MEEES and M2R STU(TUE552)

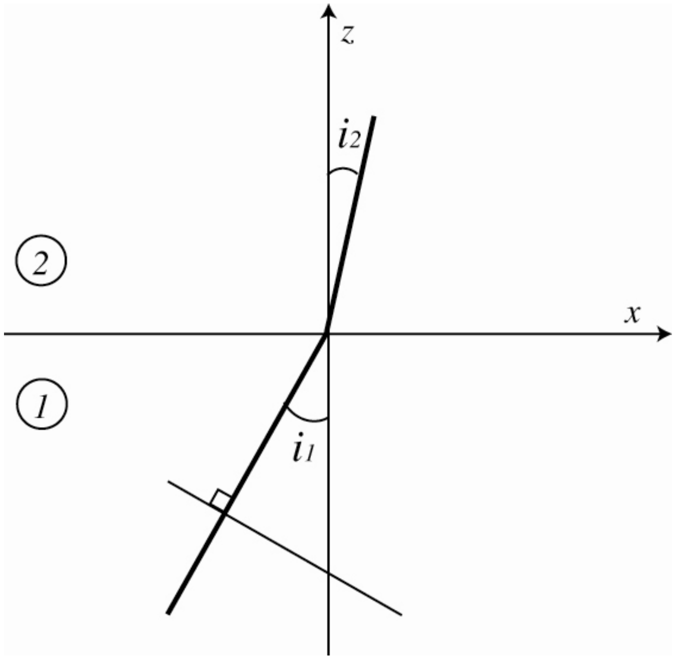
Seismology 2
(Michel Campillo)

http://www-lgit.obs.ujf-grenoble.fr/users/campillo/Master-TUE552

CRITICAL REFLEXION

$$\frac{\sin i_1}{V_1} = \frac{\sin i_2}{V_2}$$

Problem for : $V_1 < V_2$ Critical angle i_c ; $\sin i_c = \frac{V_1}{V_2}$



For $i_1 < i_c$ no geometrical interpretation ($\sin i_2 > 1$!)

$$R_{SS} = \frac{1-\alpha}{1+\alpha} = \frac{1 - \left(\frac{\frac{\mu_2}{V_2}\cos i_2}{\frac{\mu_1}{V_1}\cos i_1}\right)}{1 + \left(\frac{\frac{\mu_2}{V_2}\cos i_2}{\frac{\mu_1}{V_1}\cos i_1}\right)}$$

$$\cos i_2 = (1 - \sin^2 i_2)^{1/2}$$

$$= (1 - \frac{V_2^2}{V_1^2}\sin^2 i_1)^{1/2}$$

$$= i\sqrt{\frac{V_2^2}{V_1^2}\sin^2 i_1 - 1}$$

$$R_{SS} = \frac{1 - i\left(\frac{\frac{\mu_2}{V_2}\sqrt{\frac{V_2^2}{V_1^2}\sin^2 i_1 - 1}}{\frac{\mu_1}{V_1}\cos i_1}\right)}{1 + i\left(\frac{\frac{\mu_2}{V_2}\sqrt{\frac{V_2^2}{V_1^2}\sin^2 i_1 - 1}}{\frac{\mu_1}{V_1}\cos i_1}\right)}$$

$$R_{SS} = \frac{1 - i\tan(\Phi(i_1))}{1 + i\tan(\Phi(i_1))} = \exp(-2\,i\,\Phi(i_1))$$

$$\Rightarrow \boxed{|R_{SS}| = 1 \qquad \text{Phase shift} = 2\,\Phi(i_1)}$$

$$\boxed{\tan\Phi(i) = \frac{\mu_2}{\mu_1}\left(\frac{1 - \frac{C^2}{V_2^2}}{\frac{C^2}{V_1^2} - 1}\right)^{1/2}}$$

with C, the apparent velocity:

$$C = V_1/\sin i_1$$

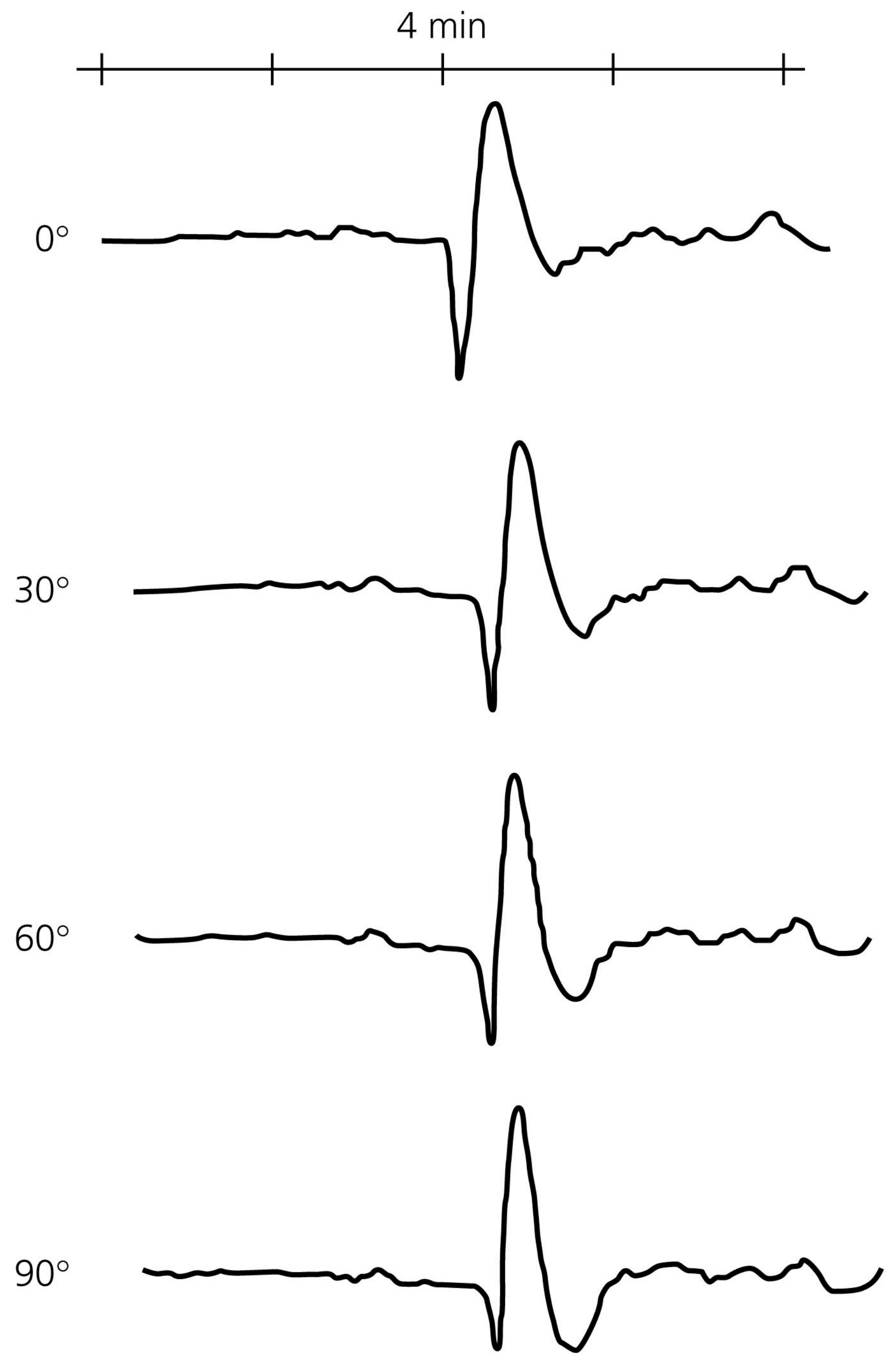
$$\exp(-ik_x r_{\beta_2} z) = \exp(-k_x r^*_{\beta_2} z)$$

$$R_{12} = \frac{\mu_1 r_{\beta_1} + i\mu_2 r^*_{\beta_2}}{\mu_1 r_{\beta_1} - i\mu_2 r^*_{\beta_2}}$$

This a complex number divided by its conjugate, so the magnitude of the reflection coefficient is one, but there is a phase shift of 2ε:

$$R_{12} = e^{i2\varepsilon} \qquad \varepsilon = \tan^{-1}\frac{\mu_2 r^*_{\beta_2}}{\mu_1 r_{\beta_1}}$$

Figure 2.6-5: Effect of phase shifts on a seismic waveform.



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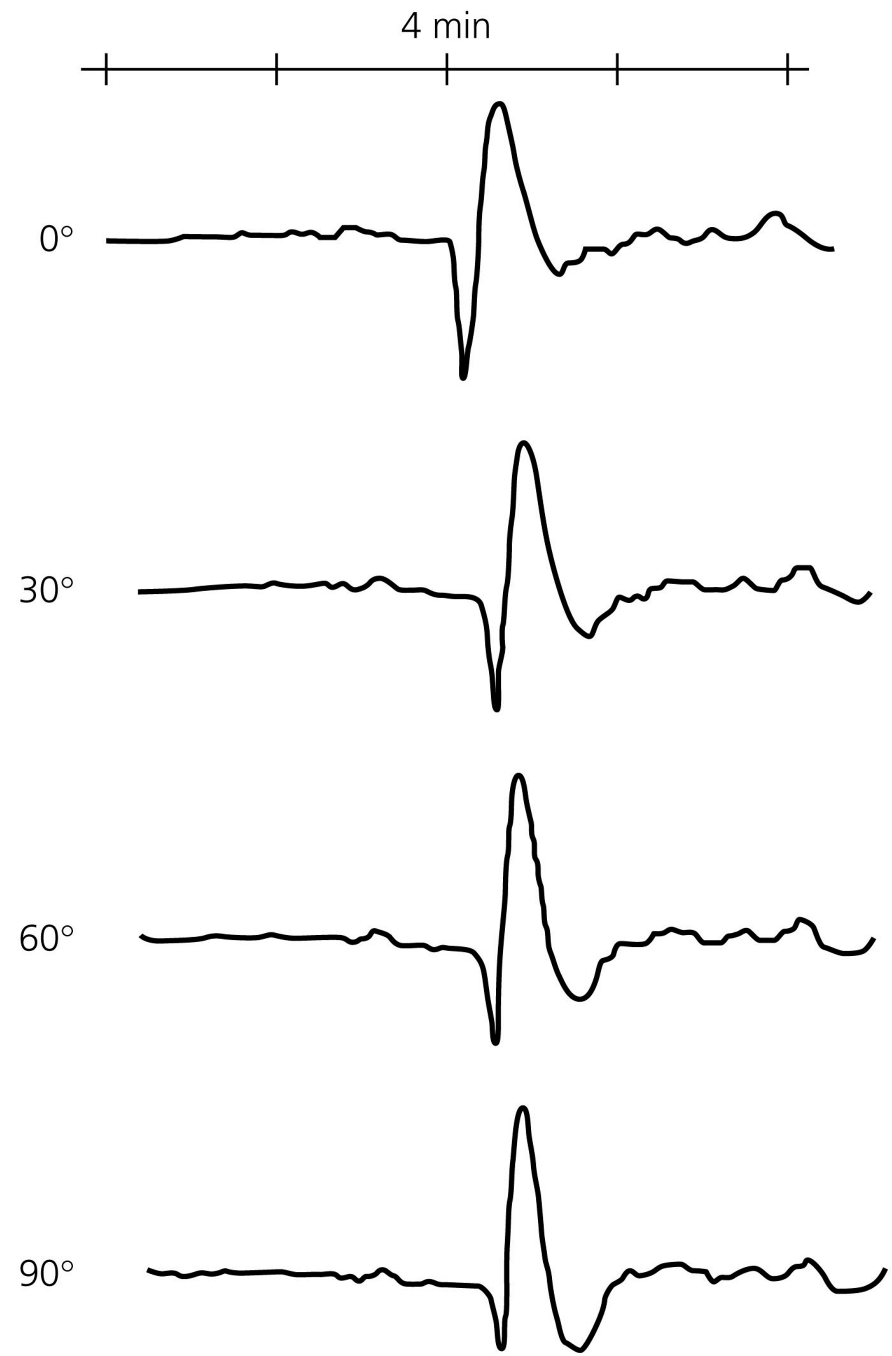
At critical incidence,

$c_x = \beta_2$, so $r^*_{\beta_2} = 0$ and $\varepsilon = 0°$

As the angle of incidence increases beyond critical, ε increases.

Figure 2.6-5: Effect of phase shifts on a seismic waveform.



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At critical incidence,

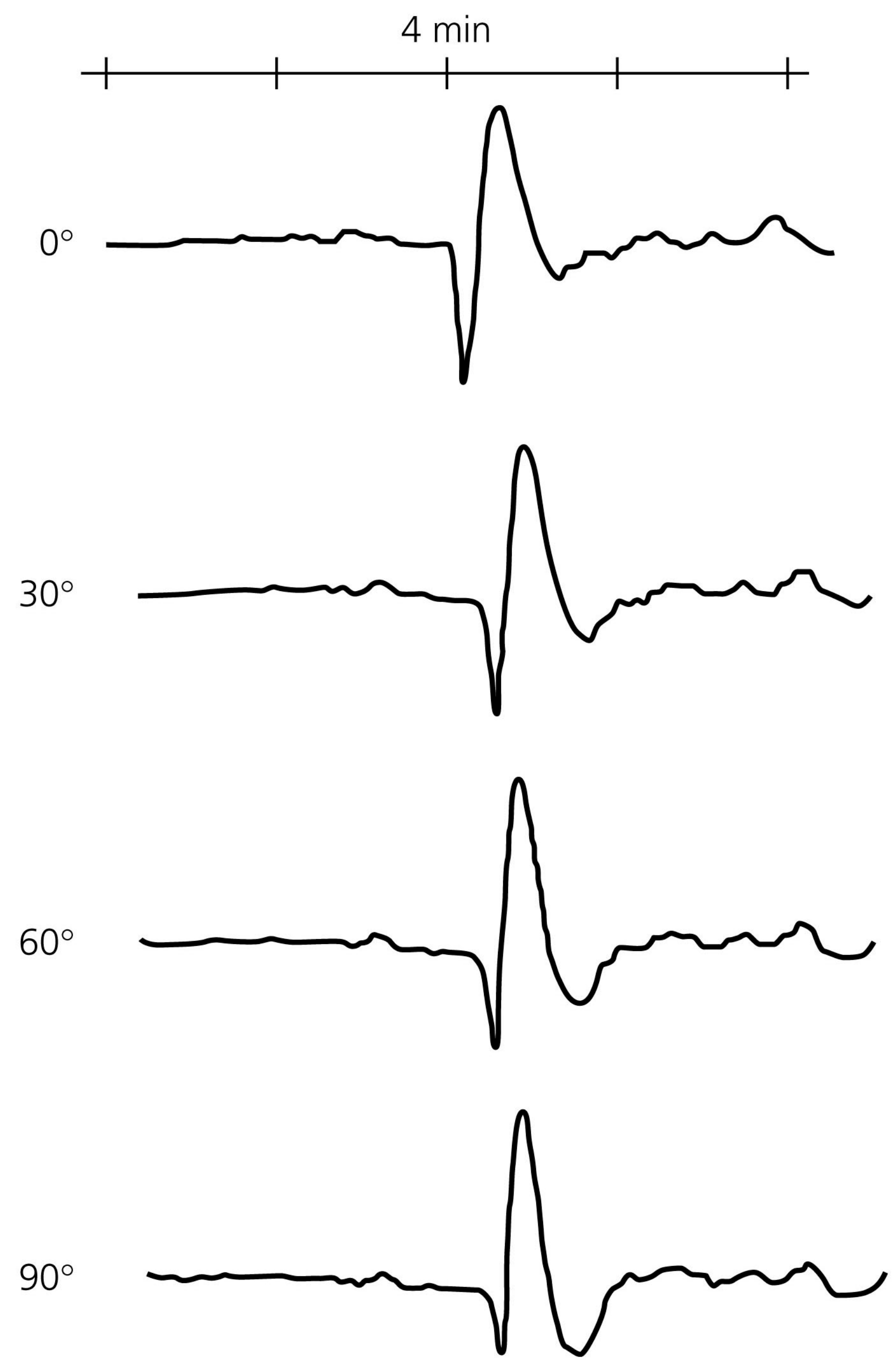
$c_x = \beta_2$, so $r^*_{\beta_2} = 0$ and $\varepsilon = 0°$

As the angle of incidence increases beyond critical, ε increases.

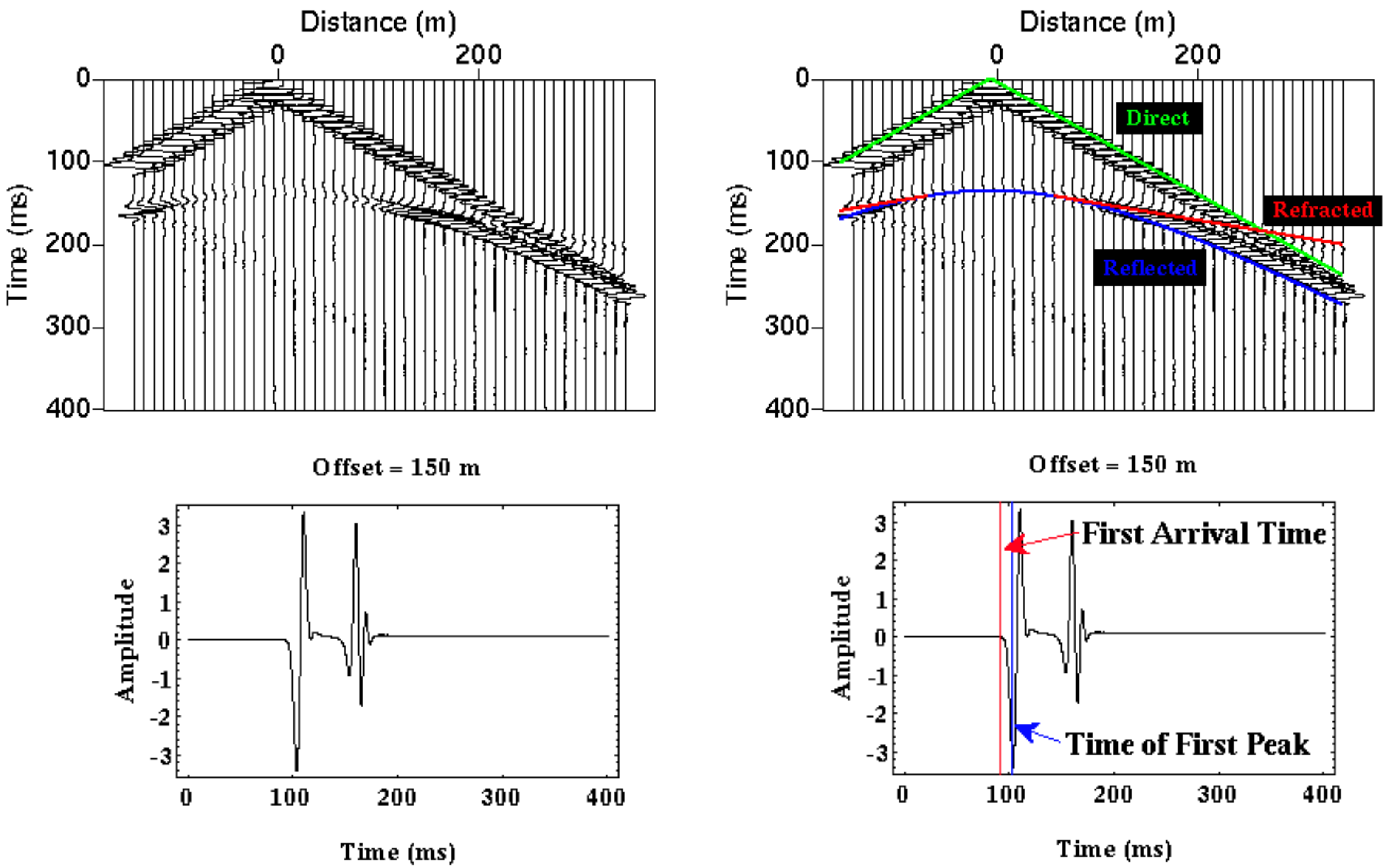
At grazing incidence, $j_1 = 90°$, we have

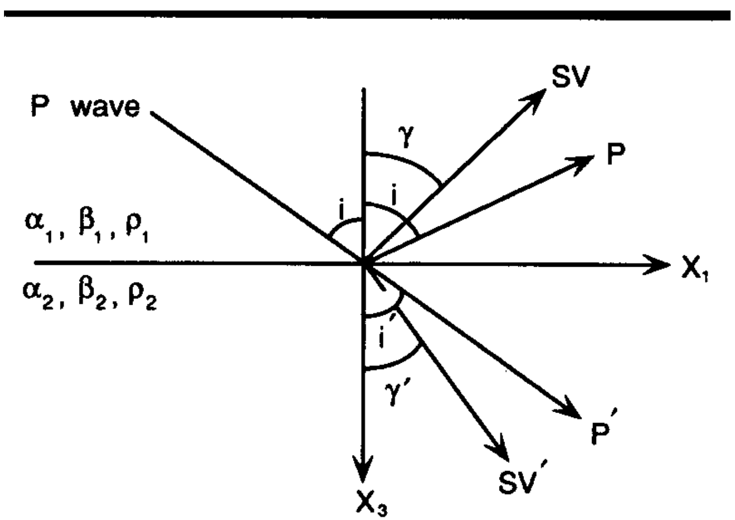
$c_x = \beta_1$, $r_{\beta_1} = 0$ and $\varepsilon = 90°$

Figure 2.6-5: Effect of phase shifts on a seismic waveform.



Formes d’ondes et temps d’arrivée





medium 1: $\phi_1 = A_1 \exp[i\omega(px_1 + \eta_1 x_3 - t)]$

$+ A_2 \exp[i\omega(px_1 - \eta_1 x_3 - t)]$

medium 2: $\phi_2 = A_3 \exp[i\omega(px_1 + \eta_2 x_3 - t)]$.

$$p \sim k_x \;\; ; \;\; \eta \sim k_z$$

TABLE 3.1 Displacement Reflection and Transmission Coefficients

Coefficient	Formula
Solid–free surface (P–SV)	
R_{PP}	$\{-[(1/\beta^2) - 2p^2]^2 + 4p^2\eta_\alpha\eta_\beta\}/A$
R_{PS}	$\{4(\alpha/\beta)p\eta_\alpha[(1/\beta^2) - 2p^2]\}/A$
R_{SP}	$\{4(\beta/\alpha)p\eta_\beta[(1/\beta^2) - 2p^2]\}/A$
R_{SS}	$\{-[(1/\beta^2) - 2p^2]^2 + 4p^2\eta_\alpha\eta_\beta\}/A$
$R_{SS}(SH)$	1
Solid–solid (P–SV)	
R_{PP}	$[(b\eta_{\alpha_1} - c\eta_{\alpha_2})F - (a + d\eta_{\alpha_1}\eta_{\beta_2})Hp^2]/D$
R_{PS}	$-[2\eta_{\alpha_1}(ab + cd\eta_{\alpha_2}\eta_{\beta_2})p(\alpha_1/\beta_1)]/D$
T_{PP}	$[2\rho_1\eta_{\alpha_1}F(\alpha_1/\alpha_2)]/D$
T_{PS}	$[2\rho_1\eta_{\alpha_1}Hp(\alpha_1/\beta_2)]/D$
R_{SS}	$-[(b\eta_{\beta_1} - c\eta_{\beta_2})E - (a + b\eta_{\alpha_2}\eta_{\beta_1})Gp^2]/D$
R_{SP}	$-[2\eta_{\beta_1}(ab + cd\eta_{\alpha_2}\eta_{\beta_2})p(\beta_1/\alpha_1)]/D$
$R_{SS}(SH)$	$\dfrac{\mu_1\eta_{\beta_1} - \mu_2\eta_{\beta_2}}{\mu_1\eta_{\beta_1} + \mu_2\eta_{\beta_2}}$
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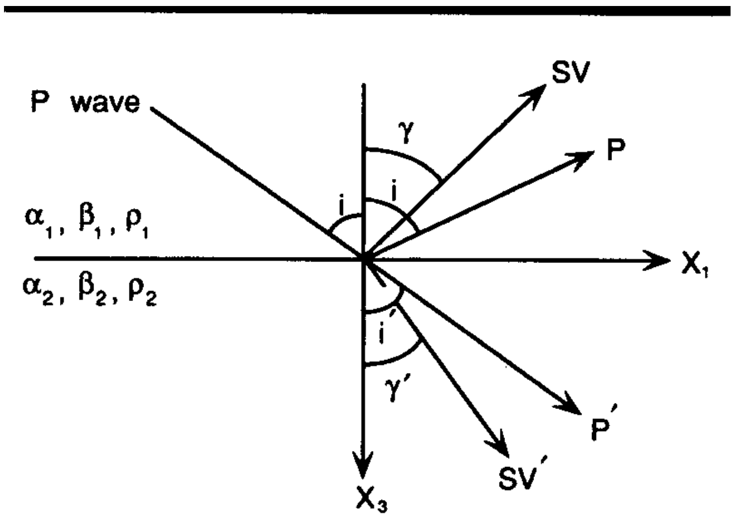
$a = \rho_2(1 - 2\beta_2^2p^2) - \rho_1(1 - 2\beta_1^2p^2)$	$E = b\eta_{\alpha_1} + c\eta_{\alpha_2}$
$b = \rho_2(1 - 2\beta_2^2p^2) - 2\rho_1\beta_1^2p^2$	$F = b\eta_{\beta_1} + c\eta_{\beta_2}$
$c = \rho_1(1 - 2\beta_1^2p^2) + 2\rho_2\beta_2^2p^2$	$G = a - d\eta_{\alpha_1}\eta_{\beta_2}$
$d = 2(\rho_2\beta_2^2 - \rho_1\beta_1^2)$	$H = a - d\eta_{\alpha_2}\eta_{\beta_1}$

$$D = EF + GHp^2$$

$$A = [(1/\beta^2) - 2p^2]^2 + 4p^2\eta_{\alpha_1}\eta_{\beta_1}$$

Lay and Wallace (1995)



medium 1: $\phi_1 = A_1 \exp[i\omega(px_1 + \eta_1 x_3 - t)] + A_2 \exp[i\omega(px_1 - \eta_1 x_3 - t)]$

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$D = EF + GHp^2$	
$A = [(1/\beta^2) - 2p^2]^2 + 4p^2\eta_{\alpha_1}\eta_{\beta_1}$	

Lay and Wallace (1995)

Ray approximation

$$\tilde{f}(\omega) = \int_{-\infty}^{+\infty} f(t) \,\mathsf{exp} - i\omega t dt$$

$$\tilde{\Phi}(\omega; \vec{x}) = \tilde{f}(\omega) A(\omega; \vec{x}) \,\mathsf{exp} - i\omega T(\vec{x})$$

f : source

A : amplitude term

T : travel time

For ω large, $A(\omega; \vec{x})$ tends to be independant of ω (when $\lambda << a$ the correlation length of the heterogeneity).

Development in $\frac{1}{\omega}$:

$$A(\omega; \vec{x}) = \sum_{j=0}^{\infty} \frac{A_j(x)}{(i\omega)^j}$$

Simplest approximation : $j = 0$ i.e. $\omega \to \infty$.

$$\tilde{\Phi}(\omega; \vec{x}) = \tilde{f}(\omega) A_0(x) \exp -i\omega T(\vec{x})$$

pour $\omega \to \infty$.

$$\Delta\Phi = \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2}$$

Example for derivative w.r to x

$$\frac{\partial \Phi}{\partial x} = \tilde{f}(\omega)\ \left(A_0(x) i\omega \frac{\partial T}{\partial x} exp^{i\omega T}\ +\ \frac{\partial A_0}{\partial x} exp^{i\omega T}\right)$$

$$\frac{\partial^2 \Phi}{\partial x^2} = \tilde{f}(\omega)\ \left(A_0(x)\ i\omega \frac{\partial T}{\partial x}\ i\omega \frac{\partial T}{\partial x}\ +\ \frac{\partial A}{\partial x} i\omega \frac{\partial T}{\partial x}\ +\ A_0\ i\omega \frac{\partial^2 T}{\partial x^2}\ +\ \frac{\partial^2 A_0}{\partial x^2}\ +\ \frac{\partial A_0}{\partial x} i\omega \frac{\partial T}{\partial x}\right) exp^{i\omega T}$$

Dominant terms (ω^2)

$$\frac{\partial^2 \Phi}{\partial x^2}\ \sim\ -\Phi\, \omega^2\ \left(\frac{\partial T}{\partial x}\right)^2$$

Derivatives in x, y, z:

$$\Delta \Phi\ =\ -\Phi\, \omega^2\ \mid \nabla T \mid^2$$

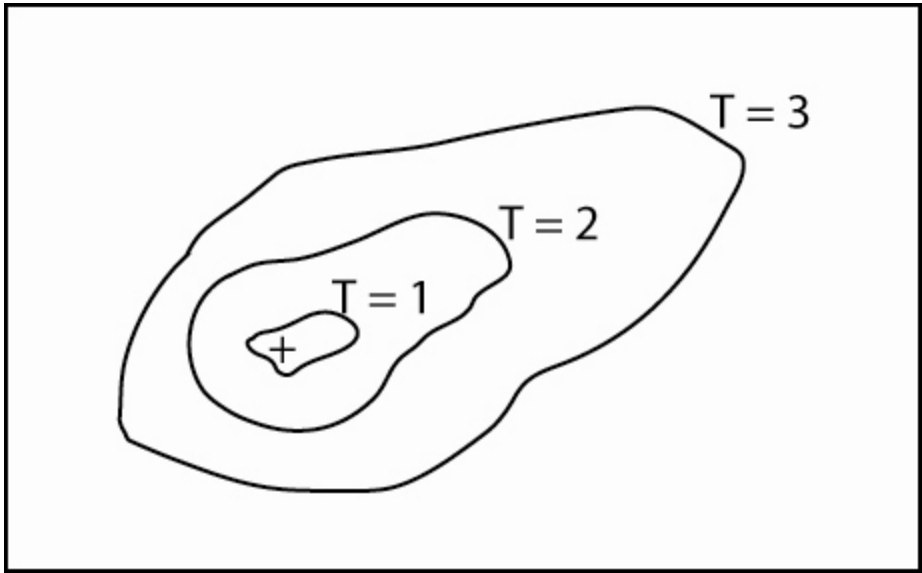
Derivatives in x, y, z:

$$\Delta\Phi \;=\; -\Phi\,\omega^2 \;|\,\nabla T\,|^2$$

and with wave equation :

$$-\Phi\,\omega^2 \;|\,\nabla T\,|^2 = \;-\frac{\omega^2}{c^2}\,\Phi$$

$$|\,\nabla T\,|^2 = \;\frac{1}{c^2}$$



Eikonal

RAY DEFINITION :

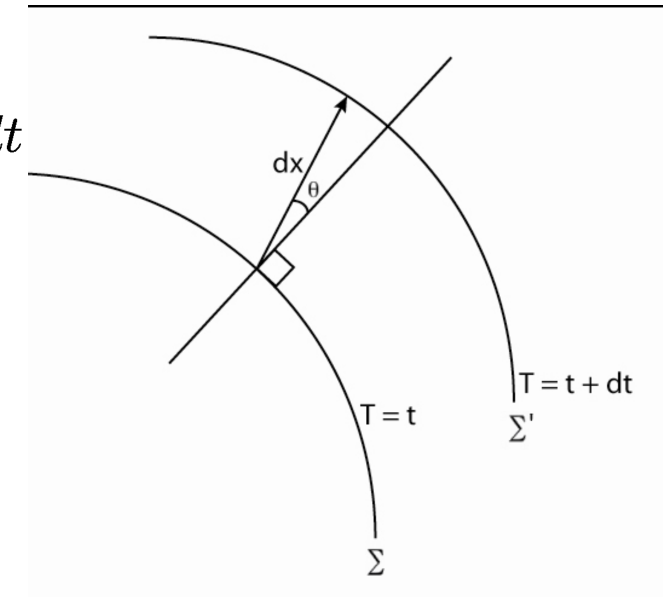
$\sum$ wave front, $\vec{x} \,/\, T(\vec{x}) = t$

$$\vec{x} + \vec{dx} \quad \in \sum \; /T \; = \; t + dt$$

$\vec{dx}$ arbitrary.

$$t + dt \; = \; T(\vec{x} + \vec{dx})$$

$$= \; T(\vec{x}) + \vec{\nabla} T . \vec{dx}$$



$$dt \; = \; \vec{\nabla} T . \vec{dx}$$

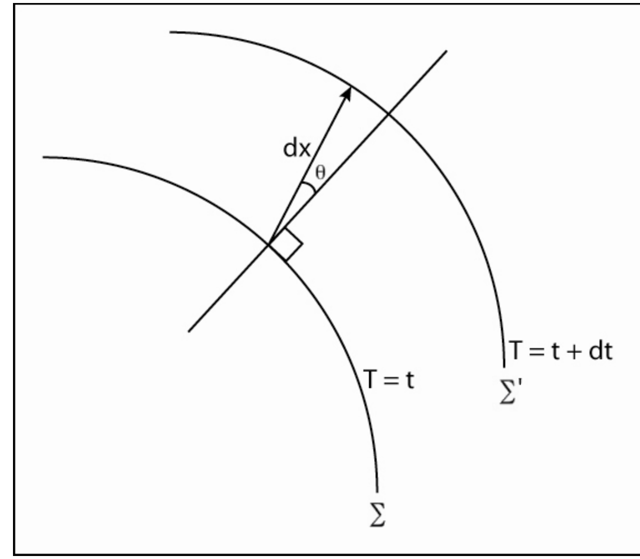
Physical meaning for particular $\vec{dx}$?

$$\vec{V} = \frac{\vec{dx}}{dt}$$

$$\vec{\nabla T} \,.\, \vec{V} \,.\, dt \;=\; \vec{\nabla T} \,.\, dx \;=\; dt$$

$$\vec{\nabla T} \,.\, \vec{V} \;=\; 1$$

$$|\, \vec{\nabla T} \,| \;.\; |\, \vec{V} \,| \;.\, cos\,\theta \;=\; 1$$



If $\theta \;=\; 0$

$|\, \vec{\nabla T} \,| \;.\; |\, \vec{V} \,| = 1$ $\qquad \Rightarrow \;\; V^2 = \dfrac{1}{|\, \vec{\nabla T} \,|^2}$

$$\Rightarrow \;\; V^2 \;=\; c^2$$

True velocity measured along trajectories perpendicular to the wave front: rays

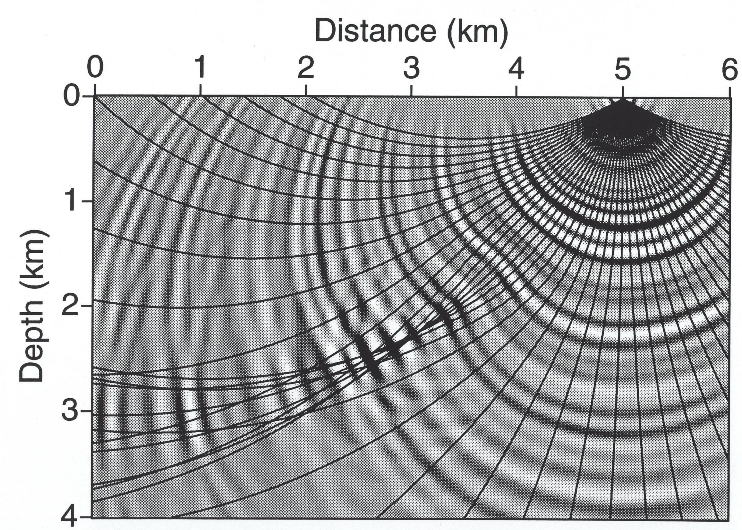
Amplitude knowing travel time

Dominant term in ω :

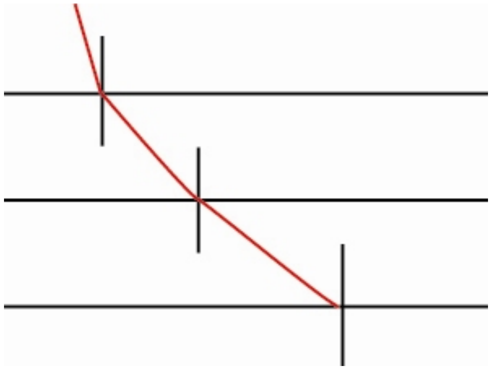
$$\frac{\partial^2 \Phi}{\partial x^2} = 2\, i\, \omega \, \frac{\partial A_0}{\partial x} \, \frac{\partial T}{\partial x} \; + \; i\omega A_0 \frac{\partial^2 T}{\partial x^2}$$

$$\Delta \Phi \; = \; 2\, i\, \omega \, \vec{\nabla} A_0 \, \vec{\nabla} T \; + \; i\, \omega \, A_0 \, \Delta T \; = \; 0$$

$$\Rightarrow 2 \vec{\nabla} A \, \vec{\nabla} T \; + \; A \, \Delta T \; = \; 0$$

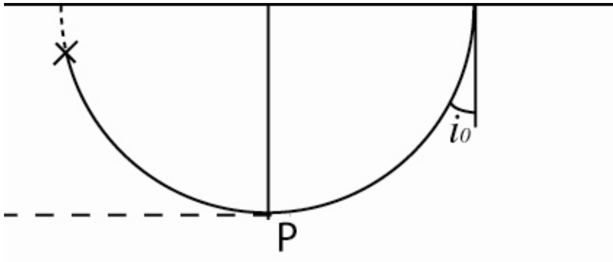


1D STRATIFICATION : V(z)



$$\frac{\sin i}{V} = \frac{\sin i'}{V'} = C^{ste} = p \geq \ 0$$

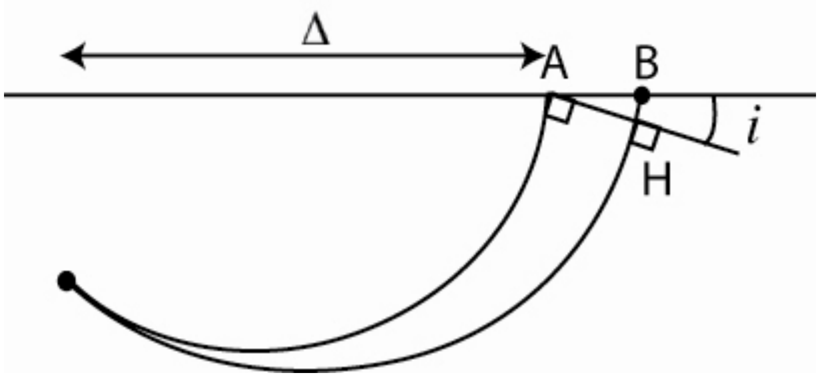
$$p = \frac{\sin i(z)}{V(z)}$$



$$V = V(h) \qquad i = \frac{\pi}{2} \ \Rightarrow \ \sin i = 1 \qquad et\ p = \frac{1}{V(h)}$$

i :incidence angle , Δ epicentral distance

$AB = \delta x$; $HB = V_0 \delta t$



$$\sin i = \frac{HB}{AB} = V \frac{\delta t}{\delta x}$$

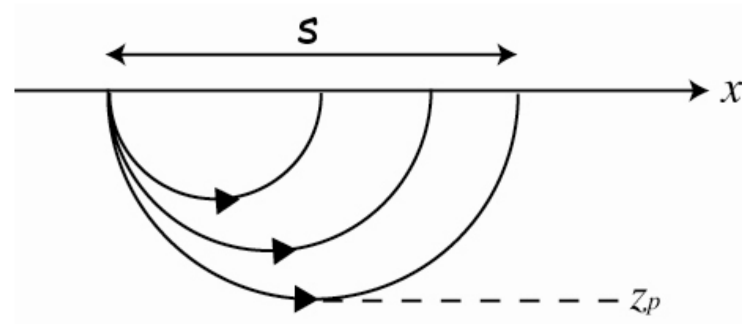
$$\Longrightarrow \qquad p = \frac{\sin i}{V} = \frac{\delta t}{\delta x}$$

$$p = \frac{dt}{d\Delta}$$

Apparent velocity

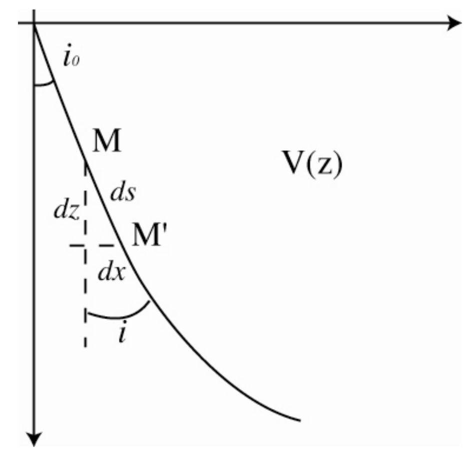
$$V_a = \frac{\partial T}{\partial x} = \frac{\sin i}{V(z)} = p$$

Emergence



$$i = \frac{\pi}{2} \quad \Rightarrow \quad \sin i = 1$$

$$p = \frac{1}{V(h)}$$

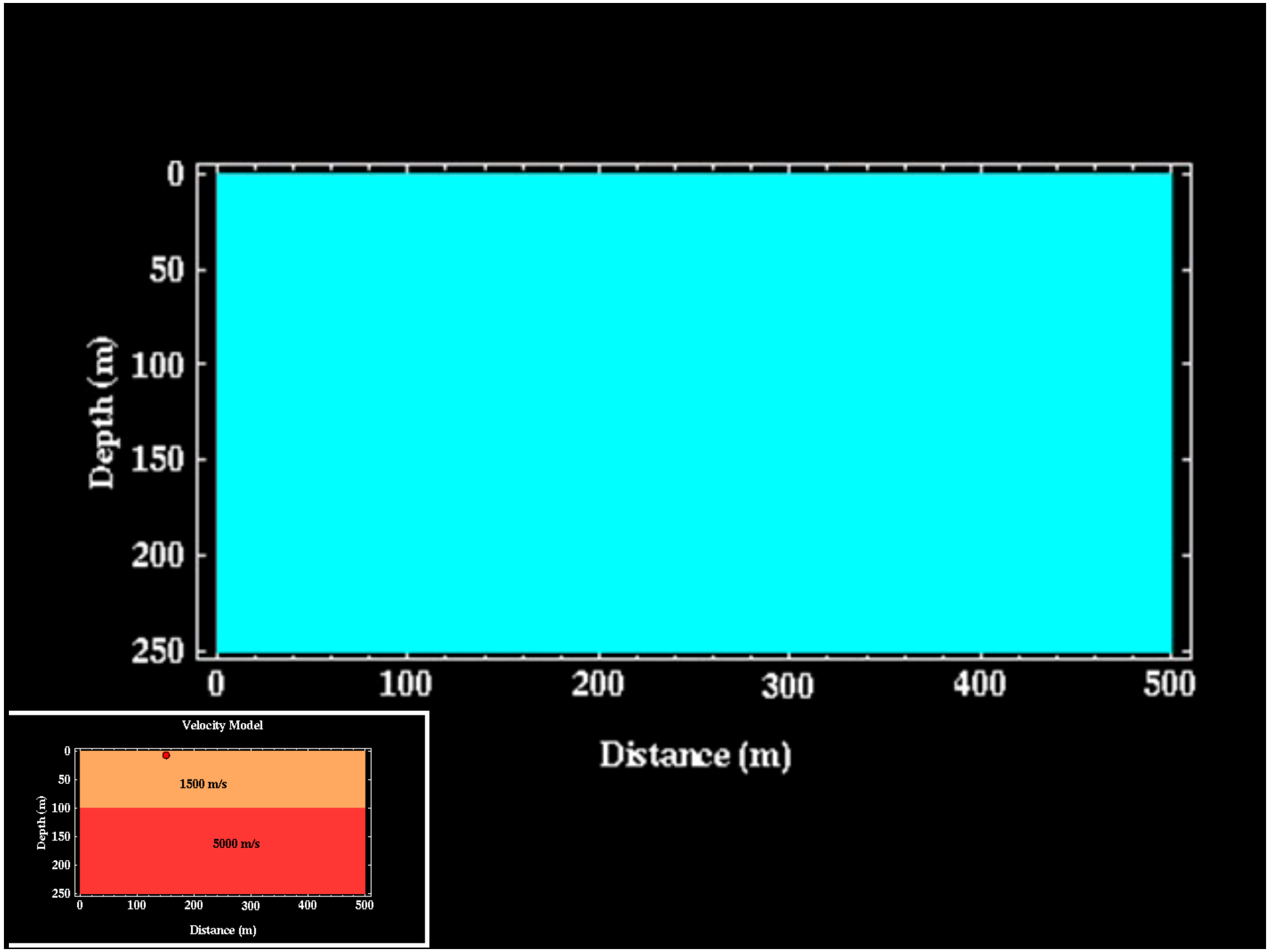


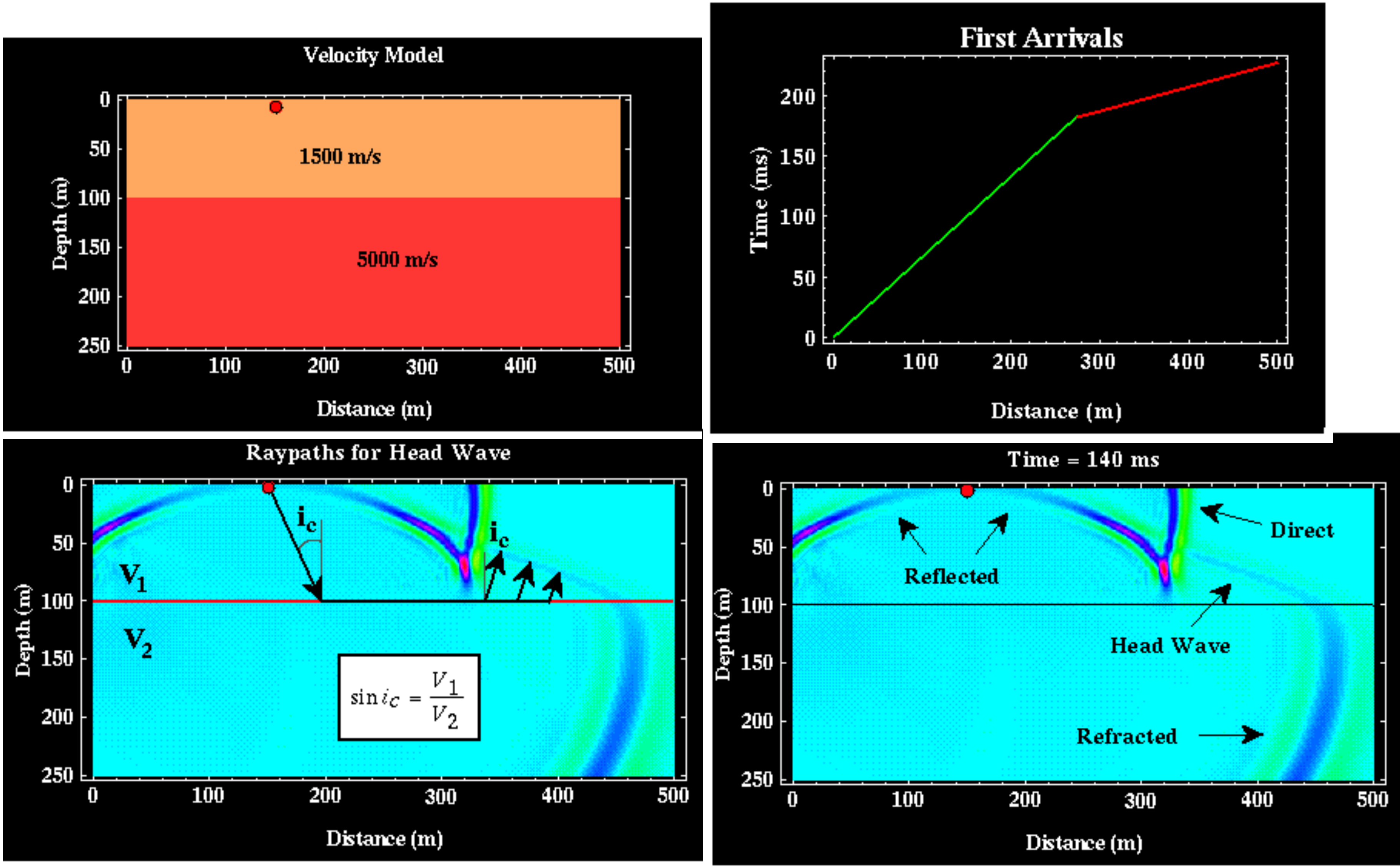
$$p = \frac{\sin i}{V} \quad \Longrightarrow \quad \tan i = \frac{pV}{\sqrt{1-p^2V^2}}$$

$$\Delta = \int_0^{\Delta} dx = 2\int_0^{z_p} dz \; \tan i = 2\int_0^{z_p} \frac{pV(z)}{\sqrt{1-p^2V^2(z)}} dz$$

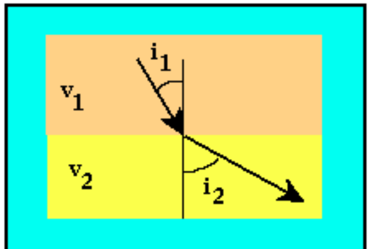
$$t = \int_0^{\Delta} \frac{ds}{V(z)} = 2\int_0^{z_p} \frac{dz}{V\cos i} = 2\int_0^{z_p} \frac{dz}{V(z)\sqrt{1-p^2V^2(z)}}$$

Low velocity layer over a half space

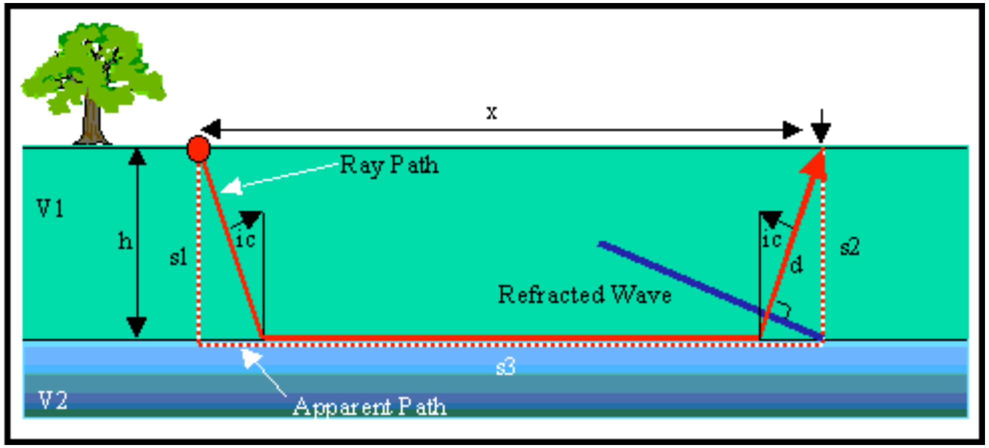


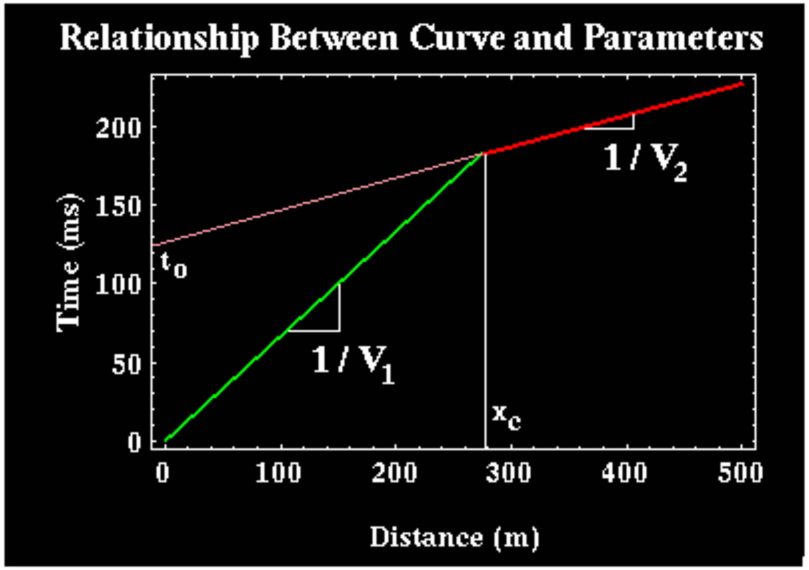

Velocity Model
1500 m/s
5000 m/s
Depth (m)
Distance (m)
First Arrivals
Time (ms)
Distance (m)
Raypaths for Head Wave
i_c
i_c
V_1
V_2
$\sin i_c = \frac{V_1}{V_2}$
Depth (m)
Distance (m)
Time = 140 ms
Direct
Reflected
Head Wave
Refracted
Depth (m)
Distance (m)

Rays and travel times



$$\frac{\sin i_1}{v_1} = \frac{\sin i_2}{v_2}$$





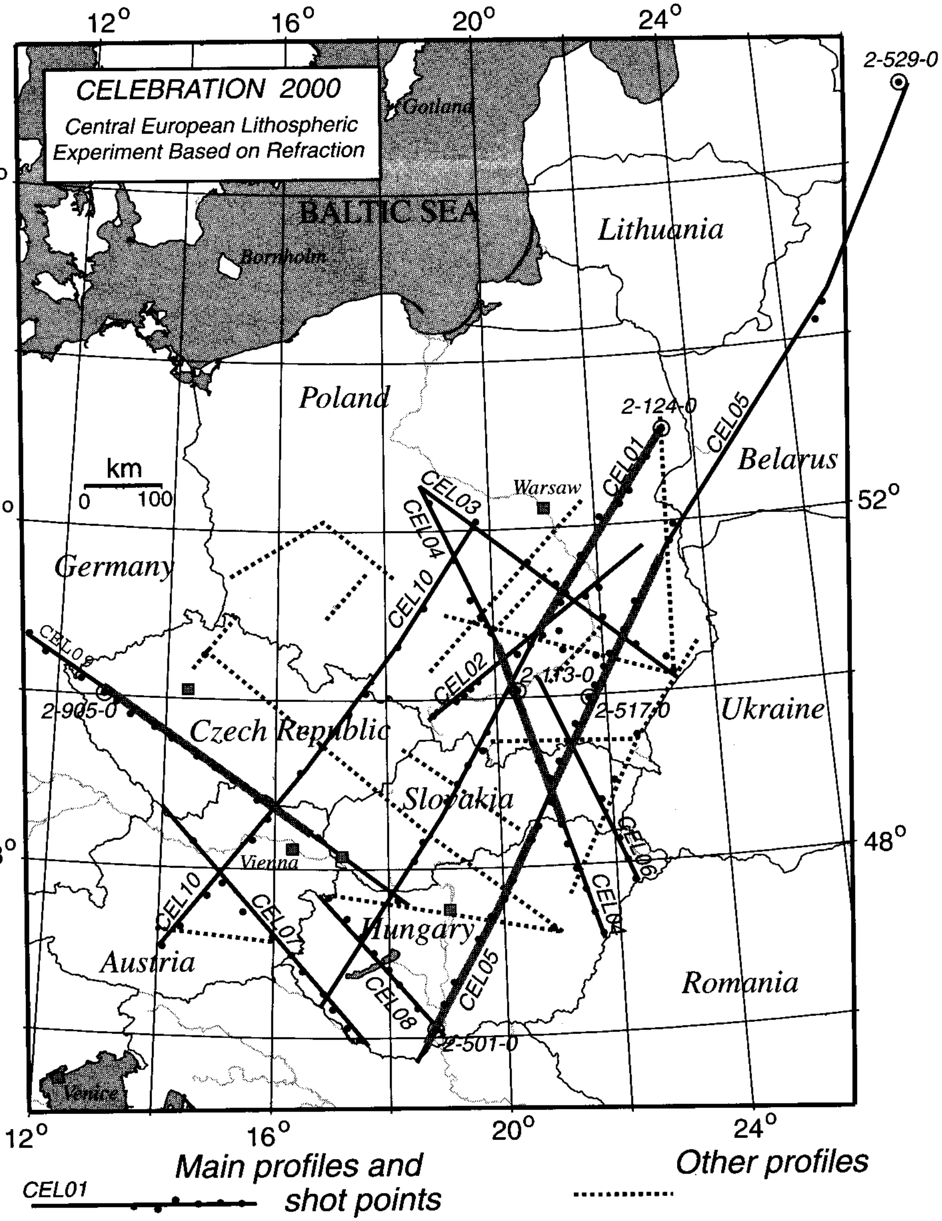
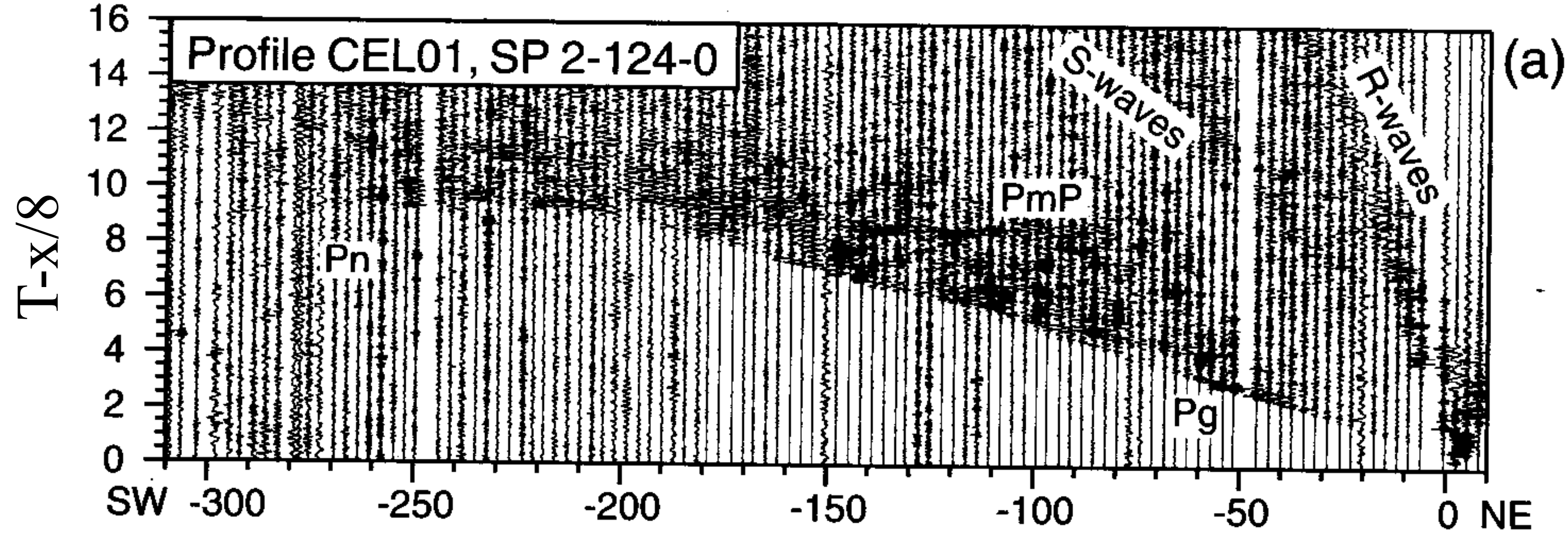
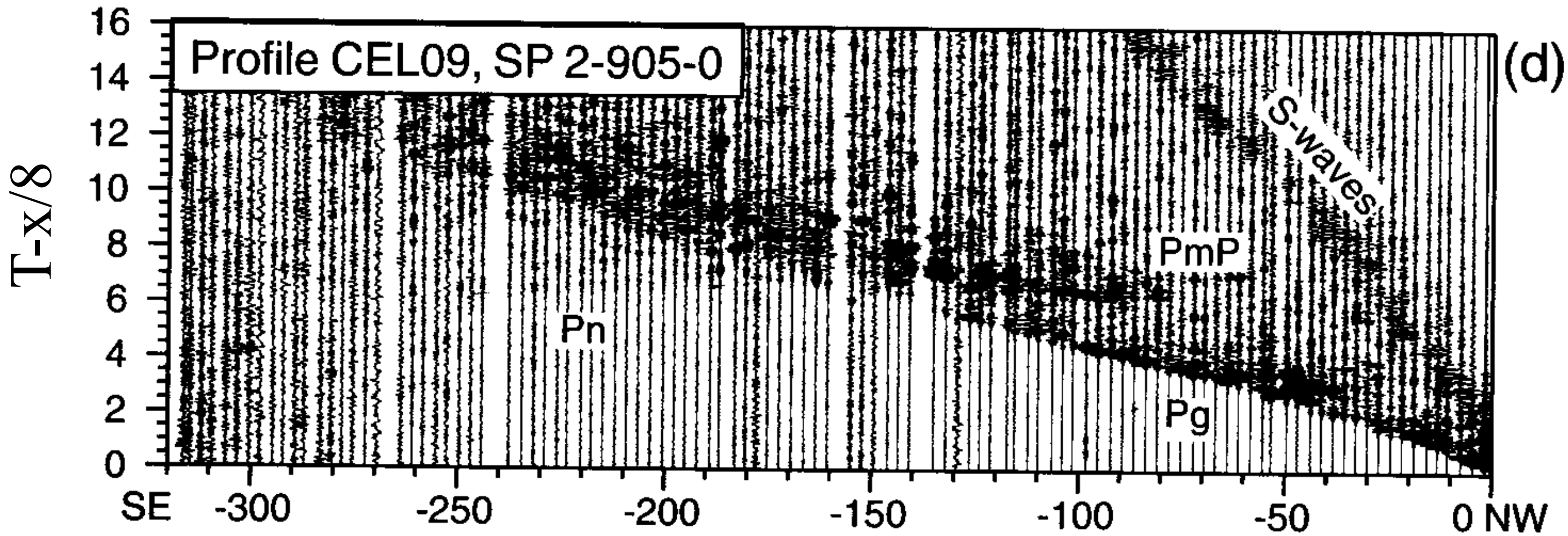


Fig. 2. Layout of the CELEBRATION 2000 seismic experiment. Thick gray lines indicate regions where the data shown in Figures 3 and 4 were recorded. Numbers next to circled dots indicate shot points that produced the data shown in these figures.


Profile CEL01, SP 2-124-0
(a)
T-x/8
S-waves
R-waves
PmP
Pn
Pg
SW
NE
16
14
12
10
8
6
4
2
0
-300
-250
-200
-150
-100
-50
0


Profile CEL09, SP 2-905-0
(d)
T-x/8
S-waves
PmP
Pn
Pg
SE
NW
16
14
12
10
8
6
4
2
0
-300
-250
-200
-150
-100
-50
0

Computer simulation of slab guided waves

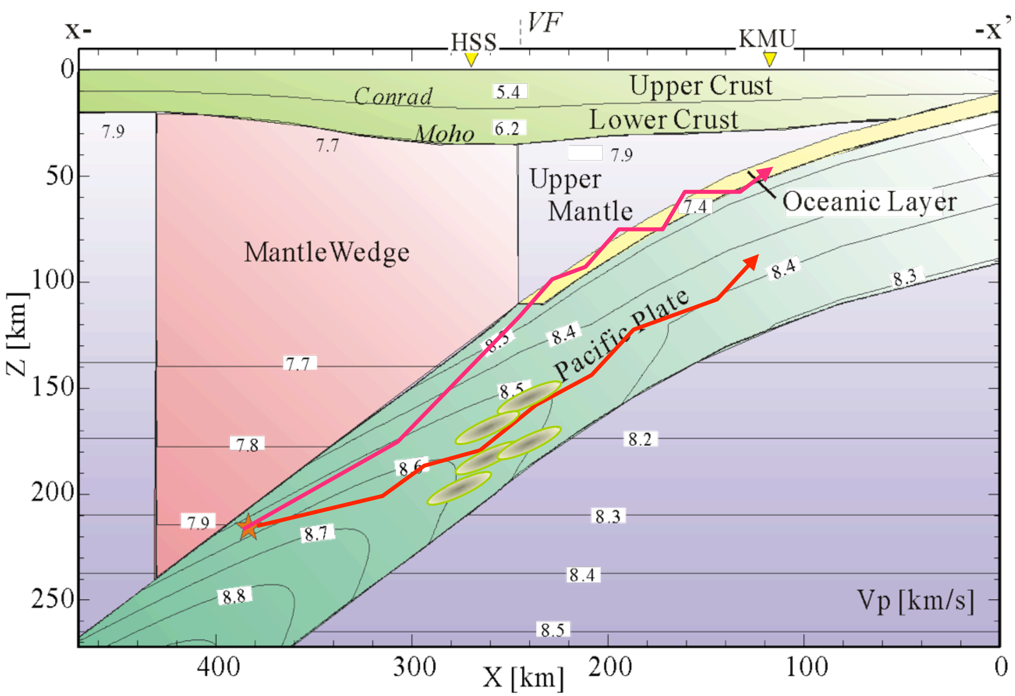
Model:

1. Trapped Signal in Low-V Oceanic Crust

Abers[2003], Martin et al.[2003], Furumura [1998]

2. Scattering Waveguide Effect in Heterogeneous Plate

Furumura and Kennett [2005] NEW

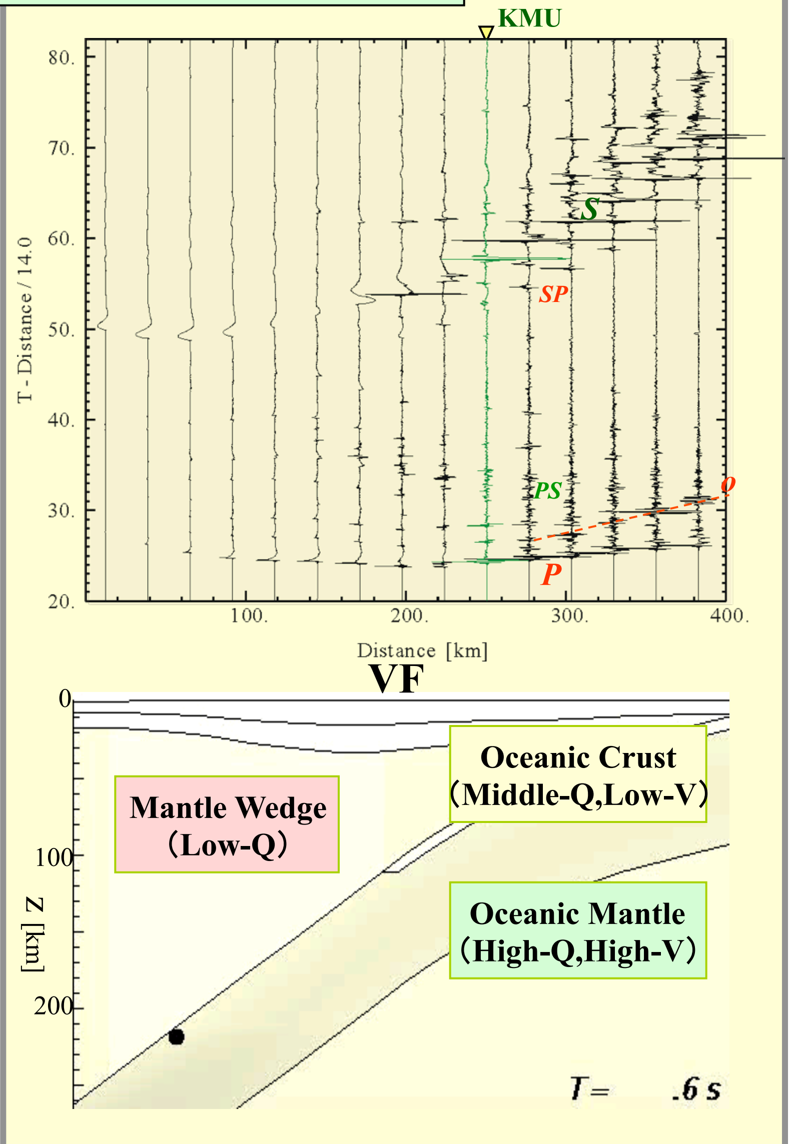


2D Simulation Model

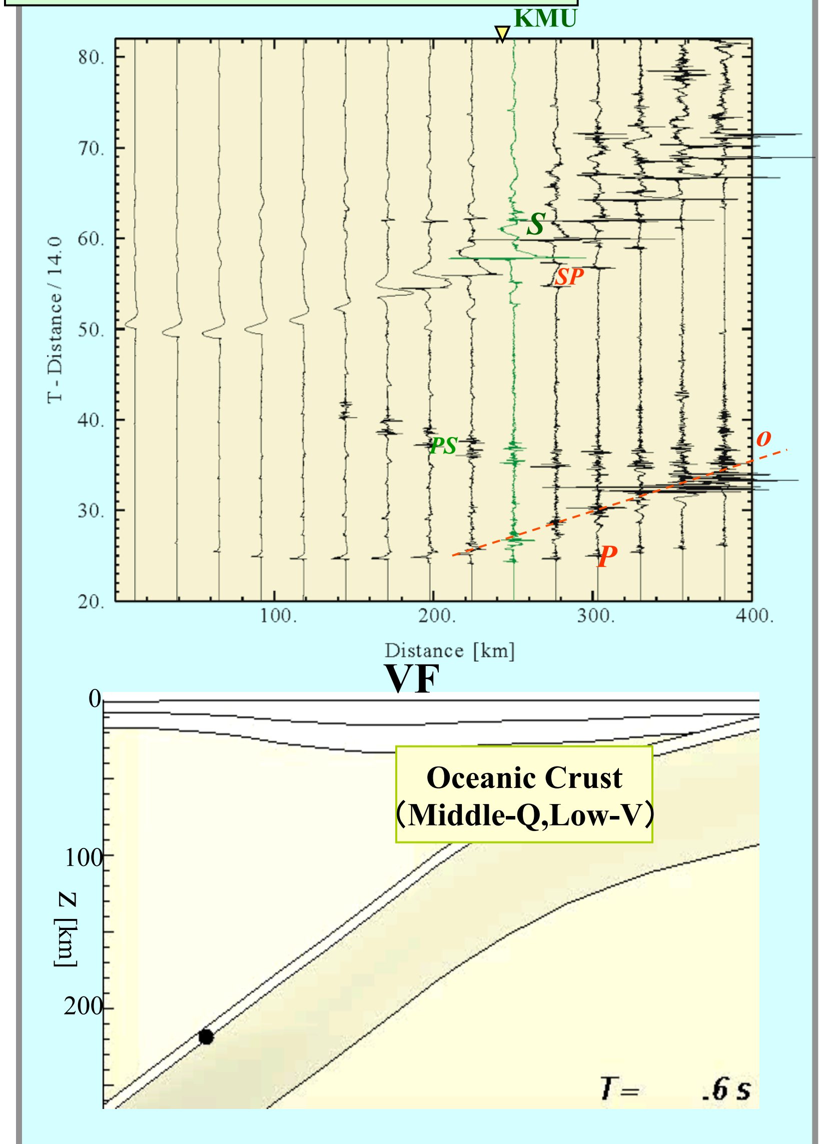
Model Size	400*280 km (Dx=60m)
Source	Point Source * Nakamura & Miyatake (2000)
Max Freq.	20 Hz *(Vs<3.2km/s)*
Scheme	Parallel FDM
Memory	4 GByte
CPU Time	60 h (Intel Xeon 16CPU)

Computer Simulation（1）: Homogeneous Plate (*iasp81)*

(a) Homogeneous Plate



(b) Extended Oceanic Crust



Observation
(KMU)