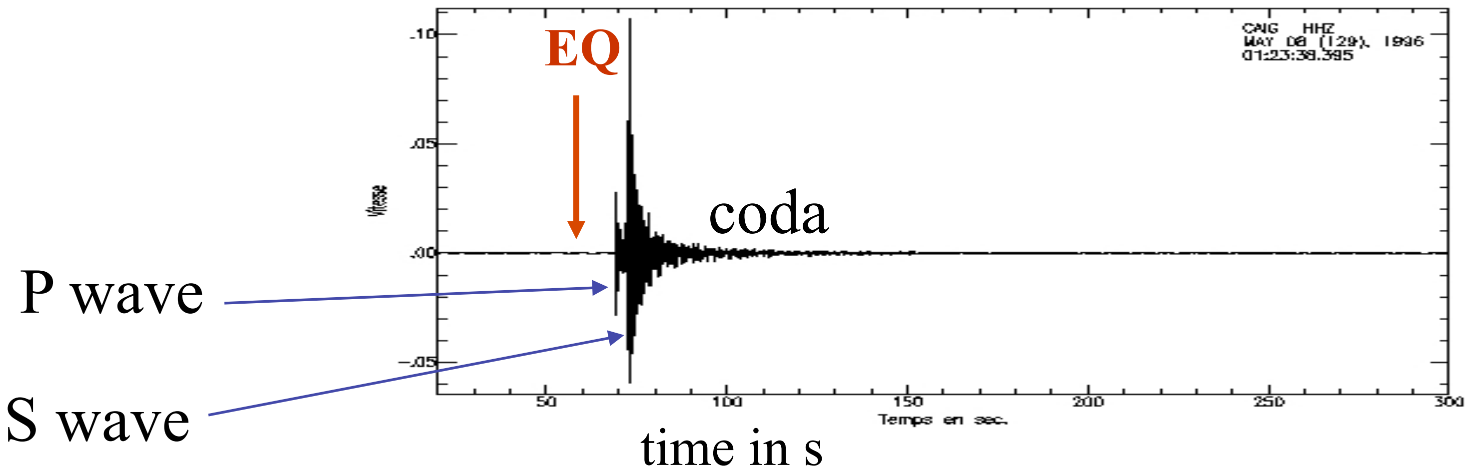
MEEES and M2R STU(TUE552)

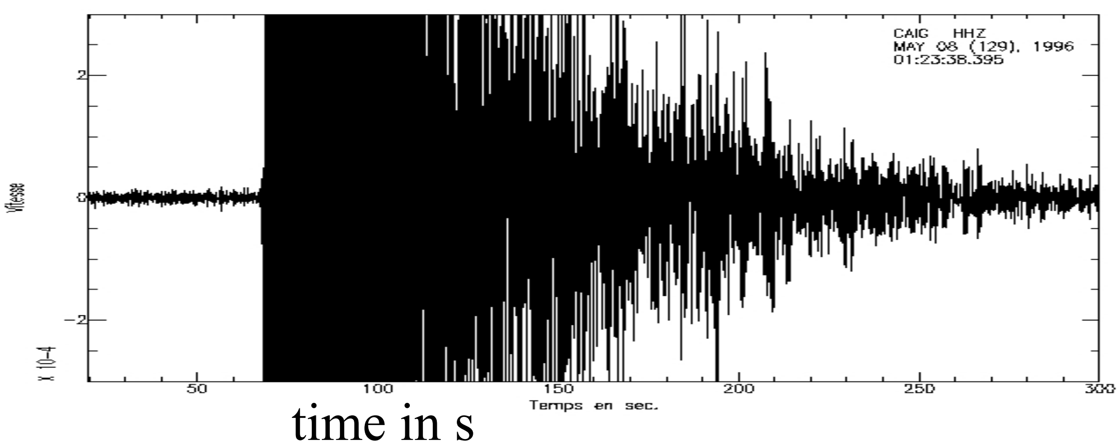
Seismology 7
(Michel Campillo)

http://www-lgit.obs.ujf-grenoble.fr/users/campillo/Master-TUE552

Example of a record of a local earthquake in the band .5-20Hz

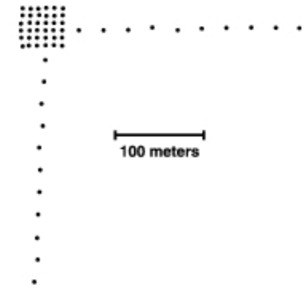


scale x 1000:



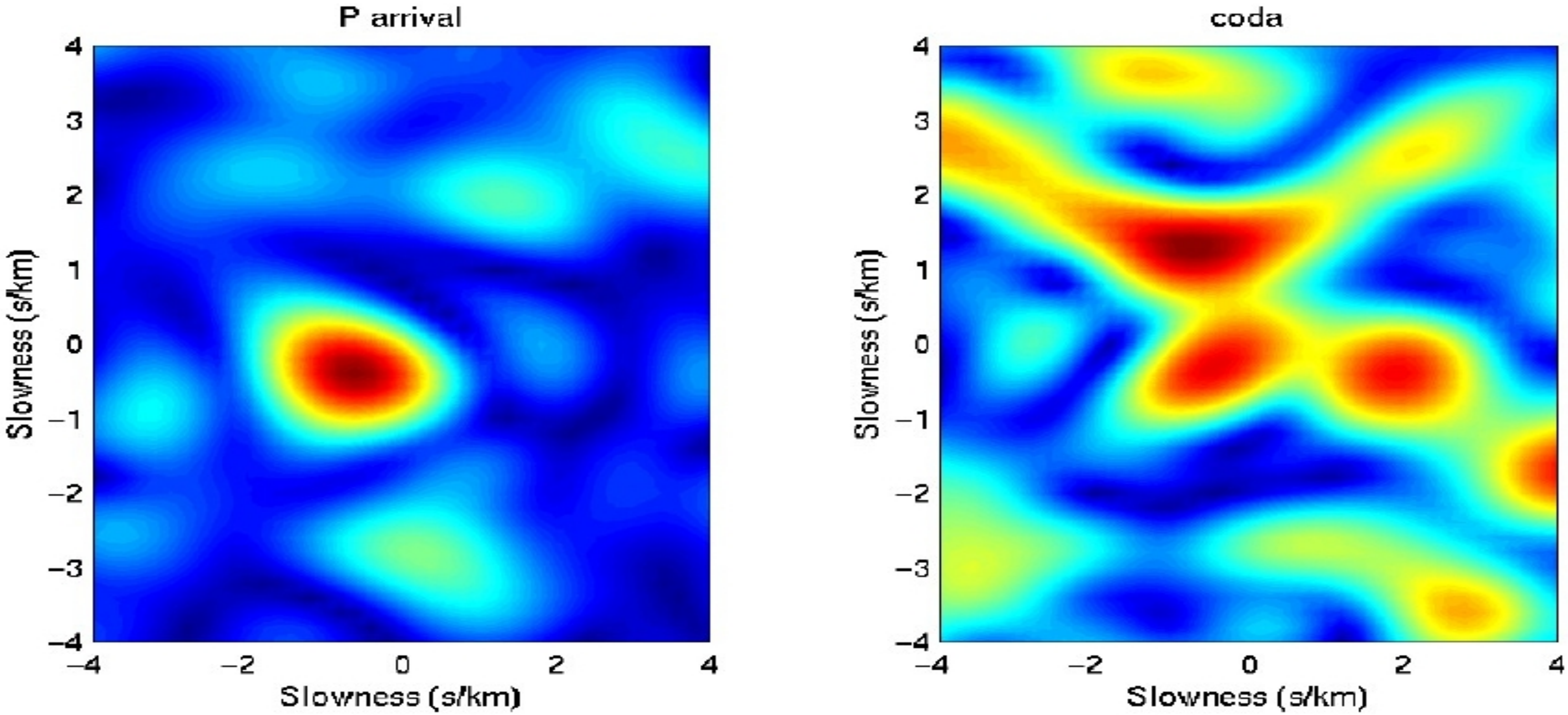
Coda: tail, end of a piece of music….

Frequency-wavenumber analysis

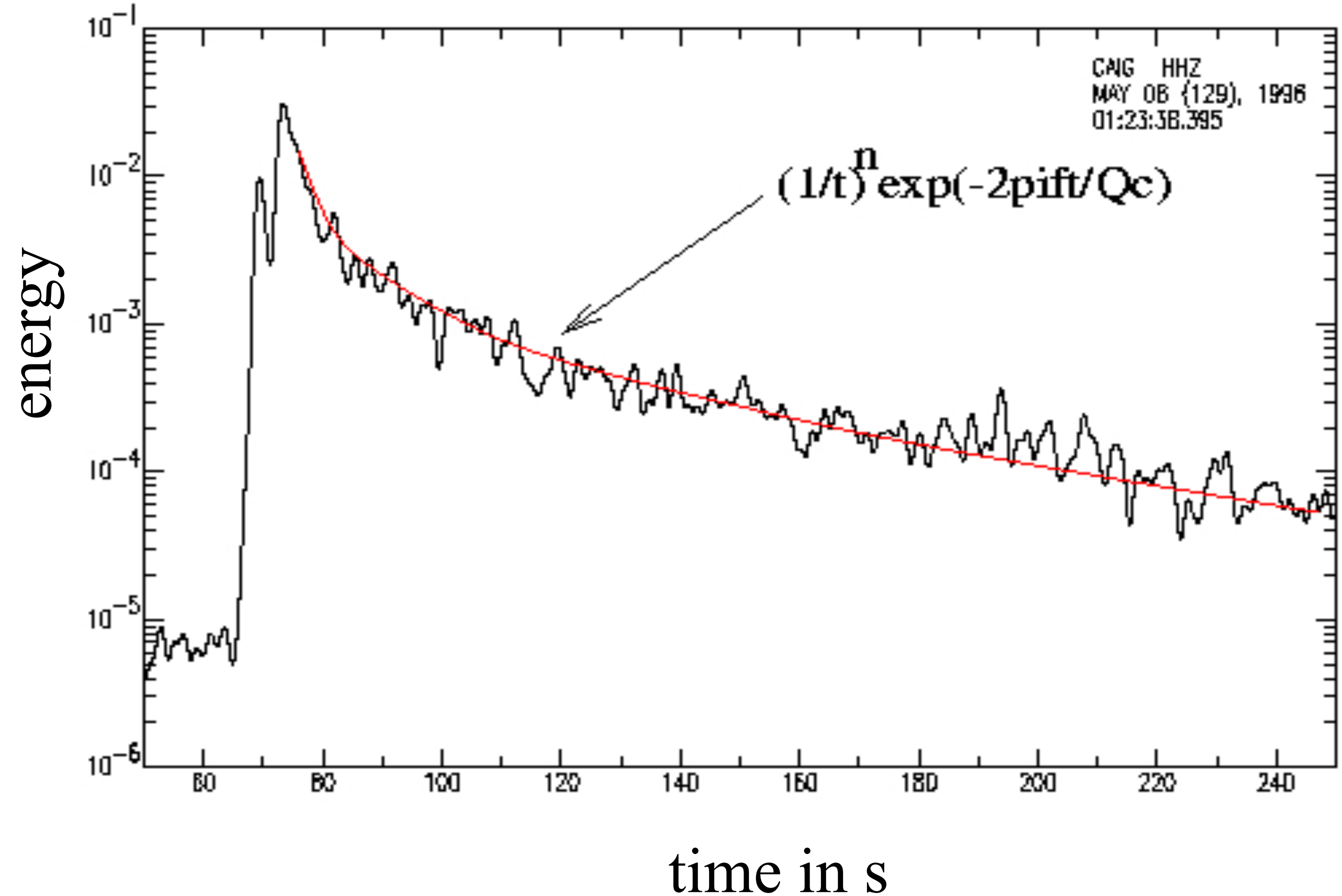


(Pinon Flat Seismometer Array)

$u(x,y) \rightarrow u(k_x,k_y)$

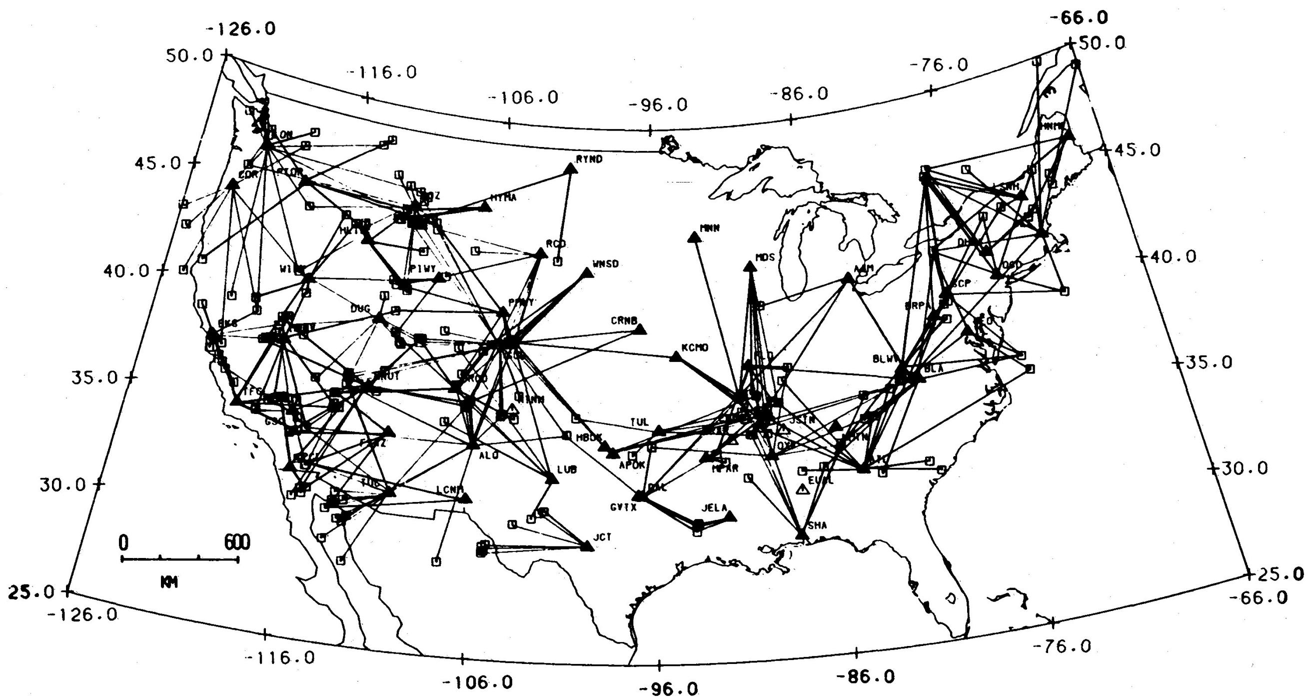


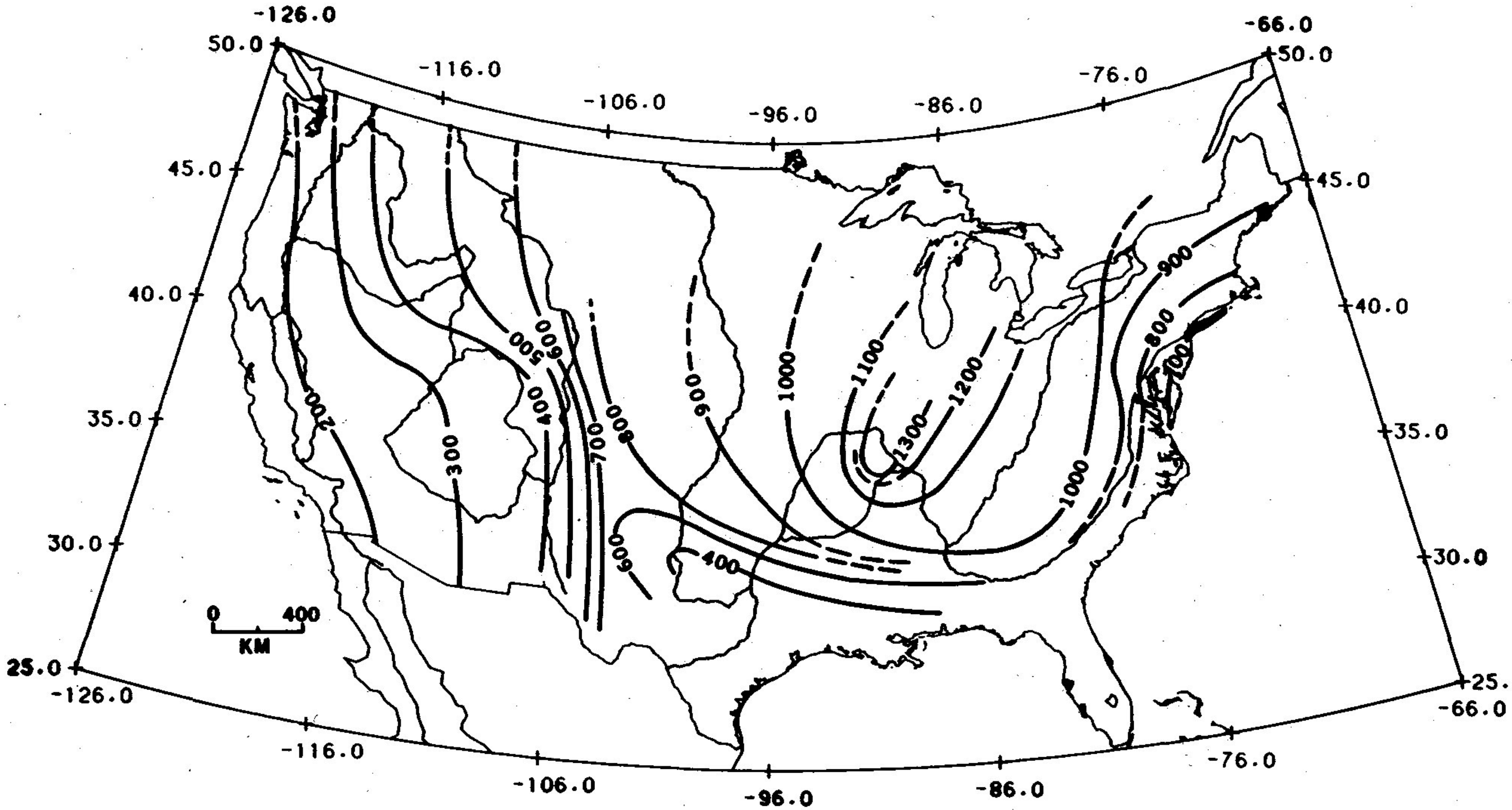
Energy decay in the coda (Aki and Chouet, 1975)



The decay is constant in a region, independently of source and receiver: Qcoda

Coda Q in US (Singh and Herrmann, 1983)




-126.0
50.0
-116.0
-106.0
-96.0
-86.0
-76.0
-66.0
50.0
45.0
45.0
40.0
40.0
35.0
35.0
30.0
30.0
25.0
-126.0
25.0
-66.0
-116.0
-106.0
-96.0
-86.0
-76.0
0
400
KM
200
300
400
500
600
700
800
900
1000
1100
1200
1300
600
400
900
800
700
1000

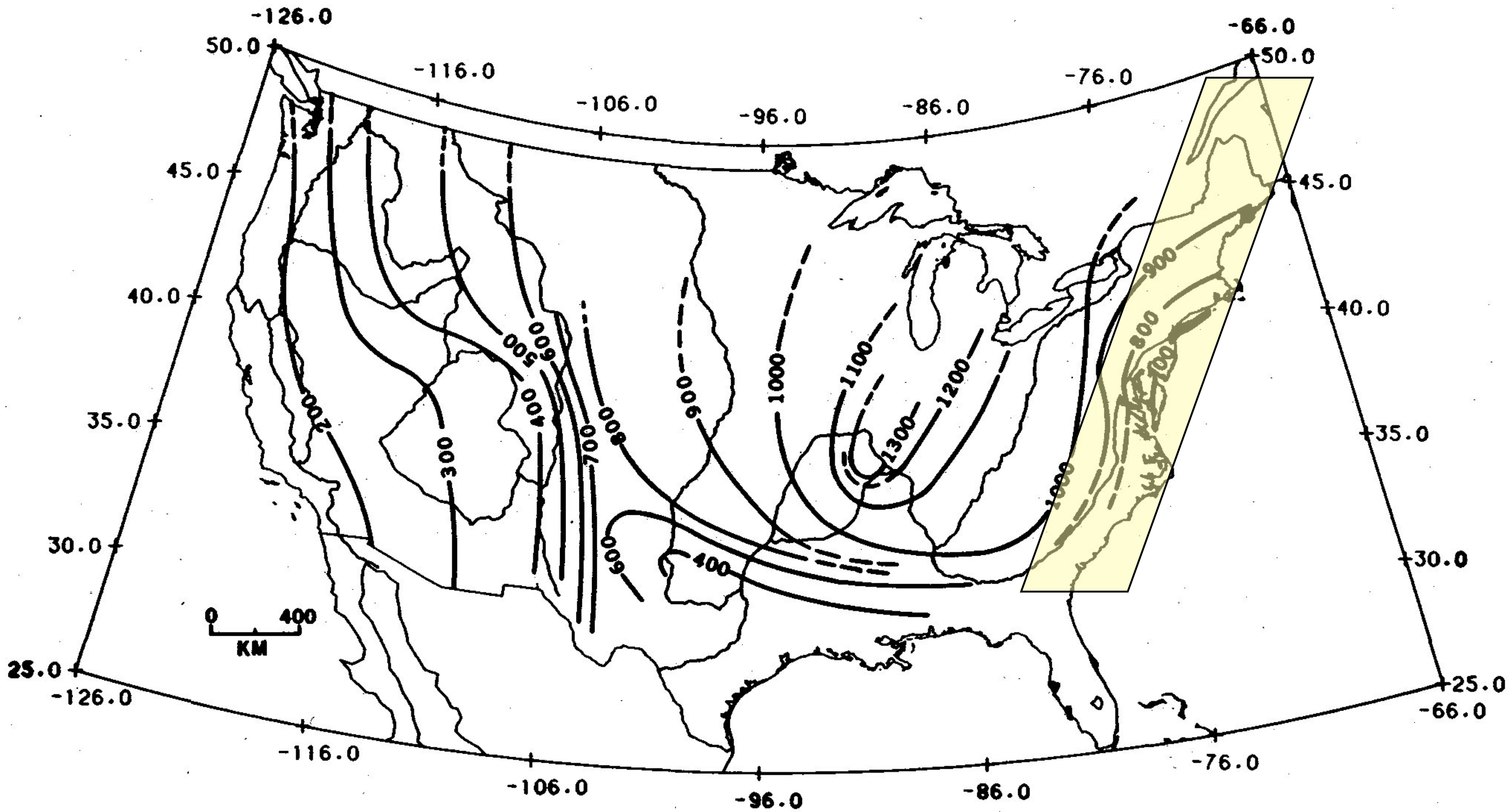


Fig. 15. Contour map of coda Q_0 for the entire continental United States.

Appalachian (Hercynian) belt : Qc ~ 600

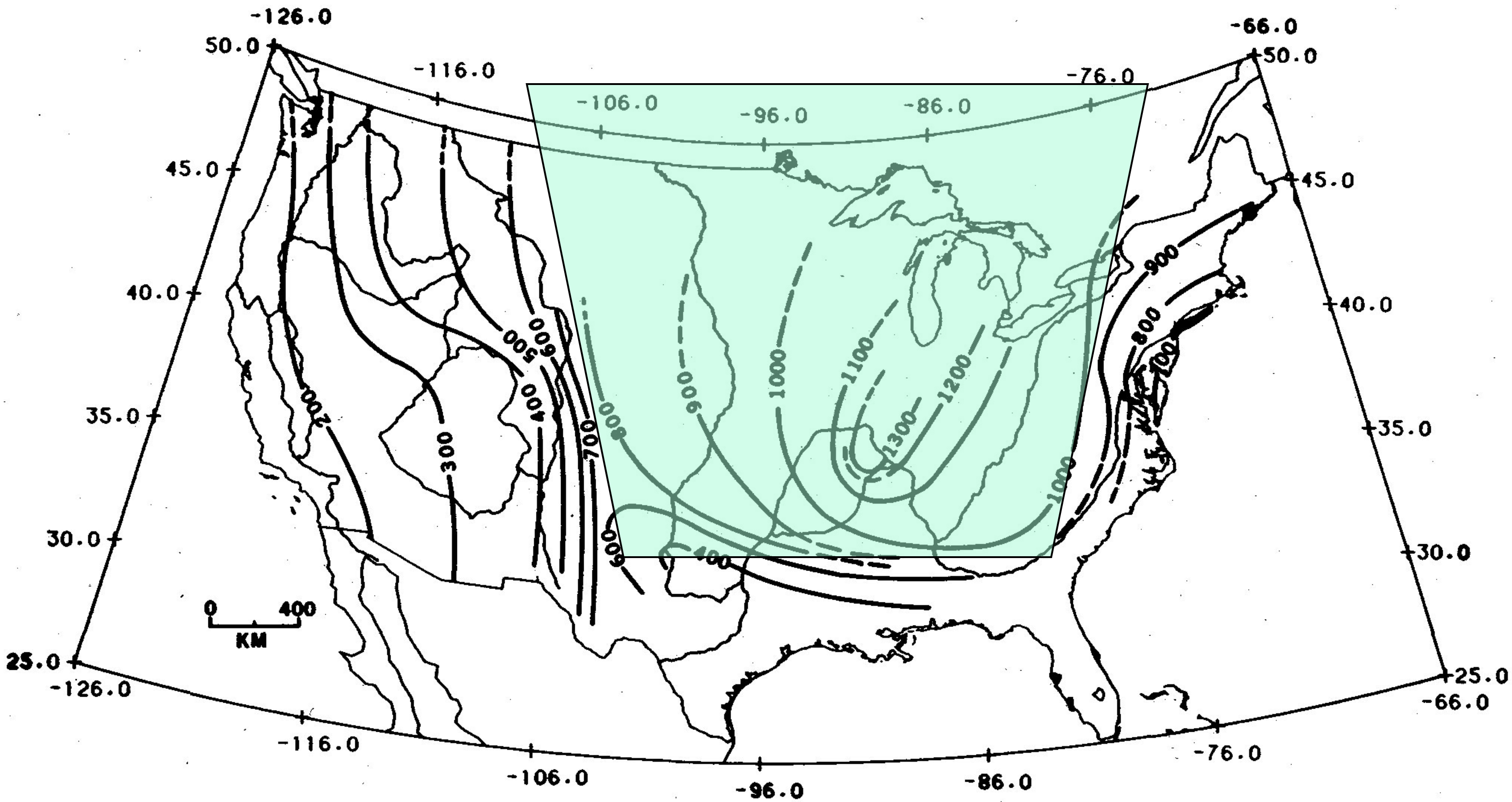


Fig. 15. Contour map of coda Q_0 for the entire continental United States.

Central shield : Qc ~ 1000

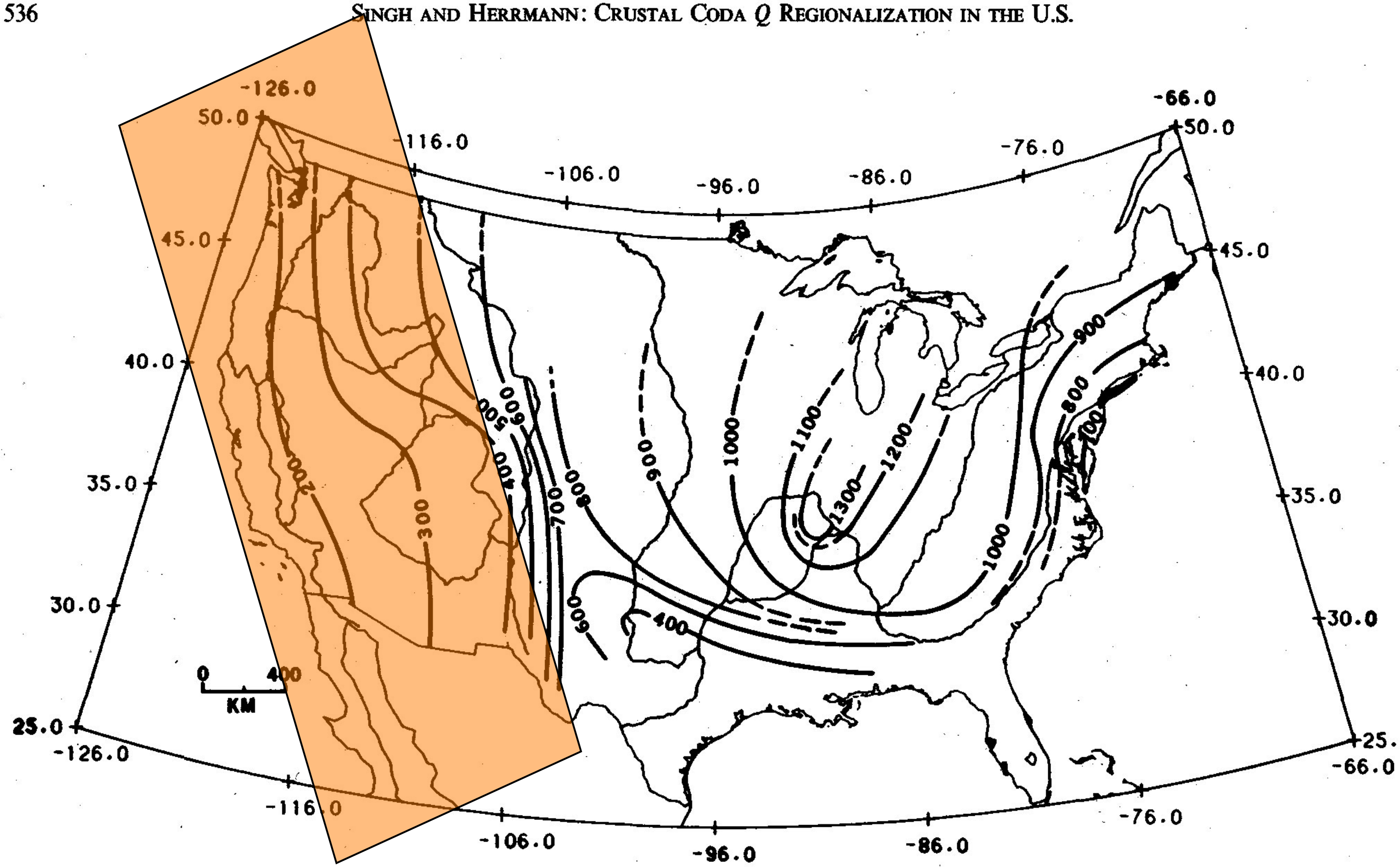
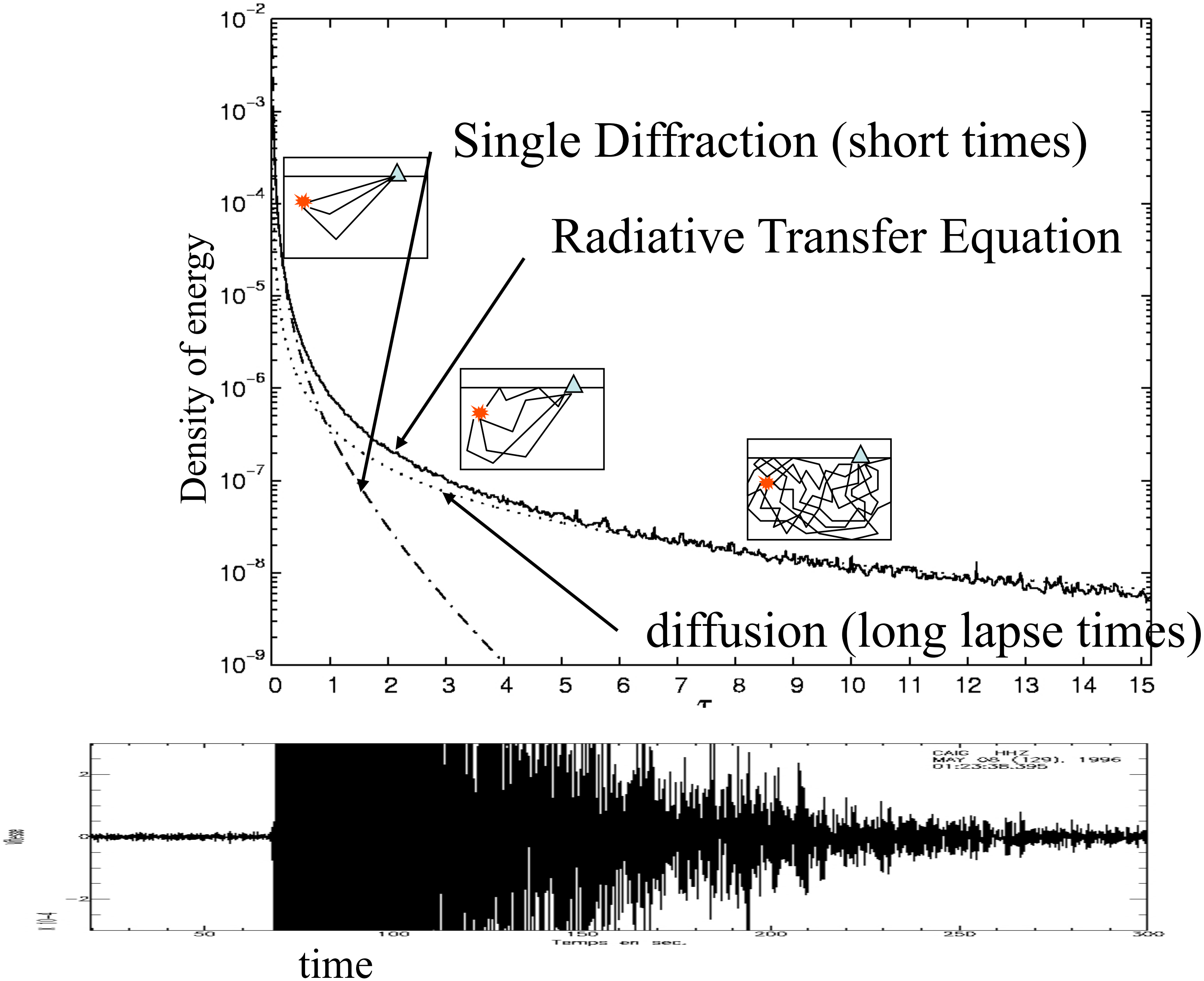


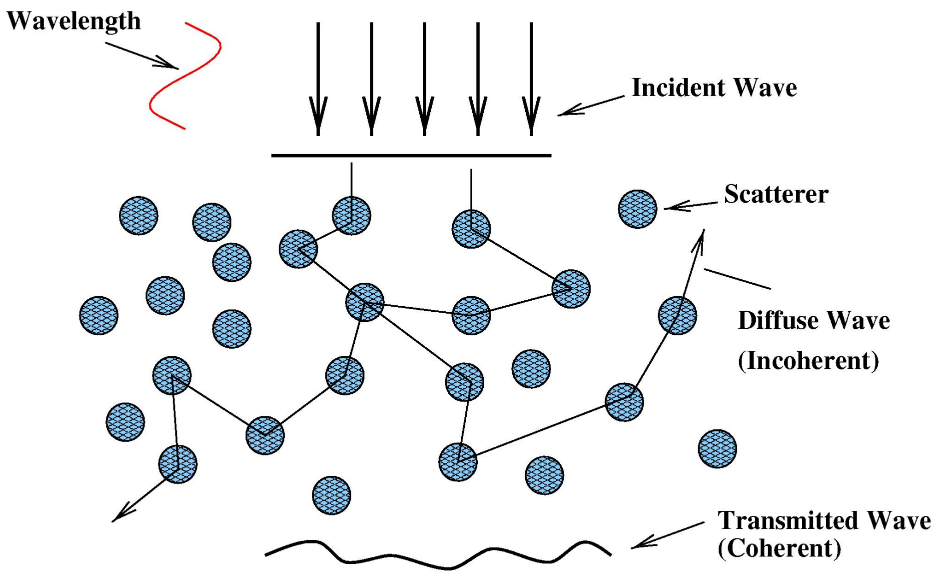
Fig. 15. Contour map of coda Q_0 for the entire continental United States.

Tectonically active western US: Qc=100-300

Propagation regimes and description of energy



Wave Propagation through Random Media



Length Scales: λ, Correlation Length, Propagation Distance

Question: Ensemble Average Response?

Precise Definition of Coherent and Incoherent Waves

The Concept of Mean Free Path

First Moment of the Green Function:

- Dyson Equation

$$\langle G \rangle = G_0 + G_0 M \langle G \rangle$$

  M, Mass Operator Describes all Possible Scattering Situations

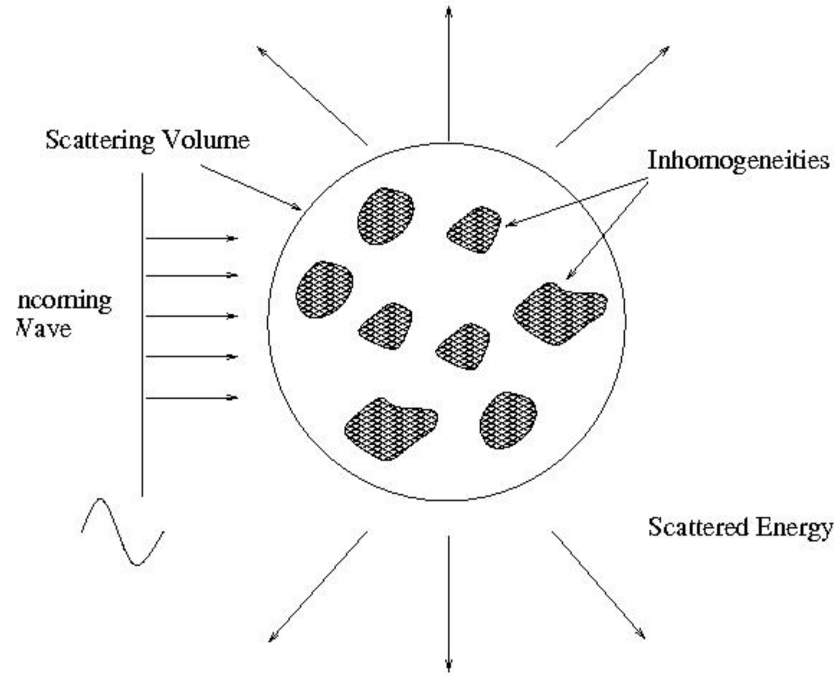
- Approximate Solution

$$\langle G(\vec{r}; \vec{r_0}) \rangle = -\frac{1}{4\pi |\vec{r} - \vec{r_0}|} e^{ik|\vec{r} - \vec{r_0}|}$$

$$k = k_0 + \frac{i}{2l}$$

$\langle G \rangle$: Coherent Field

New Length Scale: Mean Free Path of Waves $l = f(\epsilon, a, \lambda)$

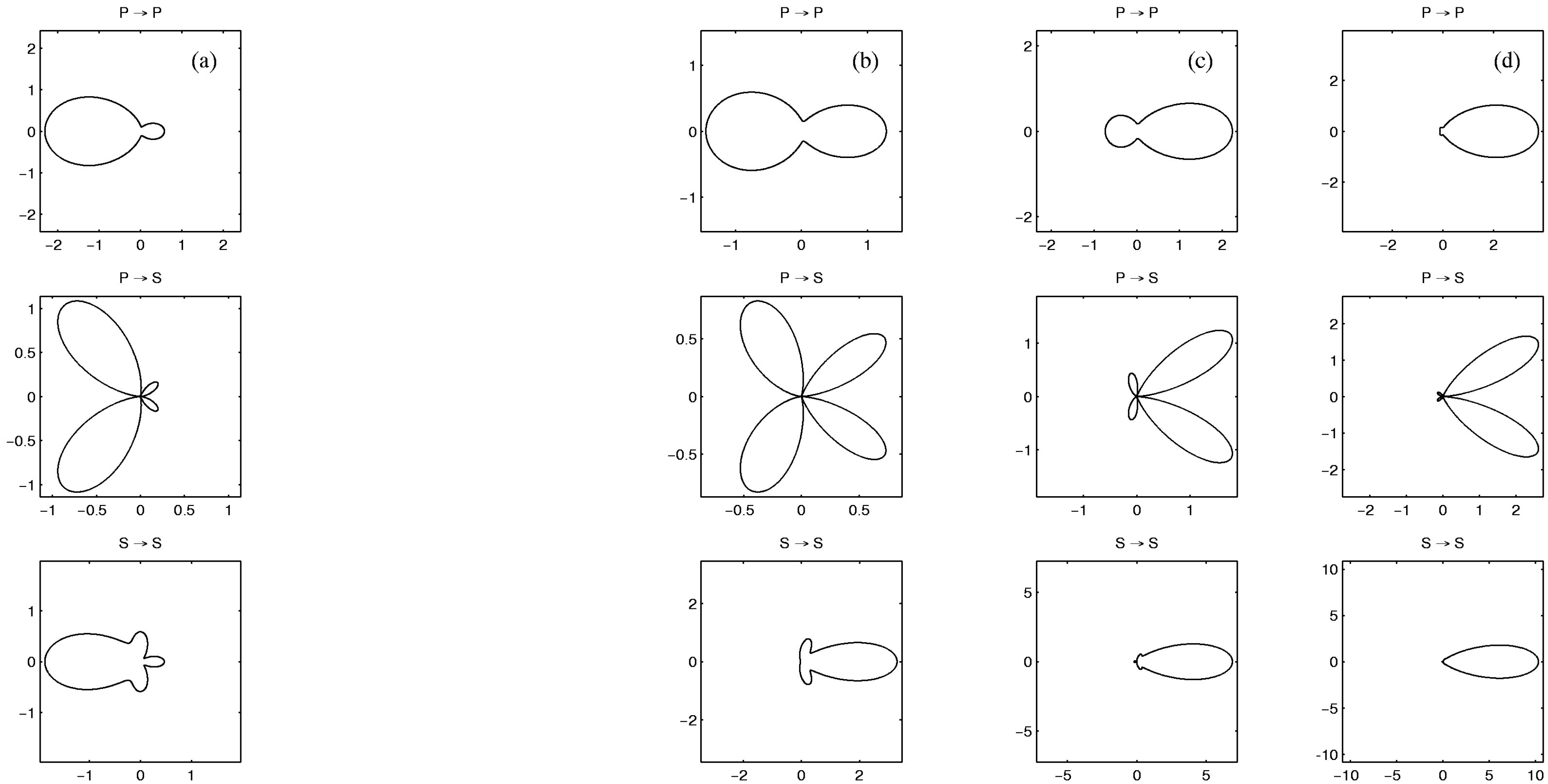


$$\frac{1}{l} = \frac{\text{Total Scattering Cross-Section}}{\text{Scattering Volume}}$$

Frequency Dependence:

- Rayleigh ($ka \ll 1$): $l \sim \omega^{-4}$
- High-Frequency ($ka > 1$): $l \sim \omega^{-2}$

Differential cross sections of scattering and conversion for a sphere of radius a



ka→0 : Rayleigh approximation

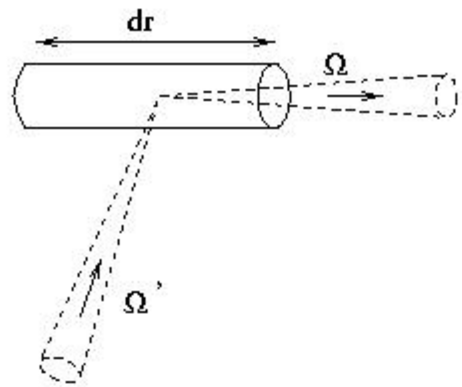
ka=1.2

ka=1.6

ka=2.0

(averaged in φ for S polarisation)

Energy balance of a beam of energy propagating a distance dr in the scattering medium



Variation of Intensity

$=$

Loss due to scattering into all space directions

$+$

Gain due to scattering from direction $\vec{\Omega}'$ to direction $\vec{\Omega}$

The Equation of Radiative Transfer

Second moment of the Green's function is governed by the Bethe-Salpeter equation:

$$\langle GG^\star \rangle = \langle G \rangle \langle G^\star \rangle + \langle G \rangle \langle G^\star \rangle K \langle GG^\star \rangle$$

K, Intensity Operator describes all scattering situations.

Neglecting recurrent scattering leads to:

$$\partial_t I(t, \vec{\Omega}, \vec{r}) + \vec{\Omega} \cdot \vec{\nabla}_r I(t, \vec{r}, \vec{\Omega}) \quad = \quad -\frac{1}{l} + \frac{1}{4\pi l} \int d\vec{\Omega}' I(t, \vec{\Omega}', \vec{r}) P(\vec{\Omega}, \vec{\Omega}')$$

Describes the transport of the incoherent part of the intensity.

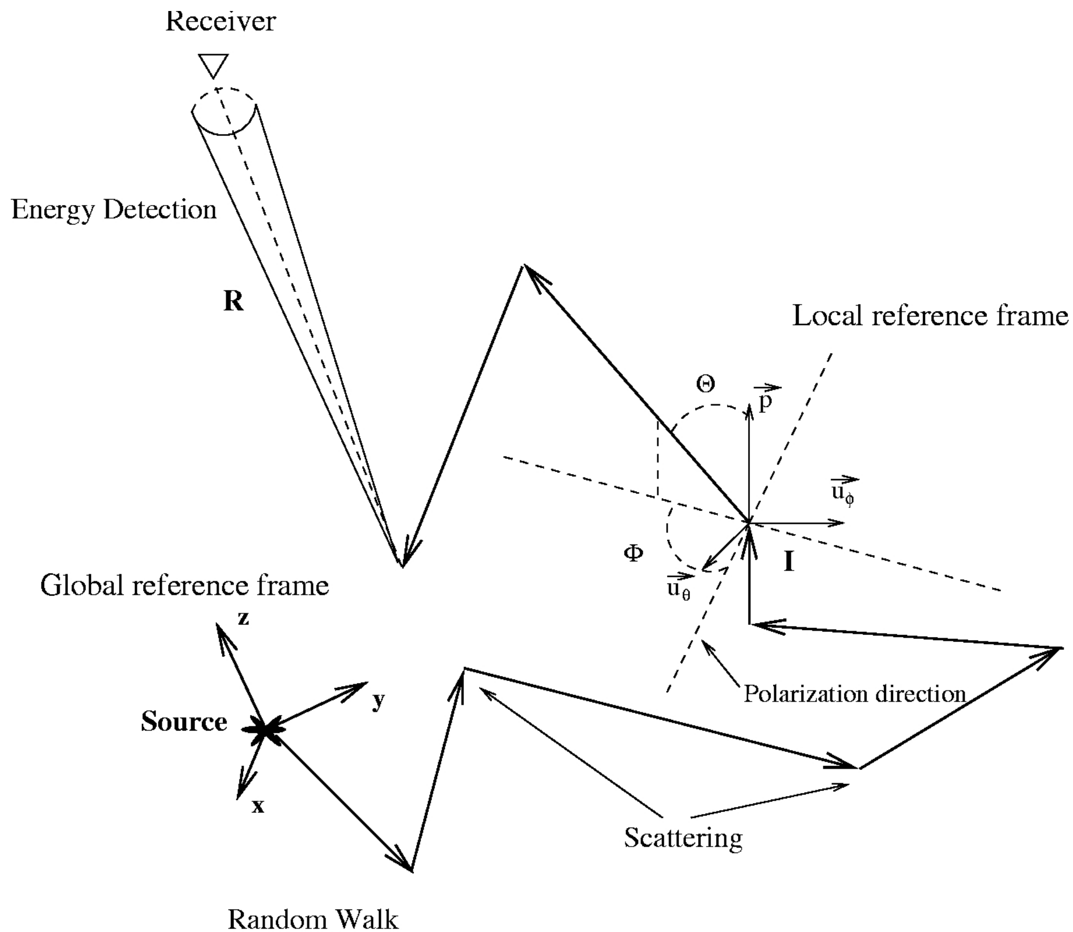
I, Specific Intensity function of space direction, time and position

$P(\vec{\Omega}, \vec{\Omega}')$, phase function (matrix) related to the power spectrum of the inhomogeneities

The radiative transfer equation

« particle analogy »

propagation under the ray theory assumptions



parameters: l_P, l_S ..., differential cross-sections

Single Scattering Approximation

The waves interact only once with the medium inhomogeneities

First term of an expansion of the intensity in a multiple scattering series:

$$I = I^0 + I^1 + \cdots + I^n + ...$$

I^0 : Coherent Intensity
I^n : Mean intensity of waves that have been scattered n times

$$I^1 \sim \frac{l}{t^2} e^{-vt/l}$$

When $vt \ll l$ reduces to the Born Approximation

The Diffusion Approximation

General Idea:

- Each scattering distributes energy over all space directions
- After several scatterings the intensity becomes almost isotropic

$$I(t, \vec{r}, \vec{\Omega}) = \text{Angularly Averaged Intensity} + \text{constant} \times \vec{J}(t, \vec{r}) \cdot \vec{\Omega}$$

The current density $\vec{J}(\vec{r}, t)$, points in the direction of maximum energy flow.
Integrating the RT Eq over all space directions leads to:

$$\partial_t \rho(t, \vec{r}) - D\nabla^2 \rho(t, \vec{r}) = \delta(t, \vec{r})$$

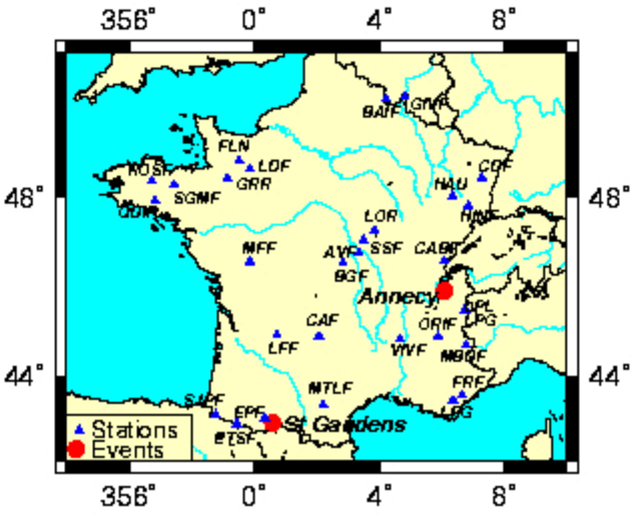
where rho is the local energy density.

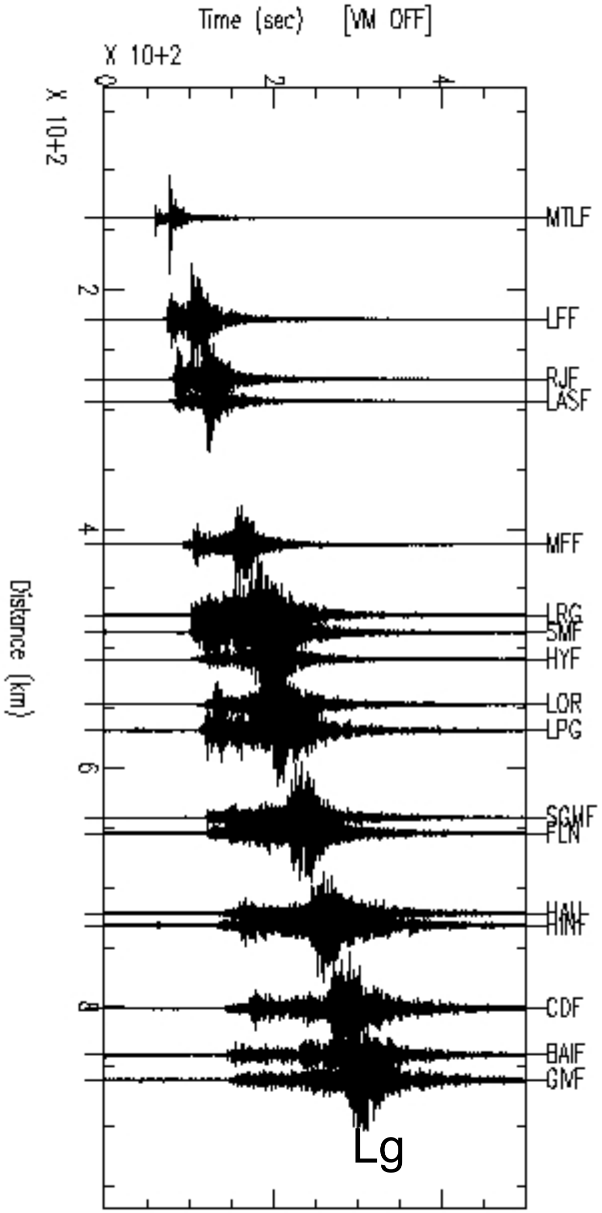
$$\rho(\boldsymbol{r}|\boldsymbol{r}', t) = \frac{1}{(4\pi D t)^{d/2}} e^{-|\boldsymbol{r}-\boldsymbol{r}'|^2/4Dt}$$

$\rho(t, \vec{r}) \sim \frac{1}{(Dt)^{3/2}}$ for large t.

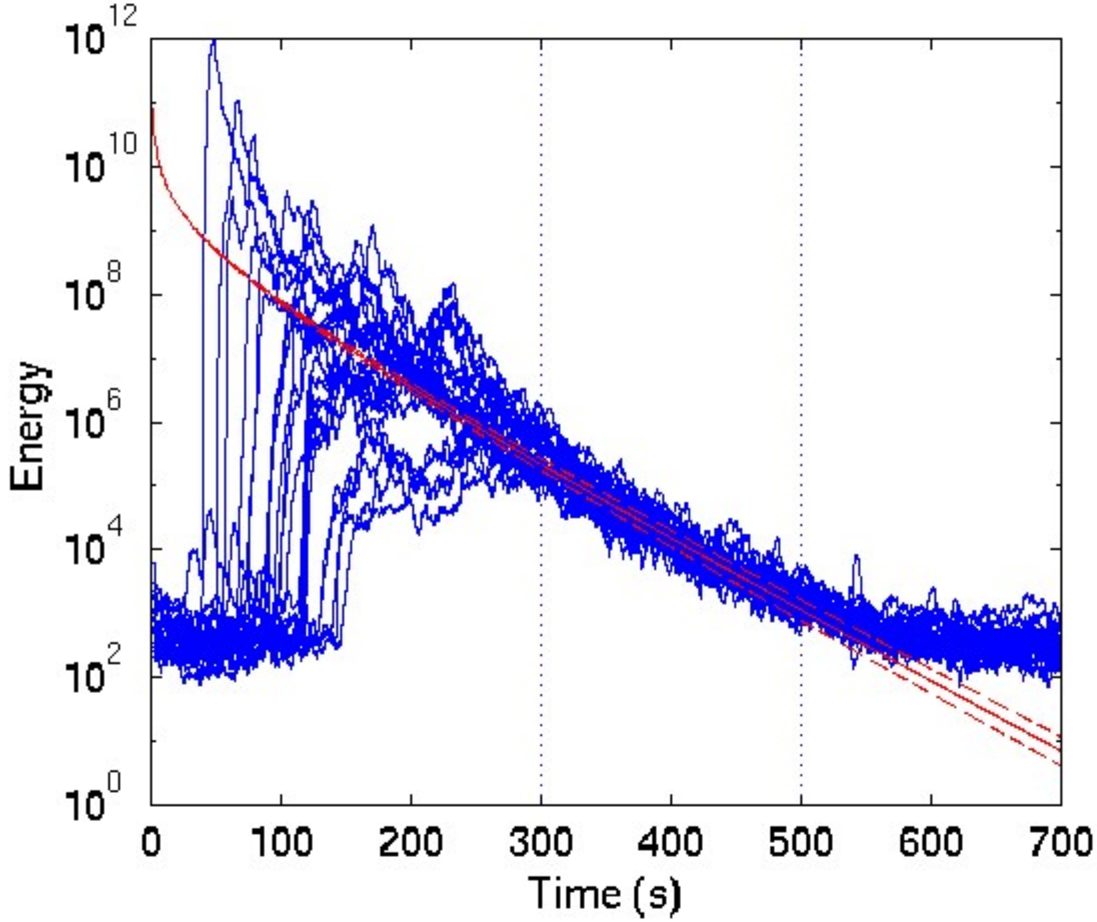
$D = vl/3$ is the diffusion constant of the waves.

Regional seismograms

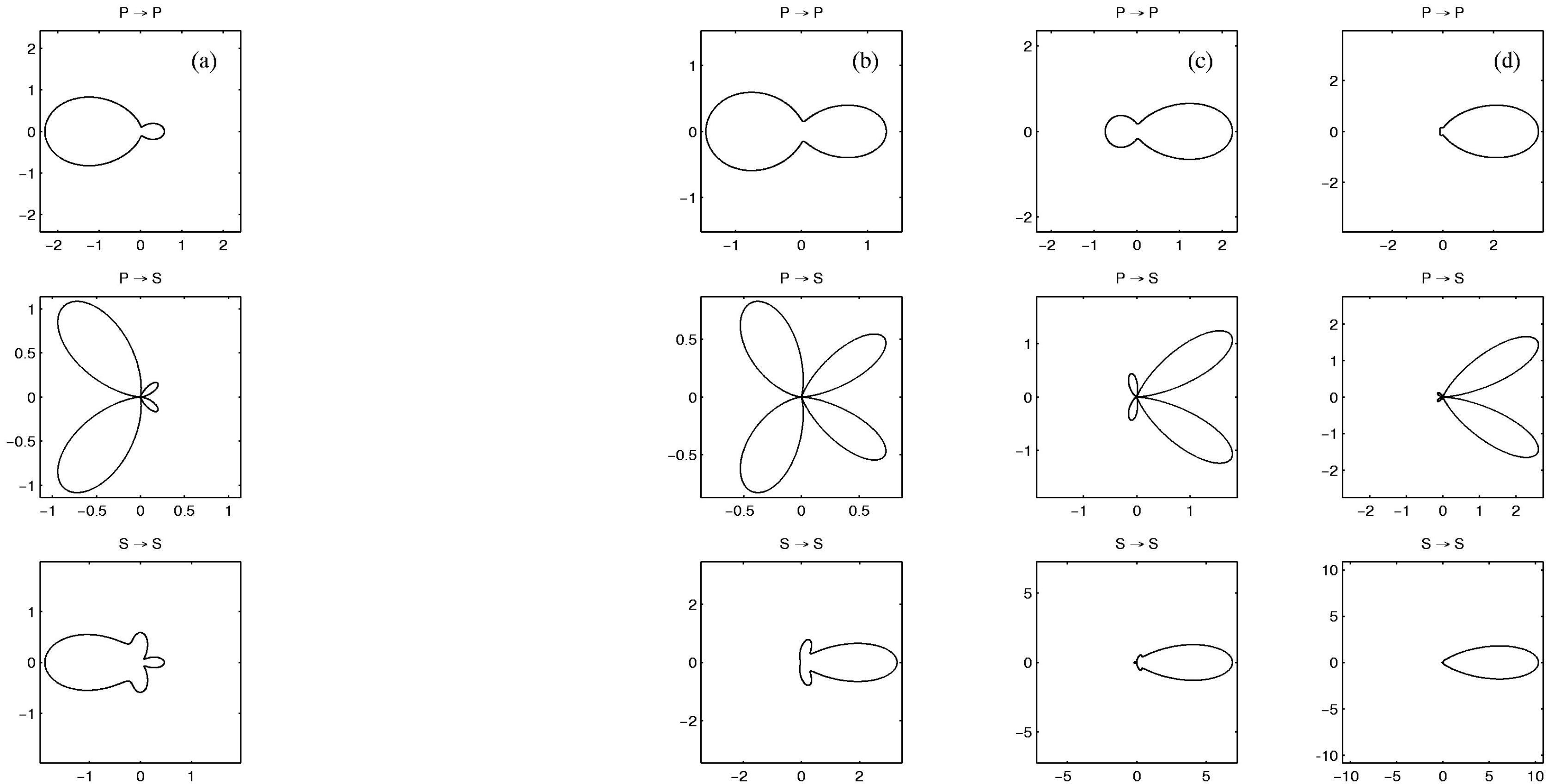




Observations at distances between 150 and 800 km!!



Differential cross sections of scattering and conversion for a sphere of radius a



ka→0 : Rayleigh approximation

ka=1.2

ka=1.6

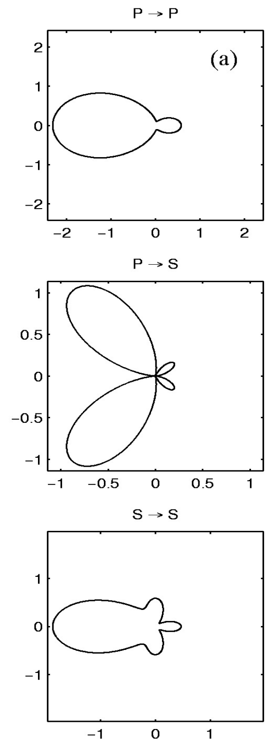
ka=2.0

(averaged in φ for S polarisation)

Because of the Reciprocity theorem, the scattering tends naturally to favour S waves

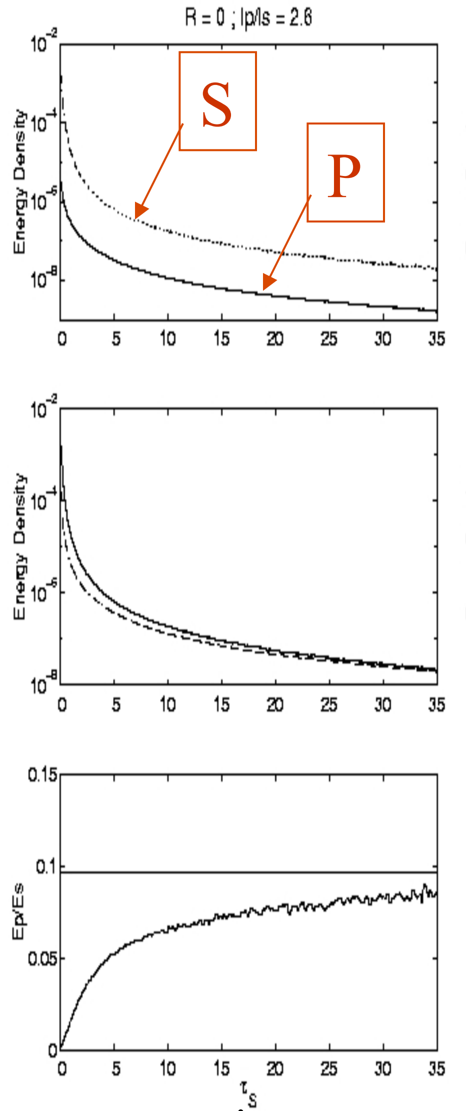
$$\frac{g_{PS}}{g_{SP}} = 2\frac{\alpha^4}{\beta^4}$$

Cross sections



ka→0 : Rayleigh approximation

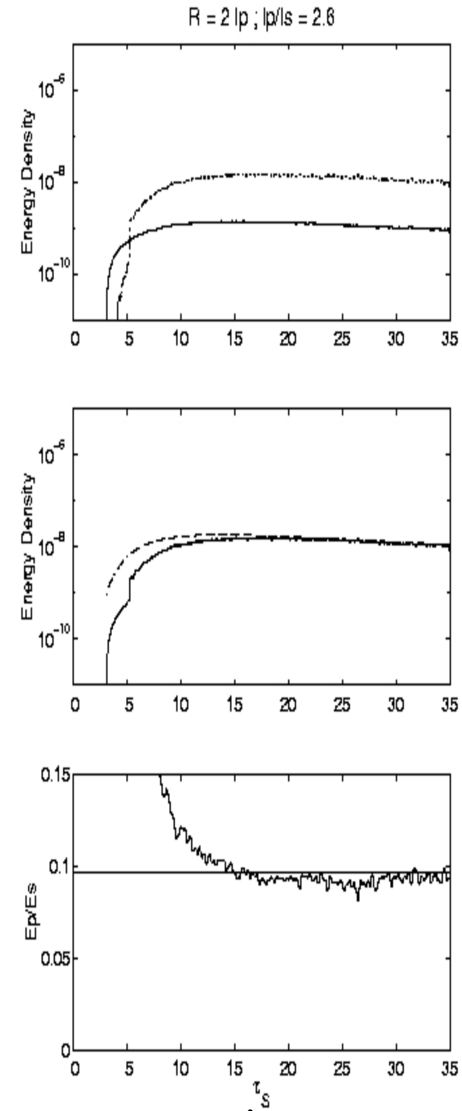
Numerical solutions of the RTE



time

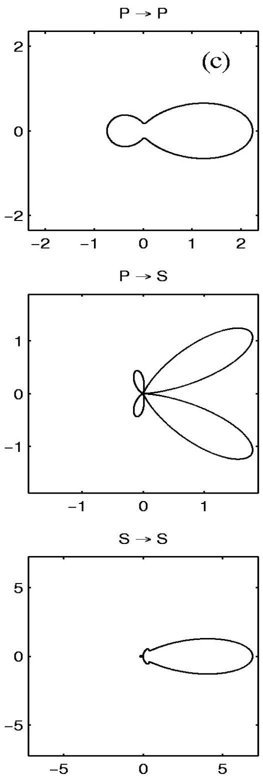
total energy
vs
diffusion app.

Energy ratio

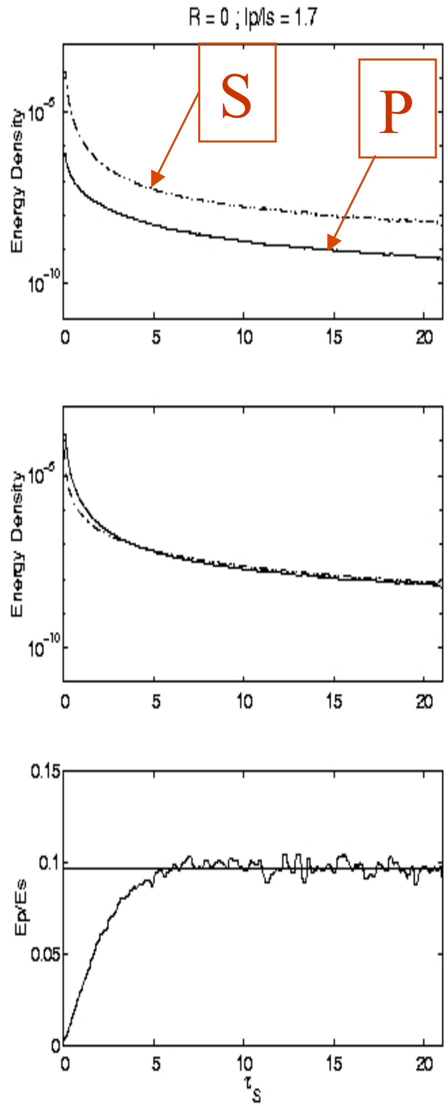


time

Cross sections



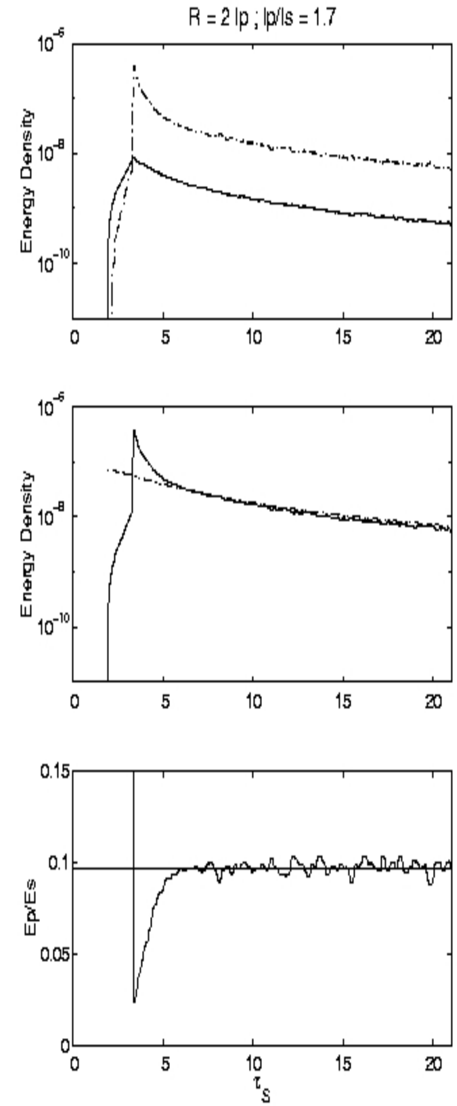
ka=1.6



time

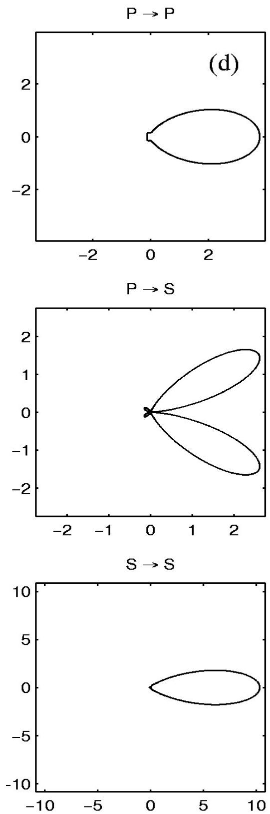
total energy
vs
diffusion app.

Energy ratio

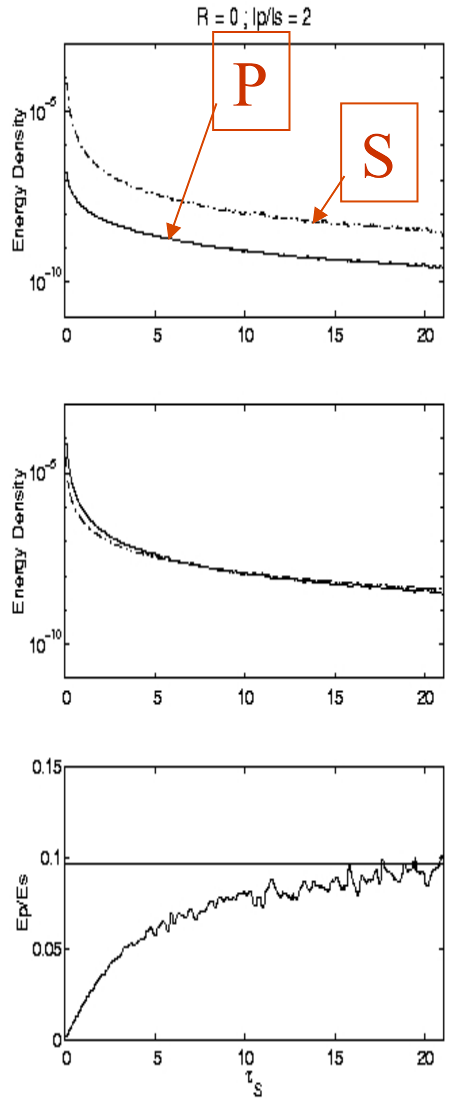


time

Cross sections



ka=2.0



time

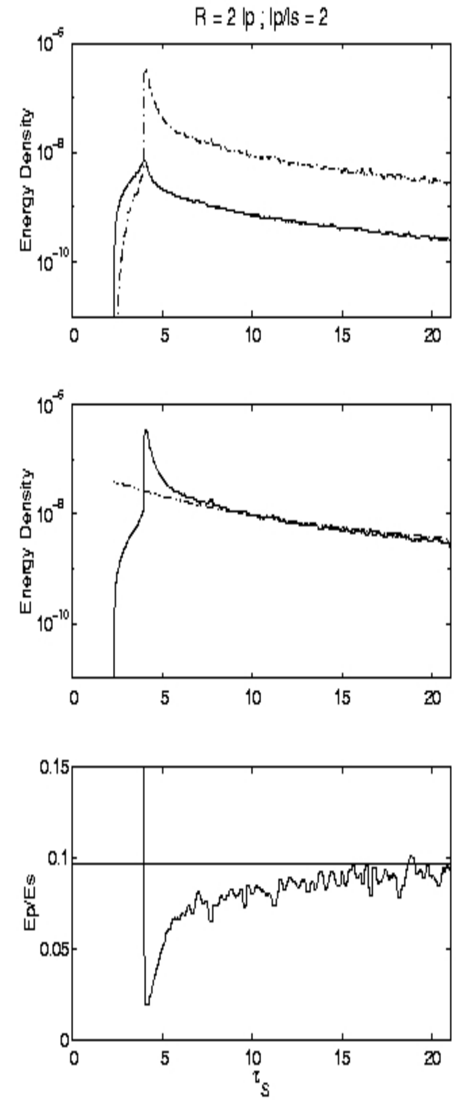
total energy

vs

diffusion app.

Energy ratio

is the same for different values of ka



time

S to P Energy ratio as a marker of the regime of scattering...

Equipartition Principle for Waves

Weaver, 1982:
In a diffuse field, all the modes are excited to equal energy

$$G_{i,j}(\vec{R},\vec{S},t)=\sum_n \varepsilon_n \Phi^n(\vec{R})\exp(-i\Omega_n t)$$

where ε_n are random independent variables (finite body)

Consequence for an infinite inhomogeneous solid:

$$\frac{E_s}{E_p} = 2\left(\frac{v_p}{v_s}\right)^3$$

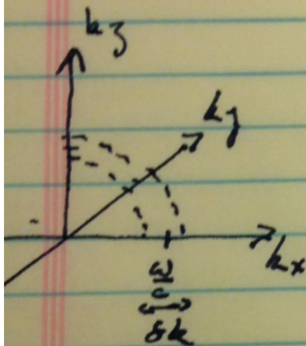
Independent of the Details of the Scattering !

Independent of the position in a full space with homogeneous reference

Partition of energy (Full elastic space)

Multiple scattering, large t
$\rightarrow$ "equipartition"
[reference medium + disorder]

Phase space of the full space elastic problem
$\rightarrow$ all propagating plane waves excited at same level of energy



Energy in a band $\omega \pm \frac{\delta\omega}{2}$

$\rightarrow$ Volume for P waves

$$\delta k = \frac{\delta\omega}{\alpha}$$

$$V_P = 4\pi \left(\frac{\omega}{\alpha}\right)^2 \frac{\delta\omega}{\alpha} = 4\pi\, \delta\omega\, \omega \frac{1}{\alpha^3}$$

Volume for each S polarisation: $V_{SH/SV} = 4\pi\, \delta\omega\, \omega \frac{1}{\beta^3}$

$$\Rightarrow V_S = 2 \times 4\pi\, \delta\omega\, \omega \frac{1}{\beta^3}$$

Equal excitation $\Rightarrow \frac{E_S}{E_P} = \frac{V_S}{V_P} = 2\frac{\alpha^3}{\beta^3}$

[Note $\lambda = \mu \Rightarrow \frac{E_S}{E_P} \sim 10.4 \Rightarrow$ see numerical simulation

Energy in an Elastic Solid

$$E = K + P + S + I$$

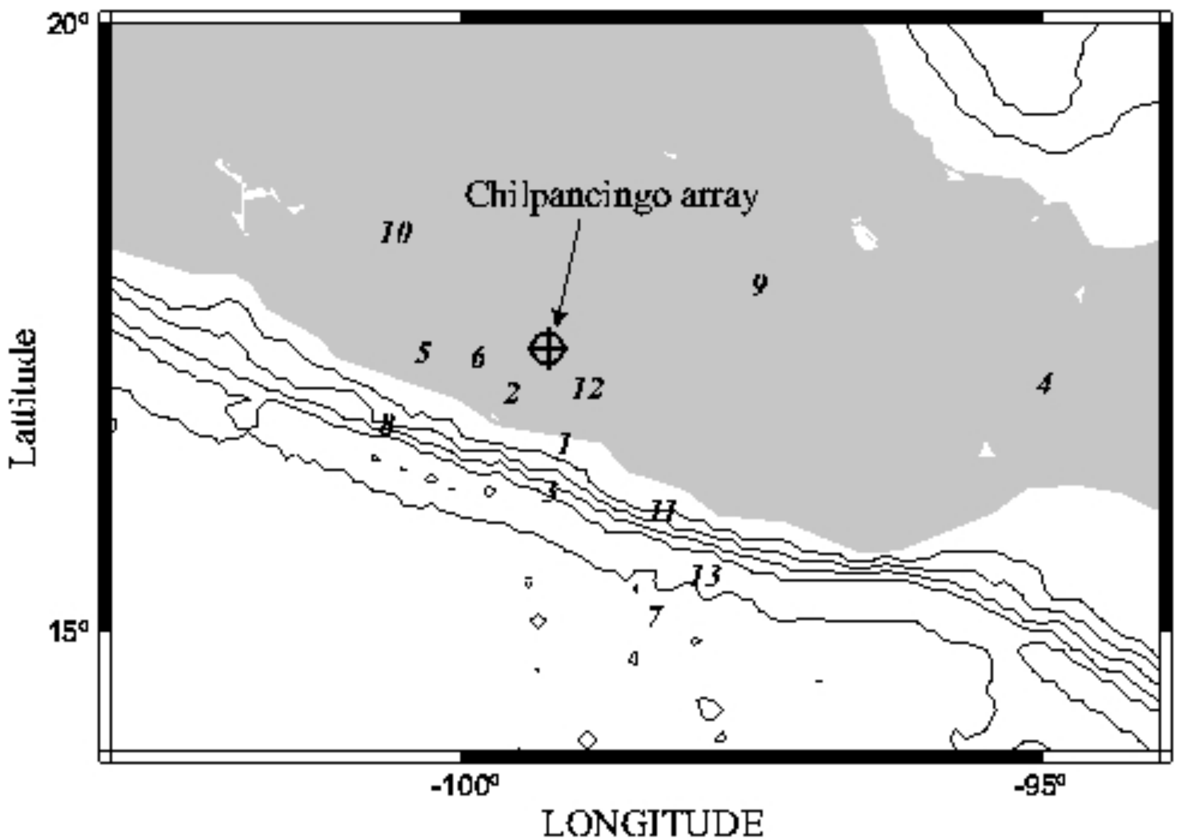
$$E = \frac{1}{2}\rho(\partial_t \mathbf{u})^2 + (\frac{\lambda}{2} + \mu)(\mathbf{div u})^2 + \frac{\mu}{2}(\mathbf{curl\ u})^2 + I$$

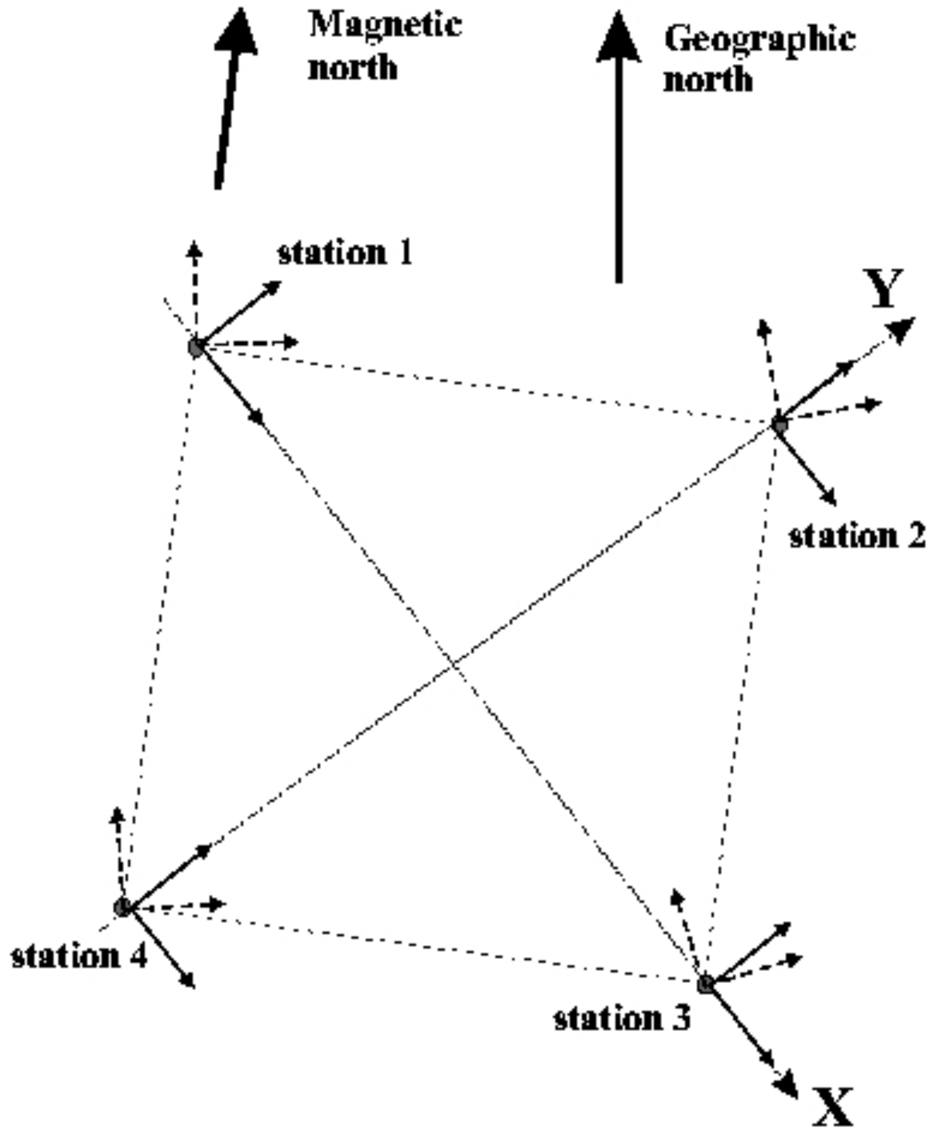
I contains mixed partial derivatives
$K = H^2 + V^2$

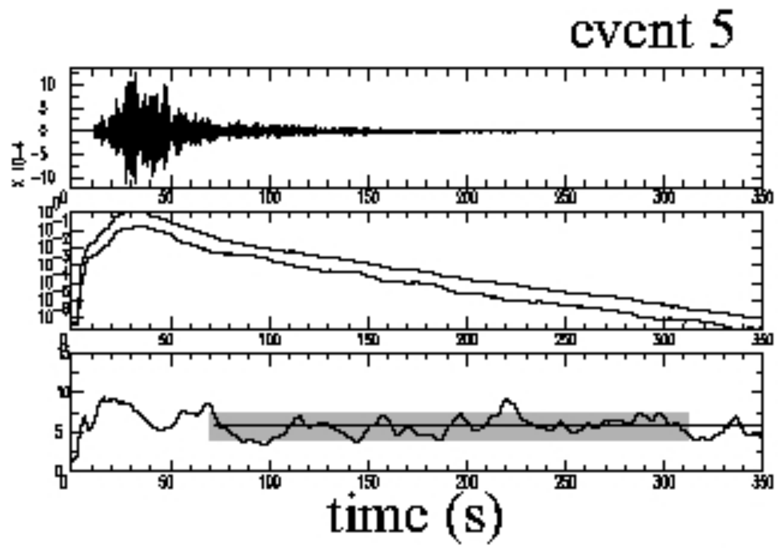
Focus on the ratios:
P/S, $K/(P+S)$,$I/(S+P)$,H^2/V^2

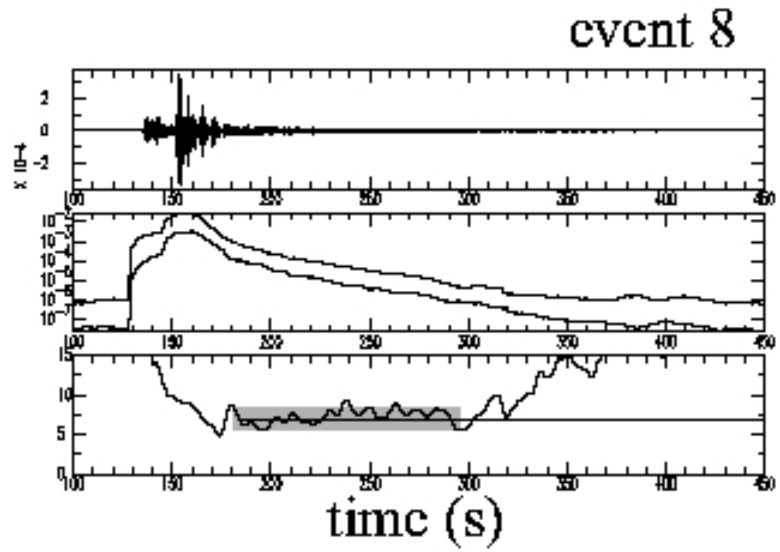
Equipartition predicts: Any Ratio of Energies Becomes Independent of Time

Measurement of the deformation energy requires evaluation of partial derivatives of the wavefield

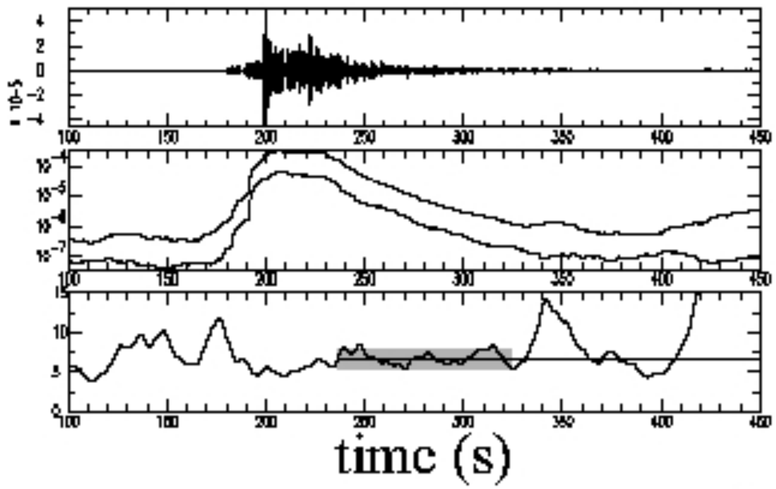

20°
15°
-100°
-95°
LONGITUDE
Latitude
Chilpancingo array
10
9
5
6
2
12
4
8
1
3
11
13
7


Magnetic north
Geographic north
station 1
Y
station 2
station 4
station 3
X

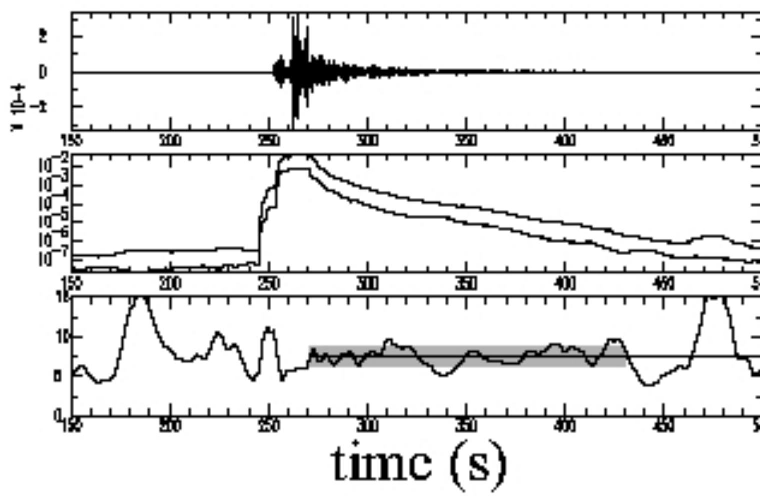

event 5
time (s)


event 8
time (s)

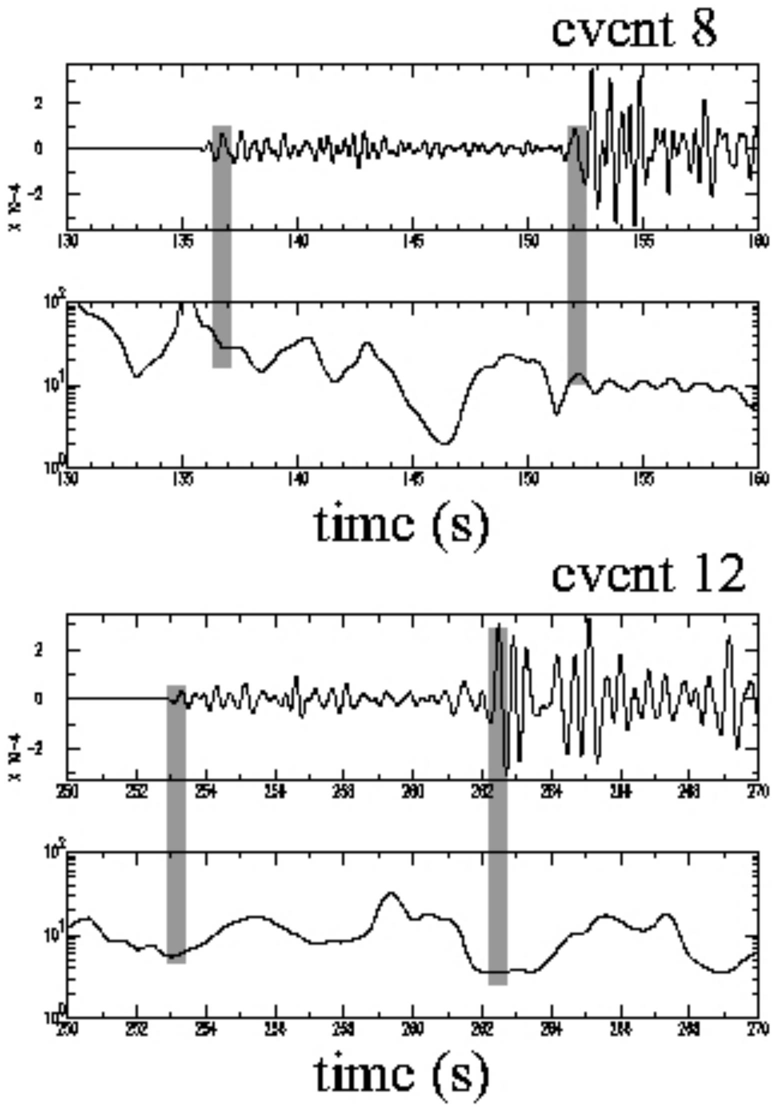
event 9

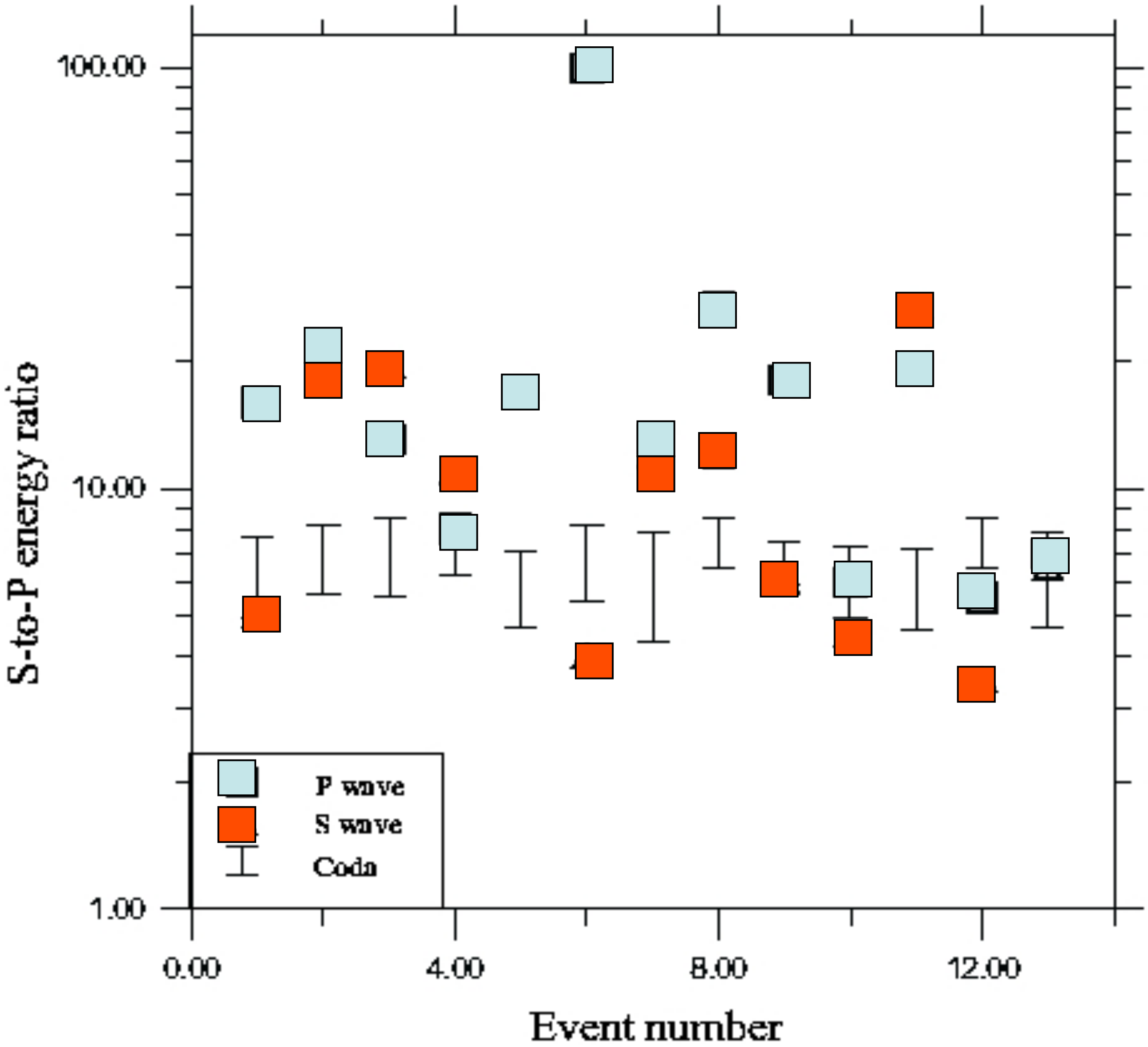


event 12



Direct waves





100.00
10.00
1.00
0.00
4.00
8.00
12.00
S-to-P energy ratio
Event number
P wave
S wave
Coda

ENERGY RATIO	**DATA**	**THEORY FULL SPACE**	**THEORY HALF SPACE BULK WAVES**	**THEORY HALF SPACE with RAYLEIGH WAVES**
S/P	7.3	10.39	9.76	7.19
K/(S+P)	0.65	1	1.19	0.534
I/(S+P)	-0.62	0	-0.336	-0.617

The Diffusion Approximation

General Idea:

- Each scattering distributes energy over all space directions
- After several scatterings the intensity becomes almost isotropic

$$I(t, \vec{r}, \vec{\Omega}) = \text{Angularly Averaged Intensity} + \text{constant} \times \vec{J}(t, \vec{r}) \cdot \vec{\Omega}$$

The current density $\vec{J}(\vec{r}, t)$, points in the direction of maximum energy flow.
Integrating the RT Eq over all space directions leads to:

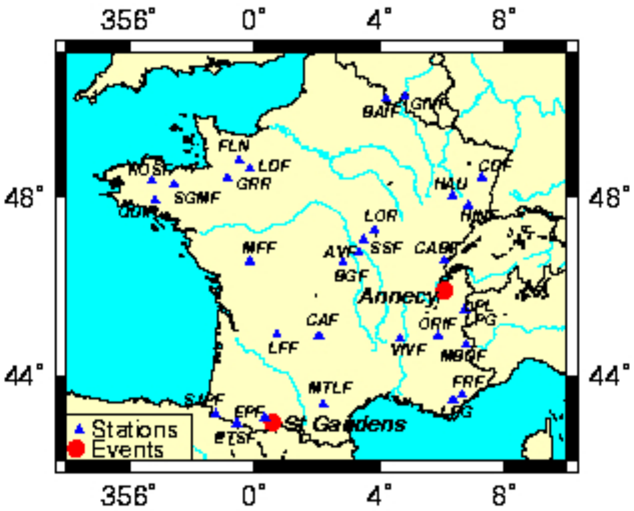
$$\partial_t \rho(t, \vec{r}) - D\nabla^2 \rho(t, \vec{r}) = S(t, \vec{r})$$

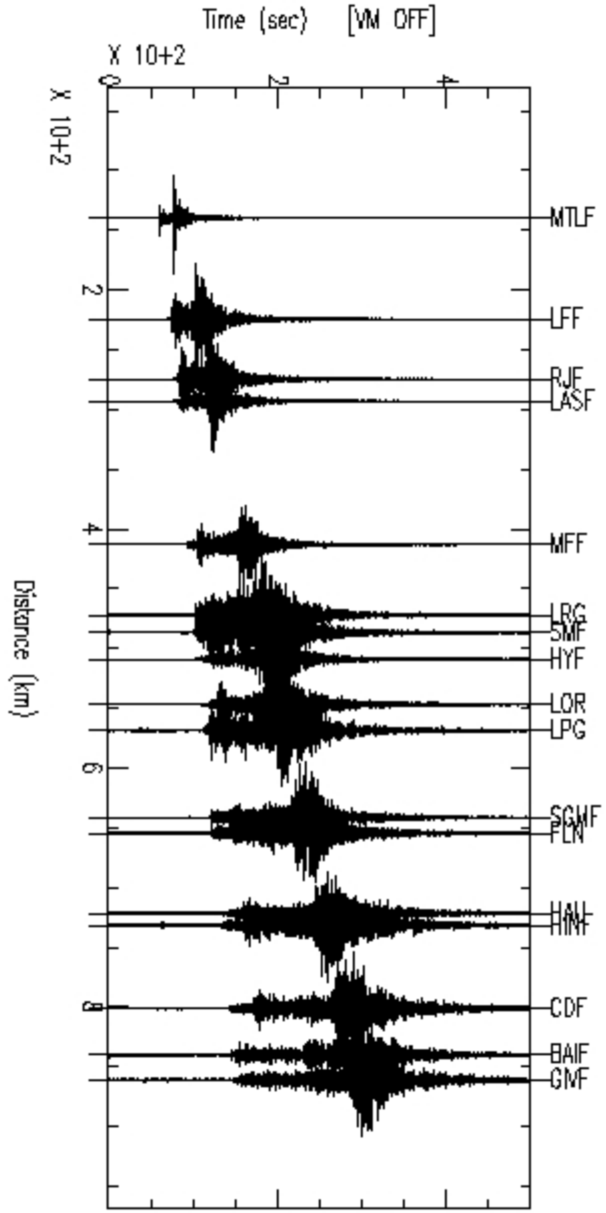
where rho is the local energy density.

$\rho(t, \vec{r}) \sim \frac{1}{(Dt)^{3/2}}$ for large t.

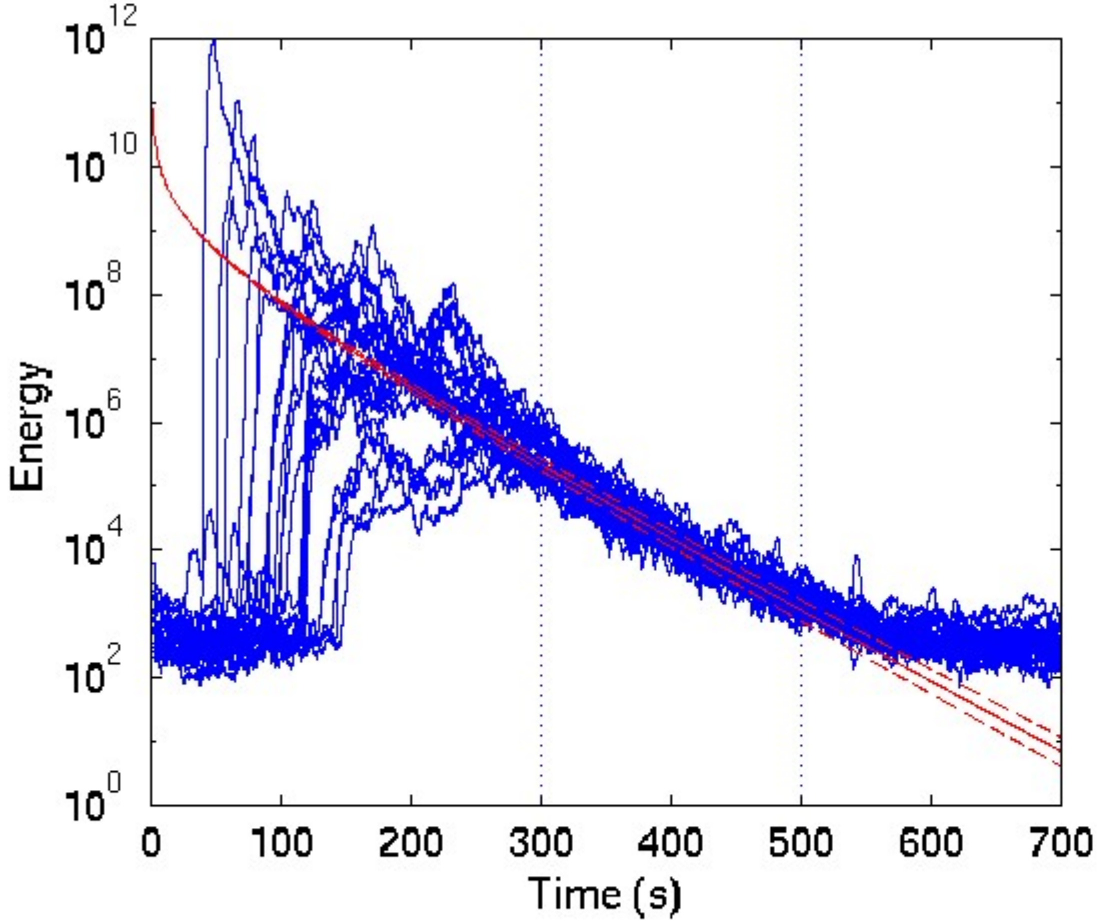
$D = vl/3$ is the diffusion constant of the waves.

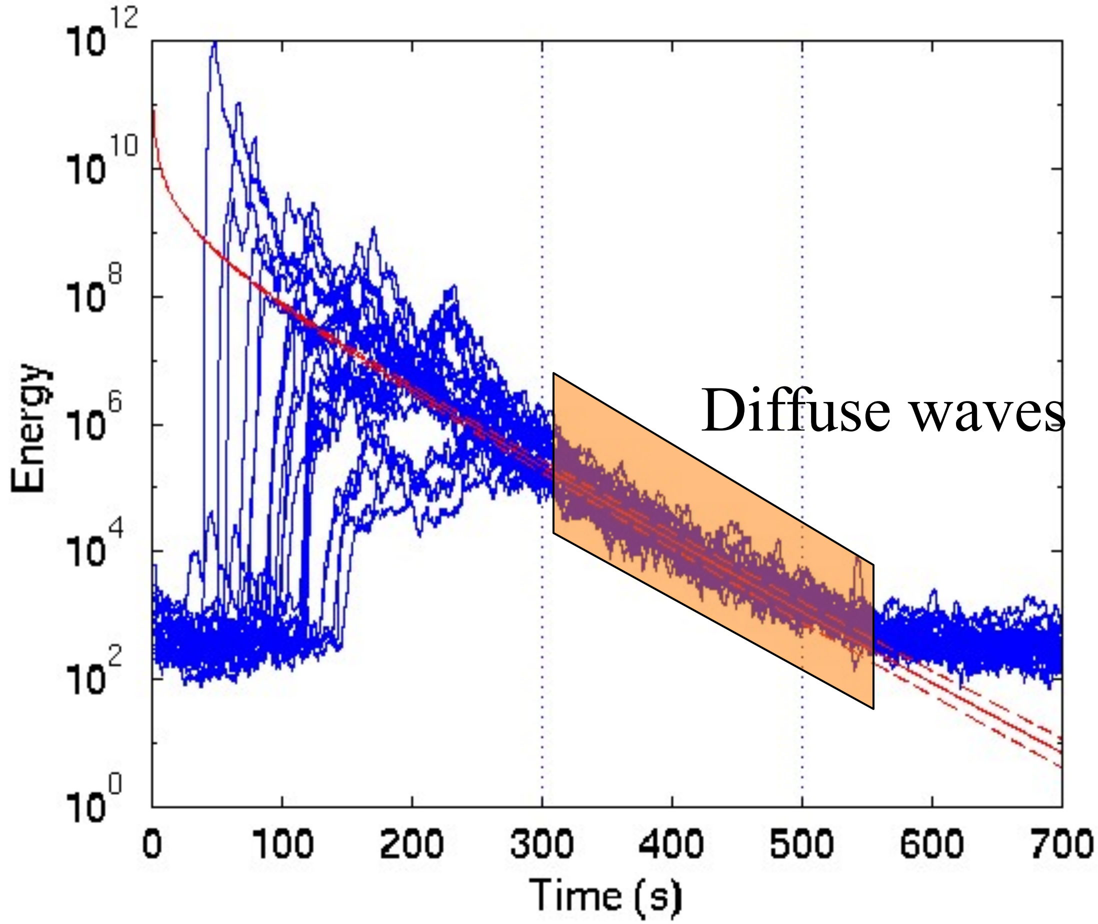
Coda of regional seismograms



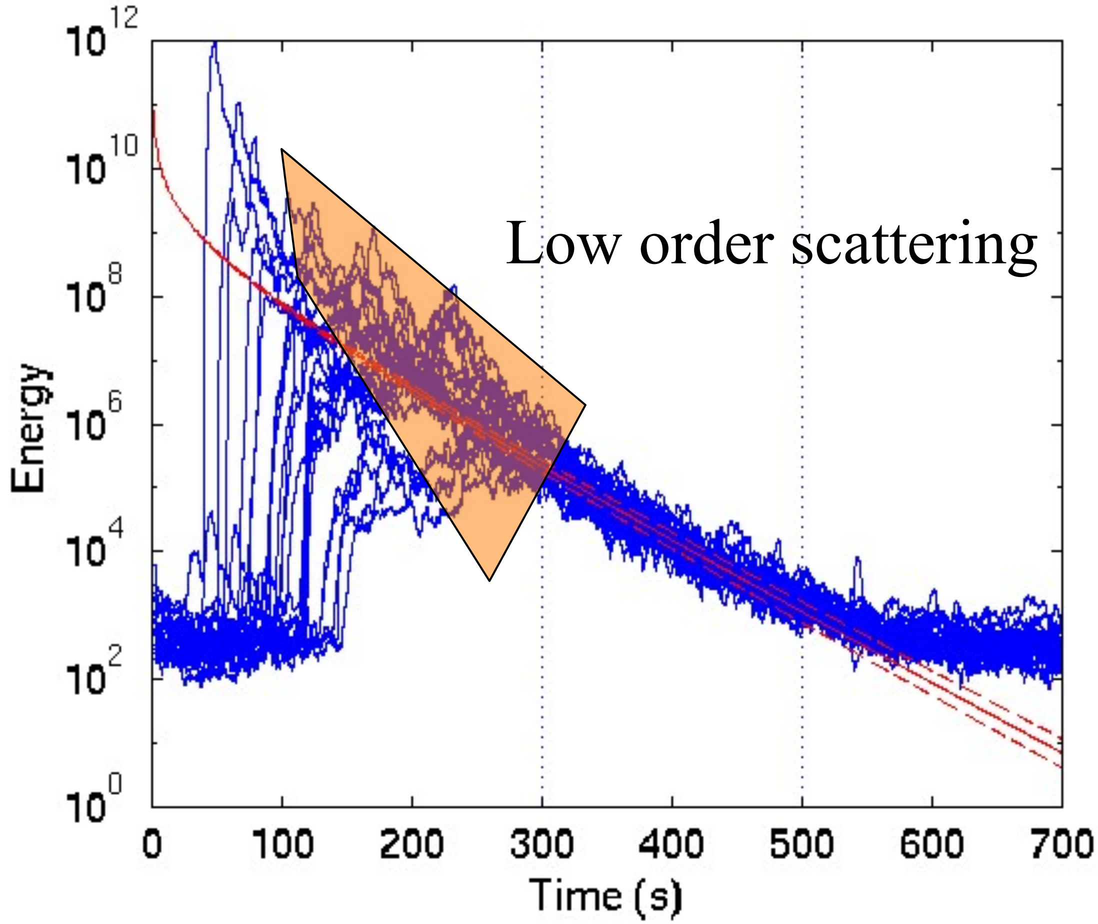


Observations at distances between 150 and 800 km!!



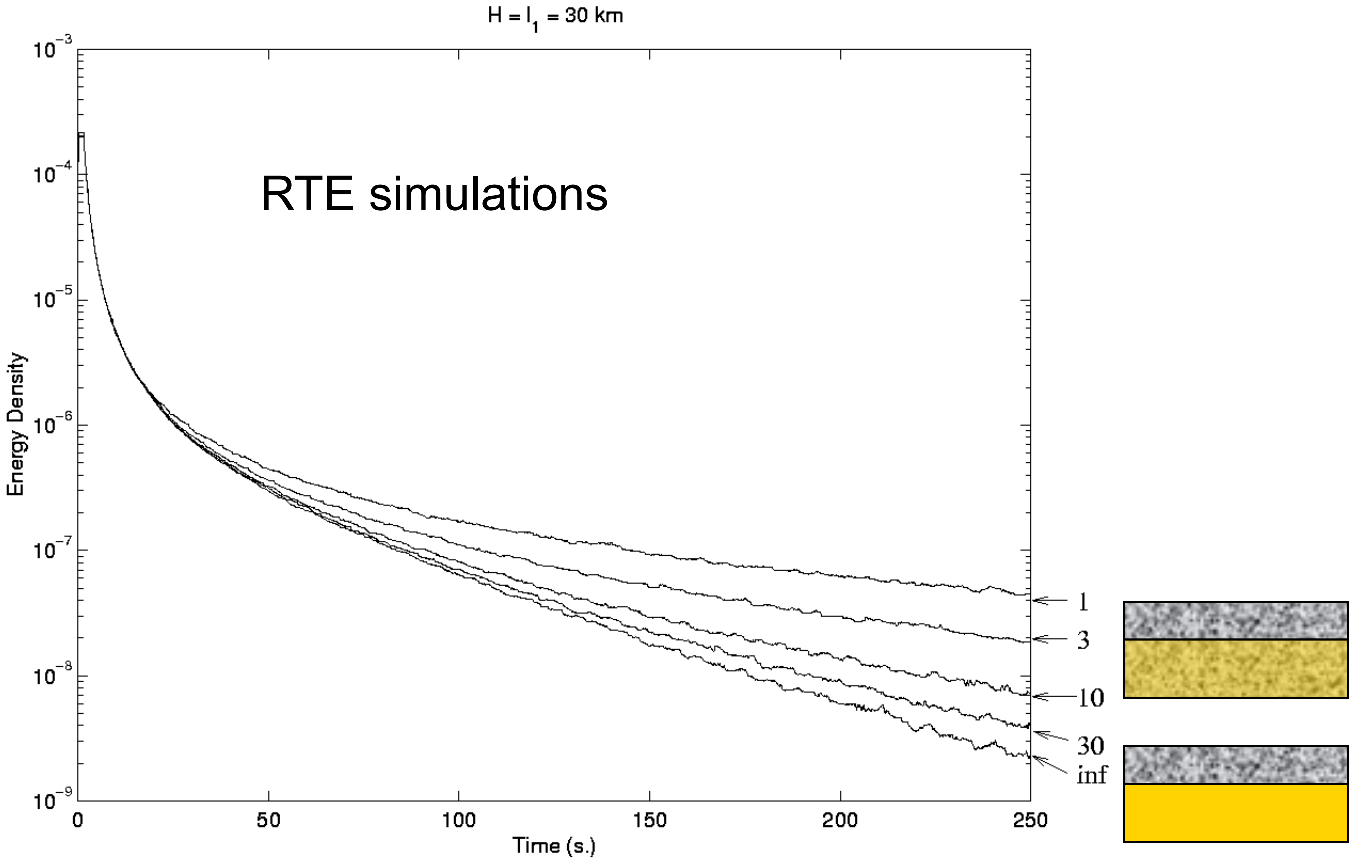


Diffuse waves
Energy
Time (s)
10^12
10^10
10^8
10^6
10^4
10^2
10^0
0
100
200
300
400
500
600
700

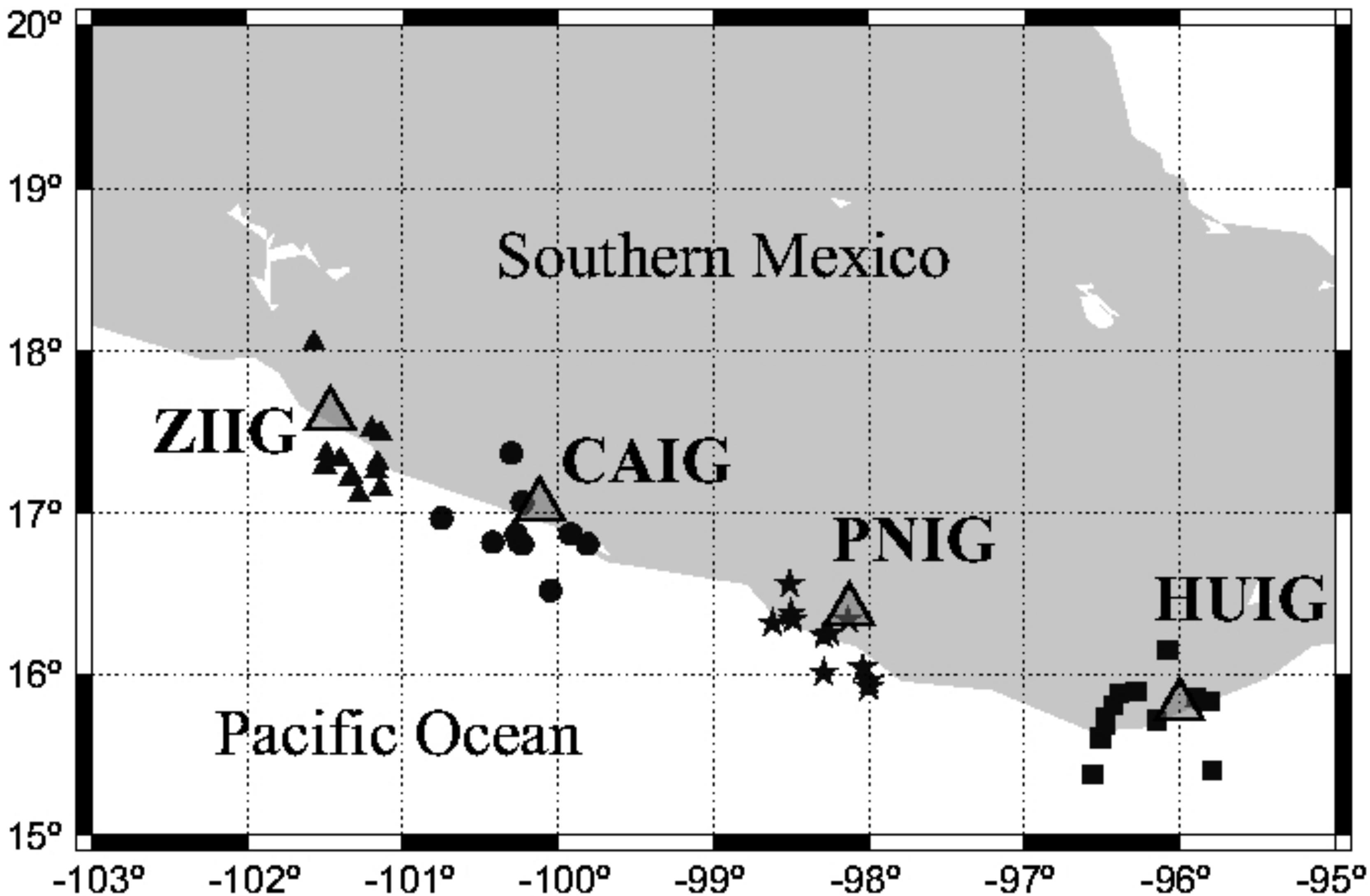


requires radiative transfer equation

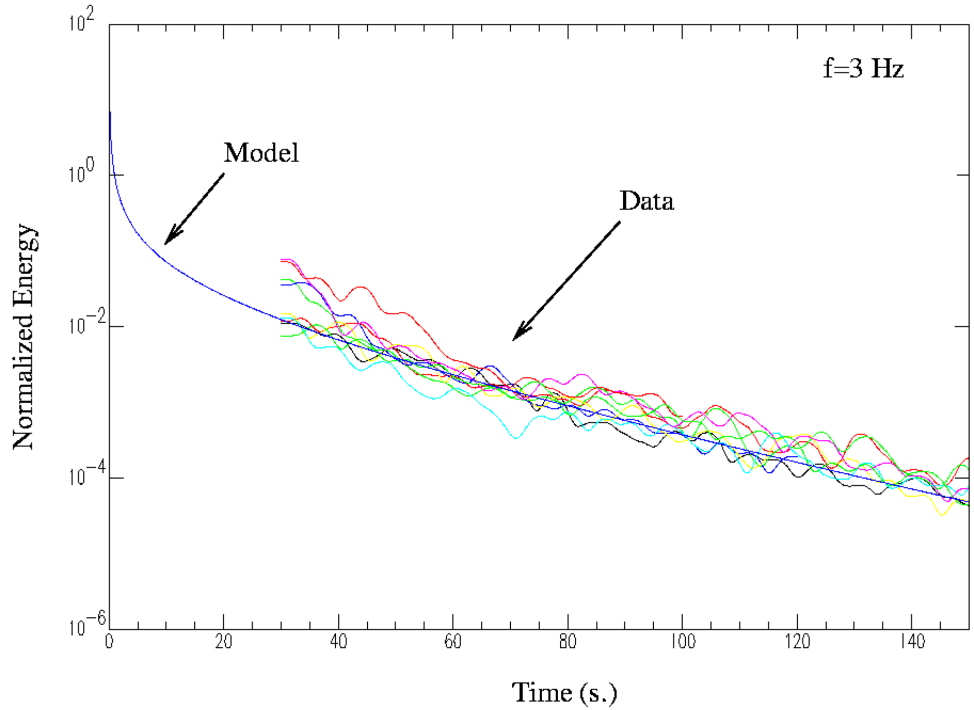
Influence of the value of mantle mean free path



Leakage of energy in the mantle



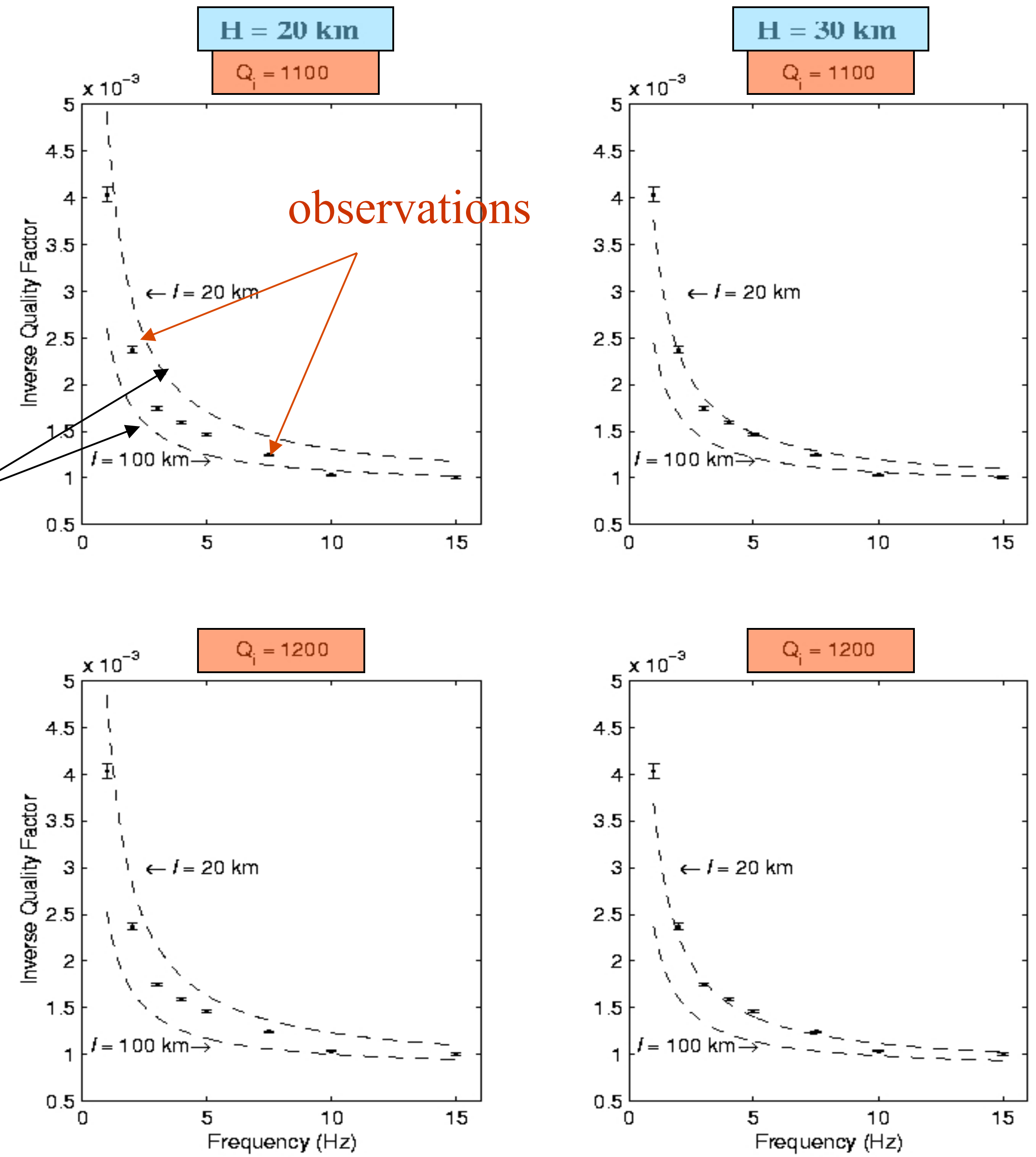
20°
19°
18°
17°
16°
15°
-103°
-102°
-101°
-100°
-99°
-98°
-97°
-96°
-95°
Southern Mexico
Pacific Ocean
ZIIG
CAIG
PNIG
HUIG



f=3 Hz
Model
Data
Normalized Energy
Time (s.)
10^2
10^0
10^−2
10^−4
10^−6
0
20
40
60
80
100
120
140

With dissipation

Q > 1000



RTE solutions

l(f) in the range 20-80 km

Energy decay in the coda (Aki and Chouet, 1975)