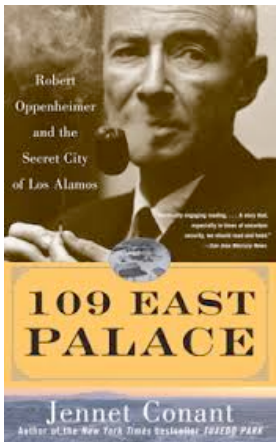


# Nonlinear elasticity: observations and implications

oppenheimer



**Paul Johnson**

*Los Alamos*

Ambient Noise Imaging and Monitoring Workshop,  
Cargese 2015

teller



Bethe



fermi



feynman

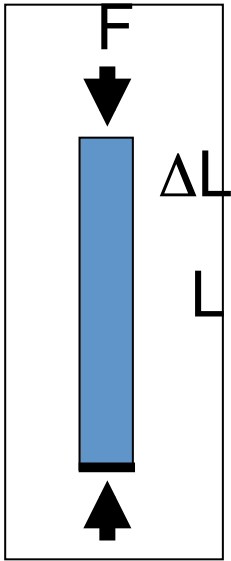


# outline

- Background on nonlinear elasticity
- Applications:
  - Slow slip laboratory, simulation
  - Slow slip Mexico
  - Widespread elastic perturbation in Japan
  - Velocity changes following the San Simeon and Parkfield earthquakes

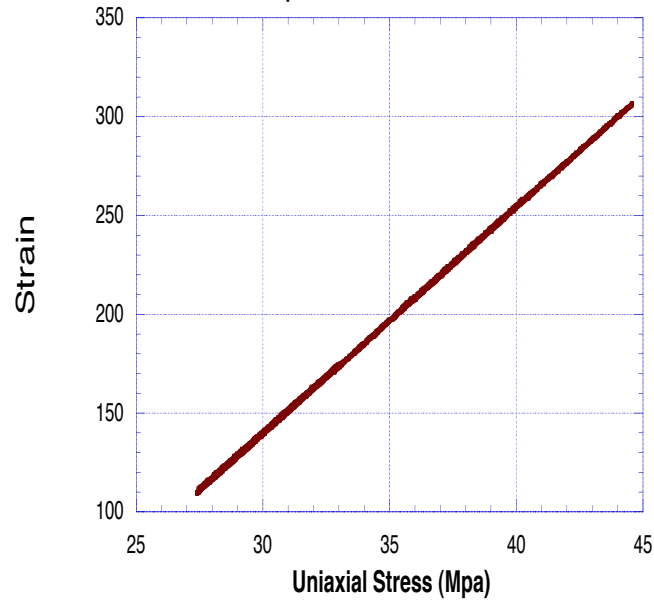
summary

# Stress-strain measurements in the lab

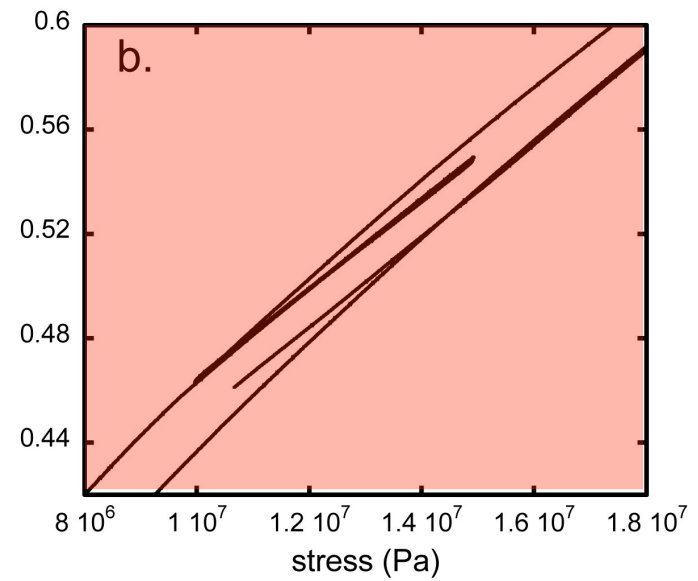
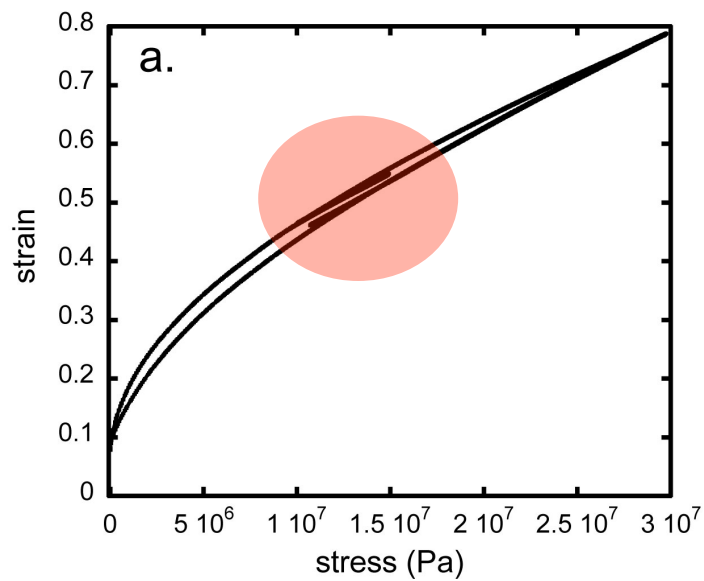


$$\epsilon = \Delta L / L$$

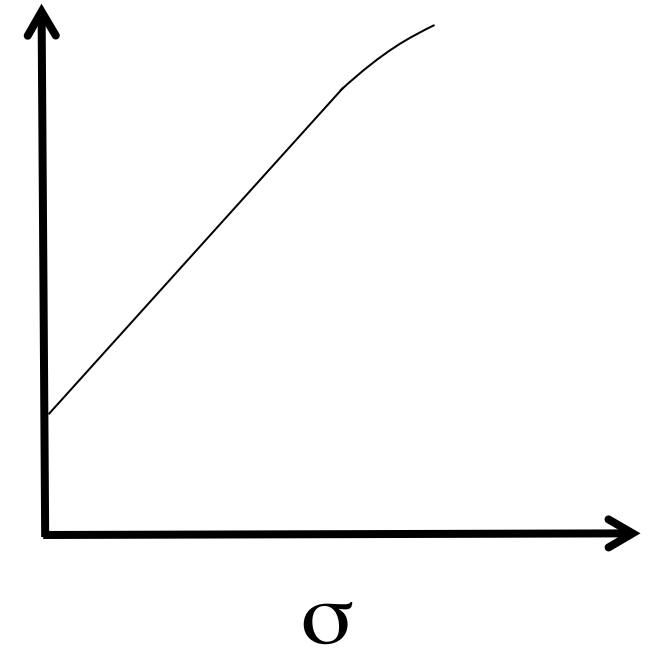
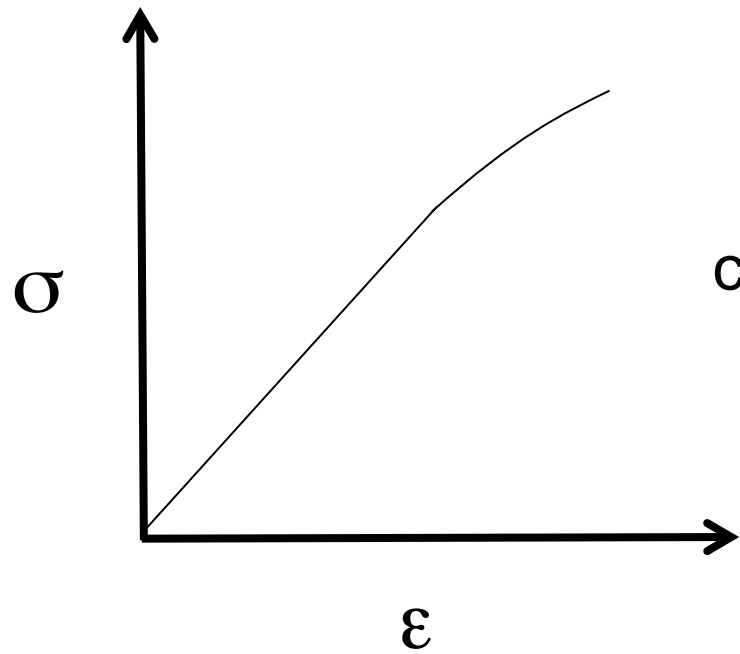
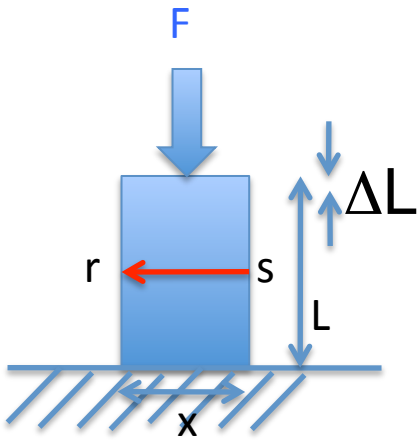
Optical Grade Glass



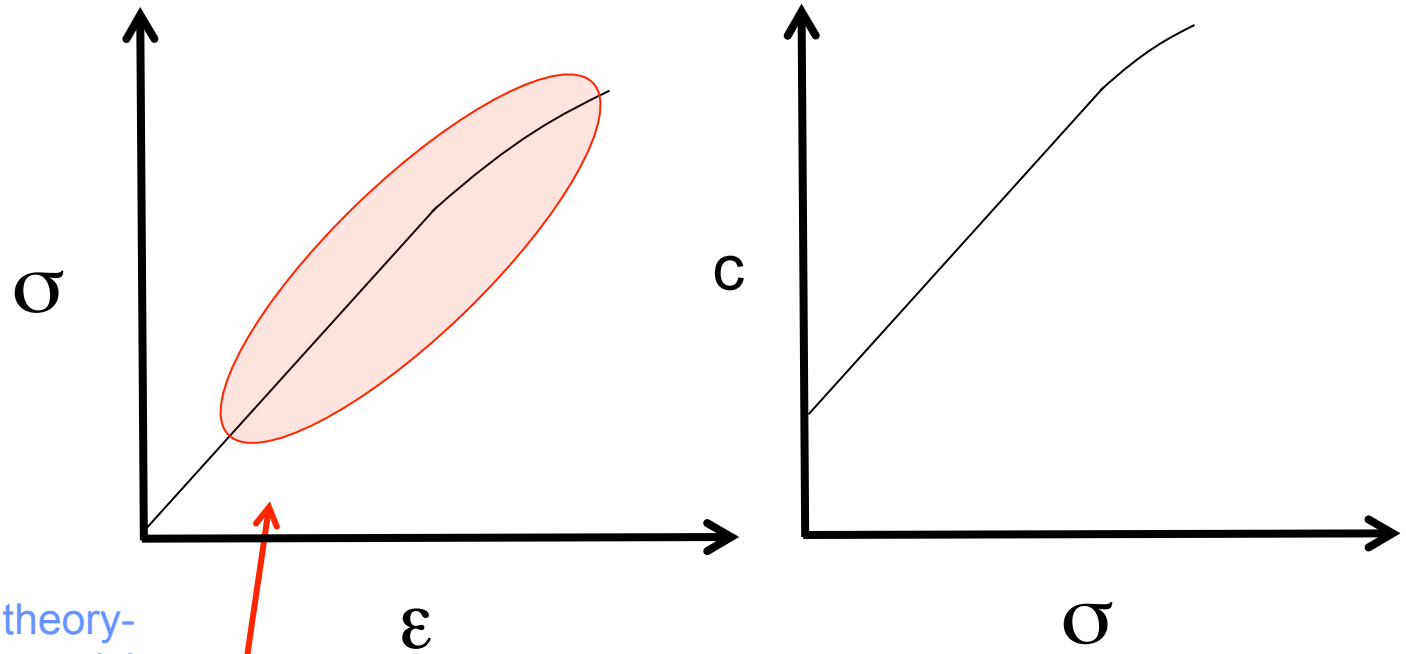
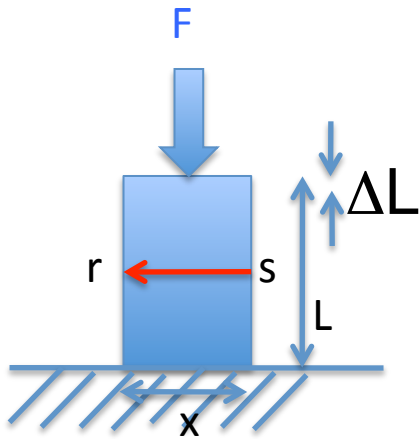
Berea sandstone



# Quasistatic and dynamic elasticity of individual crystals and many metals



# Quasistatic and dynamic elasticity of individual crystals and many metals



Thermodynamics based theory-  
Nonlinearity due to anharmonicity  
in the crystalline lattice.

$$\sigma_{dyn} = M \varepsilon_{dyn} [(1 + \beta \varepsilon_{dyn})]$$

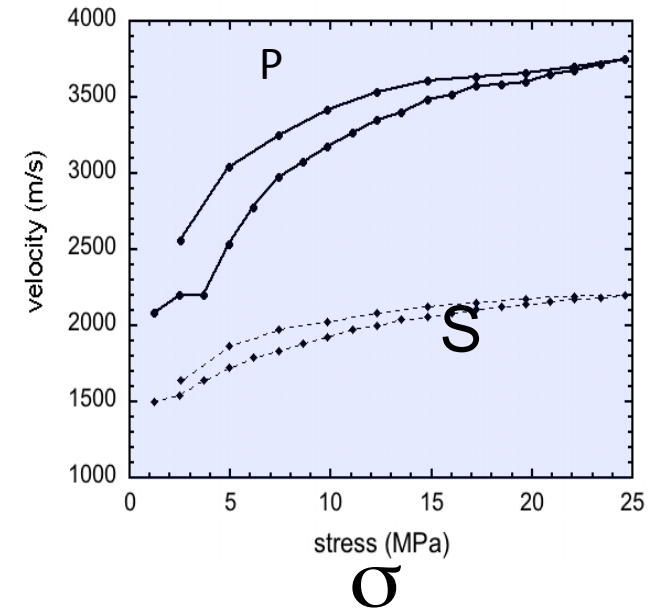
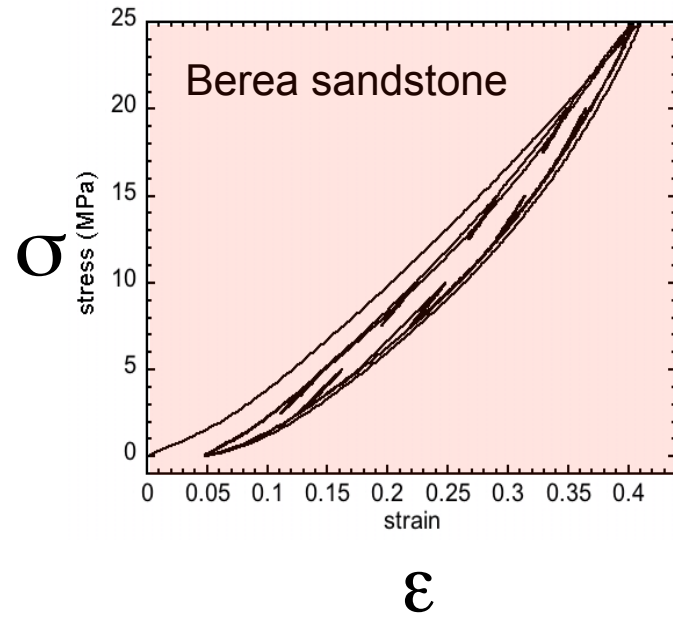
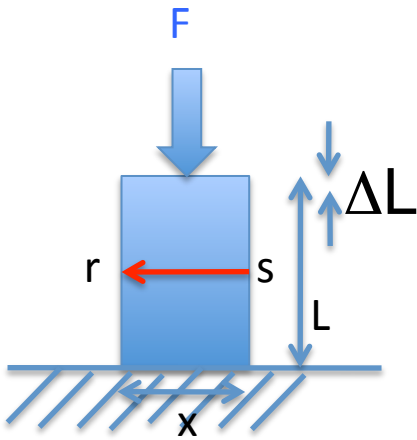
$$\beta = \frac{3}{4} + \frac{A+B+C}{2\rho c^2} = \frac{3}{2} + \frac{l+2m}{\lambda+2\nu}$$

Goldberg

Murnaghan

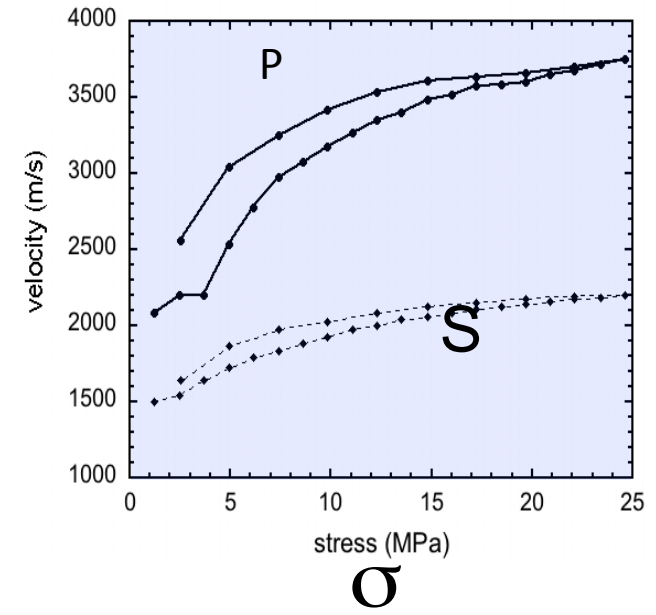
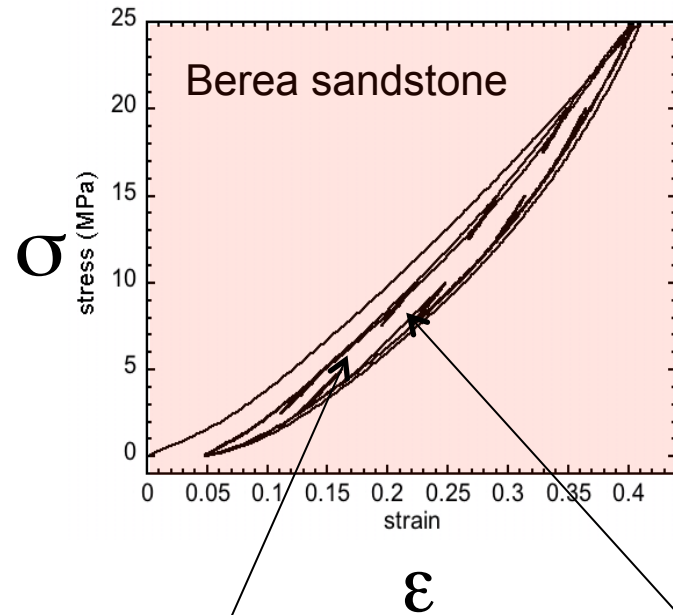
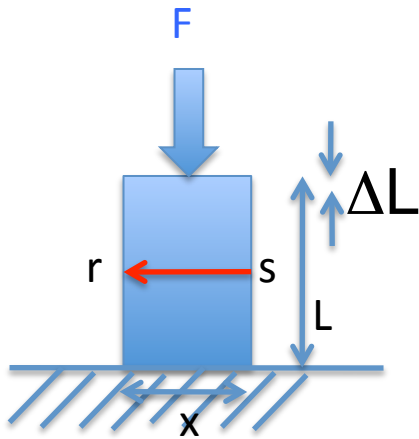
# Quasistatic elasticity of rock and 'damaged' materials

Nonlinearity much larger than metals and single crystals.....



# Elasticity of individual rock and damaged materials

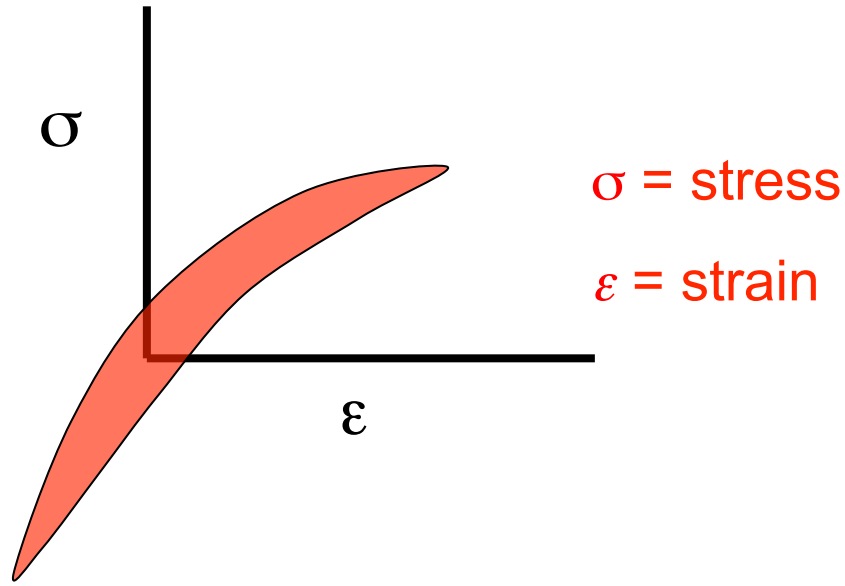
Nonlinearity much larger than metals and single crystals.....



Phenomenological model of nonlinearity and hysteresis (Preisach-Mayergoyz)

$$\sigma_{dyn} = M \varepsilon_{dyn} \left[ (1 + \beta \varepsilon_{dyn} + \delta \varepsilon_{dyn}^2 + \dots) + \left( \varepsilon - \alpha [(\Delta \varepsilon_{dyn}) \varepsilon \pm \frac{1}{2} (\varepsilon_{dyn}^2 - \Delta \varepsilon_{dyn})^2] \right) \right]$$

# Wave dynamics in rock and other earth materials

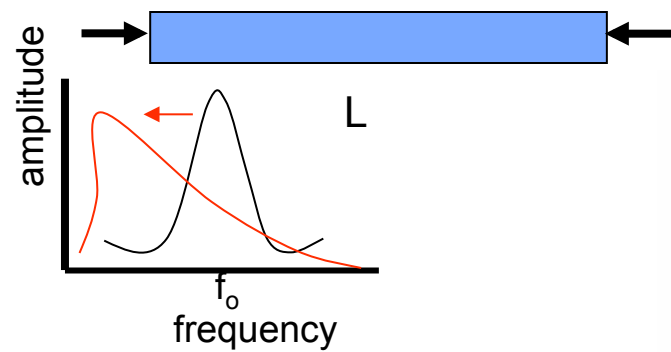


Elastic nonlinearity and hysteresis are present in dynamics when strains exceed  $\sim 5 \times 10^{-7} - 10^{-6}$  [*at low effective pressure*].

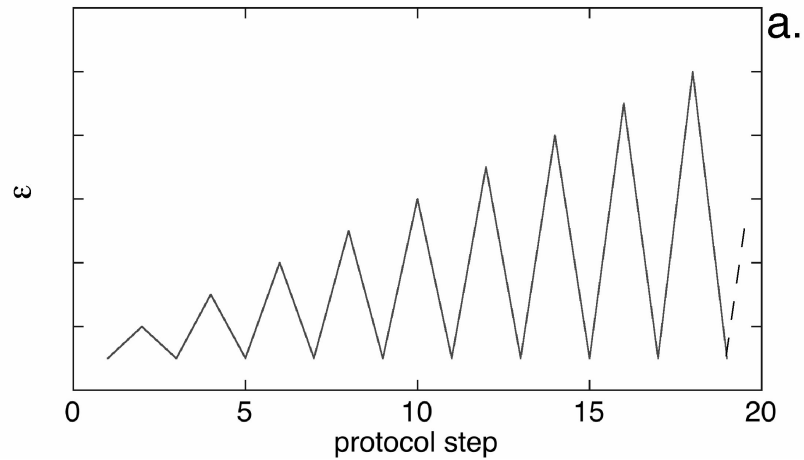
*Material dependent*



# 'Memory' of wave excitation ('conditioning')

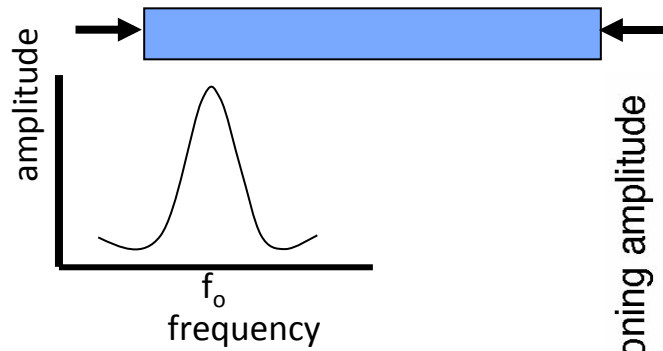


Young's mode experiments in a bar



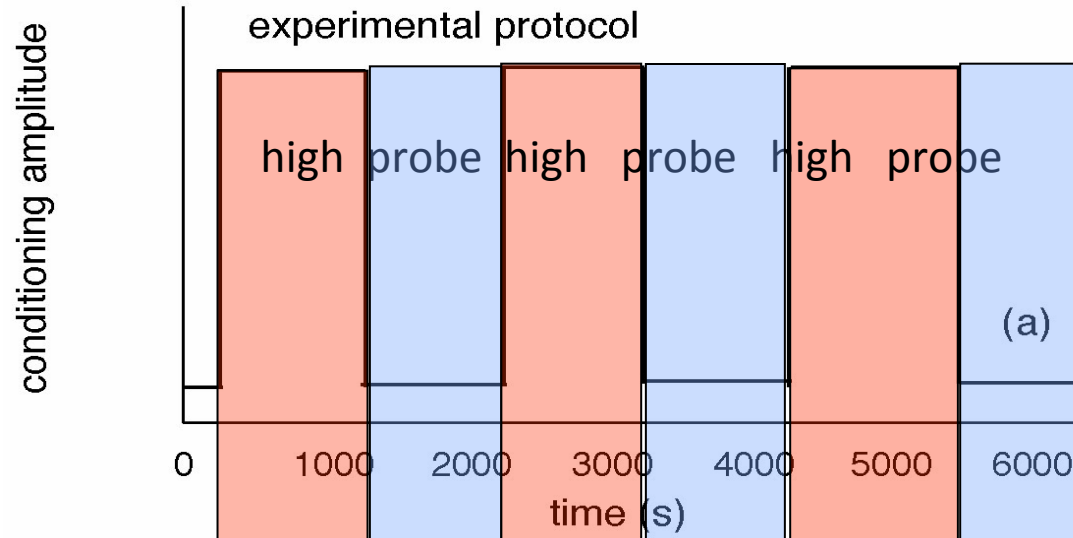
Experimental Protocol

# Regime iii: nonequilibrium slow dynamics



Slow dynamics— a type of creep

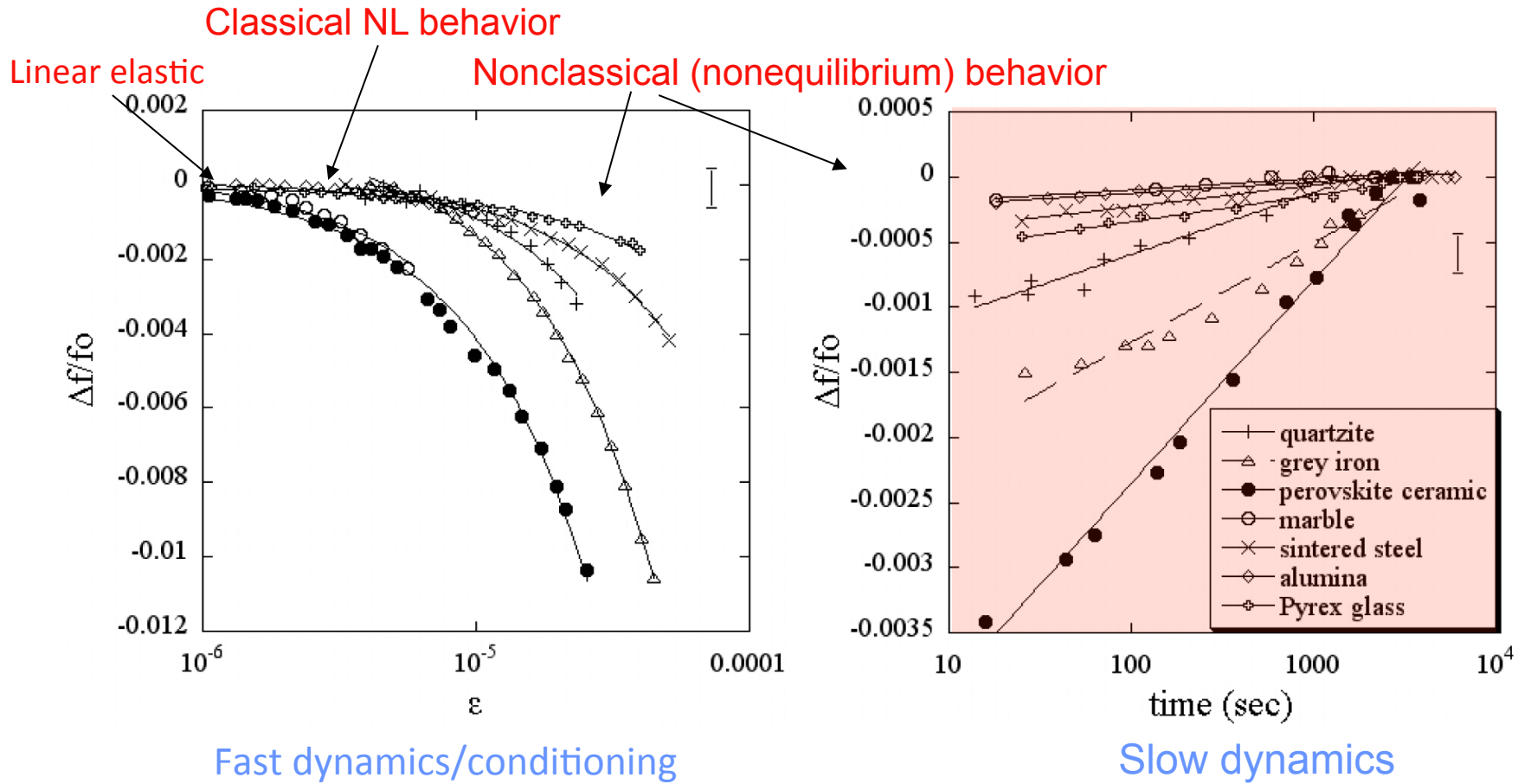
## Young's mode experiments in a bar



# Modulus 'softening' and recovery (the 'slow dynamics')



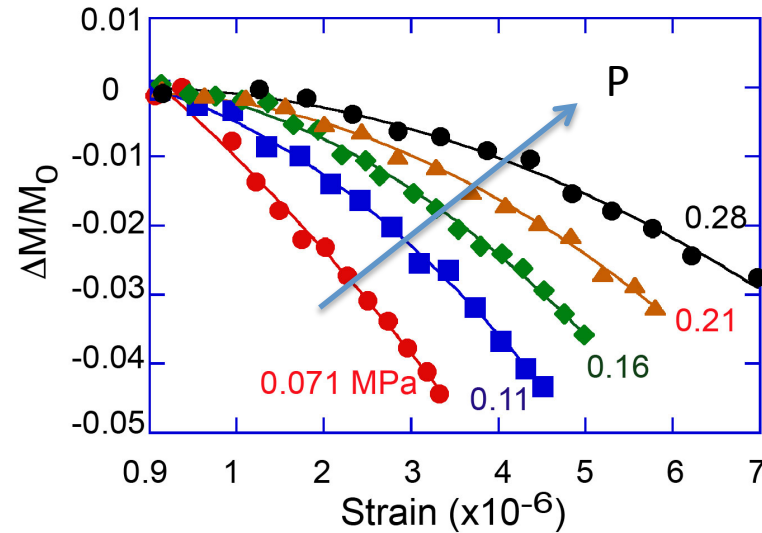
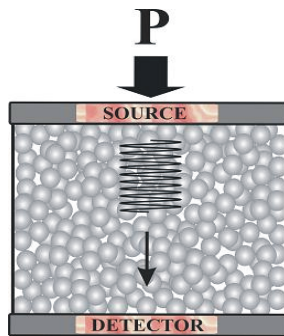
Young's mode experiments



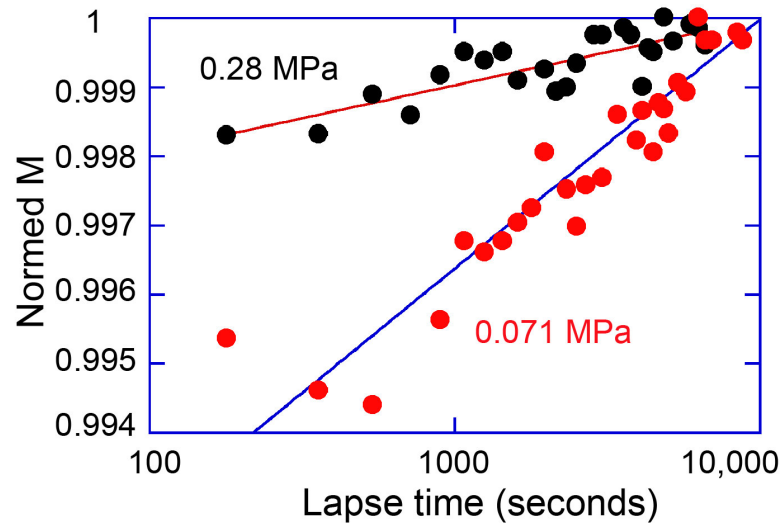
From Johnson and Sutin, JASA 2005

# Modulus 'softening' and recovery (the 'slow dynamics') in glass beads

Experiments with rock and canisters of glass beads

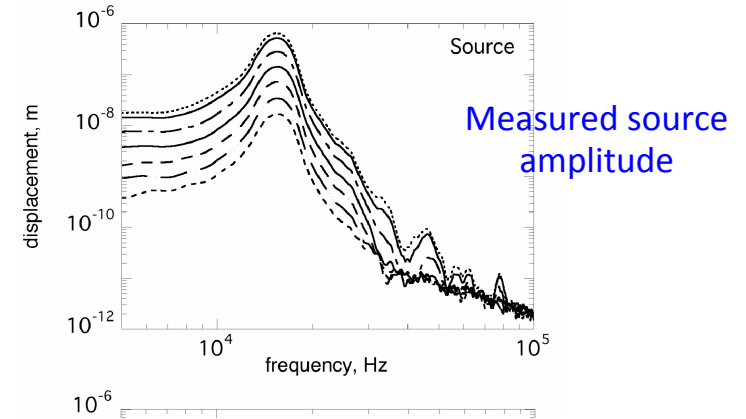
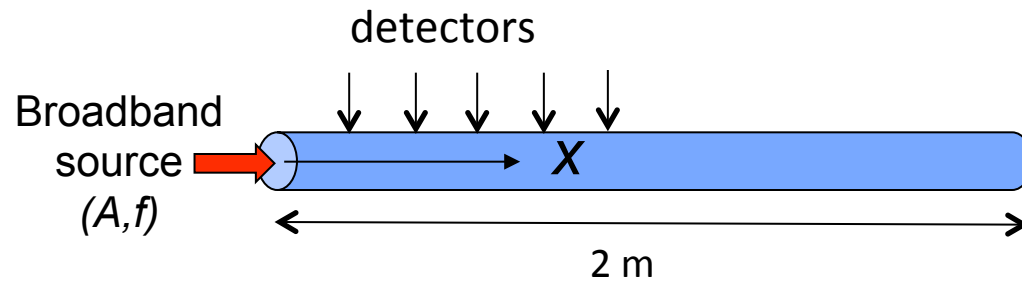


Decrease in modulus and increase in dissipation with wave amplitude.

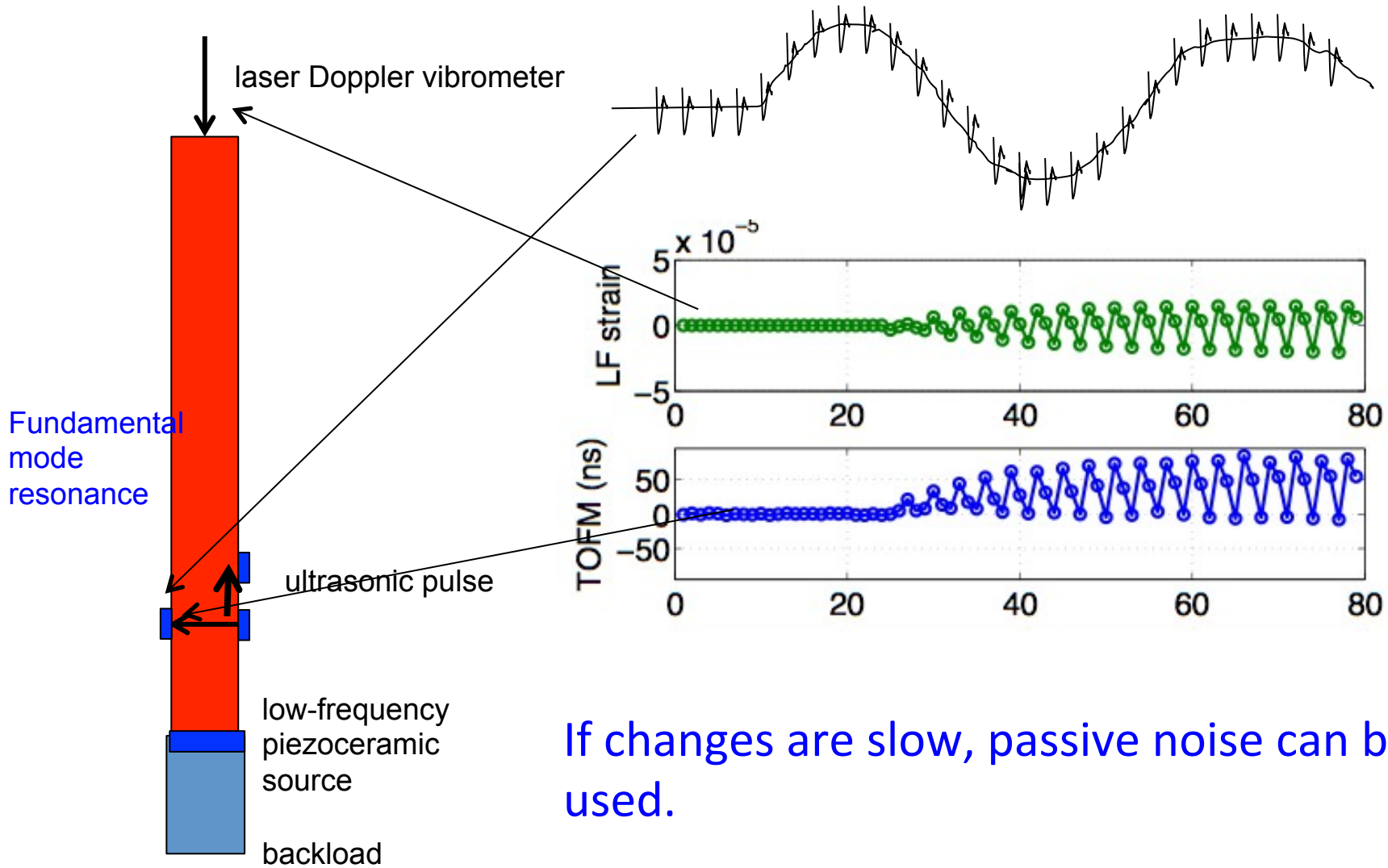


Long recovery to equilibrium (the slow dynamics)

# Nonlinear dynamical experiments: NL 'path' effects



# Dynamic acousto-elasticity (DAE)

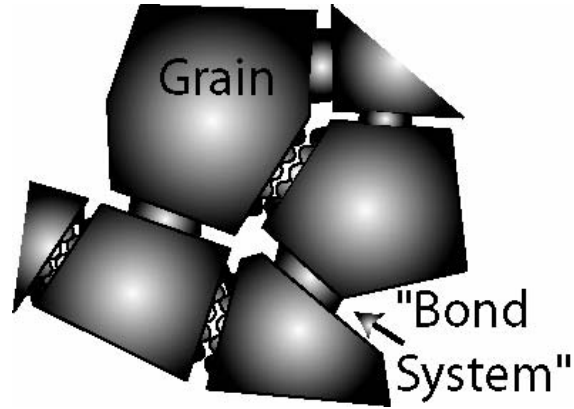


If changes are slow, passive noise can be used.

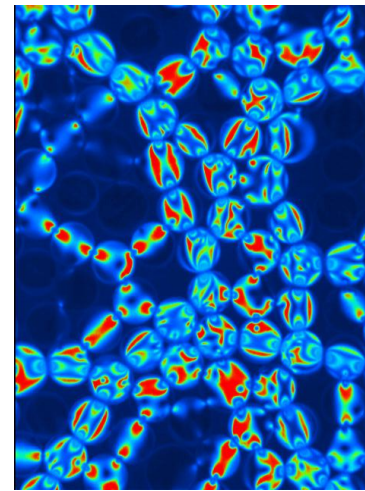
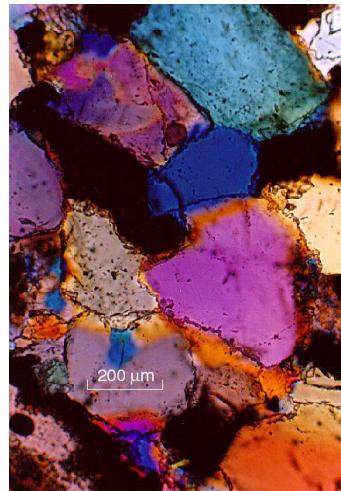
G. Renaud, J. Riviere , others.

# Physical origin of behavior

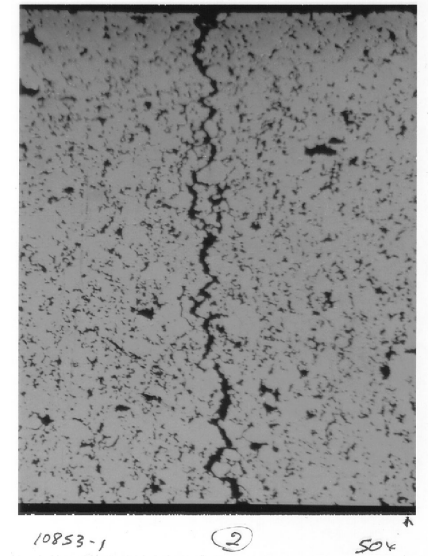
Dominant contribution is material 'damage'



"Distributed"



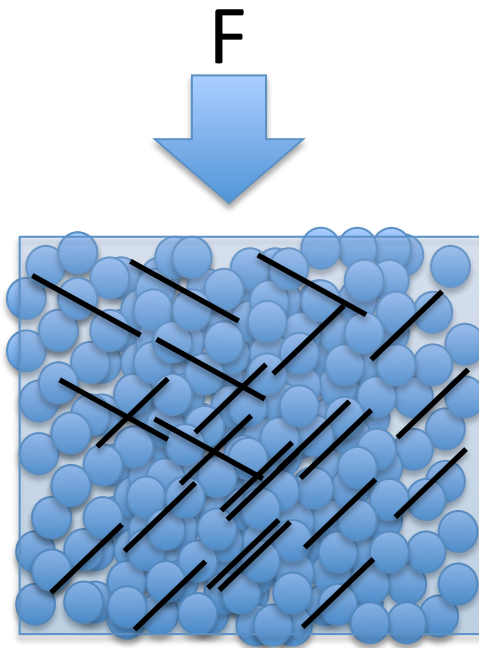
"Localized"



The grains or material around damaged region remain rigid.  
The bond system is soft.

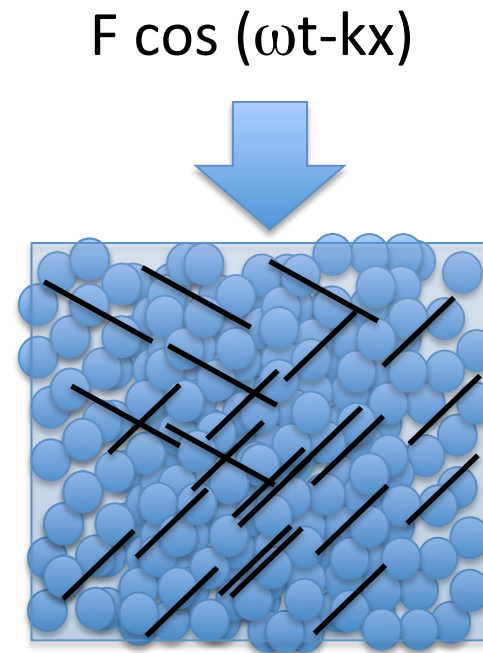
## Why does this happen?

Velocity **increase**, length decrease



Stress transferred by skeleton.  
Little activation of damage features.

Velocity **decrease**, length decrease



Stress activation of damage features, primarily by shearing



Applications: Slow slip.....

# Seismic evidence of tremors and slow-slip events

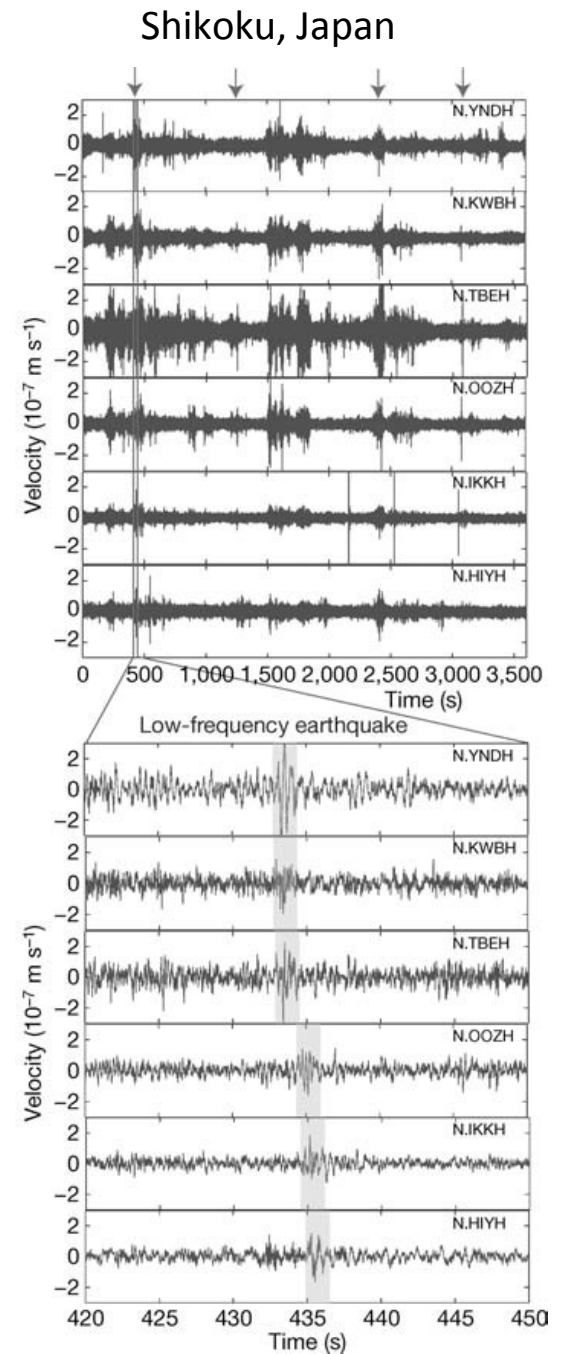
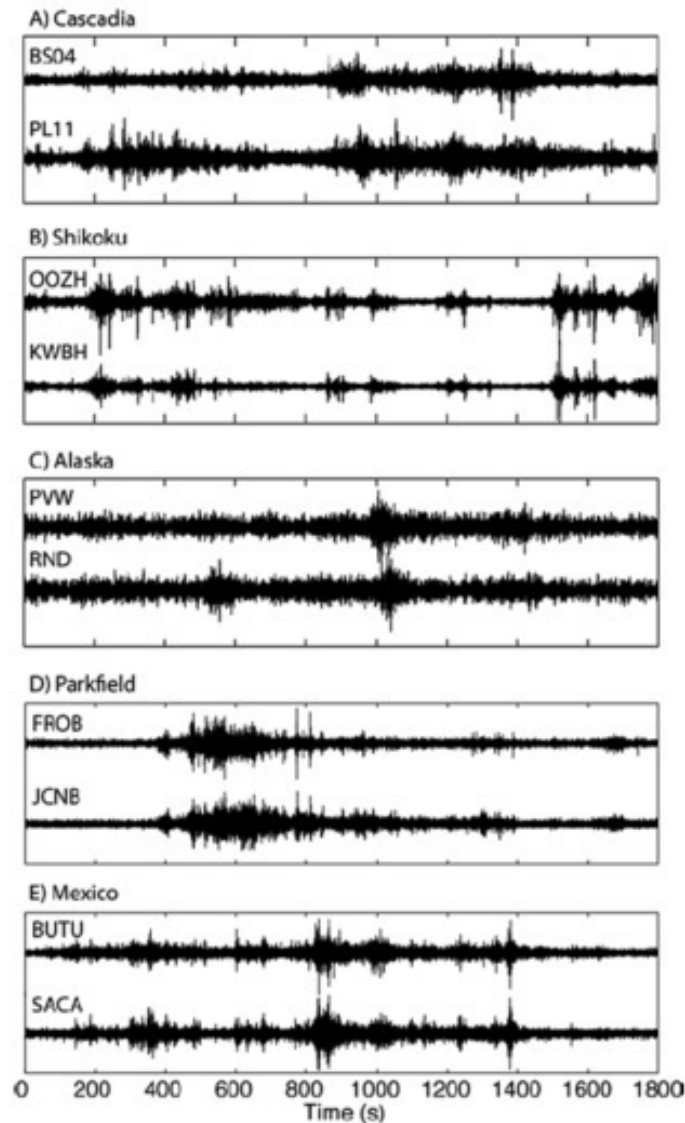
Long duration

Low Amplitude

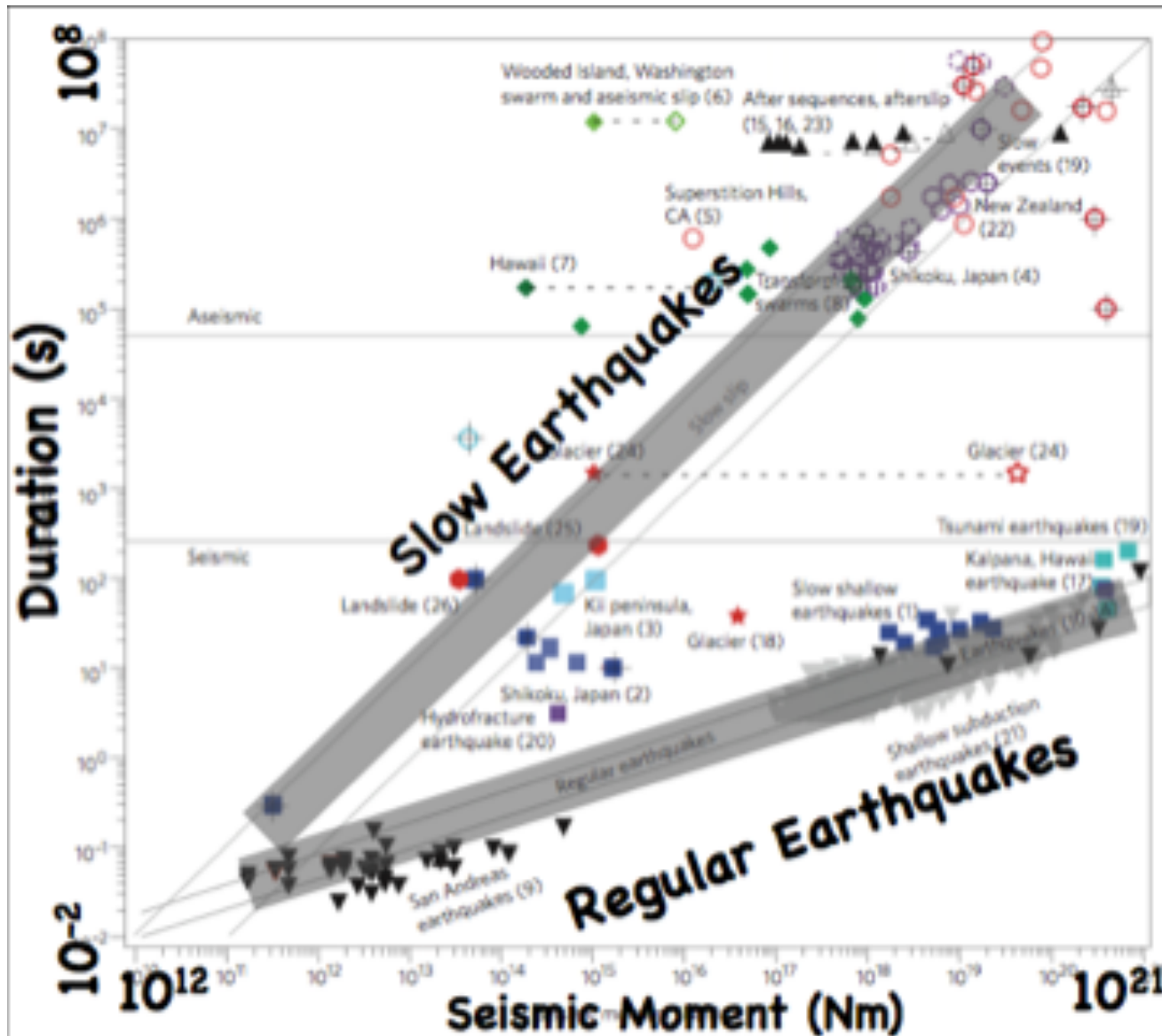
Non-impulsive seismic radiation

usually in the 1-10Hz frequency band

very difficult to distinguish from the ambient noise.



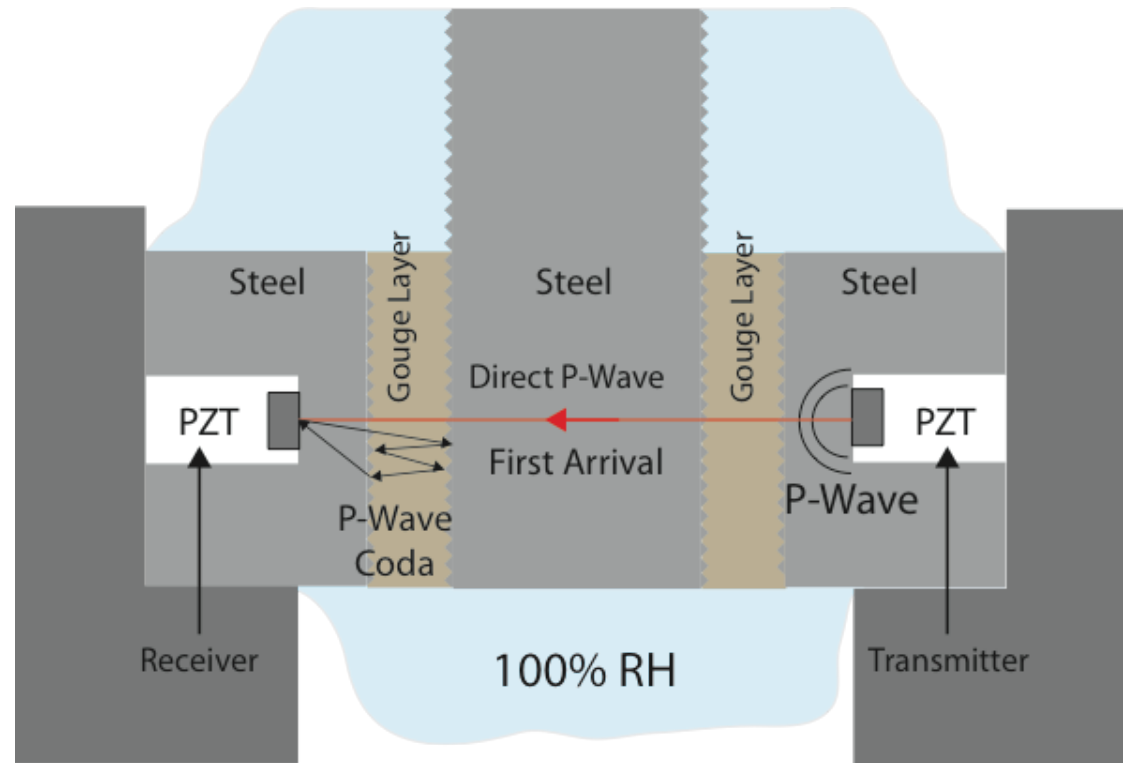
# The spectrum of fault slip behavior



Quasi  
dynamic  
processes

Regular  
earthquakes

# Scuderi et al: Ultrasonic wave speed measurements

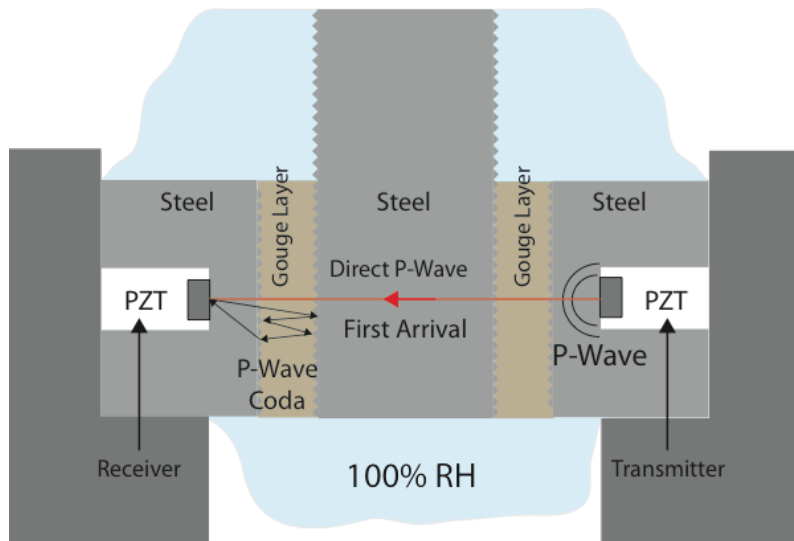


From the transmitter P-Waves are sent at a rate of 10 Hz.  
The receiver records the first 140  $\mu\text{s}$  of the waveform  
at a frequency of 50 MHz.

During the experiment, waveforms are accurately synchronized with mechanical data.

# Ultrasonic wave speed measurements

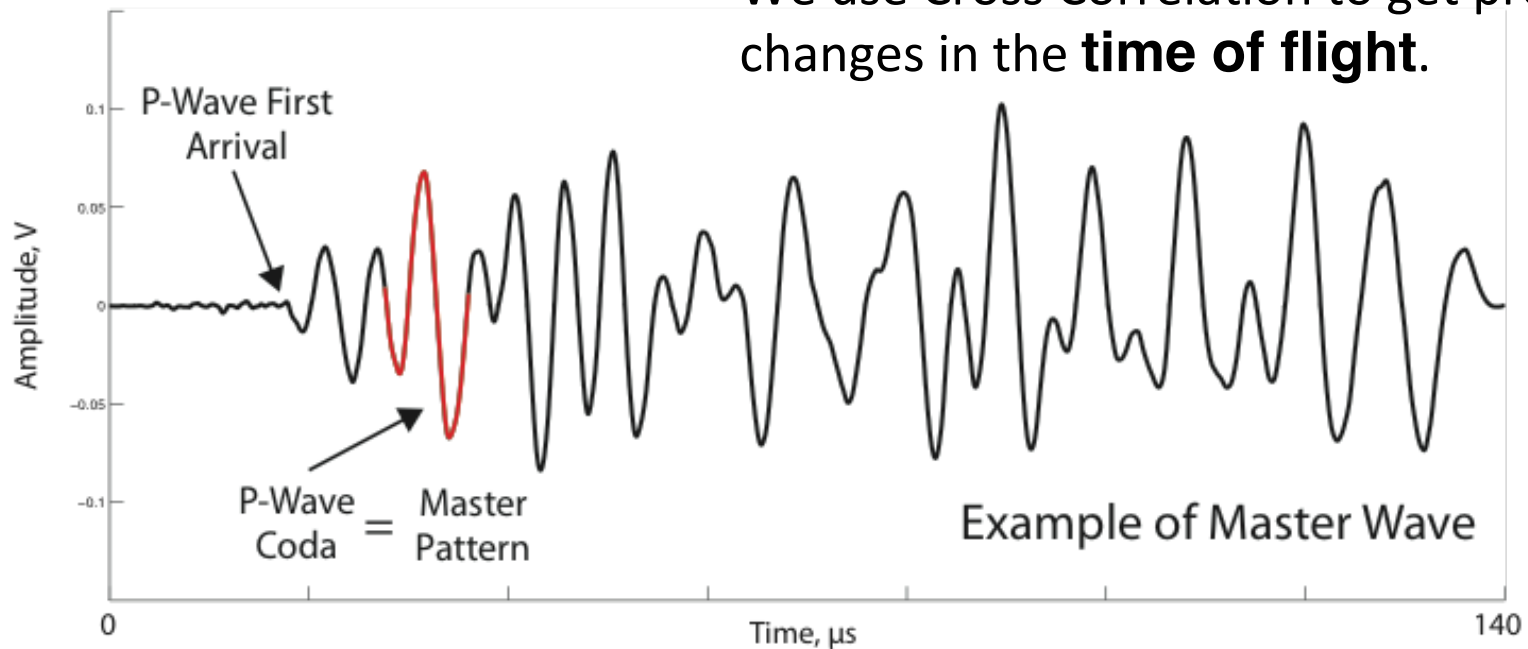
## Waveform Analysis



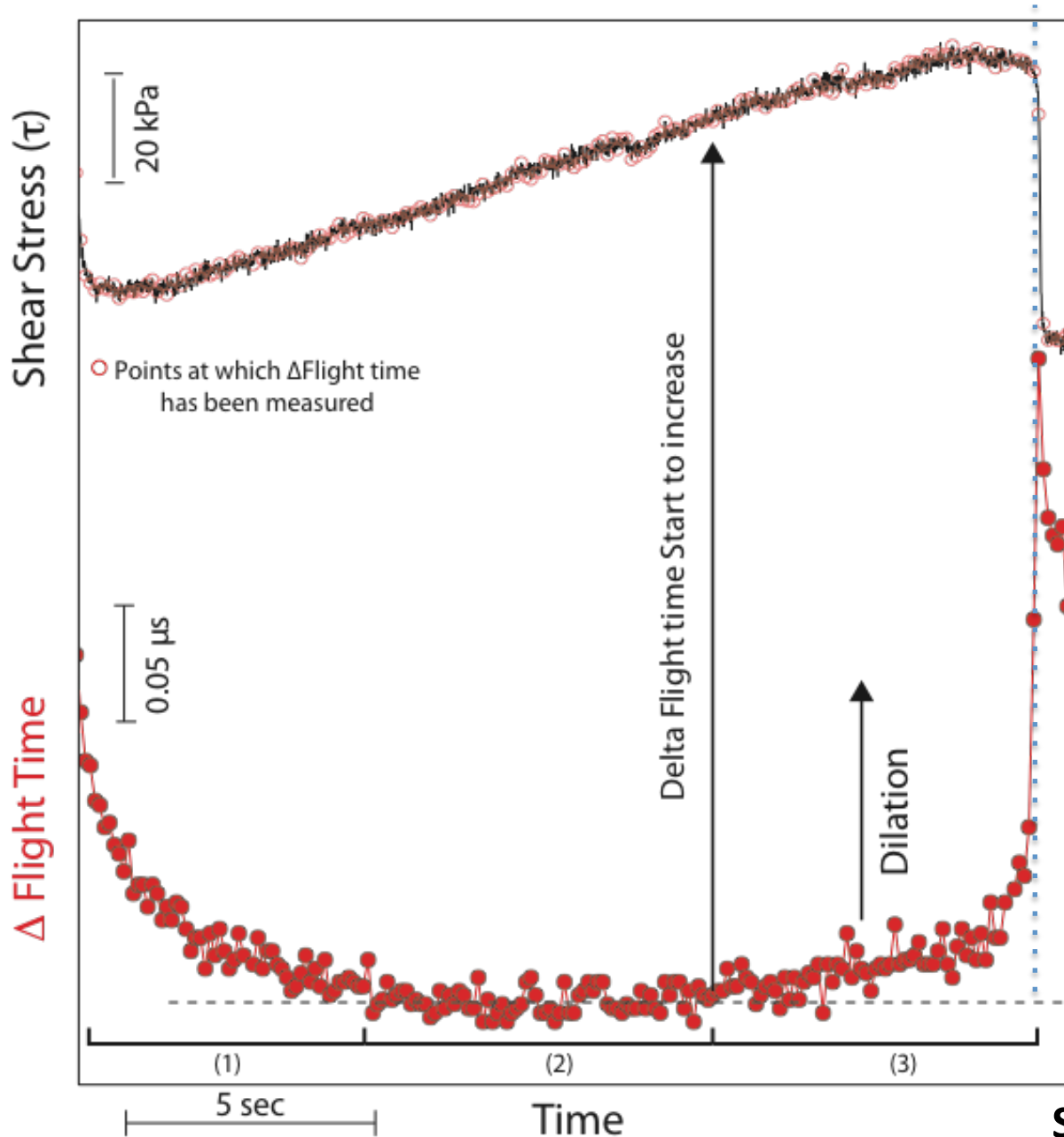
We select a master wave from a stable period of the experiment prior to the events of interest

We focus on the P-Wave coda and choose a master pattern

We use Cross Correlation to get precise changes in the **time of flight**.



# Scuderi: Details of a slow-slip and its acoustic signature



Time of flight evolves during a slow-slip cycle, showing:

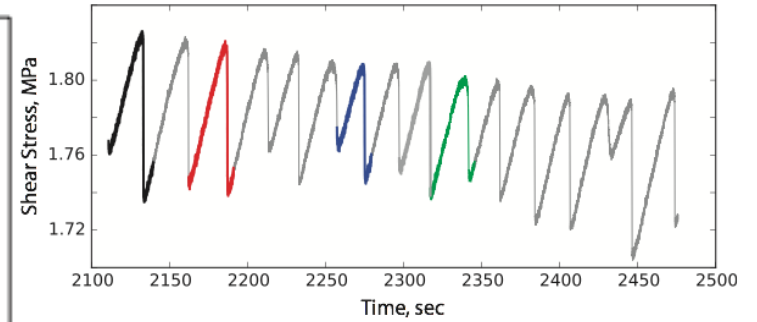
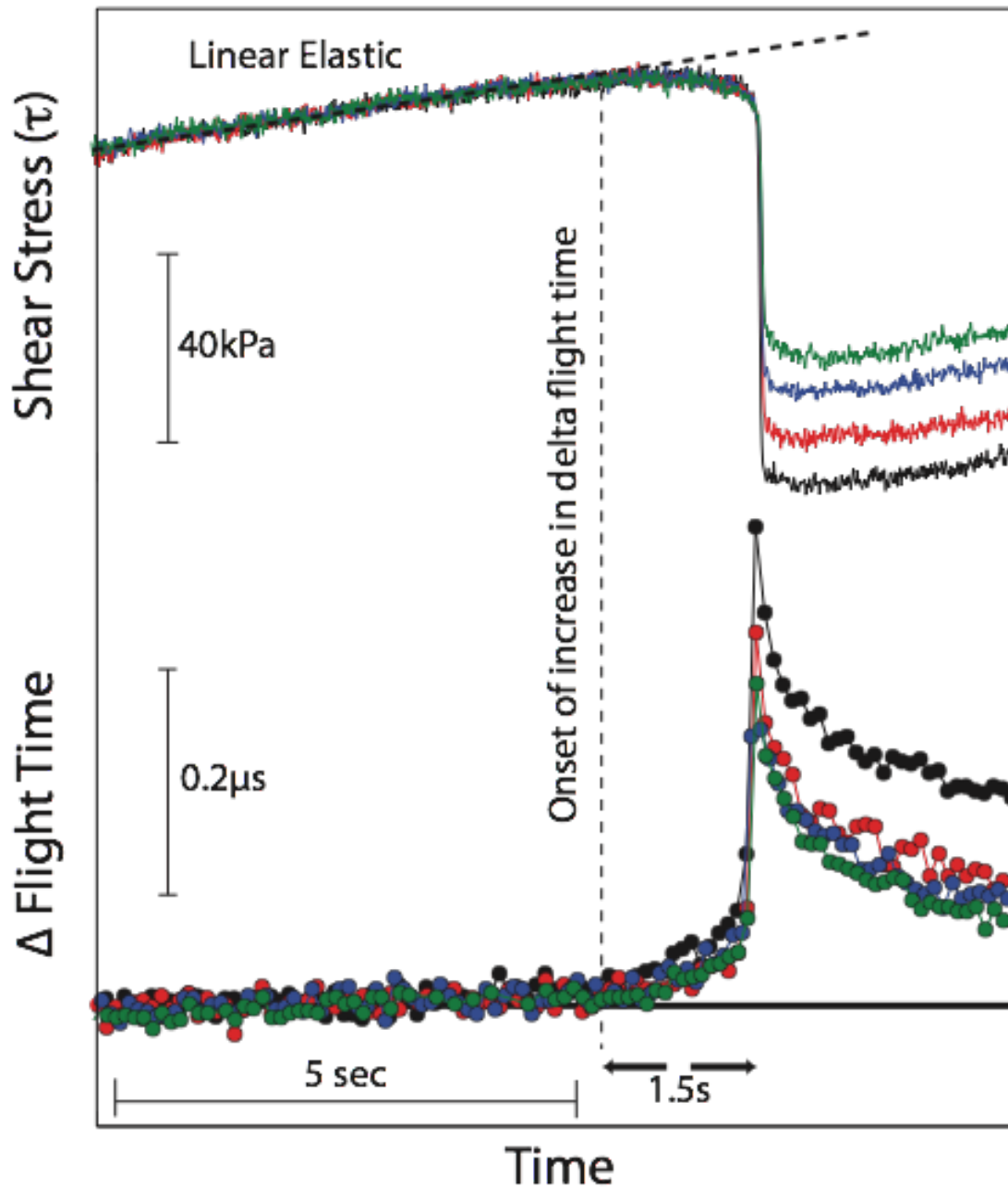
**(1)** First stage of decrease (possibly due to layer compaction).

**(2)** Stabilization to constant values.

**(3)** A gradual increase preceding the failure event and stress drop

Scuderi et al. in prep.

# Comparison between different events

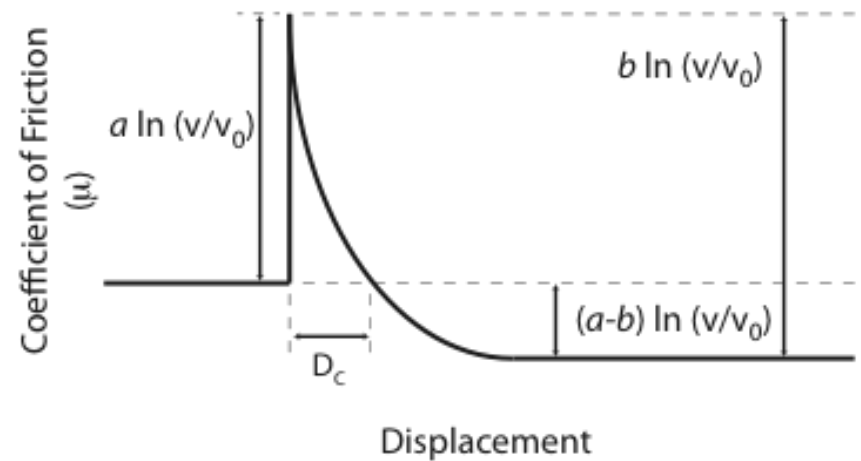
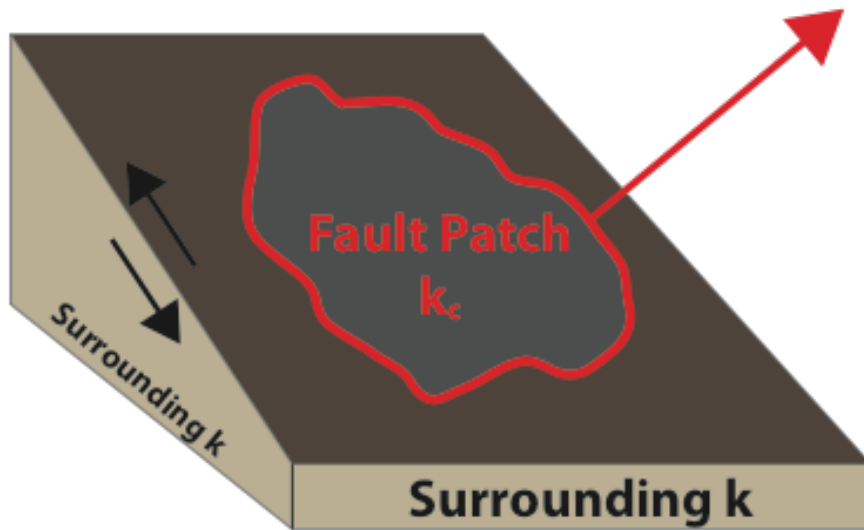


Flight time always starts to increase before the maximum strength is reached and continues during the stress drop.

Scuderi et al. in prep.

# Frictional stability along faults

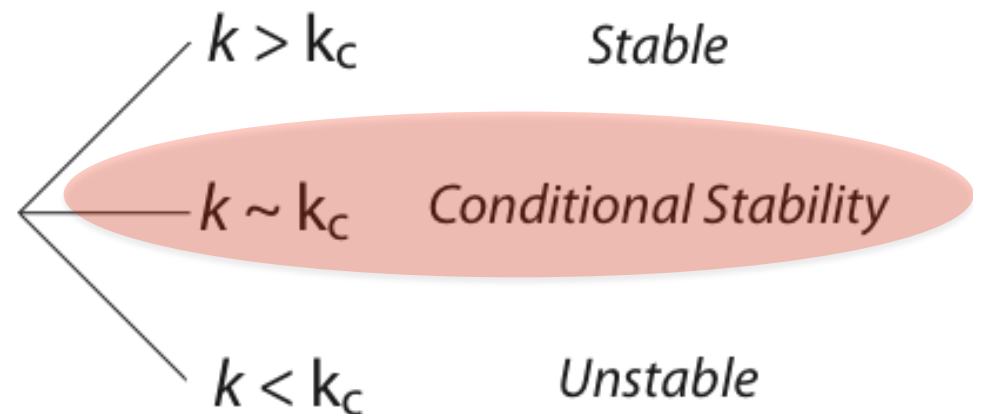
$(a-b) < 0$  Velocity Weakening (potentially unstable)



## Criterion for fault Stability

defined by the critical stiffness ( $k_c$ )

$$k_c = (\sigma_n - P_f)(b-a) / D_c$$



Scuderi et al. in prep.





Discrete element modeling of triggered slip in faults with granular gouge: application to dynamic earthquake triggering

Behrooz Ferdowsi

Discrete element modeling of triggered slip in faults with granular gouge: application to dynamic earthquake triggering

Behrooz Ferdowsi

Diss. ETH No. 21960

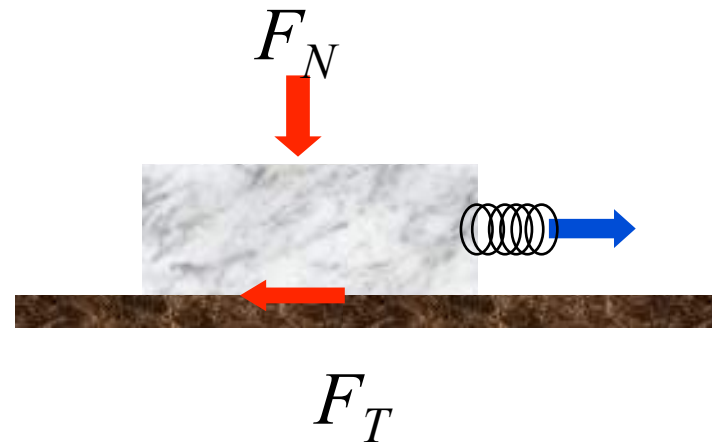
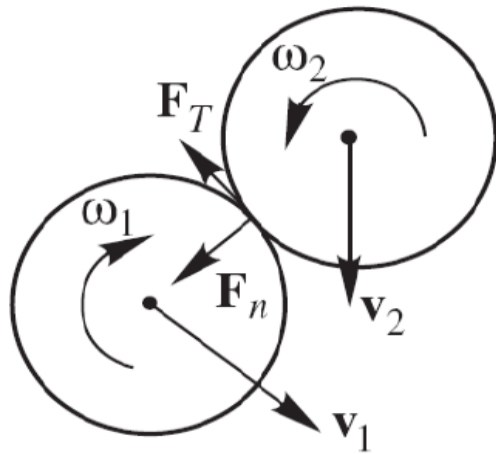
**ETH**

Eidgenössische Technische Hochschule Zürich  
Swiss Federal Institute of Technology Zurich

B. Ferdowsi, J. Carmeliet, M. Griffa, R.A. Guyer, P.A. Johnson, C. Marone

# DEM modeling

Motion of particles solving Newton's equations



□ Normal contact force

□ Tangential contact force

Coulomb friction law  $|F_T| \leq \mu \cdot |F_N|$

$\mu_s$ : static friction

$\mu_D$ : dynamic friction

# Particle kinetic energy

$$K_{lin}^i \equiv \frac{1}{2} \cdot m_i \cdot \|\dot{\vec{u}}_i\|^2$$

translational kinetic energy

$$K_{rot}^i \equiv \frac{1}{2} \cdot I_i \cdot \omega_i^2$$

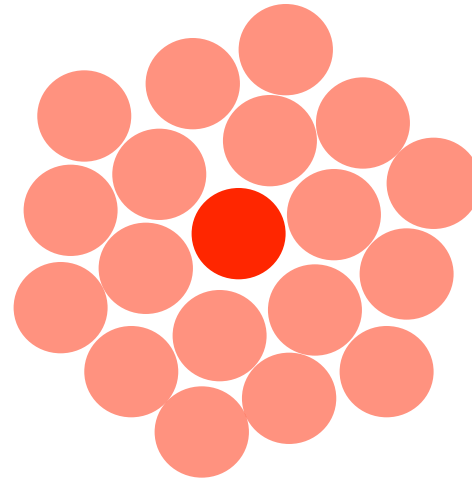
rotational kinetic energy

$m_i \equiv$  mass

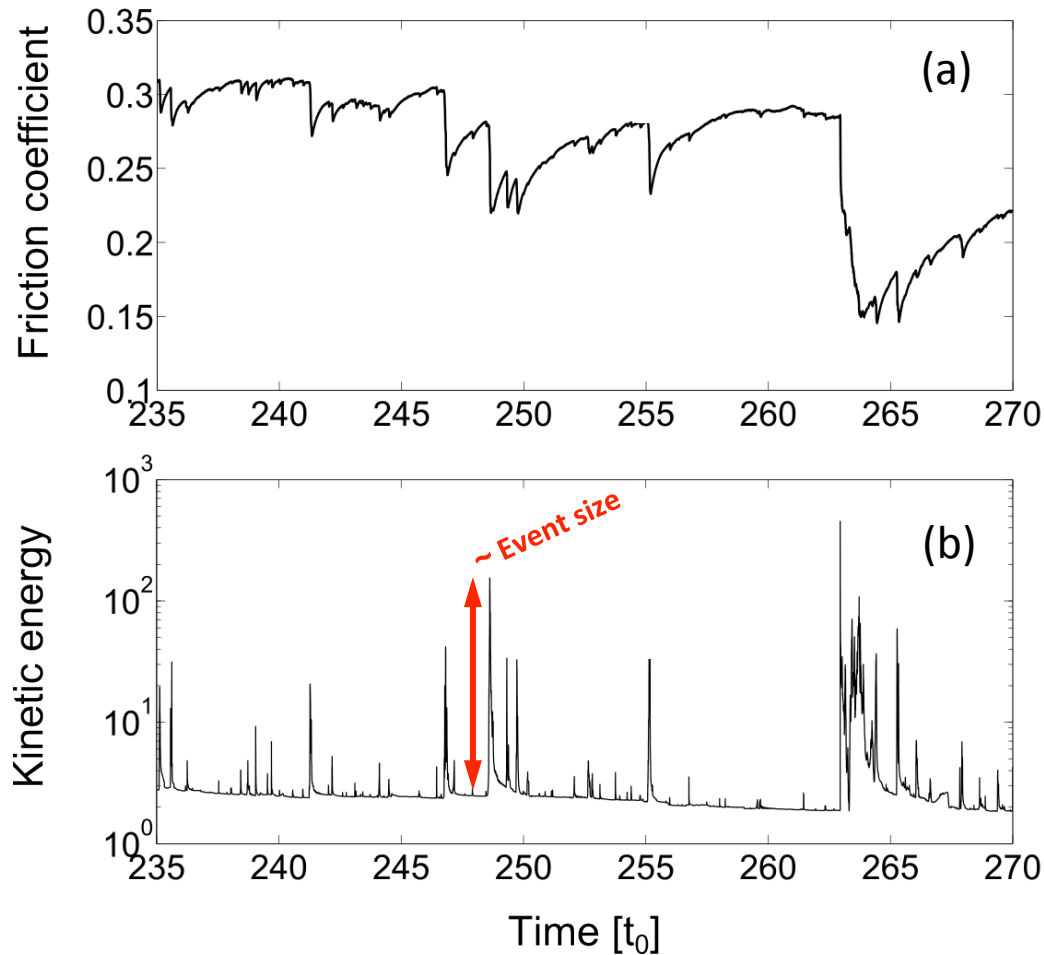
$\vec{u}_i \equiv$  displacement vector

$I_i \equiv$  moment of inertia

$\omega_i \equiv$  angular velocity



# Similarities in Stick-slip dynamics in DEM

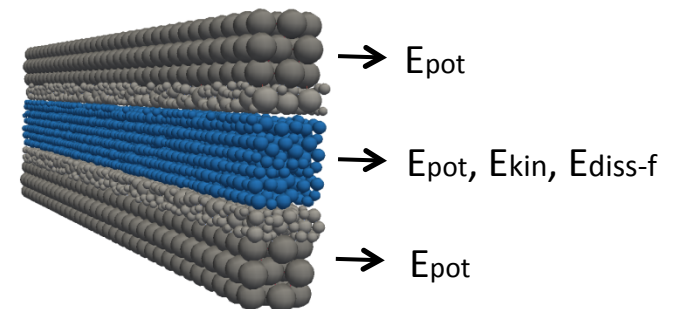


Stick-slip energy budget:

$$E_{\text{pot}} = E_{\text{kin}} \quad \sim 10\%$$

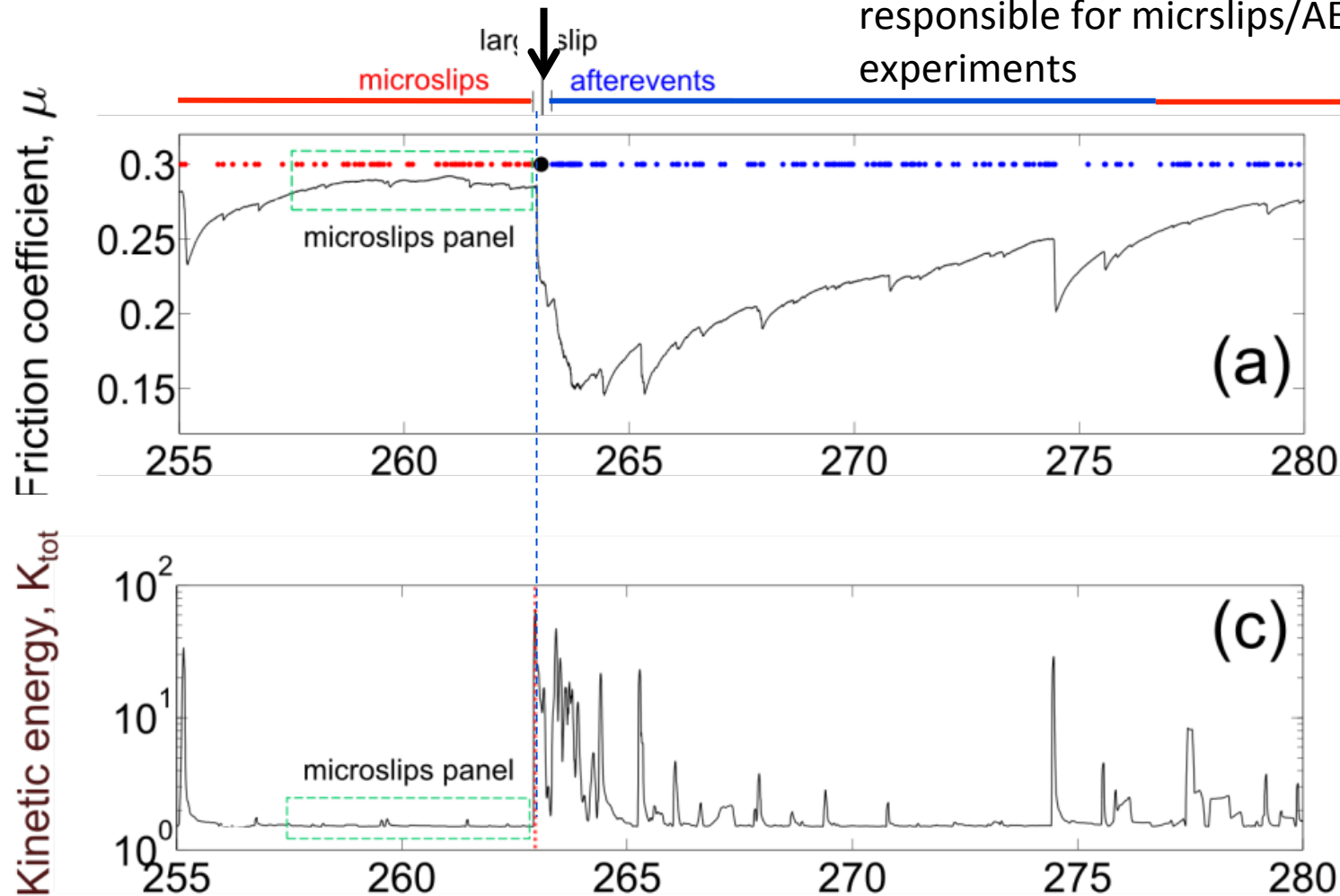
$$+ E_{\text{diss-friction}} \quad \sim 80\%$$

$$+ E_{\text{diss-viscous}}$$

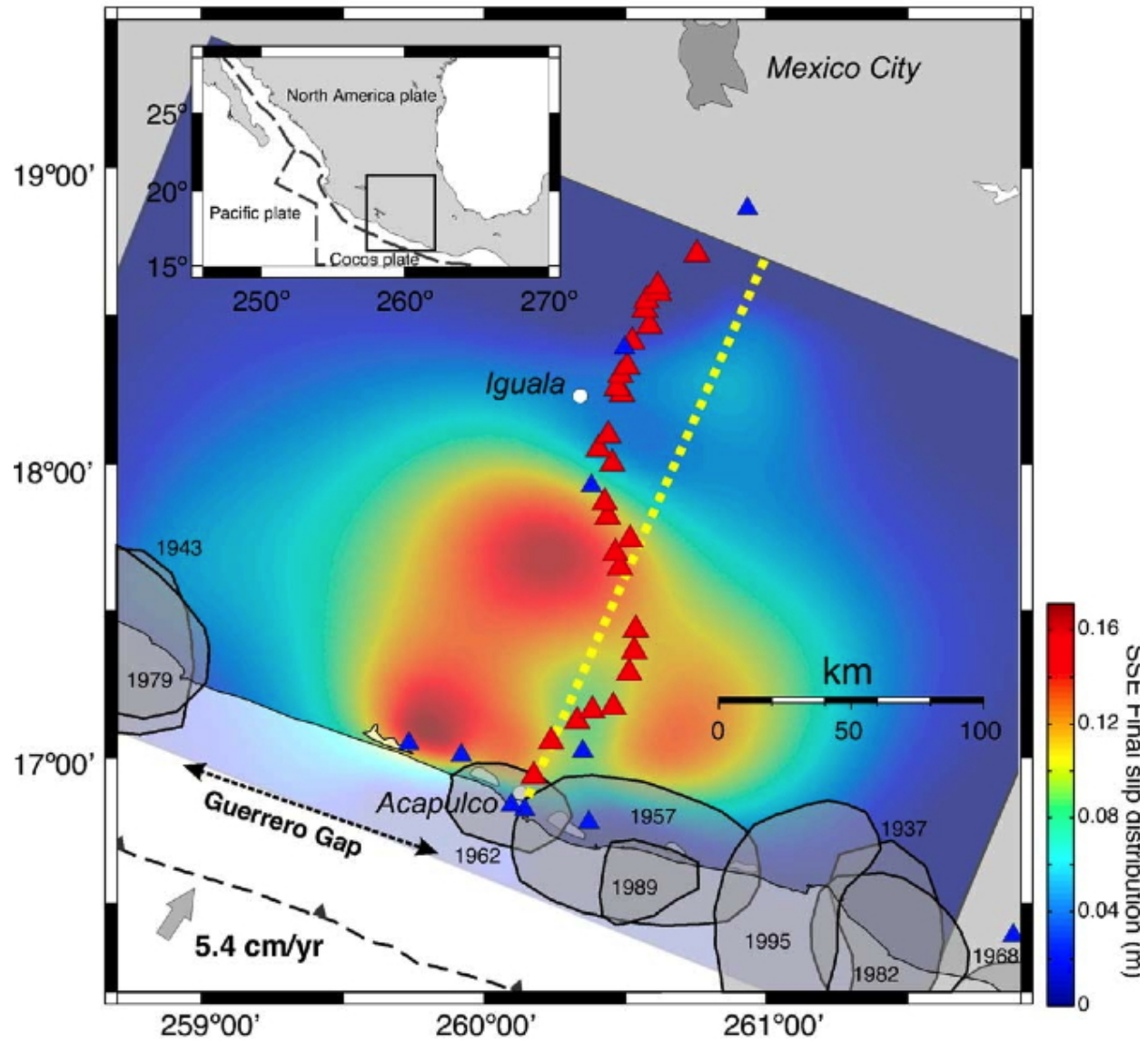


# Slips associated with kinetic energy release

We infer that grain rearrangements are responsible for microslips/AE in the lab experiments

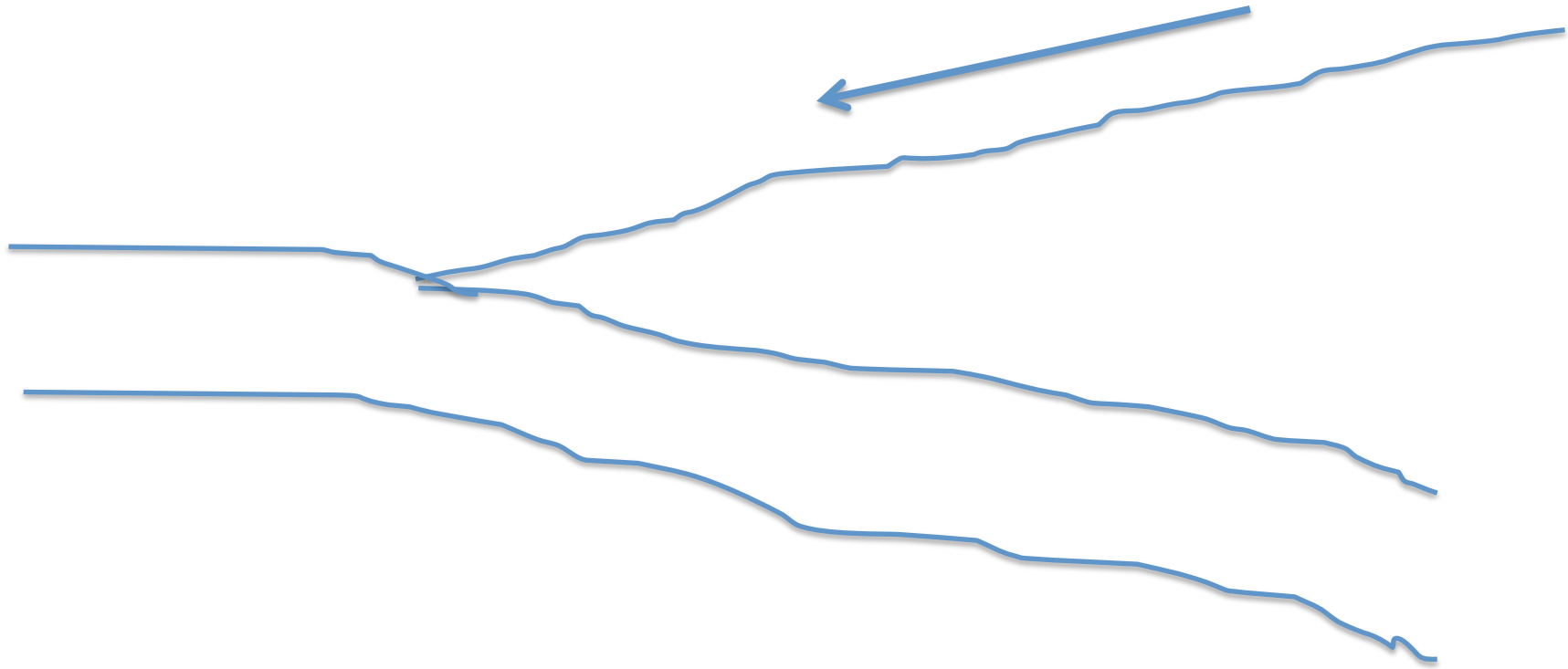


# Slow slip at Guerrero, Mexico

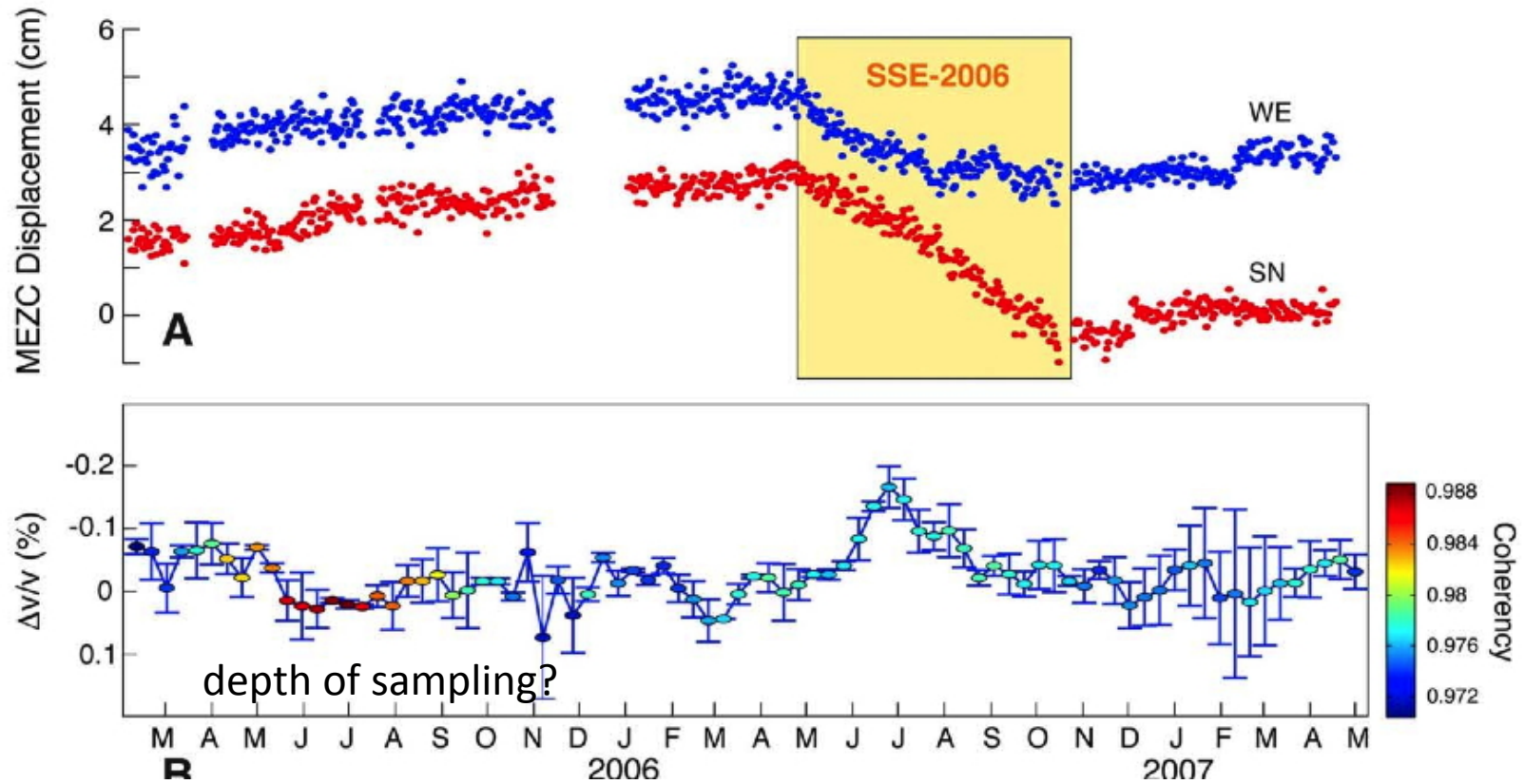


Rivet et al. GRL 2011

Slow slip



# Guerrero



Rivet, et al., GRL 2011

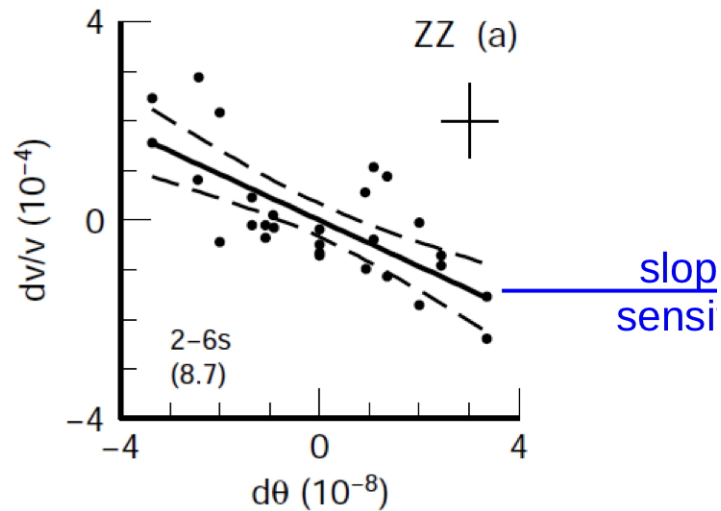
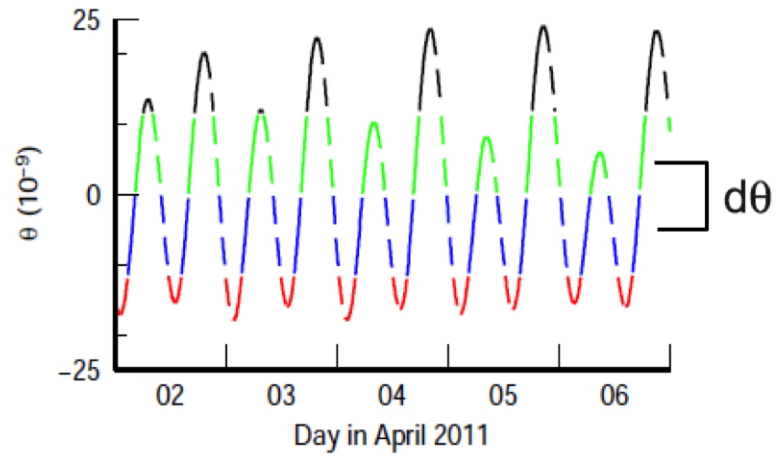


# Guerrero vs. Lab

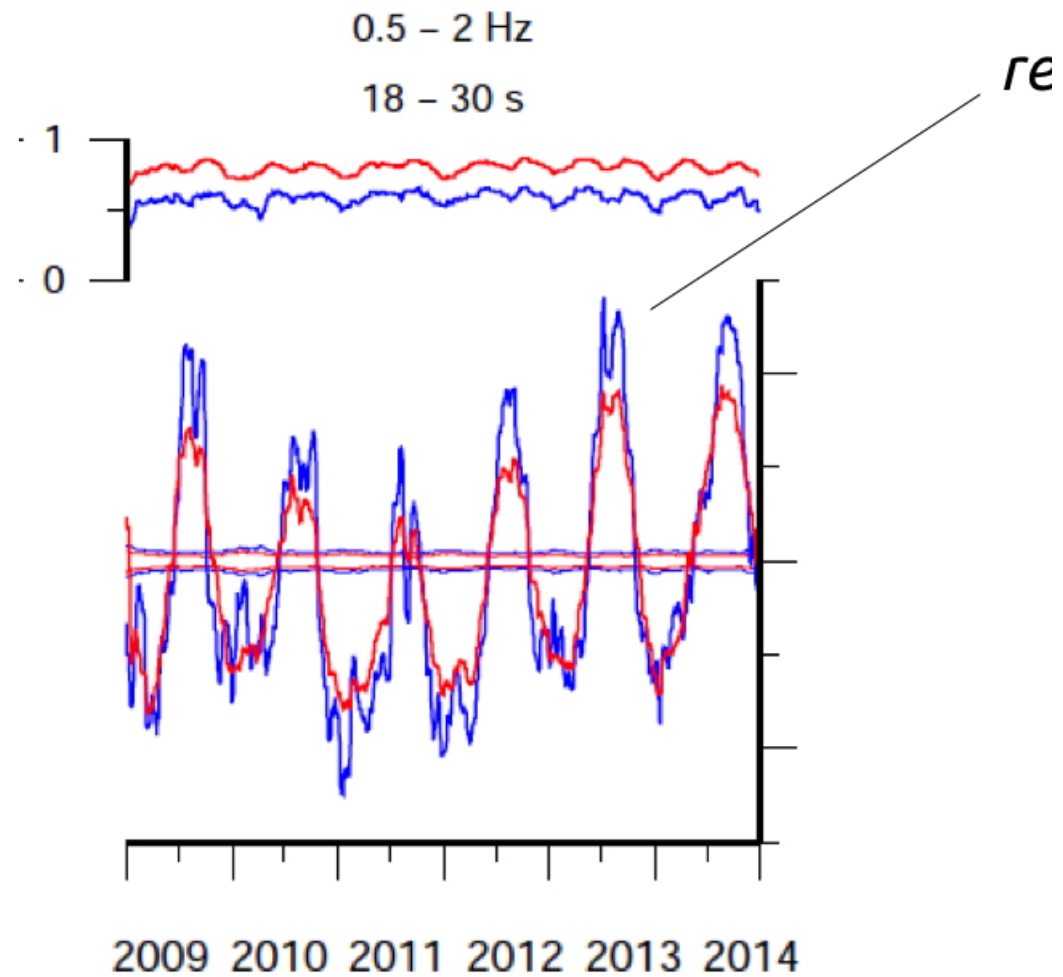
- Lab experiment in the granular layer
- Guerrero measurement in bulk material that may be highly fractured.

Similar mechanism?

# Hillers- probing earth tide elasticity



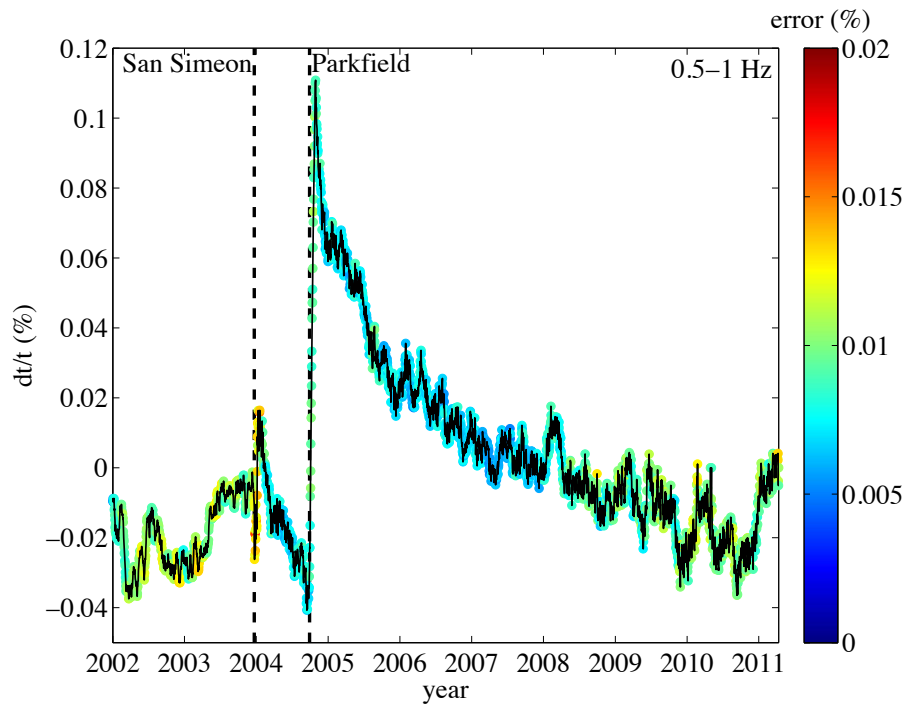
# Hillers – earth tides



Next: Parkfield Earthquake recovery study

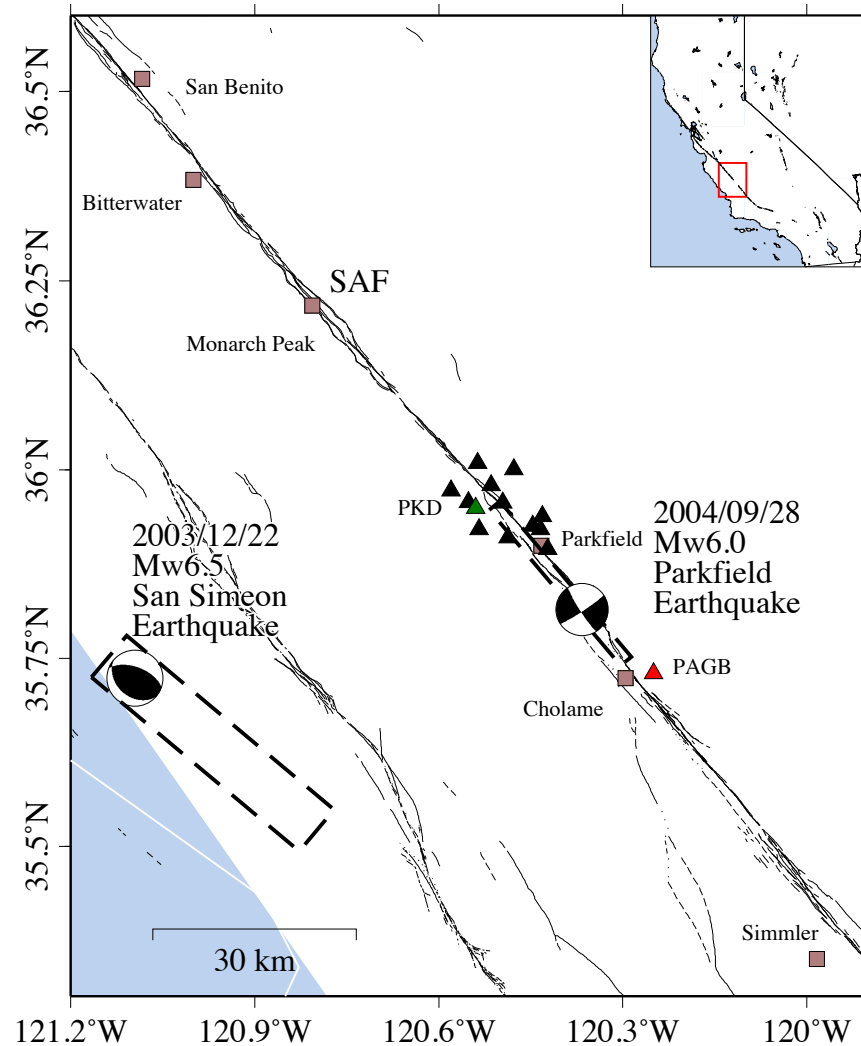
Wu et al., in preparation 2015

# Temporal Changes in Seismic Velocity

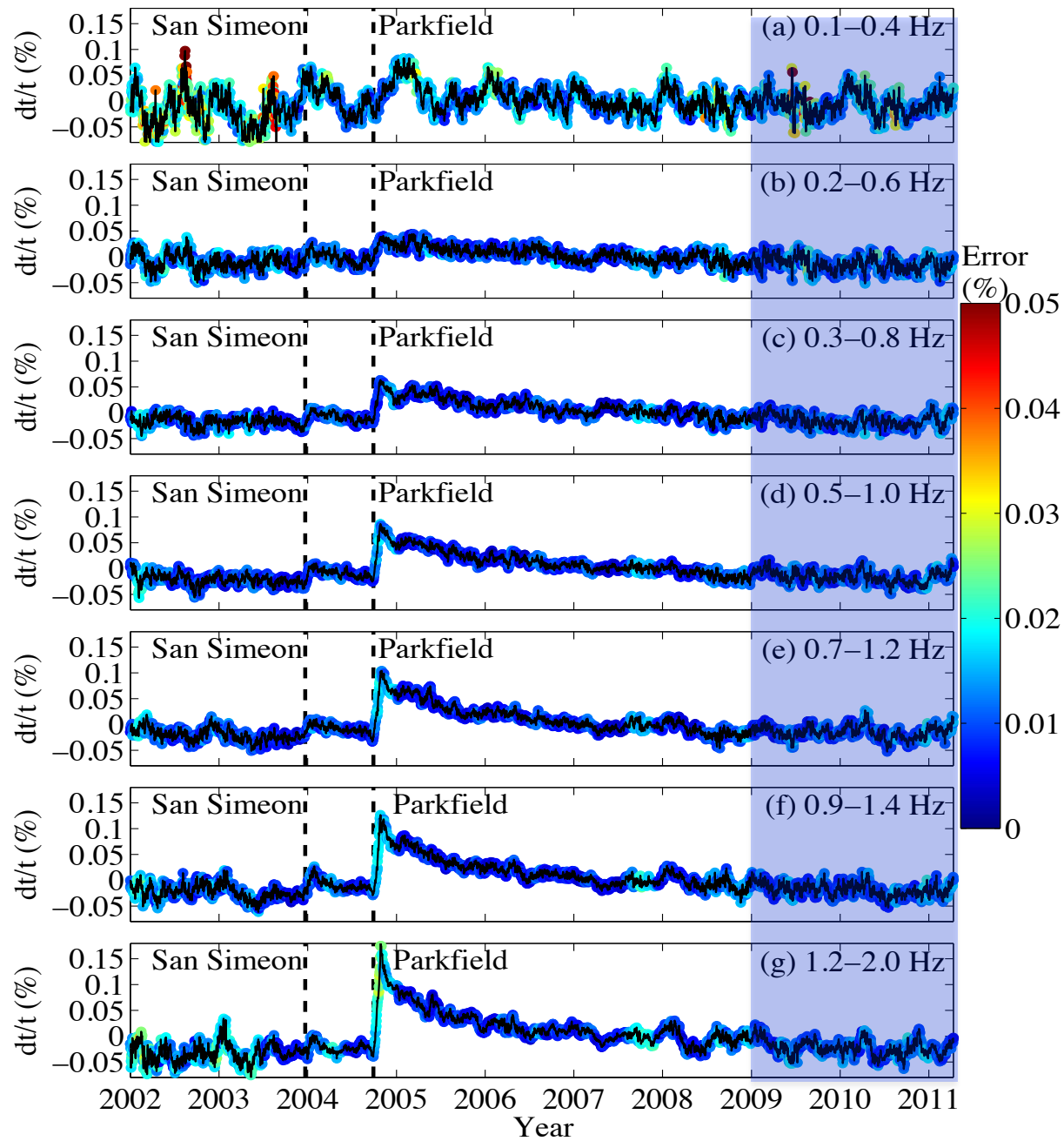


- Passive Noise Imaging
- Tracks changes in empirical Green's functions over time
- Strong shaking causes an increase in travel times due to a weakening of contacts in cracks and faults.

# How deep are the changes?

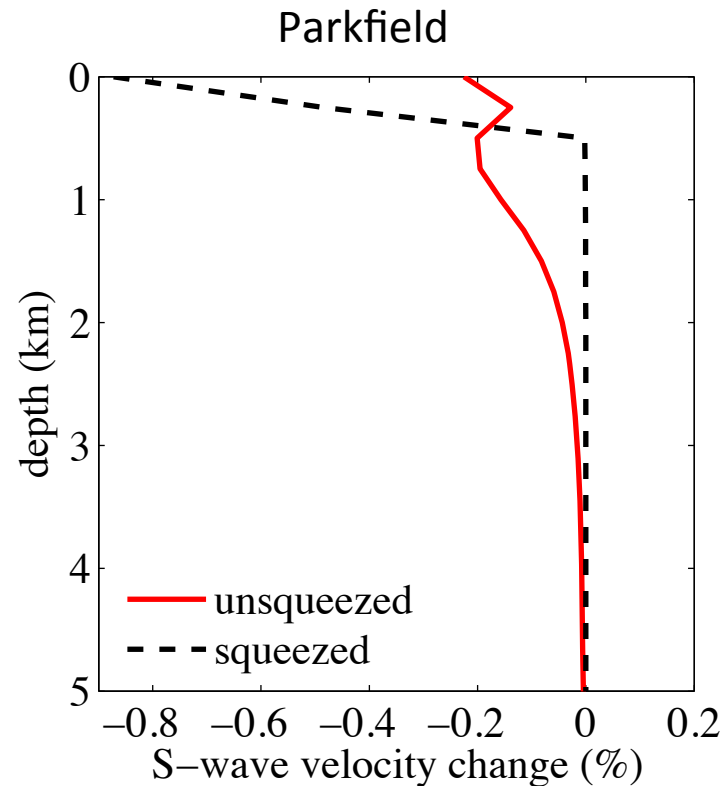
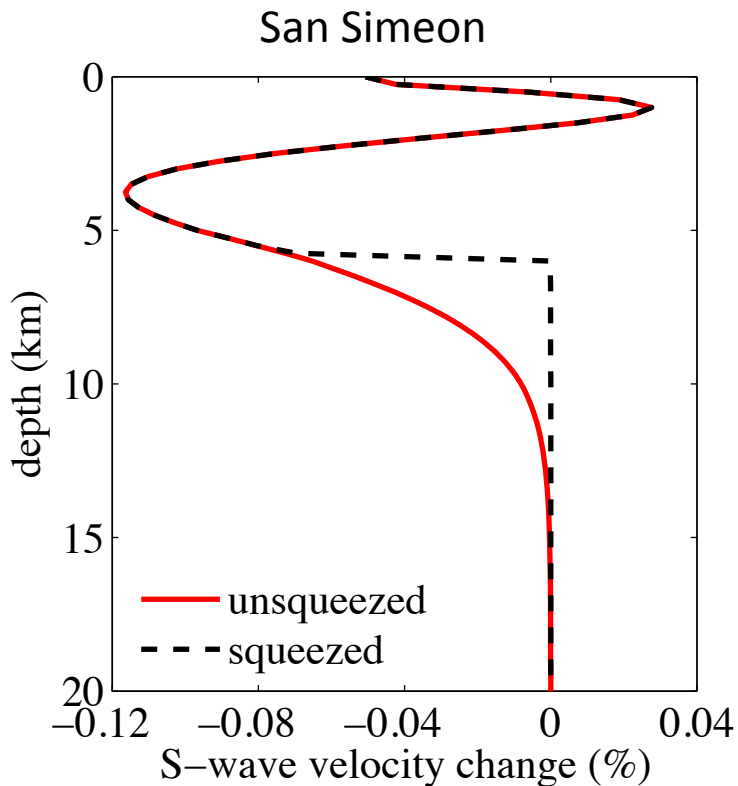


We apply passive noise in different frequency bands to find out....



# Surface wave inversion results from passive noise

The red curve is *unsqueezed* inversion result and the black curve is *squeezed* inversion result. Squeezed means we force the velocity changes to the shallower depths in the inversion, to estimate how deep the Vs must be to fit the observed phase velocity changes.





We saw an overview of Nonlinear elastic effects in earth materials, and nonlinear effects in Earth.