Accurate traveltime computation in complex anisotropic media with discontinuous Galerkin method

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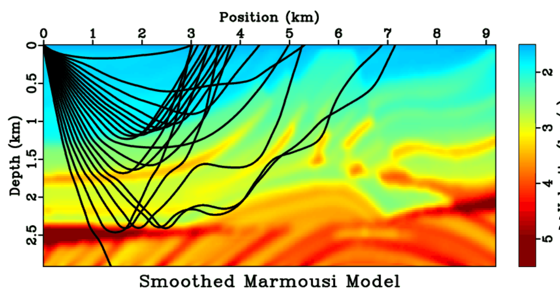
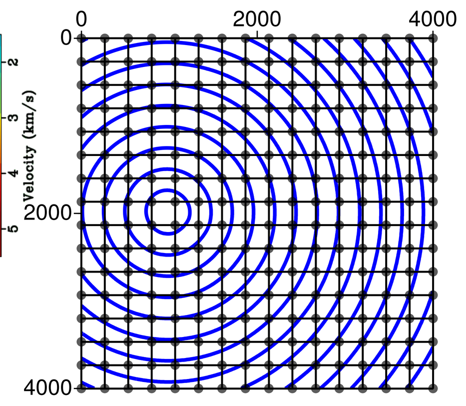
I - INTRODUCTION

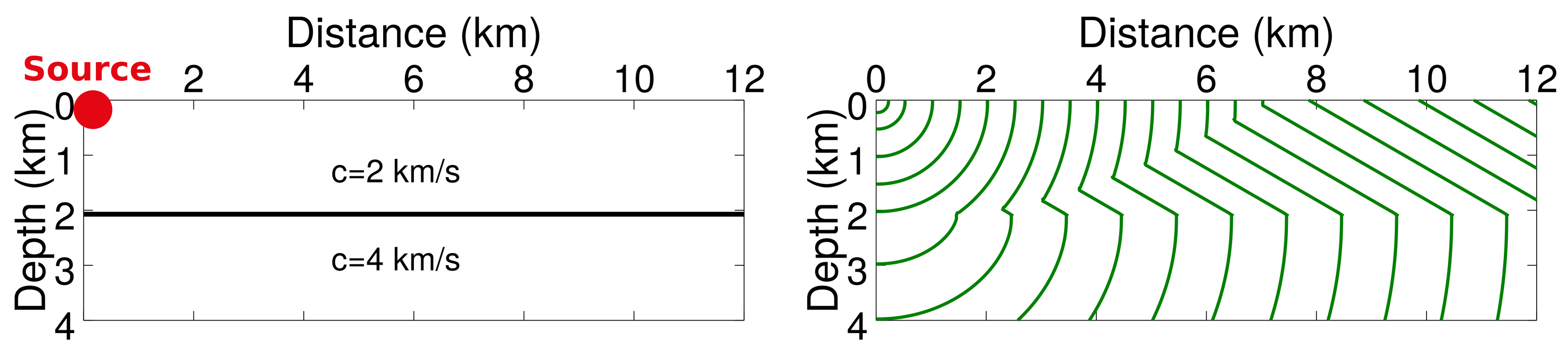
Seismic traveltimes and their spatial derivatives are key quantities for many geophysical applications:

- Source location
- Imaging and macromodel building prior to FWI/MVA:
  - * (stereo)tomography
  - * migration
- Data windowing
- Computation of seismograms





Ray-tracing methods are efficient in simple models but they are no longer reliable in complex media and/or when a uniform sampling of the medium is required, where **Eulerian methods perform better**.



Traveltime is continuous but singularities occur in the derivatives, even in smooth areas of the model, due to the non-linearity of the first-arrival Eulerian problem.

Existing Eikonal solvers:

- Mostly finite-difference based
- Mostly first-order convergent: high-order is costly
- Not suitable for topography handling
- Recently extended to anisotropy, either iteratively or by explicitly solving quartics

Main goal of this study:

- Design a highly accurate Eikonal solver suitable for anisotropy and topography handling.

Main ingredient:

- State-of-the-art discontinuous Galerkin techniques

II - EIKONAL

In an **isotropic medium**, Eikonal rules the behavior of traveltime T(**x**) with respect to the wave velocity c(**x**). The non-linear partial differential equation writes

$$\boxed{(\nabla T)^2(\mathbf{x}) = \frac{1}{c^2(\mathbf{x})}\ .}$$

In a 2D **transversely isotropic** (TI) medium, the Eikonal equation is derived from the Christoffel's equation and under the acoustic approximation, yielding

$$\boxed{aT_x^2T_z^2 + bT_x^2 + cT_z^2 - 1 = 0,}$$

with 3 parameters related to the Thomsen's parameters of the medium:

$$a(\mathbf{x}) = -2V_p^4(\epsilon-\delta),\ b(\mathbf{x}) = (1+2\epsilon)V_p^2,\ c(\mathbf{x}) = V_p^2.$$

The **tilted** (TTI) case is obtained by introducing the local tilt angle θ through the local rotation

$$T_x \leftarrow T_x\cos\theta + T_z\sin\theta,$$
$$T_z \leftarrow T_z\cos\theta - T_x\sin\theta.$$

Traveltimes are computed as the steady-state solution of a dynamic Hamiltonian formulation (Osher (1993)). The generic Hamilton-Jacobi equation writes

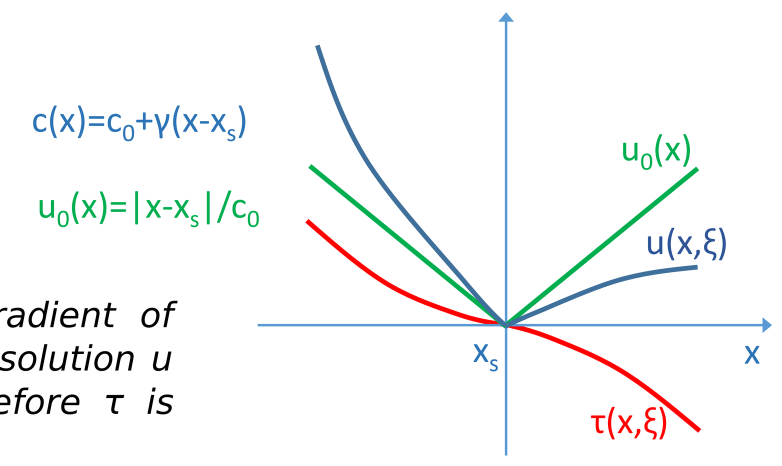
$$\frac{\partial u}{\partial \xi} + \mathcal{H}(\nabla u, \mathbf{x}) = 0.$$

The **additive factorization** is performed in order to reduce the point-source effect and gain accuracy. The numerical solution is decomposed into two additive factors:

- a suitable reference solution u0(x),
- the numerical perturbation τ(x,ξ) which has to satisfy the factored Hamilton-Jacobi equation.

The point-source singularity is thus carried by the reference solution. The perturbation is smooth at the source point and the source no longer pollutes the numerical solution.

In a 1D medium with a constant gradient of velocity, the upwind singularity of the solution u at the source is carried by u_0. Therefore τ is smooth at the source.

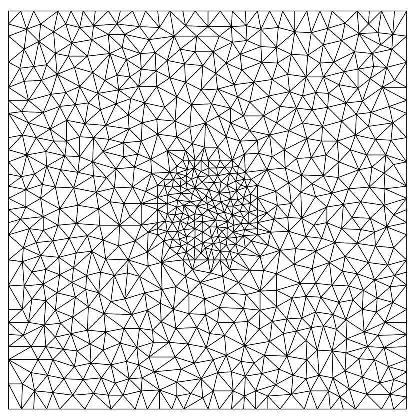


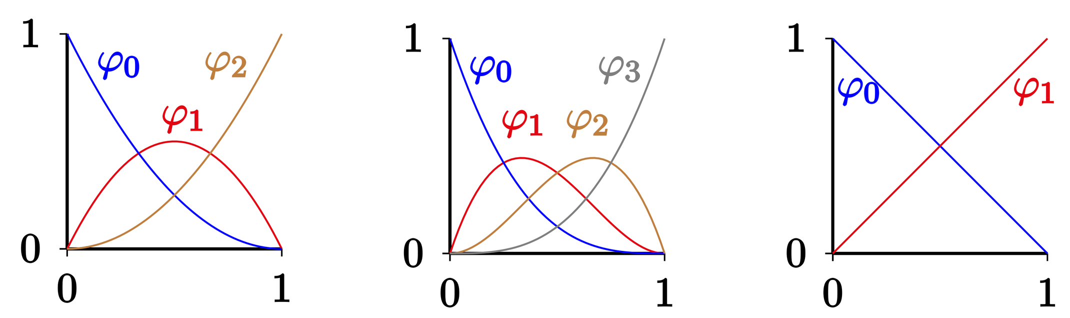
III - COMPUTATIONAL METHOD

Discontinuous Galerkin discretization of the domain with either triangular or quadrangular elements, structured or unstructured meshes.

Different elements may have different sizes.

Approximation spaces P_i are local and may differ among elements.





→ *h- and p-adaptivity*

The local weak formulation is the following:

Find $u_h(\mathbf{x},\xi)$ such that:

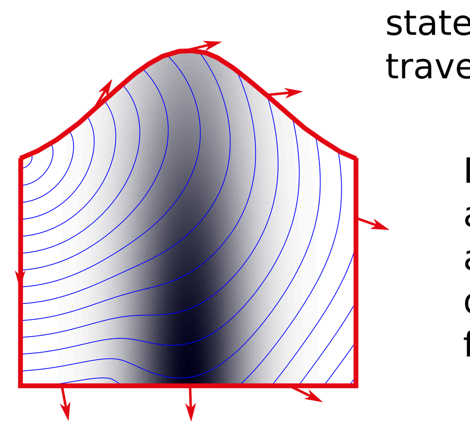
$$\int_{K_i}\Big(\partial_\xi u_h(\mathbf{x},\xi) + \mathcal{H}(\nabla_{\mathbf{x}} u_h(\mathbf{x},\xi),\mathbf{x})\Big)v_h(\mathbf{x})d\mathbf{x} + \underbrace{\int_{\partial K_i}\widehat{u_h}(\mathbf{x},\xi)v_h(\mathbf{x})d\mathbf{x}}_{\text{flux term}} = 0$$

holds for every $v_{h|K_i} \in \mathcal{P}_i$ and every $1 \le i \le N$.

with suitable flux terms designed by Cheng and Wang (2014); Le Bouteiller et al. (2017).

Integrals are estimated by quadrature rules.

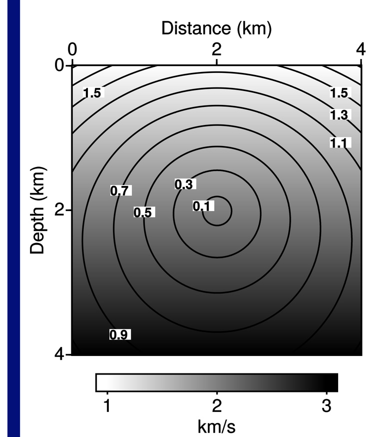
The partial derivative in the first integral allows for integration in evolution parameter ξ with a Runge-Kutta scheme. The steady-state must be reached in to obtain traveltimes.



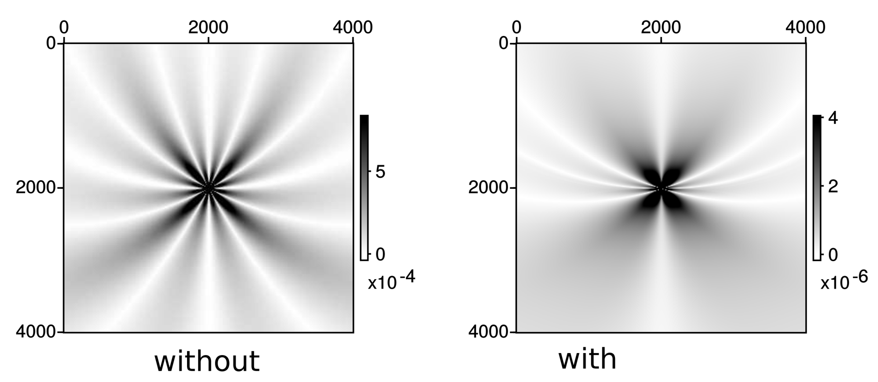
Domain boundaries and topography are handled by the weak formulation: an outgoing boundary condition is designed by the use of an additional flux term.

IV - NUMERICAL RESULTS

Isotropic constant gradient of velocity case study

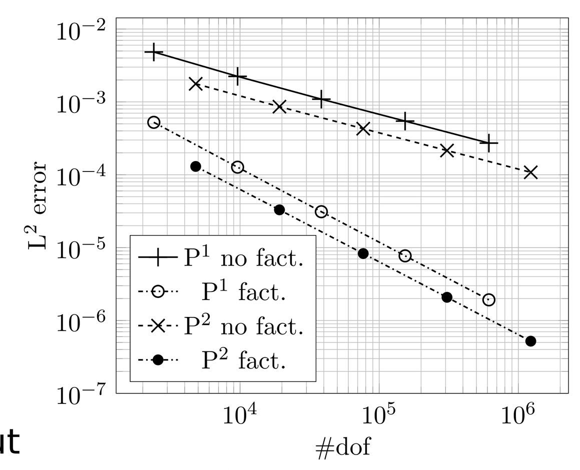


Analytical traveltimes



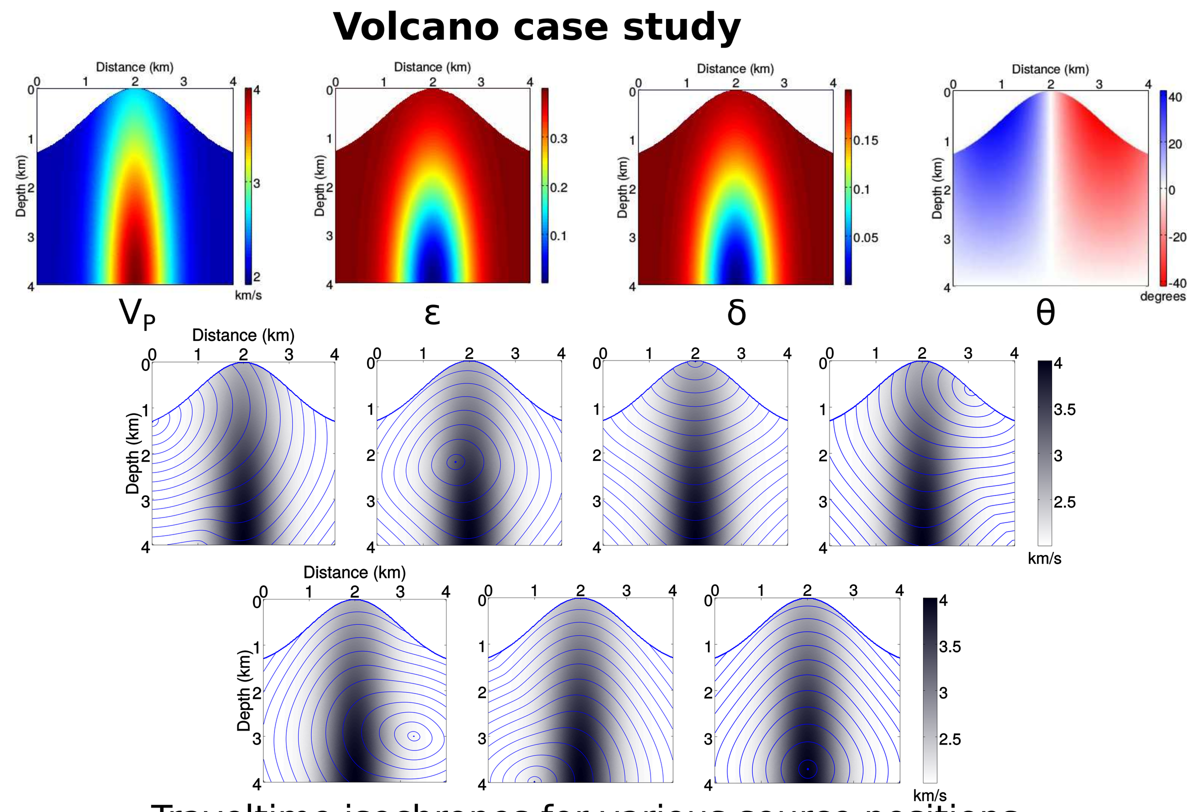
Relative error patterns without and with factorization

$c(z) = c_0 + \gamma_{CGV}(z - z_0)$
$c_0 = 2$ km.s^{-1}
$z_0 = 2$ km
$\gamma_{CGV} = 0.5$ s^{-1}
$c_{min} = 1$ km.s^{-1}
$c_{max} = 3$ km.s^{-1}



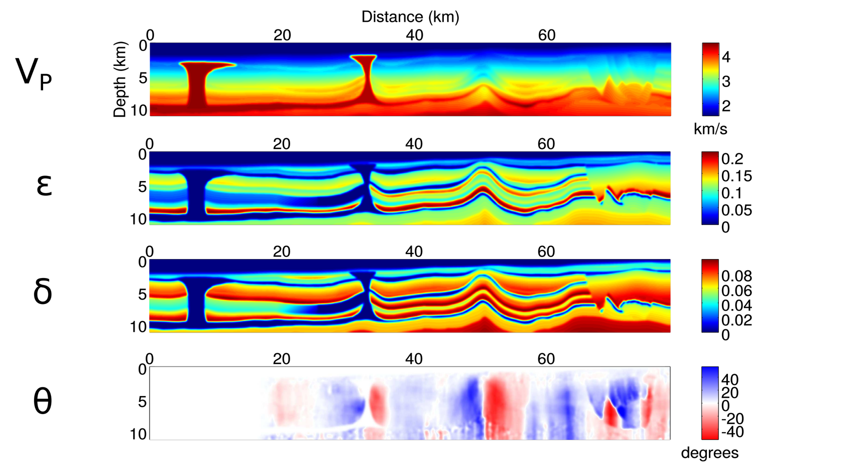
L^2 integrated error:
1st-order convergence without factorization, 2nd-order convergence with factorization

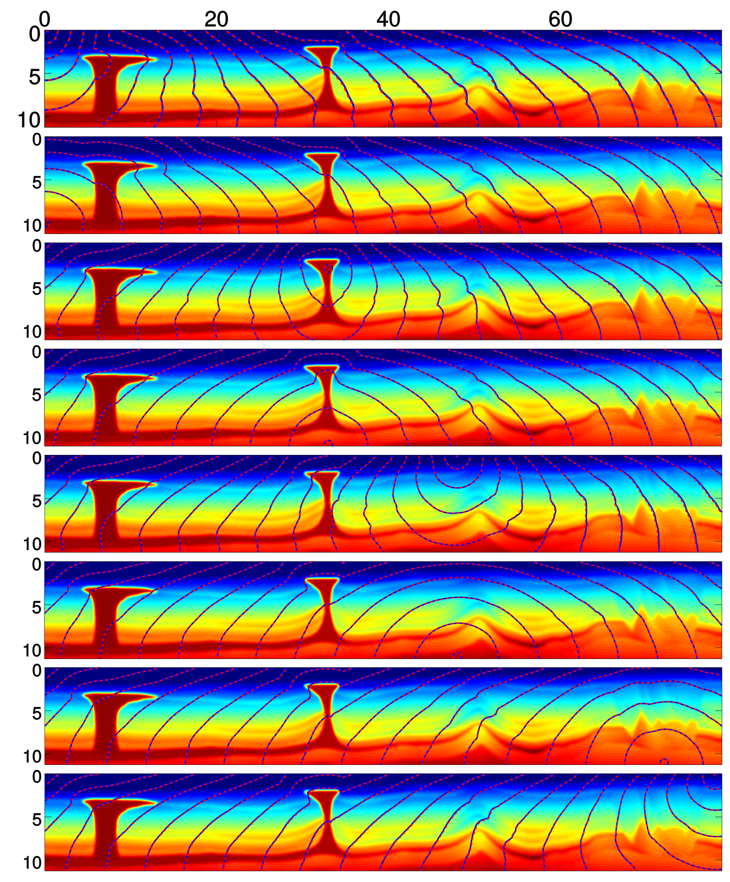
Volcano case study



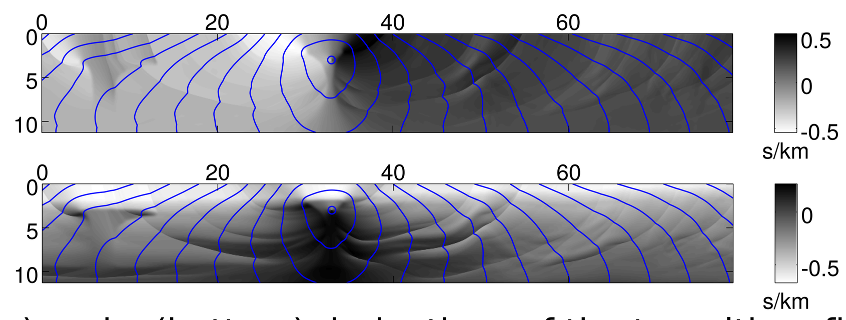
Traveltime isochrones for various source positions.
316000 degrees of freedom.

BP 2007 TTI realistic model





Traveltime isochrones for various source positions.
Results from a FD code superimposed.
537600 degrees of freedom.



x-(top) and z-(bottom) derivatives of the traveltime field.
Blue isochrones superimposed.

V - CONCLUSIONS AND PERSPECTIVES

Conclusions

We develop an **accurate and efficient solver for TTI traveltimes in complex media**. Flexibility is provided by the discontinuous Galerkin discretization for **topography handling**. However, the computational cost is significantly higher than standard finite-difference-based methods, due to the dynamic HJ framework and the related Courant-Friedrichs-Lewy condition.

Perspectives

Fast Sweeping Method: promising approach for fast steady state computation
Transport equation solver based on the same engine for computation of **amplitudes**
3D (no expected theoretical difficulties)
Other classes of anisotropy: **Orthorhombic**...

Target applications

Migration (accurate solution and derivatives)
Stereotomography (uniform sampling)

VI - REFERENCES

Cheng, Y. and Wang, Z. (2014). A new discontinuous Galerkin finite element method for directly solving the Hamilton–Jacobi equations. *Journal of Computational Physics*, 268:134–153.

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VII - ACKNOWLEDGMENTS

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