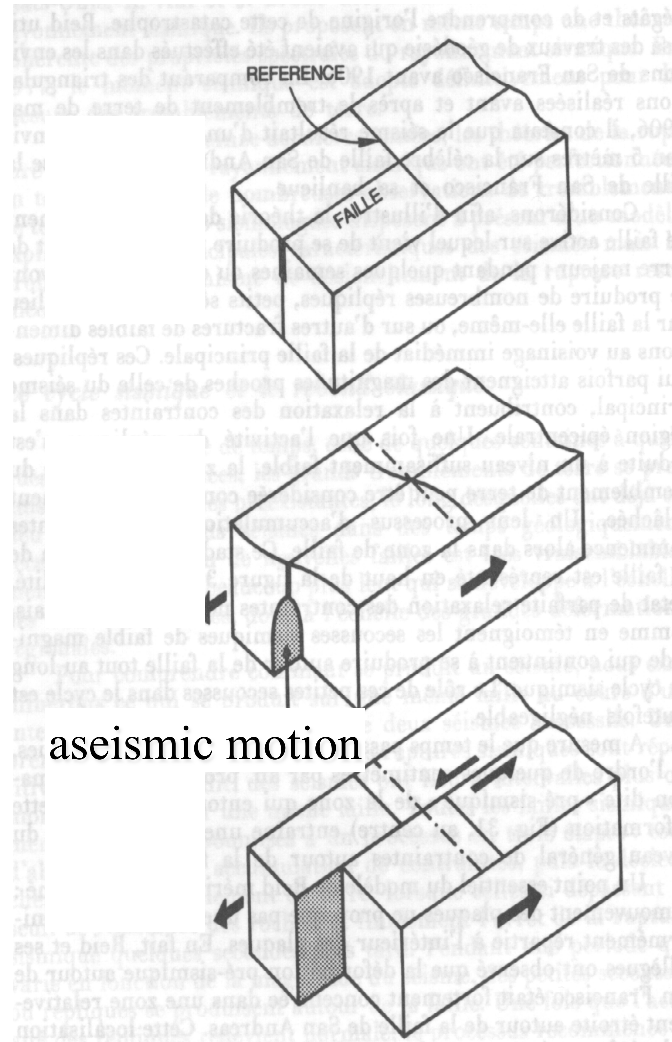
MEEES and M2R STU(TUE552)

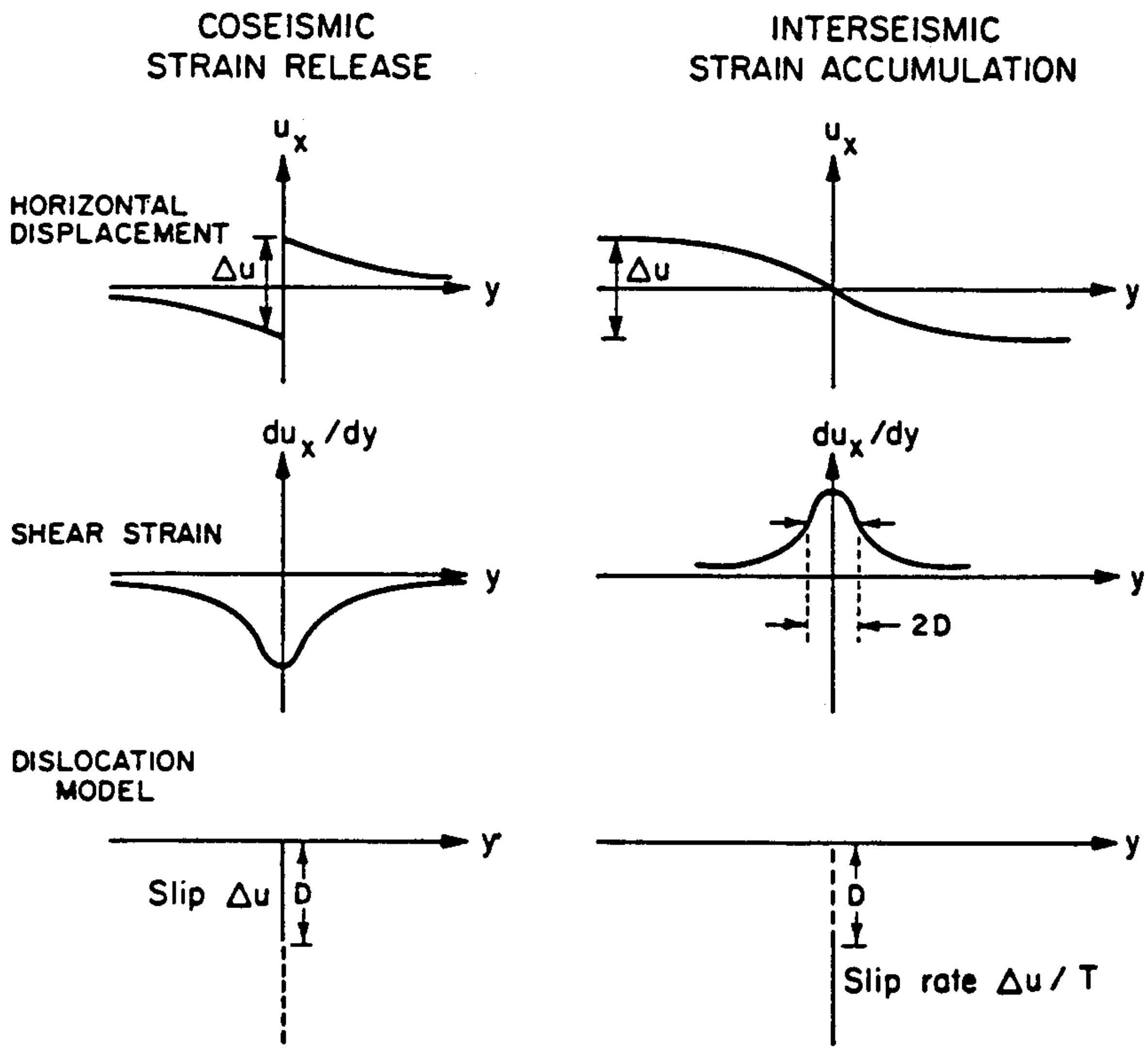
Seismology 1
(Michel Campillo)

http://www-lgit.obs.ujf-grenoble.fr/users/campillo/Master-TUE552

Plate motions, faults and « elastic rebound »



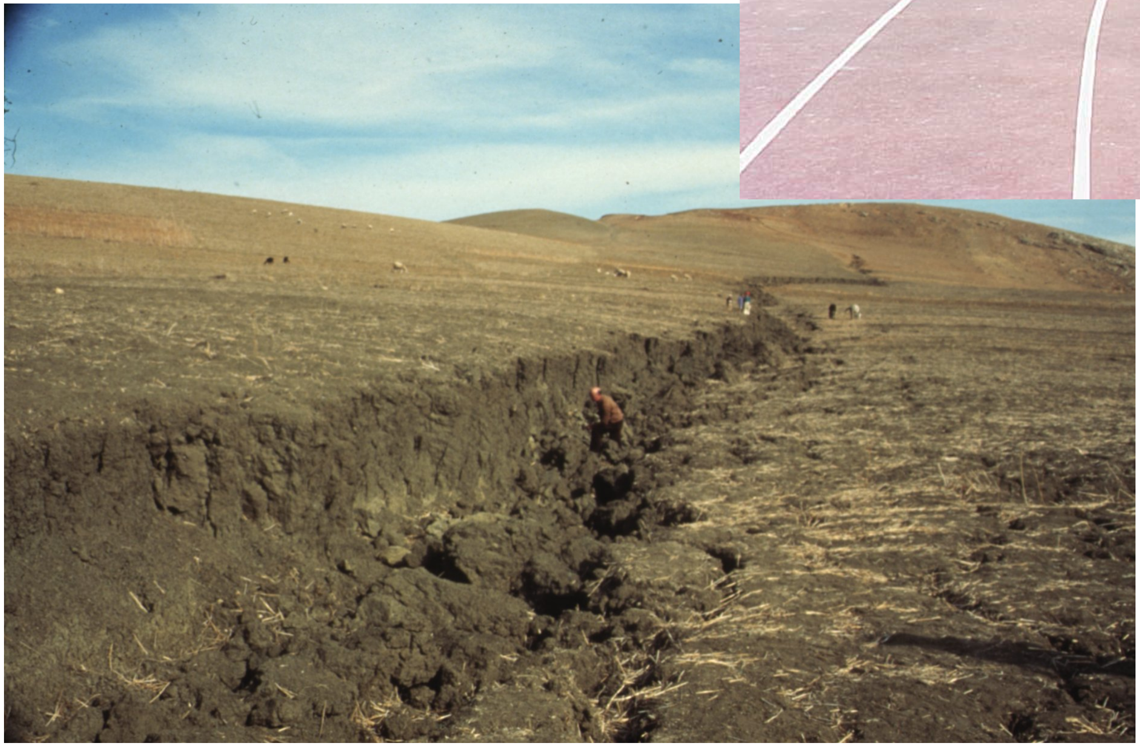
aseismic motion



Thatcher, 1986

Brittle behavior:

Taiwan, 2001



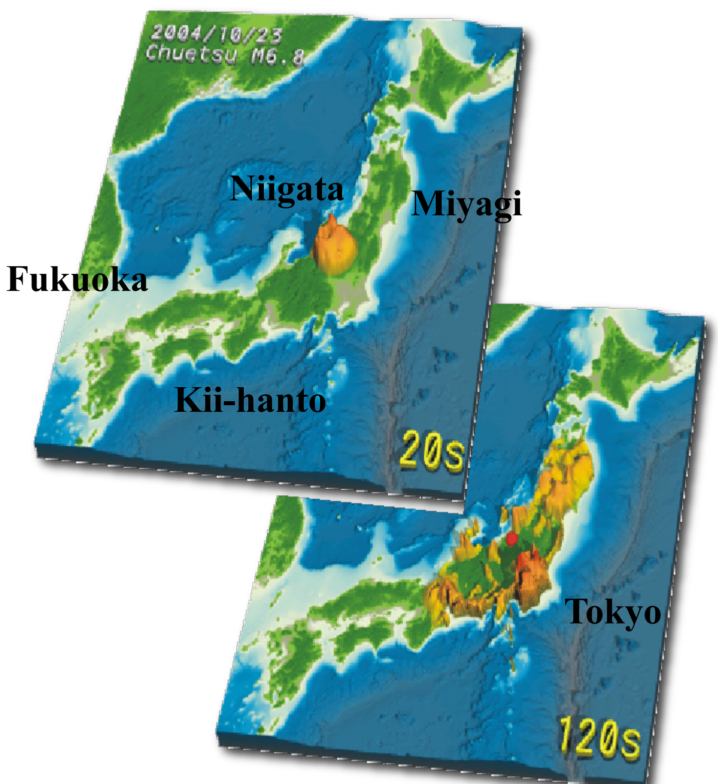
Algeria, 1980

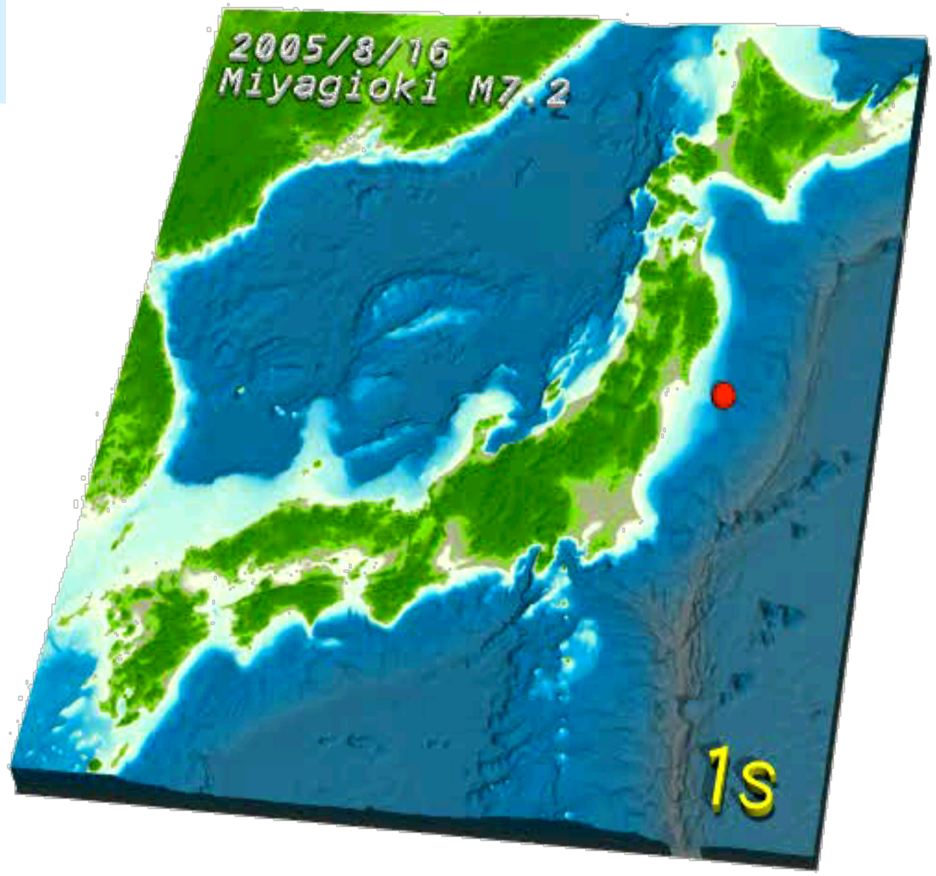
II. 10 Years after the Kobe earthquake
Dense Seismic Array

Nation Wide Strong Motion Network:
K-NET, KiK-net, 1800 Stations, NIED

Seismic Wave Propagation for Recent Earthquakes

Visualized Seismic Wavefield:
record from a Nation-Wide Seismic
Network of over 1800 stations





Recent Large Earthquakes:

2005 Miyagi-ken Oki (M7.2)
2004 Kii-hanto Nanto Oki（M7.6）
2005 Fukuoka-ken Seiho Oki（M7.0）
2004 Niigata-ken Chuetsu （M6.8）

EQ = dislocation

Dislocation= non elastic process

Point dislocation has an elastic equivant system of body forces

EQ = Double couple of forces

Radiation of EQ is represented by radiation of a system of forces

Radiation of a single impulsive point source= Green's function

Radiation of EQ is computed from a combination of Green's function

Green's function is the building block of earthquake simulation

Example of solutions

Laplace (scalar) equation

$$\frac{\partial^2 g}{\partial t^2} - c^2 \Delta^2 g = \delta(\vec{x})\delta(t)$$

$$\Rightarrow g(\vec{x}, t) = \frac{1}{4\pi c^2} \frac{1}{|\vec{x}|} \delta(t - \frac{|\vec{x}|}{c})$$

$$\frac{\partial^2 g}{\partial t^2} - c^2 \Delta^2 g = \delta(\vec{x} - \vec{\xi})\delta(t - \tau)$$

$$\Rightarrow \ g(\overrightarrow{x}, t) = \frac{1}{4\pi c^2} \frac{1}{\left|\vec{x} - \vec{\xi}\right|} \delta(t - \tau - \frac{\left|\vec{x} - \vec{\xi}\right|}{c})$$

$$\frac{\partial^2 g}{\partial t^2} - c^2 \Delta^2 g = \delta(\vec{x} - \vec{\xi}) f(t)$$

$$\Rightarrow \quad g(\overrightarrow{x}, t) = \frac{1}{4\pi c^2} \frac{1}{\left|\vec{x} - \vec{\xi}\right|} f\left(t - \frac{\left|\vec{x} - \vec{\xi}\right|}{c}\right)$$

Extended force

$$\frac{\partial^2 g}{\partial t^2} - c^2 \Delta^2 g = \frac{\phi(\vec{x}, t)}{\rho}$$

$$\Rightarrow \quad g(\overrightarrow{x}, t) = \frac{1}{4\pi c^2 \rho} \int\int\int_V \frac{\phi(\vec{\xi}, t - \frac{\left|\vec{x} - \vec{\xi}\right|}{c})}{\left|\vec{x} - \vec{\xi}\right|} dV(\vec{\xi})$$

Elasticity and elastic Green function

Strain:

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

Stress:

$$\tau_{ij} = c_{ijpq} e_{pq}$$

with c_{ijpq} are the elastic constants.
It reduces for an isotropic body to:

$$\tau_{ij} = \lambda \delta_{ij} e_{kk} + 2\mu e_{ij}$$

where λ and μ are the Lamé coefficients.

Equation of motion:

$$\rho \ddot{u}_i = \tau_{ij,j}$$

In a homogeneous space:

$$\rho \frac{\partial^2 \overrightarrow{u}}{\partial t^2} = (\lambda + 2\mu) \overrightarrow{grad}(div(\overrightarrow{u})) - \mu \overrightarrow{curl}(\overrightarrow{curl}(\overrightarrow{u}))$$

Potential decomposition:

$$\begin{aligned}\overrightarrow{u} &= \overrightarrow{grad}(\varphi) \quad + \quad \overrightarrow{curl}(\overrightarrow{\psi}) \\ &= \overrightarrow{u_P} \quad + \quad \overrightarrow{u_S}\end{aligned}$$

If we can find $\vec{\chi}$ satisfying the Laplace equation:

$$\vec{\Delta}\vec{\chi} = \overrightarrow{u}$$

we obtain

$$\varphi = div(\vec{\chi}) \; and \; \overrightarrow{\psi} = \overrightarrow{curl}(\vec{\chi})$$

$$\begin{aligned}(\varphi, \overrightarrow{u_P})\text{: compressional waves with velocity } \alpha &= \sqrt{\frac{\lambda + 2\mu}{\rho}} \\ (\overrightarrow{\psi}, \overrightarrow{u_S})\text{: shear waves with velocity } \beta &= \sqrt{\frac{\mu}{\rho}}\end{aligned}$$

Green function

$$G_{in}(\overrightarrow{x}, 0; \overrightarrow{\xi}, \tau)$$

Displacement produced in $(\overrightarrow{x}, t)$ in direction i by an impulse point force in $\overrightarrow{\xi}$ in direction n at $t = \tau$:

$$\rho \frac{\partial}{\partial t^2} G_{in} = \delta_{in} \delta(\overrightarrow{x} - \overrightarrow{\xi}) \delta(t - \tau) + \frac{\partial}{\partial x_j} [c_{ijkl} \frac{\partial}{\partial x_j} G_{kn}]$$

The GF contains all the informations about the response of the Earth to an arbitrary source

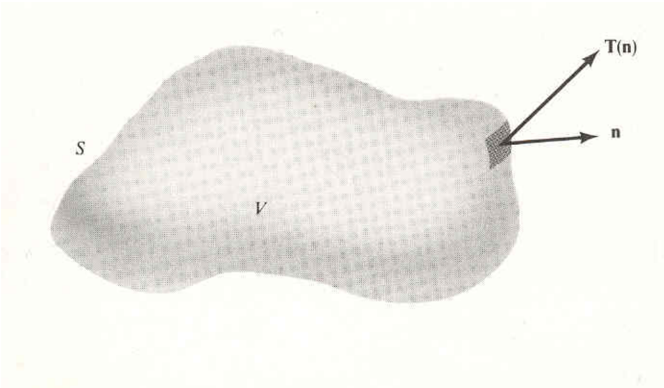
Reciprocity theorem:

With homogeneous conditions (either $\overrightarrow{u}$ or $\overrightarrow{T}$ equals 0 on S):

$$G_{nm}(\overrightarrow{\xi_2}, \tau; \overrightarrow{\xi_1}, 0) = G_{mn}(\overrightarrow{\xi_1}, \tau; \overrightarrow{\xi_2}, 0)$$

Uniqueness theorem

Elastic body of volume V limited by S
$\overrightarrow{u}(\overrightarrow{x}, t)$ in V is uniquely defined after time t_0 by:
-the initial values of $\overrightarrow{u}$ and $\overrightarrow{\dot{u}}$ in V
-the values of the body forces for $t > t_0$,
-the tractions over any part S_1 of S $(t > t_0)$
-the displacements over S_2 , $S = S_1 + S_2$.



Representation theorem

$$
\begin{aligned}
u_n(\overrightarrow{x}, t) &= \int_{-\infty}^{\infty} d\tau \iiint_V f_i(\overrightarrow{\xi}, \tau) G_{in}(\overrightarrow{\xi}, t-\tau; \overrightarrow{x}, 0) dV(\overrightarrow{\xi}) \\
&+ \int_{-\infty}^{\infty} d\tau \iint_S (G_{in}(\overrightarrow{\xi}, t-\tau; \overrightarrow{x}, 0) T_i(\overrightarrow{u}(\overrightarrow{\xi}, \tau)) \\
&- u_i(\overrightarrow{\xi}, t) c_{ijkl}(\overrightarrow{\xi}) G_{kn,l}(\overrightarrow{\xi}, t-\tau; \overrightarrow{x}, 0)) dS(\overrightarrow{\xi})
\end{aligned}
$$

Solving the elastodynamic equation….

In a homogeneous space and an applied force

$$\rho \frac{\partial^2 \vec{u}}{\partial t^2} = (\lambda + 2\mu)\overrightarrow{grad}(div(\vec{u})) - \mu\overrightarrow{curl}(\overrightarrow{curl}(\vec{u})) + \vec{f}$$

$$\frac{\partial^2 \varphi}{\partial t^2} = \alpha^2 \Delta \varphi$$
$$\frac{\partial^2 \vec{\psi}}{\partial t^2} = \beta^2 \Delta \vec{\psi}$$

Green function (Laplace equation):

$$\frac{\partial^2 \varphi}{\partial t^2} = \alpha^2 \Delta \varphi + \delta(\vec{x})\delta(t)$$

$$\varphi(\vec{x},t) = \frac{1}{4\pi\alpha^2}\frac{1}{x}\delta(t - \frac{x}{\alpha})$$

Potential decomposition of the source term $\vec{f}(\delta(t),0,0)$:

$$\vec{f} = \overrightarrow{grad}(\varphi_f) \quad + \quad \overrightarrow{curl}(\vec{\psi}_f)$$

$$\varphi_f = div(\vec{W})$$
$$\vec{\psi}_f = -\overrightarrow{curl}(\vec{W})$$

with

$$\vec{W}(\vec{x},t) = -\frac{\delta(t)}{4\pi}\frac{\vec{x}_1}{x}$$

Two equations for potentials:

$$\frac{\partial^2 \varphi}{\partial t^2} = \alpha^2 \Delta \varphi + \varphi_f$$
$$\frac{\partial^2 \vec{\psi}}{\partial t^2} = \beta^2 \Delta \vec{\psi} + \vec{\psi}_f$$

The potential decomposition of a impulse force is extended in space:
$\varphi_f(\overrightarrow{\xi}), \overrightarrow{\psi_f}(\overrightarrow{\xi}) \Rightarrow$ **Near field terms**

Green function for the homogeneous space

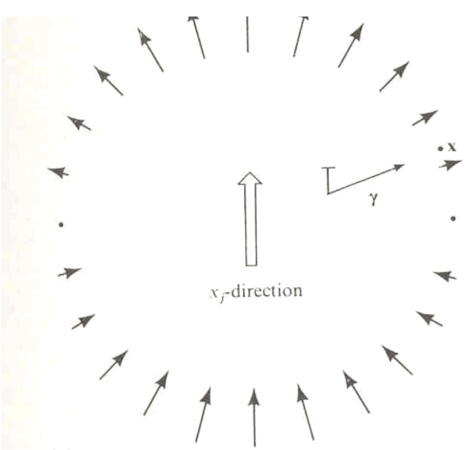
$$\gamma_i = \frac{x_i}{x} = \frac{\partial x}{\partial x_i}$$

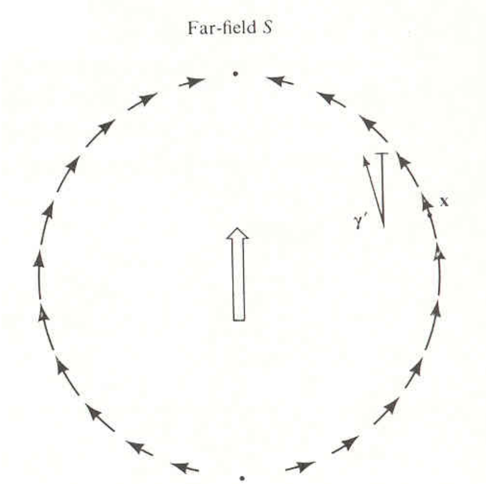
$$\begin{aligned} G_ij(\overrightarrow{x}, t; \overrightarrow{0}, 0) &= \frac{1}{4\pi\rho}(3\gamma_i\gamma_j - \delta_{ij})\frac{1}{x^3}\int_{\frac{x}{\alpha}}^{\frac{x}{\beta}} \tau\delta(t-\tau)d\tau \\ &+ \frac{1}{4\pi\rho\alpha^2}\gamma_i\gamma_j\frac{1}{x}\delta(t-\frac{x}{\alpha}) \\ &- \frac{1}{4\pi\rho\beta^2}(\gamma_i\gamma_j - \delta_{ij})\frac{1}{x}\delta(t-\frac{x}{\beta}) \end{aligned}$$

Near field

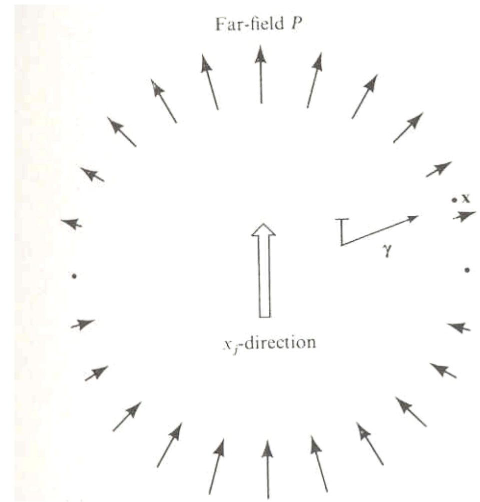
Far field P

Far field S





P far field term: $$\frac{1}{4\pi\rho\alpha^2}\gamma_i\gamma_j\frac{1}{x}\delta(t-\frac{x}{\alpha})$$



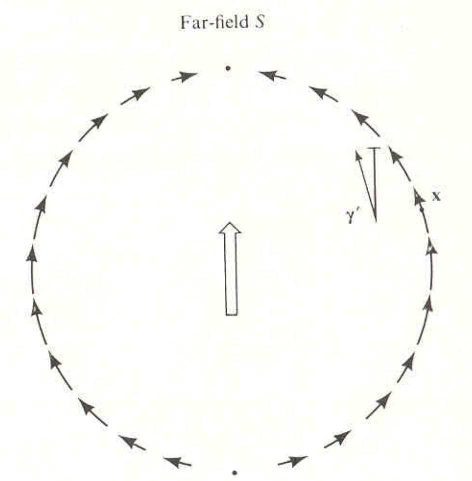
-spherical wave expansion: 1/r

-causality

-waveshape = time dependance of the applied force

- longitudinal polarization

S far field term: $$- \frac{1}{4\pi\rho\beta^2}(\gamma_i\gamma_j - \delta_{ij})\frac{1}{x}\delta(t - \frac{x}{\beta})$$



-spherical wave expansion : 1/r

-causality

-waveshape = time dependance of the applied force

- transversal polarization

The potential decomposition of a impulse force is extended in space:
$\varphi_f(\overrightarrow{\xi}), \overrightarrow{\psi_f}(\overrightarrow{\xi}) \Rightarrow$ **Near field terms**

Green function for the homogeneous space

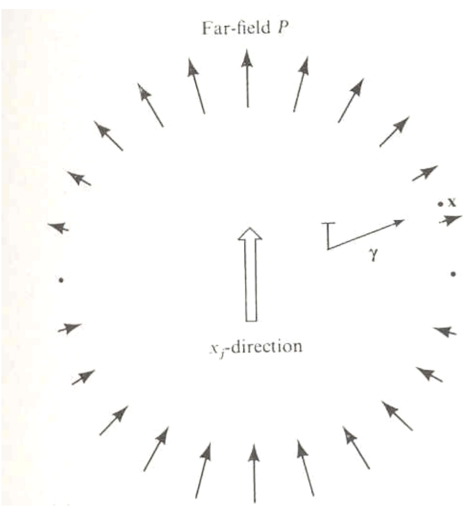
$$\gamma_i = \frac{x_i}{x} = \frac{\partial x}{\partial x_i}$$

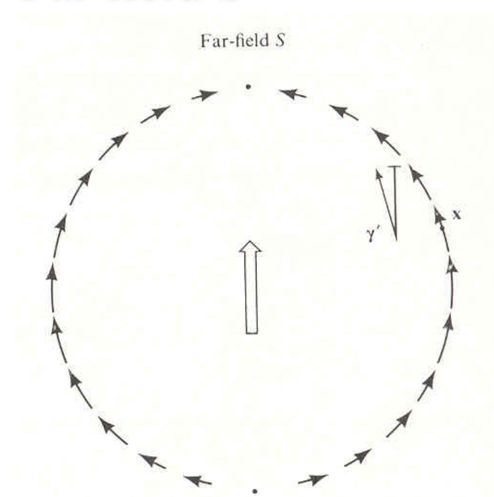
$$\begin{aligned} G_i j(\overrightarrow{x}, t; \overrightarrow{0}, 0) &= \frac{1}{4\pi\rho}(3\gamma_i\gamma_j - \delta_{ij})\frac{1}{x^3}\int_{\frac{x}{\alpha}}^{\frac{x}{\beta}} \tau\delta(t-\tau)d\tau \\ &+ \frac{1}{4\pi\rho\alpha^2}\gamma_i\gamma_j\frac{1}{x}\delta(t-\frac{x}{\alpha}) \\ &+ \frac{1}{4\pi\rho\beta^2}(\gamma_i\gamma_j - \delta_{ij})\frac{1}{x}\delta(t-\frac{x}{\beta}) \end{aligned}$$

Near field

Far field P

Far field S





Near field term:

$$\frac{1}{4\pi\rho}(3\gamma_i\gamma_j - \delta_{ij})\frac{1}{x^3}\int_{\frac{x}{\alpha}}^{\frac{x}{\beta}} \tau\delta(t-\tau)d\tau$$

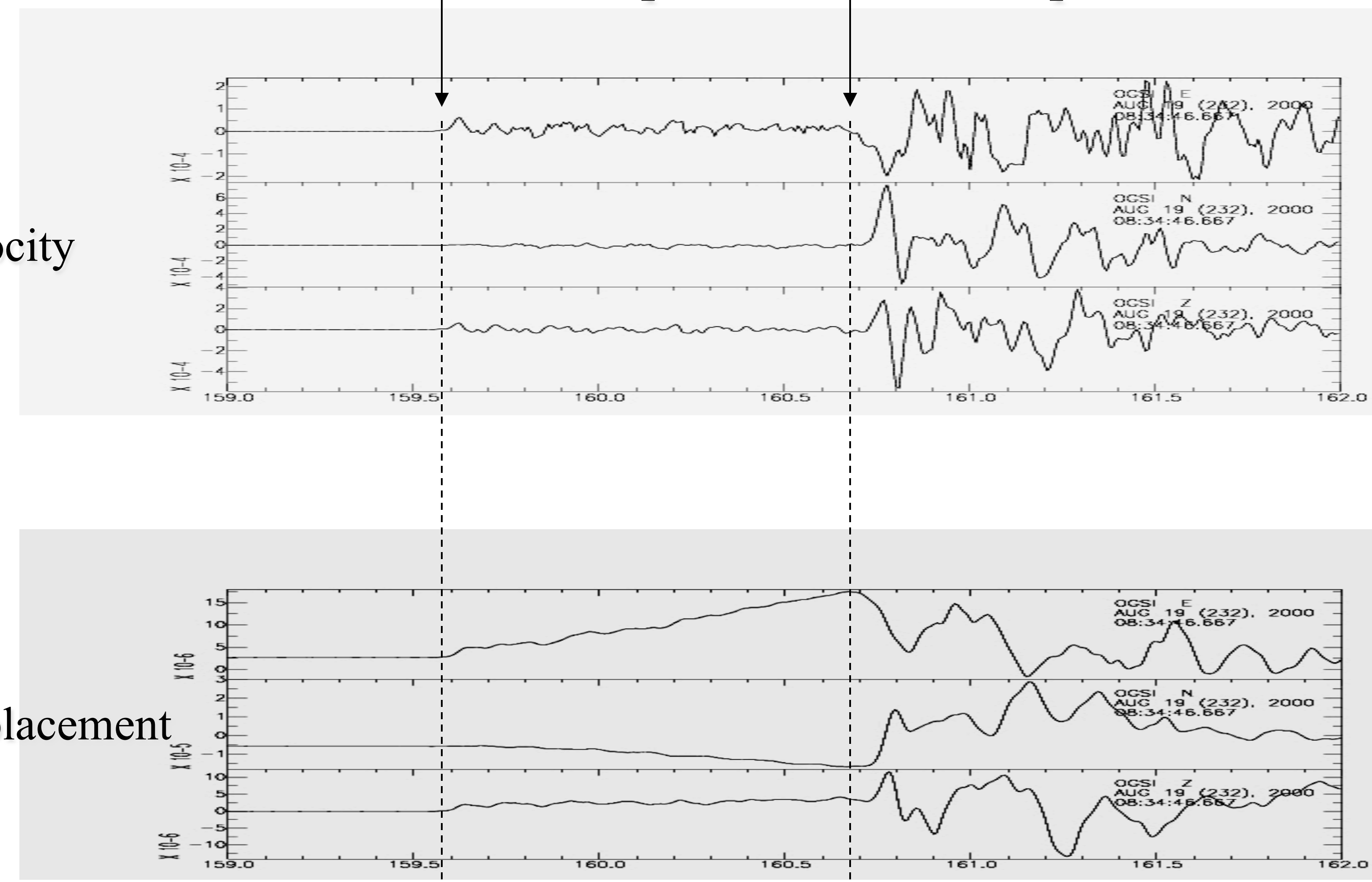
- P and $S \rightarrow$ polarization
- decay in $1/r^2$
- wave shape changing with distance

Earthquake in Samoens recorded at a distance of 5km

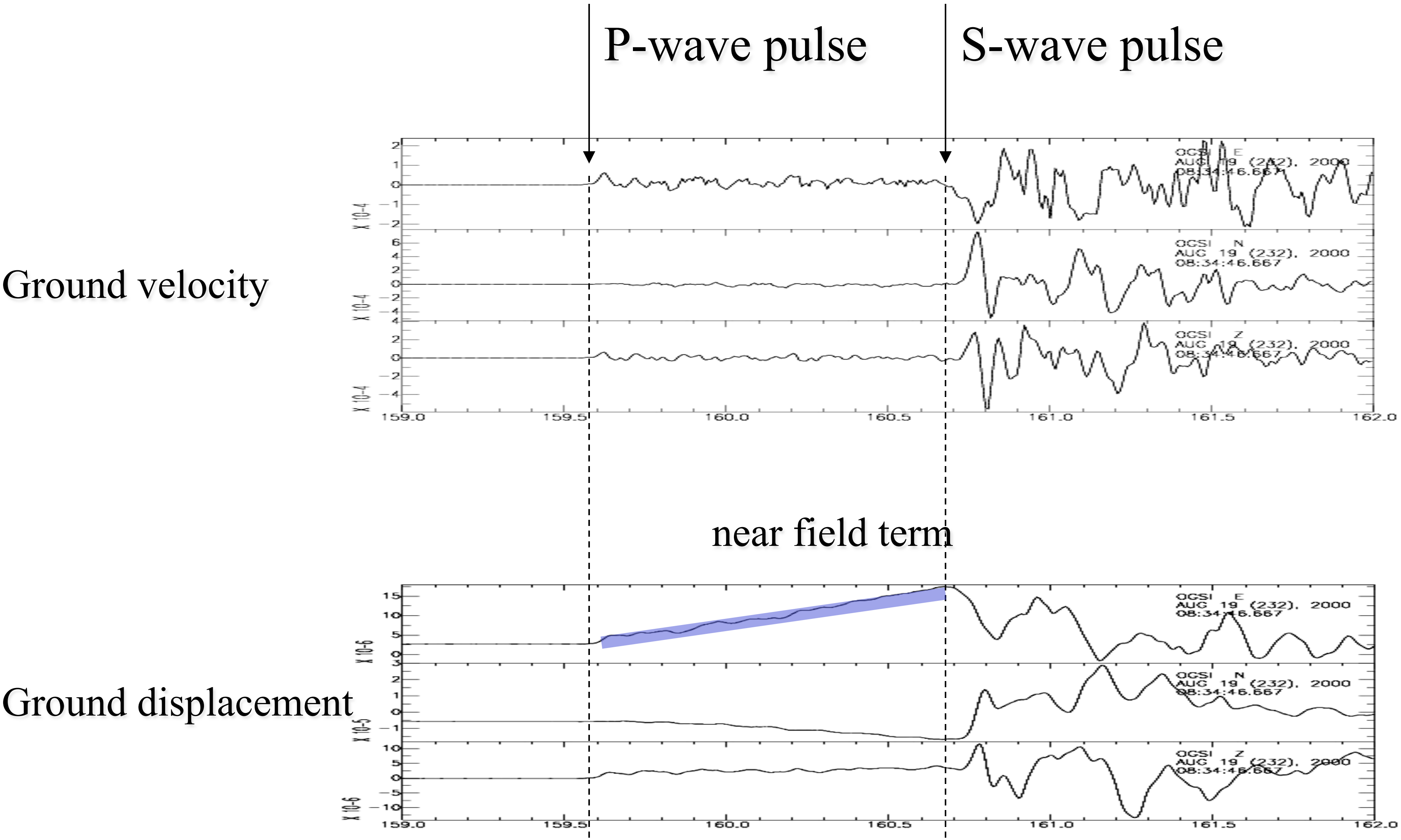
P-wave pulse S-wave pulse

Ground velocity

Ground displacement



Earthquake in Samoens recorded at a distance of 5km



Green's functions in the spectral domain

THE 2D SCALAR CASE

The Green function in the frequency domain is :

$$G_{22} = \frac{1}{i4\rho}\,\frac{H_0^{(2)}(kr)}{\beta^2},$$

$G_{22}(\mathbf{x}, \mathbf{y}, \omega) = 1/4\mathrm{i}\mu\,[J_0(kr) - \mathrm{i}Y_0(kr)]$,

where $Y_0\,(kr)$ = Neumann function of zero order and μ = shear modulus.

$J_0(kr)$ is proportional to the imaginary part of the Green function.

$$G_{22}(\mathbf{x},\mathbf{y},t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_{22}(\mathbf{x},\mathbf{y},\omega)\exp(\mathbf{i}\omega t)\,d\omega = \frac{1}{2\pi\mu}\,\frac{H(t - r/\beta)}{\sqrt{t^2 - r^2/\beta^2}}$$

Applications:

Physical origins of the distance decay of motions:

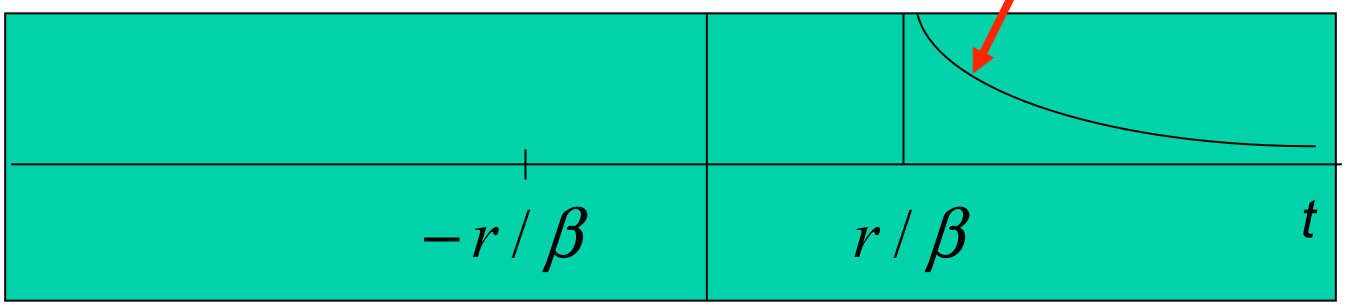
1) Elastic space: form of the decay? Approximation? Depth dependance?

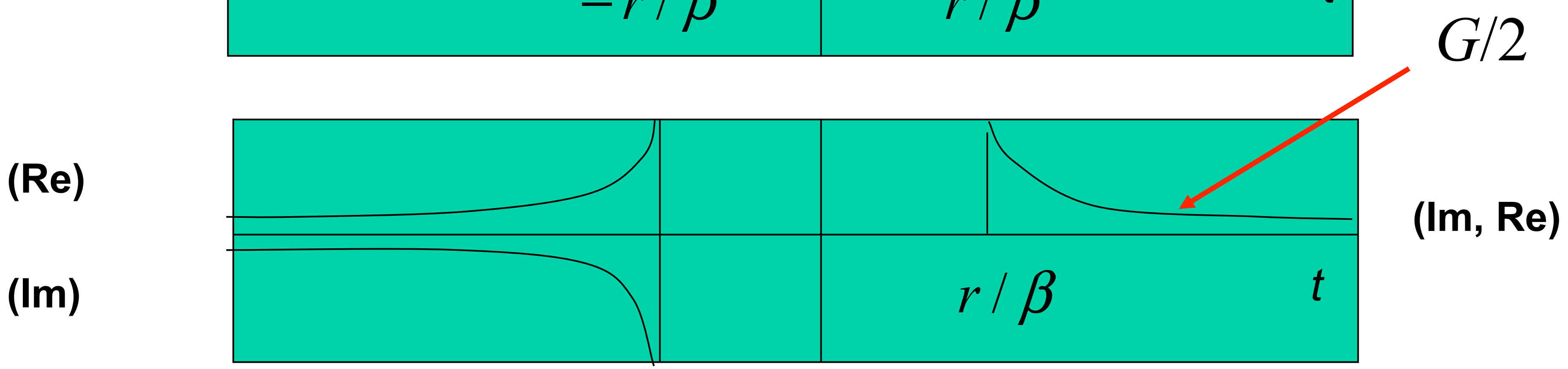
2)Attenuation: loss per cycle?

Causality

$$G = \frac{1}{4\mathrm{i}\mu} H_0^{(2)}\left(\frac{\omega r}{\beta}\right)$$

$$G = \frac{1}{2\pi\mu} \frac{H\left(t - r/\beta\right)}{\sqrt{t^2 - r^2/\beta^2}}$$





THE 2D VECTORIAL (P-SV) GREEN FUNCTION

$$\beta^2 \frac{\partial^2 u_i}{\partial x_j \partial x_j} + (\alpha^2 - \beta^2)\frac{\partial^2 u_j}{\partial x_i \partial x_j} = \frac{\partial^2 u_i}{\partial t^2}$$

$$G_{ij} = \frac{1}{i8\rho}\left[\delta_{ij}A - (2\gamma_i\gamma_j - \delta_{ij})B\right] \quad i, j = 1, 3,$$

$$A = \frac{H_0^{(2)}(qr)}{\alpha^2} + \frac{H_0^{(2)}(kr)}{\beta^2},$$

$$B = \frac{H_2^{(2)}(qr)}{\alpha^2} - \frac{H_2^{(2)}(kr)}{\beta^2},$$

THE 3D VECTORIAL GREEN FUNCTION

$$\beta^2 \frac{\partial^2 u_i}{\partial x_j \partial x_j} + (\alpha^2 - \beta^2)\frac{\partial^2 u_j}{\partial x_i \partial x_j} = \frac{\partial^2 u_i}{\partial t^2}$$

$$G_{ij}(\mathbf{x}, \mathbf{y}) = \frac{1}{4\pi\mu r}\left[f_2\delta_{ij} + (f_1 - f_2)\gamma_i\gamma_j\right]$$

where f_1 and f_2 are Stokes' functions, and they are given by

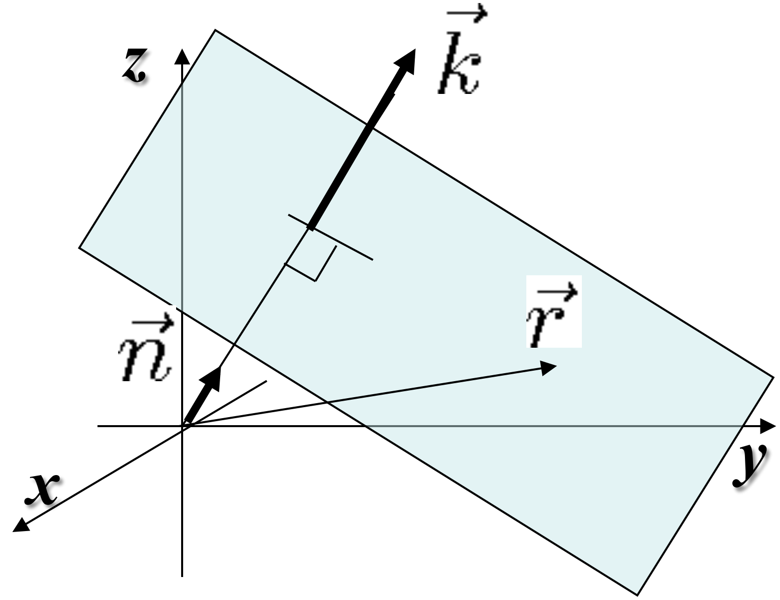
$$f_1 = (\beta^2/\alpha^2)\left[1 - \mathrm{i}2/(qr) - 2/(qr)^2\right]\exp(-\mathrm{i}qr) + \left[\mathrm{i}2/(kr) + 2/(kr)^2\right]\exp(-\mathrm{i}kr)$$

$$f_2 = (\beta^2/\alpha^2)\left[\mathrm{i}/(qr) + 1/(qr)^2\right]\exp(-\mathrm{i}qr) + \left[1 - \mathrm{i}/(kr) - 1/(kr)^2\right]\exp(-\mathrm{i}kr)$$

Useful solutions....

Plane waves

Cartesian coordinates:
$\frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = \Delta f$



Plane wave propagating in the direction of unitary vector $\vec{n}$:

$$F = f(\vec{n}\vec{r} - vt) + g(\vec{n}\vec{r} + vt)$$

Equiphase surface is a plane defined by [$\vec{n}\vec{r} - vt =$constant]

With harmonic wave of angular frequency $\omega = 2\pi f$

$F = f_0 exp(i(\vec{k}\vec{r} - \omega t)$ avec $\vec{k} = \vec{n}\frac{\omega}{v}$ wave vector.

Weyl integral:

Let us consider a scalar equation, as for a potential ϕ, with velocity c:

$$\frac{\partial^2 \phi}{\partial t^2} = c^2 \Delta \phi$$

Harmonic plane waves:

$$P = \phi_0 \exp(-i\omega t + i\vec{k}\vec{x})$$

are solutions as far as:

$$|\vec{k}| = \frac{\omega}{c}.$$

Note that the 3 components of k are not independant.

The Green function is the solution of the harmonic equation with an impulse source :

$$\frac{\partial^2 \phi}{\partial t^2} = c^2 \Delta \phi + 4\pi c^2 \delta(\vec{x}) \exp -i\omega t$$

(Note that its solution is a spherical wave: $\frac{1}{R} \exp -i\omega(t - R/c)$.)

For a plane wave of wavenumber $\vec{k}$, we get:

$$\phi(\vec{k}, t) = \frac{4\pi c^2}{c^2 |\vec{k}|^2 - \omega^2} \exp -i\omega t$$

By use of the regular Fourier transform:

$$\phi(\vec{x}, t) = \frac{1}{(2\pi)^3} \int\int\int \phi(\vec{k}, t) \exp\left(i\vec{k}\vec{x}\right) d\vec{k} \exp -i\omega t$$

we get

$$G(\vec{x}, t) = \frac{\exp -i\omega t}{2\pi^2} \int\int\int \frac{\exp i\vec{k}\vec{x}}{|\vec{k}|^2 - \omega^2/c^2} \, d\vec{k}$$

Integration over k_z.

We must evaluate:

$$I = \int \frac{\exp i(k_x x + k_y y) \exp i(k_z z)}{k_x^2 + k_y^2 + k_z^2 - \omega^2/c^2} dk_z$$

There is a singularity for:

$$k_z = (\omega^2/c^2 - k_x^2 - k_y^2)^{-1/2} = \nu$$

that is with $k_z \in \Re$ if $c \in \Re$.

If we consider the presence of absorption, c becomes complex and the singularities are moved in the complex plane. (Another practical possibility is to allow a small imaginary part for the frequency ω).

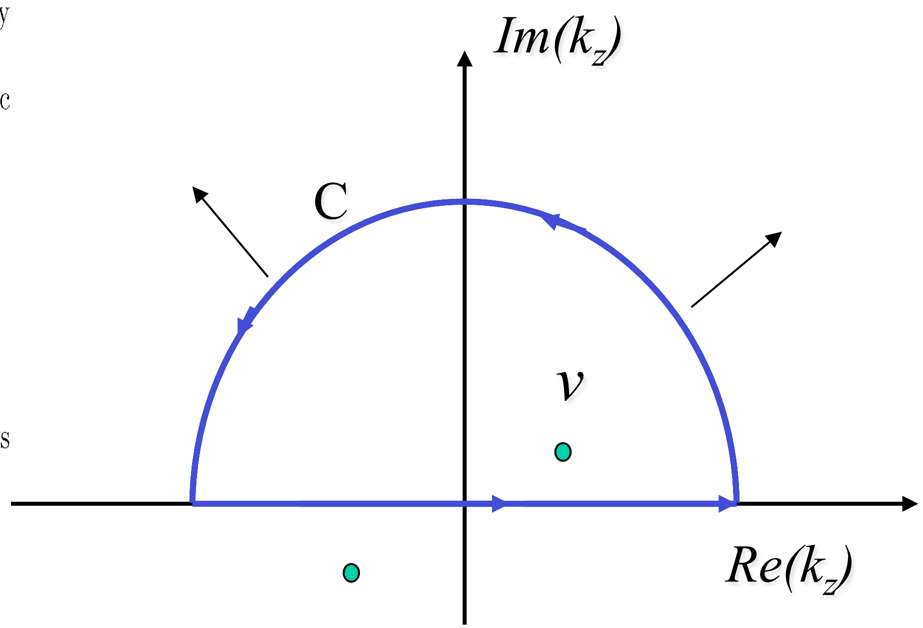
We evaluate I using the theorem of the residues. We choose an ad hoc contour of integration C.

$$\int_C \bullet\, dl = \int_{\Re} \bullet\, dk_z + \oint_{C\infty} \bullet\, dl = 2i\pi R$$

with R the residue associated with the singularity in $k_z = \nu$.

Note that by moving the half circle C to infinity, we ensure that its contribution is 0 and therefore:

$$I = 2i\pi R$$



The **residue theorem**, sometimes called **Cauchy's Residue Theorem**[1], in complex analysis is a powerful tool to evaluate line integrals of analytic functions over closed curves and can often be used to compute real integrals as well. It generalizes the Cauchy integral theorem and Cauchy's integral formula.

The statement is as follows. Suppose U is a simply connected open subset of the complex plane, and $a_1,\ldots,a_n$ are finitely many points of U and f is a function which is defined and holomorphic on $U\setminus\{a_1,\ldots,a_n\}$. If γ is a rectifiable curve in U which bounds the a_k, but does not meet any and whose start point equals its endpoint, then

$$\oint_\gamma f(z)\,dz = 2\pi i \sum_{k=1}^n \mathrm{I}(\gamma, a_k)\mathrm{Res}(f, a_k).$$

If γ is a Jordan curve, $\mathrm{I}(\gamma, a_k) = 1$ and so

$$\oint_\gamma f(z)\,dz = 2\pi i \sum_{k=1}^n \mathrm{Res}(f, a_k).$$

Here, $\mathrm{Res}(f, a_k)$ denotes the residue of f at a_k, and $\mathrm{I}(\gamma, a_k)$ is the winding number of the curve γ about the point a_k. This winding number is an integer which intuitively measures how many times the curve γ winds around the point a_k; it is positive if γ moves in a counter clockwise ("mathematically positive") manner around a_k and 0 if γ doesn't move around a_k at all.

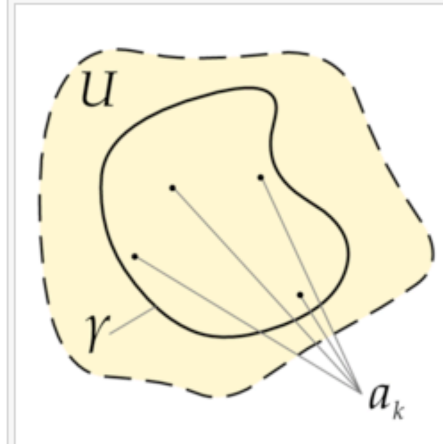


Illustration of the setting.

In order to evaluate real integrals, the residue theorem is used in the following manner: the integrand is extended to the complex plane and its residues are computed (which is usually easy), and a part of the real axis is extended to a closed curve by attaching a half-circle in the upper or lower half-plane. The integral over this curve can then be computed using the residue theorem. Often, the half-circle part of the integral will tend towards zero as the radius of the half-circle grows, leaving only the real-axis part of the integral, the one we were originally interested in.

In mathematics, the **Laurent series** of a complex function $f(z)$ is a representation of that function as a power series which includes terms of negative degree. It may be used to express complex functions in cases where a Taylor series expansion cannot be applied. The Laurent series was named after and first published by Pierre Alphonse Laurent in 1843. Karl Weierstrass may have discovered it first in 1841 but did not publish it at the time.

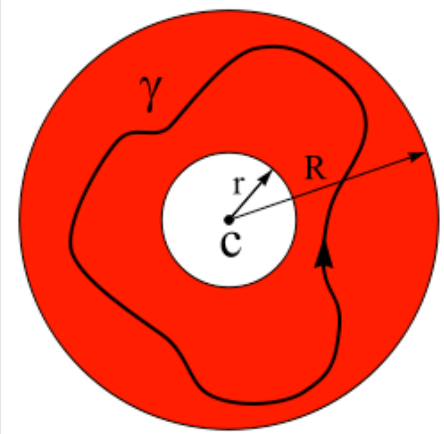
The Laurent series for a complex function $f(z)$ about a point c is given by:

$$f(z) = \sum_{n=-\infty}^{\infty} a_n (z-c)^n$$

where the a_n are constants, defined by a line integral which is a generalization of Cauchy's integral formula:

$$a_n = \frac{1}{2\pi i} \oint_\gamma \frac{f(z)\,dz}{(z-c)^{n+1}}.$$

The path of integration γ is counterclockwise around a closed, rectifiable path containing no self-intersections, enclosing c and lying in an annulus A in which $f(z)$ is holomorphic (analytic). The expansion for $f(z)$ will be valid anywhere inside this annulus. The annulus is shown in red in the diagram on the right, along with an example of a suitable path of integration labelled γ. In practice, this formula is rarely used because the integrals are difficult to evaluate; instead, one typically pieces together the Laurent series by combining known Taylor expansions. The numbers a_n and c are most commonly taken to be complex numbers, although there are other possibilities, as described below.



A Laurent series is defined with respect to a particular point c and a path of integration γ. The path of integration must lie in an annulus (shown here in red) inside of which $f(z)$ is holomorphic (analytic).

$$I = 2i\pi R$$

We have to make a choice for the sign of ν.

$$\nu = \pm(\sqrt{\omega^2/c^2 - k_x^2 - k_y^2}) = (\nu_R + i\nu_I)$$

with $\exp i\nu z \rightarrow 0$ when $|z| \rightarrow \infty$

The two cases $z > 0$ and $z < 0$ reduce to the form:

$$\exp i\nu|z|,\ \nu = \sqrt{\omega^2/c^2 - k_x^2 - k_y^2}$$

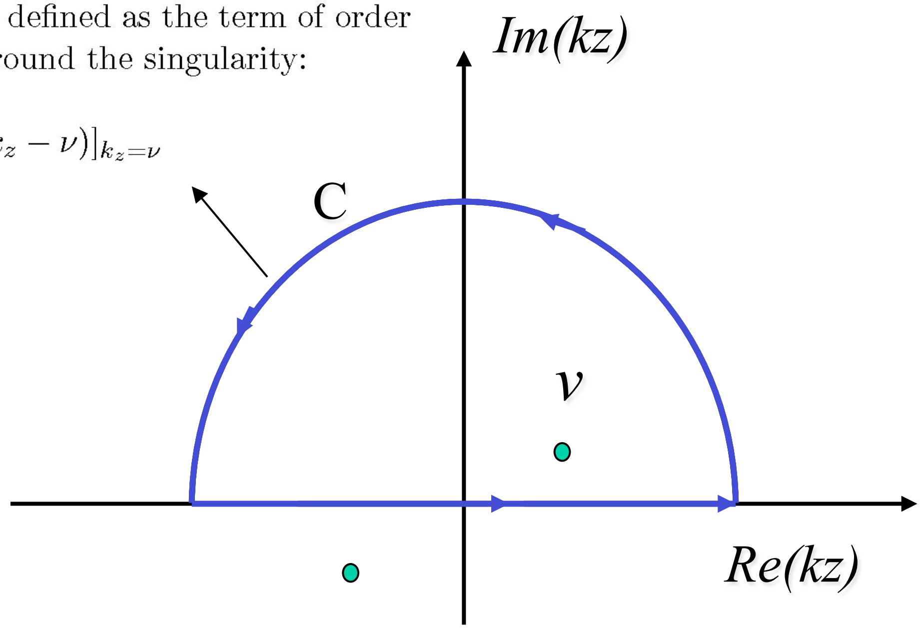
The integrand is:

$$f(k_z) = \frac{\exp i(k_x x + k_y y)\exp i(k_z z)}{k_x^2 + k_y^2 + k_z^2 - \omega^2/c^2} = \frac{\exp i(k_x x + k_y y)\exp i(k_z z)}{(k_z + \nu)(k_z - \nu)}$$

The singularity is of order 1. The residue is defined as the term of order -1 of the Laurent expansion of the integrand around the singularity:

$$A_{-1} = \frac{1}{(m-1)!}\frac{d^{m-1}}{dk_z^{m-1}} f(k_z)(k_z - \nu)]_{k_z=\nu}$$

with $m = 1$ in our case.



$$R = A_{-1} = f(k_z)(k_z - \nu)]|_{k_z=\nu} = \frac{\exp i(k_x x + k_y y) \exp i(\nu|z|)}{2\nu}$$

Inserting in I, we get the Green function in the form:

$$G(\vec{x}, t) = \frac{\exp -i\omega t}{2\pi} i \int\int \frac{\exp i(k_x x + k_y y) \exp i(\nu|z|)}{\nu} dk_x dk_y$$

We end with the Weyl integral, which shows the expansion of the Green function in terms of plane waves:

$$\boxed{\frac{1}{R} \exp -i\omega(t - R/c) = \frac{\exp -i\omega t}{2\pi} i \int\int \frac{\exp i(k_x x + k_y y) \exp i(\nu|z|)}{\nu} dk_x dk_y}$$

with

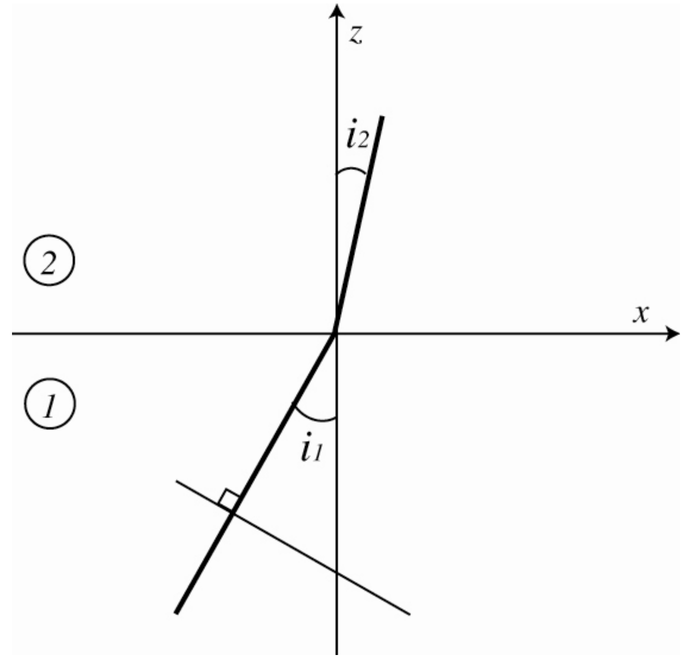
$$\nu = \sqrt{\omega^2/c^2 - k_x^2 - k_y^2}$$

.

Propagation in flat layers:

Integration: reflectivity, discrete wavenumber, ……

PLANE WAVE SH 2D :



$\vec{u} = (u_1, u_2, u_3)$ et $u_2 = v$

Incident wave

$$v_i = v_0 = \exp i(\omega t - k_x x - k_z z)$$

Resulting waves?

reflected : $v_R = v_0 \, R \, \exp i(\omega t - i k_x^{'} x + k_z^{'} z)$

transmitted : $v_T = v_0 \, T \, \exp i(\omega t - i k_x^{''} x - k_z^{''} z)$

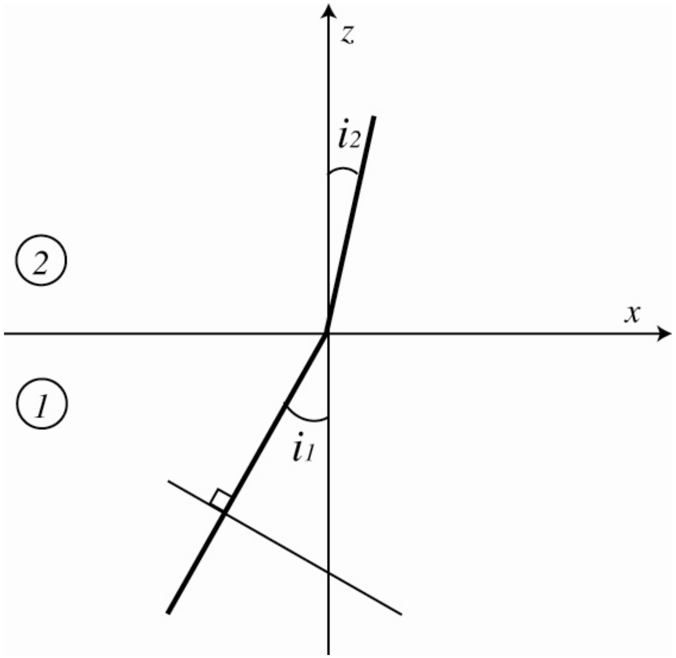
$\Rightarrow$ remarque : valeur de k?

$$\frac{\omega^2}{c^2} = k_x^2 + k_z^2$$

$\Rightarrow$ Conditions aux limites :

en $z = 0$; $\forall x$ $\quad v_i + v_r = v_t$

$$\tau_i + \tau_r = \tau_t$$



displacement

$$\exp(-ik_x x) + R\exp(-ik_x' x) = T\exp(-ik_x'' x) \qquad \forall x \Rightarrow k_x = k_x' = k_x''$$

$$k_x = \frac{\omega}{v_1}\sin i_1 \quad ; \quad k_x'' = \frac{\omega}{v_2}\sin i_2$$

$$\Rightarrow \frac{\sin i_1}{v_1} = \frac{\sin i_2}{v_2}$$

$$1 + R = T$$

stress

$$\mu_1(-ik_z + iRk_z) = \mu_2(-ik_z''T)$$

$$1 - R = \frac{\mu_2\, k_z''}{\mu_1\, k_z} T = \frac{\mu_2 \cos i_2 \frac{\omega}{V_2}}{\mu_1 \cos i_1 \frac{\omega}{V_1}} T = \alpha\, T$$

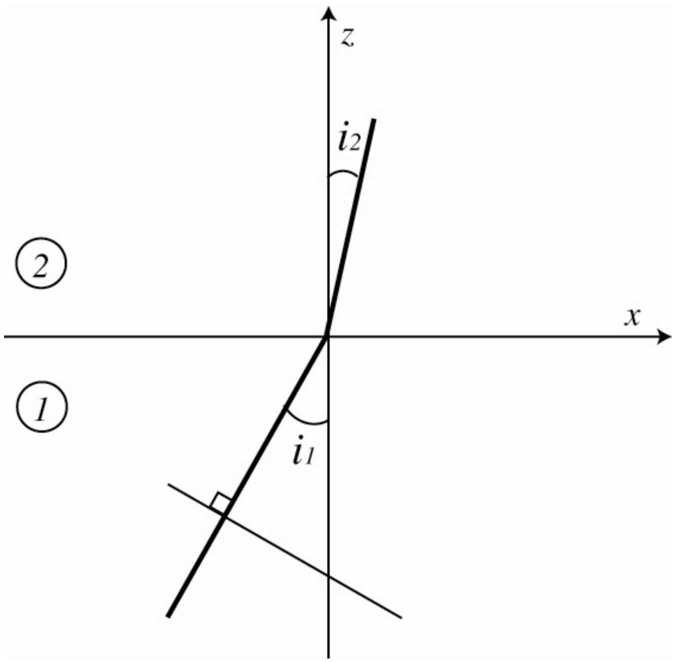
$$2 = T(1+\alpha) \qquad \Rightarrow \qquad T = \frac{2}{1+\alpha}$$

$$2R = T(1-\alpha) = 2\,\frac{1-\alpha}{1+\alpha} \qquad \Rightarrow \qquad R = \frac{1-\alpha}{1+\alpha}$$

CRITICAL REFLEXION

$$\frac{\sin i_1}{V_1} = \frac{\sin i_2}{V_2}$$

Problem for : $V_1 < V_2$ Critical angle i_c ; $\sin i_c = \frac{V_1}{V_2}$



For $i_1 < i_c$ no geometrical interpretation ($\sin i_2 > 1$!)

$$R_{SS} = \frac{1-\alpha}{1+\alpha} = \frac{1 - \left(\frac{\frac{\mu_2}{V_2}\cos i_2}{\frac{\mu_1}{V_1}\cos i_1}\right)}{1 + \left(\frac{\frac{\mu_2}{V_2}\cos i_2}{\frac{\mu_1}{V_1}\cos i_1}\right)}$$

$$\cos i_2 = (1 - \sin^2 i_2)^{1/2}$$

$$= (1 - \frac{V_2^2}{V_1^2}\sin^2 i_1)^{1/2}$$

$$= i\sqrt{\frac{V_2^2}{V_1^2}\sin^2 i_1 - 1}$$

$$R_{SS} = \frac{1 - i\left(\frac{\frac{\mu_2}{V_2}\sqrt{\frac{V_2^2}{V_1^2}\sin^2 i_1 - 1}}{\frac{\mu_1}{V_1}\cos i_1}\right)}{1 + i\left(\frac{\frac{\mu_2}{V_2}\sqrt{\frac{V_2^2}{V_1^2}\sin^2 i_1 - 1}}{\frac{\mu_1}{V_1}\cos i_1}\right)}$$

$$R_{SS} = \frac{1 - i\tan(\Phi(i_1))}{1 + i\tan(\Phi(i_1))} = \exp(-2\,i\,\Phi(i_1))$$

$$\Rightarrow \boxed{|R_{SS}| = 1 \qquad \text{Phase shift} = 2\,\Phi(i_1)}$$

$$\boxed{\tan\Phi(i) = \frac{\mu_2}{\mu_1}\left(\frac{1 - \frac{C^2}{V_2^2}}{\frac{C^2}{V_1^2} - 1}\right)^{1/2}}$$

with C, the apparent velocity:

$$C = V_1/\sin i_1$$

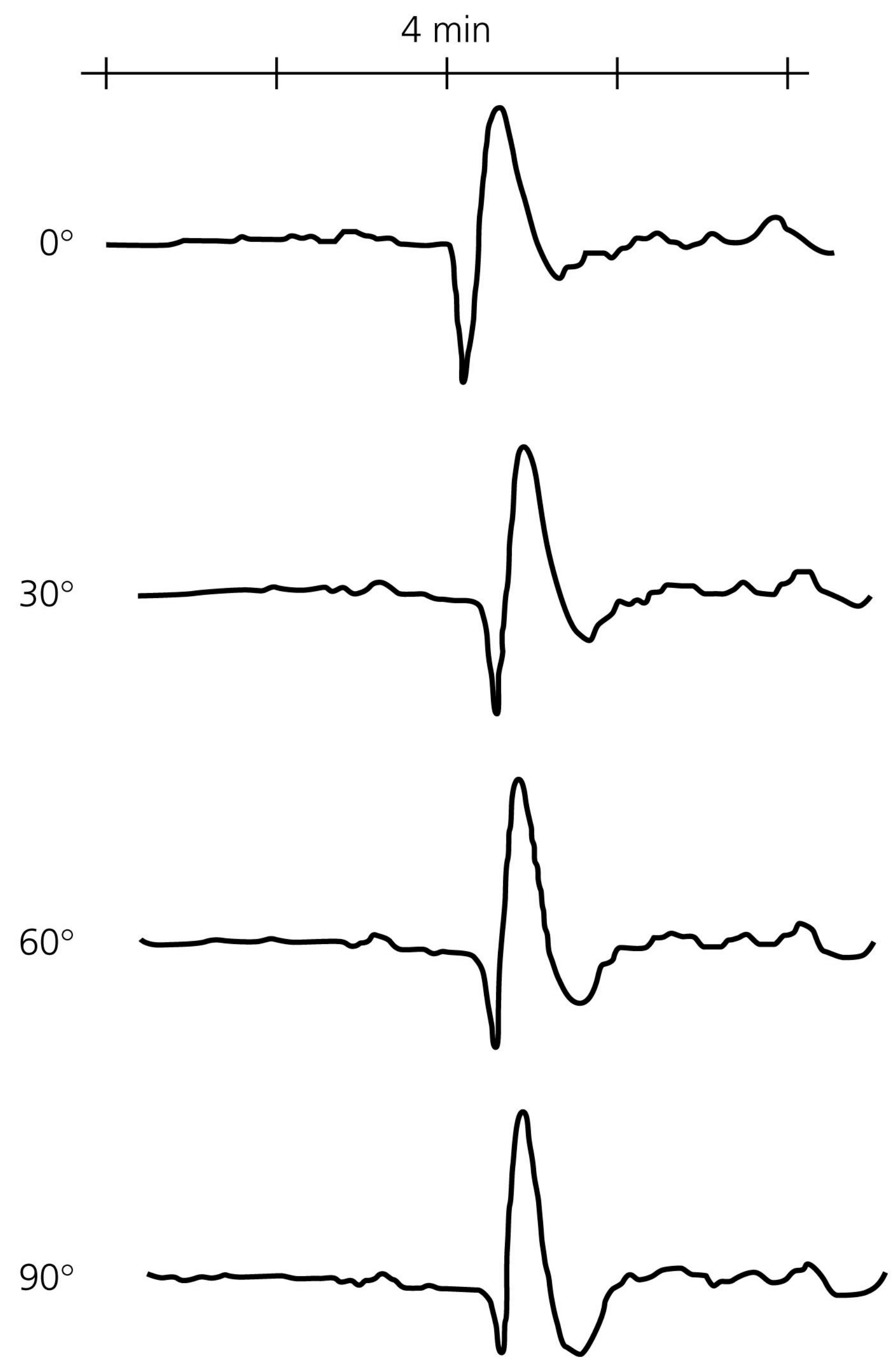
$$\exp(-ik_x r_{\beta_2} z) = \exp(-k_x r^*_{\beta_2} z)$$

$$R_{12} = \frac{\mu_1 r_{\beta_1} + i\mu_2 r^*_{\beta_2}}{\mu_1 r_{\beta_1} - i\mu_2 r^*_{\beta_2}}$$

This a complex number divided by its conjugate, so the magnitude of the reflection coefficient is one, but there is a phase shift of 2ε:

$$R_{12} = e^{i2\varepsilon} \qquad \varepsilon = \tan^{-1} \frac{\mu_2 r^*_{\beta_2}}{\mu_1 r_{\beta_1}}$$

Figure 2.6-5: Effect of phase shifts on a seismic waveform.



$$\exp(-ik_x r_{\beta_2} z) = \exp(-k_x r^*_{\beta_2} z)$$

$$R_{12} = \frac{\mu_1 r_{\beta_1} + i\mu_2 r^*_{\beta_2}}{\mu_1 r_{\beta_1} - i\mu_2 r^*_{\beta_2}}$$

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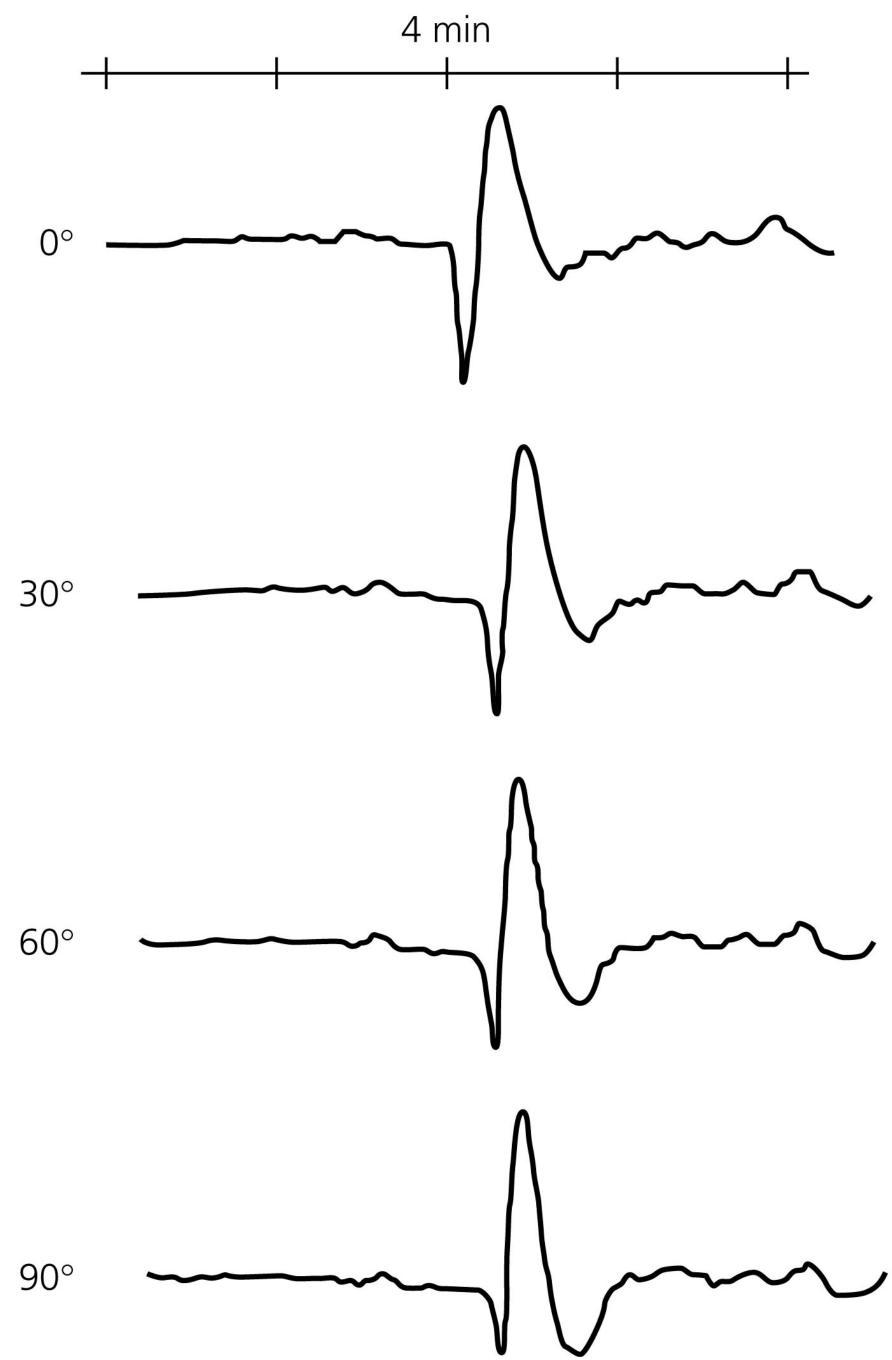
$$R_{12} = e^{i2\varepsilon} \qquad \varepsilon = \tan^{-1} \frac{\mu_2 r^*_{\beta_2}}{\mu_1 r_{\beta_1}}$$

At critical incidence,

$c_x = \beta_2$, so $r^*_{\beta_2} = 0$ and $\varepsilon = 0°$

As the angle of incidence increases beyond critical, ε increases.

Figure 2.6-5: Effect of phase shifts on a seismic waveform.



$$\exp(-ik_x r_{\beta_2} z) = \exp(-k_x r^*_{\beta_2} z)$$

$$R_{12} = \frac{\mu_1 r_{\beta_1} + i\mu_2 r^*_{\beta_2}}{\mu_1 r_{\beta_1} - i\mu_2 r^*_{\beta_2}}$$

This a complex number divided by its conjugate, so the magnitude of the reflection coefficient is one, but there is a phase shift of 2ε:

$$R_{12} = e^{i2\varepsilon} \qquad \varepsilon = \tan^{-1} \frac{\mu_2 r^*_{\beta_2}}{\mu_1 r_{\beta_1}}$$

At critical incidence,

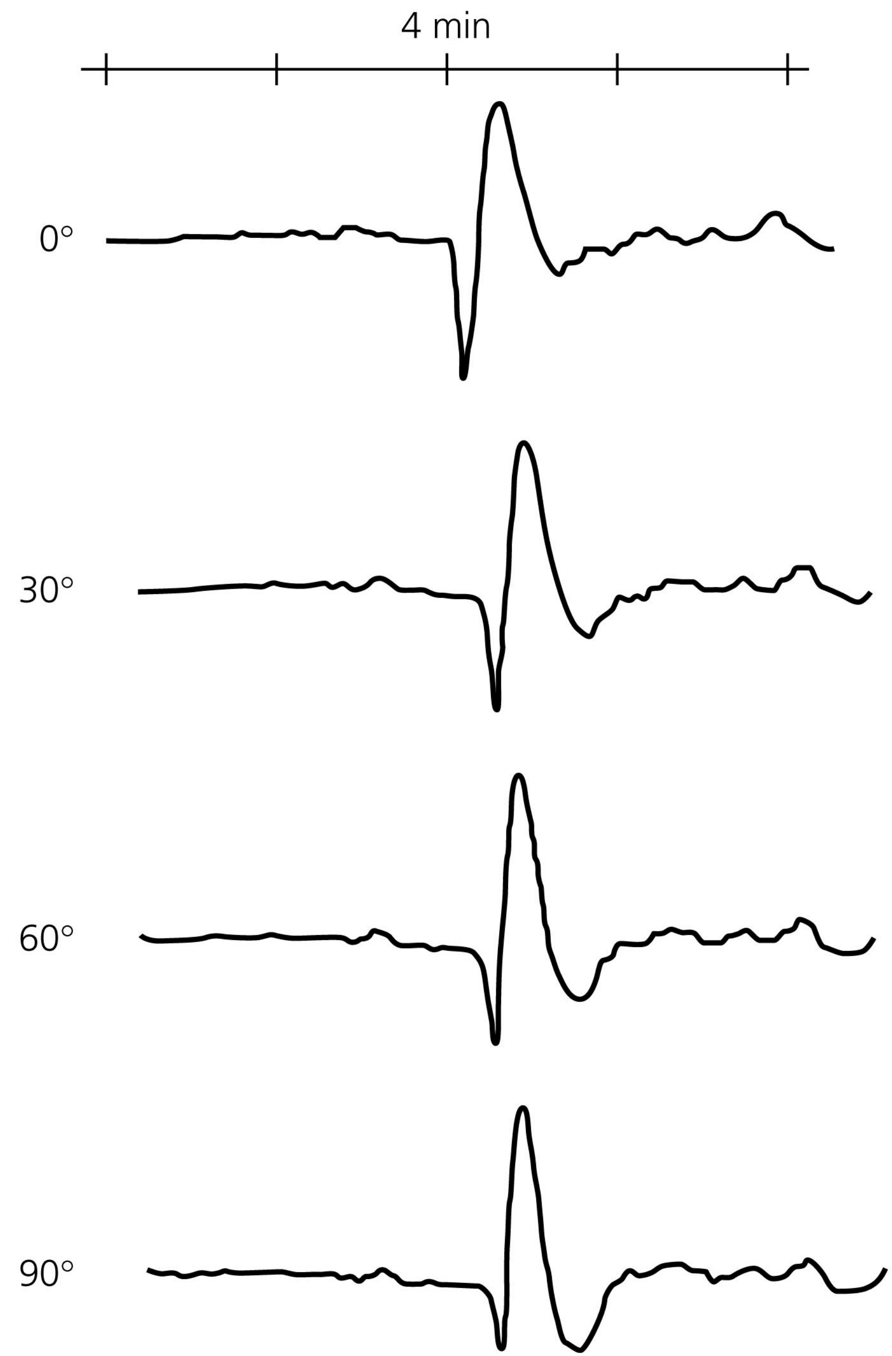
$c_x = \beta_2$, so $r^*_{\beta_2} = 0$ and $\varepsilon = 0°$

As the angle of incidence increases beyond critical, ε increases.

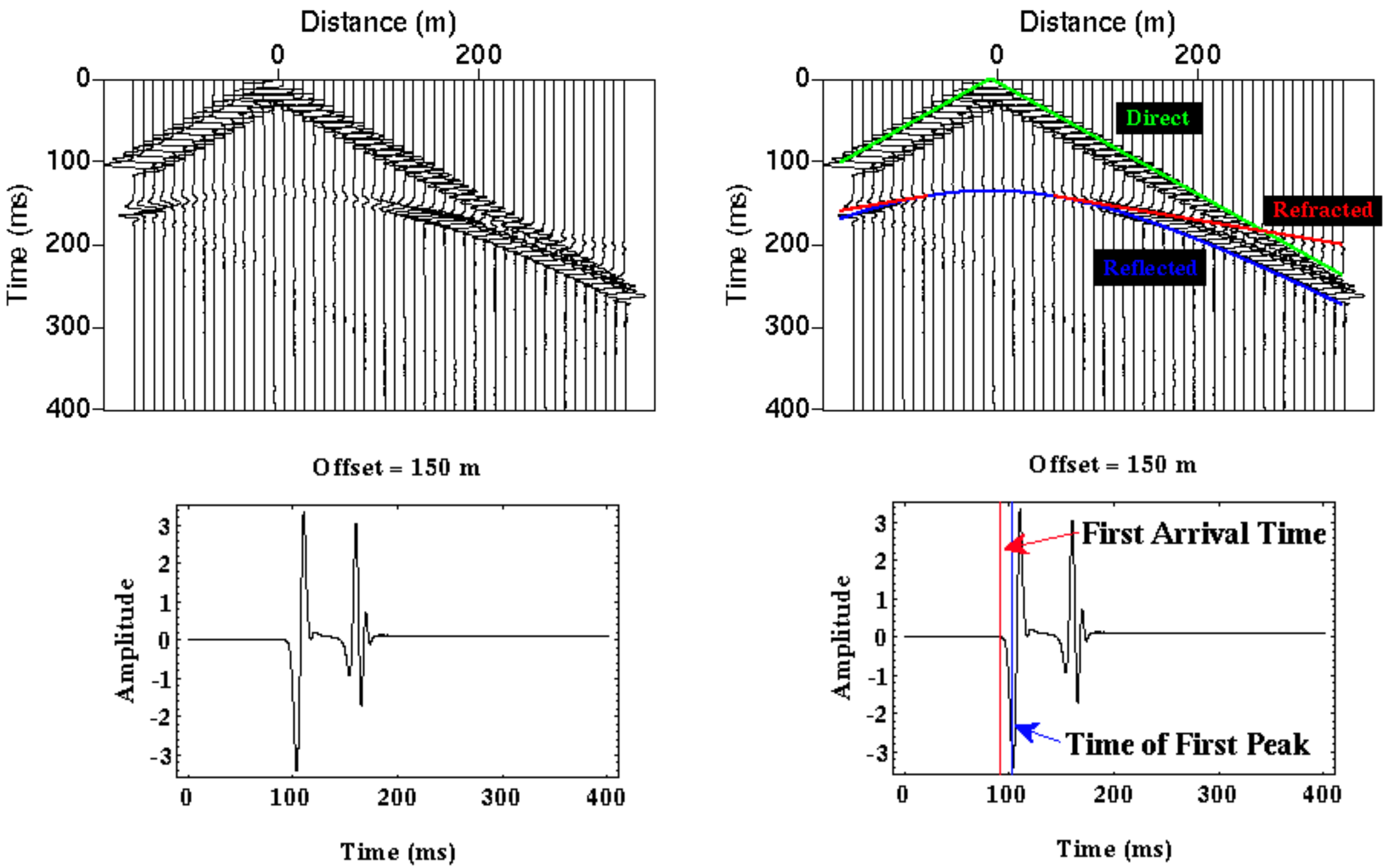
At grazing incidence, $j_1 = 90°$, we have

$c_x = \beta_1$, $r_{\beta_1} = 0$ and $\varepsilon = 90°$

Figure 2.6-5: Effect of phase shifts on a seismic waveform.



Formes d' ondes et temps d' arrivée



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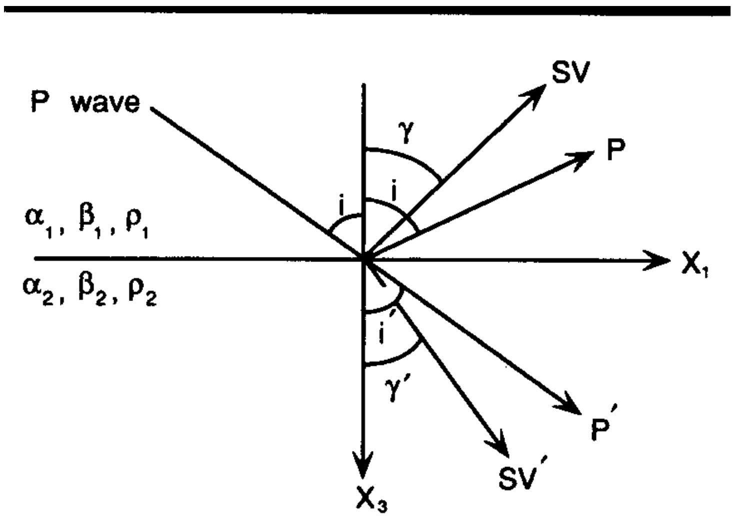


medium 1: $\phi_1 = A_1 \exp[i\omega(px_1 + \eta_1 x_3 - t)] + A_2 \exp[i\omega(px_1 - \eta_1 x_3 - t)]$

medium 2: $\phi_2 = A_3 \exp[i\omega(px_1 + \eta_2 x_3 - t)].$

$$p \sim k_x \;\; ; \;\; \eta \sim k_z$$

TABLE 3.1 Displacement Reflection and Transmission Coefficients

Coefficient	Formula
Solid–free surface (P–SV)	
R_{PP}	$\{-[(1/\beta^2) - 2p^2]^2 + 4p^2\eta_\alpha\eta_\beta\}/A$
R_{PS}	$\{4(\alpha/\beta)p\eta_\alpha[(1/\beta^2) - 2p^2]\}/A$
R_{SP}	$\{4(\beta/\alpha)p\eta_\beta[(1/\beta^2) - 2p^2]\}/A$
R_{SS}	$\{-[(1/\beta^2) - 2p^2]^2 + 4p^2\eta_\alpha\eta_\beta\}/A$
$R_{SS}(SH)$	1
Solid–solid (P–SV)	
R_{PP}	$[(b\eta_{\alpha_1} - c\eta_{\alpha_2})F - (a + d\eta_{\alpha_1}\eta_{\beta_2})Hp^2]/D$
R_{PS}	$-[2\eta_{\alpha_1}(ab + cd\eta_{\alpha_2}\eta_{\beta_2})p(\alpha_1/\beta_1)]/D$
T_{PP}	$[2\rho_1\eta_{\alpha_1}F(\alpha_1/\alpha_2)]/D$
T_{PS}	$[2\rho_1\eta_{\alpha_1}Hp(\alpha_1/\beta_2)]/D$
R_{SS}	$-[(b\eta_{\beta_1} - c\eta_{\beta_2})E - (a + b\eta_{\alpha_2}\eta_{\beta_1})Gp^2]/D$
R_{SP}	$-[2\eta_{\beta_1}(ab + cd\eta_{\alpha_2}\eta_{\beta_2})p(\beta_1/\alpha_1)]/D$
$R_{SS}(SH)$	$\dfrac{\mu_1\eta_{\beta_1} - \mu_2\eta_{\beta_2}}{\mu_1\eta_{\beta_1} + \mu_2\eta_{\beta_2}}$
$T_{SS}(SH)$	$\dfrac{2\mu_1\eta_{\beta_1}}{\mu_1\eta_{\beta_1} + \mu_2\eta_{\beta_2}}$
$a = \rho_2(1 - 2\beta_2^2p^2) - \rho_1(1 - 2\beta_1^2p^2)$	$E = b\eta_{\alpha_1} + c\eta_{\alpha_2}$
$b = \rho_2(1 - 2\beta_2^2p^2) - 2\rho_1\beta_1^2p^2$	$F = b\eta_{\beta_1} + c\eta_{\beta_2}$
$c = \rho_1(1 - 2\beta_1^2p^2) + 2\rho_2\beta_2^2p^2$	$G = a - d\eta_{\alpha_1}\eta_{\beta_2}$
$d = 2(\rho_2\beta_2^2 - \rho_1\beta_1^2)$	$H = a - d\eta_{\alpha_2}\eta_{\beta_1}$

$$D = EF + GHp^2$$

$$A = [(1/\beta^2) - 2p^2]^2 + 4p^2\eta_{\alpha_1}\eta_{\beta_1}$$

Lay and Wallace (1995)

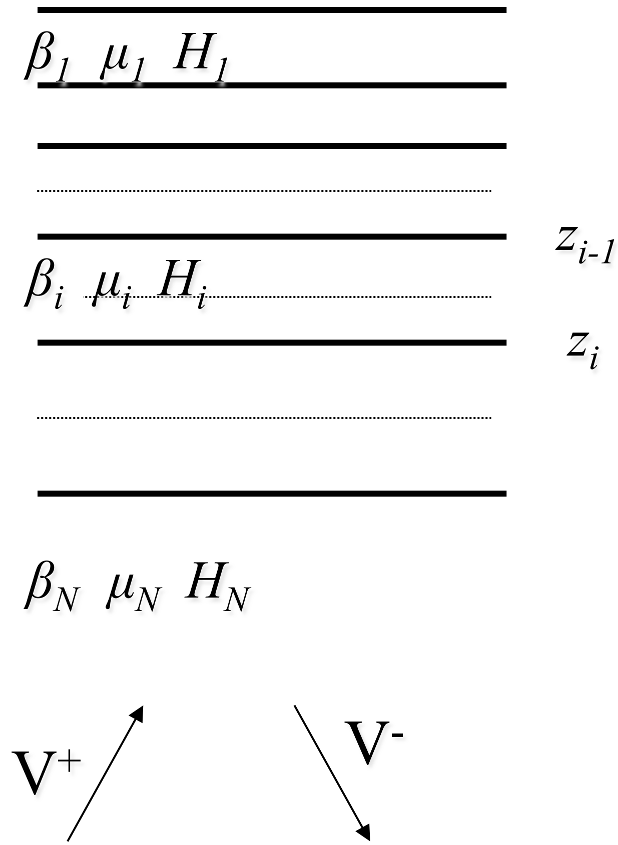
Plane waves in layered structures

'Physical propagator': Thomson Haskell method

A stack of N layers (1: upper layer; n: half space)

Consider a plane SH-wave incident from below with amplitude V_{inc}.

In each layer we separate up-going and down-going waves:



$$V_i = V_i^+ + V_i^-$$

$$V_i^+ = B_i^+ \exp(-i\omega t + ik_x x + ik_y y - ik_z^i z)$$
$$V_i^- = B_i^- \exp(-i\omega t + ik_x x + ik_y y + ik_z^i z)$$

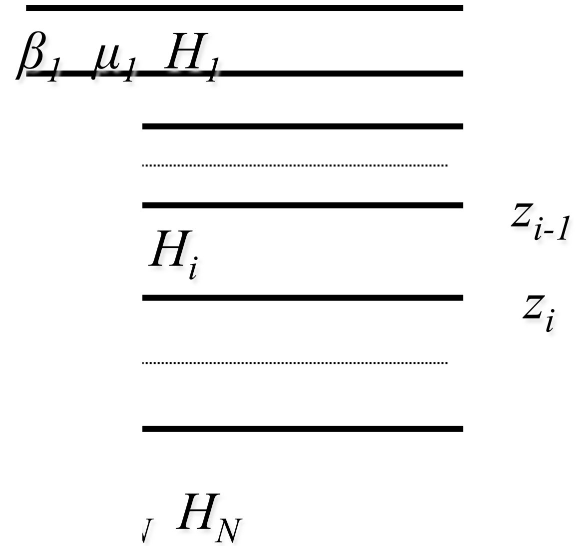
with:

$$k_z^i = (\sqrt{\omega^2/\beta_i^2 - k_x^2 - k_y^2})$$

Let us define two vectors, S the displacement-stress vector, and Φ the up-going-down going displacement vector:

$$S_i = (V_i, \tau_i)^T$$
$$\Phi_i = (V_i^+, V_i^-)^T$$

$$S_i = (V_i, \tau_i)^T$$
$$\Phi_i = (V_i^+, V_i^-)^T$$



$$S_i = \begin{pmatrix} 1 & 1 \\ i\mu_i k_z^i & -i\mu_i k_z^i \end{pmatrix} \Phi_i = T_i \Phi_i$$

We can write the propagation accross the layer:

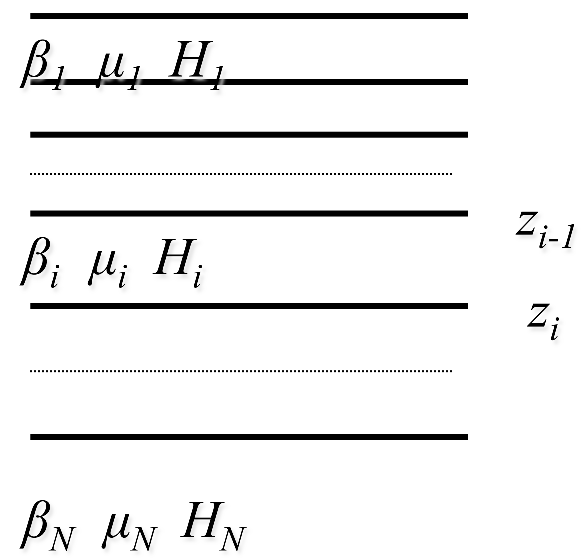
$$\Phi_i(z_i) = \begin{pmatrix} \exp(ik_z^i H_i) & 0 \\ 0 & \exp(-ik_z^i H_i) \end{pmatrix} \begin{pmatrix} V_i^+(z_{i-1}) \\ V_i^-(z_{i-1}) \end{pmatrix} = E_i \Phi_i(z_{i-1})$$

$$\Phi_i(z_i) = E_i \Phi_i(z_{i-1}) = E_i T_i^{-1} S_i(z_{i-1})$$

$$S_i(z_i) = T_i \Phi_i(z_i) = T_i E_i T_i^{-1} S_i(z_{i-1}) = G_i S_i(z_{i-1})$$

where G is the *propagator.*

S is continuous across the interface: $S_i(z_i) = S_{i+1}(z_i)$



$$S_i(z_i) = G_i S_i(z_{i-1})$$

S is continuous across the interface: $S_i(z_i) = S_{i+1}(z_i)$

$$S_N(z_{N-1}) = S_{N-1}(z_{N-1}) = G_{N-1}S_{N-1}(z_{N-2}) =$$
$$G_{N-1}S_{N-2}(z_{N-2}) = G_{N-1}G_{N-2}S_{N-2}(z_{N-3}) = ...$$
$$G_{N-1}G_{N-2}G_{N-3}.....G_1S_1(0) = \mathbf{G}S_1(0)$$

Displacement at the free surface V_0 in function of the incident field V_{inc}.

$$\Phi_N(z_{N-1}) = T^{-1}\mathbf{G}S_1(0) = \mathbf{R}S_1(0)$$

Free surface condition:

$$S_1(0) = (V_0, 0)^T$$

$$\begin{pmatrix} V_{inc} \\ V_{refl} \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} V_0 \\ 0 \end{pmatrix} \tag{1}$$

Therefore:

$$V_{inc} = R_{11}V_0$$

and

$$V_{refl} = \frac{R_{21}}{R_{11}}V_{inc}$$

These coefficients are easily computed when no vanishing waves are involved.

Computer simulation of slab guided waves

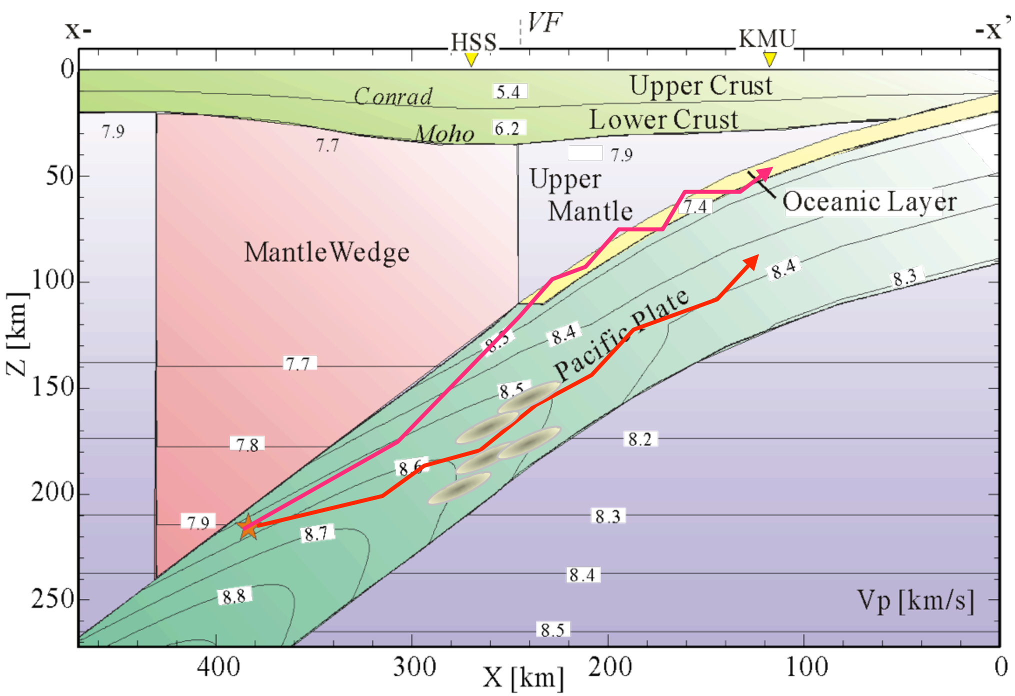
Model:

1. Trapped Signal in Low-V Oceanic Crust

Abers[2003], Martin et al.[2003], Furumura [1998]

2. Scattering Waveguide Effect in Heterogeneous Plate

Furumura and Kennett [2005] NEW

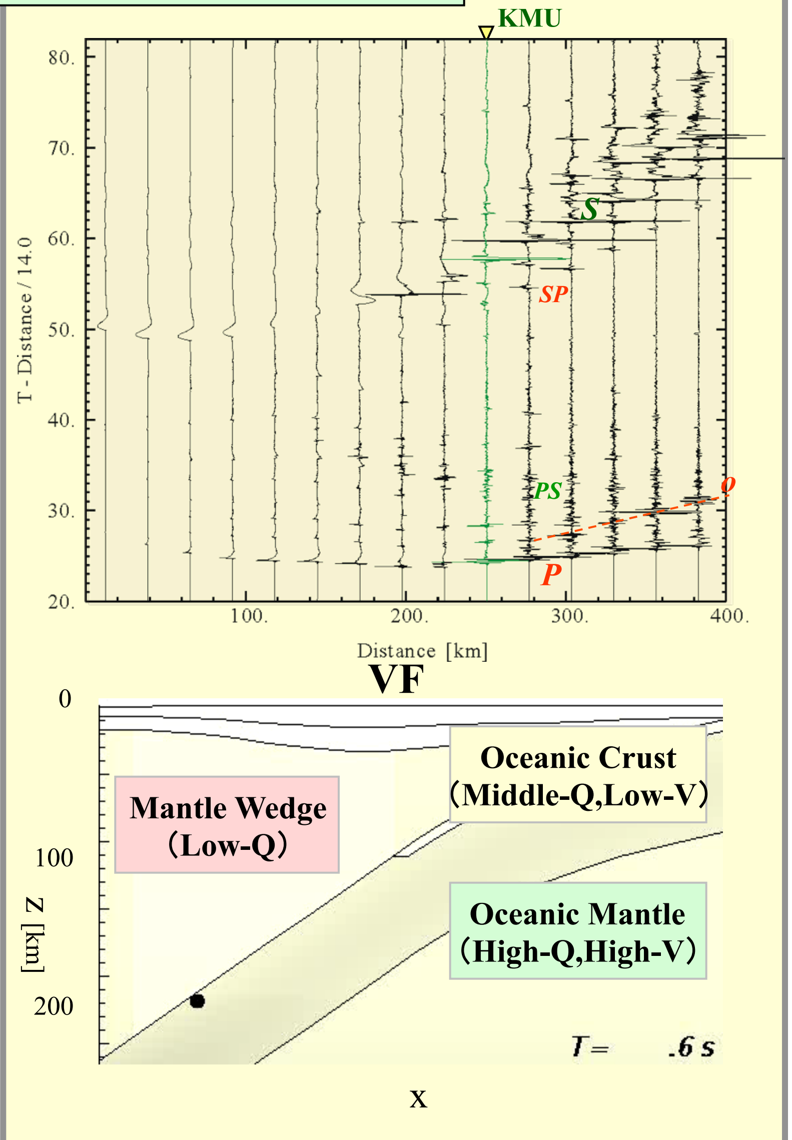


2D Simulation Model

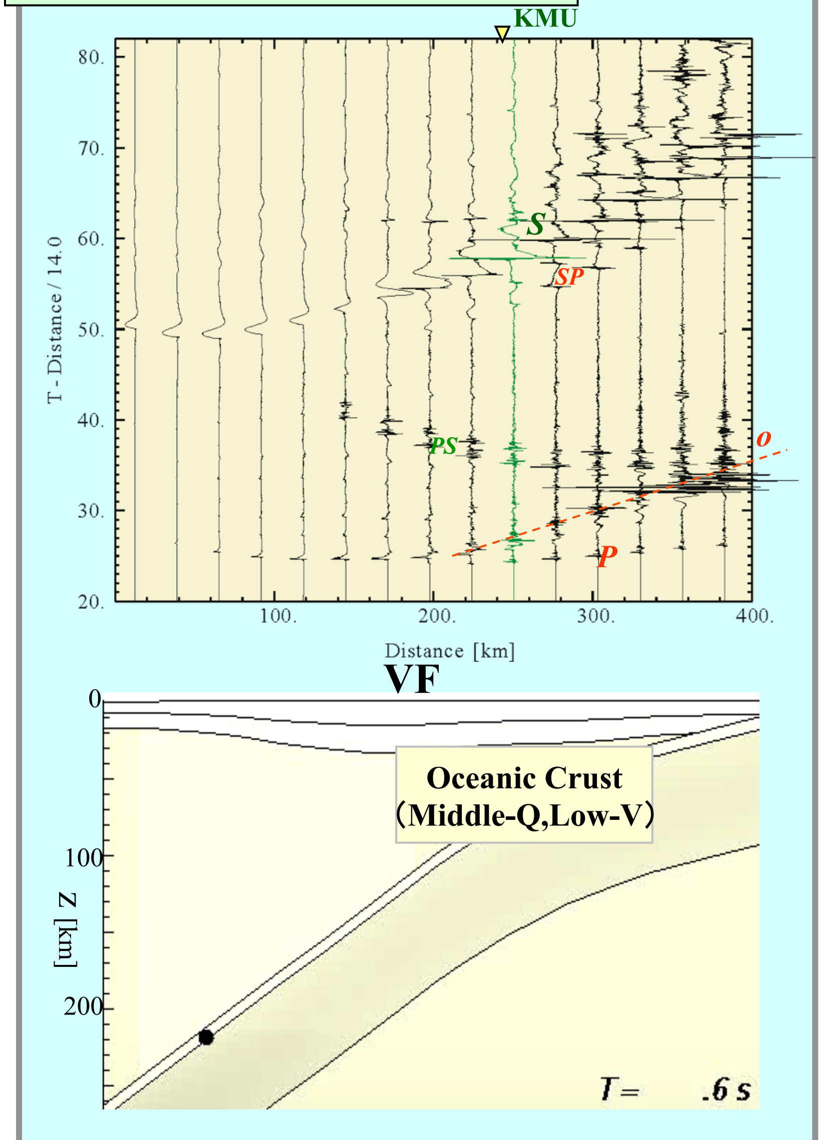
Model Size	400*280 km (Dx=60m)
Source	Point Source * Nakamura & Miyatake (2000)
Max Freq.	20 Hz *(Vs<3.2km/s)*
Scheme	Parallel FDM
Memory	4 GByte
CPU Time	60 h (Intel Xeon 16CPU)

Computer Simulation（1）: Homogeneous Plate (*iasp81)*

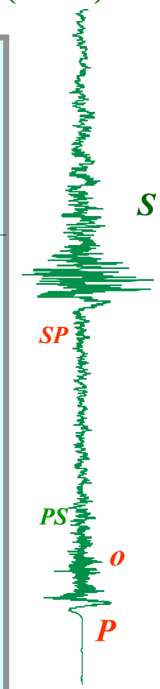
(a) Homogeneous Plate

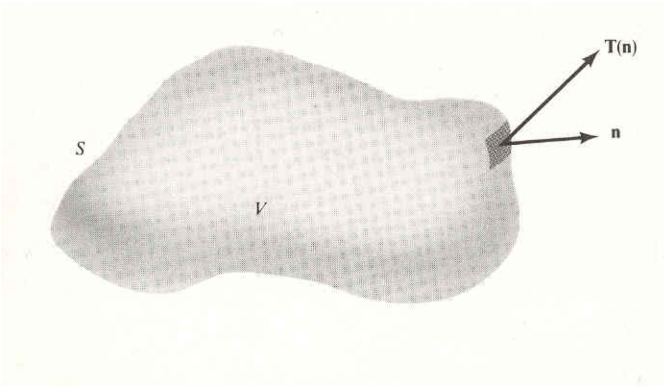


(b) Extended Oceanic Crust



Observation
(KMU)





Representation theorem

$$\begin{aligned}
u_n(\overrightarrow{x},t) &= \int_{-\infty}^{\infty} d\tau \iiint_V f_i(\overrightarrow{\xi},\tau) G_{in}(\overrightarrow{\xi},t-\tau;\overrightarrow{x},0)\, dV(\overrightarrow{\xi}) \\
&+ \int_{-\infty}^{\infty} d\tau \iint_S (G_{in}(\overrightarrow{\xi},t-\tau;\overrightarrow{x},0) T_i(\overrightarrow{u}(\overrightarrow{\xi},\tau)) \\
&- u_i(\overrightarrow{\xi},t) c_{ijkl}(\overrightarrow{\xi}) G_{kn,l}(\overrightarrow{\xi},t-\tau;\overrightarrow{x},0))\, dS(\overrightarrow{\xi})
\end{aligned}$$