Ambient Seismic Vibrations in Seismology and Earthquake Engineering: Some Energy Considerations

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Outline of Presentation

Introduction → Motivation, Site effects, Site Characterization
Diffuse Fields ⟷ Multiple Diffraction
Equipartition of Energy in Dynamic Elasticity
- **Full Space, Half-Space → Experimental Verification**

Correlation Type Representation Theorem
Green's Function from the Average of Correlations
- **Dispersion Curves for Tomography of Alluvial Valleys**

Energy Densities from Average of Auto-Correlations
- **Deterministic Partition of Energy**
- **Ambient Seismic Noise and H/V Spectral Ratios**
- **Fast Calculation of H/V and Inversion (1D, 3D)**

Conclusions

Cargese, May 2015

Introduction

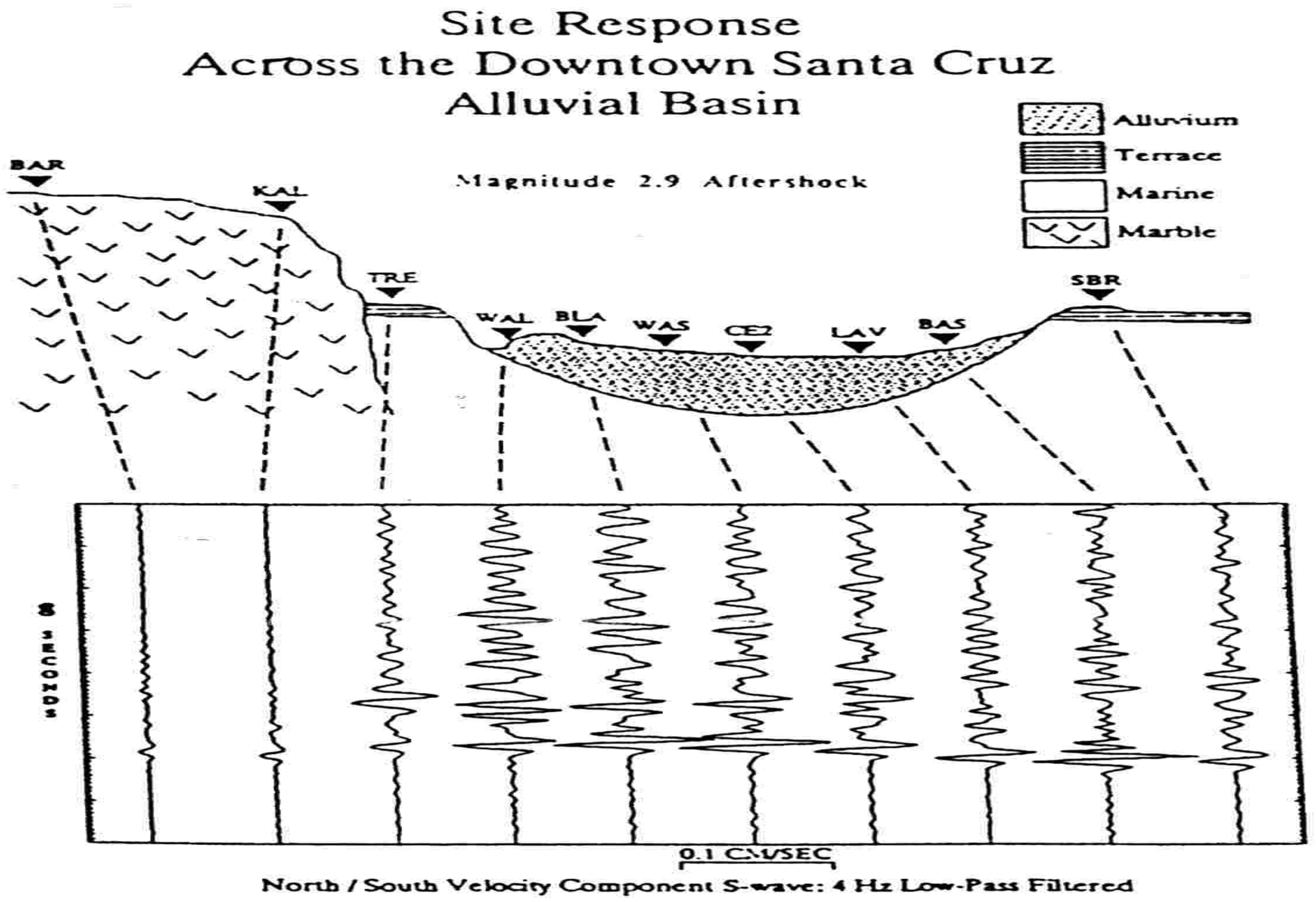
The last decade witnessed the specular advent of **ambient seismic vibrations** as a powerful tool for imaging Earth structure at many different scales.

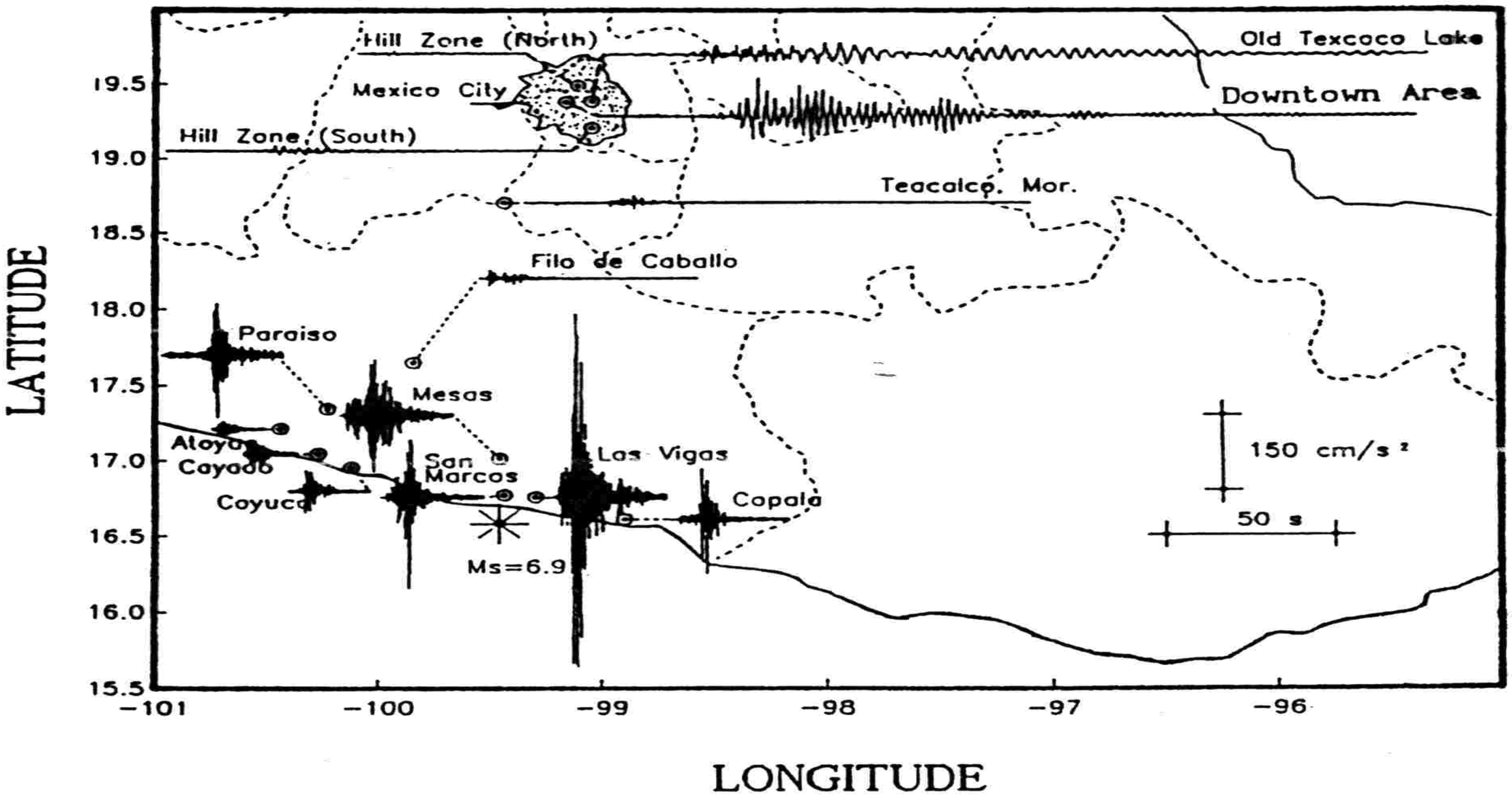
All this lead to improvements of methods that utilize ambient noise. They are now applied to more data sets with better theoretical understanding and are useful for a wide range of applications in **Seismology** including time-dependent imaging for sundry physical processes like underwater acoustics, helioseismology, and structural health monitoring, to cite a few.

These advances powered many applications in **Earthquake Engineering** as well. In addition to improved performance of noise-based imaging, innovative applications emerged to establish both the dominant period and the velocity structure in order to compute seismic response.

Loma-Prieta (U.S.A.) Earthquake M = 7.1

October 17th, 1989





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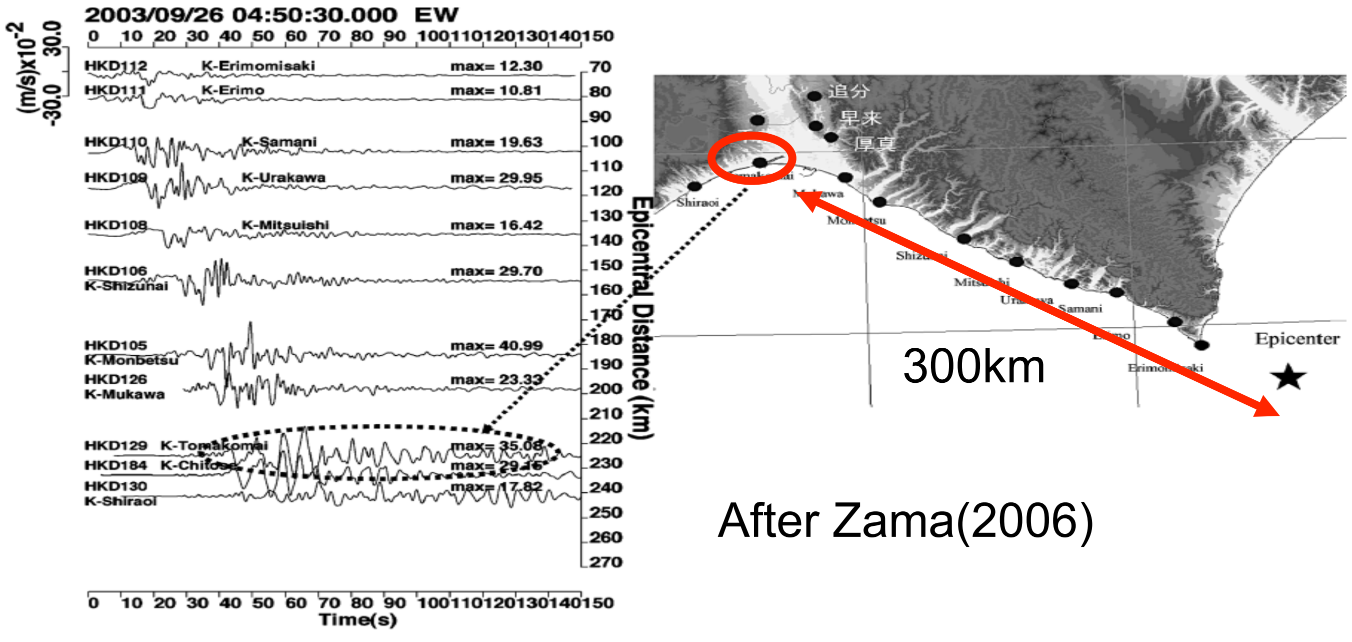

Fig. 2 Velocity waveforms at K-net stations from Erimo to Tomakomai during the 2003 Tokachi-oki earthquake

2003年 十勝沖地震により苫小牧市の製油所で発生したナフサタンクの全面火災
Full-surface fire in an oil refinery's naphtha tank in Tomakomai after the 2003 Tokachi-oki earthquake

2003 TOKACHI-OKI EARTHQUAKE Mw 8.3

Significant long-period ground motions of very long duration were recorded in the Yufutsu basin. Oil storage tanks suffered great damage at Tomakomai because of sloshing associated with ground motions.

The deep extent of the sedimentary basin generated long-period ground motions with a duration of several hundred seconds.

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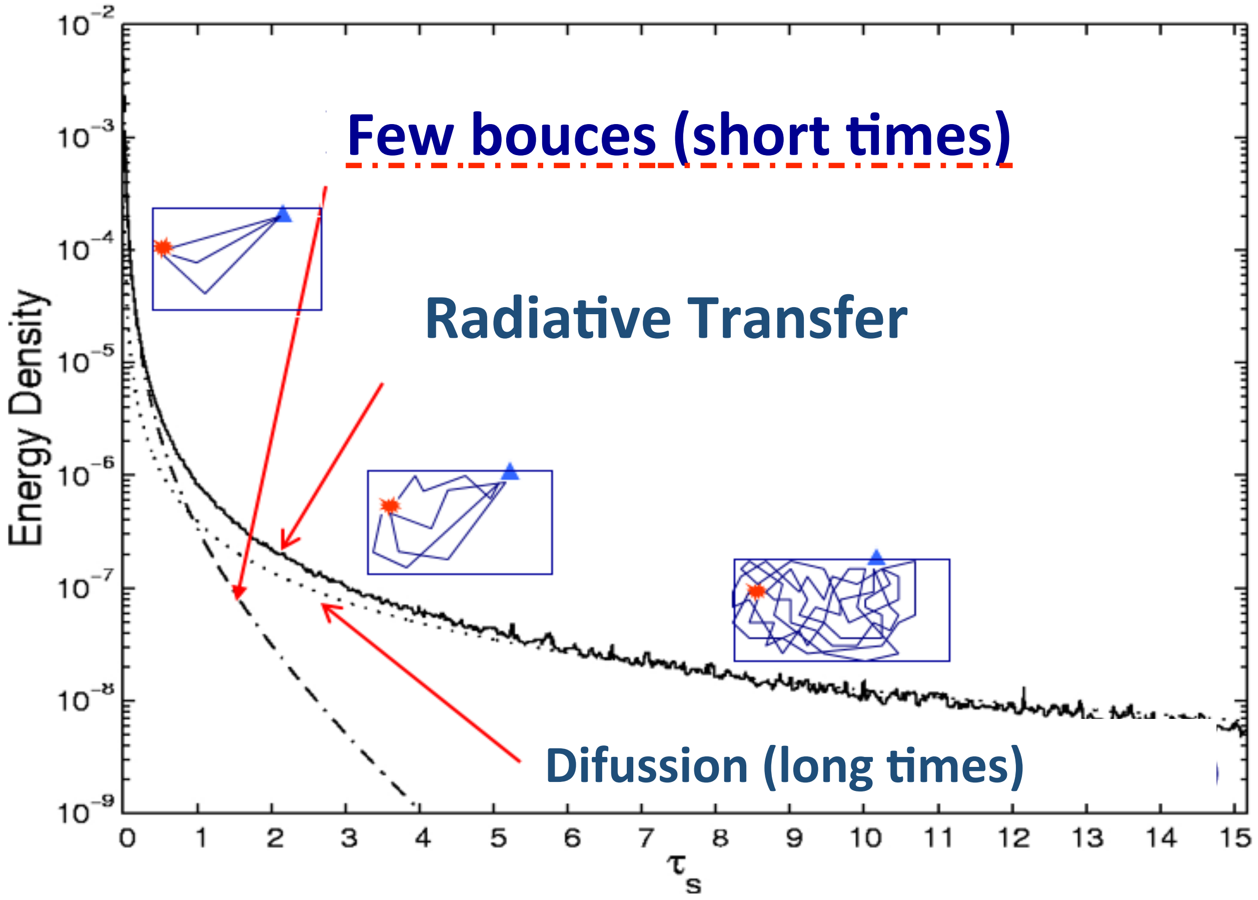
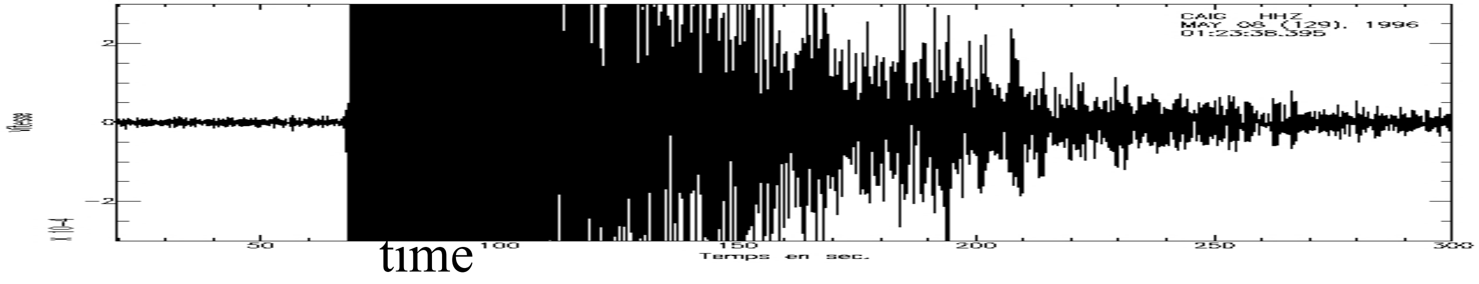
Propagation regimes & Energy Density decay





Radiative Transfer Theory

Originated in astrophysics by **Chandrasheckar** & others (40´s & 50's). Introduced to sesmology by **R.S. Wu** (1985) and developepend by **K. Aki** and **Y. Zeng**, **H. Sato**, **K. Mayeda**, **M. Campillo**, **L. Margerin**, **A. Gusev** and others.

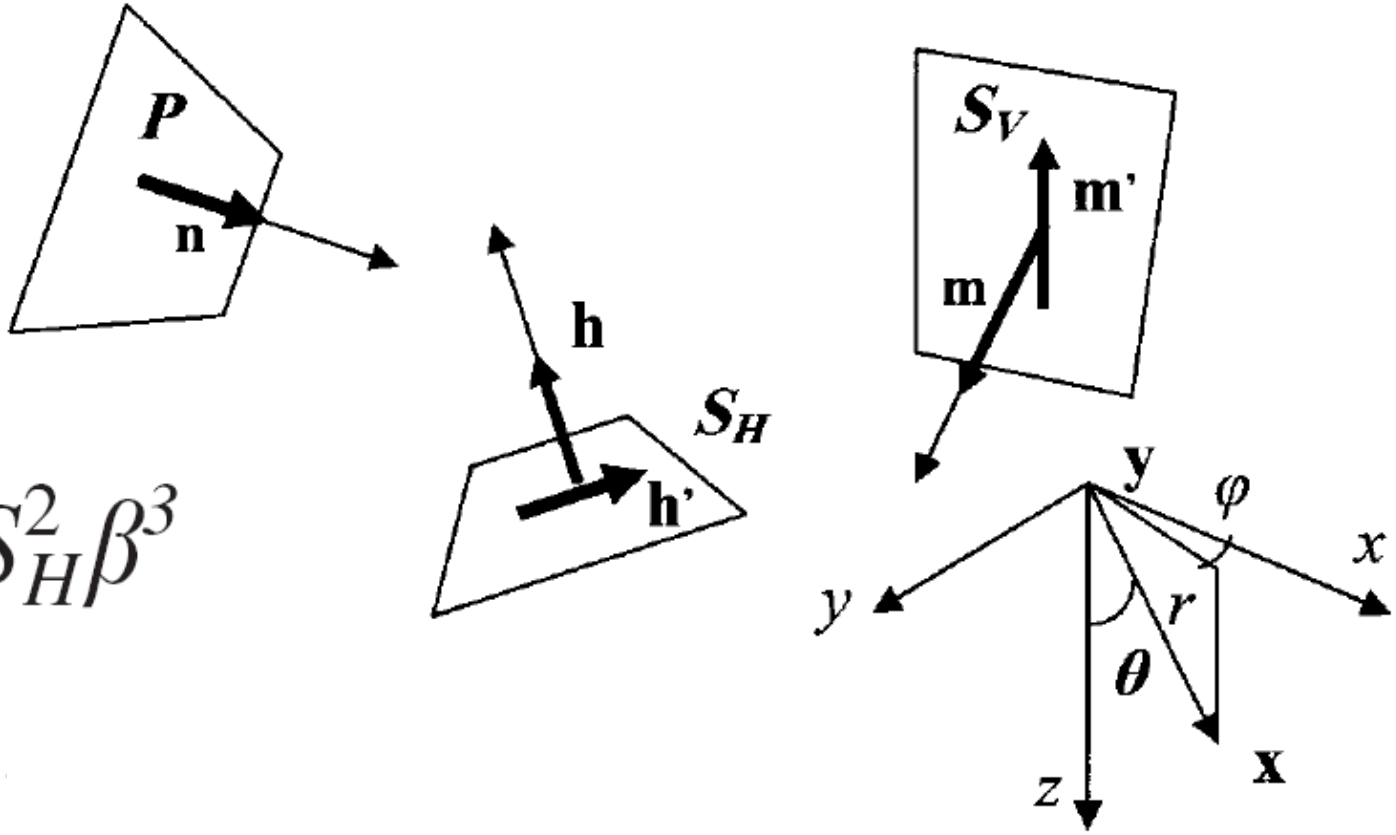
- **Sato H.** & **M. Fehler** (1998). *Wave propagation and scattering in the heterogeneous Earth*, Academic Press, Cambridge, Mass. **2nd Edition..!**
- **Dmowska R., H. Sato** & **M. Fehler** (2008) (Eds) Vol. 50 of *Advances in Geophysics*, Academic Press, Cambridge, Mass.

$$\frac{E_S}{E_P} = \frac{\alpha^2}{\beta^2} \quad \text{in 2D;} \qquad \frac{E_S}{E_P} = \frac{2\alpha^3}{\beta^3} \quad \text{in 3D}$$

Predictions for Elastic Diffuse Fields

Multiple Scattering → Energy Equipartition

$$\left\langle u_i(\mathbf{x}_\mathbf{A},\omega)u_j^*(\mathbf{x}_\mathbf{B},\omega)\right\rangle = -2\pi E_S k^{-3}\,\mathrm{Im}\left[G_{ij}(\mathbf{x}_\mathbf{A},\mathbf{x}_\mathbf{B},\omega)\right]$$



$$P^2\alpha^3 = S_V^2\beta^3 = S_H^2\beta^3$$

$$E_S/E_P = 2\alpha^3/\beta^3$$

Sánchez-Sesma and Campillo (2006) BSSA

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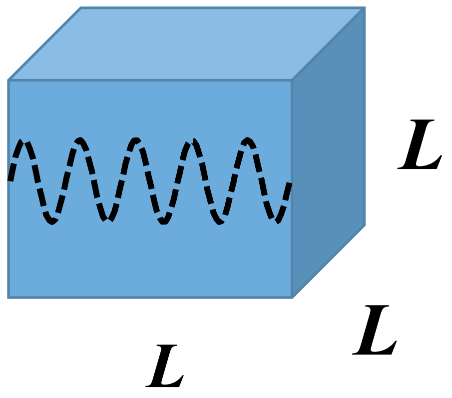
Energy Equipartition Principle
Infinite Space, Volume modes



Assume stationary P waves inside a finite region (e.g. A cube of side L >> the wave lenght) and Dirichlet boundary conditions:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = \frac{1}{\alpha^2}\frac{\partial^2 \Phi}{\partial t^2} \qquad \Phi = 0$$

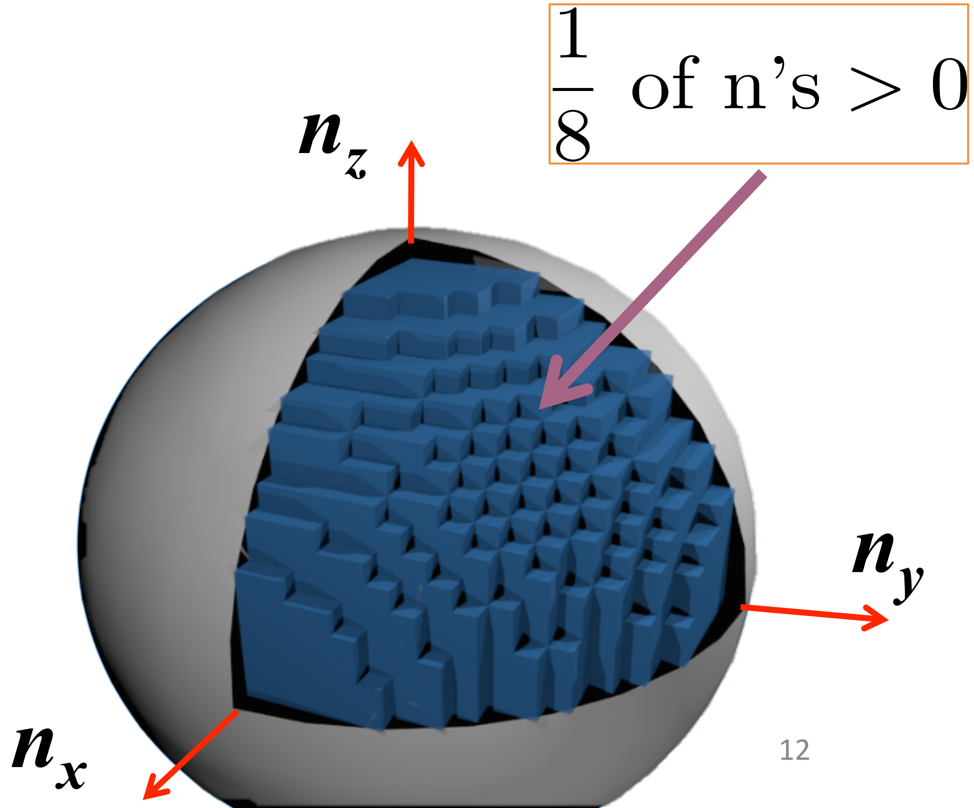
Then we can admit a modal solution of the form,

$$\Phi = A \sin\left(\frac{n_x \pi x}{L}\right) \sin\left(\frac{n_y \pi y}{L}\right) \sin\left(\frac{n_z \pi z}{L}\right) \sin(\omega t)$$

with

$$n_x^2 + n_y^2 + n_z^2 = \left[\frac{L\omega}{\pi\alpha}\right]^2$$

Modal Sphere



According to the Principle of Equipartition of Energy, the energy associated to every state is proportional to the density of modes at a given frequency.

$$n_P = \frac{1}{8} \times \frac{4\pi}{3} \left(\frac{L\omega}{\pi\alpha} \right)^3 = \frac{\pi}{6} (n_x^2 + n_x^2 + n_x^2)^{3/2}$$

$$dn_P = (1/2\pi^2)\omega^2 \alpha^{-3} V d\omega$$

$$dn_S = 2(1/2\pi^2)\omega^2 \beta^{-3} V d\omega,$$

In a diffuse field the energy associated to elastic waves in a volume is arranged according to the **Principle of Equipartition**.

The energy of each state is proportional to the density of modes in a given frequency band.

The ratio of the S and P wave energy is:

$$\frac{E_S}{E_P} = \frac{q \times dn_S}{q \times dn_P} = 2\alpha^3/\beta^3 = 2R^3$$

Weaver (1982)

$$n_P = \frac{1}{8} \times \frac{4\pi}{3} \left(\frac{L\omega}{\pi\alpha} \right)^3 = \frac{\pi}{6} (n_x^2 + n_x^2 + n_x^2)^{3/2}$$

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The ratio of the S and P wave energy is:

Proportionality constant

$$\frac{E_S}{E_P} = \frac{q \times dn_S}{q \times dn_P} = 2\alpha^3/\beta^3 = 2R^3$$

Weaver (1982)

$$n^{1D}\frac{\lambda}{2} = L \Rightarrow n^{1D} = \frac{L\omega}{\pi c} \Rightarrow dn^{1D} = \frac{1}{\pi c} L\, d\omega,$$

$$n^{2D} = \frac{1}{4}\pi\left(\frac{L\omega}{\pi c}\right)^2 \Rightarrow dn^{2D} = \frac{1}{2\pi c^2}\omega A d\omega,\ \ A = L^2$$

$$n^{3D} = \frac{1}{8}\frac{4\pi}{3}\left(\frac{L\omega}{\pi c}\right)^3 \Rightarrow dn^{3D} = \frac{1}{2\pi^2 c^3}\omega^2 V d\omega, V = L^3$$

$$\xi_P = \frac{1}{1+2R^3}\xi$$

$$\xi_{SH} = \frac{R^3}{1+2R^3}\xi$$

$$\xi_{SV} = \frac{R^3}{1+2R^3}\xi$$

Weaver (1982)

$$\xi_1 = \frac{1}{3}\xi_P + \frac{1}{6}\xi_{SV} + \frac{1}{2}\xi_{SH} = \frac{1}{3}\xi$$

$$\xi_2 = \frac{1}{3}\xi_P + \frac{1}{6}\xi_{SV} + \frac{1}{2}\xi_{SH} = \frac{1}{3}\xi$$

$$\xi_3 = \frac{1}{3}\xi_P + \frac{2}{3}\xi_{SV} = \frac{1}{3}\xi$$

Sánchez-Sesma & Campillo (2006)

$$\xi_P + \xi_{SH} + \xi_{SV} = \xi = \xi_1 + \xi_2 + \xi_3$$

Half-Space, Rayleigh's Modes

Assume now stationary Rayleigh waves associated to the free-surface (for instance, a square with side L >> wave length).

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \frac{1}{c_R^2}\frac{\partial^2 w}{\partial t^2}$$

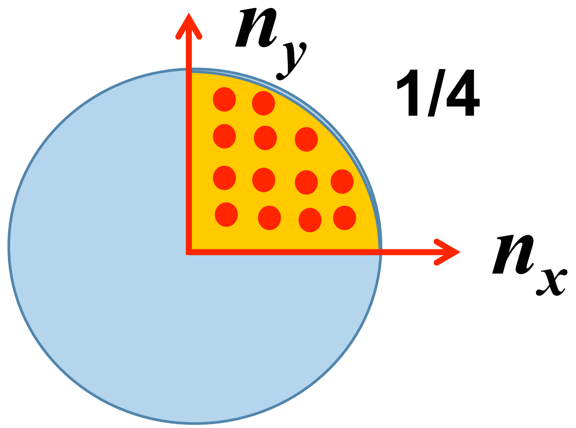
Consider also Dirichlet boundary conditions (w=0 en |x|=|y|=0,L)

$$w = \text{sen}\frac{n_x \pi x}{L} \times \text{sen}\frac{n_y \pi y}{L} \times \text{sen}\,\omega t \rightarrow n_x^2 + n_y^2 = \left[\frac{L\omega}{\pi c_R}\right]^2$$

Modal Circle

$$n_R = \frac{\pi}{4}(n_x^2 + n_x^2) = \frac{\pi}{4}\left(\frac{L\omega}{\pi c_R}\right)^2 = \frac{1}{4\pi}\frac{\omega^2 A}{c_R^2}$$

$$dn_R = \frac{1}{2\pi}\frac{\omega A}{c_R^2}d\omega$$



$$E_S = 2\frac{\alpha^3}{\beta^3}E_P = 2R^3E_P$$

$$E_R = q \times dn_R = \frac{q}{2\pi}\times\frac{\omega A}{c_R^2}d\omega$$

$$E = E_P + E_S = (1+2R^3)E_P$$

$$q = \xi_P \times 2\pi^2\frac{\alpha^3}{\omega^2}\times\frac{1}{d\omega}$$

$$\xi = E/V, \xi_P = E_P/V, \xi_S = E_S/V, \xi_R = E_R/A$$

$$\xi_P = \frac{1}{1+2R^3}\xi$$

$$\xi_S = \frac{2R^3}{1+2R^3}\xi$$

$$\xi_R = \frac{\pi\beta}{\omega}\left(\frac{\beta}{c_R}\right)^2\frac{R^3}{1+2R^3}\xi$$

Weaver (1985)
Perton *et al.* (2009)

3D

$$E_S = 2\frac{\alpha^3}{\beta^3}E_P = 2R^3E_P$$

$$E_R = q \times dn_R = \frac{q}{2\pi} \times \frac{\omega A}{c_R^2}d\omega$$

$$E = E_P + E_S = (1+2R^3)E_P$$

$$q = \xi_P \times 2\pi^2\frac{\alpha^3}{\omega^2} \times \frac{1}{d\omega}$$

Proportionality constant

$$\xi = E/V, \xi_P = E_P/V, \xi_S = E_S/V, \xi_R = E_R/A$$

$$\xi_P = \frac{1}{1+2R^3}\xi$$

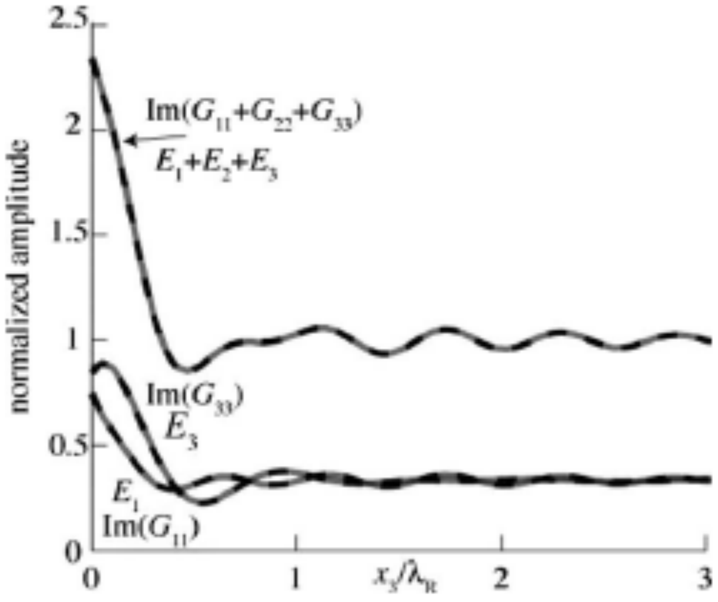
$$\xi_S = \frac{2R^3}{1+2R^3}\xi$$

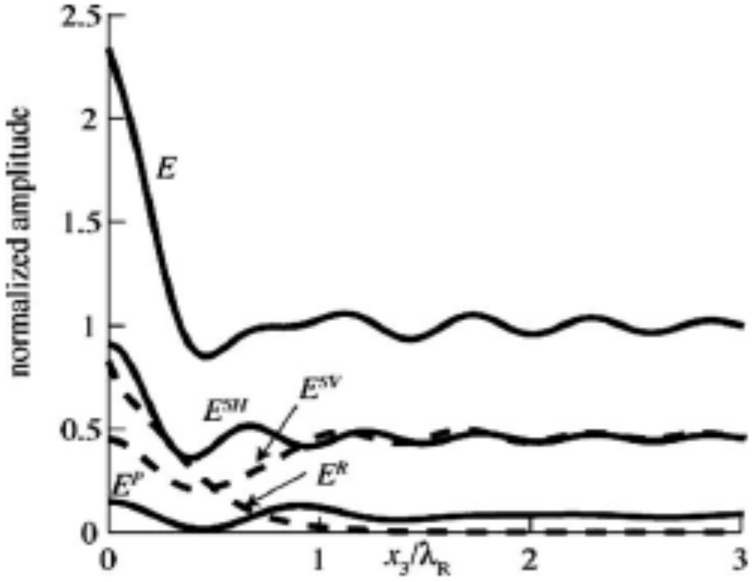
$$\xi_R = \frac{\pi\beta}{\omega}\left(\frac{\beta}{c_R}\right)^2\frac{R^3}{1+2R^3}\xi$$

Weaver (1985)
Perton *et al.* (2009)

3D

Elastic Half-space (3D)



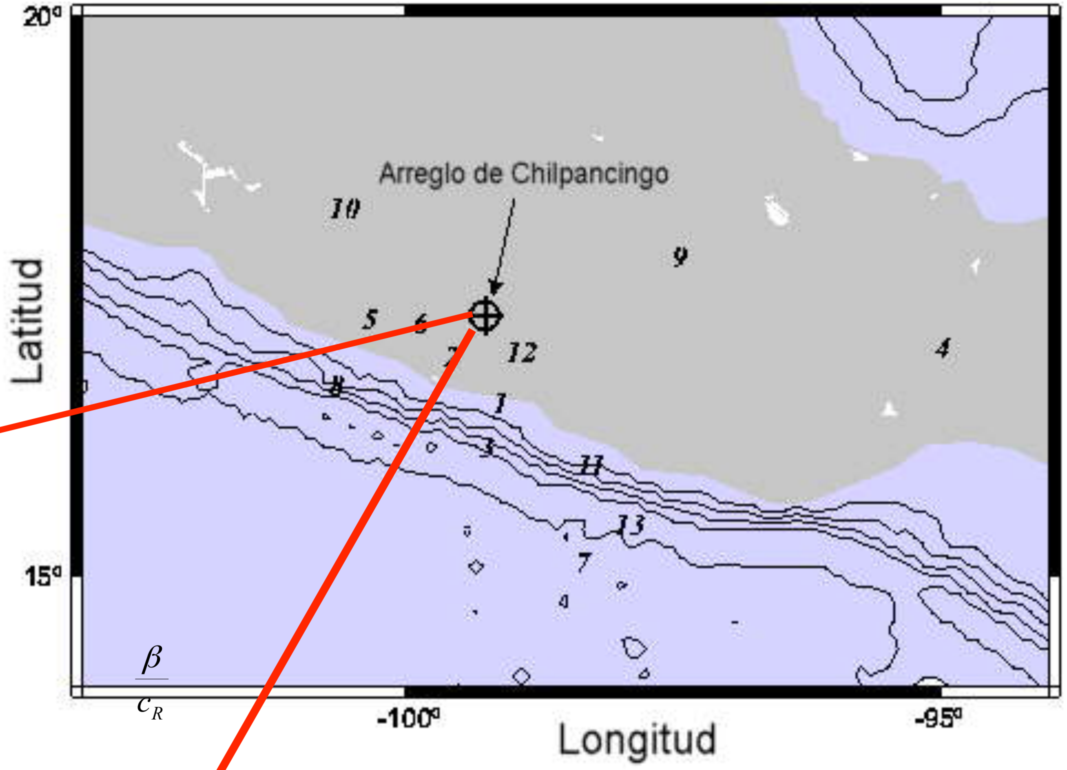


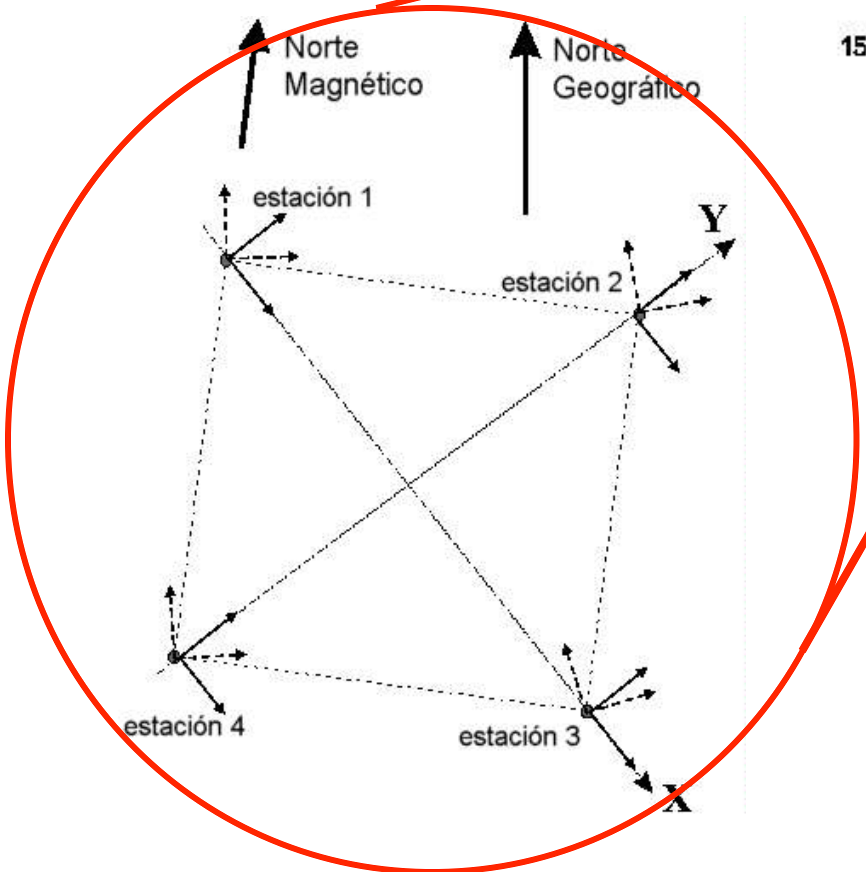
Perton *et al* (2009)

This result shows two ways the equipartition can occur: (1) *à la* Maxwell or (2) *à la* Weaver

The Principle of Equipartition of Energy in Elastodynamics

Experimental Verification

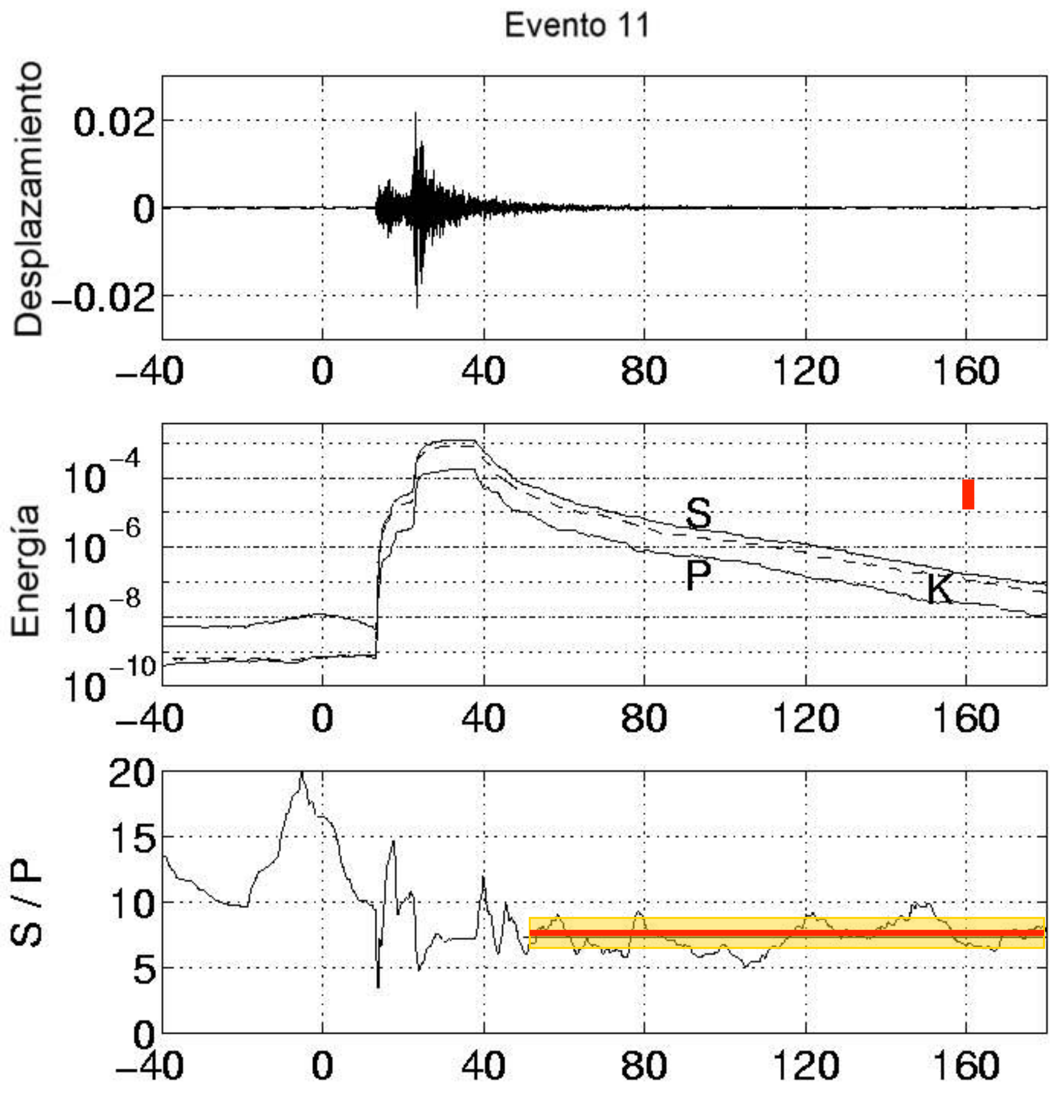




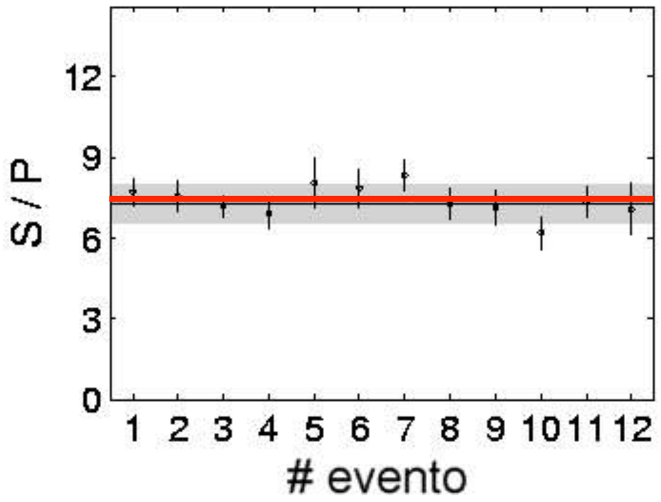
50 m aperture

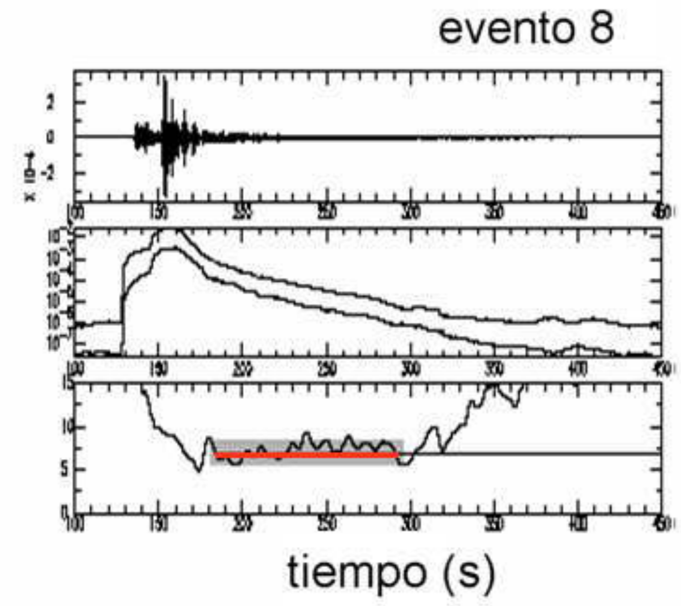
Campillo et al. (1999); Shapiro et al. (2000)

Experimental Verification



Hennino *et al.* (2001)





Energy ratio	Data $z = 0$	Theory $z = 0$	Theory $z = \infty$	Theory Rayleigh only $z = 0$	Theory Bulk only $z = 0$
S/P	7.30 ± 0.72	7.19	10.39	6.460	9.76
$K/(S + P)$	0.65 ± 0.08	0.534	1	0.268	1.19
$I/(S + P)$	-0.62 ± 0.03	-0.167	0	-1.464	-0.336
H^2/V^2	2.56 ± 0.36	1.774	2	0.464	4.49
X^2/Y^2	0.60 ± 0.20	1	1	1	1

Hennino *et al.* (2001)

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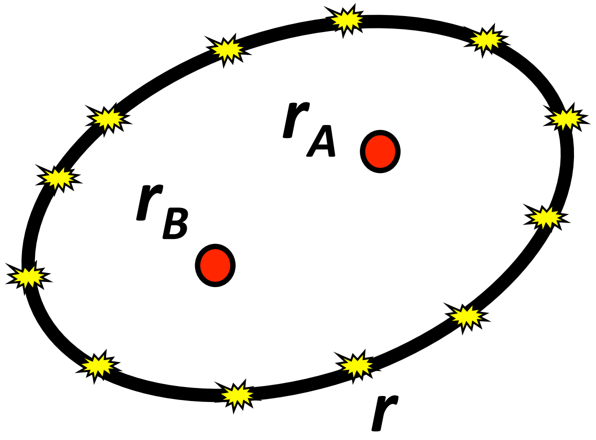
Correlation type Representation Theorem

$$2i\,\mathrm{Im}\left[G_{ij}(\mathbf{r}_A,\mathbf{r}_B)\right] = -\oint \left\{ G_{il}(\mathbf{r}_A,\mathbf{r})T^*_{lj}(\mathbf{r},\mathbf{r}_B) - G^*_{jl}(\mathbf{r}_B,\mathbf{r})T_{li}(\mathbf{r},\mathbf{r}_A) \right\} dS$$



Weaver & Lobkis (2004), Wapenaar (2004), Van Manen, Curtis & Robertson (2006)

$$\frac{E_S}{E_P} = \begin{cases} \dfrac{\alpha^2}{\beta^2} \text{ in } 2D \\ \dfrac{2\alpha^3}{\beta^3} \text{ in } 3D \end{cases}$$

¡ Equipartition !

$$\left\langle u_i(\mathbf{x_A},\omega)u^*_j(\mathbf{x_B},\omega)\right\rangle = -2\pi E_S k^{-3}\,\mathrm{Im}\left[G_{ij}(\mathbf{x_A},\mathbf{x_B},\omega)\right]$$

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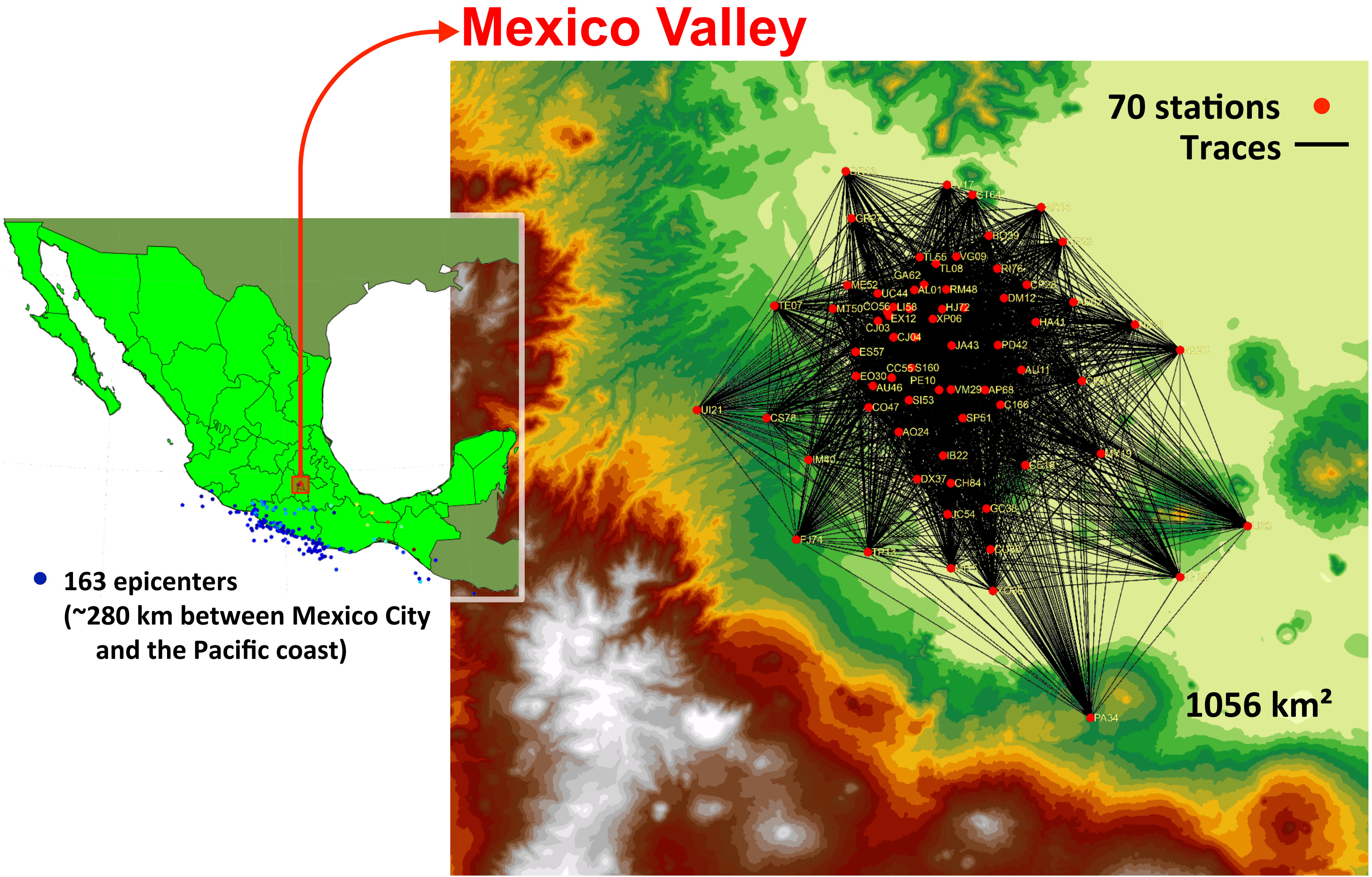
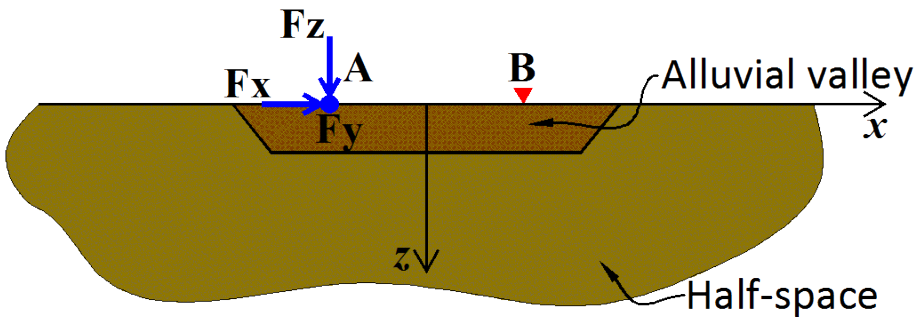
SURFACE WAVE TOMOGRAPHY

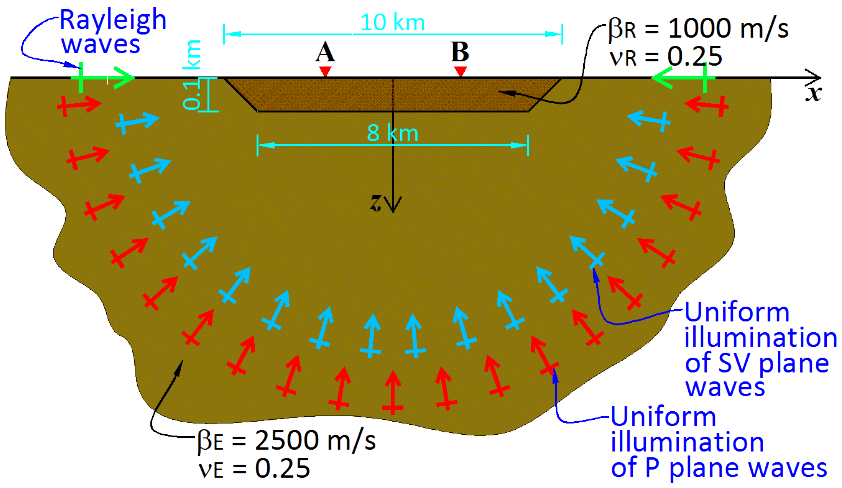


3 WAYS FOR RETRIEVING THE GREEN'S FUNCTION

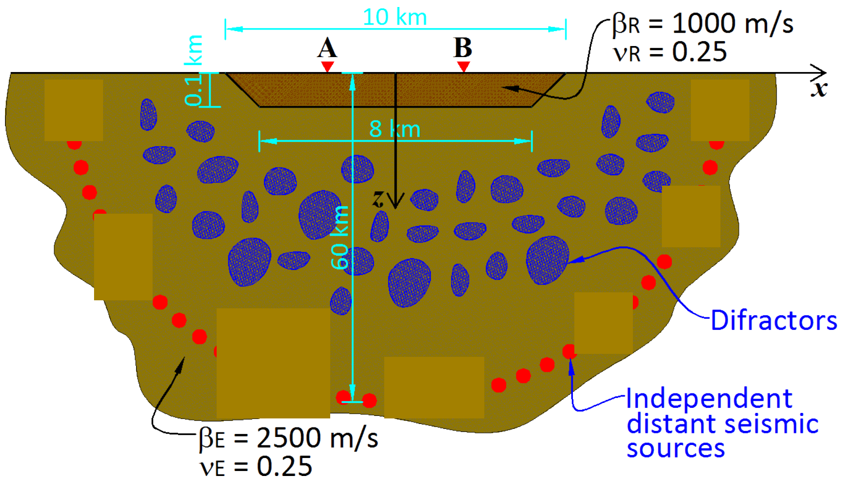
By definition: Impulsive Response



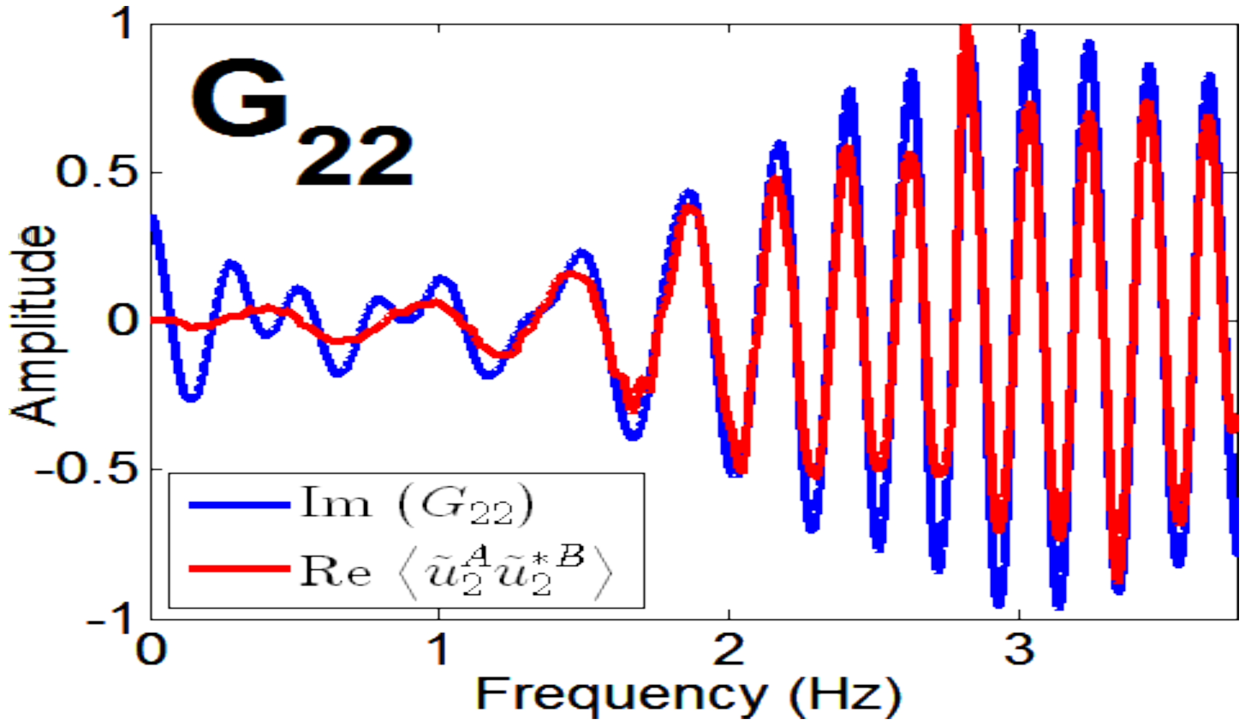
Plane waves: An equipartitioned cocktail of P,SV and Rayleigh waves



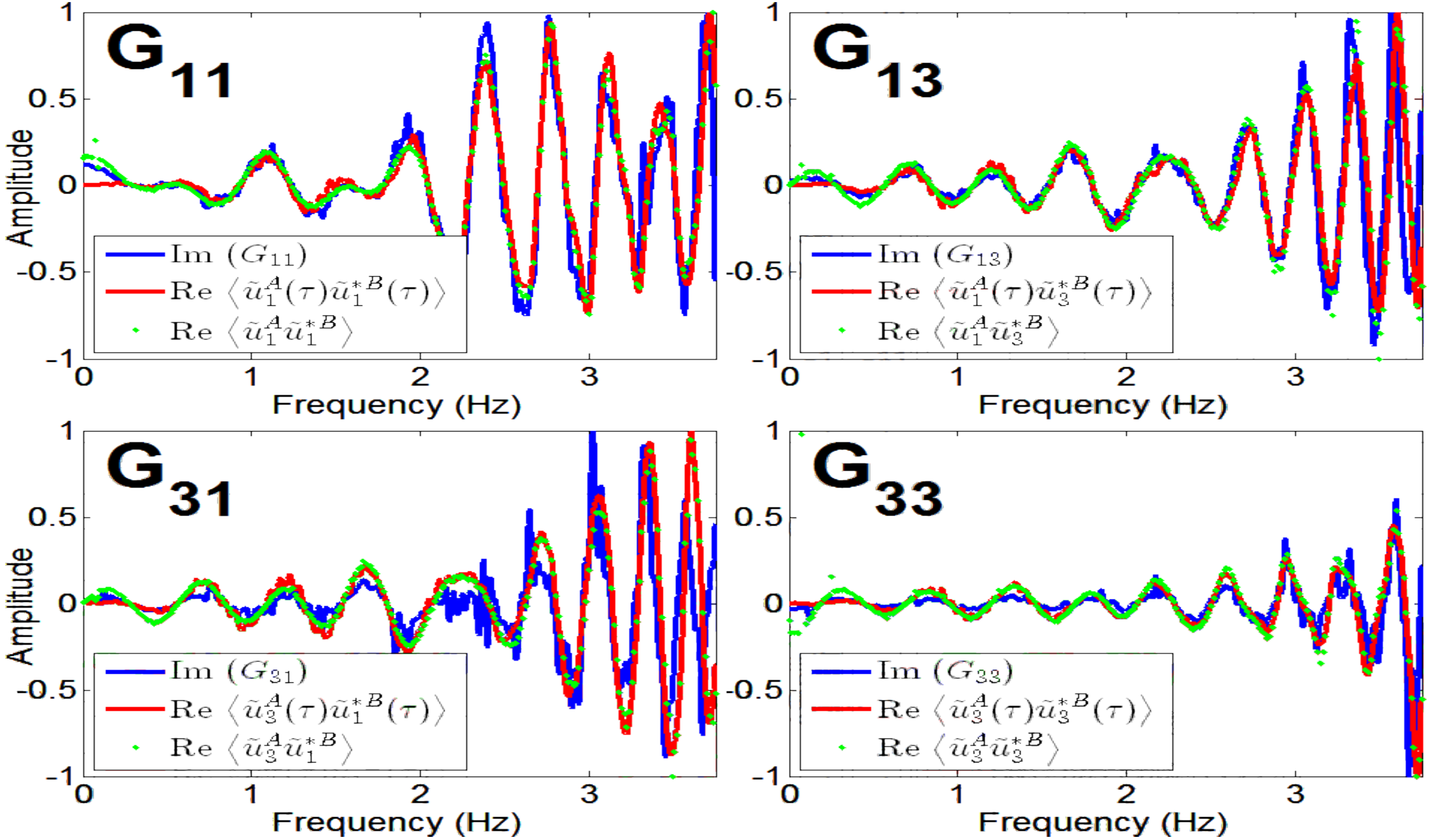
Independent distant seismic sources and lots of randomly placed difractors



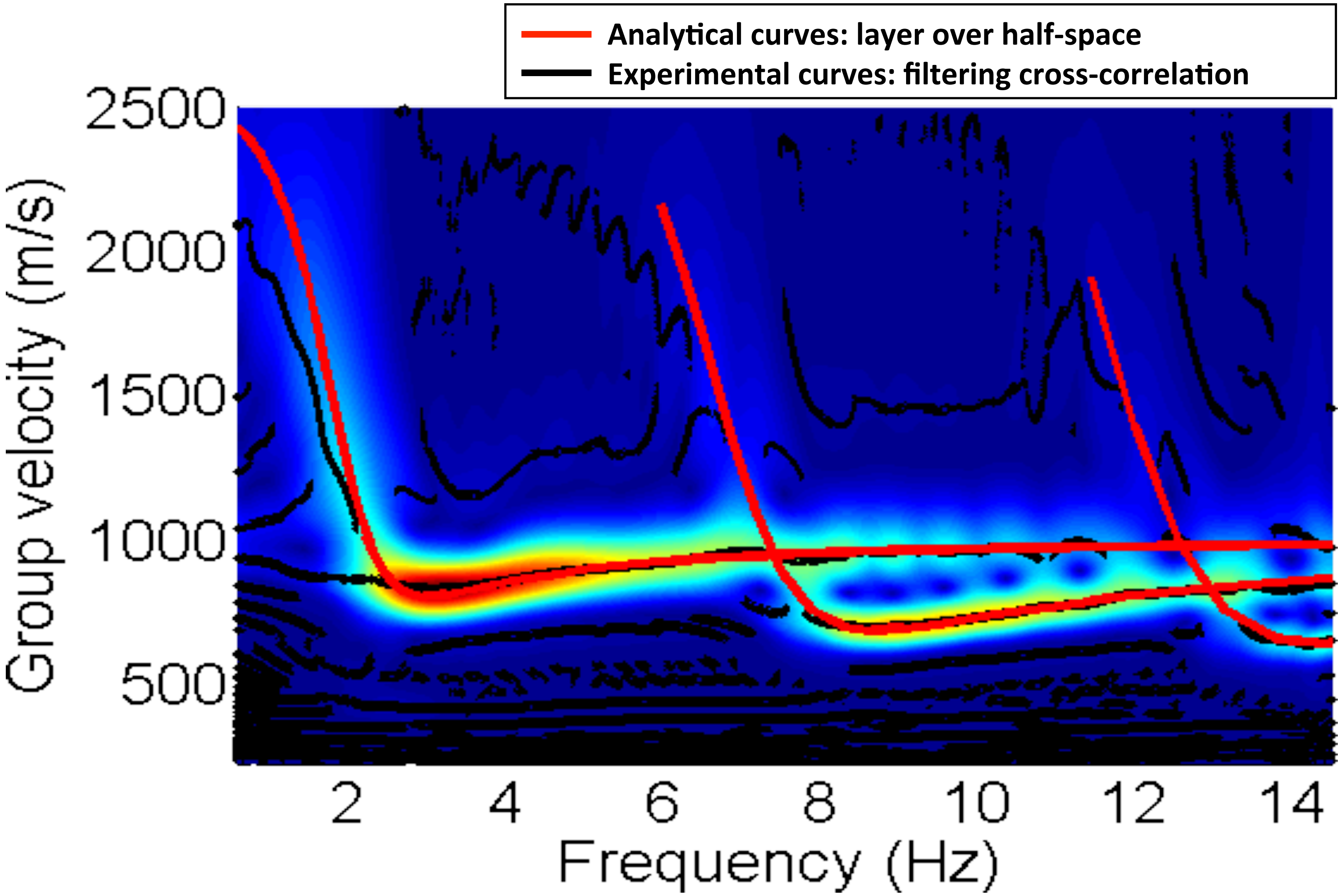
Green's Function G_{22} (SH case)



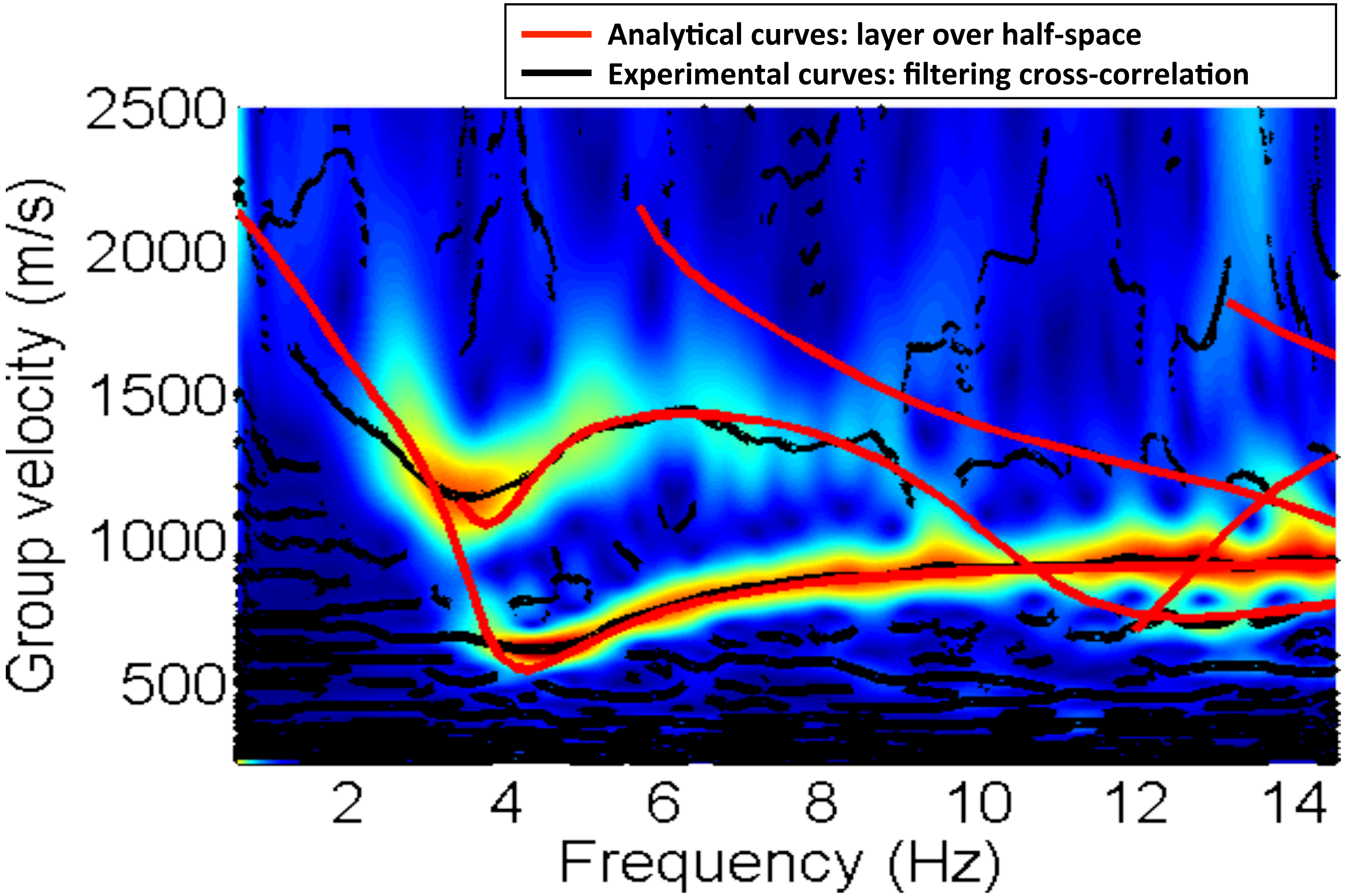
Green's Function G_{ij} (P-SV case)



LOVE WAVES DISPERSION CURVES



Rayleigh waves Dispersion Curves



Green's function from the average of correlations

$$\left\langle u_i(\mathbf{x}_\mathrm{A},\omega)u_j^*(\mathbf{x}_\mathrm{B},\omega)\right\rangle = -2\pi\, E_S k^{-3}\,\mathrm{Im}\left[G_{ij}(\mathbf{x}_\mathrm{A},\mathbf{x}_\mathrm{B},\omega)\right]$$

3D

$$\mathrm{Im}\left[G_{ij}(\mathbf{x}_\mathrm{A},\mathbf{x}_\mathrm{B},\omega)\right] = -\frac{\omega}{2\pi\rho\beta^{3}S^{2}}\left\langle u_i(\mathbf{x}_\mathrm{A},\omega)u_j^*(\mathbf{x}_\mathrm{B},\omega)\right\rangle$$

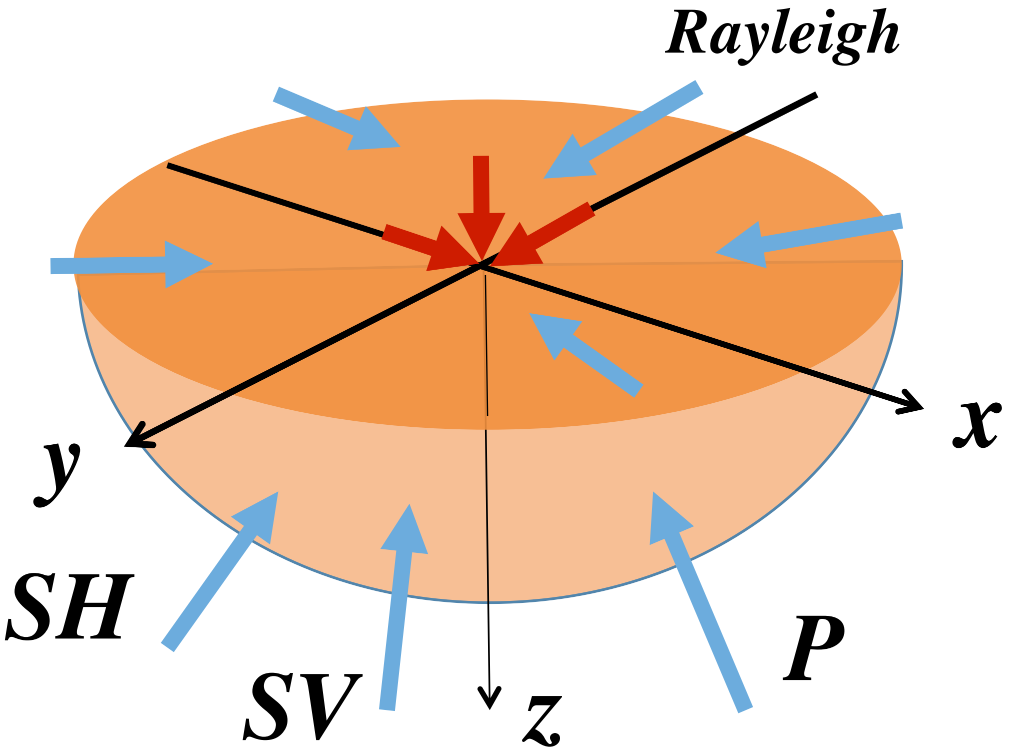
Green's Function G_{ij} (Lamb, 1904; Chao 1960)

$$\xi_P = \frac{1}{1+2R^3}\xi,$$

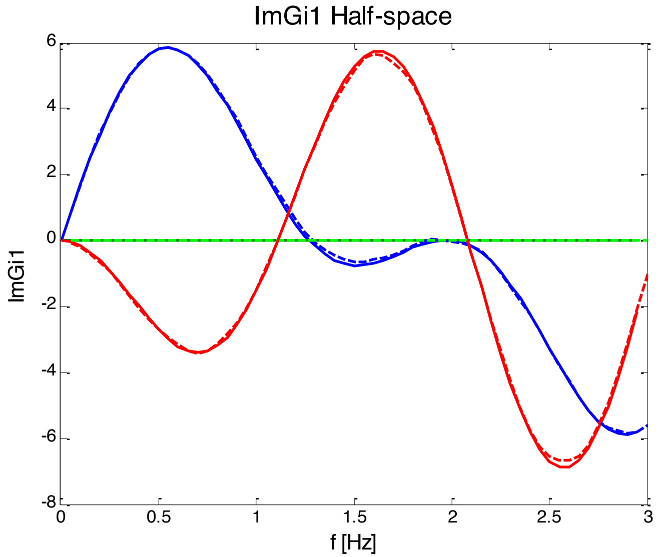
$$\xi_{SV} = \frac{R^3}{1+2R^3}\xi,$$

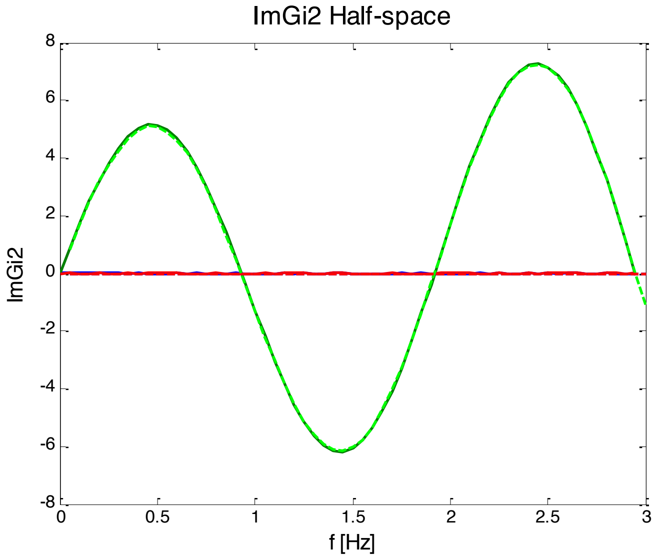
$$\xi_{SH} = \frac{R^3}{1+2R^3}\xi,$$

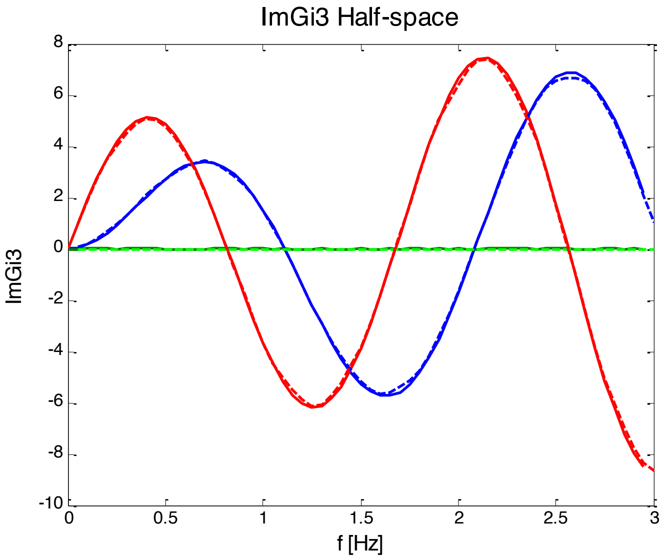
$$\xi_R = \frac{\pi\beta}{\omega}\left(\frac{\beta}{c_R}\right)^2\frac{R^3}{1+2R^3}\xi$$



GREEN'S FUNCTION G_{ij} (LAMB, 1904; CHAO 1960)





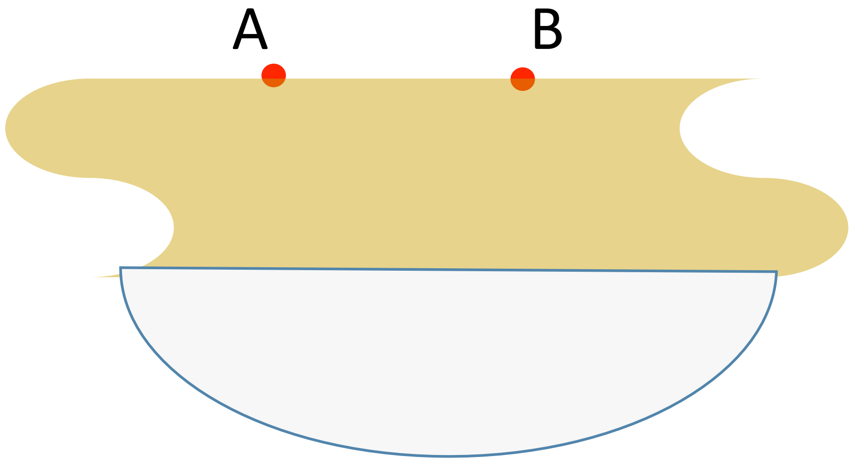


$$\mathrm{Im}\left[G_{ij}(\mathbf{x_A},\mathbf{x_B},\omega)\right] = -\frac{\omega}{2\pi\rho\beta^3 S^2}\left\langle u_i(\mathbf{x_A},\omega)u_j^*(\mathbf{x_B},\omega)\right\rangle$$

$$G_{r3}(r,z,\omega) = \frac{1}{2\pi\mu}\int_0^\infty \frac{(k^2-\nu^2)e^{-i\gamma z} + 2\gamma\nu\, e^{-i\nu z}}{(k^2-\nu^2)^2 + 2\gamma\nu\, k^2} k^2 J_1(kr)\,dk$$

LAMB (1904)

GREEN'S FUNCTION G_{22} (2D, STRATIFIED MEDIUM)



Layer over half-space

Antiplane (or SH)case

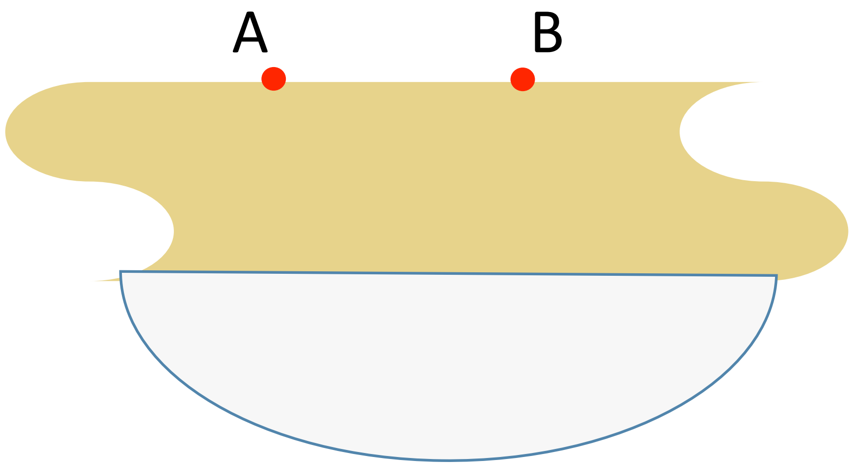
See poster by Perton and Sánchez-Sesma

The density of SH states per unit area around a frecuency band centered in ω can be written as $dn_S = \frac{1}{2\pi}\frac{\omega}{\beta^2}\, d\omega$

Thus, energy density per unit area is $\zeta = q\, dn_S = q\frac{1}{2\pi}\frac{\omega}{\beta^2}\, d\omega$.

Then the proporcionality constant is $q = \zeta \frac{2\pi\beta^2}{\omega}\frac{1}{d\omega}$.

For stratified médium the number of states of Love waves in mode *m* per unit lenght is



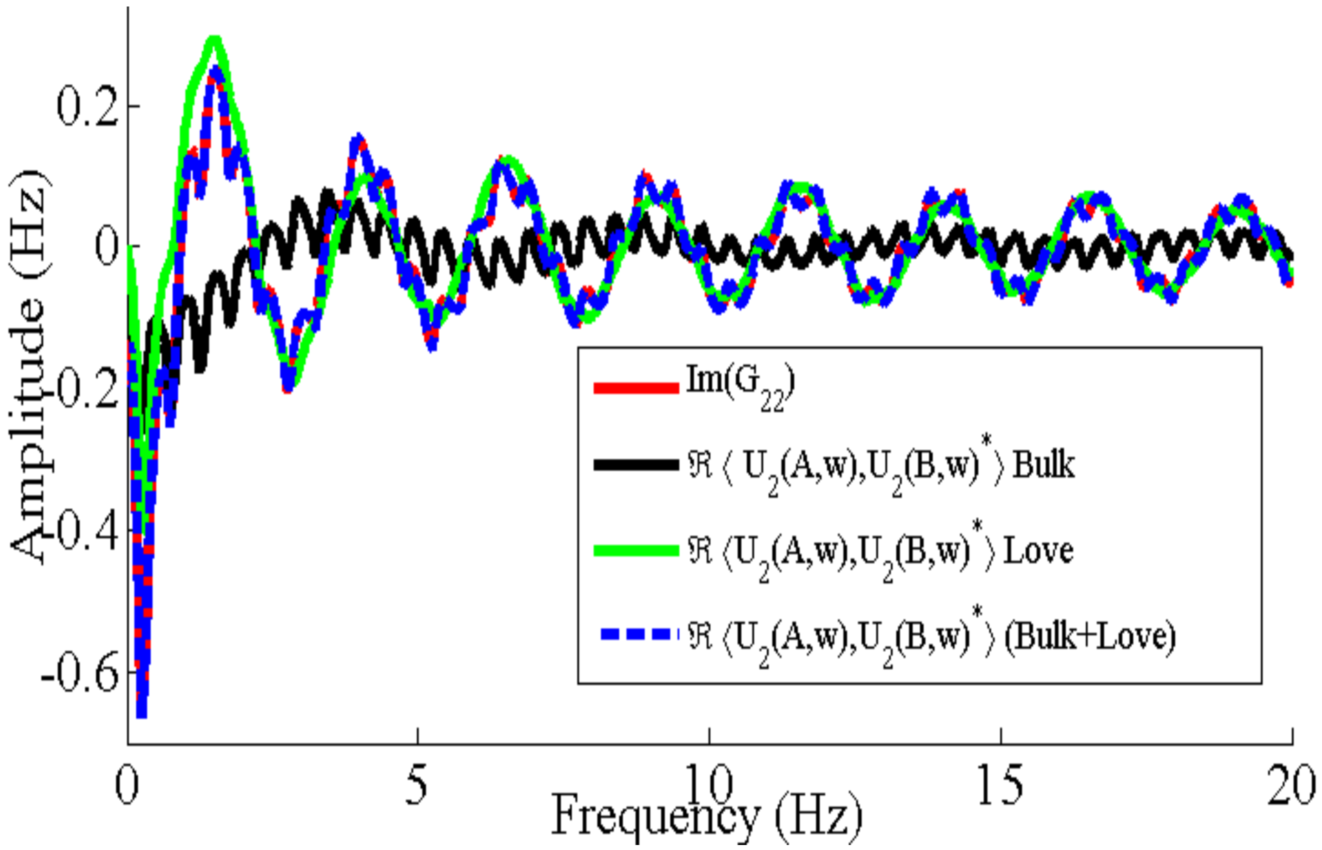
$$n_{L_m} = \frac{1}{\pi}\frac{\omega}{C_m} = \frac{1}{\pi}k_m$$

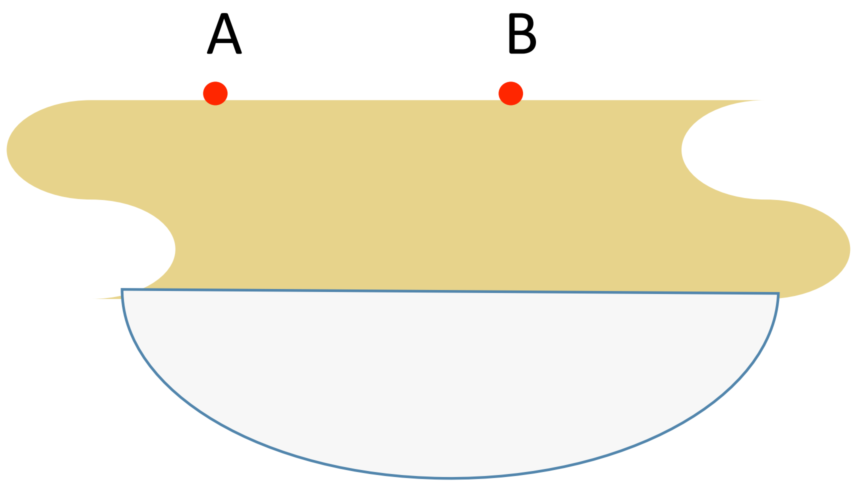
The density of states in a frequency band $d\omega$ around ω is

$$dn_{L_m} = \frac{1}{\pi}\frac{\partial k_m}{\partial \omega}d\omega = \frac{1}{\pi}\frac{1}{U_m}d\omega,$$

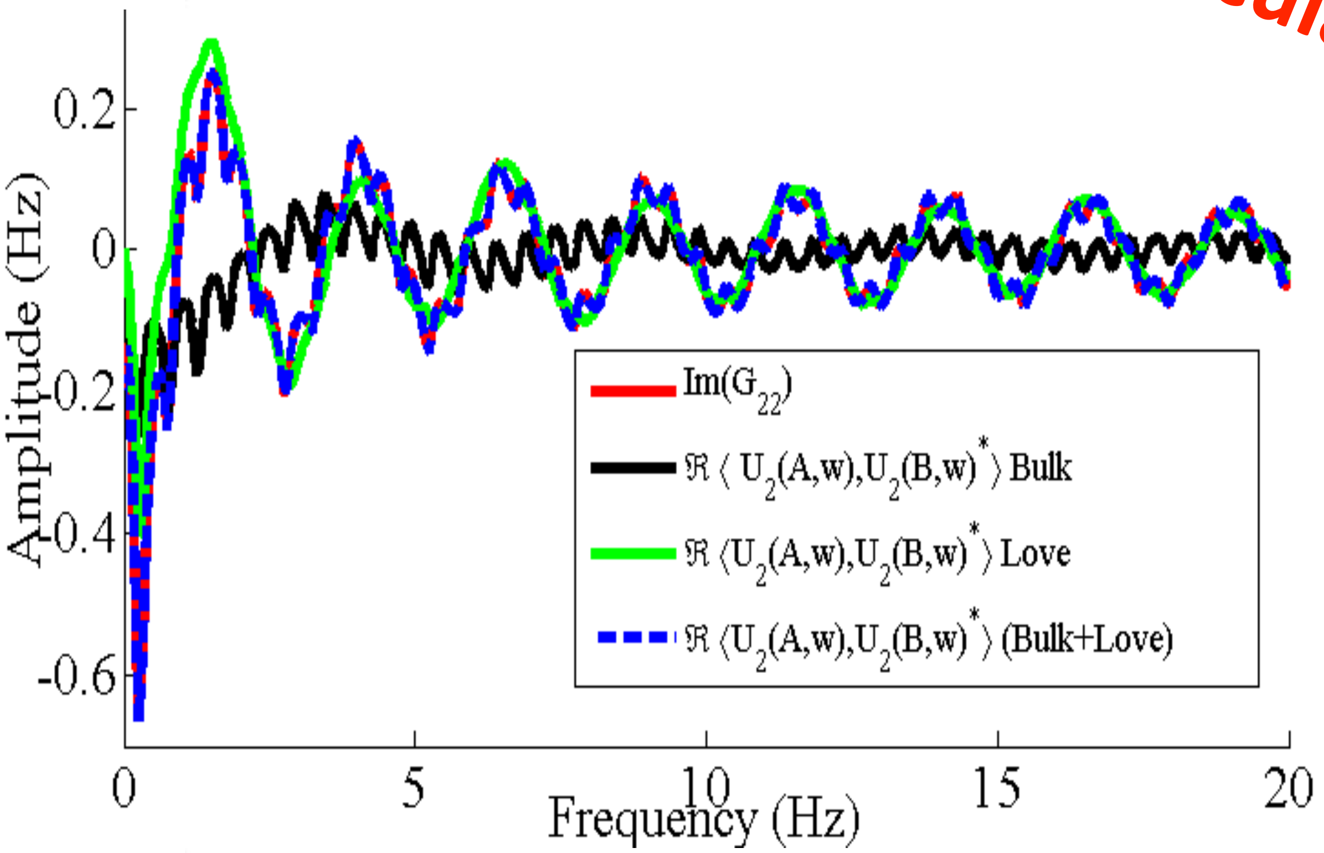
where U_m = group velocity of mode *m*. Then, the energy density per unit length of mode *m* is given by

$$\zeta_{L_m} = q\,dn_{L_m} = q\frac{1}{\pi}\frac{1}{U_m}d\omega = \left(\zeta\frac{2\pi\beta^2}{\omega}\frac{1}{d\omega}\right)\frac{1}{\pi}\frac{1}{U_m}d\omega = \frac{2\beta^2}{\omega U_m}\zeta$$

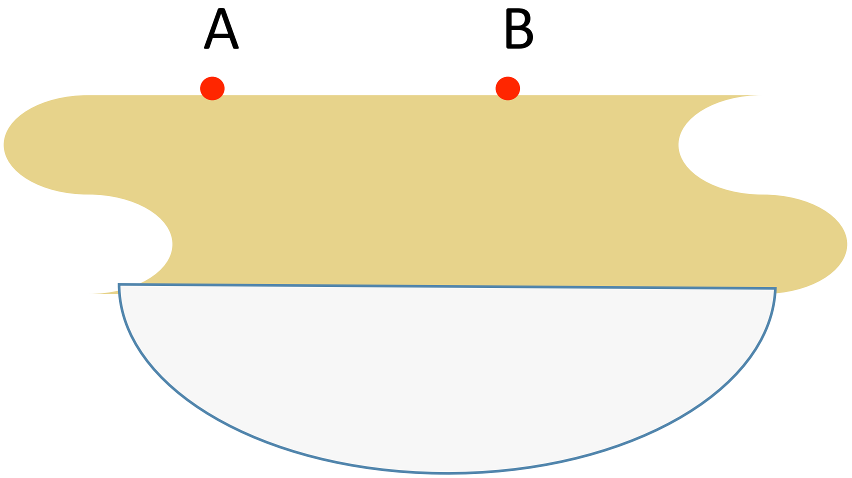




Perton and Sánchez-Sesma (2015)



Fast Calculation !



Perton and Sánchez-Sesma (2015)

Outline of Presentation

Introduction → Motivation, Site effects, Site Characterization
Diffuse Fields ← Multiple Diffraction
Equipartition of Energy in Dynamic Elasticity
    Full Space, Half-Space → Experimental Verification
Correlation Type Representation Theorem
Green's Function from the Average of Correlations
    Dispersion Curves for Tomography of Alluvial Valleys
Energy Densities from Average of Auto-Correlations
    Deterministic Partition of Energy
    Ambient Seismic Noise and H/V Spectral Ratios,
    Fast Calculation of H/V and Inversion (1D, 3D)
Conclusions

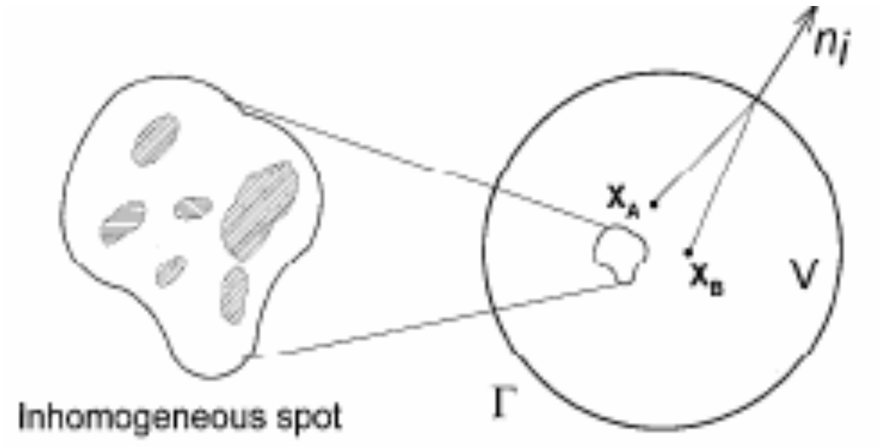
Cargese, May 2015

$$E(\mathbf{x}) = \rho\omega^2 \left\langle u_m(\mathbf{x}) u_m^*(\mathbf{x}) \right\rangle = -4\pi\mu E_S k^{-1} \times \mathrm{Im}[G_{mm}(\mathbf{x},\mathbf{x})]$$

Directional Energy Density (DED). It is the Imaginary part of Green’s function at source

$$\mathrm{Re}[G_{11}(\mathbf{x},\mathbf{x};\omega) \times i\omega \left| e^{i\omega t} \right|] = \omega\,\mathrm{Im}\left[G_{11}(\mathbf{x},\mathbf{x};\omega)\right]$$

Proportional to the power transmited to the medium by the unit harmonic force



From the Representation Theorem

Impossible d'afficher l'image. Votre ordinateur manque peut-être de mémoire pour ouvrir l'image ou l'image est endommagée. Redémarrez l'ordinateur, puis ouvrez à nouveau le fichier. Si le x rouge est toujours affiché, vous devrez peut-être supprimer l'image avant de la réinsérer.

Impossible d'afficher l'image. Votre ordinateur manque peut-être de mémoire pour ouvrir l'image ou l'image est endommagée. Redémarrez l'ordinateur, puis ouvrez à nouveau le fichier. Si le x rouge est toujours affiché, vous devrez peut-être supprimer l'image avant de la réinsérer.

Stokes (1849)

Sánchez-Sesma *et al.* (2008)

AS A CONSEQUENCE OF THE IDENTITY

Energy ⟷ ***Green's Function***

$$E_1 + E_2 + E_3 = A \times \mathrm{Im}[G_{kk}(\mathbf{x},\mathbf{x})] = E_P + E_S$$

$$E_1 = \rho\omega^2 \langle u_1^2 \rangle \propto \mathrm{Im}[G_{11}(\mathbf{x},\mathbf{x})]$$

$$E_2 = \rho\omega^2 \langle u_2^2 \rangle \propto \mathrm{Im}[G_{22}(\mathbf{x},\mathbf{x})]$$

$$E_3 = \rho\omega^2 \langle u_3^2 \rangle \propto \mathrm{Im}[G_{33}(\mathbf{x},\mathbf{x})]$$

Directional Energy Densities (DEDs)

Perton et al. (2009)

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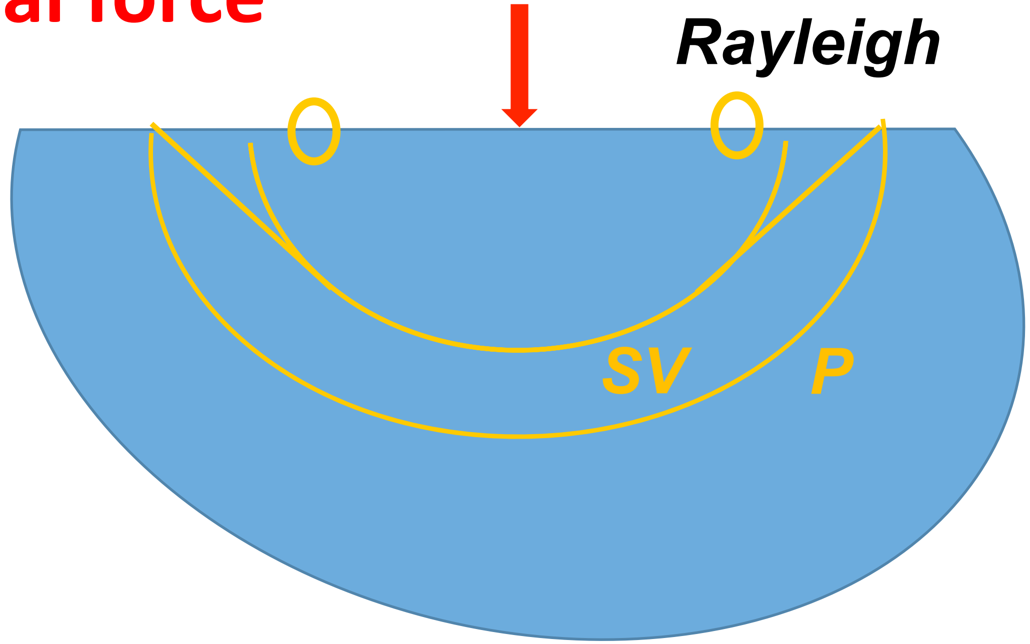
Cargese, May 2015

Deterministic Partition of Energy

Vertical force



Lamb(1904)

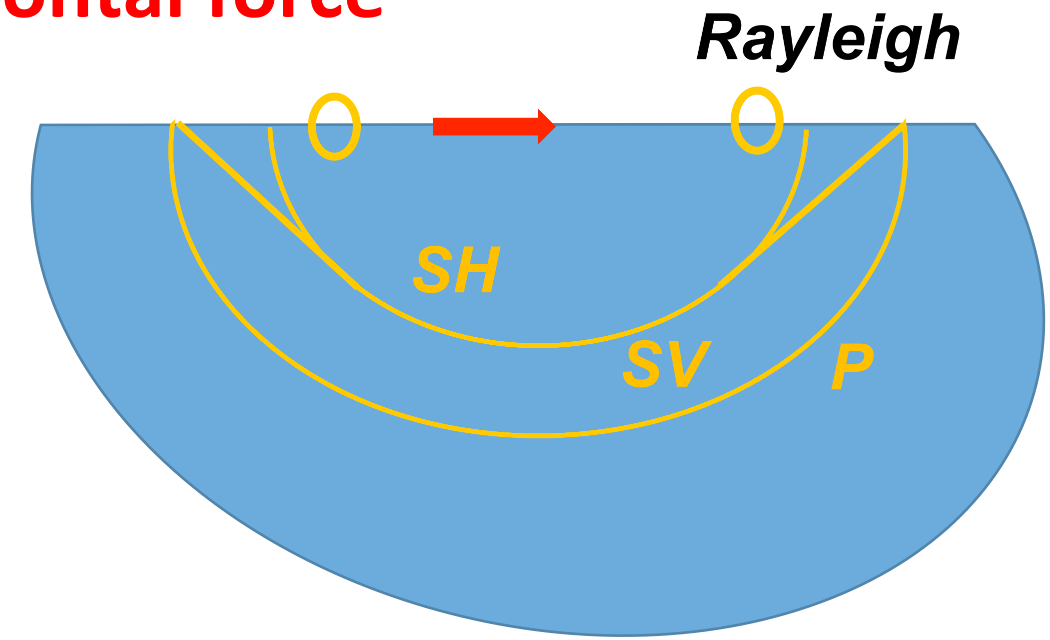
Miller & Pursey (1955)

Weaver (1985)

SH=0 % R=67% SV=26% P=7%

Deterministic Partition of Energy

Horizontal force



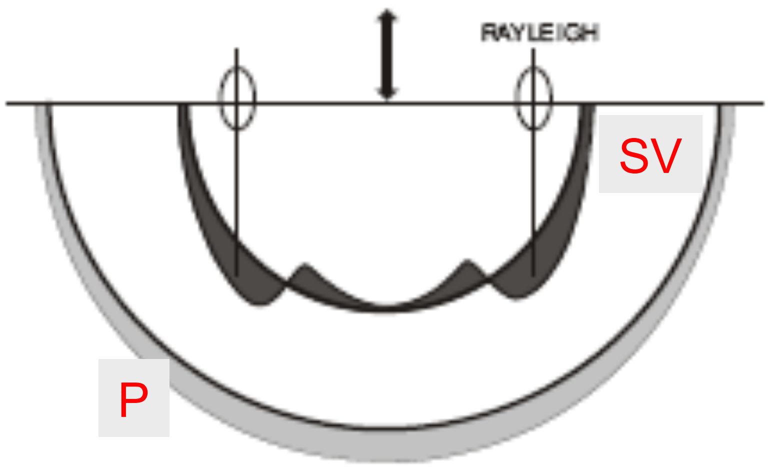
Chao(1960)

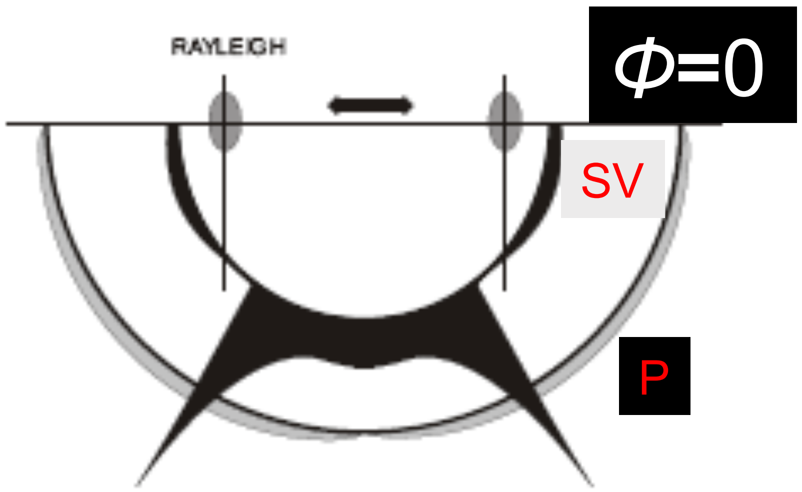
Cherry (1962)

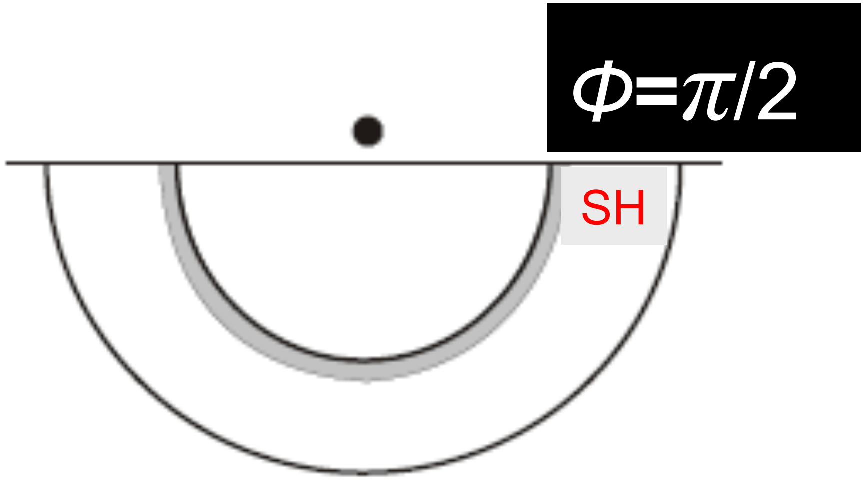
Weaver (1985)

SH=60%   R=18%   SV=16% P=6%

Sánchez-Sesma et al (2011) BSSA







Sánchez-Sesma *et al.* (2011) BSSA

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Cargese, May 2015

A Theory for H/V

With **Directional Energy Densities** one can compute the H/V ratio as:

$$[H/V](\mathbf{x};\omega) = \sqrt{\frac{E_1(\mathbf{x};\omega) + E_2(\mathbf{x};\omega)}{E_3(\mathbf{x};\omega)}}$$

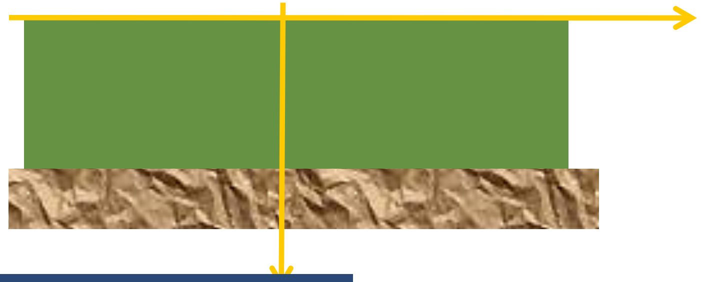
measurements ←→ system properties

$$[H/V](\mathbf{x};\omega) = \sqrt{\frac{\mathrm{Im}[G_{11}(\mathbf{x},\mathbf{x};\omega)] + \mathrm{Im}[G_{22}(\mathbf{x},\mathbf{x};\omega)]}{\mathrm{Im}[G_{33}(\mathbf{x},\mathbf{x};\omega)]}}$$

Sánchez-Sesma et al. (2011)
3D problem (BW & SW)

Kawase et al. (2011)
1D problem (BW)

$$\nabla^2 G + k^2 G = -\frac{\delta(|\mathbf{x} - \xi|)}{\rho c^2}$$



$$G = \frac{1}{iH\rho c^2} \sum_{m=0}^{\infty} K_m^{-1} \exp(-iK_m r) \cos\frac{\Omega_m z_0}{c} \cos\frac{\Omega_m z}{c}, \quad \text{2D}$$

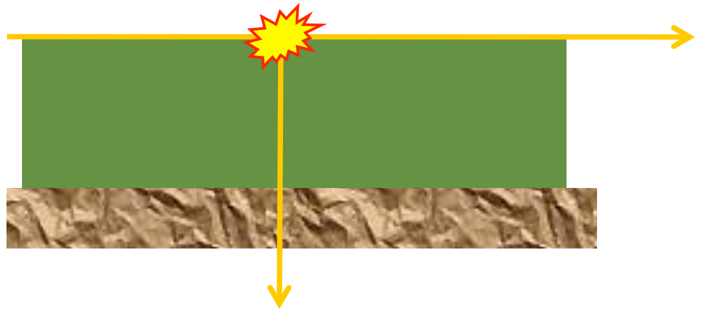
$$G = \frac{1}{i2H\rho c^2} \sum_{m=0}^{\infty} H_0^{(2)}(K_m r) \cos\frac{\Omega_m z_0}{c} \cos\frac{\Omega_m z}{c}, \quad \text{3D}$$

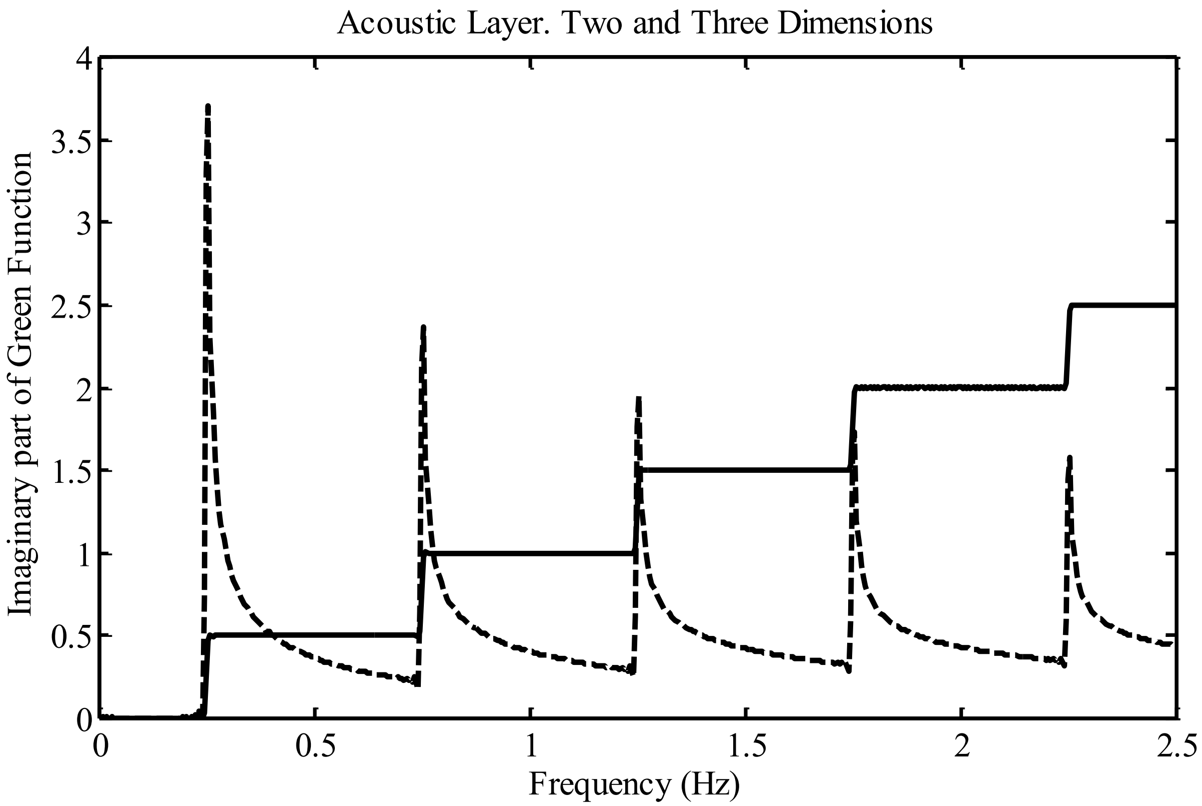
$$K_m = c^{-1}\sqrt{\omega^2 - \Omega_m^2}, \qquad \Omega_m = \frac{(2m+1)\pi c}{2H}, \qquad r = |x - x_0|$$

$$\operatorname{Im}[G(0,0;\omega)] = \frac{1}{2H\rho c} \sum_{m=0}^{\infty} \frac{H(\omega - \Omega_m)}{\sqrt{\omega^2 - \Omega_m^2}}, \qquad \text{2D}$$

$$\operatorname{Im}[G(0,0;\omega)] = \frac{1}{2H\rho c^2} \sum_{m=0}^{\infty} H(\omega - \Omega_m), \qquad \text{3D}$$

$$\nabla^2 G + k^2 G = -\frac{\delta(|\mathbf{x} - \xi|)}{\rho c^2}$$





Computation of ImG$_{11}$, and ImG$_{33}$ by an integral on the radial wavenumber

$$\mathrm{Im}\left[G_{11}(r,0,0;0;\omega)\right]=\mathrm{Im}\left[\underbrace{\frac{\mathrm{i}}{4\pi}\int_0^{+\infty} f_{SH}(k)\left[J_0(kr)+J_2(kr)\right]dk}_{SH}+\underbrace{\frac{\mathrm{i}}{4\pi}\int_0^{+\infty} f_{PSV}^{H}(k)\left[J_0(kr)-J_2(kr)\right]dk}_{PSV}\right]$$

$$\mathrm{Im}\left[G_{33}(r,0,0;0;\omega)\right]=\mathrm{Im}\left[\frac{\mathrm{i}}{2\pi}\int_0^{+\infty} f_{PSV}^{V}(k)J_0(kr)dk\right]$$

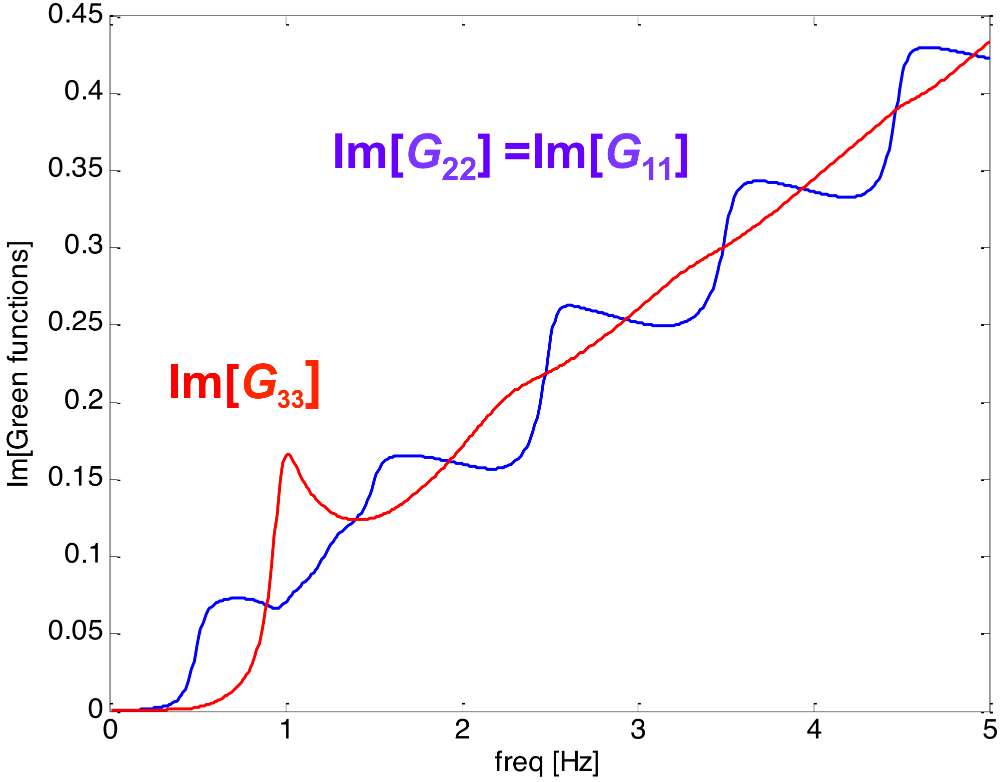
$$f_{PSV}^{V}(k)=-\frac{[GN-LH]}{[NK-LM]},\quad f_{PSV}^{H}(k)=\frac{[RM-SK]}{[NK-LM]},\quad f_{SH}(k)=\frac{(J_L)_{12}-(J_L)_{22}}{(J_L)_{21}-(J_L)_{11}}$$

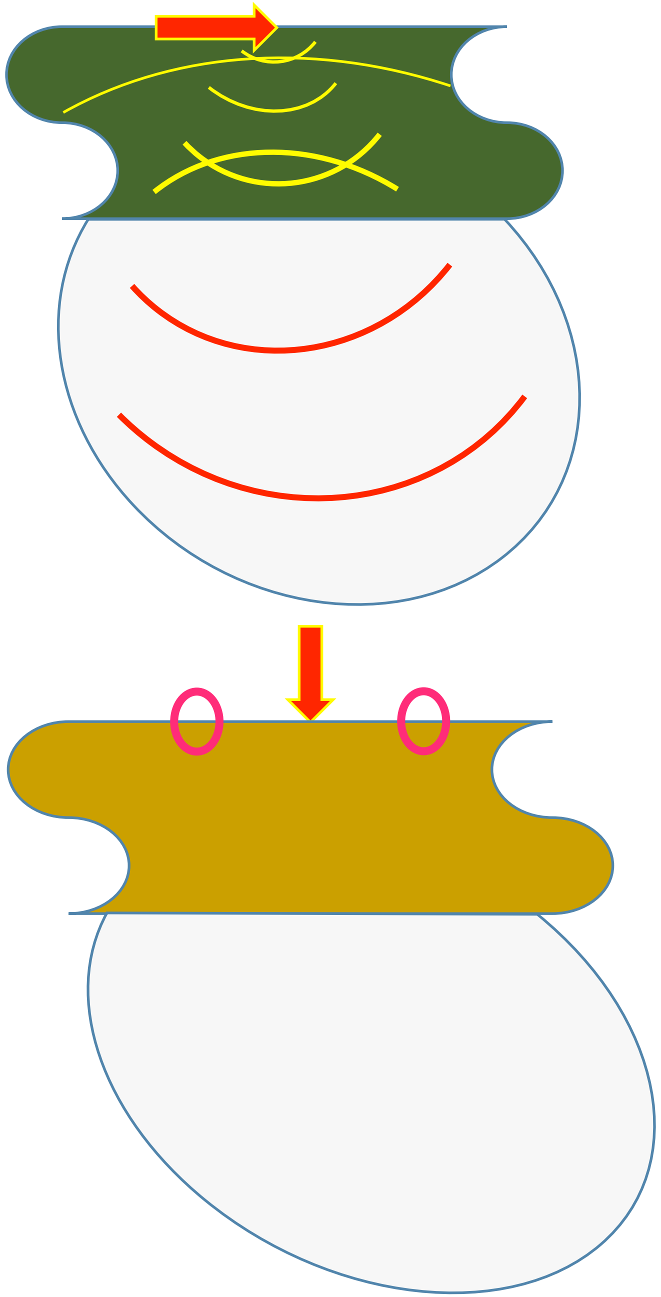
Harkrider (1964)

Layer over Half-space

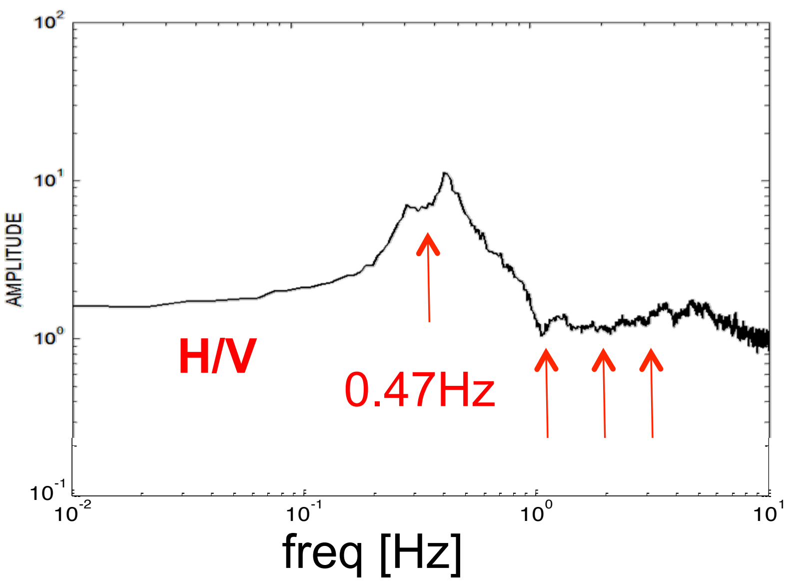
$\mathrm{Im}[G_{11}(0,0,\omega)]$, $\mathrm{Im}[G_{33}(0,0,\omega)]$

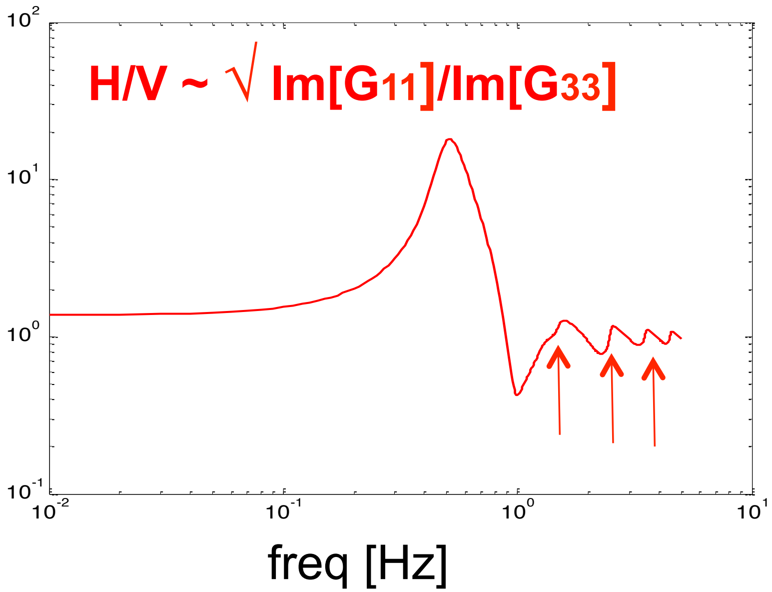
3D Solution

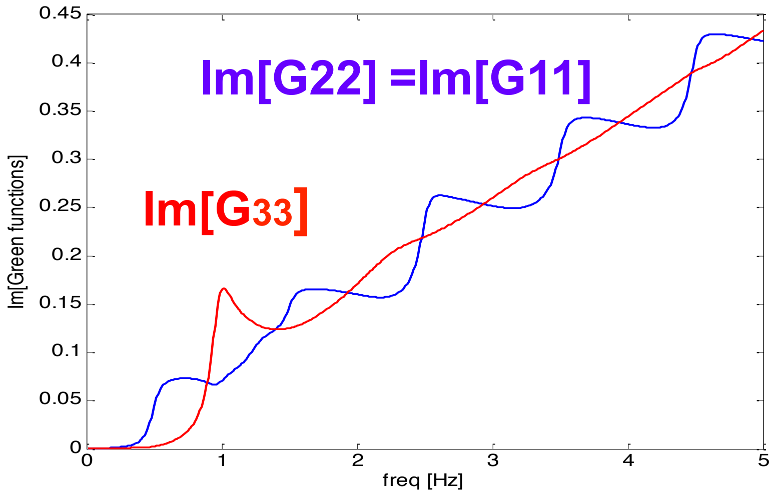




The Texcoco Experiment







Sánchez-Sesma et al. (2011)

3D Effect !

Layer over half-space

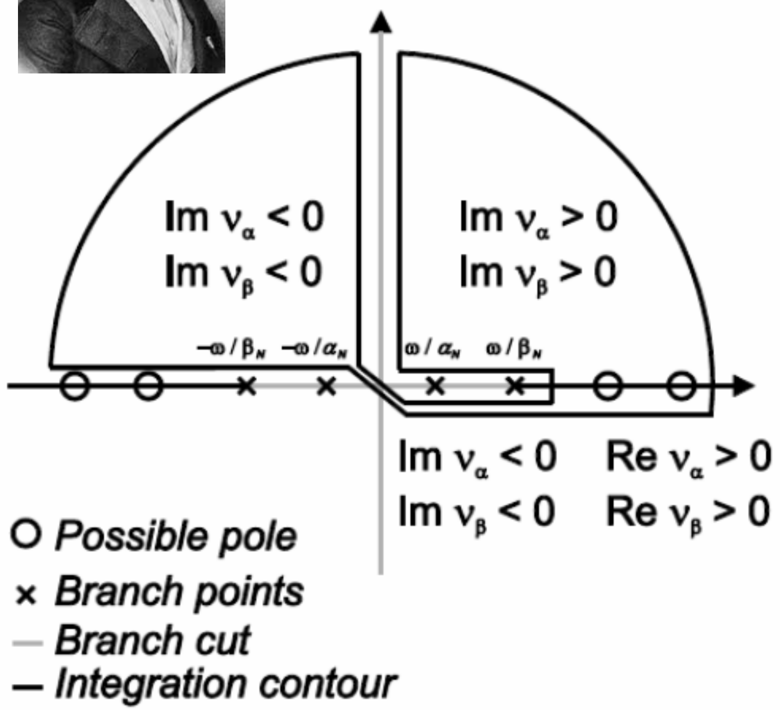
See poster by Piña et al...!!!

$$\nu_{\beta_N} = \sqrt{k^2 - (\omega / \beta_N)^2}$$

$$\nu_{\alpha_N} = \sqrt{k^2 - (\omega / \alpha_N)^2}$$

$$\mathrm{Im}[G_{11}^{PSV}(0;0;\omega)] = \mathrm{Im}[G_{22}^{PSV}(0;0;\omega)] = -\frac{1}{4}\sum_{m\in \mathrm{RAYLEIGH}} A_{Rm}\chi_m^2 + \frac{1}{4\pi}\int_0^{\omega/\beta_N} \mathrm{Re}\left[f_{PSV}^{H}(k)\right]_{4^{th}\,qu} dk$$

$$\mathrm{Im}[G_{11}^{SH}(0;0;\omega)] = \mathrm{Im}[G_{22}^{SH}(0;0;\omega)] = -\frac{1}{4}\sum_{m\in \mathrm{LOVE}} A_{Lm} + \frac{1}{4\pi}\int_0^{\omega/\beta_N} \mathrm{Re}\left[f_{SH}(k)\right]_{4^{th}\,qu} dk$$

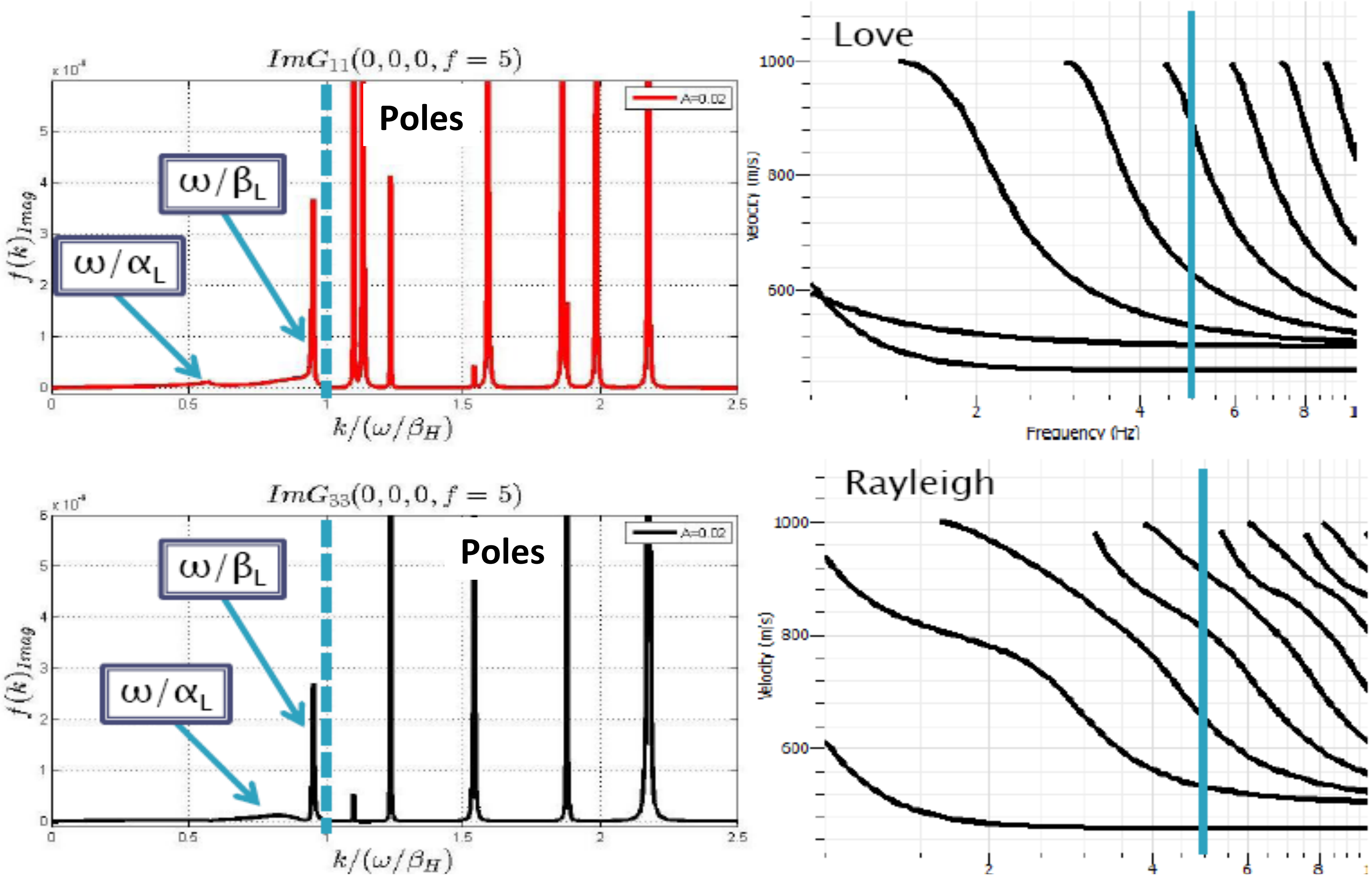


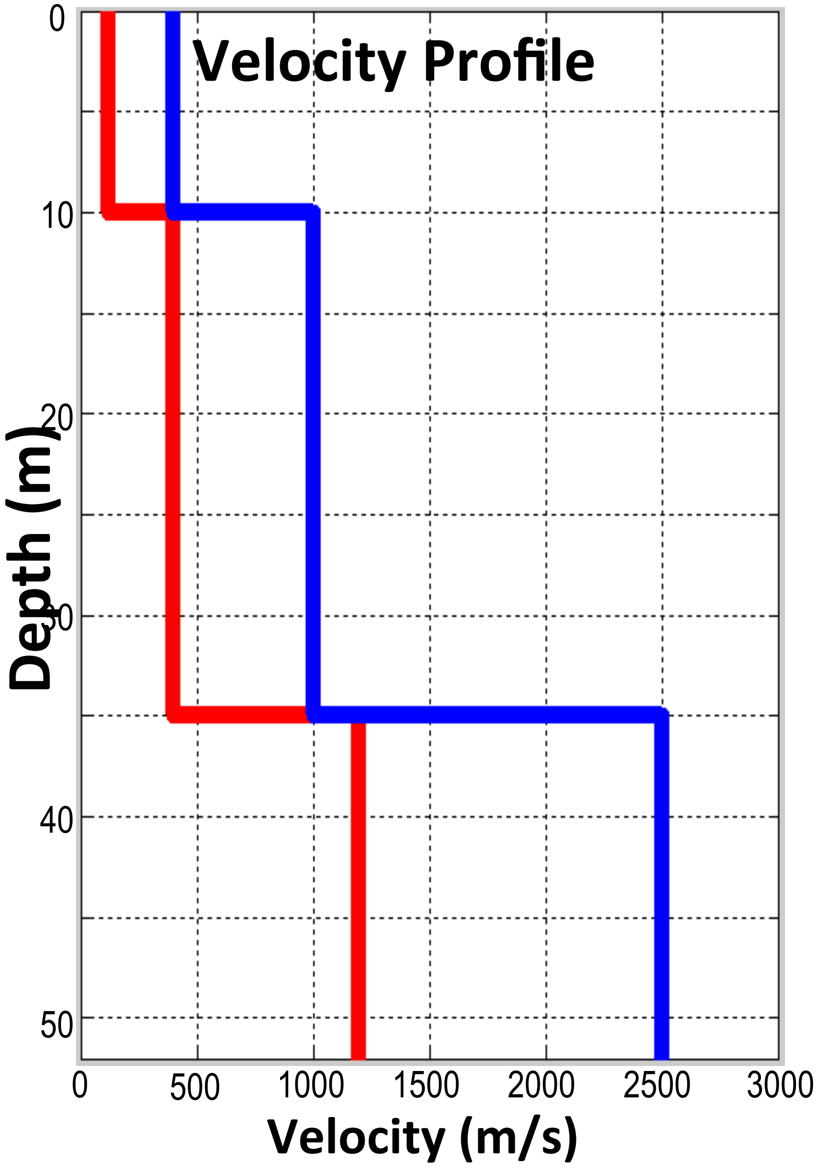
$$\mathrm{Im}[G_{33}(0;0;\omega)] = -\frac{1}{2}\sum_{m\in \mathrm{RAYLEIGH}} A_{\mathrm{Rm}} + \frac{1}{2\pi}\int_0^{\omega/\beta_N} \mathrm{Re}\left[f_{PSV}^{V}(k)\right]_{4^{th}\,qu} dk\,,$$

Fast computation of $\mathrm{Im}[G_{ij}(0,0,\omega)]$
with Cauchy Residue Theorem
An oportunity to speed up inversion

García-Jerez et al (2013)

Poles localization – Dispersion Curves (Piña *et al.* 2015)

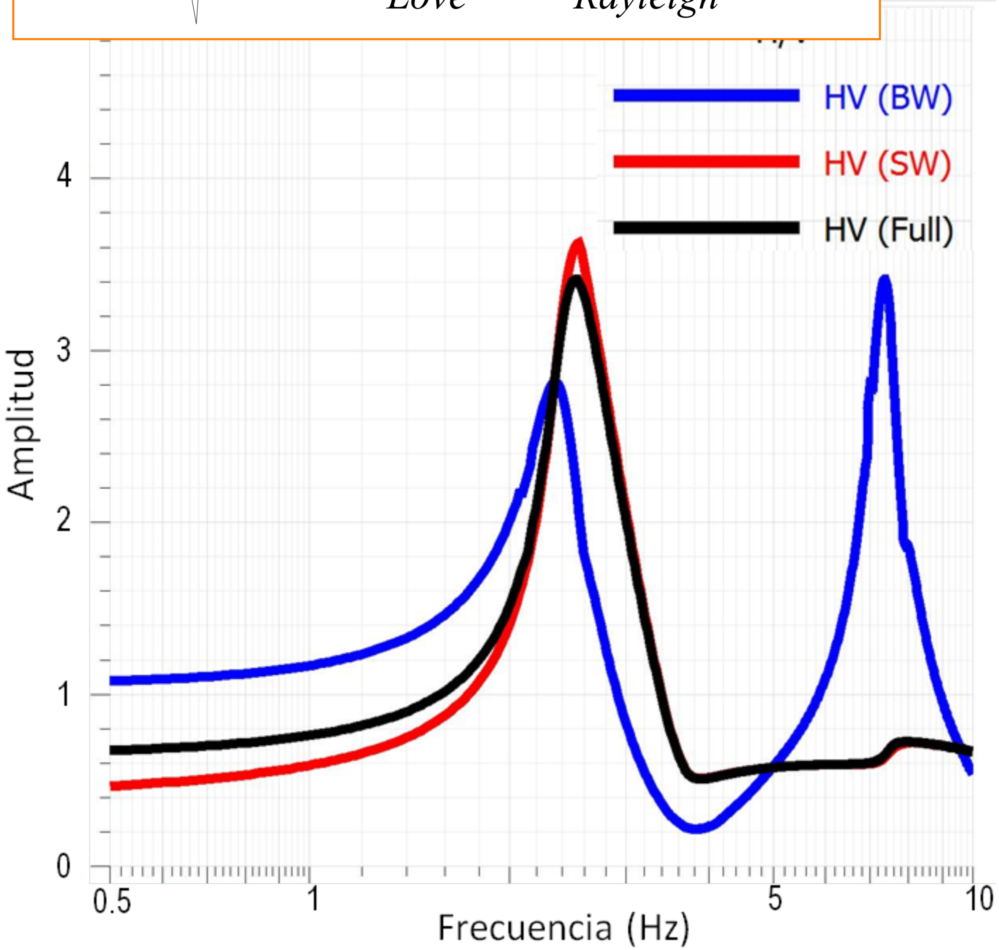




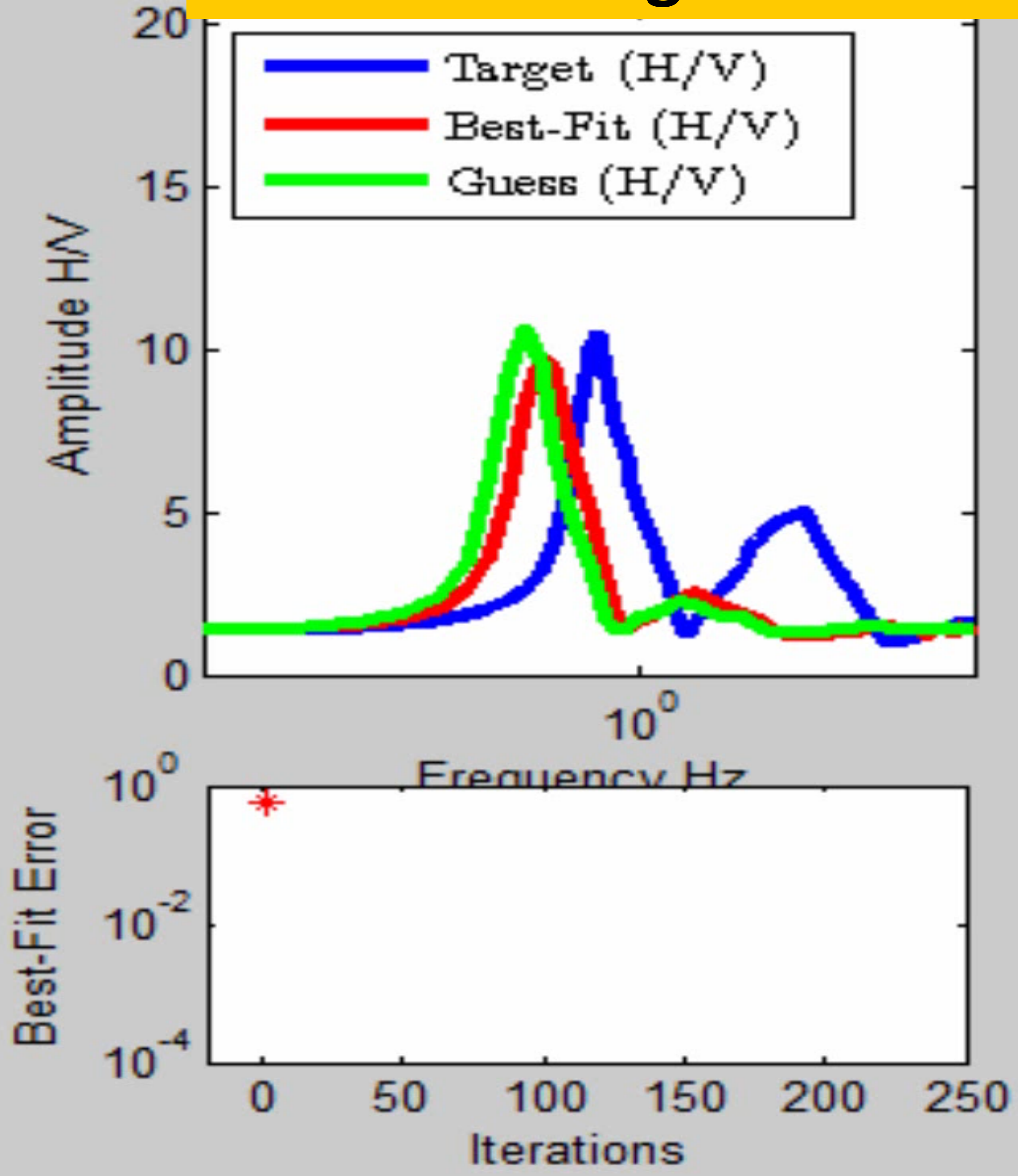
Direct Problem

Inverse Problem

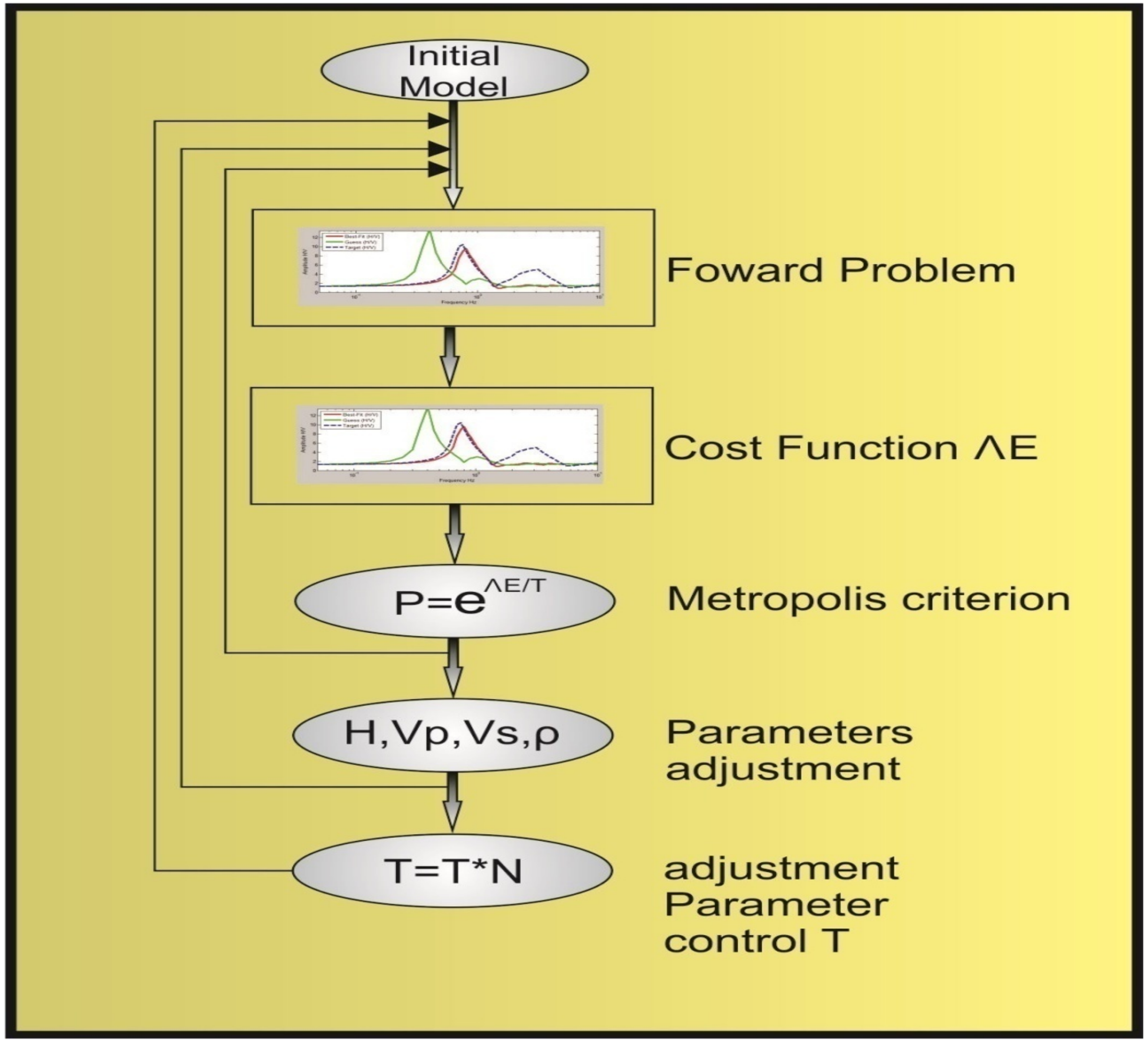
$$\frac{H}{V} = \sqrt{\frac{H_{body}^2 + H_{Love}^2 + H_{Rayleigh}^2}{V_{Love}^2 + V_{Rayleigh}^2}}$$



Inversion using Simulated Annealing (Piña et al., 2015)



Global optimization using simulated annealing



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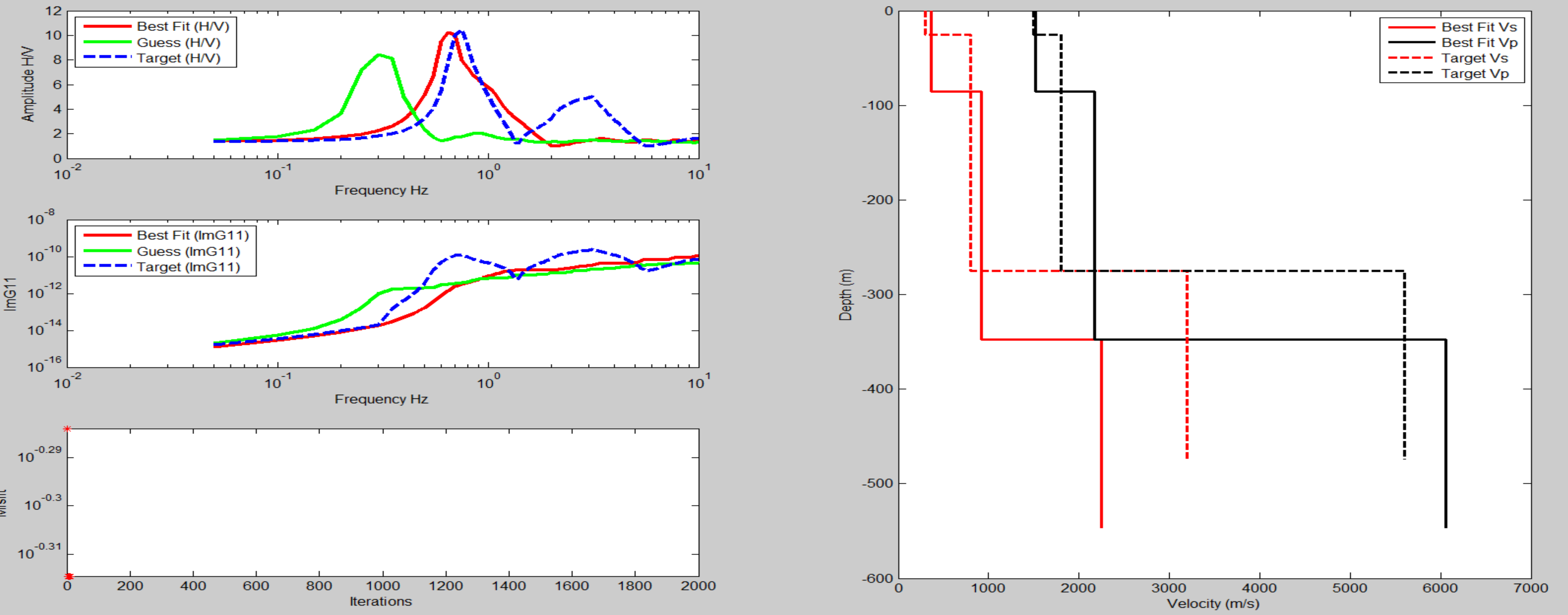
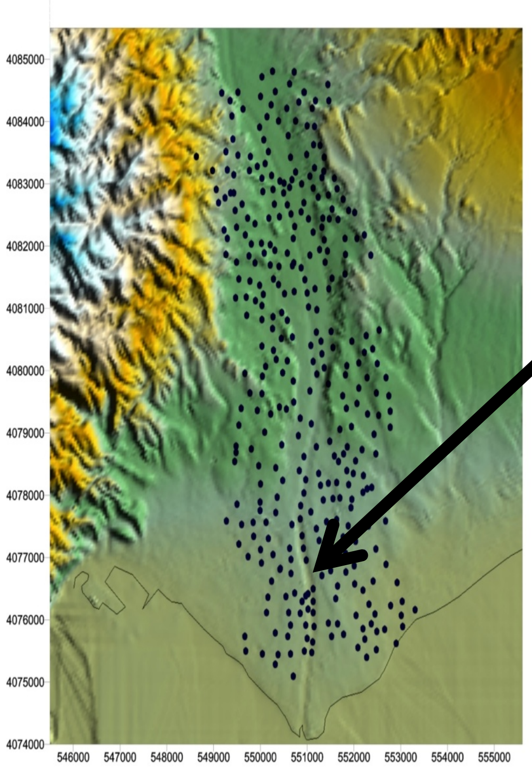
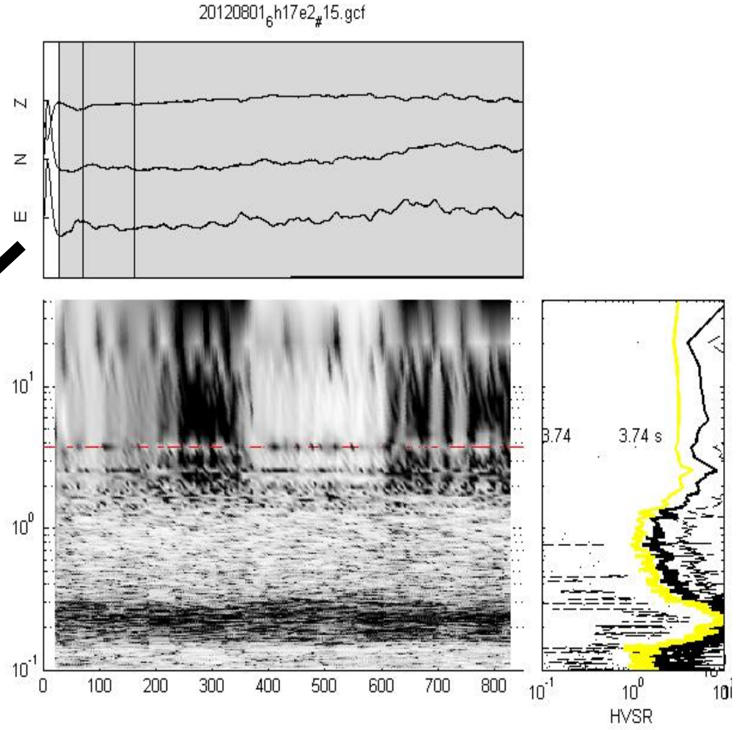


Application to site effect characterization at Almería, Andarax River, Spain

Stability of H/V

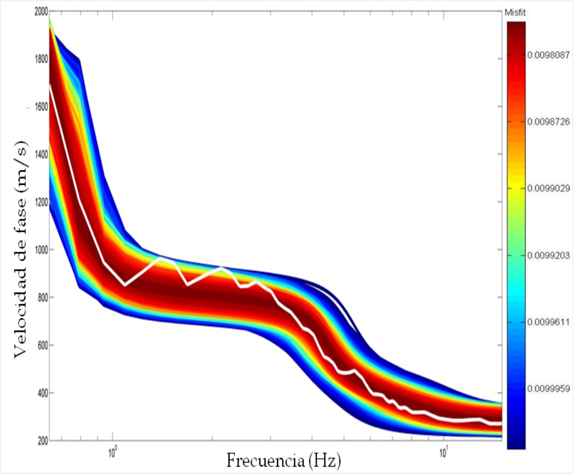


Stations distribution

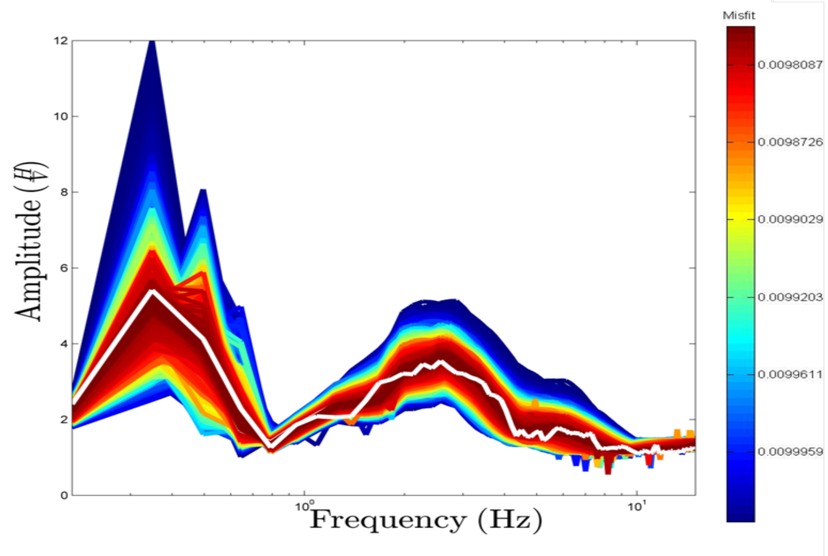


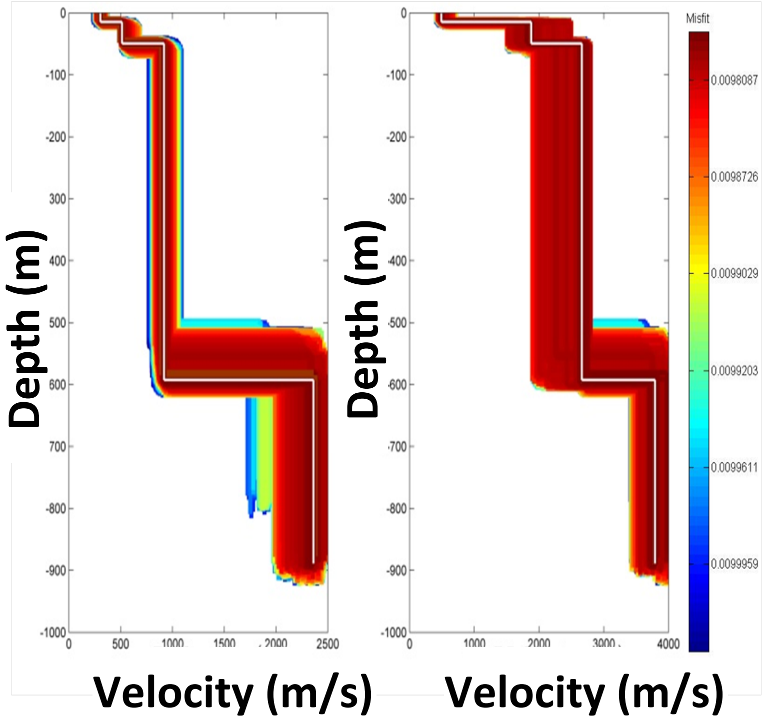
FTAN H/V
(from Dr. E. Carmona)

Application to site effect characterization at Almería, Andarax River, Spain

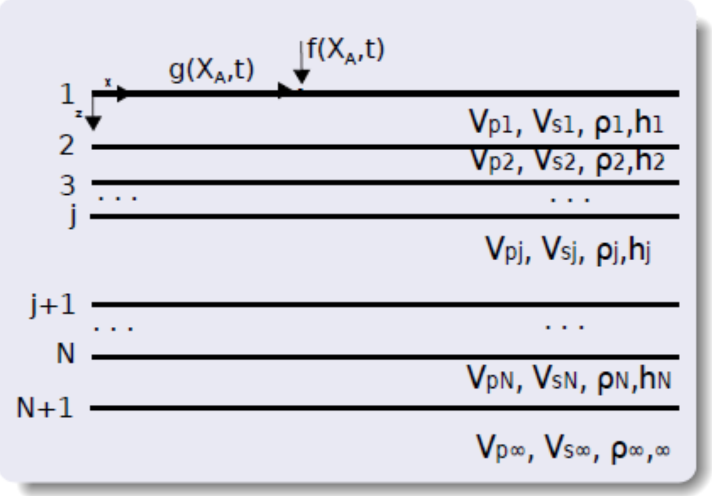


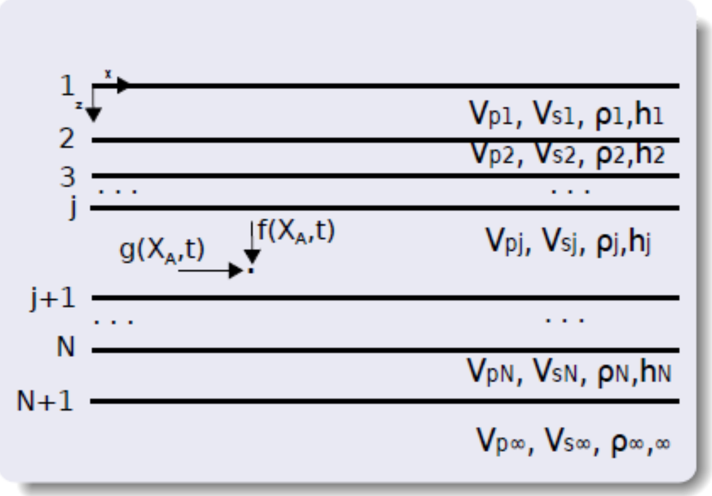
SPAC - 5 vertex, Rmax 450m





Theory for H/V(*f*, *z*)





Assumption: Diffuse wavefield

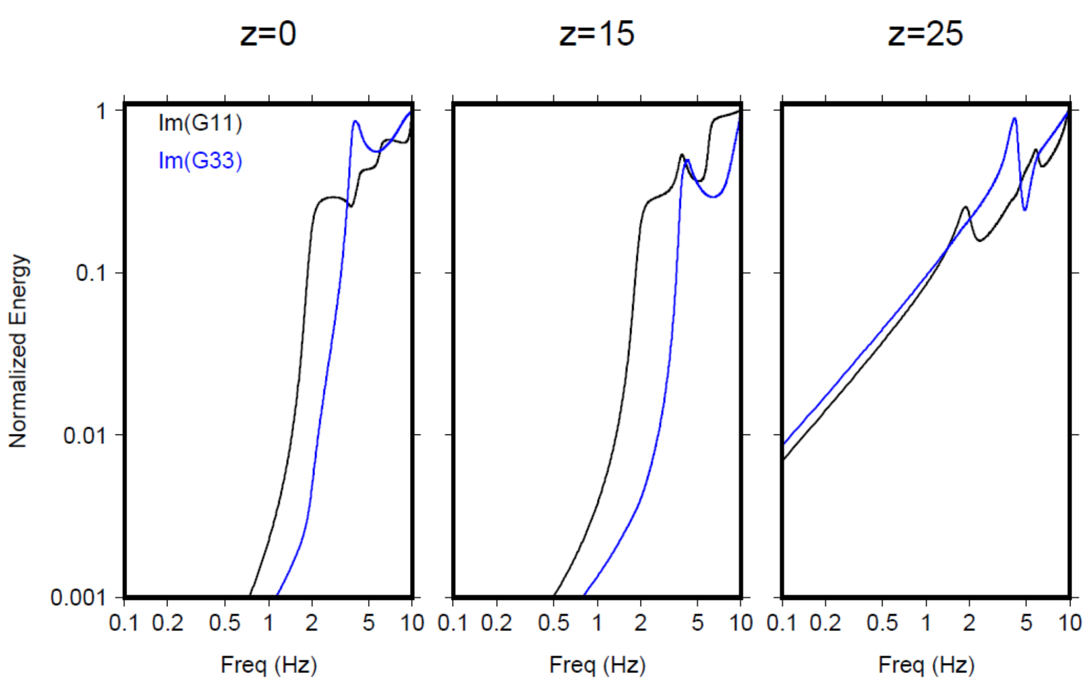
$$\frac{H}{V}(f,z) = \sqrt{\frac{Im[G_{11}(\mathbf{x},\mathbf{x};f)] + Im[G_{22}(\mathbf{x},\mathbf{x};f)]}{Im[G_{33}(\mathbf{x},\mathbf{x};f)]}}$$

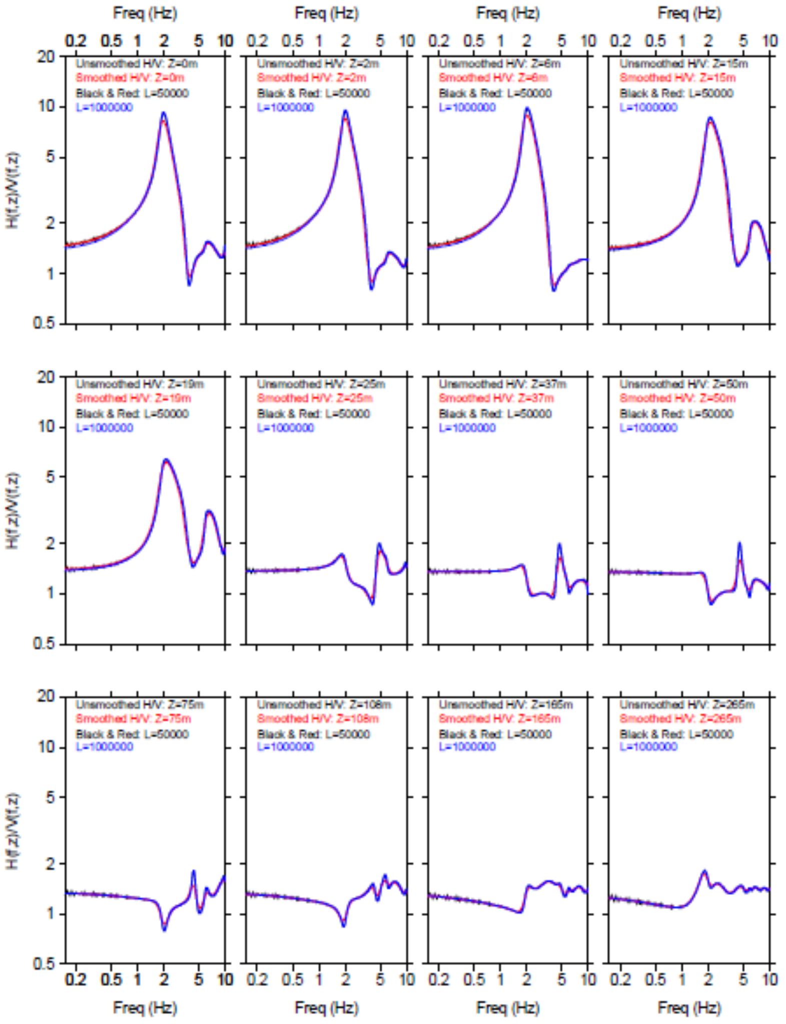
(Sanchez-Sesma et al., 2011)

Theory for H/V(*f*, *z*)

Lontsi et al. (2015)

$h(m)$	$Vp(m/s)$	$Vs(m/s)$	$\rho(kg/m^3)$
25	500	200	1900
∞	2000	1000	2500





Theory for H/V(*f*, *z*)

Lontsi et al. (2015)

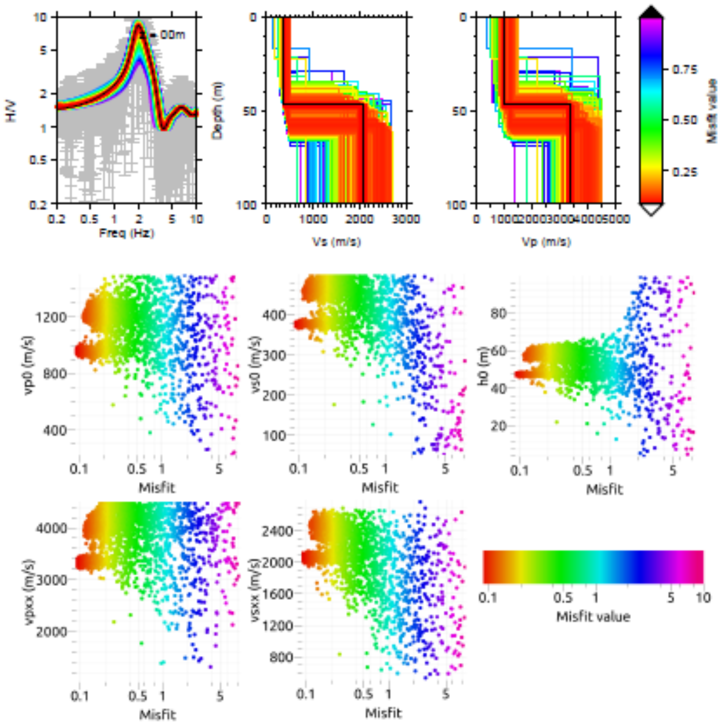


Figure: Synth. full H/V(f,z=0) inversion

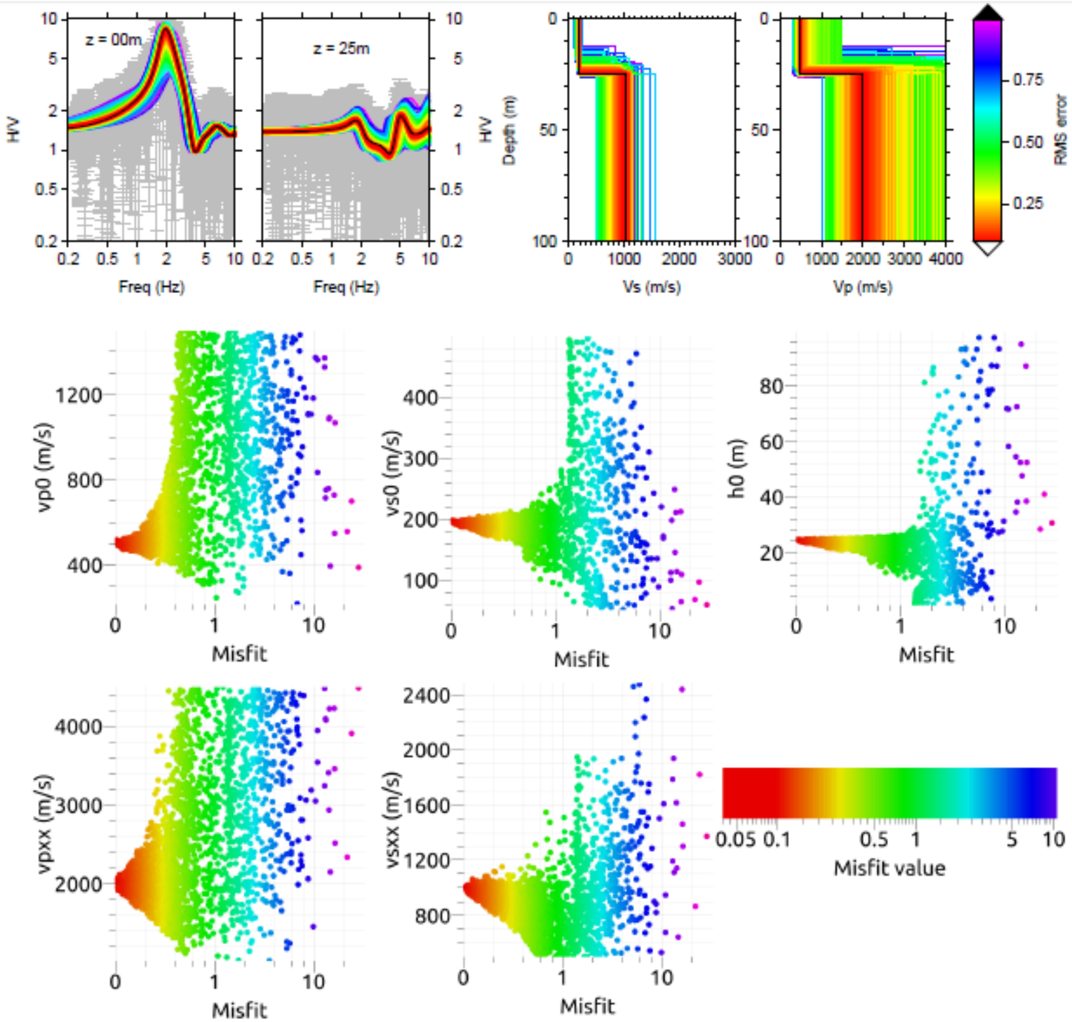
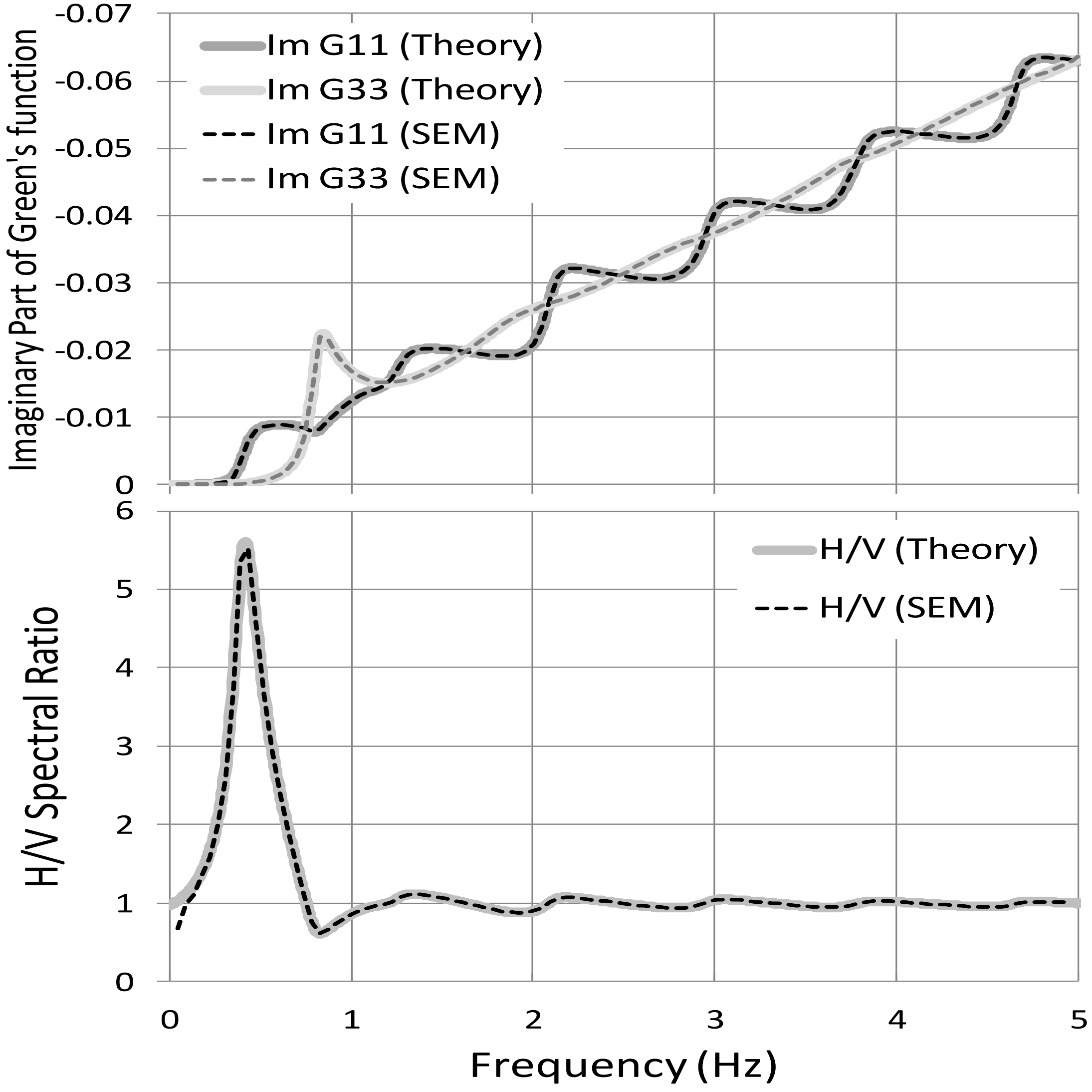


Figure: Synth. full H/V(f,z=0)+H/V(f,z=25m) inversion

H/V with lateral irregularity

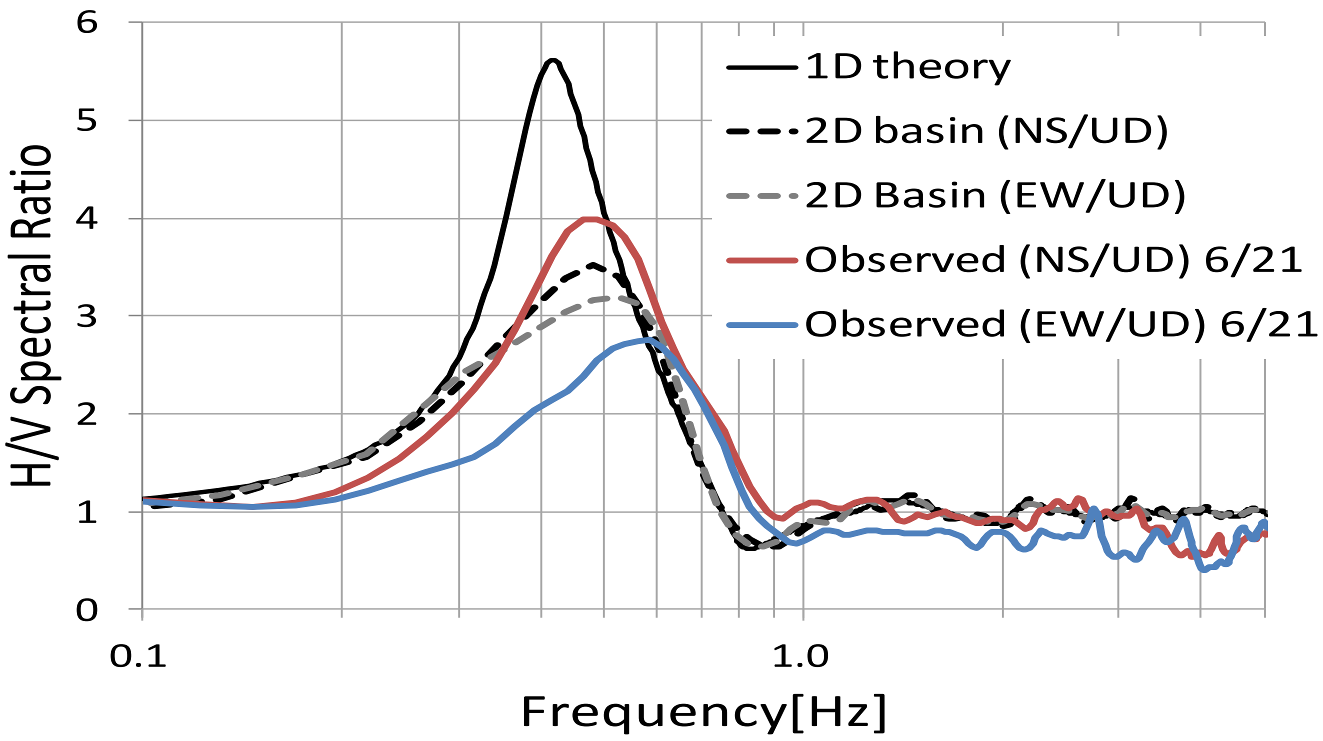
Verification of 1D results with numerical modeling



- **Imaginary Parts of Green's function**
  - **Good agreement between theory and SEM for G11 y G33**
- **Spectral Ratio H/V**
  - **Excellent agreement between theory and SEM**

Matsushima et al. (2014)

H/V with lateral irregularity



- Spectral ratios H/V
  - The efects of lateral irregularity are clear in NS/UD and EW/UD
  - Peak Amplitudes
    - NS/UD > EW/UD
  - Peak Frequency
    - NS/UD < EW/UD
  - Qualitative Agreement

Matsushima et al. (2014)

Cargese, May 2015

Comments and Conclusions (1)

The Principle of Equipartition of Energy allows characterization of diffuse fields. In seismology a key issue is Multiple Scattering.

We examined the Properties of the Equipartition in a full-space, a half-space and we mentioned the experimental verification.

We reviewed retrieval of the Green's function from the average of correlations in a diffuse field (coda, noise) or for earthquakes with dominance of body waves. Generalized diffuse field.

Deterministic G_{ij} with equipartitioned plane wave cocktails.

Comments and Conclusions (2)

Directional Energy Densities from autocorrelation averages.

- **Deterministic partition of Energy**
- **H/V ratios for site charaterization → Site Effects**
  - **Noise (microtremors)→ fast calculation of H/V → inversion**
  - **Incoming body waves (EQ) → fast calculation → inversion**
  - **Measurement at depth (MT)→ fast calculation → inversion**
  - **H/V near lateral irregularities→ Dipping Layers**
  - **With appropriate data processing H/V may be used to assess Site Effects in Strong Ground Motion.**

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Thank you ☺..!
For your Attention