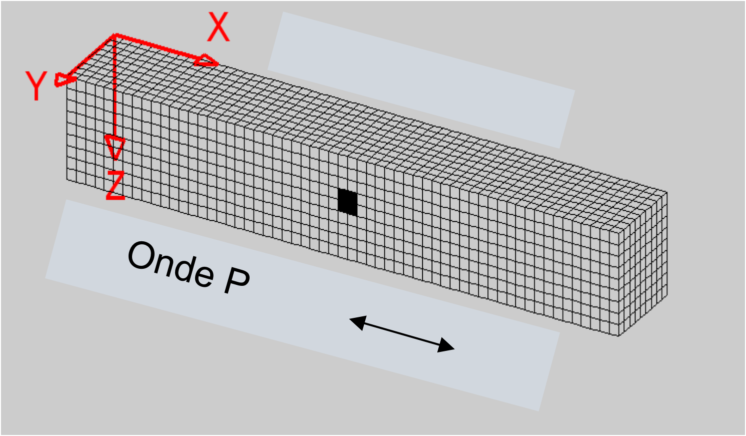
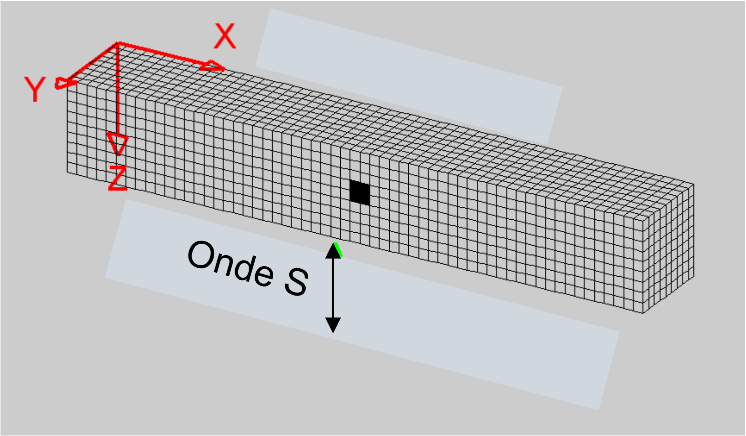
MEEES and M2R STU

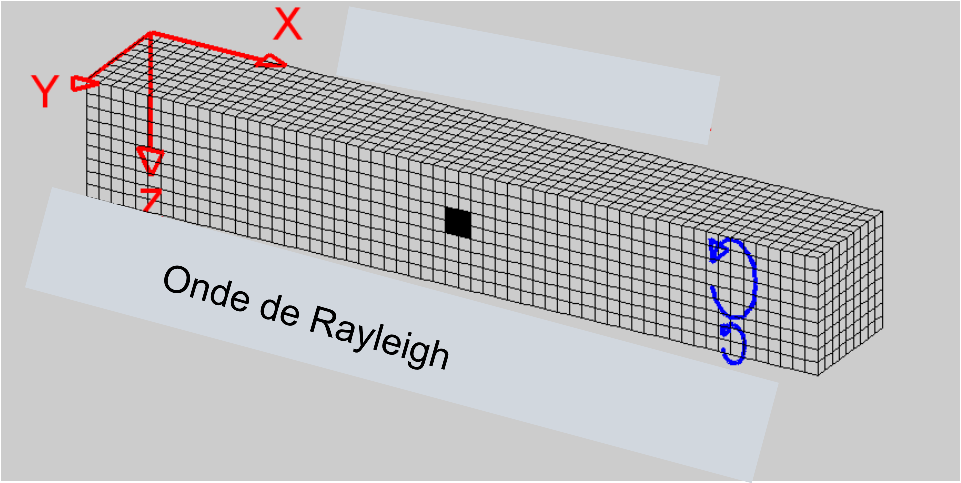
Seismology (Michel Campillo)

P and S bulk waves



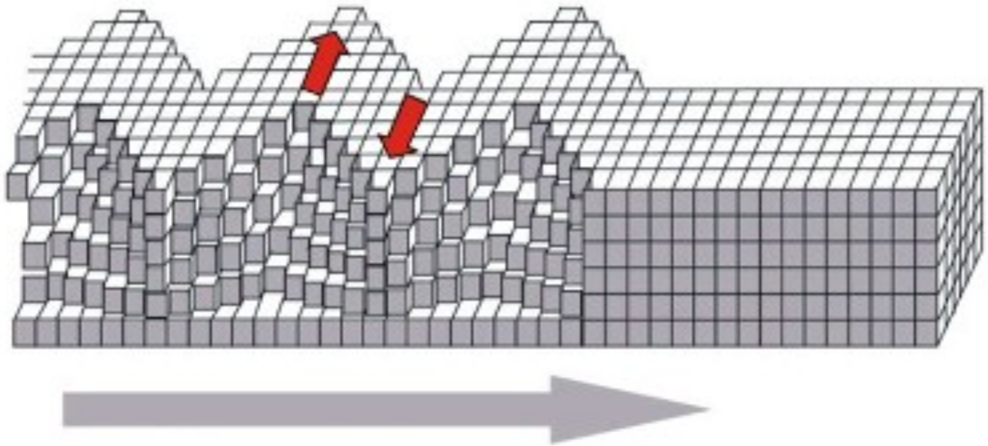


Surface waves: Rayleigh waves

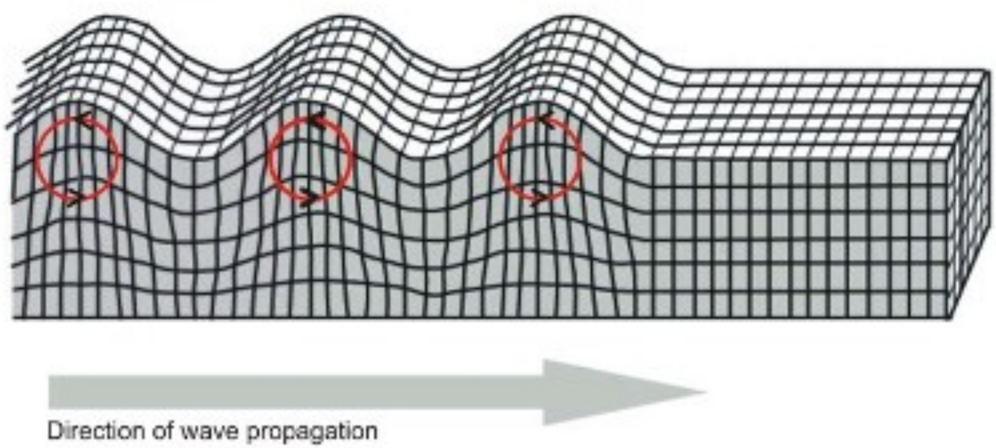


Surface waves

Love wave

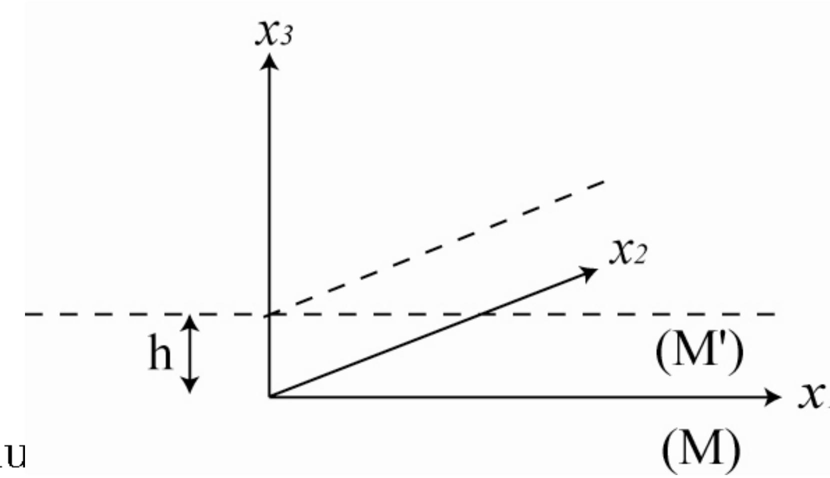


Rayleigh wave



LOVE WAVES :

SH surface waves are observed
→ only for stratified media



- u_2 (SH)
- M' : no constraint on the limit of the solu

⇒

$$u_2 = C' \exp(ik(-s'z + x - ct)) + F' \exp(ik(s'z + x - ct))$$

- s' is not necessarily imaginary
- M : ⇒

$$u_2 = C \exp(ik(-sz + x - ct))$$

- s is necessarily imaginary

Boundary conditions : continuity for $z = 0$ stress $= 0$ for $z = h$

$$C = C' + F'$$

$$\mu s C = \mu' s' (C' - F')$$

$$C' \exp(-iks'h) = F' \exp(iks'h)$$

Determinant $= 0$:

$$i\mu s + \mu' s' \tan(ks'h) = 0$$

$$\mu(1 - \frac{c^2}{\beta^2})^{1/2} - \mu'(\frac{c^2}{\beta'^2} - 1)^{1/2} \tan(kh(\frac{c^2}{\beta'^2} - 1)^{1/2}) = 0$$

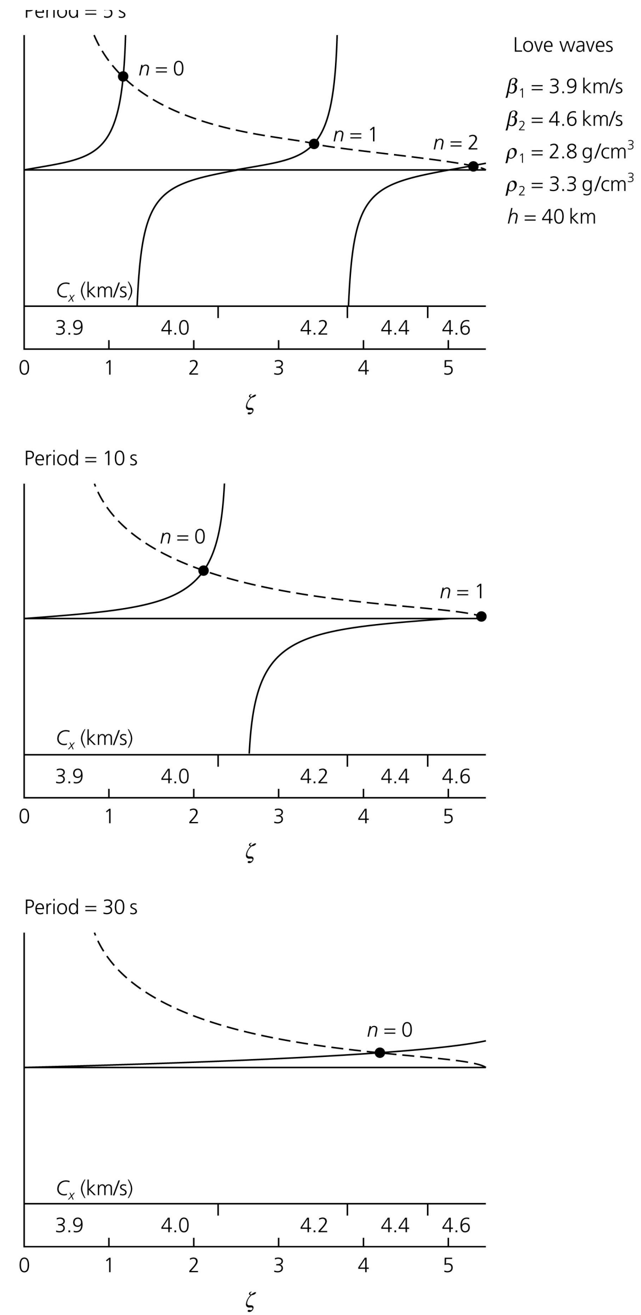
Love wave dispersion equation.

s imaginary $\Rightarrow\ c\ <\ \beta$
dispersion equation $\Rightarrow$ s' $\in\ \Re\ \ \Rightarrow$ c $\ >\ \beta'$

$$\Longrightarrow \beta'\ <\ c\ <\ \beta$$

Love wave for $\beta'\ <\ \beta$.

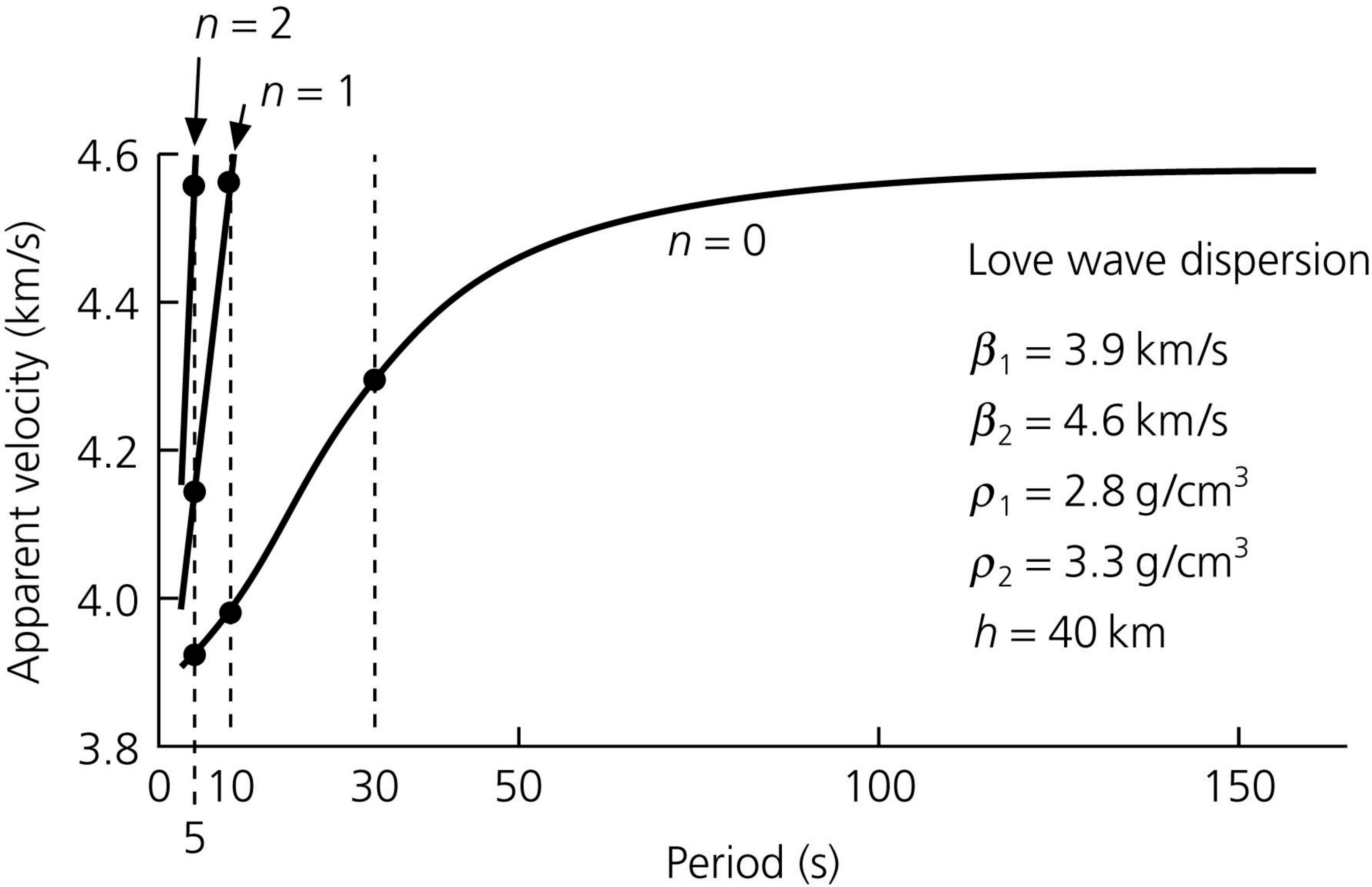
$$\mu(1-\frac{c^2}{\beta^2})^{1/2}-\mu'(\frac{c^2}{\beta'^2}\;-1)^{1/2}\tan(kh(\frac{c^2}{\beta'^2})-1)^{1/2})=0$$



$$\xi=(\frac{h}{c})(\frac{c^2}{\beta'^2}\;-1)^{1/2}$$

$$\tan(\omega\xi)=(\frac{\mu(1-\frac{c^2}{\beta^2})^{1/2}}{\mu'})(\frac{h}{c\xi})$$

Figure 2.7-9: Dispersion curves for Love waves in a layer over a halfspace.

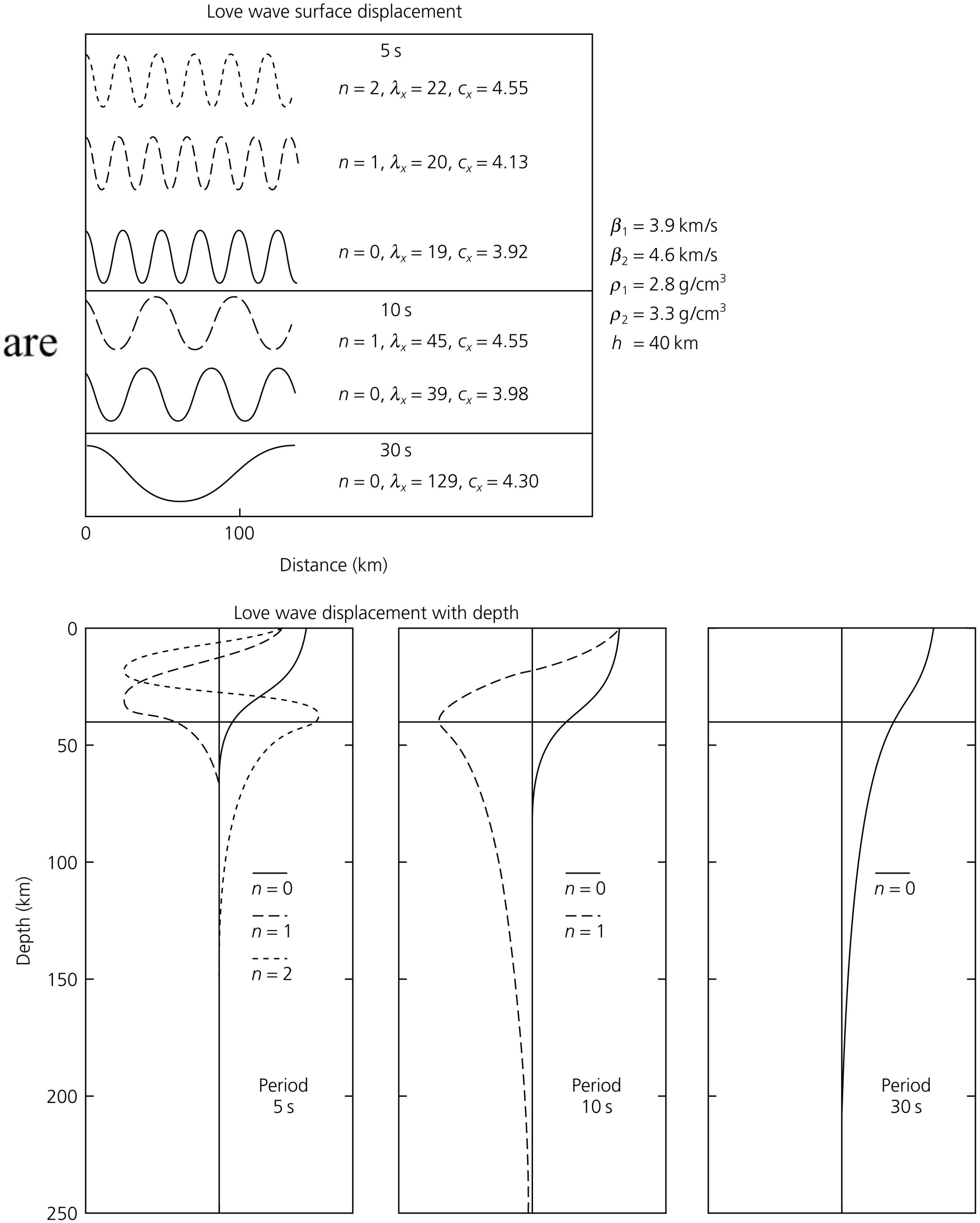


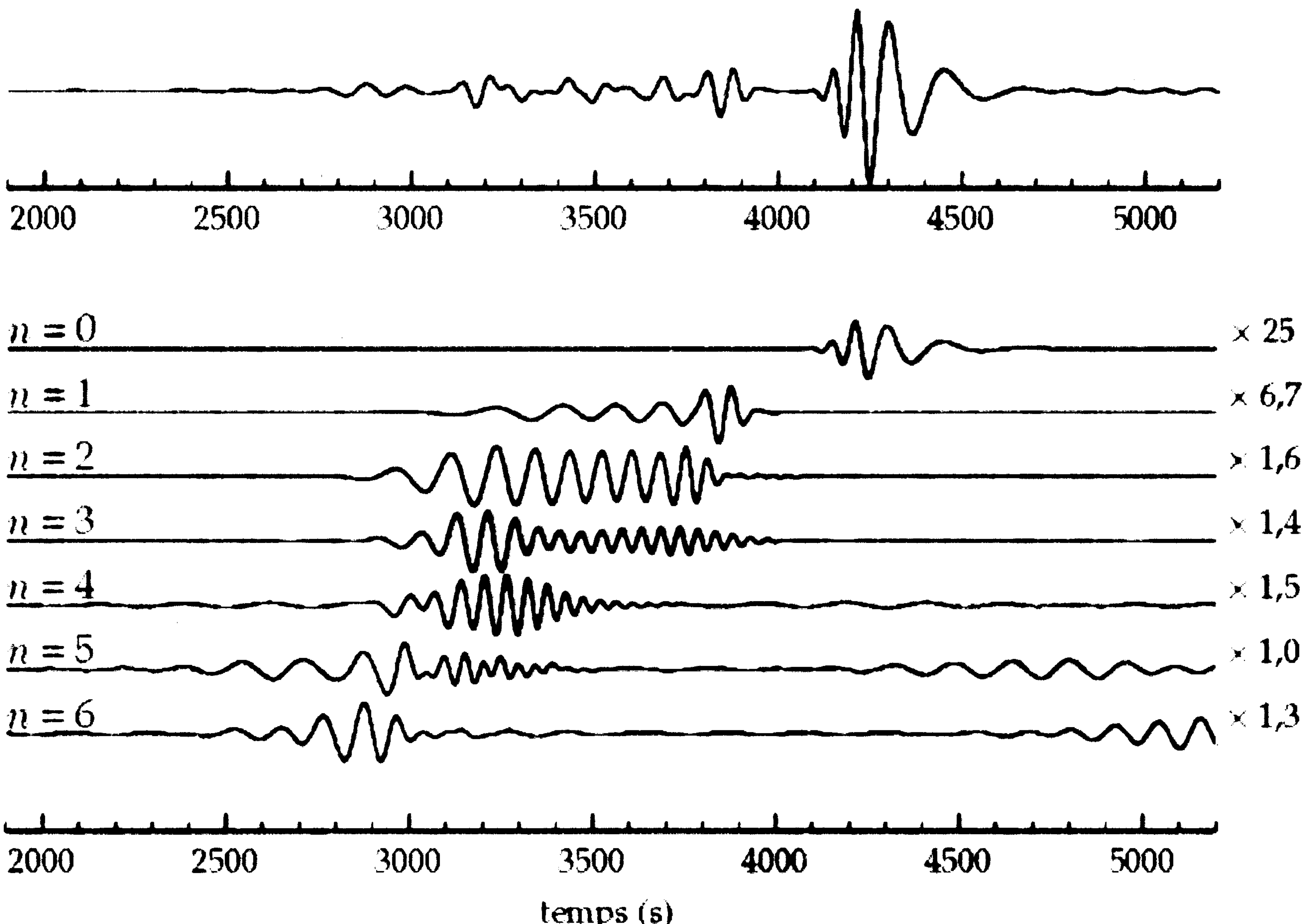
Dispersion occurs because longer-period waves "see" more of the halfspace, and travel at faster velocities.

Figure 2.7-10: Displacements of for Love waves in a layer over a halfspace.

The wave oscillates as $\cos(k_x r_{\beta_1} z)$ in the layer, but decays exponentially as $\exp(-k_x r^*_{\beta_2} z)$ in the half-space.

The vertical sensitivities of the modes are the eigenfunctions.





2000
2500
3000
3500
4000
4500
5000
n = 0
× 25
n = 1
× 6,7
n = 2
× 1,6
n = 3
× 1,4
n = 4
× 1,5
n = 5
× 1,0
n = 6
× 1,3
2000
2500
3000
3500
4000
4500
5000
temps (s)

Dispersive signals

Fourier transform: $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}\, dt$

Inverse Fourier transform: $f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)e^{i\omega t}\, d\omega$

$F(\omega) = A(\omega)e^{i\phi(\omega)}$

with a magnitude, $A(\omega) = |\ F(\omega)\ |$, and phase, $\phi(\omega)$.

So the Fourier transform represents a time series by two real functions of angular frequency: the *amplitude spectrum,* $A(\omega)$, and the *phase spectrum,* $\phi(\omega)$.

The displacements are: $u(x, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A(\omega) \exp i[\omega t - k(\omega)x + \phi_i(\omega)]d\omega$

The phase has two parts (propagation and initial phase): $\Phi(\omega) = \omega t - k(\omega)x + \phi_i(\omega)$

The phase velocity $c(\omega) = \omega/k(\omega)$ describes wave surfaces of constant phase (individual peaks).

On a seismogram recorded at a distance x from the earthquake at time t after the earthquake, the phase has three terms:

$$\Phi(\omega) = [\omega t - k(\omega)x] + \phi_i(\omega) + 2n\pi$$

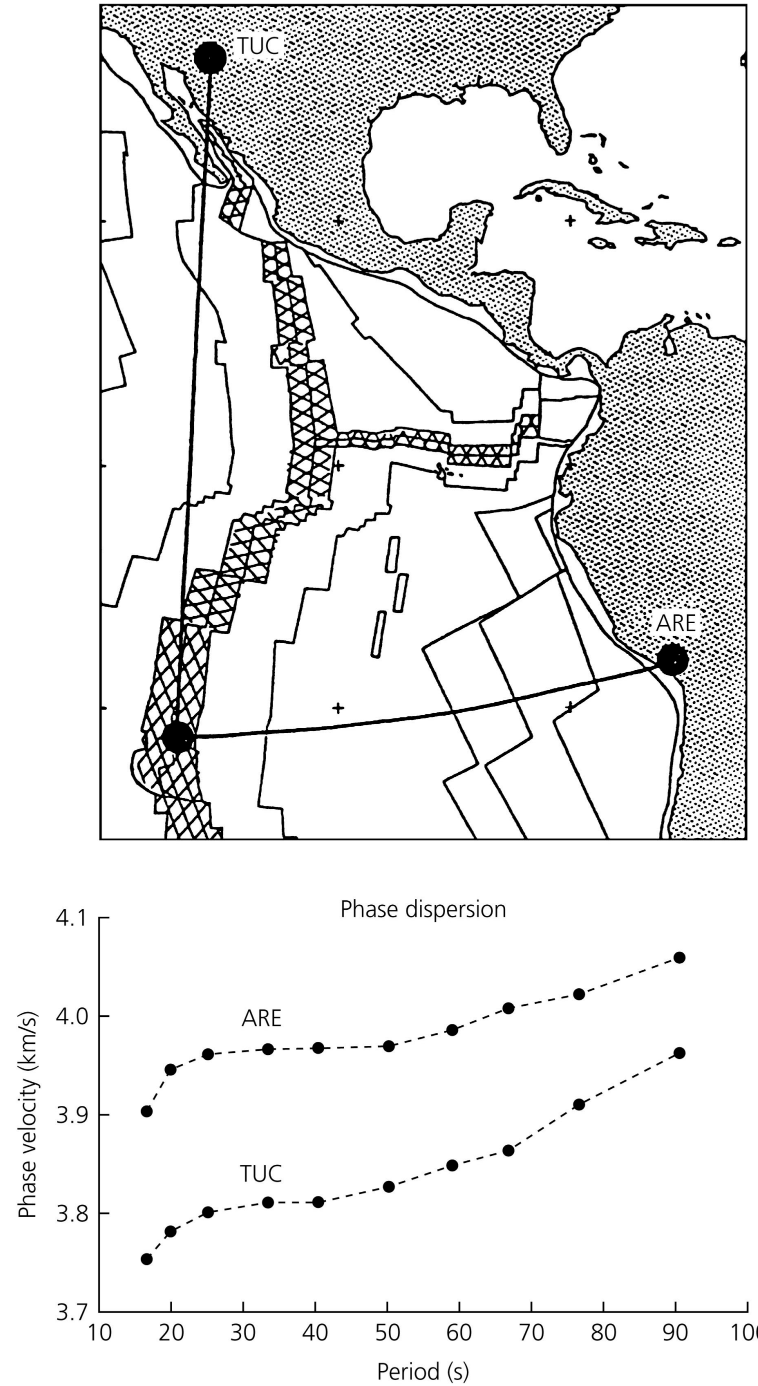
$$= [\omega t - \omega x/c(\omega)] + \phi_i(\omega) + 2n\pi$$

$\omega t - k(\omega)x$ is the phase due to the propagation of the wave in time and space.

$\phi_i(\omega)$ includes the initial phase at the earthquake and any phase shift introduced by the seismometer.

$2n\pi$ reflects the periodicity of the complex exponential, because adding an integral multiple of 2π to the argument yields the same value.

Figure 2.8-6: Example of Rayleigh wave phase velocities for ocean lithosphere.



Demonstration: sum two harmonic waves with slightly different angular frequencies and wavenumbers:

$$u(x, t) = \cos(\omega_1 t - k_1 x) + \cos(\omega_2 t - k_2 x)$$

$$\omega_1 = \omega + \delta\omega \qquad \omega_2 = \omega - \delta\omega \qquad \omega \gg \delta\omega$$

$$k_1 = k + \delta k \qquad k_2 = k - \delta k \qquad k \gg \delta k$$

Add the two cosines:

$$u(x, t) = \cos(\omega t + \delta\omega t - kx - \delta kx)$$
$$+ \cos(\omega t - \delta\omega t - kx + \delta kx)$$
$$= 2\cos(\omega t - kx)\cos(\delta\omega t - \delta kx)$$

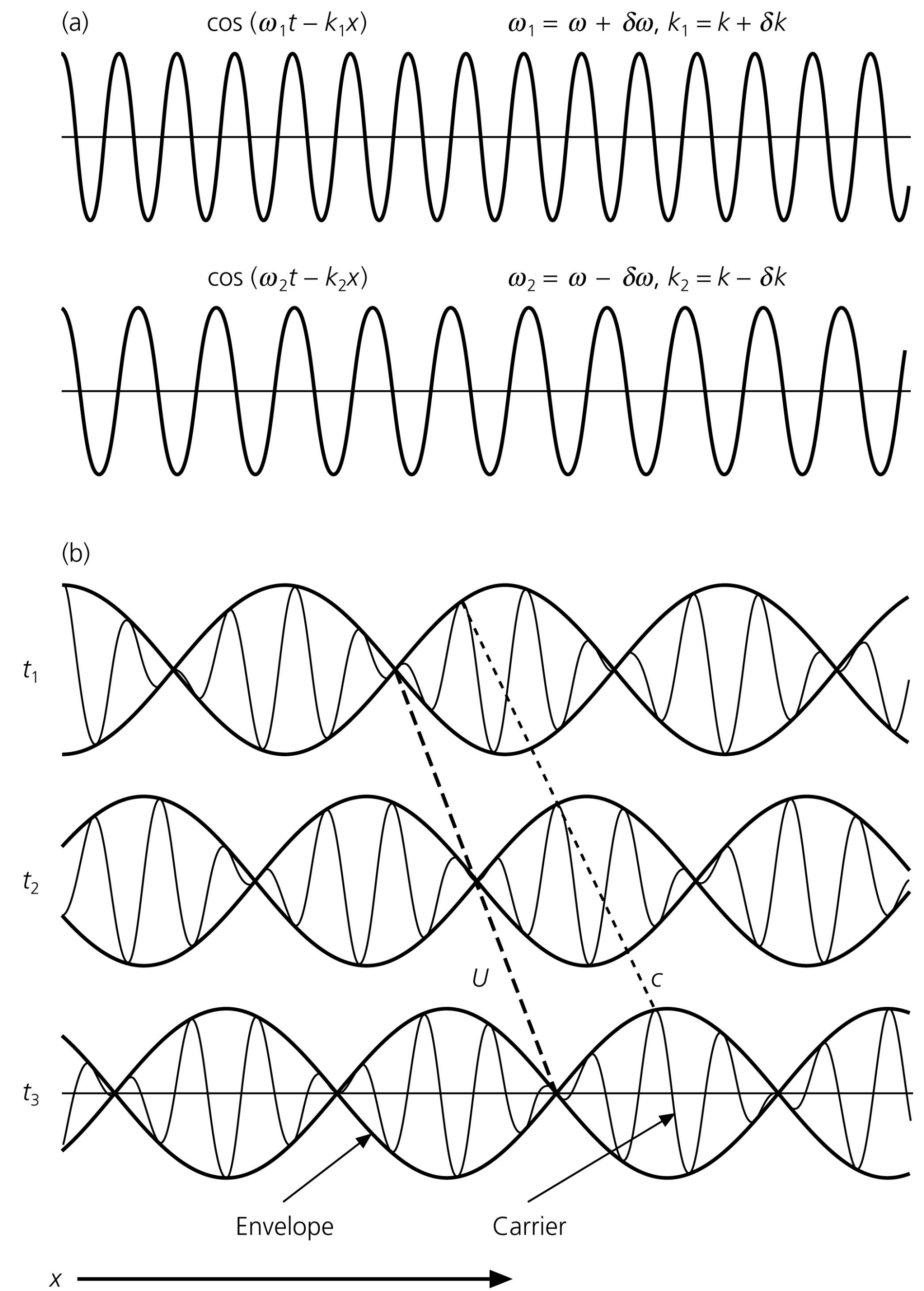
The envelope (beat) has a *group velocity*:

$U = \delta\omega/\delta k$

The individual peaks move with a *phase velocity*:

$c = \omega/k$

Figure 2.8-1: Demonstration of group and phase velocities for the sum of two sine waves.



To find the group velocity of energy propagation in the angular frequency band between $\omega_0 - \Delta\omega$ and $\omega_0 + \Delta\omega$, first approximate the wavenumber $k(\omega)$ by the first term of a Taylor series about ω_0:

$$k(\omega) \approx k(\omega_0) + \frac{dk}{d\omega}\Big|_{\omega_0} (\omega - \omega_0)$$

This gives: $$u(x, t) \approx \frac{1}{2\pi} \int_{\omega_0 - \Delta\omega}^{\omega_0 + \Delta\omega} A(\omega) \exp\left[i \left(\omega t - k(\omega_0)x - \frac{dk}{d\omega}\Big|_{\omega_0} (\omega - \omega_0)x + \phi_i(\omega) \right) \right] d\omega$$

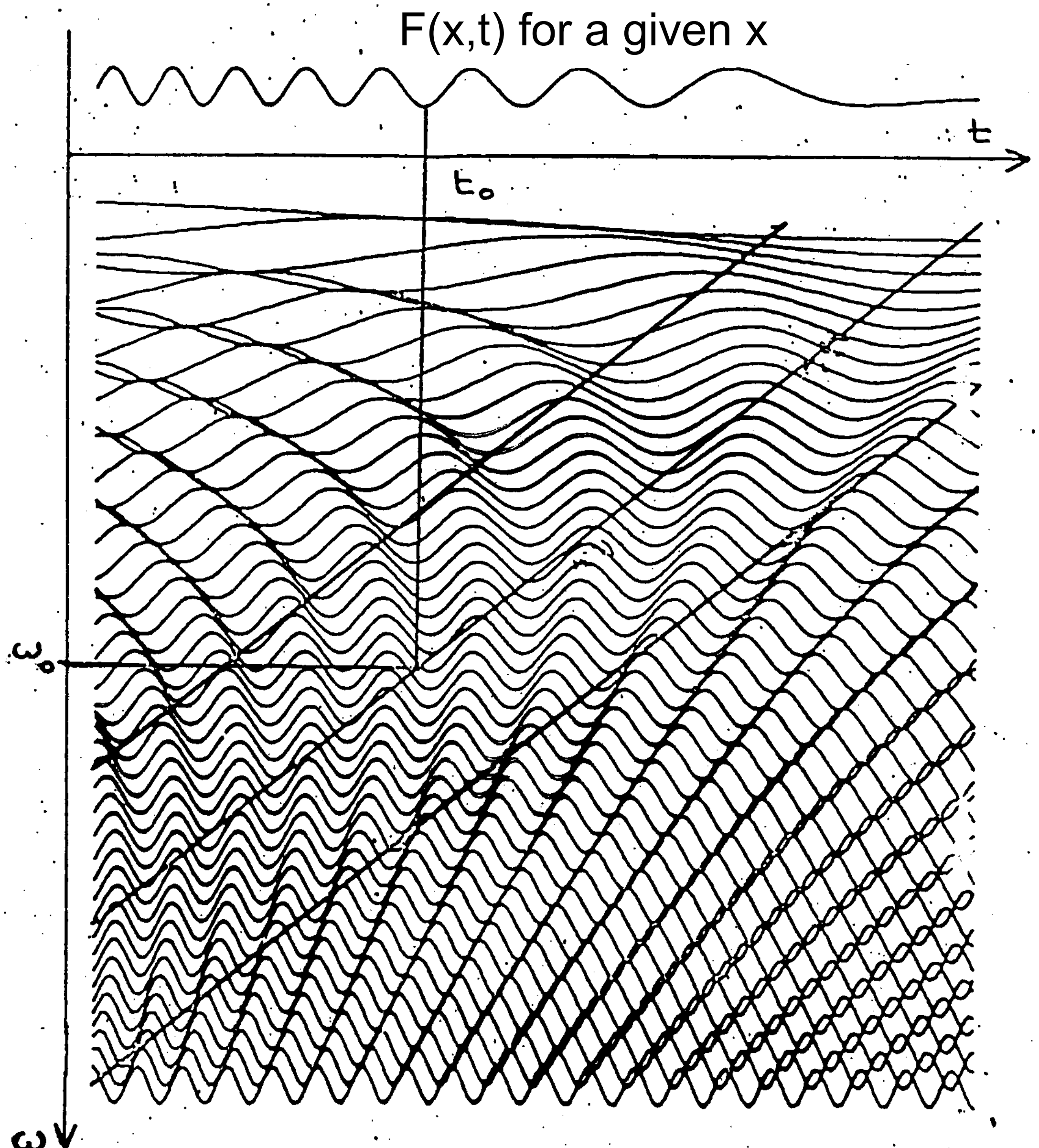
$$u(x, t) \approx \frac{1}{2\pi} \int_{\omega_0 - \Delta\omega}^{\omega_0 + \Delta\omega} A(\omega) \exp\left[i \left((\omega - \omega_0)(t - \frac{dk}{d\omega}\Big|_{\omega_0} x) + (\omega_0 t - k(\omega_0)x) + \phi_i(\omega) \right) \right] d\omega$$

Compare to the simple situation of two cosine waves:

$$u(x, t) = 2\cos(\omega t - kx)\cos(\delta\omega t - \delta kx)$$

Similar to the cosine waves, the group velocity is defined as $U(\omega) = \dfrac{d\omega}{dk}$

Stationnary phase

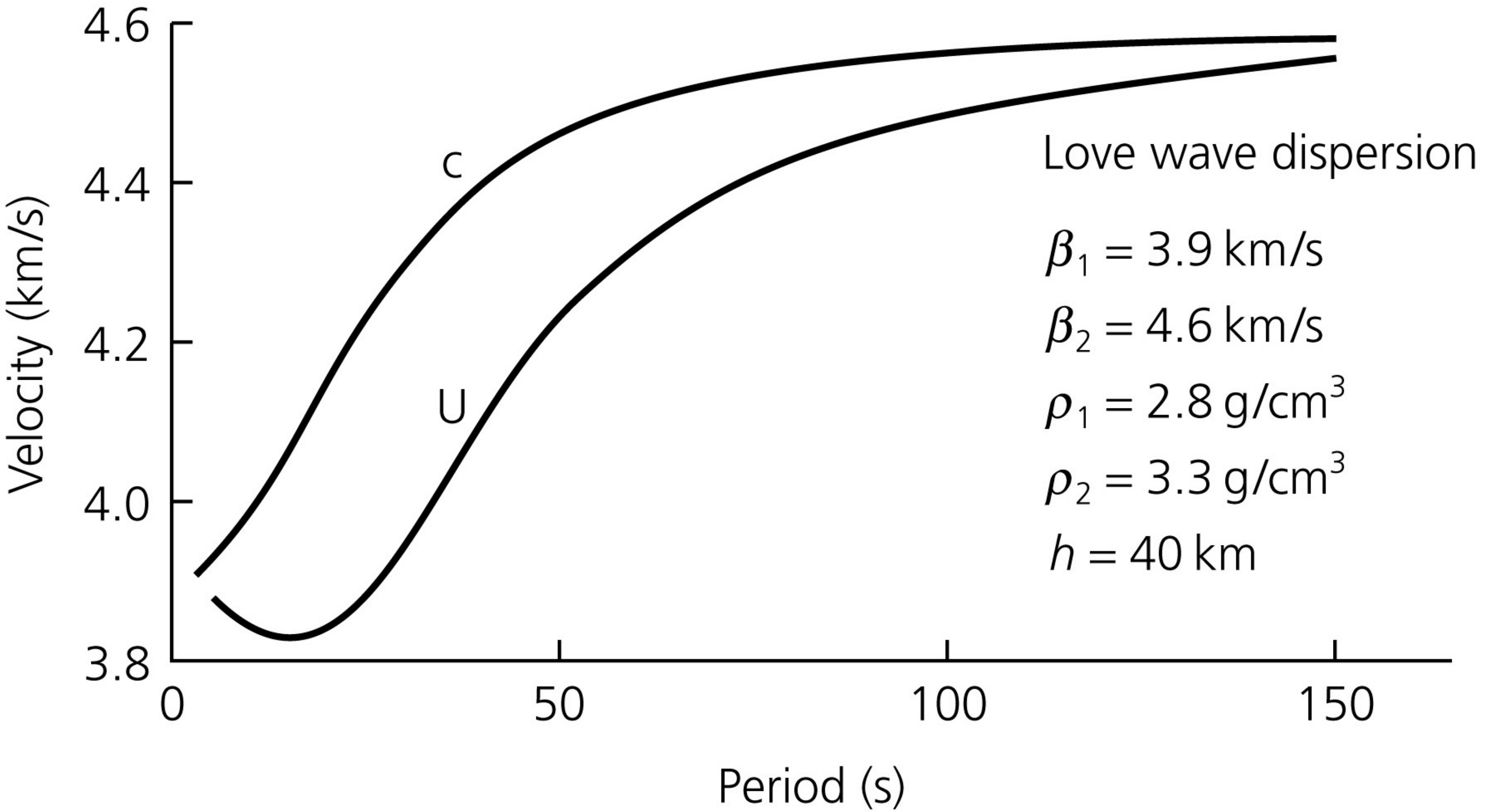


$$\frac{\partial(\omega t - kx)}{\partial \omega} = 0$$

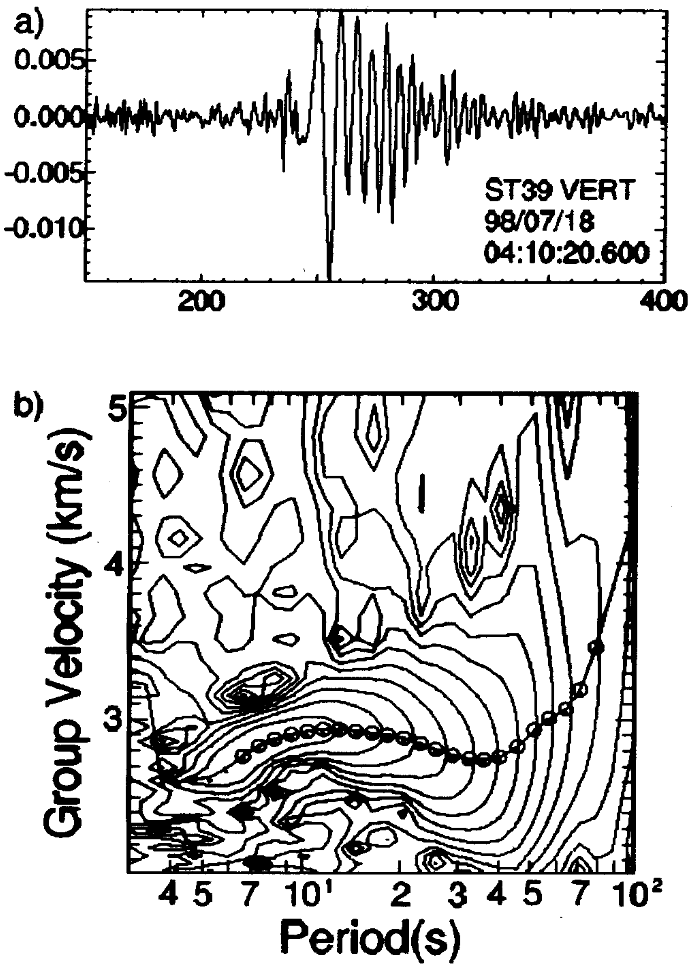
$$t - \frac{\partial k}{\partial \omega} x = 0$$

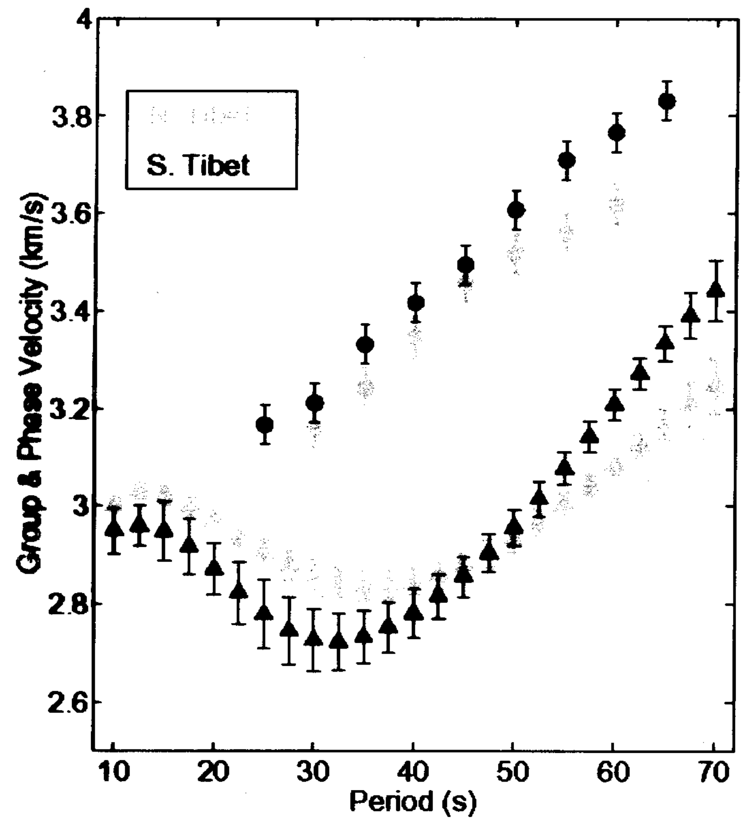
$$U = \frac{\partial \omega}{\partial k}$$

Figure 2.8-2: Fundamental mode Love wave group and phase velocities.



$$U = \frac{d\omega}{dk} = \frac{d(ck)}{dk} = c + k\,\frac{dc}{dk} = c - \lambda\,\frac{dc}{d\lambda}$$


a)
0.005
0.000
-0.005
-0.010
ST39 VERT
98/07/18
04:10:20.600
200
300
400
b)
Group Velocity (km/s)
5
4
3
4 5 7 10¹ 2 3 4 5 7 10²
Period(s)


S. Tibet
Group & Phase Velocity (km/s)
4
3.8
3.6
3.4
3.2
3
2.8
2.6
10 20 30 40 50 60 70
Period (s)