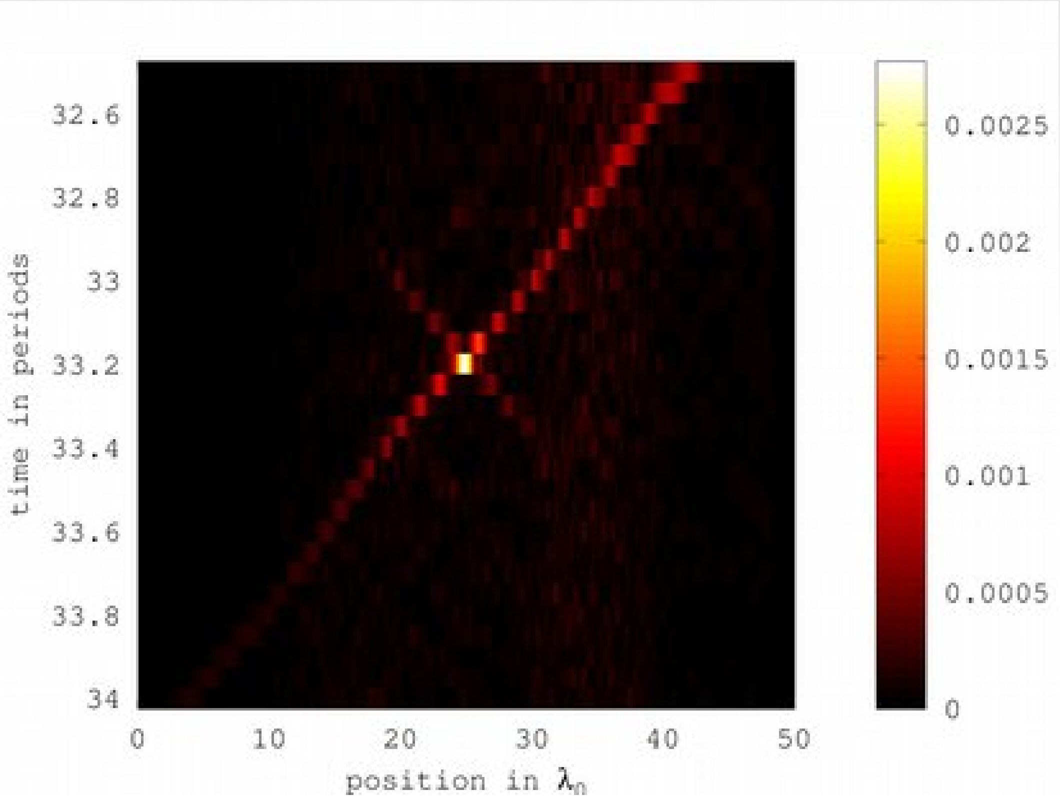
Single noise souce



Numerical validation



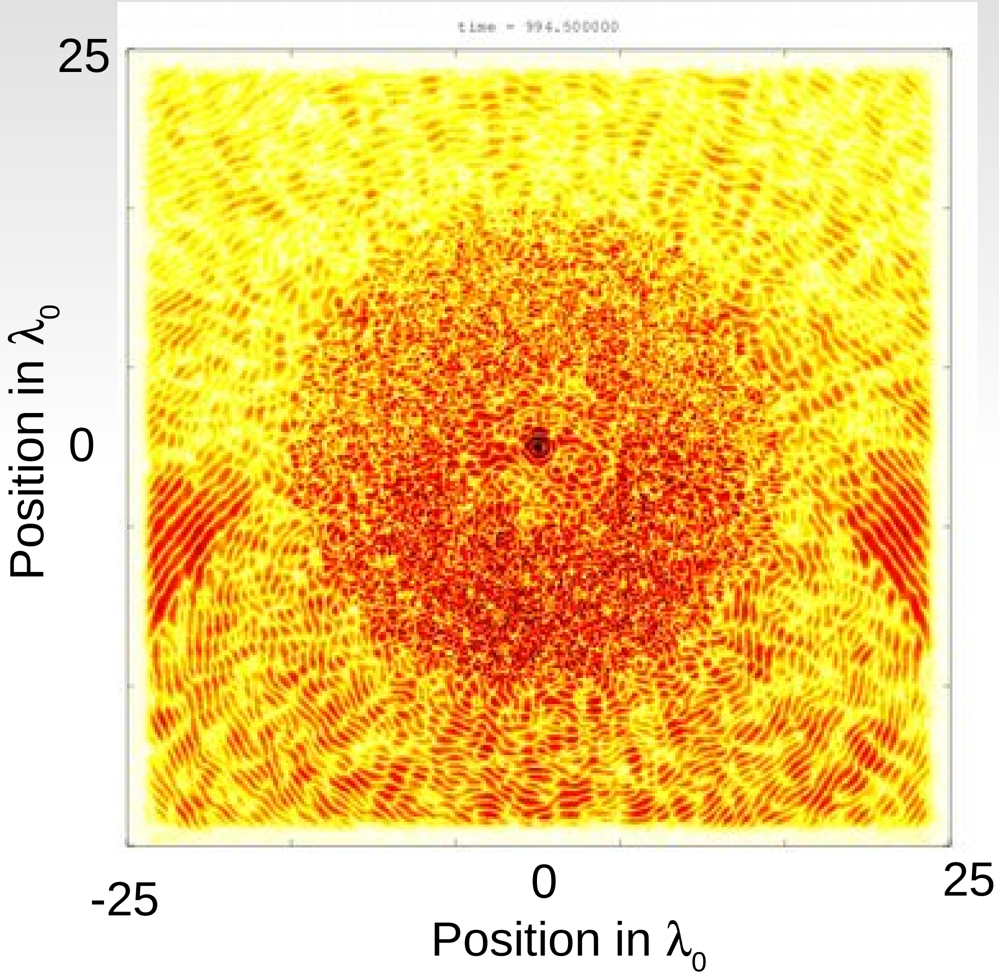
<C(A,B,t)> over a vertical that goes by A

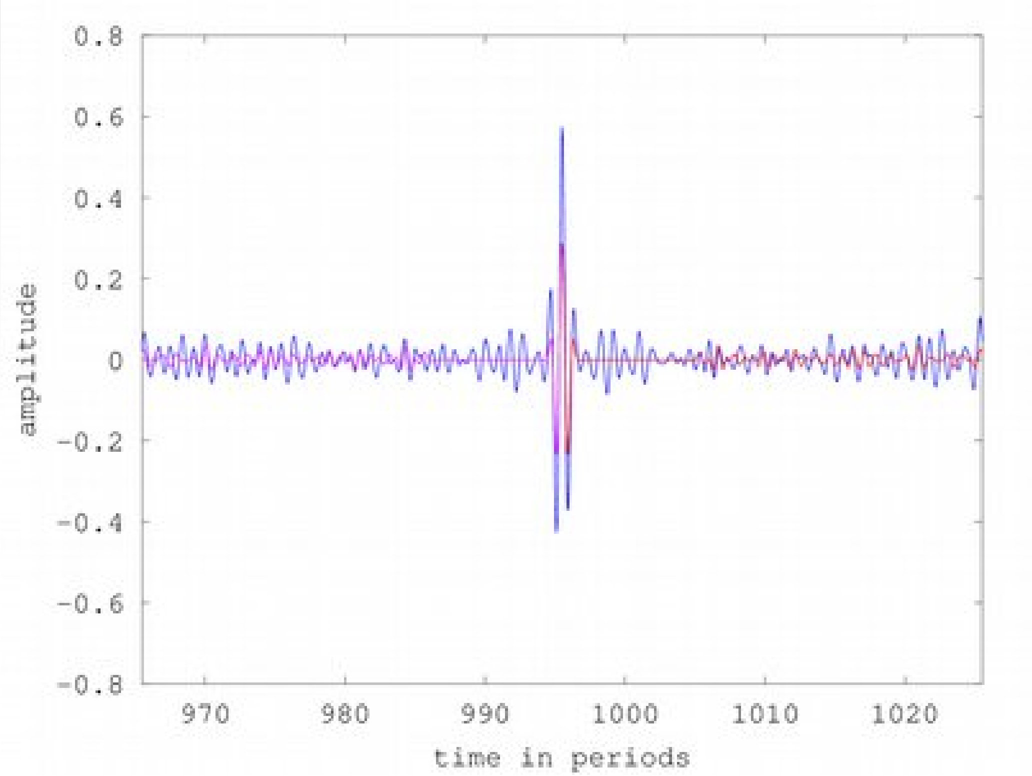


2.3 - NCF in multiple scattering media :

fluctuations

1 noise source on single realization





Estimation of the Green Function on a single realization

Self-averaging process?

- Time Reversal or NCF are wideband processes of bandwidth B. *Frequency correlation length* $df \sim 1/(2\pi t_T)$ *(Thouless time)*

Averaging over B/df "realizations". Gaussian correlations. Self averaging process

P Blomgren JASA2002, Derode PRE 2001, Larose2006 Geophysics , ...

BUT

$$C_{AB} = \frac{-l_a}{k_0} \Im G(A,B) \Rightarrow \delta\langle C_{AB}^2 \rangle = -\left(\frac{l_a}{k_0}\right)^2 \delta\langle \Im G(A,B)^2 \rangle \propto \langle I(A,B) \rangle$$

Correlation fluctuations equal to the fluctuations of Im G

δC_{AB} and δI

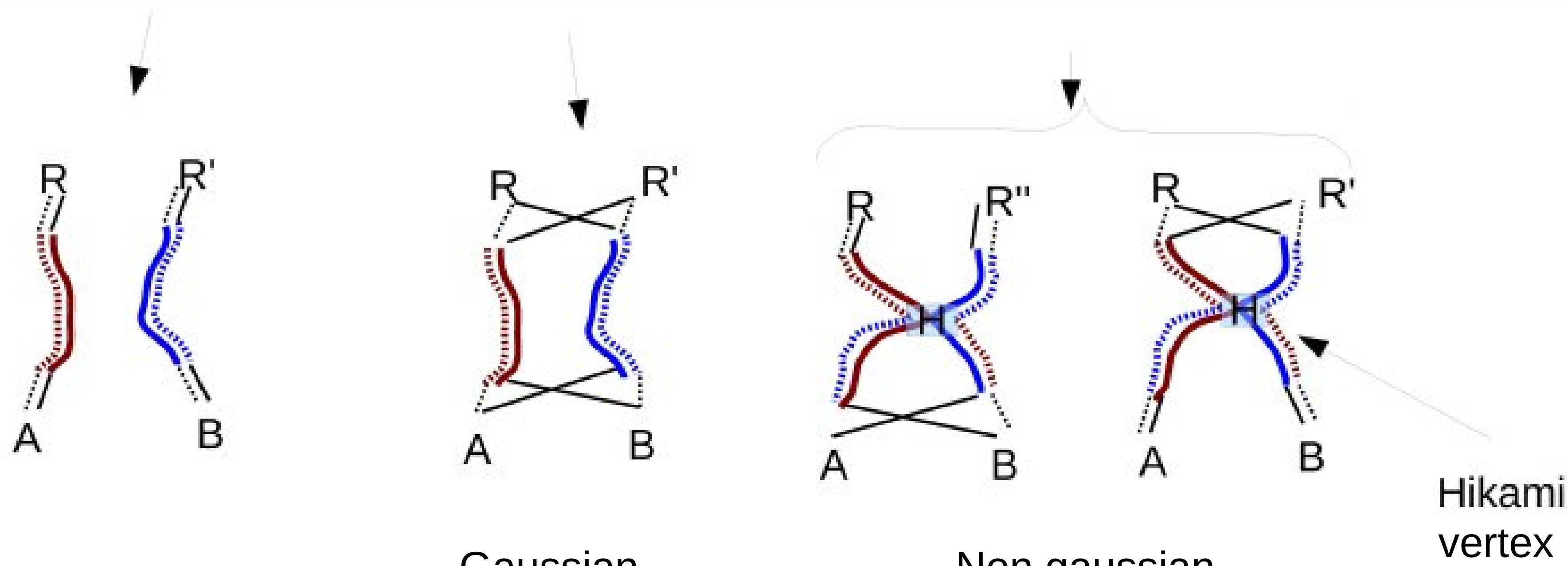
Both depends on average value of the product of **4 green's functions**

Intensity correlation

$$I(R,A)=|G(R,A)|^2$$

$$\langle I(R,A)I(R',B)\rangle=$$

$$\langle I(R,A)\rangle\langle I(R',B)\rangle + C_1 + C_2 + C_3$$



Gaussian Fluctuations & Short range

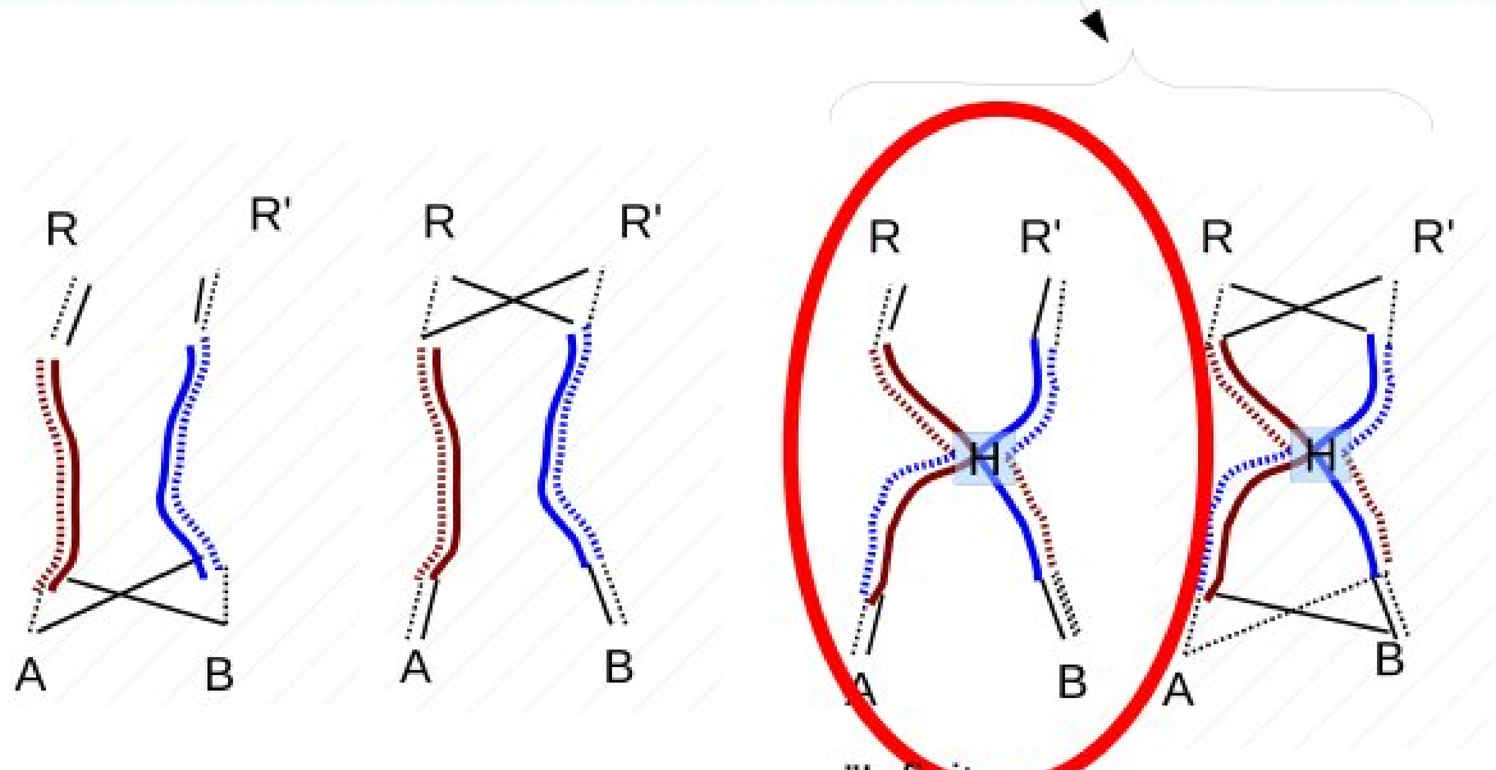
Non gaussian fluctuations & long range

Fluctuations of NCF

$$C(A,B) = \oint G(R,A)\, G^*(R,B)\, dR$$

$$\langle |C(A,B)^2| \rangle =$$

$$|\langle C(A,B) \rangle^2| \quad + \quad \gamma_1 \quad + \quad \gamma_2 \quad + \quad \gamma_3$$



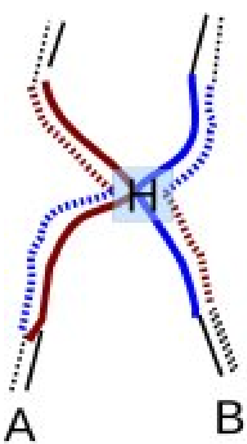
"Infinite range fluctuations"

Infinite range

$$2\int_V \nabla L(\mathbf{r}_A, s)\nabla L(\mathbf{r}_B, s)d^d s = \frac{K}{D}\left[L(\mathbf{r}_A,\mathbf{r}_B) + L(\mathbf{r}_B,\mathbf{r}_A)\right]$$

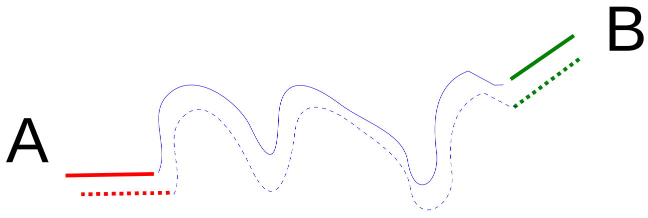
$\|\mathbf{r}_A - \mathbf{r}_B\| \gg l_e$

$\int\int$



$d^3R d^3R'$

μ

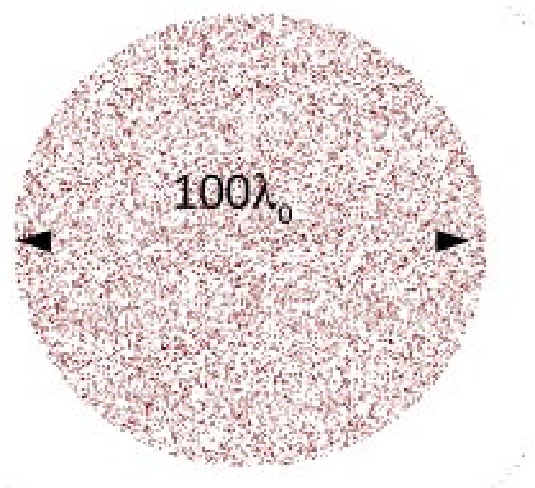


When $\|r_A - r_B\| \gg \ell$

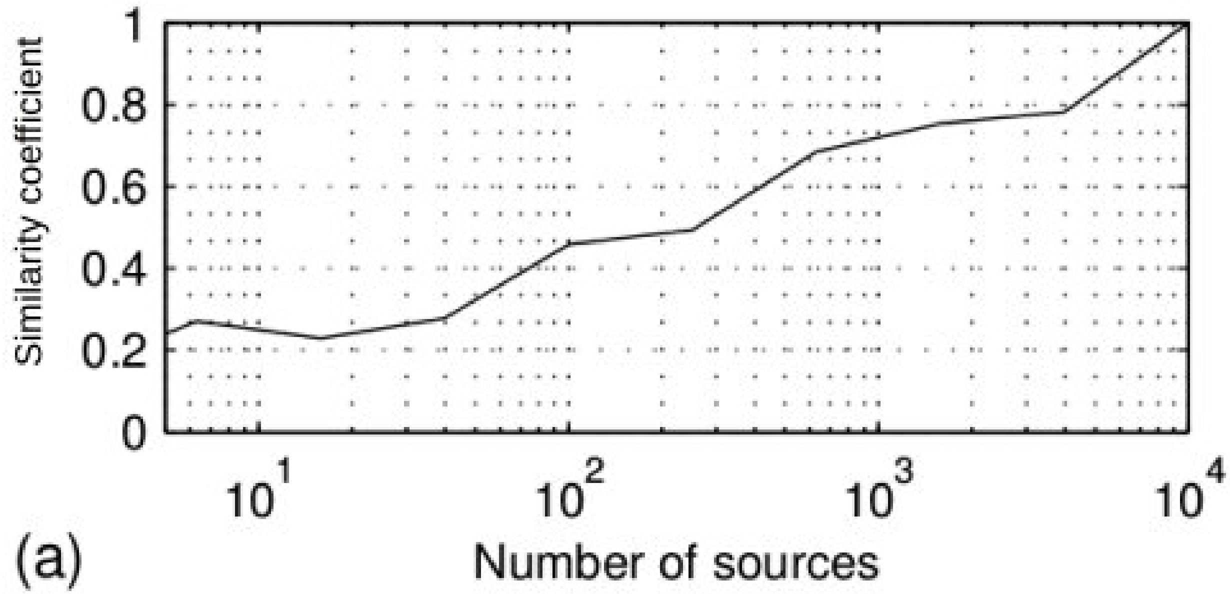
$$\langle C_{AB}^2\rangle \propto \langle \Im G(A,B)^2\rangle \propto \langle I(A,B)\rangle$$

Numerical validation

Numerical model –
10000 coupled dipoles



$l_a \sim 63\lambda_0$
$l_e \sim 2\lambda_0$



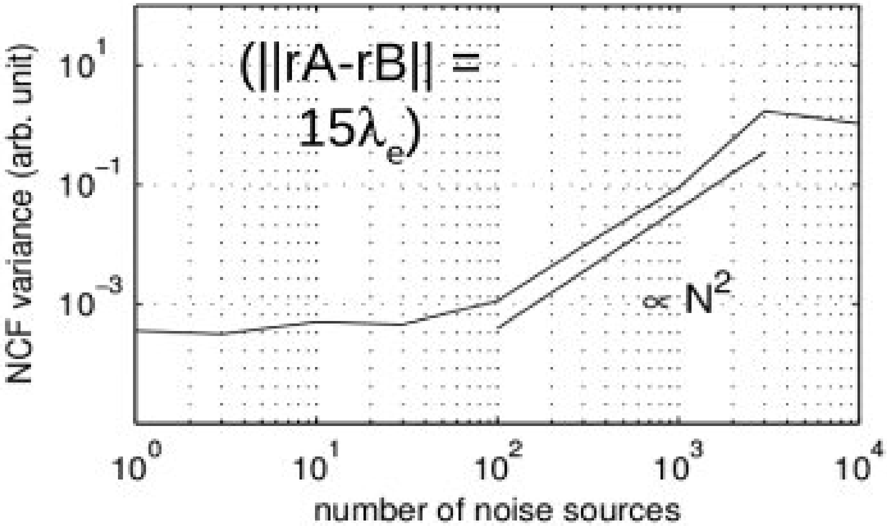
Similarity coefficient between the correlation with respect to the number of noise source ($||r_A - r_B|| = 15\lambda_e$)

When Number of sources = 10000 → Similarity =1

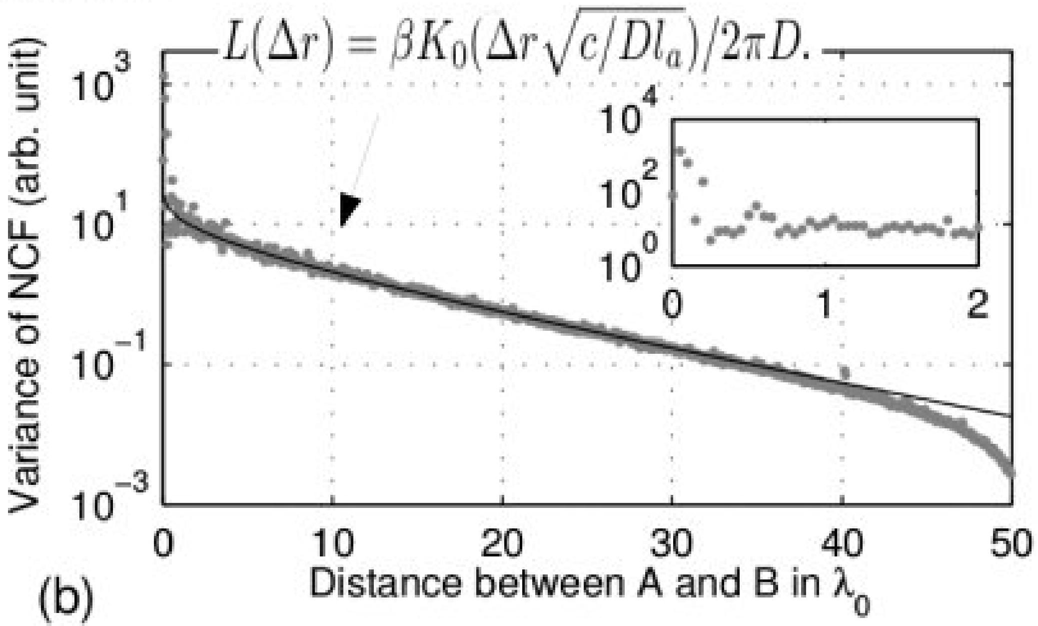
Numerical validation

$\gamma_2 \propto N^2$ — *N* number of sources

500 disorder realizations



Variance with respect to the noise sources



Comparaison between the variance of the NCF with respect to $||\mathbf{r}_A - \mathbf{r}_B||$

Good agreement when $\mathbf{r}_A <> \mathbf{r}_B$

Variance when A=B

Divergence comes from integration close to A

→ Must take into account local disorder

$\left(C_{AA}=I_A\right)$ → Fluctuations of C_{AA} equal fluctuations of I_A

Works of Tiggelen & Skipetrov PRE 2006 & Cazé et al. PRA 2010 on the **local density of states**

Density of states

Density of states

$$\sum_n \delta(\omega-\omega_n)$$

Local Density of States

$$\rho(\boldsymbol{r}_A)=\sum_n |\Phi_n(\boldsymbol{r}_A)|^2 \delta(\omega-\omega_n)$$

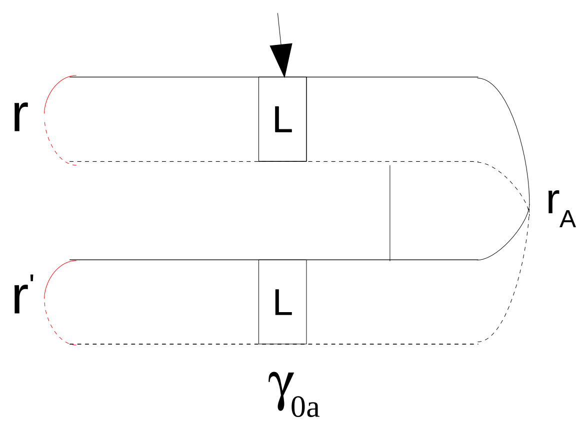
$$\rho(\boldsymbol{r}_A) \propto \Im G(A,A) \propto \int I(\boldsymbol{r},\boldsymbol{r}_A)\, d^3 r$$

$$\frac{\langle [\delta\rho(\boldsymbol{r}_A)]^2 \rangle}{\langle \rho(\boldsymbol{r}_A) \rangle} = C_0$$

Tiggelen & Skipetrov, PRE, 2006

$$\langle |C_{AA}{}^2| \rangle = |\langle C_{AA} \rangle^2| \quad + \quad \gamma_0$$

$$\gamma_0 \propto C_0$$



The fluctuation of C_{AA} is governed by C_0

Cross density of states

$$\rho(\boldsymbol{r}_A) = \sum_n |\Phi_n(\boldsymbol{r}_A)|^2 \delta(\omega - \omega_n)$$

Local Density of States

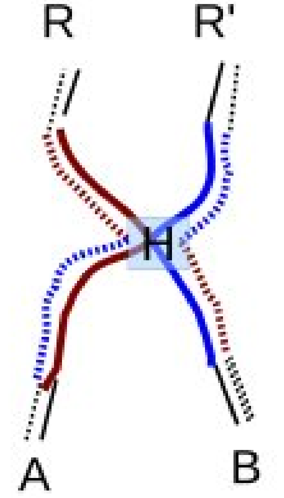
$$\rho(\boldsymbol{r}_A, \boldsymbol{r}_B) = \sum_n |\Phi_n(\boldsymbol{r}_A)\Phi_n^*(\boldsymbol{r}_B)| \delta(\omega - \omega_n)$$

Cross Density of States

$$\rho(\boldsymbol{r}_A, \boldsymbol{r}_B) \propto \Im G(A,B) \propto C(A,B)$$

$$\langle[\delta\rho(\boldsymbol{r}_A, \boldsymbol{r}_B)]^2\rangle \propto \delta \Im G(A,B)^2 \propto \gamma_2$$

Cazé et al. - Akkermans and Montambaux' book



γ_0 (C_0) and γ_2 are the two limits of the fluctuations of the cross-density of states

Conclusion

• Provide a better view of Green's function extraction in multiple scattering media

• Explain from diffusion approximation how scatterers play the rôle of secondary source for the **averaged** NCF

• The NCF is **not** a self-avergaing quantity

• For large number of sources, the fluctuations are dominated by non-Gaussian correlations

• Other contributions C0, C00, etc can contributes to the fluctuations of the NCF (not shown here)

• Extension in the time domain to retreive Michel's results, C3 may be also analyzed with this approach

Most of this work in de Rosny & Davy, EPL, 2014

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