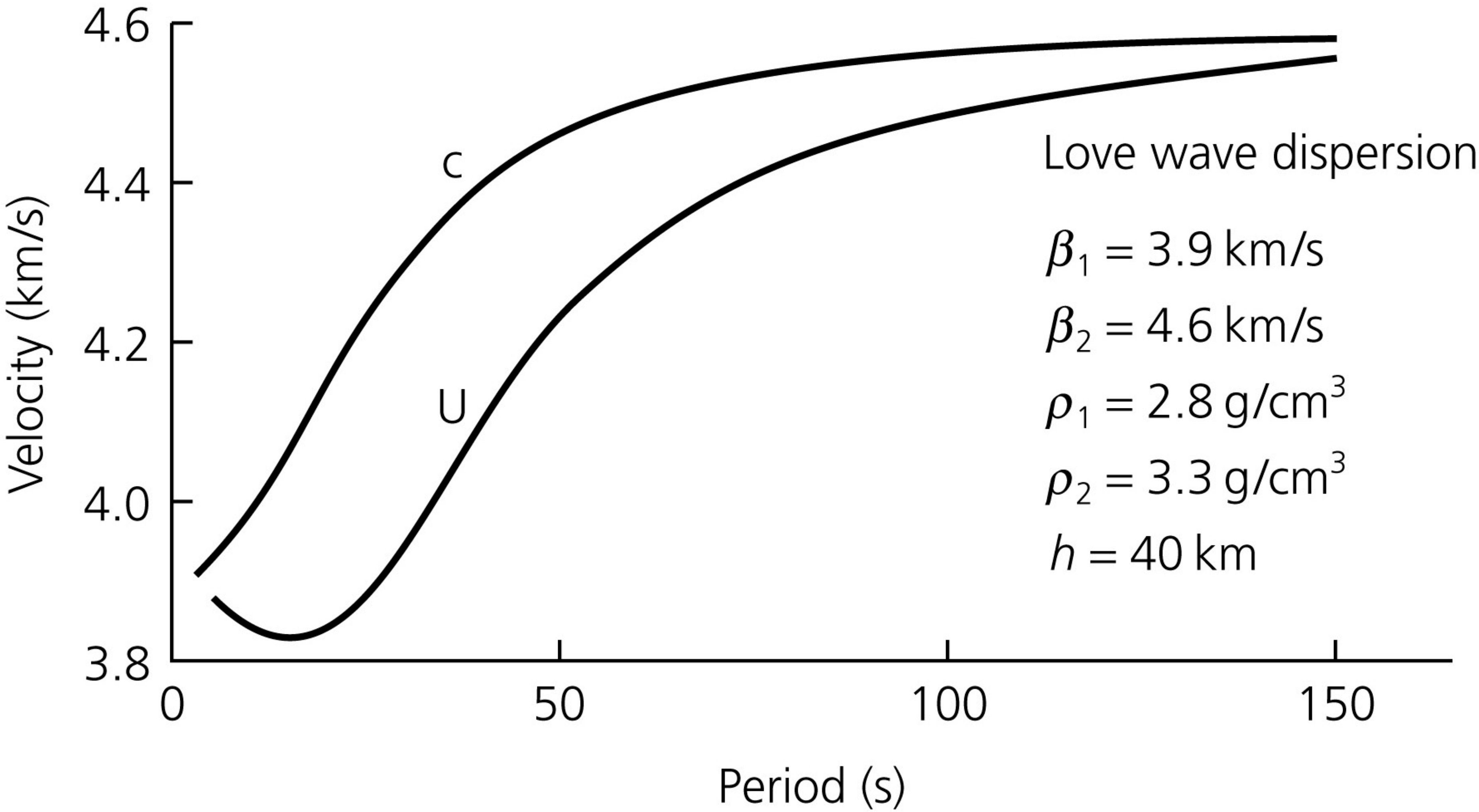
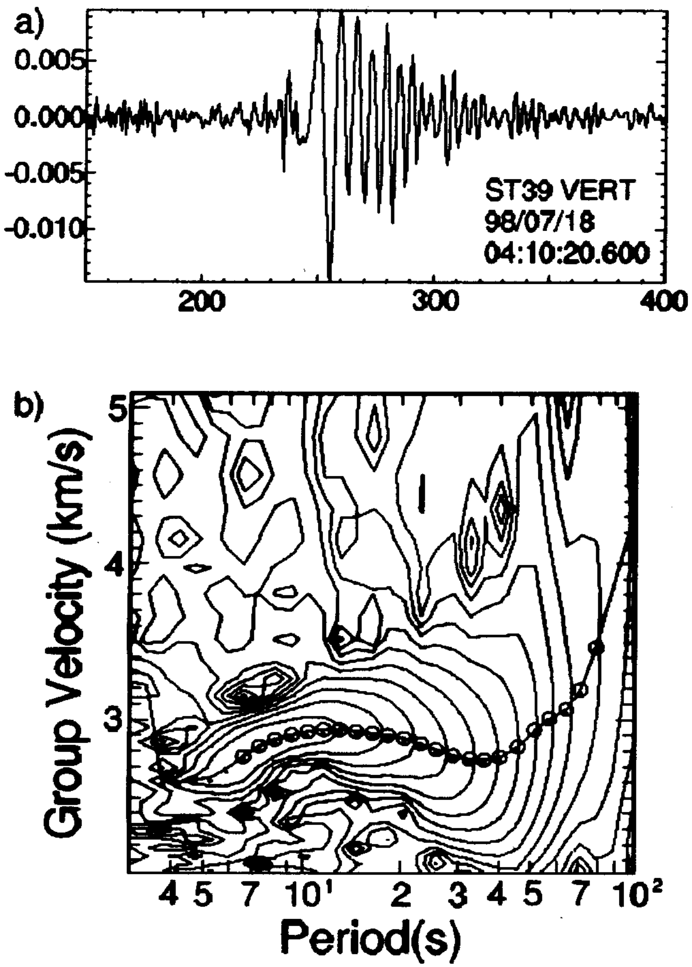
MEEES and M2R STU

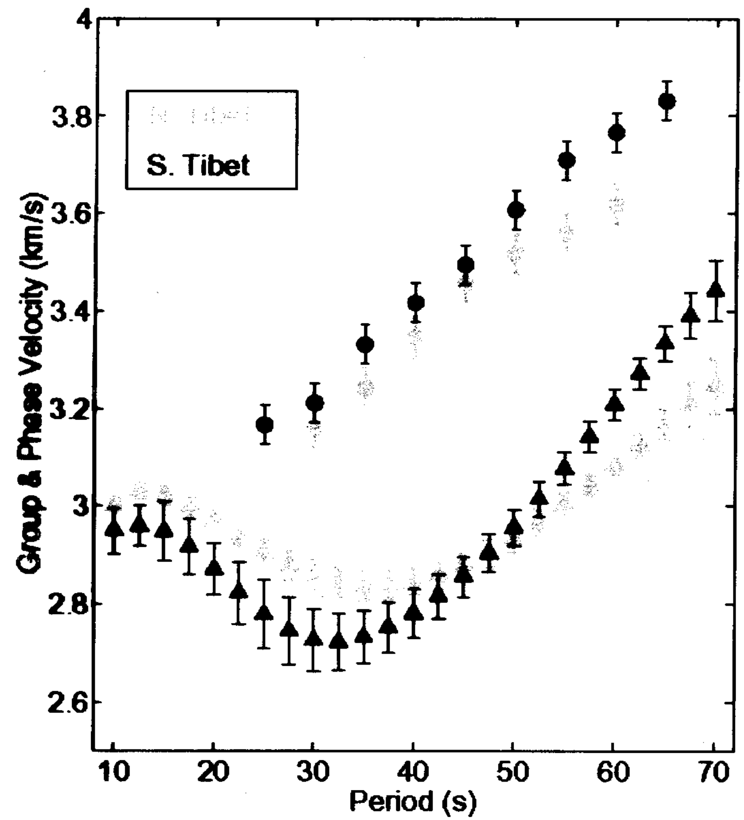
Seismology (Michel Campillo)

Figure 2.8-2: Fundamental mode Love wave group and phase velocities.



$$U = \frac{d\omega}{dk} = \frac{d(ck)}{dk} = c + k\,\frac{dc}{dk} = c - \lambda\,\frac{dc}{d\lambda}$$


a)
ST39 VERT
98/07/18
04:10:20.600
b)
Group Velocity (km/s)
Period(s)


S. Tibet
Group & Phase Velocity (km/s)
Period (s)

Fitting observations with a layered model…..

Sensitivity of phase velocity to the velocity of the medium at depth.

Relation with eigenfunctions. The case of Love waves

h is the eigenfunction of a mode in the reference medium : $\rho(z)$,$\mu(z)$,$\beta(z)$

$$h = h(\rho, \mu, k, \omega)$$

In a perturbed medium $(\rho + \delta\rho, \mu + \delta\mu)$:

$$h' = h + \delta h = h(\rho + \delta\rho, \mu + \delta\mu, k + \delta k, \omega)$$

We use stationarity properties of energy quantities to find the change of phase velocity associated with a small change of the properties of the medium.

The Lagrangian $L =$ *kinetic energy* - *potential energy.*

For elastic motions:

$$L = \frac{\rho}{2}(\dot{u}_i)^2 - [\frac{\lambda}{2}(e_{kk})^2 + \mu e_{ij}e_{ij}]$$

Love wave:

$$u_2 = h(k, z, \omega)exp(i(kx - \omega t))$$

After integration over one cycle:

$$\langle L \rangle = \frac{1}{4}\rho\omega^2 h^2 - \frac{1}{4}\mu[k^2 h^2 + (\frac{\partial h}{\partial z})^2]$$

Integrating L over z, we introduce the energy integrals I_i:

$$H = \int_0^\infty \langle L \rangle dz = \frac{1}{2}\{\omega^2 I_1 - k^2 I_2 - I_3\}$$

with

$$I_1 = \frac{1}{2}\int_0^\infty \rho(z) h^2(z) dz$$

$$I_2 = \frac{1}{2}\int_0^\infty \mu(z) h^2(z) dz$$

$$I_3 = \frac{1}{2}\int_0^\infty \mu(z) (\frac{\partial h}{\partial z})^2 dz$$

Properties of H

Equation of motion

$$\omega^2 \rho h + \frac{\partial}{\partial z}(\mu \frac{\partial h}{\partial z}) - k^2 \mu h = 0$$

$\times h$ et integration over z :

$$\int_0^\infty [\omega^2 \rho h^2 + h \frac{\partial}{\partial z}(\frac{\mu \partial h}{\partial z}) - k^2 \mu h^2] dz = 0$$

$$\int_0^\infty [\omega^2 \rho h^2 - k^2 \mu h^2 - \mu (\frac{\partial h}{\partial z})^2] dz + [h \mu \frac{\partial h}{\partial z}]_0^\infty = 0$$

$h \longrightarrow 0$ for $z \longrightarrow \infty$
$\frac{\partial h}{\partial z} = 0$ at the free surface

$$I_1 = \frac{1}{2}\int_0^\infty \rho(z) h^2(z) dz$$

$$I_2 = \frac{1}{2}\int_0^\infty \mu(z) h^2(z) dz$$

$$I_3 = \frac{1}{2}\int_0^\infty \mu(z) (\frac{\partial h}{\partial z})^2 dz$$

$$\omega^2 I_1 - k^2 I_2 - I_3 = 0$$

$H = \int_0^\infty \langle L \rangle dz = 0$ for an eigenfunction.

Properties of H

perturbation of h:

$$\delta H = \frac{1}{2}\{\omega^2 \delta I_1 - k^2 \delta I_2 - \delta I_3\}$$

$$\delta H = \int_0^\infty \omega^2 \rho(z) h(z) \delta h(z) dz \quad - \quad \int_0^\infty k^2 \mu(z) h(z) \delta h(z) dz$$
$$- \quad \int_0^\infty \mu(z) \frac{\partial h}{\partial z} \frac{\partial \delta h}{\partial z} dz$$

Integration by part and conditions for $z = 0$ and $z \to \infty$

$$\delta H = \int_0^\infty [\omega^2 \rho(z) h(z) - k^2 \mu(z) h(z) + \frac{\partial}{\partial z}(\mu(z) \frac{\partial h}{\partial z})] \delta h(z) dz$$

equation of motion $\rightarrow = 0$

$$\delta H = 0$$

$$\omega^2 \delta I_1 - k^2 \delta I_2 - \delta I_3 = 0$$

H is *stationary.*

Perturbation ~~of~~ h.

$$H = 1/2(\omega^2 I_1 - k^2 I_2 - I_3) = 0$$

$$\delta H = \frac{1}{2}(\omega^2 \delta I_1 - k^2 \delta I_2 - \delta I_3)$$

$$I_1 = \frac{1}{2}\int \rho h^2 dz \Rightarrow \delta I_z = \frac{\partial I}{\partial h} \cdot \delta h(z) \text{ for } z.$$

$$\Rightarrow \delta I_1 = \frac{1}{2}\int \rho(z)\, 2h(z)\, \delta h(z)\, dz$$

$$I_3 = \frac{1}{2}\int \mu \left(\frac{\partial h}{\partial z}\right)^2 dz$$

for z fixed: $\delta\left[\left(\frac{\partial h}{\partial z}\right)^2\right] = \delta f$

$$= \left(\frac{\partial(h+\delta h)}{\partial z}\right)^2 - \left(\frac{\partial h}{\partial z}\right)^2 = \left(\frac{\partial h}{\partial z} + \frac{\partial \delta h}{\partial z}\right)^2 - \left(\frac{\partial h}{\partial z}\right)^2$$

$$= \left(\frac{\partial h}{\partial z}\right)^2 + \left(\frac{\partial \delta h}{\partial z}\right)^2 + 2\frac{\partial h}{\partial z}\frac{\partial \delta h}{\partial z} - \left(\frac{\partial h}{\partial z}\right)^2.$$

2nd order. $\Rightarrow \sim 2\frac{\partial h}{\partial z}\frac{\partial \delta h}{\partial z}$

Perturbations: $\rho(z), \mu(z) \rightarrow \rho(z) + \delta\rho(z), \mu(z) + \delta\mu(z)$

$$h + \delta h = h(\rho + \delta\rho, \mu + \delta\mu, k + \delta k, \omega)$$

$H(h + \delta h) = 0$:

$$\begin{aligned} \omega^2 \int_0^\infty (\rho + \delta\rho)(h + \delta h)^2 dz &= (k + \delta k)^2 \int_0^\infty (\mu + \delta\mu)(h + \delta h)^2 dz \\ &+ \int_0^\infty (\mu + \delta\mu)(\frac{\partial(h + \delta h)}{\partial z})^2 dz \end{aligned} \quad (1)$$

Noting $H(h) = 0$ and neglecting the terms of second order as $\delta\mu\delta h$ or $(\delta h)^2$:

$$\begin{aligned} \omega^2 (\int_0^\infty (\delta\rho h^2 + 2\rho h \delta h) dz &= k^2 \int_0^\infty (\delta\mu h^2 + 2\mu h \delta h) dz \\ &+ 2k\delta k \int_0^\infty \mu h^2 dz + \int_0^\infty \delta\mu (\frac{\partial(h)}{\partial z})^2 dz \\ &+ \int_0^\infty 2\mu \frac{\partial h}{\partial z} \frac{\partial \delta h}{\partial z} dz \end{aligned}$$

Considering the stationarity of H:

$$\int_0^\infty \omega^2 \delta\rho h^2 dz = k^2 \int_0^\infty (\delta\mu h^2) dz + 2k\delta k \int_0^\infty \mu h^2 dz + \int_0^\infty \delta\mu (\frac{\partial(h)}{\partial z})^2 dz$$

$$\frac{\delta C}{C} = -\frac{\delta k}{k} = \frac{\int_0^\infty \delta\mu (k^2 h^2 + (\frac{\partial(h)}{\partial z})^2) dz - \int_0^\infty \delta\rho\omega^2 h^2 dz}{2k^2 \int_0^\infty \mu h^2 dz}$$

$\rightarrow$ a relation between variation of phase velocity and perturbation of the medium: the base of linear inversion of dispersion curves.

Perturbation of ω -

$$\omega^2 I_1 - k^2 I_2 - I_3 = 0$$

$\omega \to \omega + \partial\omega$

$$2\omega I_1 + \omega^2 \frac{\delta I}{\partial \omega} - 2k \frac{dk}{d\omega} I_2$$

$$- k^2 \frac{\delta I_2}{\partial \omega} - \frac{\delta I_3}{\delta \omega} = 0$$

perturbation ω for I_1 = pertn in k.

$\to$ Stationarity* wrt k: $\omega^2 \delta I_1 - k^2 \delta I_2 - \delta I_3 = 0$

$$\omega^2 \frac{\partial I_1}{\delta \omega} \delta\omega - k^2 \frac{\partial I_2}{\partial \omega} \delta\omega - \frac{\partial I_3}{\partial \omega} \delta\omega = 0$$

$$\Rightarrow \quad 2\omega I_1 - 2k \frac{\partial k}{\partial \omega} I_2 = 0$$

$$2\omega I_1 = 2k \frac{\partial k}{\partial \omega} I_2$$

$$\frac{\partial \omega}{\partial k} = \frac{k}{\omega} \frac{I_2}{I_1}$$

$$u = \frac{1}{c} \frac{I_2}{I_1}$$

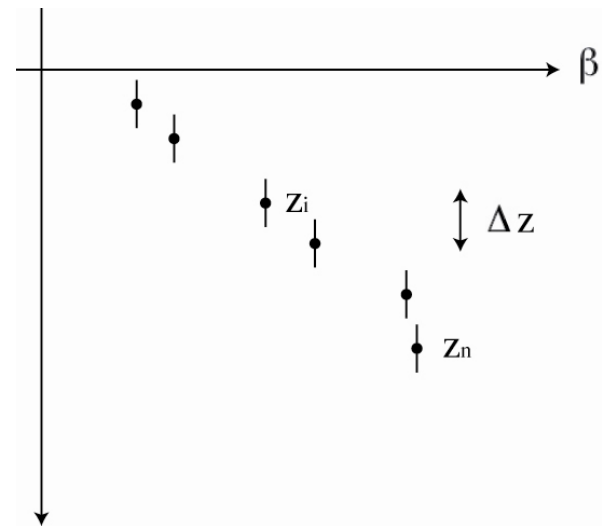
Note I_i depends only on h

Imaging (lithosphere → alluvium)

Measured phase velocity: $C_{obs}(T_j)$

Starting model $M_0(\beta\ (z_i))$

parametrization $z - \Delta z$..



$$\Rightarrow\ C_0\ (T) \quad , \quad \frac{\partial C_0(T)}{\partial \beta(z_i)}$$

→ new model M_1

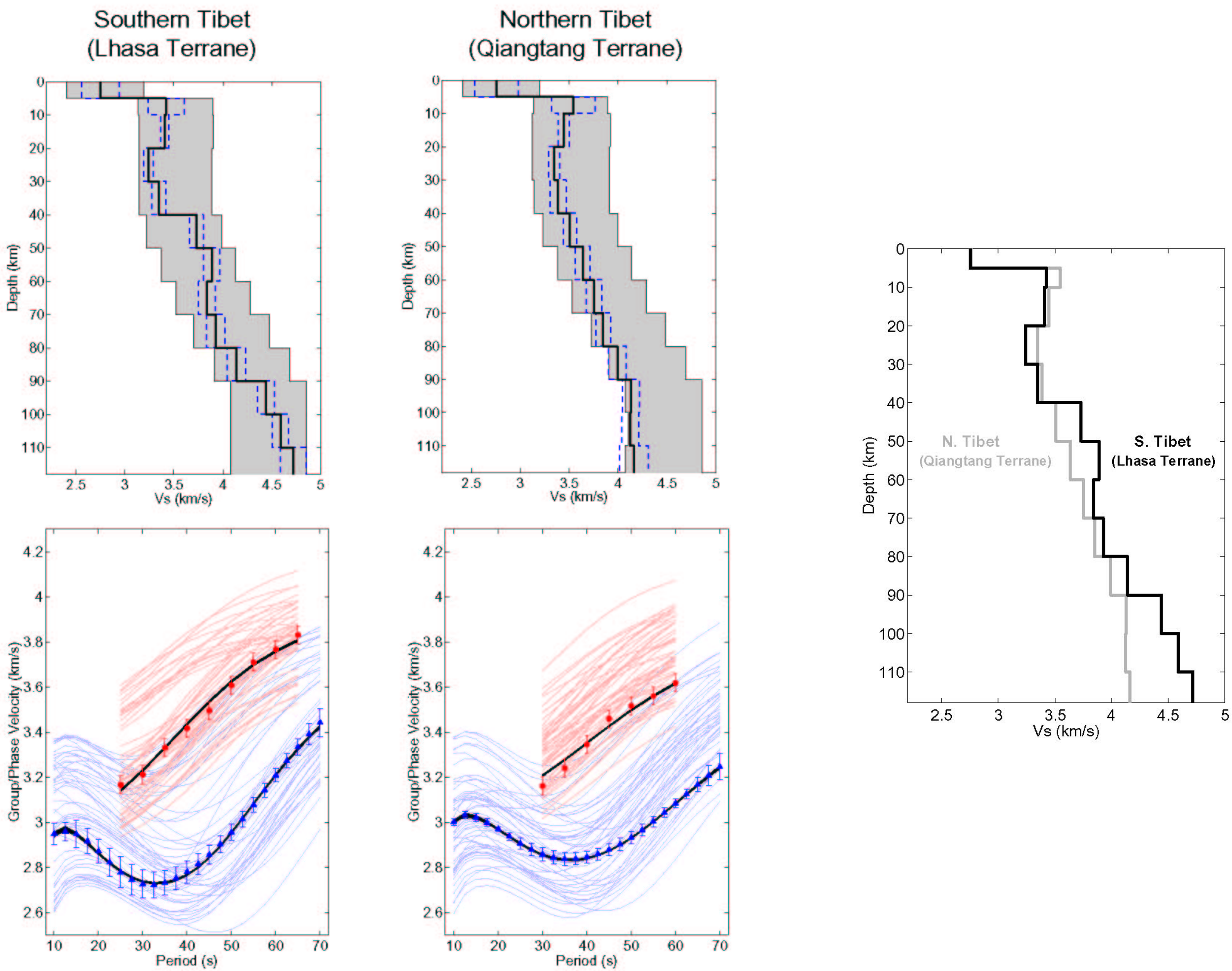
$$C_1(T) = C_0(T) + \sum_i \frac{\partial C_0(T)}{\partial \beta(z_i)} \qquad \delta\beta(z_i)$$

• Find $\delta\beta(z_i)$ such as $\sum_j \|C_1(T_j) - C_{obs}(T_j)\|^2$ is minimal

$\Rightarrow$ iteration $\Rightarrow$ $C_k(T)$, $\frac{\partial C_k}{\partial \beta(z_i)}$

• Find $\partial\beta(z_i)^k$ / $\sum_j \|C_k(T_j) - C_{obs}(T_j)\|^2$ minimal

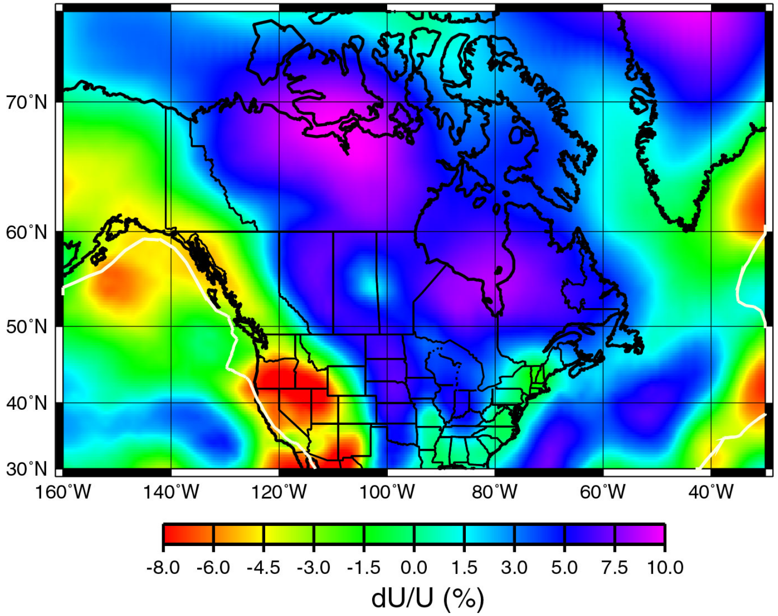
→ model M_{k+1} → convergence



Southern Tibet
(Lhasa Terrane)
Northern Tibet
(Qiangtang Terrane)
Depth (km)
Vs (km/s)
Group/Phase Velocity (km/s)
Period (s)
N. Tibet
(Qiangtang Terrane)
S. Tibet
(Lhasa Terrane)

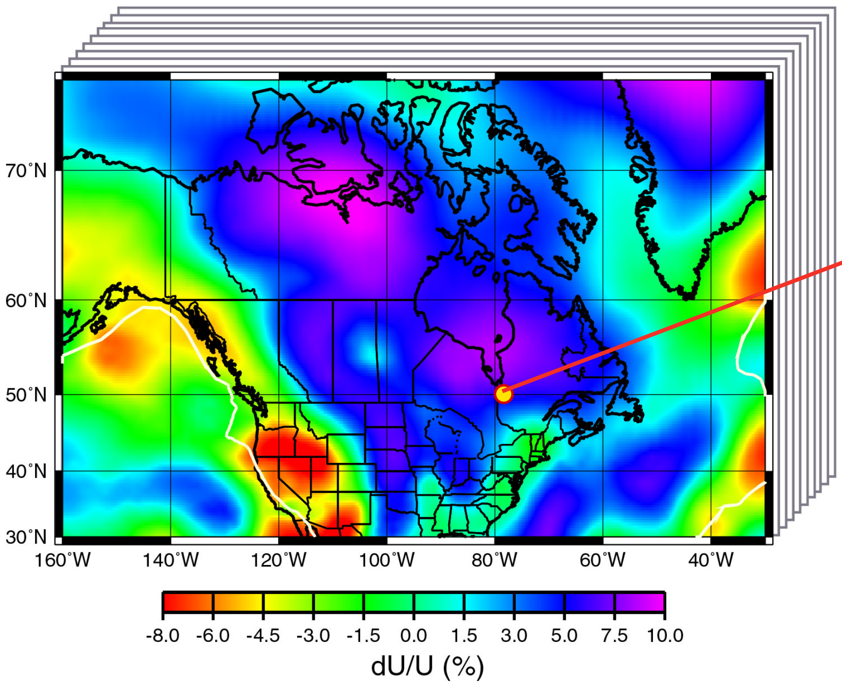
Dispersion map

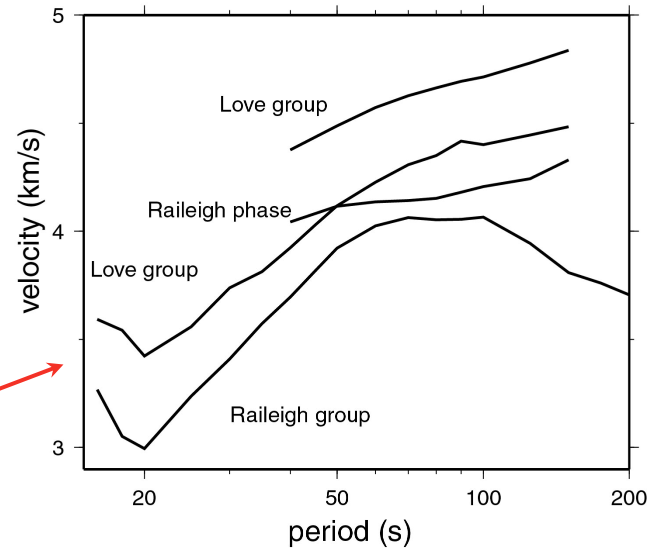
Rayleigh group velocity (100 s)



Local dispersion curves

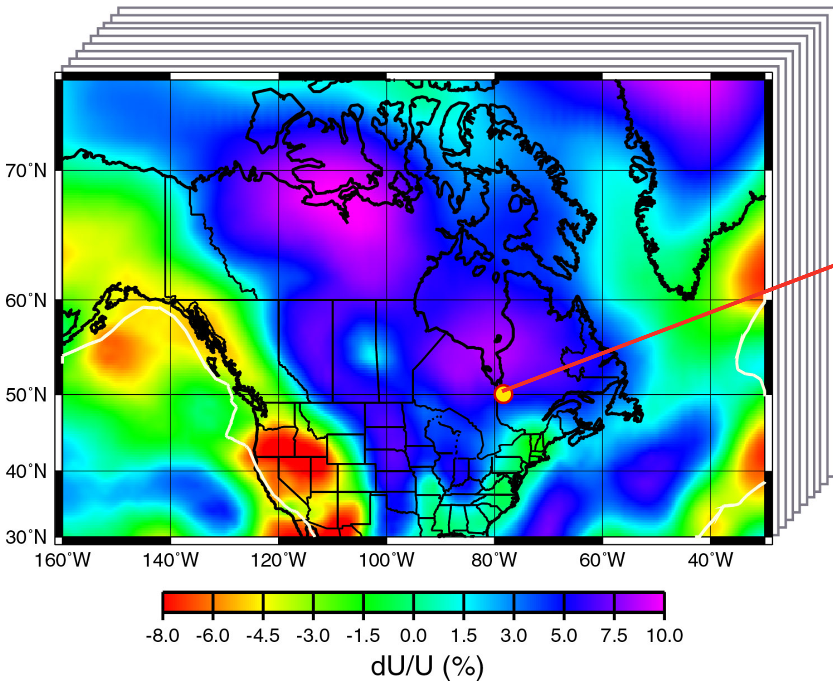
Set of maps at different periods

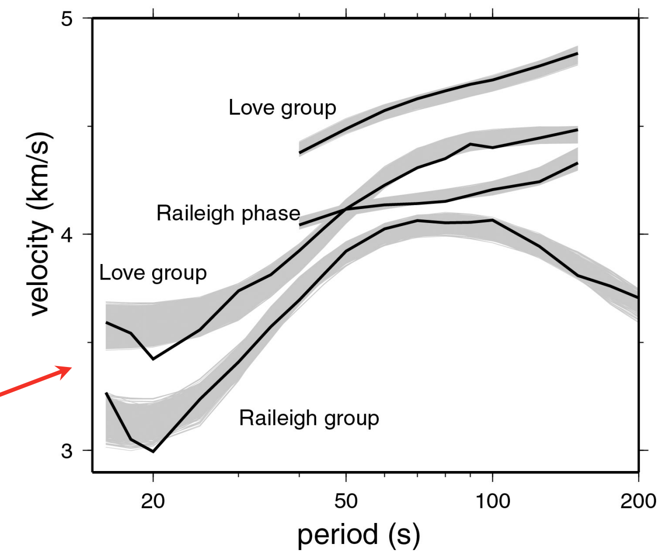




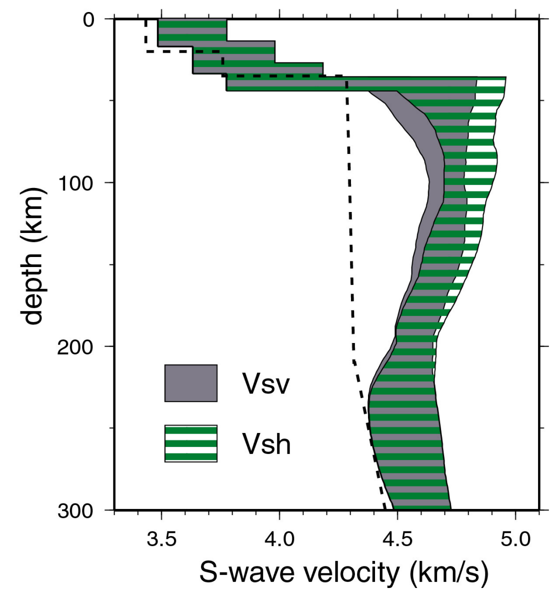
Inversion of dispersion curves

Set of maps at different periods

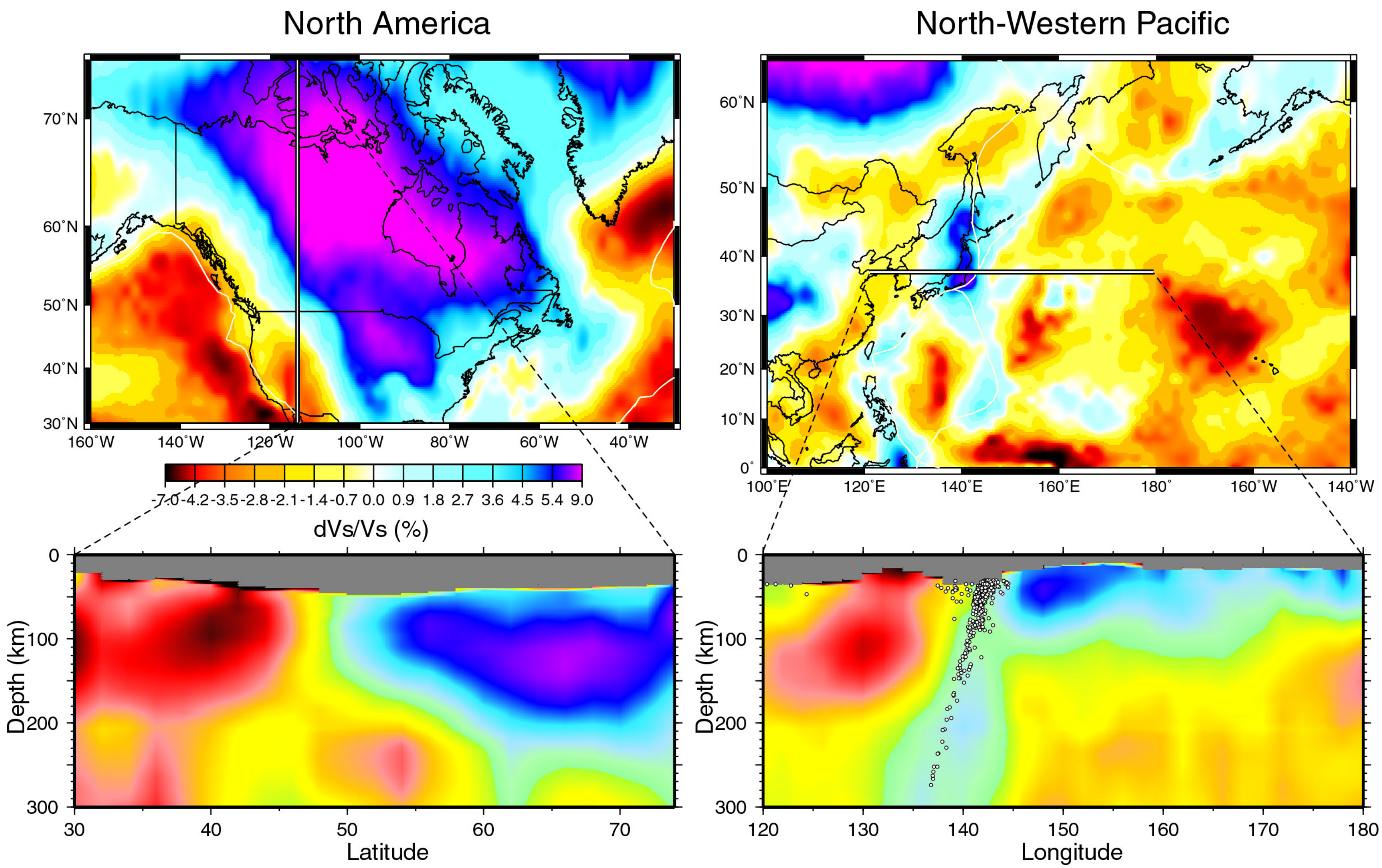




Local model



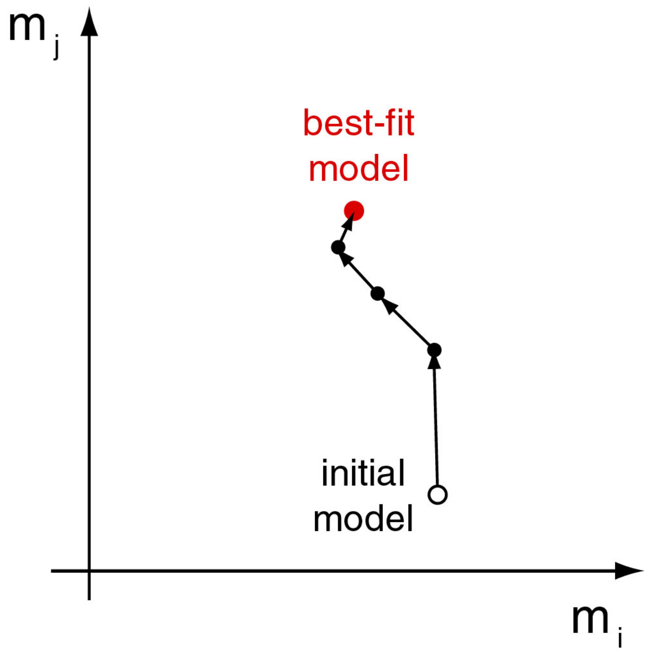
3D Vs Model



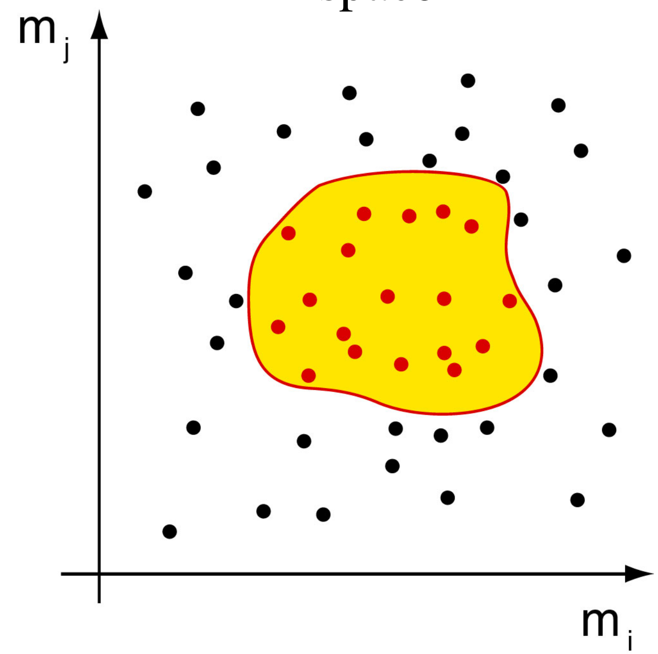
North America
North-Western Pacific
70˚N
60˚N
50˚N
40˚N
30˚N
160˚W
140˚W
120˚W
100˚W
80˚W
60˚W
40˚W
60˚N
50˚N
40˚N
30˚N
20˚N
10˚N
0˚
100˚E
120˚E
140˚E
160˚E
180˚
160˚W
140˚W
-7.0 -4.2 -3.5 -2.8 -2.1 -1.4 -0.7 0.0 0.9 1.8 2.7 3.6 4.5 5.4 9.0
dVs/Vs (%)
Depth (km)
0
100
200
300
30
40
50
60
70
Latitude
120
130
140
150
160
170
180
Longitude

Monte-Carlo inversion

Linearized Inversion



Monte-Carlo Inversion : Random sampling of model space



'Best' model
Resolution analysis
Unicity? Local minima?

Set of acceptable models
Statistics
Non unicity