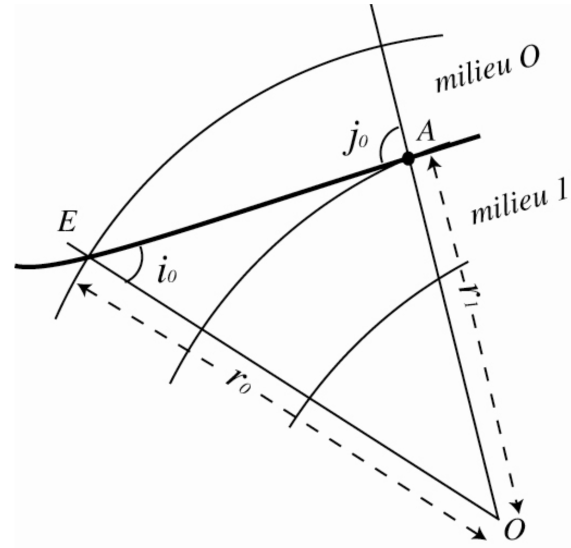
MEEES and M2R STU

Seismology (Michel Campillo)

RADIAL STRATIFICATION :



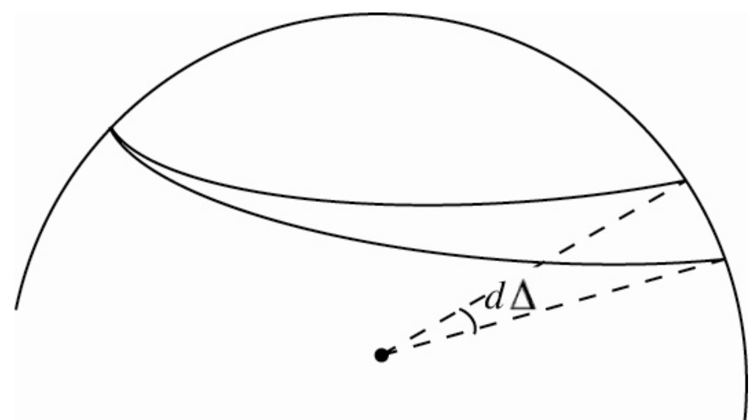
$$\Rightarrow \quad r_0 \, \sin i_0 \; = \; r_1 \, \sin j_0$$

$$\frac{\sin j_0}{V_0} = \frac{\sin i_1}{V_1}$$

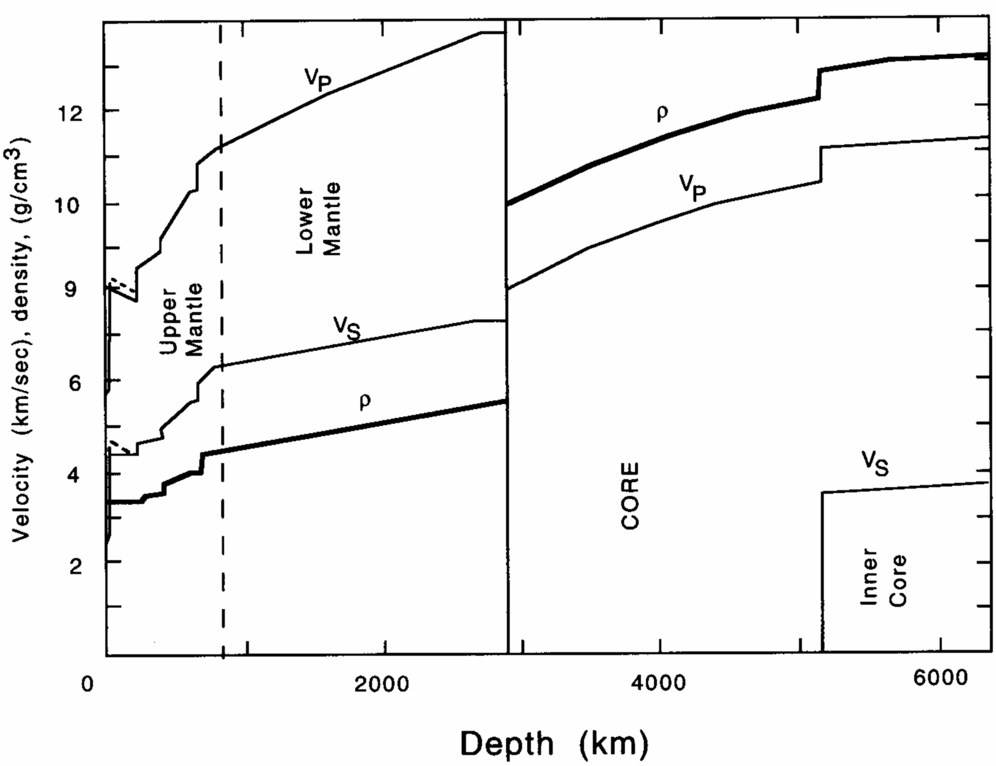
$$\Rightarrow \quad \frac{r_0}{r_1} \frac{\sin i_0}{V_0} = \frac{\sin i_1}{V_1}$$

Invariant (parameter) $\boxed{p \; = \; \frac{r \, \sin i(r)}{V(r)}}$

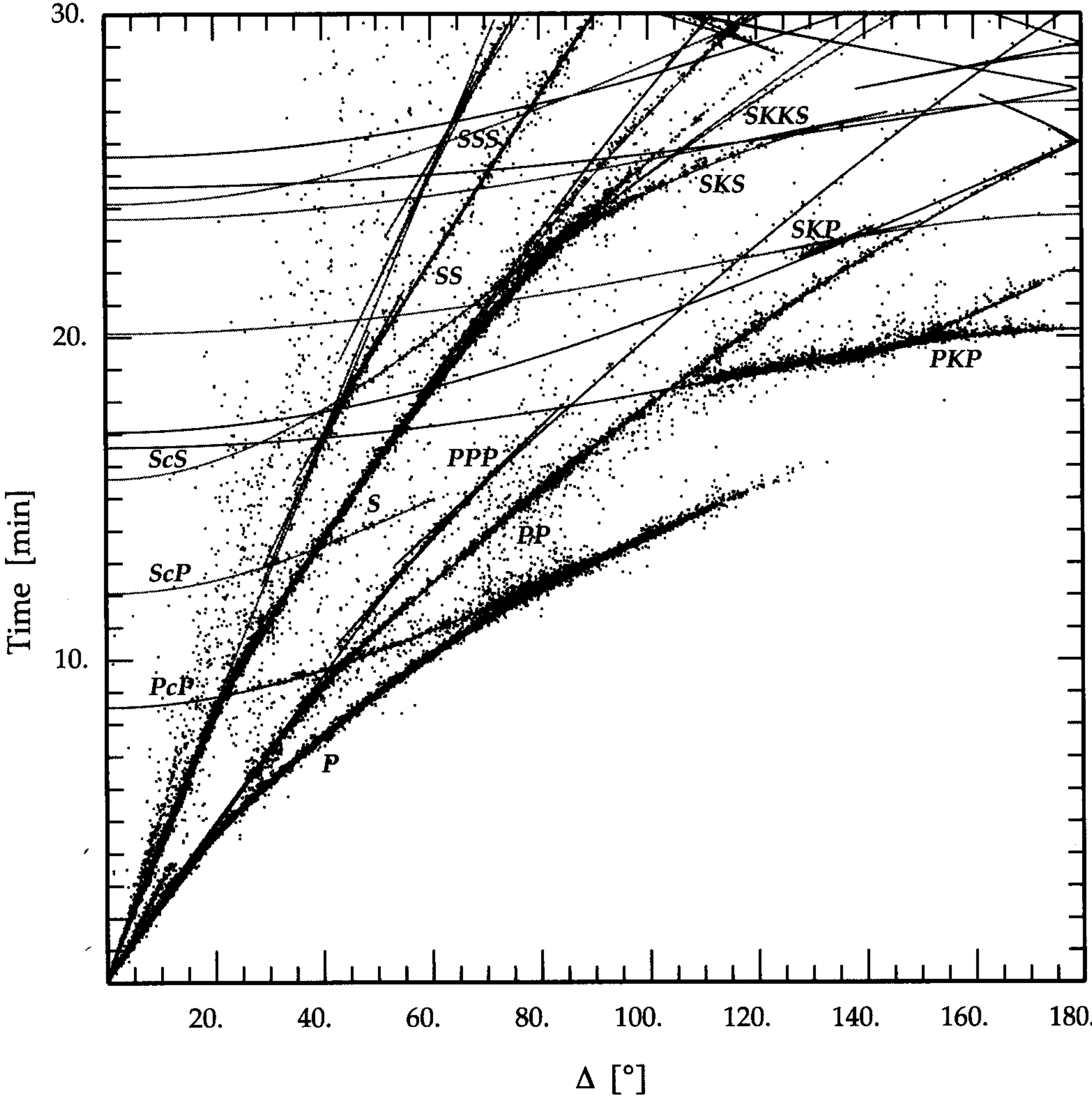
$$\boxed{p \; = \; \frac{dt}{d\Delta}} \qquad (s \,/rad)$$



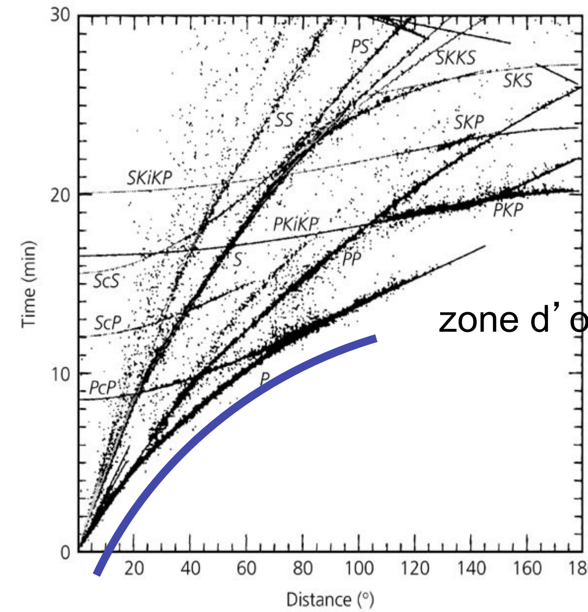
$$V_{ap}^{-1} = \frac{dt}{d\Delta} = \frac{r_0 \, d\Delta \, \sin i_0}{V_0} \frac{1}{d\Delta} = \frac{r_0 \sin i_0}{V_0} = p$$



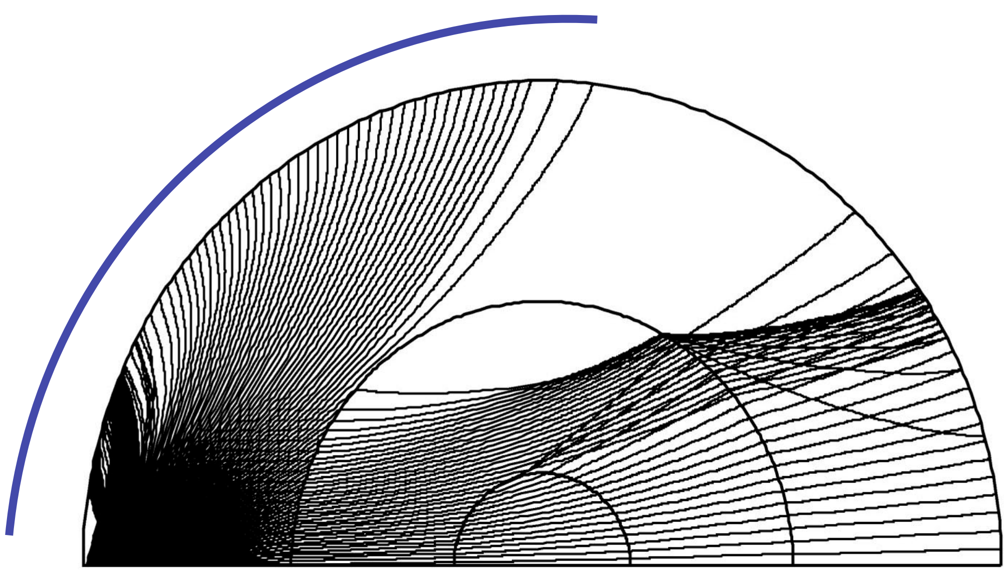
Velocity (km/sec), density, (g/cm3)
2
4
6
8
10
12
0
2000
4000
6000
Depth (km)
Upper Mantle
Lower Mantle
VP
VS
ρ
CORE
Inner Core



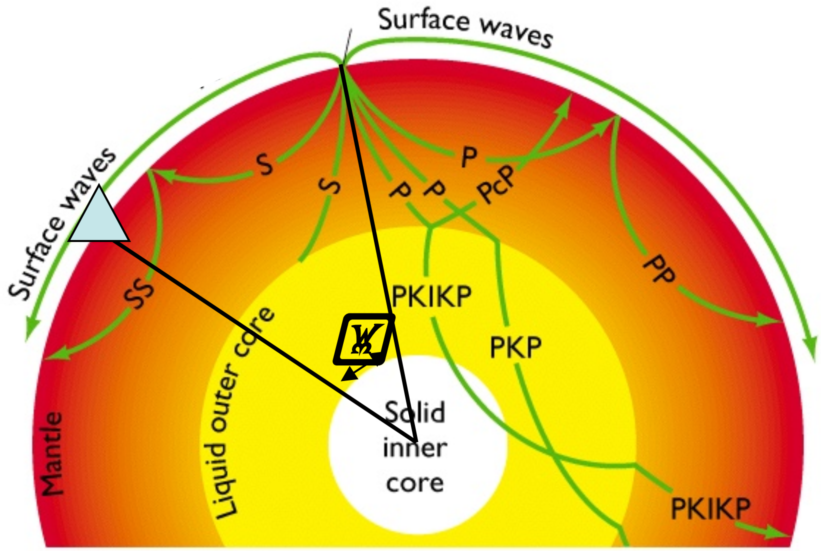
30.
20.
10.
Time [min]
20.
40.
60.
80.
100.
120.
140.
160.
180.
Δ [°]
SKKS
SSS
SKS
SKP
SS
PKP
ScS
PPP
S
PP
ScP
PcP
P



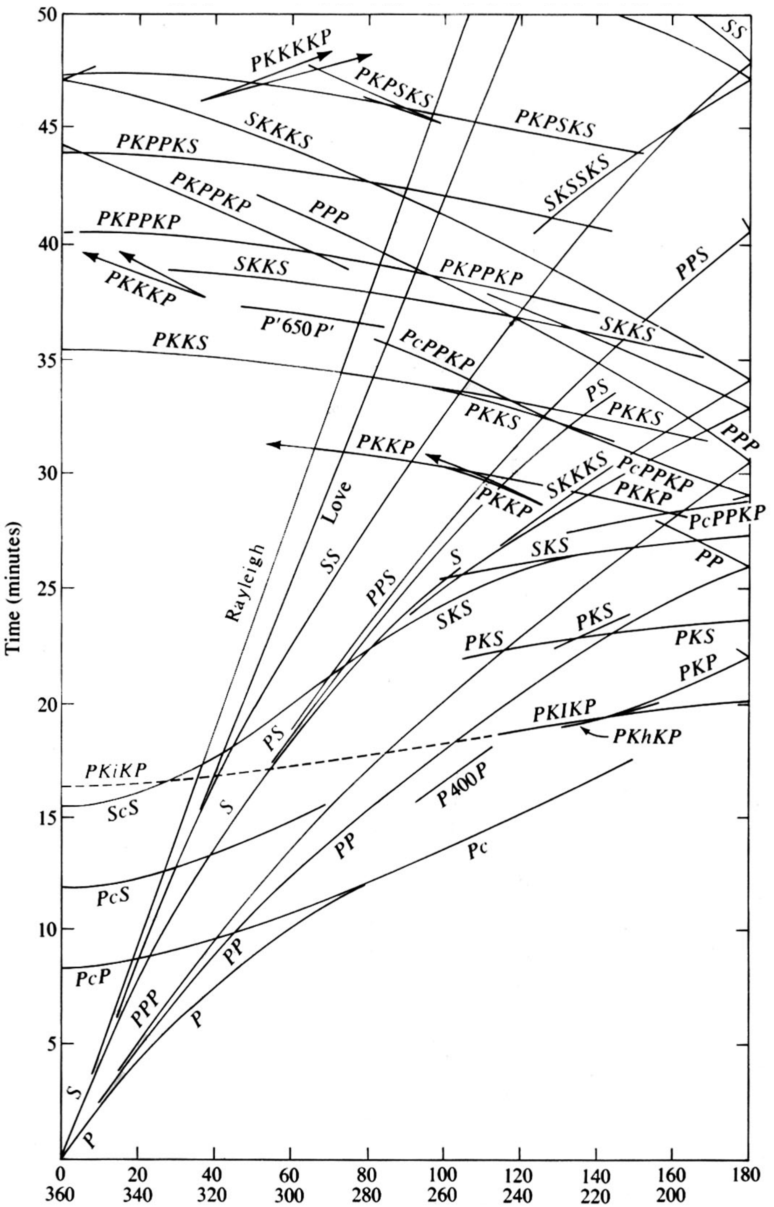
zone d’ombre



Oldham, 1906: the core

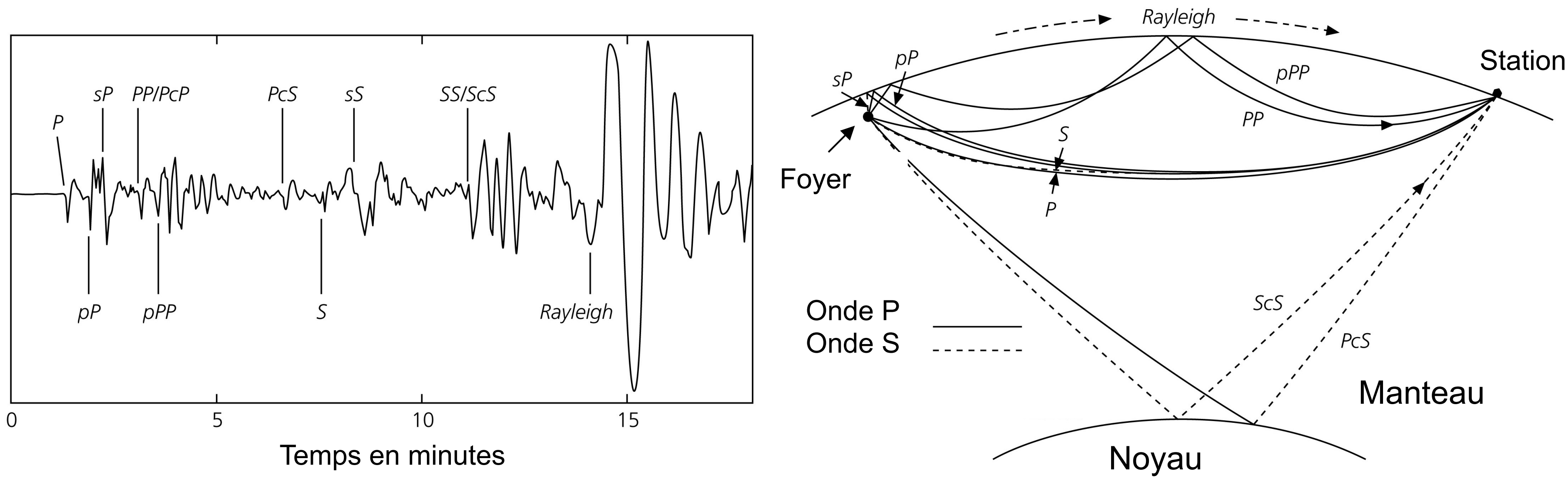


Temps de trajet

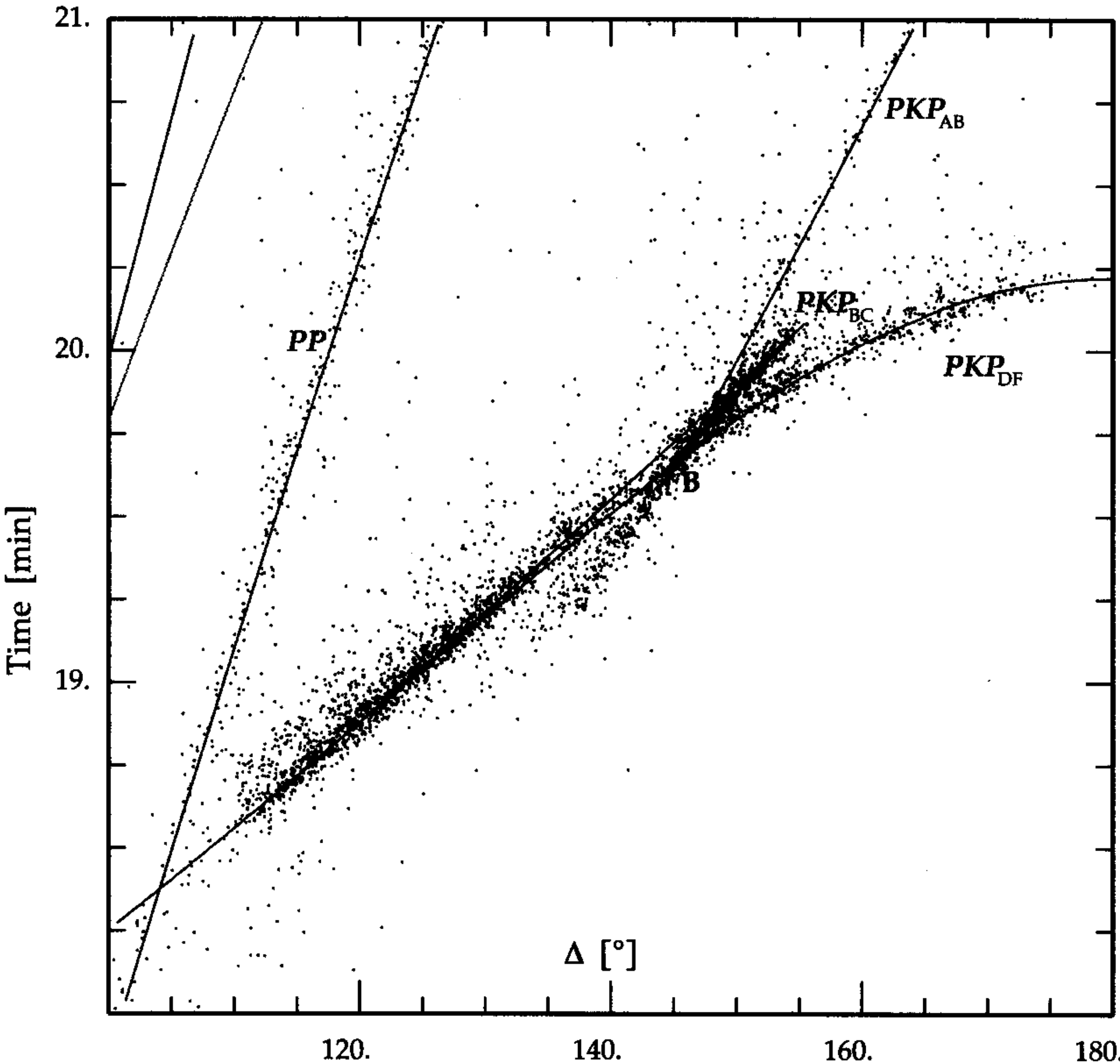


Distance angulaire en °

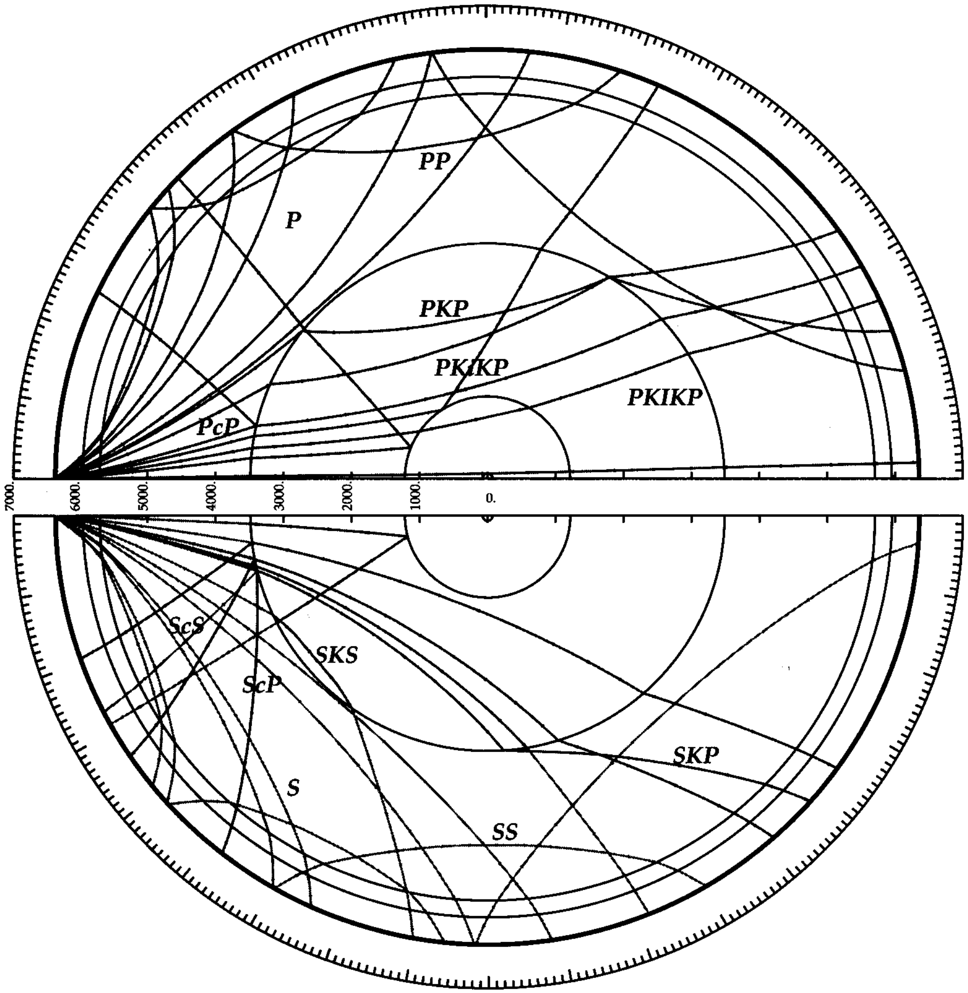
Webservice ttimes



distance=4500km



21.
20.
19.
Time [min]
Δ [°]
120.
140.
160.
180.
PP
PKP_AB
PKP_BC
PKP_DF
B



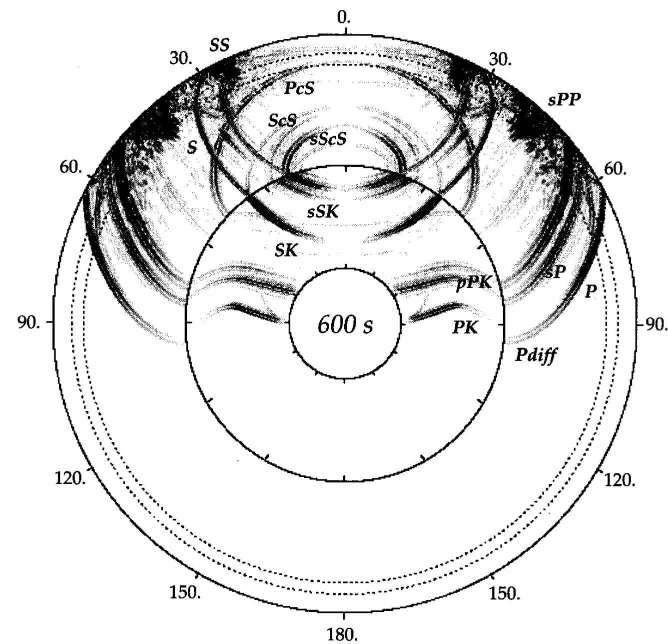
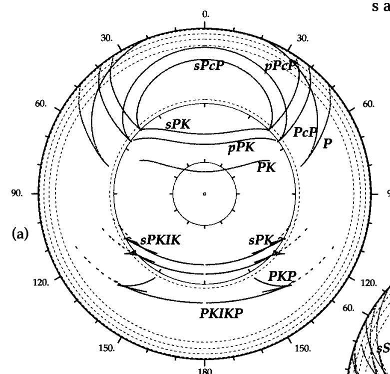
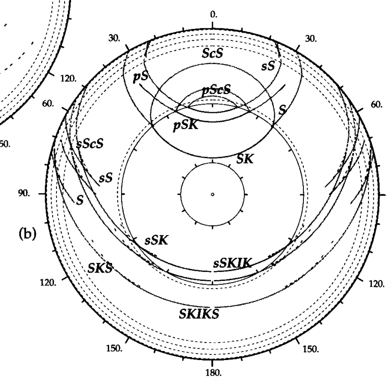
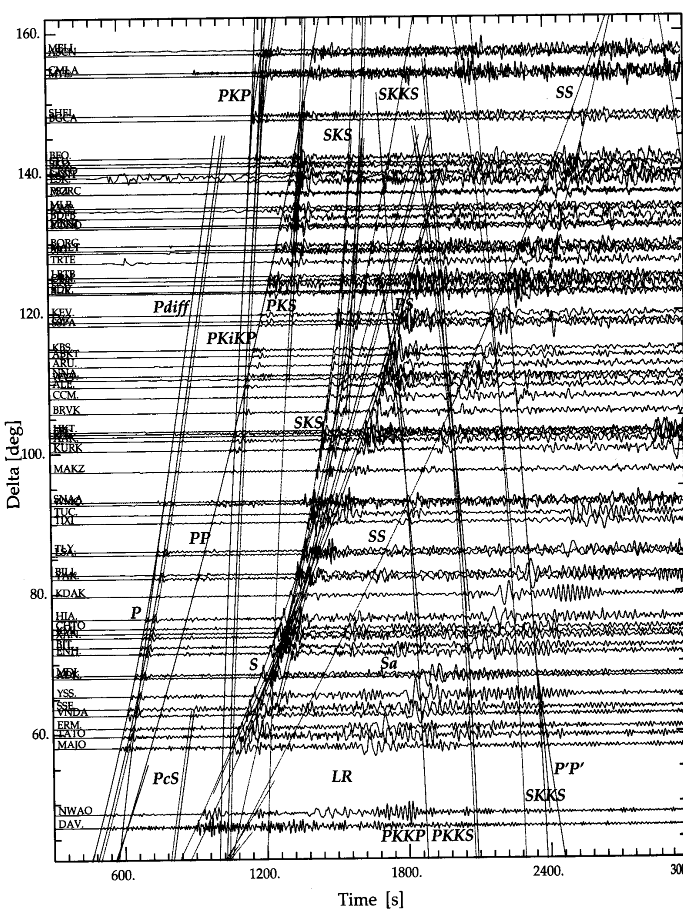


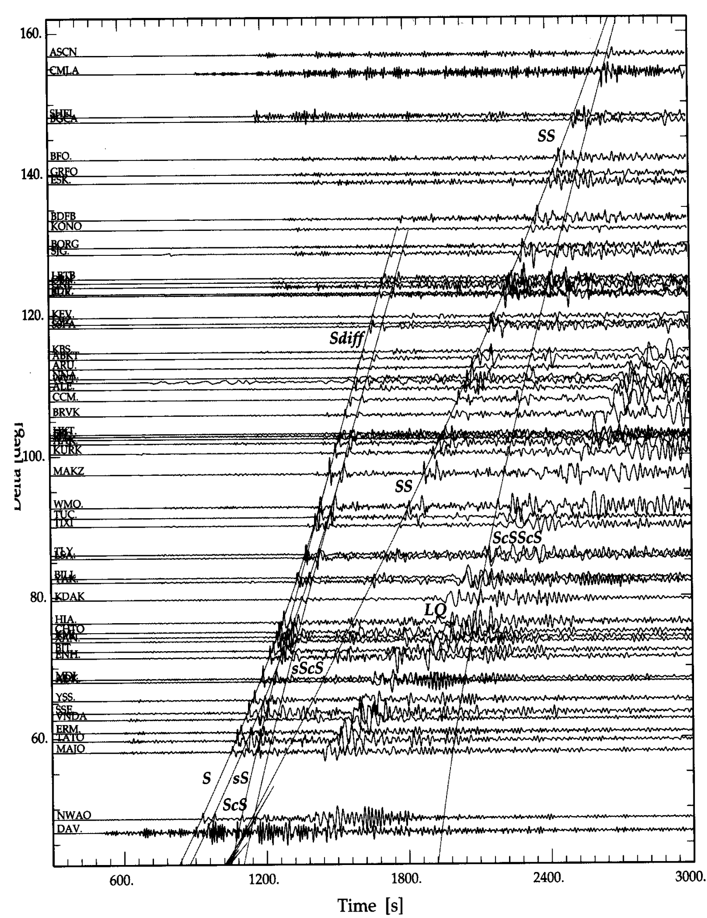
Figure 5.12. A snapshot of the seismic wavefield from a source at 260 km deep propagating in the AK135 model at 600 s after source initiation. The P wave contribution is shown in dark gray and the SV waves in light gray. The discontinuities at 410 km and 660 km are indicated by dotted lines and the core boundaries with solid lines.

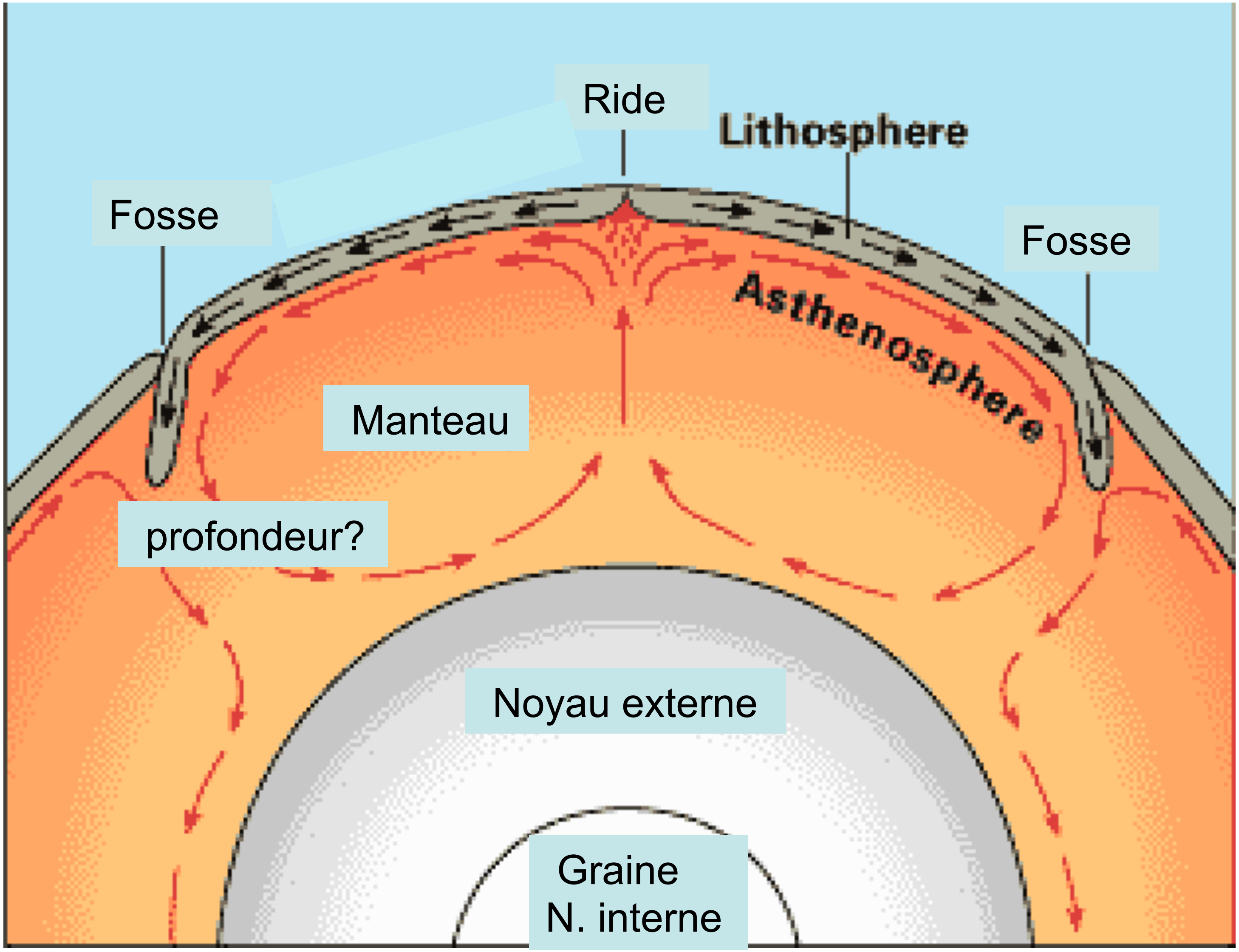
Figure 5.11. Wavefronts for the major phases from an event at 260 km depth (a) P phases at 480 s and 960 s, (b) S phases at 600 s and 1200 s.





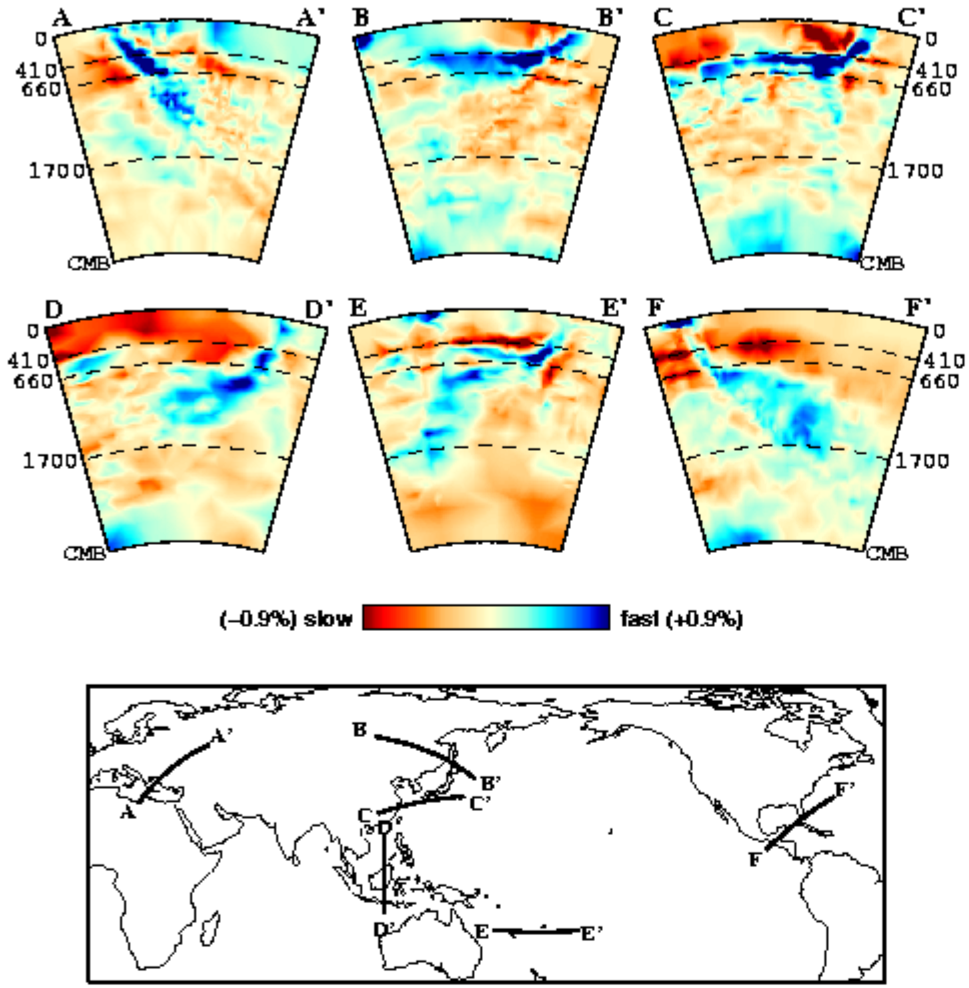






Ride
Lithosphere
Fosse
Fosse
Asthenosphere
Manteau
profondeur?
Noyau externe
Graine
N. interne

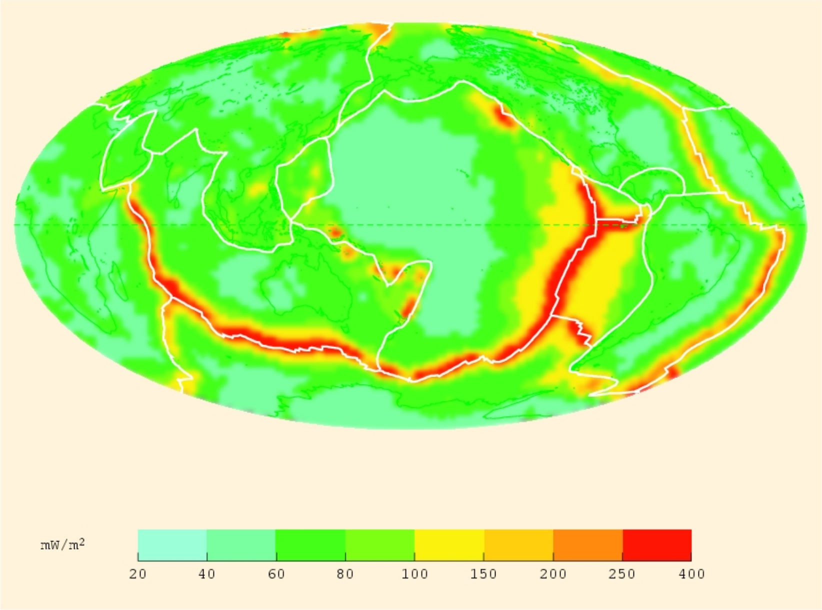
Subductions



Velocity images
+slow➔+warm

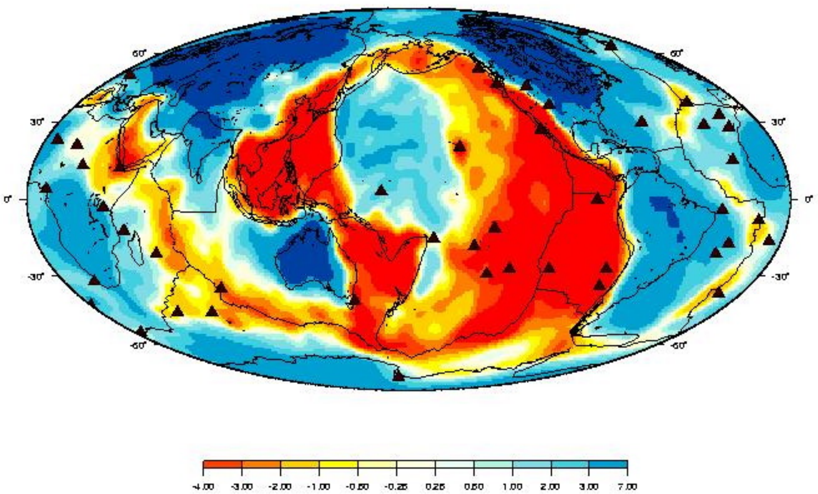
Van der Hilst et al., 1998

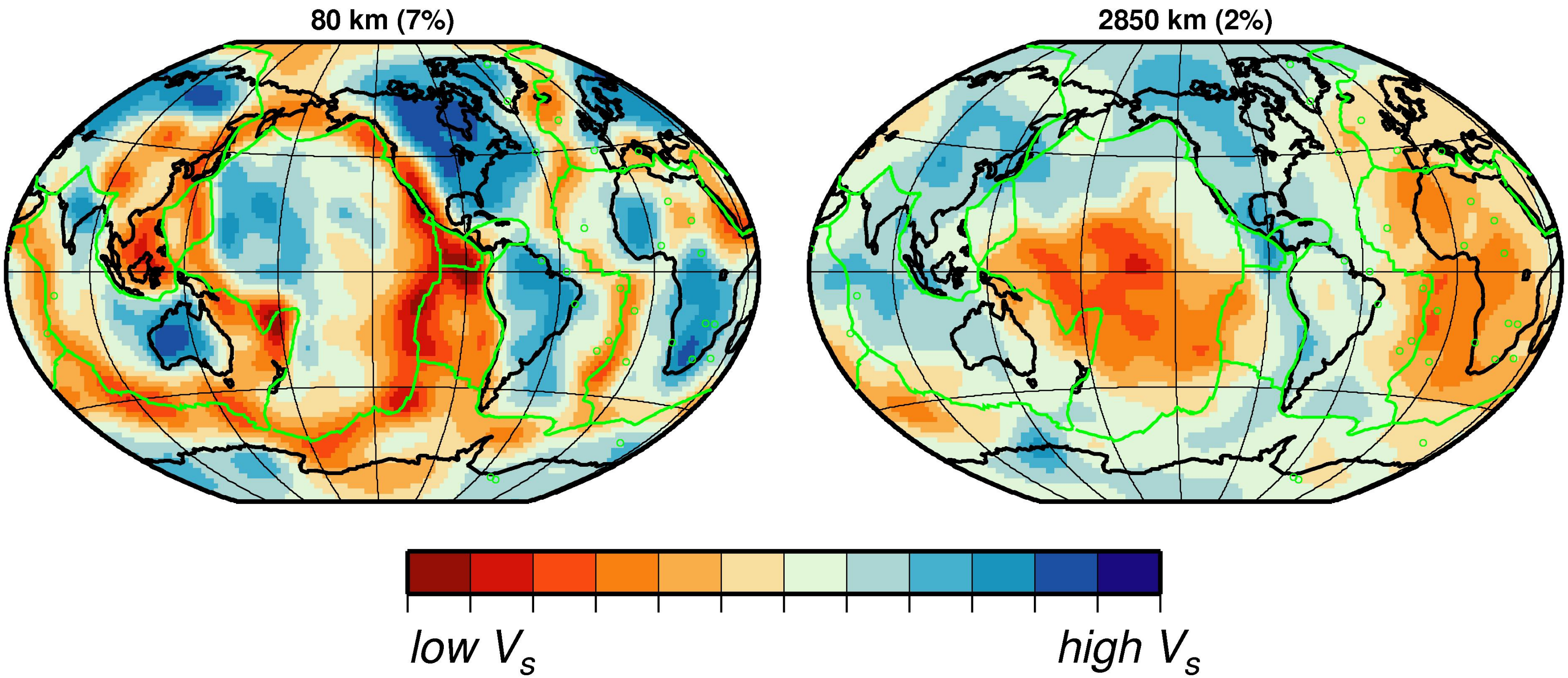
Ridge and heat flux



Flux de chaleur

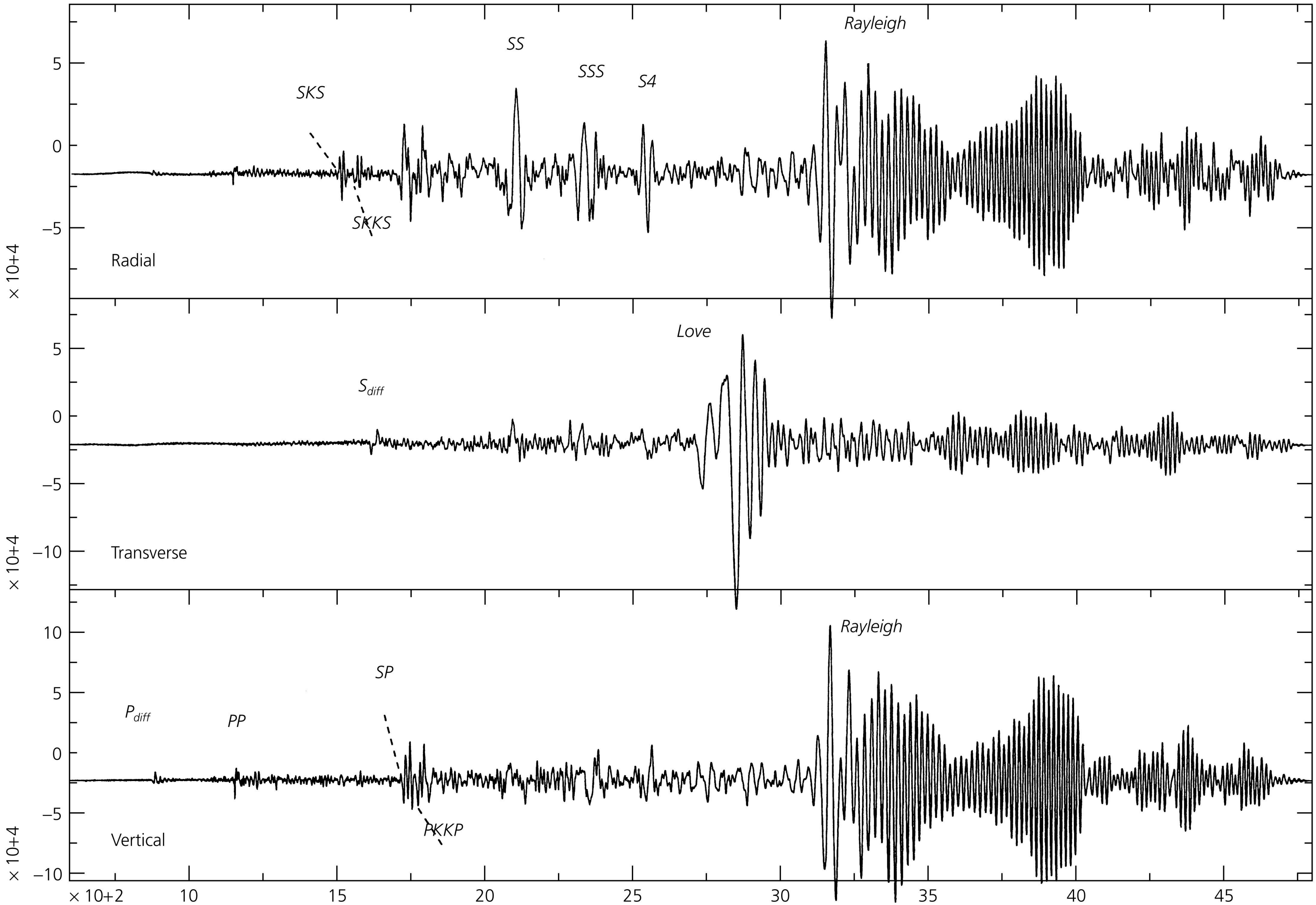
Anomalies de Vitesses d'ondes S - Profondeur=100 km





80 km (7%)
2850 km (2%)
low V_s
high V_s

Figure 2.7-1: Seismograms recorded at a distance of 110°, showing surface waves.



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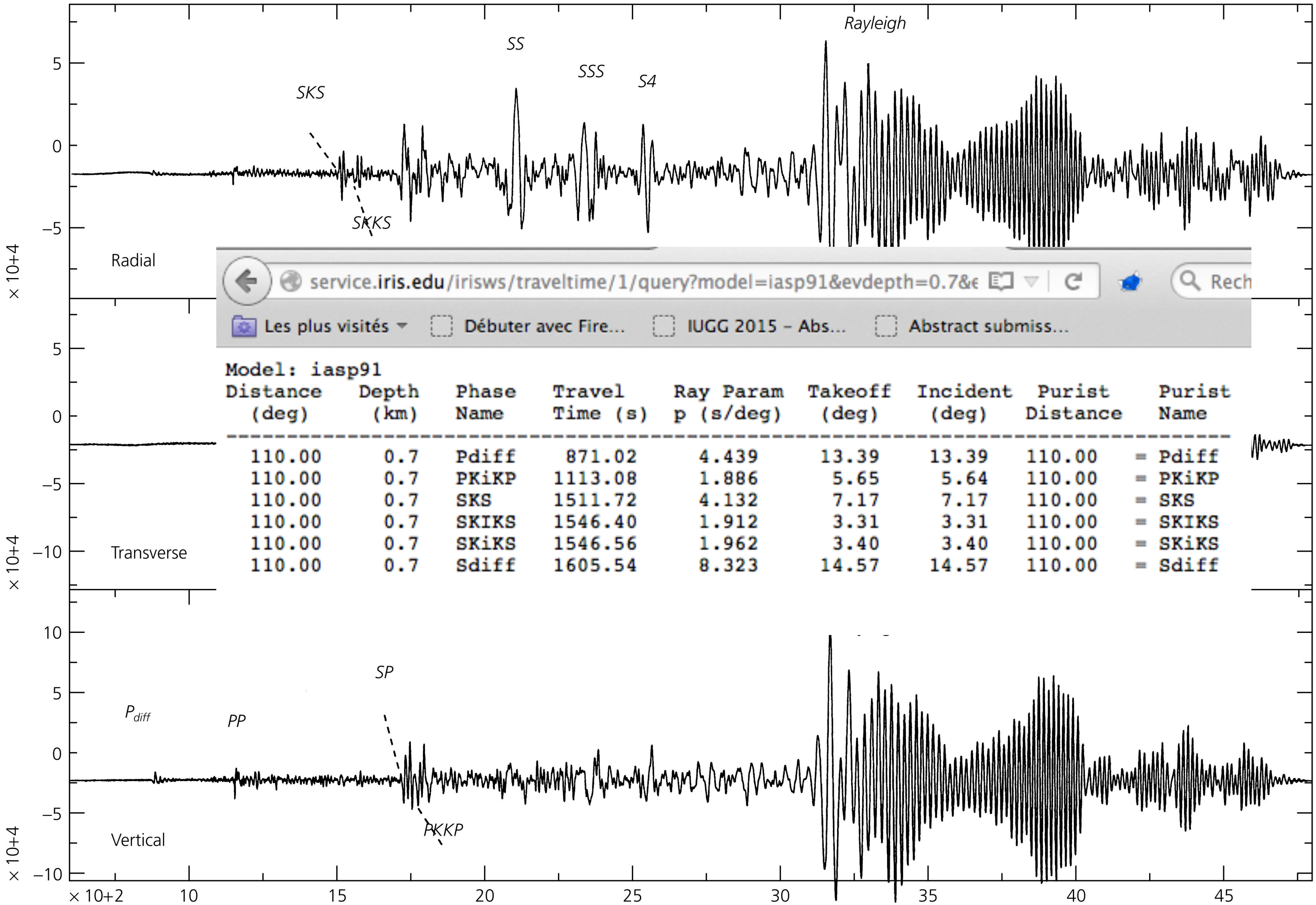
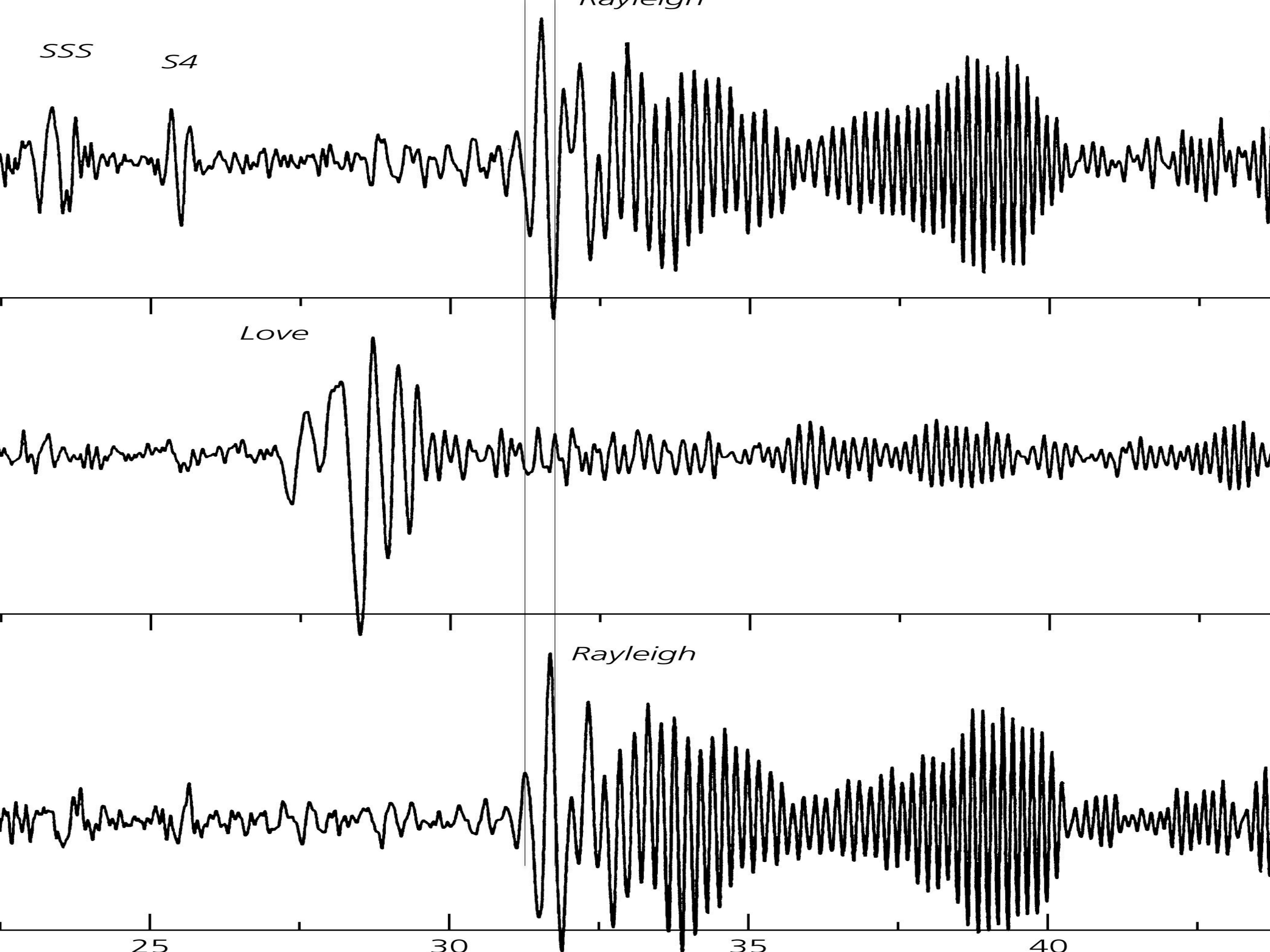
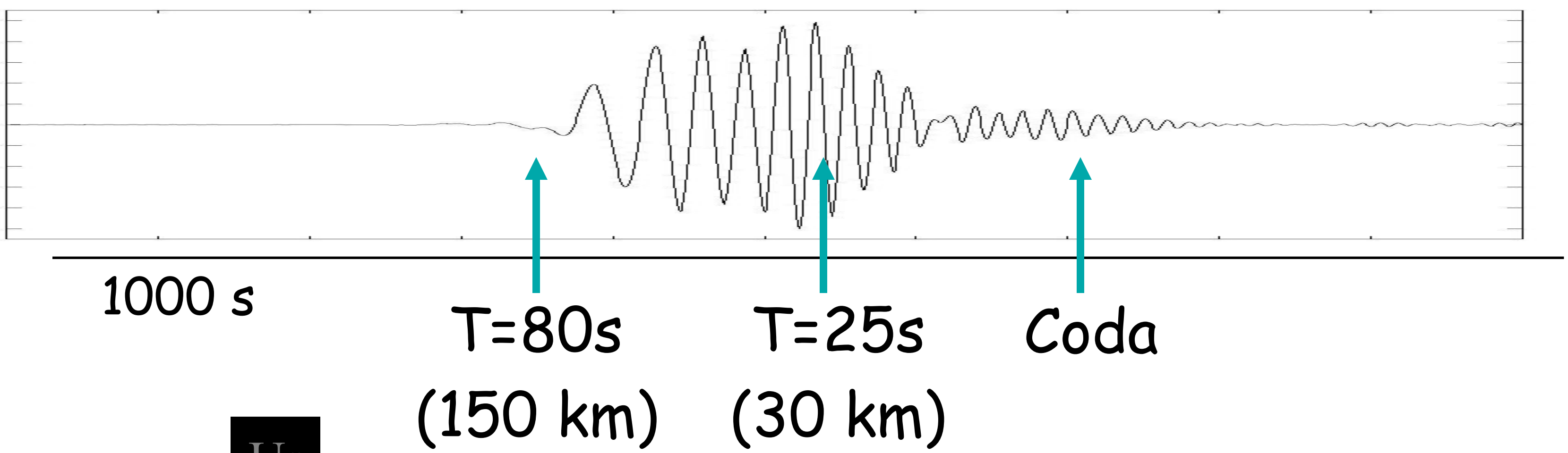


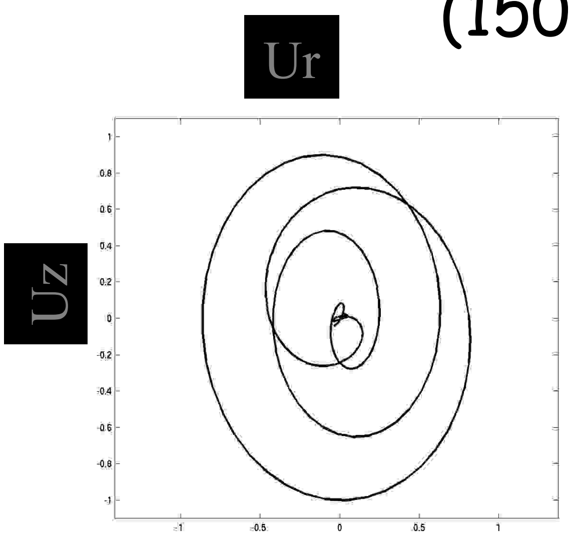
Model: iasp91

Distance (deg)	Depth (km)	Phase Name	Travel Time (s)	Ray Param p (s/deg)	Takeoff (deg)	Incident (deg)	Purist Distance	Purist Name
110.00	0.7	Pdiff	871.02	4.439	13.39	13.39	110.00	= Pdiff
110.00	0.7	PKiKP	1113.08	1.886	5.65	5.64	110.00	= PKiKP
110.00	0.7	SKS	1511.72	4.132	7.17	7.17	110.00	= SKS
110.00	0.7	SKIKS	1546.40	1.912	3.31	3.31	110.00	= SKIKS
110.00	0.7	SKiKS	1546.56	1.962	3.40	3.40	110.00	= SKiKS
110.00	0.7	Sdiff	1605.54	8.323	14.57	14.57	110.00	= Sdiff



Rayleigh
SSS
S4
Love
Rayleigh
25
30
35
40





Rayleigh: elliptic polarisation

Figure 2.7-2: Geometry for Love and Rayleigh wave motions.

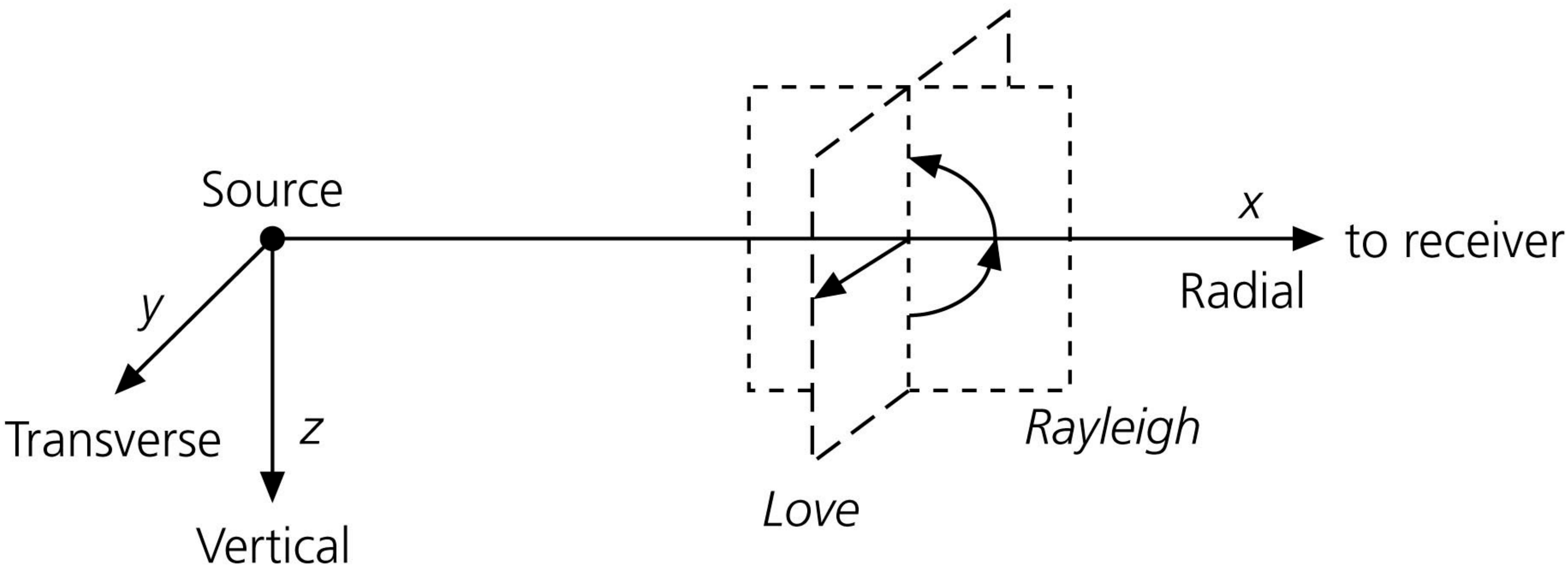


Figure 2.7-4: Six-hour stacked IDA record section.

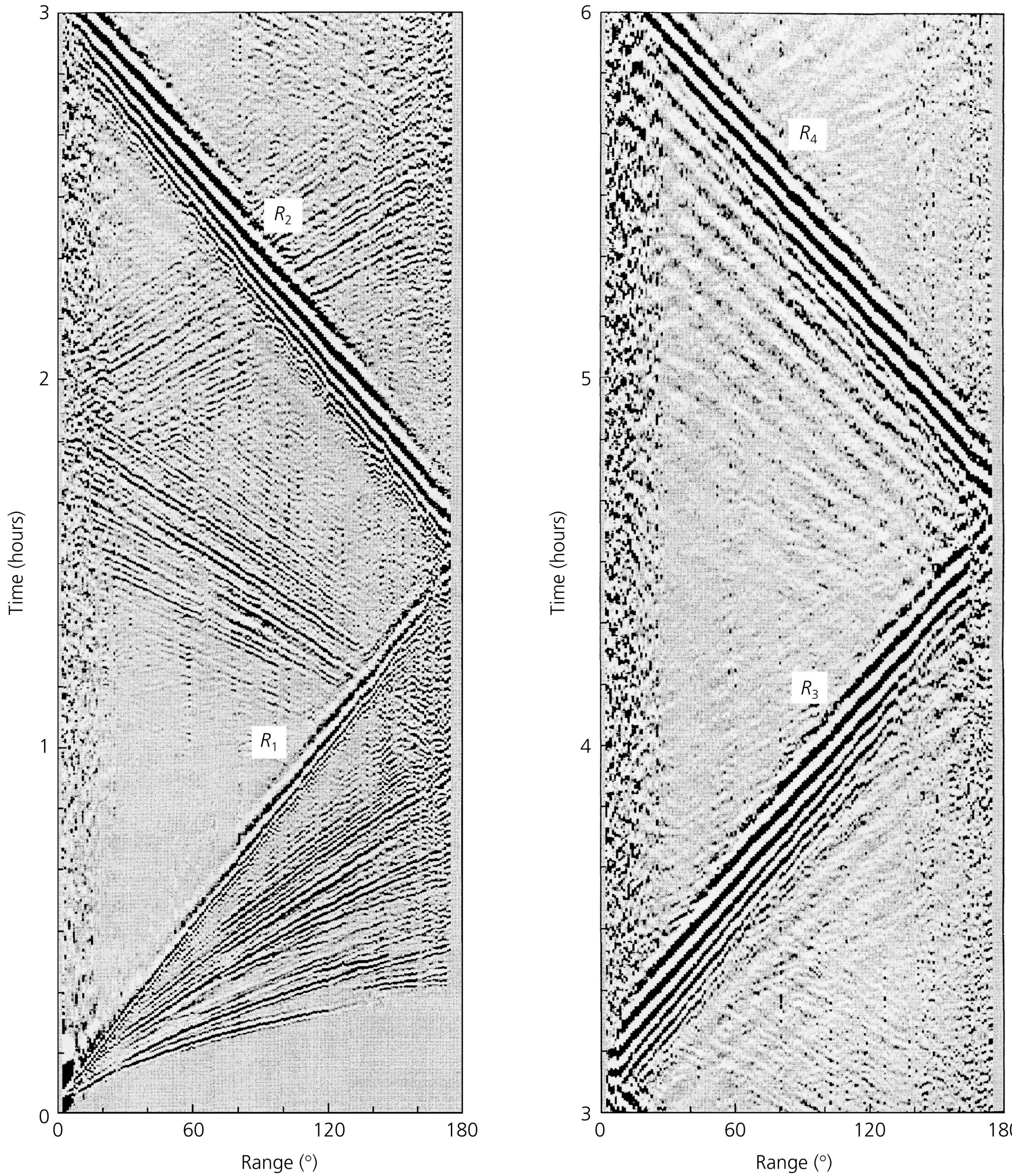
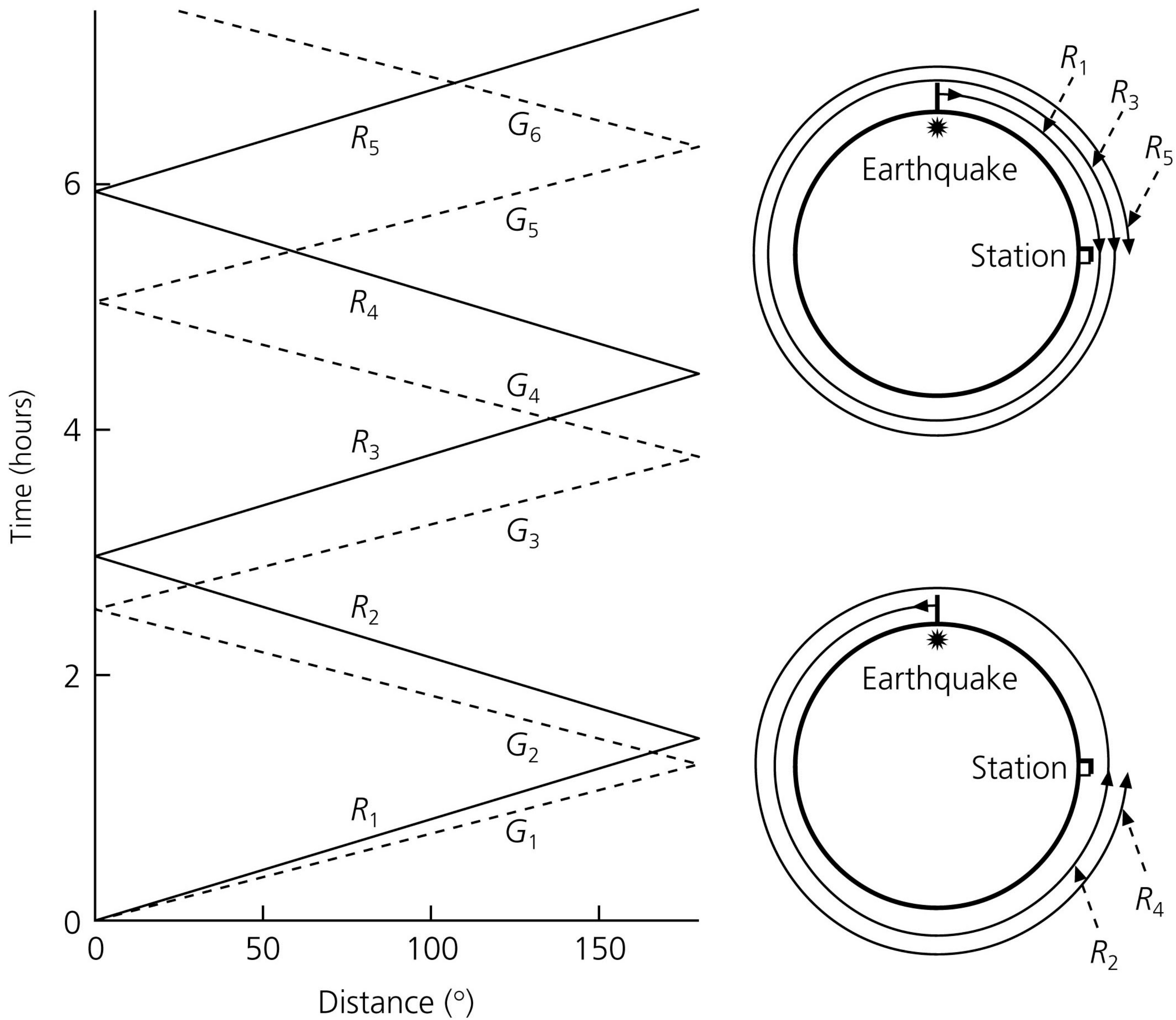


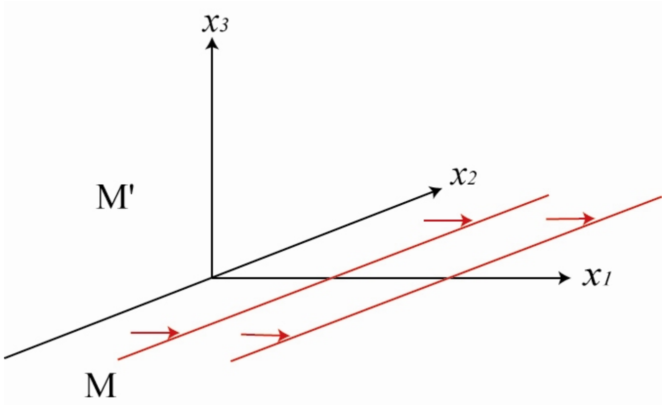
Figure 2.7-3: Multiple surface waves circle the earth.



Waves at the interface of two elastic media

2 half spaces $\quad M \quad \alpha \quad \beta \quad \rho$

$\qquad\qquad\qquad\quad M' \quad \alpha' \quad \beta' \quad \rho'$



2D problem $\qquad \frac{\partial \bullet}{\partial y} = 0$

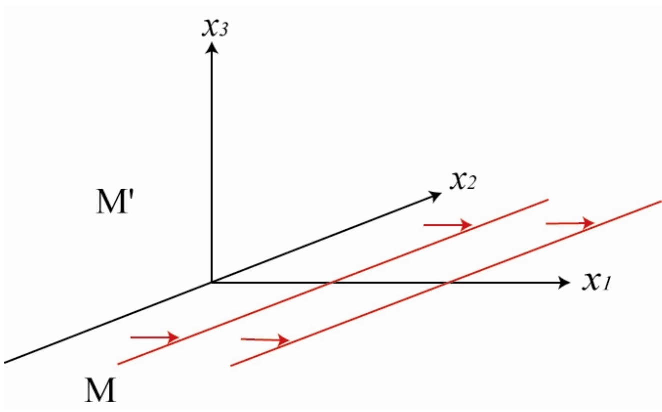
$$u_1 = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial z}$$

$$u_2$$

$$u_3 = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x}$$

$$\phi = P \qquad\qquad \psi = SV \qquad\qquad u_2 = SH$$

$$\frac{\partial^2 \phi}{\partial t^2} = \alpha^2 \ \nabla^2 \ \phi \qquad ; \qquad \frac{\partial^2 \psi}{\partial t^2} = \beta^2 \ \nabla^2 \ \psi \qquad ; \qquad \frac{\partial^2 u_2}{\partial t^2} = \beta^2 \ \nabla^2 \ u_2$$



Forms of the harmonic solutions in M:

$$\phi = f(z)\,\exp(i\,k\,(x - ct))$$

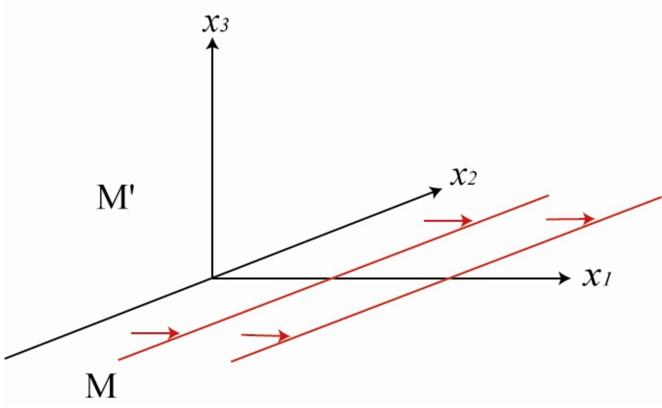
$$\psi = g(z)\,\exp(i\,k\,(x - ct))$$

$$u_2 = h(z)\,\exp(i\,k\,(x - ct))$$

in M' $\Rightarrow$ f' , g' , h'

f, g, h, f', g', h' are found from the wave equations

Example: h'



$$\frac{d^2\, h'}{dz^2} + \; h'\, k^2 \left(\frac{c^2}{\beta'^2} - 1 \right) = 0$$

$$h'(z) = C' \exp\left(-i\, k \left(\frac{c^2}{\beta'^2} - 1 \right)^{1/2} z \right) + F' \exp\left(i\, k \left(\frac{c^2}{\beta'^2} - 1 \right)^{1/2} z \right)$$

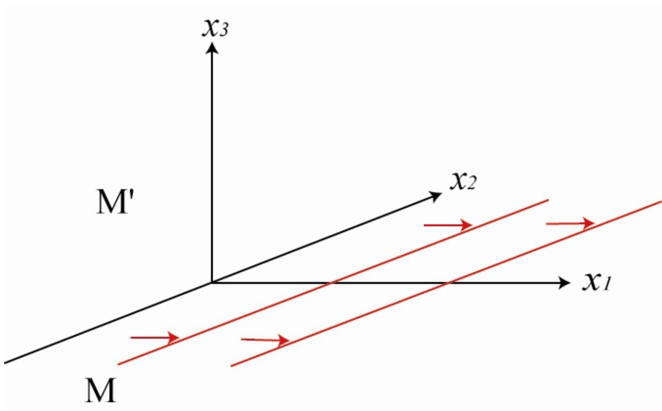
C', F' : constants.

$$s' = \left(\frac{c^2}{\beta'^2} - 1 \right)^{1/2} \quad ; \quad s = \left(\frac{c^2}{\beta^2} - 1 \right)^{1/2}$$

$$r' = \left(\frac{c^2}{\alpha'^2} - 1 \right)^{1/2} \quad ; \quad r = \left(\frac{c^2}{\alpha^2} - 1 \right)^{1/2}$$

$\Rightarrow$

$$h'(z) = C' \exp\left(-i\, k\, s'\, z \right) + F' \exp\left(i\, k\, s'\, z \right)$$



Constraints on the form of the solutions .

$\Rightarrow$ no oscillation and decay at ∞

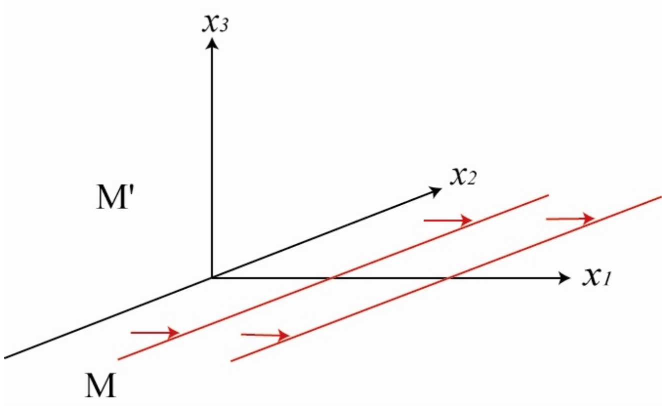
$\Rightarrow$ exponential with negative real argument

$\Rightarrow$ r, s, r', s' imaginary

$$c < \beta \ \text{ et } \ c < \beta'$$

$\Rightarrow C' = 0$

$\Rightarrow$ General Forms



in M :

$$\phi = A \exp\,(i\,k\,(-r\,z + x - c\,t))$$

$$\psi = B \exp\,(i\,k\,(-s\,z + x - c\,t))$$

$$u_2 = C \exp\,(i\,k\,(-s\,z + x - c\,t))$$

in M’

$$\phi' = D' \exp\,(i\,k\,(-r'\,z + x - c\,t))$$

$$\psi' = E' \exp\,(i\,k\,(-s'\,z + x - c\,t))$$

$$u_2' = F' \exp\,(i\,k\,(-s'\,z + x - c\,t))$$

(RAYLEIGH WAVES)

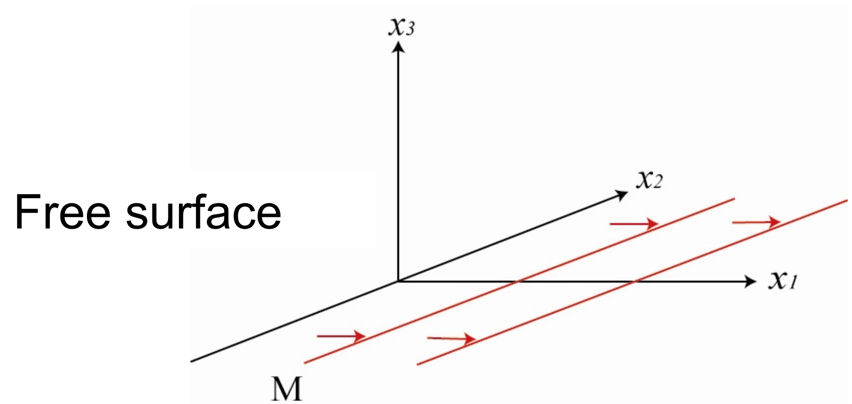
Waves at the surface of an elastic half-space.

P-SV case

exercise: SH?

$$\begin{array}{l} u_1 \\ u_2 = 0 \\ u_3 \end{array}$$

$$\Rightarrow \frac{\partial \bullet}{\partial y} = 0$$



$$\Rightarrow \quad \vec{u}^S = \vec{rot}\,\vec{\psi} \quad \Rightarrow$$

$$\vec{u}^S = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \wedge \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = \begin{pmatrix} \frac{\partial \psi_3}{\partial y} - \frac{\partial \psi_2}{\partial z} \\ 0 \\ \frac{\partial \psi_2}{\partial x} - \frac{\partial \psi_1}{\partial y} \end{pmatrix}$$

$$u_1^S = -\frac{\partial \psi_2}{\partial z}$$

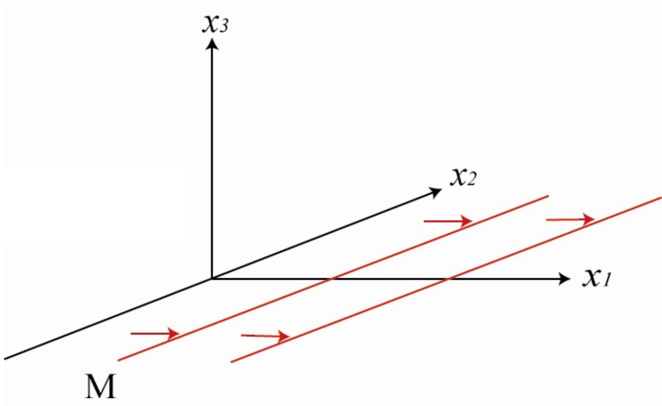
$$u_3^S = +\frac{\partial \psi_2}{\partial x}$$

$\Rightarrow \vec{\psi}$ can be replaced by $\psi_2 = \psi$

$$u_1 = \frac{\partial}{\partial x}\phi - \frac{\partial \psi}{\partial z}$$

$$u_3 = \frac{\partial}{\partial z}\phi - \frac{\partial \psi}{\partial x}$$

• Harmonic solution propagating along x



$$\phi = f(z)\ \exp\left(ik(x-ct)\right)$$

$$\psi = g(z)\ \exp\left(ik(x-ct)\right)$$

• $\Rightarrow f(z) = f_0 \exp\left(-k(1-\frac{c^2}{\alpha^2})^{1/2}\, z\right) + f_1 \exp\left(+k(1-\frac{c^2}{\alpha^2})^{1/2}\, z\right)$

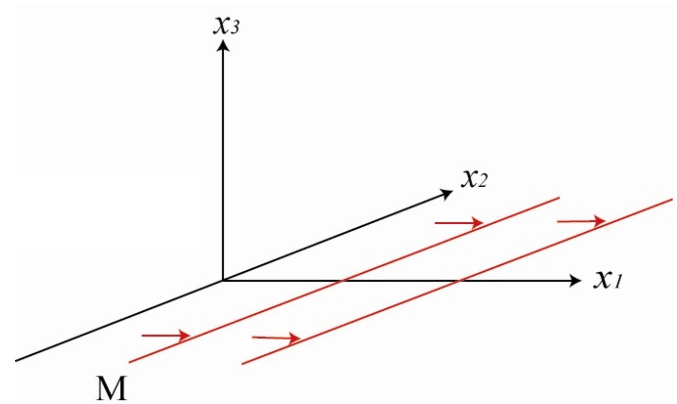
$\Rightarrow g(z) = g_0 \exp\left(-k(1-\frac{c^2}{\beta^2})^{1/2}\, z\right) + g_1 \exp\left(+k(1-\frac{c^2}{\beta^2})^{1/2}\, z\right)$

$\longrightarrow g(z) \longrightarrow$ Exponential decay for $z \longrightarrow \infty$

$\longrightarrow (1-\frac{c^2}{\alpha^2})^{1/2}$ and $(1-\frac{c^2}{\beta^2})^{1/2}$ are real numbers.

$\Rightarrow c < \alpha$ and $c < \beta$

f_1 and g_1 $=0 \leftrightarrow f$ and $g \longrightarrow 0$ for $z \longrightarrow \infty$



$$\phi = f_0 \exp\left(-k(1-\frac{c^2}{\alpha^2})^{1/2}\, z\right) \exp(ik(x-ct))$$

$$\psi = g_0 \exp\left(-k(1-\frac{c^2}{\beta^2})^{1/2}\, z\right) \exp(ik(x-ct))$$

Free surface condition
for z=0 :

$$\tau_{33} = 0$$

$$\tau_{31} = 0$$

$$\tau_{32} = 0$$

$$\tau_{32} = \mu \ \varepsilon_{32} \leftarrow u_2 = 0 \qquad (\frac{\partial \bullet}{\partial y} = 0)$$

$$\tau_{31} = \mu \ \varepsilon_{31} \ = \ \mu(\frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x})$$

$$\tau_{33} = \lambda(\varepsilon_{11} + \varepsilon_{33}) + 2\mu\varepsilon_{33}$$

$$= \lambda(\frac{\partial u_1}{\partial x} + \frac{\partial u_3}{\partial z}) + 2\mu\frac{\partial u_3}{\partial z}$$

$$\tau_{31} = \mu(\frac{\partial^2}{\partial x \partial z}\phi - \frac{\partial^2}{\partial z^2}\psi + \frac{\partial^2}{\partial x \partial z}\phi + \frac{\partial^2 \psi}{\partial x^2})$$

$$= \mu(2\frac{\partial^2}{\partial x \partial z}\phi - \frac{\partial^2}{\partial z^2}\psi + \frac{\partial^2 \psi}{\partial x^2}) = 0$$

$$\tau_{33} = \lambda(\frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \psi}{\partial x \partial z} + \frac{\partial^2 \phi}{\partial z^2} + \frac{\partial^2 \psi}{\partial x \partial z}) + 2\mu\frac{\partial^2 \phi}{\partial z^2} + 2\mu\frac{\partial^2 \psi}{\partial x \partial z}$$

$$\tau_{33} = \lambda(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} + 2\mu\frac{\partial^2 \phi}{\partial z^2} + 2\mu\frac{\partial^2 \psi}{\partial x \partial z}) = 0$$

$s = (1 - \frac{c^2}{\beta^2})^{1/2}$ et $r = (1 - \frac{c^2}{\alpha^2})^{1/2}$

$$\tau_{31} = 0; z = 0 \quad \Rightarrow \quad 2\frac{\partial^2 \phi}{\partial x \partial z} - \frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^2 \psi}{\partial x^2} = 0$$

$$\Rightarrow 2f_0(-ir) - g_0(s^2 + 1) = 0$$

$$\tau_{33} = 0; z = 0$$

$$\Rightarrow f_0(\alpha^2(r^2 - 1) + 2\beta^2) + 2\beta^2 g_0(-is) = 0$$

$\rightarrow$ System of linear equations

$$-2irf_0 - (s^2 + 1)g_0 = 0$$

$$(\alpha^2(r^2 - 1) + 2\beta^2)f_0 + (-2is\beta^2)g_0 = 0$$

$\exists$ solution , $\forall$ amplitude $\rightarrow$ determinant $= 0$

$$4rs\beta^2 = (s^2+1)(\alpha^2(r^2-1)+2\beta^2)$$

Dispersion relation (no frequency dependance in this case)

$$\beta^2 4(1-\frac{c^2}{\alpha^2})^{1/2}\ (1-\frac{c^2}{\beta^2})^{1/2} = (2-\frac{c^2}{\beta^2})(\alpha^2(-\frac{c^2}{\alpha^2})+2\beta^2)$$

$$\boxed{4(1-\frac{c^2}{\alpha^2})^{1/2}\ (1-\frac{c^2}{\beta^2})^{1/2} = (2-\frac{c^2}{\beta^2})^2}$$

$\lambda = \mu \quad \Rightarrow \quad \alpha^2 = 3\beta^2$

$$\boxed{4(1-\frac{c^2}{3\beta^2})^{1/2}\ (1-\frac{c^2}{\beta^2})^{1/2} = (2-\frac{c^2}{\beta^2})^2}$$

For an example, assume a Poisson solid ($\alpha^2/\beta^2 = 3$), so that the determinant becomes

$$(c_x^2/\beta^2)[c_x^6/\beta^6 - 8\ c_x^4/\beta^4 + (56/3)c_x^2/\beta^2 - 32/3] = 0$$

This has roots $c_x^2/\beta^2 = 04, 0, 2 + 2/\sqrt{3}\ (\approx 3.155), 2 - 2/\sqrt{3}\ (\approx 0.845)$.

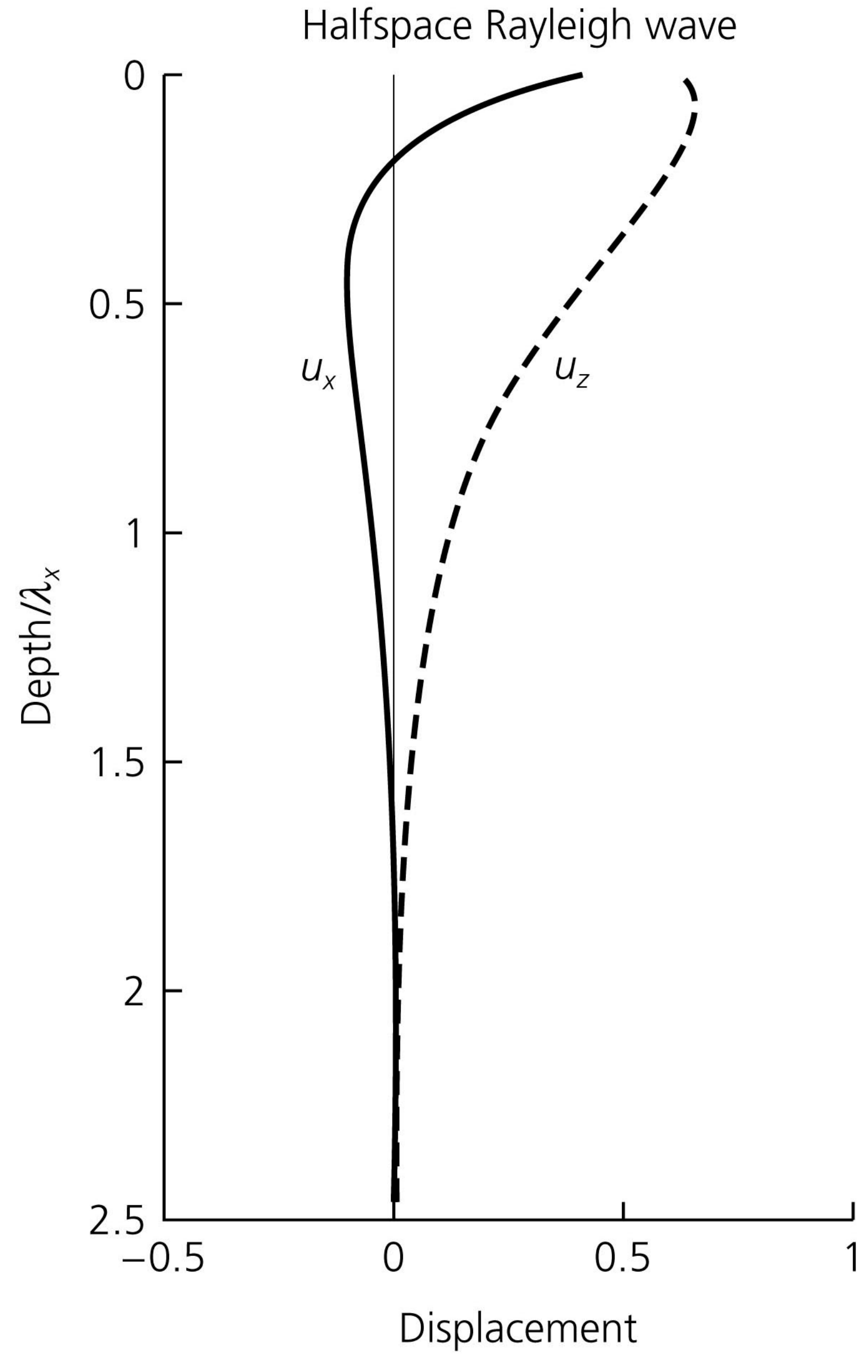
Only the last root satisfies $c_x < \beta$, the condition for waves to be trapped at the surface.

The apparent velocity of the Rayleigh wave in a halfspace that is a homogeneous Poisson solid is $c_x = (2 - 2/\sqrt{3})\beta = 0.92\ \beta$, slightly less than the shear velocity.

$$u_x = Ak_x \sin(\omega t - k_x x)[\exp(-0.85\ k_x z) - 0.58 \exp(-0.39\ k_x z)]$$

$$u_z = Ak_x \cos(\omega t - k_x x)[-0.85 \exp(-0.85\ k_x z) + 1.47 \exp(-0.39\ k_x z)]$$

Figure 2.7-5: Rayleigh wave displacements as a function of depth.



$$u_x = Ak_x \sin(\omega t - k_x x)[\exp(-0.85\ k_x z) - 0.58\ \exp(-0.39\ k_x z)]$$

$$u_z = Ak_x \cos(\omega t - k_x x)[-0.85\ \exp(-0.85\ k_x z) + 1.47\ \exp(-0.39\ k_x z)]$$

At the surface, $z = 0$:

$$u_x = 0.42\ Ak_x \sin(\omega t - k_x x)$$

$$u_z = 0.62\ Ak_x \cos(\omega t - k_x x)$$

Figure 2.7-5: Rayleigh wave displacements as a function of depth.

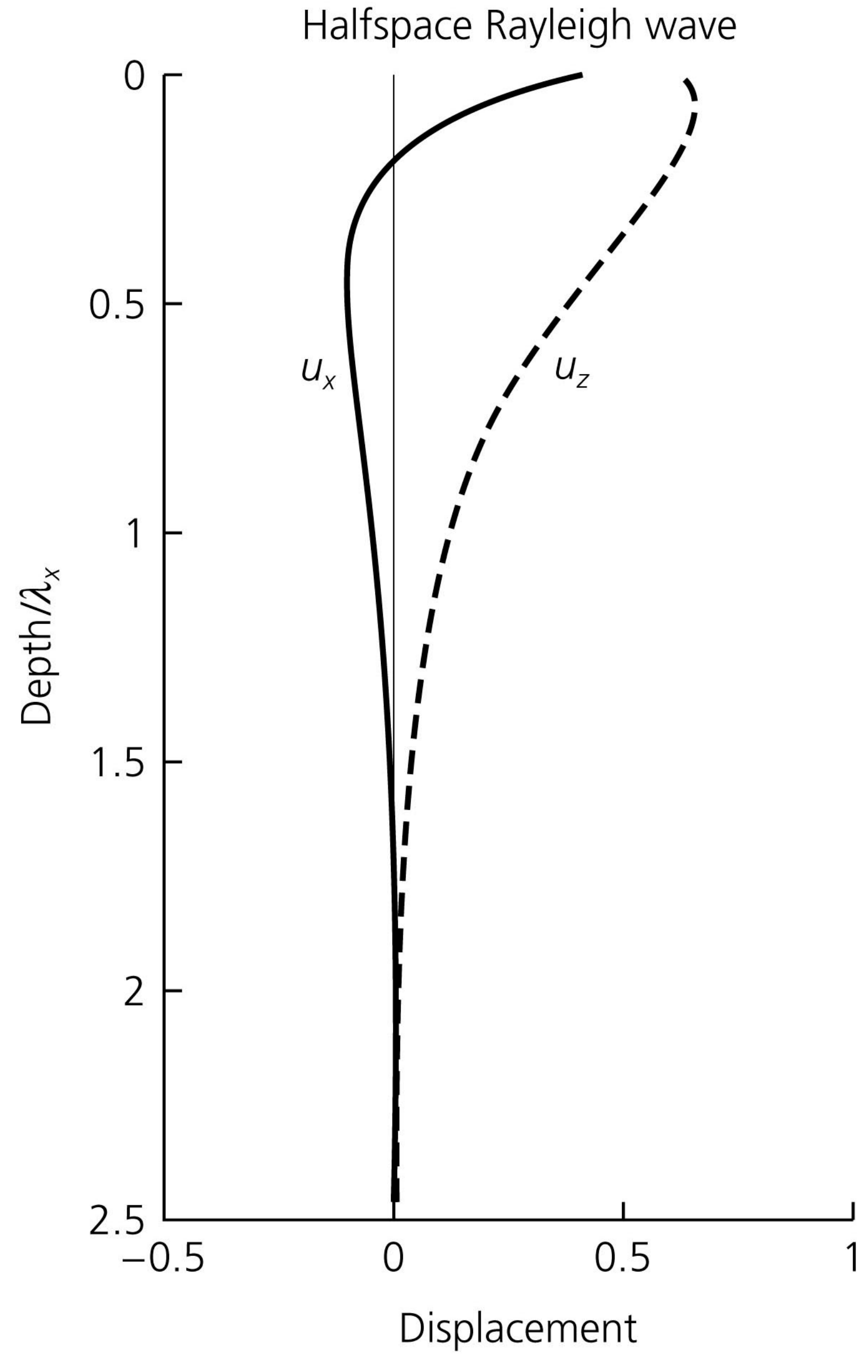
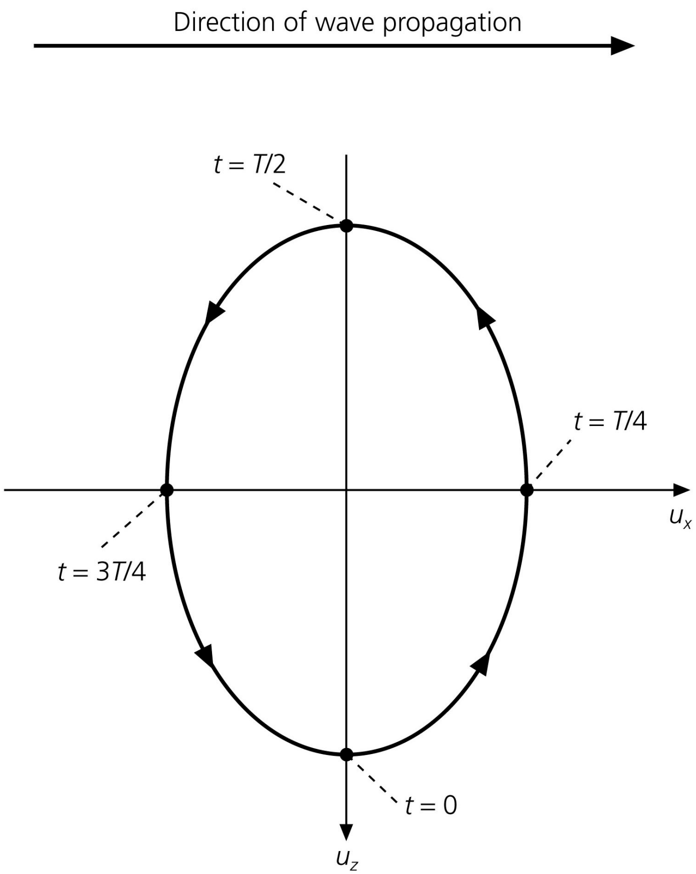
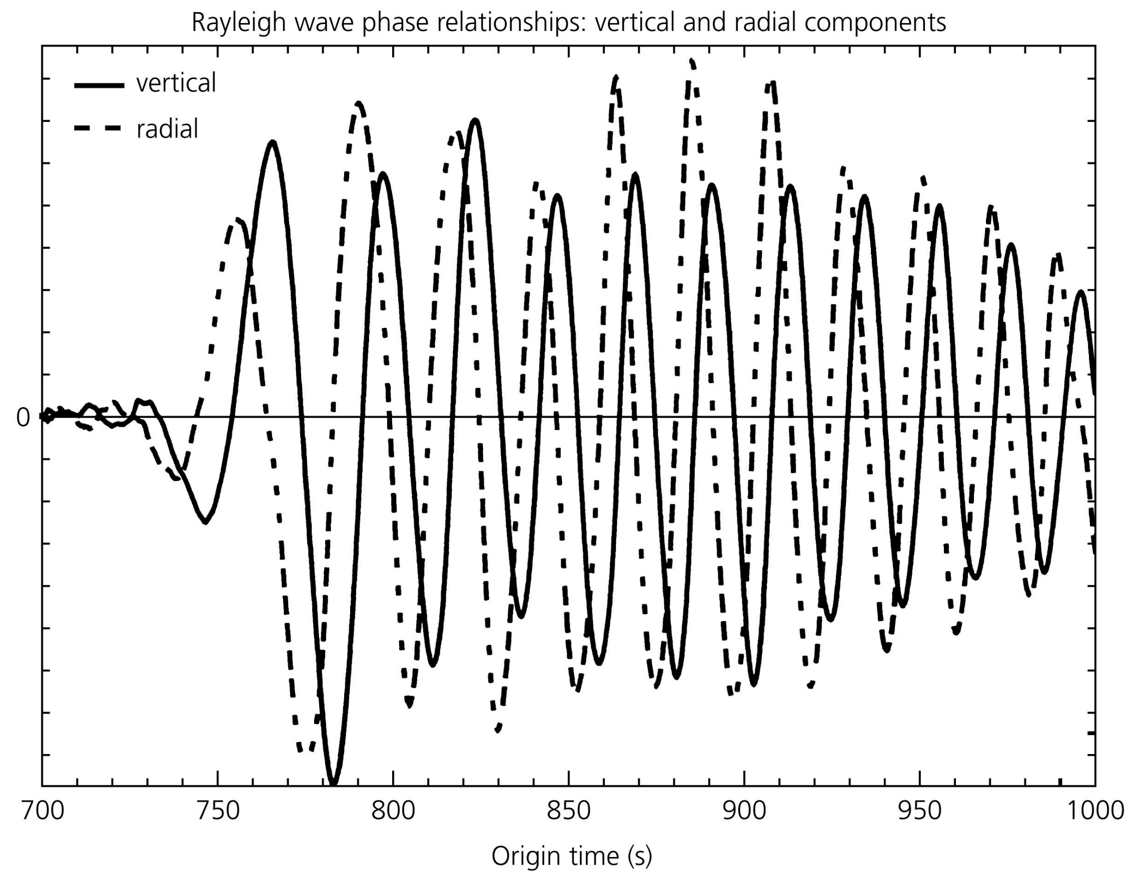


Figure 2.7-6: Horizontal and vertical Rayleigh wave motions.





Half space Green function

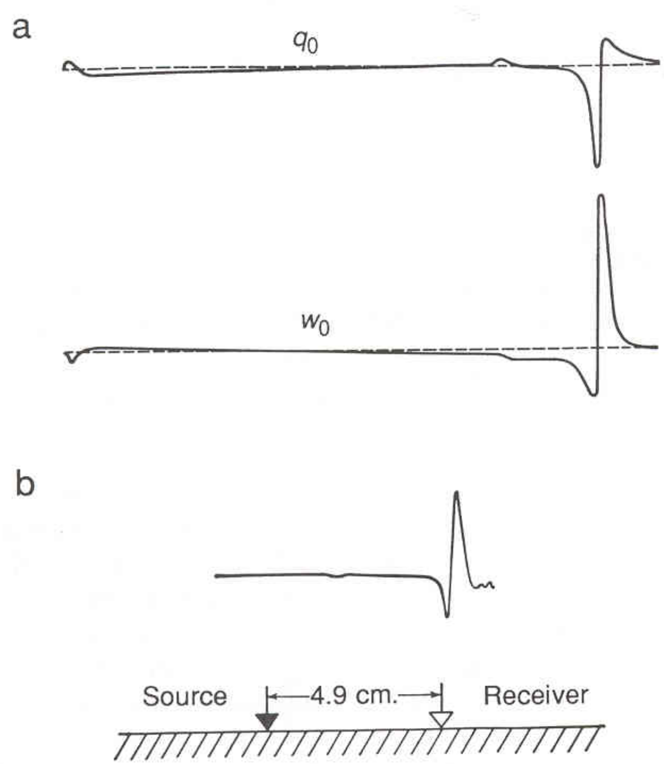


FIGURE 4.B1.1 (a) Radial (q_0) and vertical (w_0) surface ground motions calculated by Lamb (1904) for an impulsive vertical force on the surface. (b) An experimentally recorded vertical ground motion for a vertical point source. The largest motion in each case corresponds to the undispersed Rayleigh pulse. (From Ewing *et al.*, 1957).