Spectral-element numerical modeling for acoustic and elastic wave propagation

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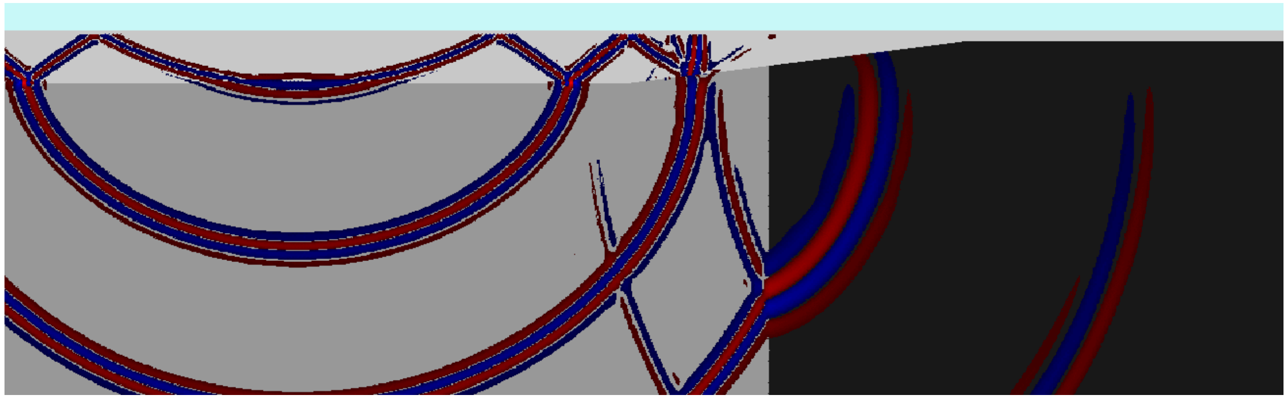
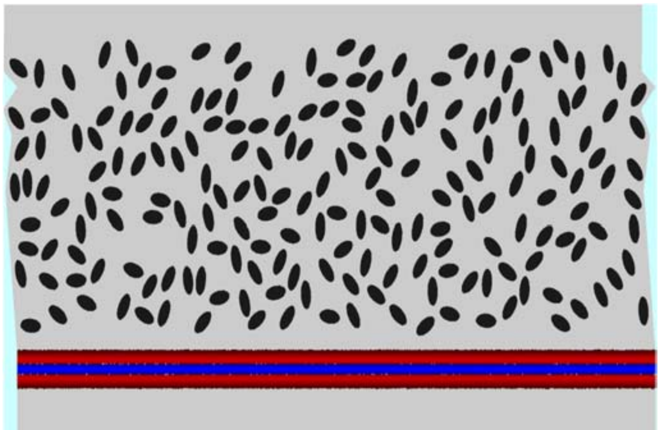
Note: the SPECFEM3D source code is freely available open source at http://www.geodynamics.org

Application domains

Earthquakes



Ocean acoustics



Non destructive testing

Brief history of numerical methods

Acoustic or seismic wave equation: tremendous increase of computational power
$\Rightarrow$ development of numerical methods for accurate modeling in complex 3D models has been a continuous effort in the last 40 years.

Finite-difference methods: Yee 1966, Chorin 1968, Alterman and Karal 1968, Madariaga 1976, Virieux 1986, Moczo et al, Olsen et al..., difficult for boundary conditions, surface waves, topography, full Earth; but can be improved: Virieux, Moczo et al., Lombard and Piraux…

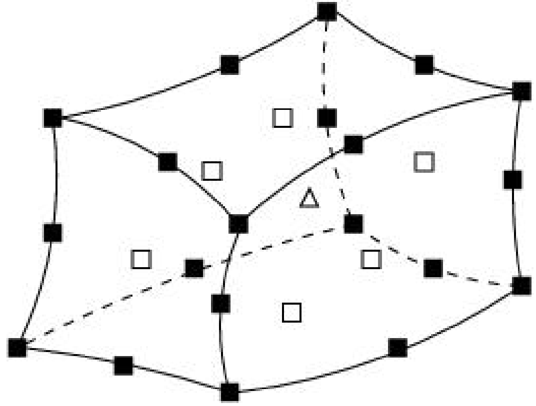
Boundary-element or boundary-integral methods: Kawase 1988, Sanchez-Sesma et al. 1991…, homogeneous layers, expensive in 3D but can be made faster (fast multipoles, e.g. Bonnet, Semblat, Chaillat et al.)

Spectral and pseudo-spectral methods: Carcione 1990…, smooth media, difficult for boundary conditions, difficult on parallel computers

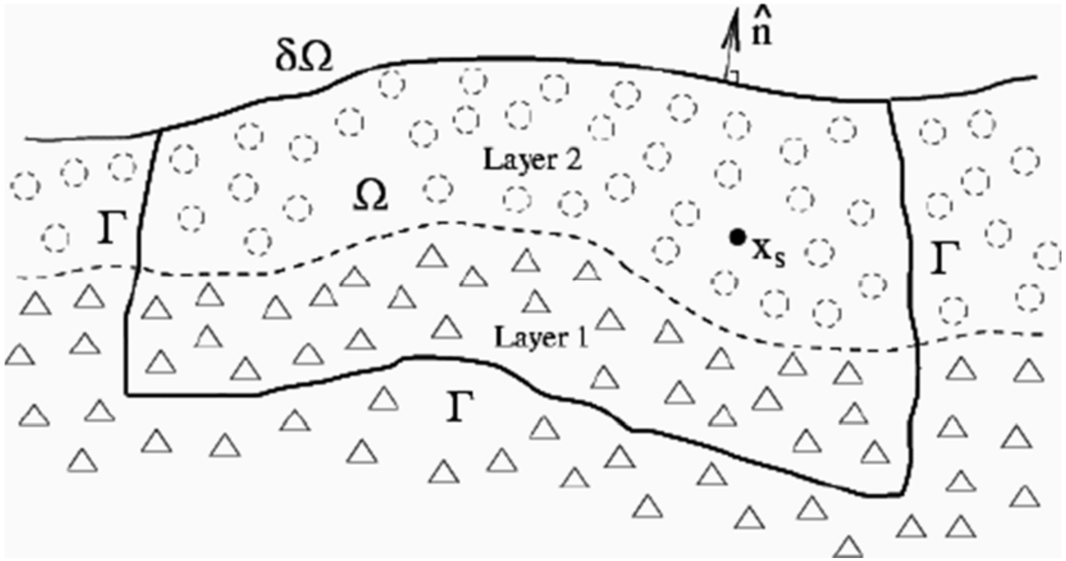
Classical finite-element methods: Lysmer and Drake 1972, Marfurt 1984, Bielak et al 1998…, linear systems, large amount of numerical dispersion

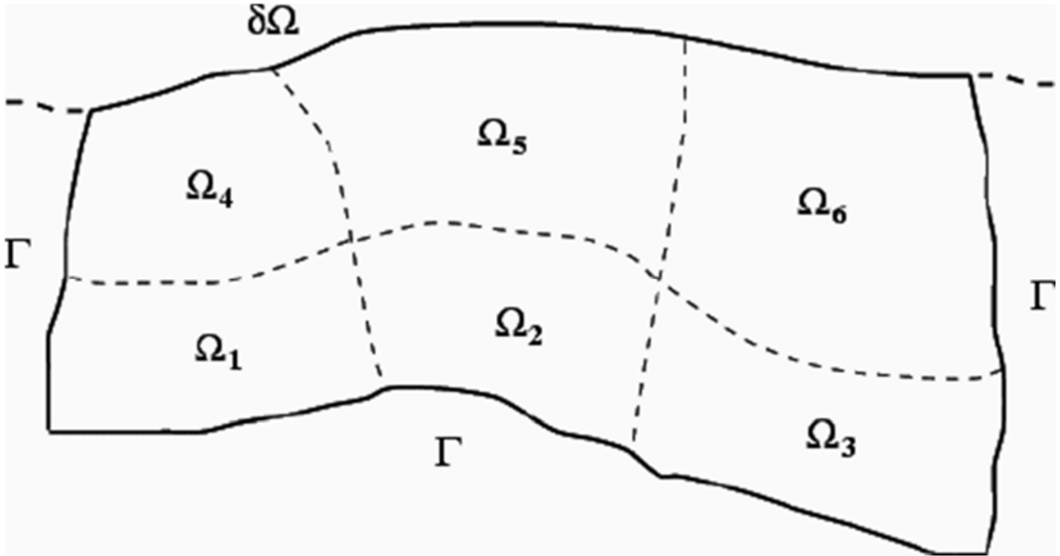
Let us combine the advantages of the last two.

Spectral-Element Method



- Developed in Computational Fluid Dynamics (Patera 1984)
- Accuracy of a pseudospectral method, flexibility of a finite-element method; continuous Galerkin, can be made discontinuous (DG) if needed
- Extended by Komatitsch and Tromp, Chaljub et al.
- Large curved “spectral” finite-elements with high-degree polynomial interpolation
- Mesh honors the main discontinuities (velocity, density) and topography
- Very efficient on parallel computers, no linear system to invert (diagonal mass matrix)





Equations of Motion (solid)

Differential or *strong* form (e.g., finite differences):

$$\boxed{\rho\,\partial_t^2\mathbf{u} = \nabla\cdot\sigma + \mathbf{f}}$$

We solve the integral or *weak* form:

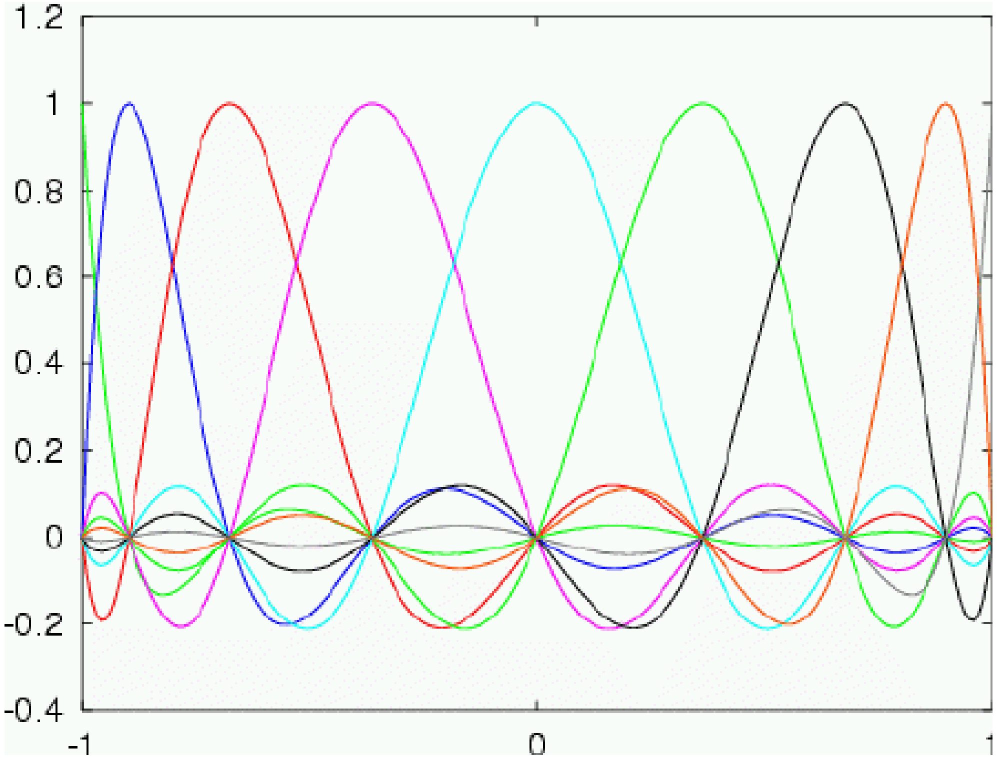
$$\boxed{\int \rho\,\mathbf{w}\cdot\partial_t^2\mathbf{u}\,\mathrm{d}^3\mathbf{r} = -\int \nabla\mathbf{w} : \sigma\,\mathrm{d}^3\mathbf{r}}$$

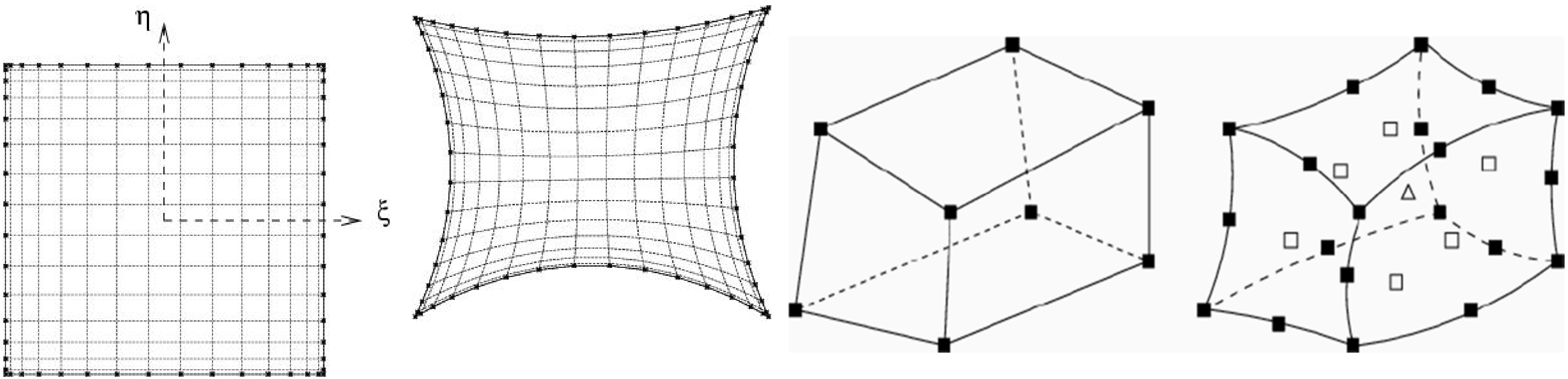
$$\boxed{+\mathbf{M} : \nabla\mathbf{w}(\mathbf{r}_\mathrm{s})S(t) - \int_{\mathrm{F-S}} \mathbf{w}\cdot\sigma\cdot\hat{\mathbf{n}}\,\mathrm{d}^2\mathbf{r}}$$

+ attenuation (memory variables) and ocean load

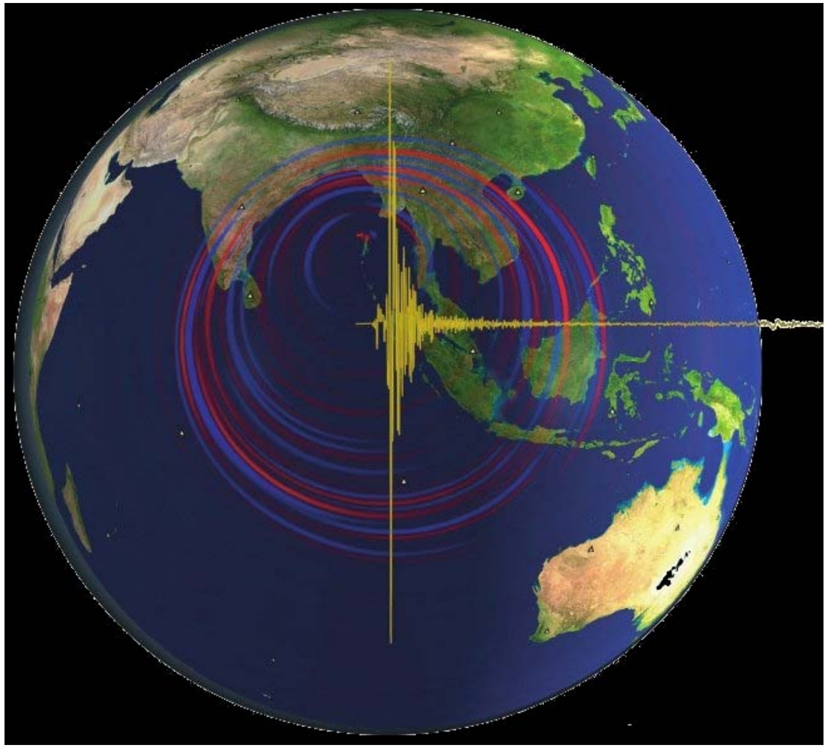
Finite Elements

- High-degree pseudospectral finite elements
- N = 4 to 8 usually
- *Strictly* diagonal mass matrix
- No linear system to invert





SPECFEM3D software package



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Goal: modeling (linear) acoustic or seismic wave propagation in complex models

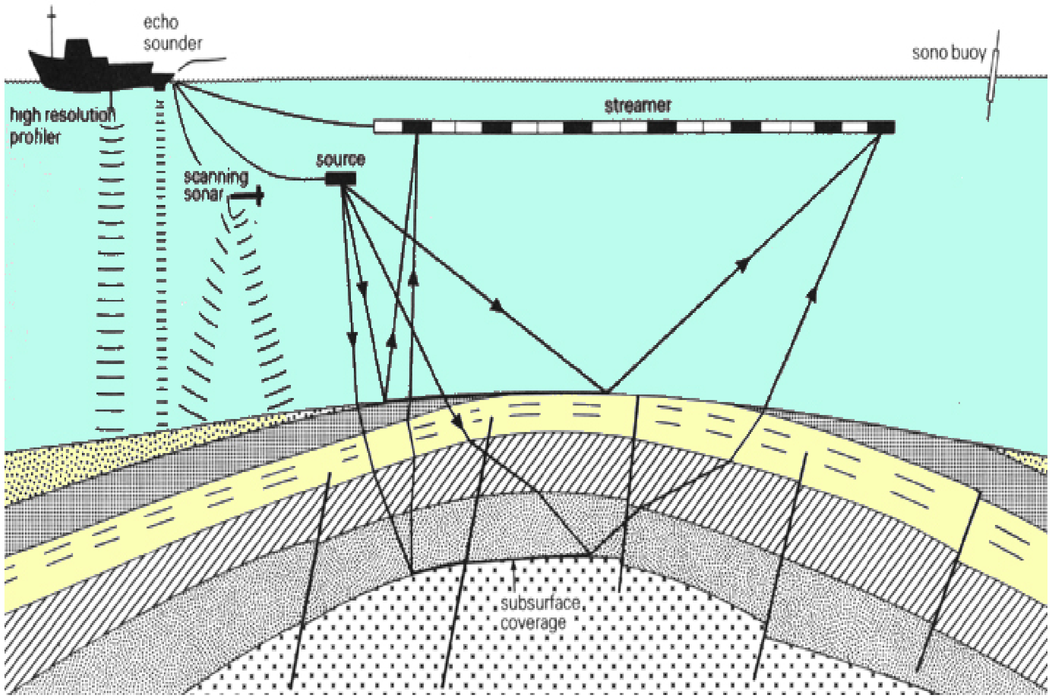
The SPECFEM3D source code is open (GNU GPL v2)

Mostly developed by Dimitri Komatitsch and Jeroen Tromp at Harvard University, Caltech and Princeton (USA) and later University of Pau (France) since 1996.

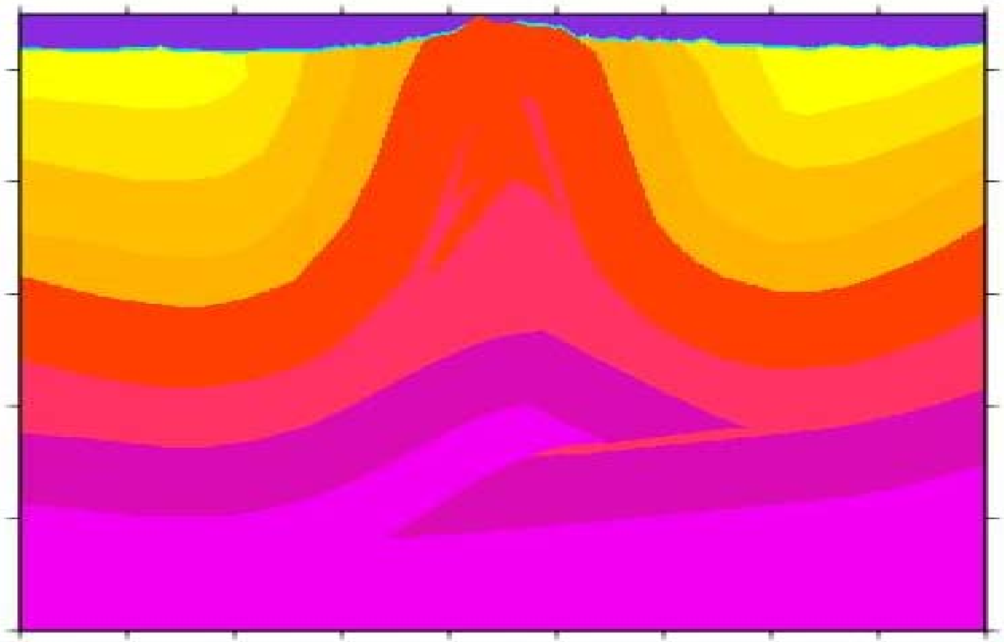
Improved with CNRS (Marseille, France), INRIA (Pau, France), the Barcelona Supercomputing Center (Spain) and University of Basel (Switzerland).

Oil industry applications

TOTAL S.A.



Offshore



In foothill regions

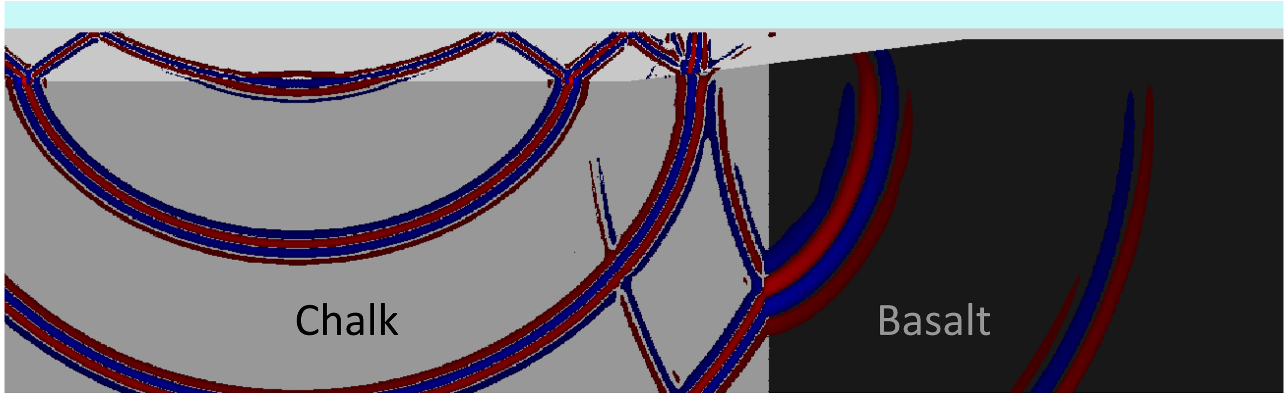
- Elastic wave propagation in complex 3D structures,
- Often fluid / solid problems: many oil fields are located offshore (deep offshore, or shallower).
- Anisotropic rocks, geological faults, cracks, bathymetry / topography…
- Thin weathered zone / layer at the surface $\Rightarrow$ model dispersive surface waves.

Ocean acoustics

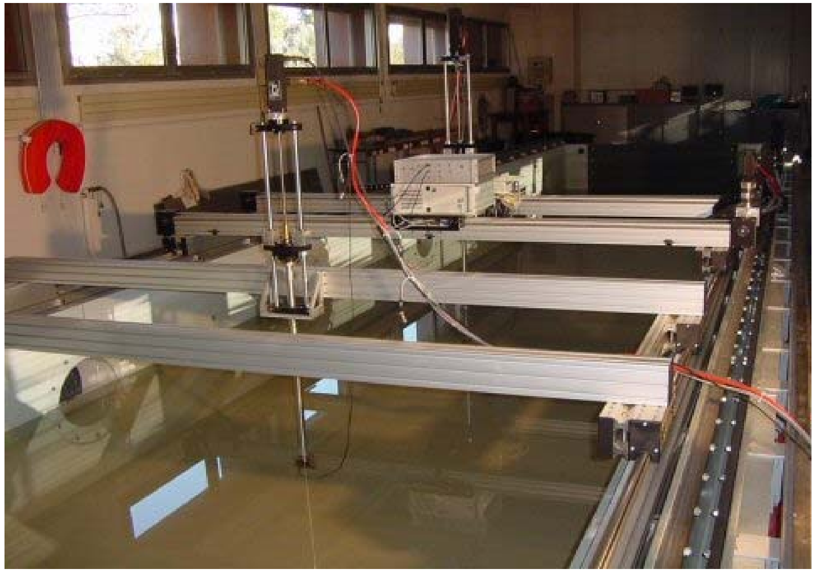
Numerical simulation

Wave propagation across an impedance discontinuity.

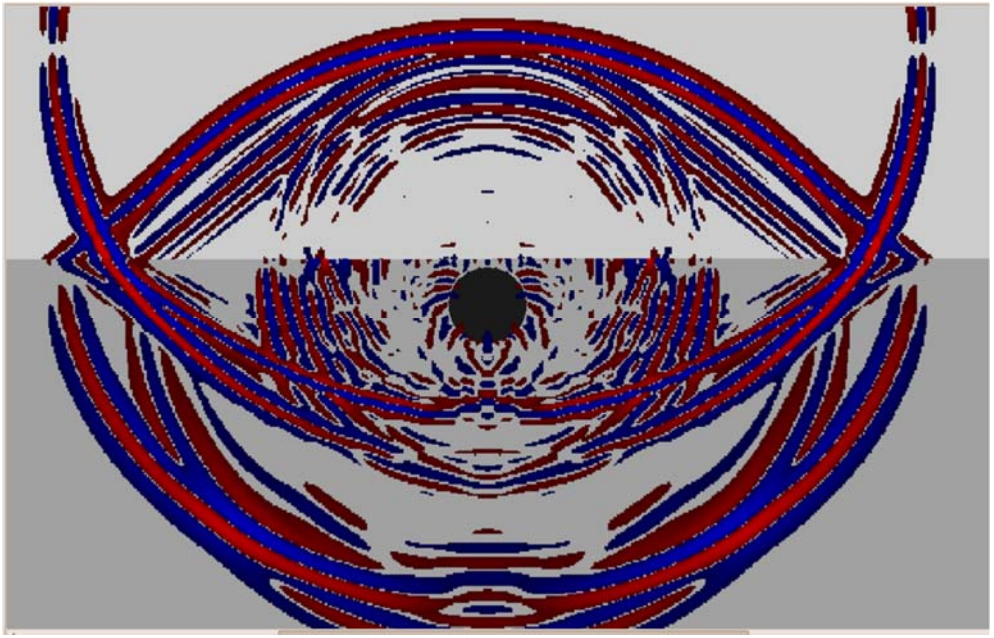
Influence on interface waves.



Experiments performed in tanks



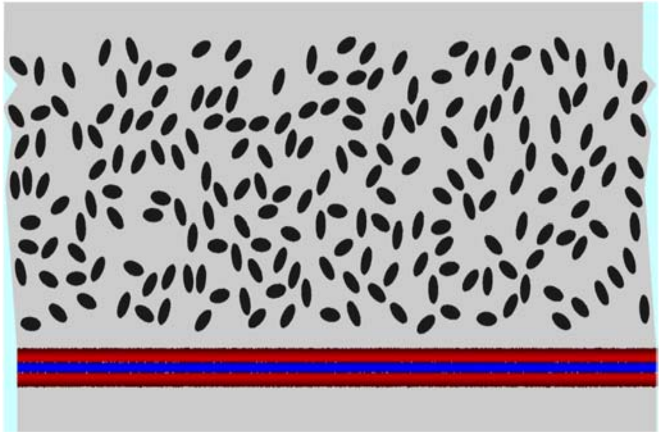
Experimental tanks in Marseille

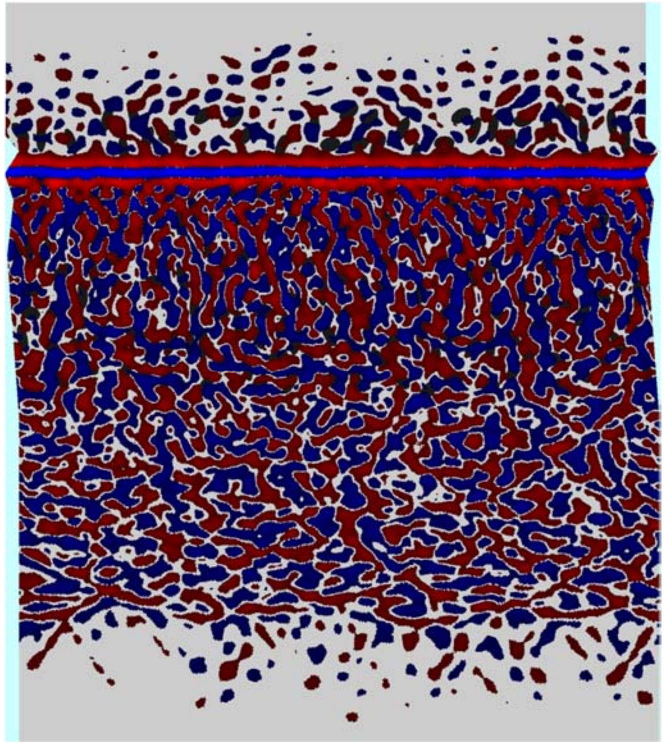


Experiments in known environment / setup

Perform experimental benchmarks

Non destructive testing of materials





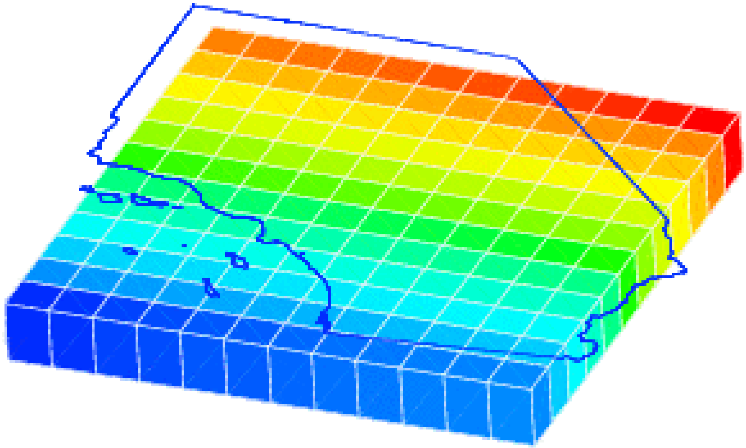
Collaboration with LCND in Aix-en-Provence, France.

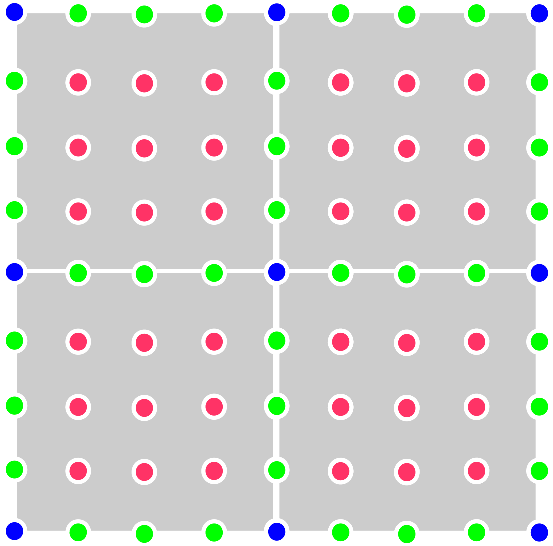
Currently at LCND: Physical modeling based on diffusion functions for objects of complex shape, cracks or multiple cavities in concrete, metals, or composite materials. Experiments on samples.

Very accurate calculations without homogenization can validate (or not) these diffusion functions and extend them beyond their domain of validity.

Reliable modeling of the “coda” part of the signal, which contains useful information on the medium.

High-performance & GPU computing





Parallel calculations based on non-blocking message passing (MPI), overlapping communication with calculations.

GPU cards:
Why are they so powerful for scientific computing?
Compute all pixels simultaneously, massive multithreading.

⇒ GPU computing: code is complex to rewrite, but large speedup can be obtained (but it is difficult to define speedup).

Adjoint methods for tomography and imaging

$$\chi_1(\mathbf{m}) = \frac{1}{2}\sum_{r=1}^{N_r}\int_0^T w_r(t)||\mathbf{s}(\mathbf{x}_r, t; \mathbf{m}) - \mathbf{d}(\mathbf{x}_r, t)||^2 \, \mathrm{d}t,$$

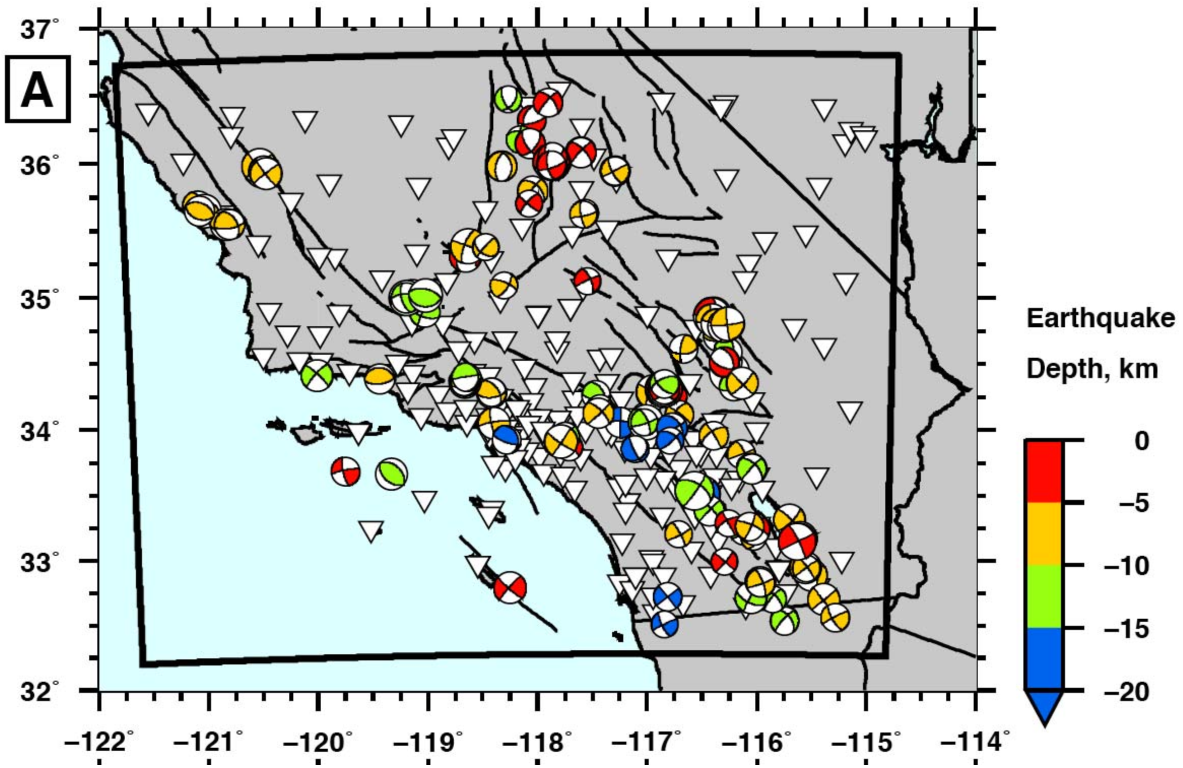
$$\delta\chi_1 = \int_V [K_\rho(\mathbf{x})\,\delta\ln\rho(\mathbf{x}) + K_\mu(\mathbf{x})\,\delta\ln\mu(\mathbf{x}) + K_\kappa(\mathbf{x})\,\delta\ln\kappa(\mathbf{x})]\,\mathrm{d}^3\mathbf{x},$$

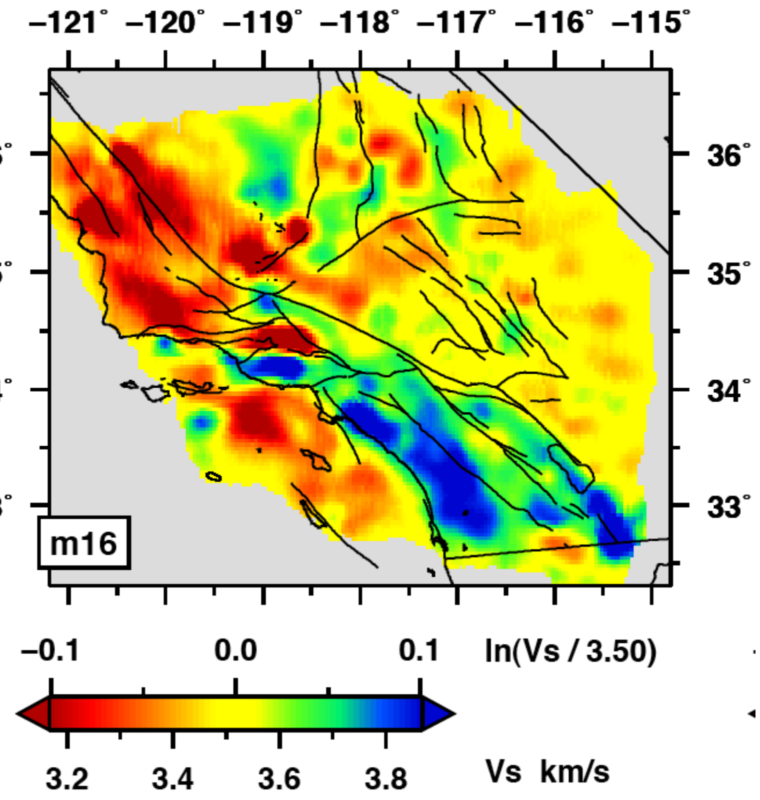
$$K_\kappa(\mathbf{x}) = -\int_0^T \kappa(\mathbf{x})\,[\nabla\cdot\mathbf{s}^\dagger(\mathbf{x}, T-t)]\,[\nabla\cdot\mathbf{s}(\mathbf{x}, t)]\,\mathrm{d}t,$$

Theory: A. Tarantola, Talagrand and Courtier.
‘Banana-Donut’ kernels (Tony Dahlen et al., Princeton)
Close to time reversal (Mathias Fink et al.) but not identical, thus interesting developments to do.

Idea: apply this to tomography of the full Earth (current project with Princeton University, USA), and in acoustic tomography: ocean acoustics, non destructive testing.

Tape et al. (2009): 143 earthquakes used in inversion





Depth 10 km

- 3 simulations per earthquake per iteration
- 16 iterations
- 6,864 simulations
- 168 processor cores per simulation
- 45 minutes of wall-clock time per simulation
- 864,864 processor core hours

Princeton, USA

Conclusions

- On modern computers, large 3D full-waveform forward modeling problems can be solved at high resolution for acoustic / elastic / viscoelastic / poroelastic / seismic waves
- The Legendre spectral-element method (SEM) is very efficient for that, in particular on modern parallel computers
- Inverse (adjoint) tomography / imaging problems can also be studied, although the cost is still high
- Useful in different industries in addition to academia: oil and gas, ocean acoustics / sonars, non destructive testing (concrete, composite media, fractures, cracks)
- Hybrid (GPU) computing is useful to solve inverse problems in seismic wave propagation and imaging