

Seismic Interferometry and Beyond

Kees Wapenaar

Cargese Workshop 2015

Contents

- Part I: Seismic interferometry by cross-correlation
- Part II: Seismic interferometry by multi-dimensional deconvolution
- Part III: Beyond seismic interferometry

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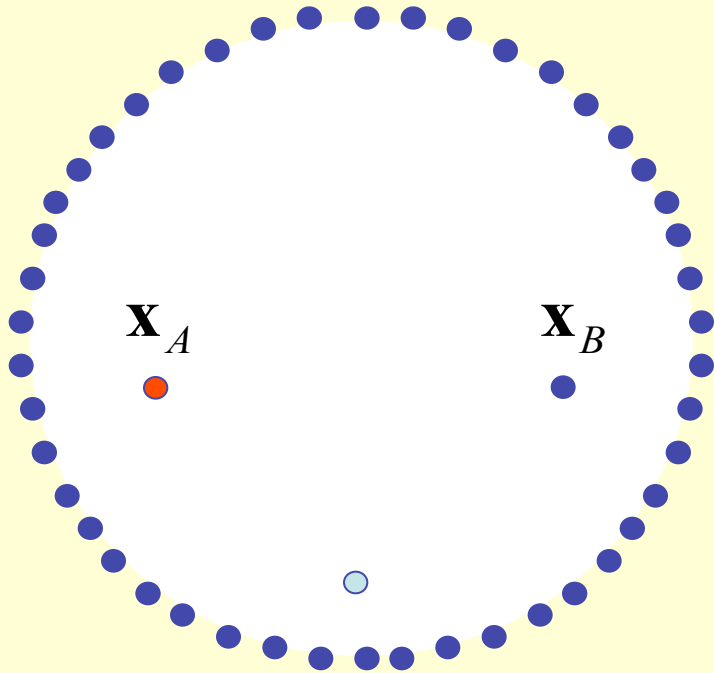
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Part III is also available as E-lecture:

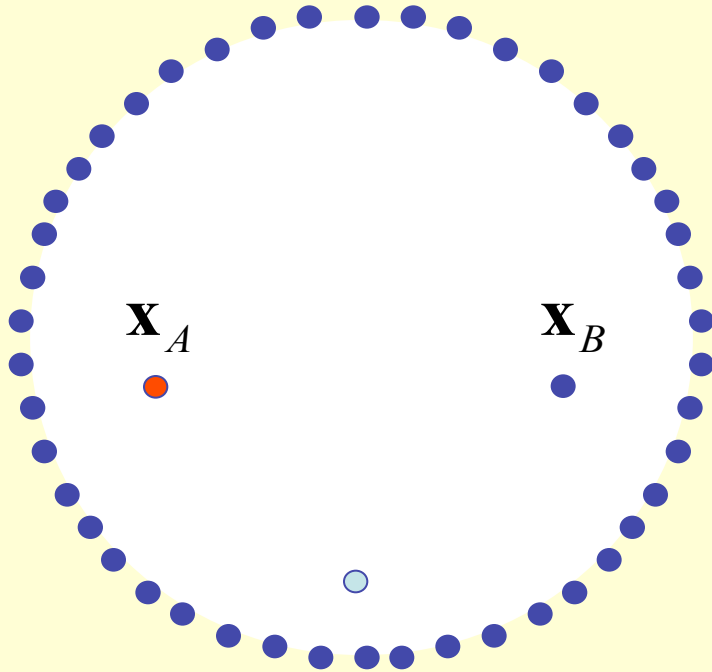
<https://youtu.be/DrpagMsK09M>

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$$G_0(\mathbf{x}_B, \mathbf{x}_A, t) - G_0(\mathbf{x}_B, \mathbf{x}_A, -t) \approx -\frac{2}{\rho c} \frac{\partial}{\partial t} \int_S G_0(\mathbf{x}_B, \mathbf{x}, t) * G_0(\mathbf{x}_A, \mathbf{x}, -t) d^2 \mathbf{x}$$

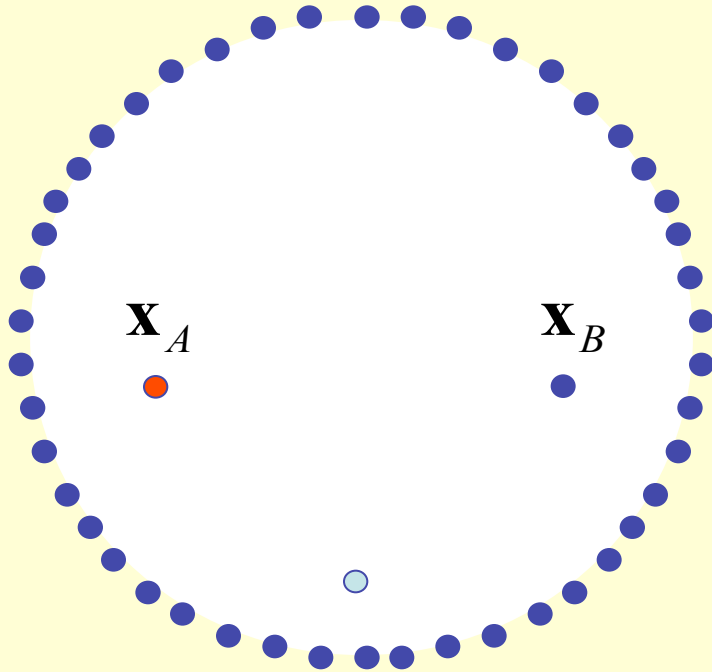


Representation proposed in the context of image-resolution analysis:

Porter, R. P., 1970, Diffraction-limited, scalar image formation with holograms of arbitrary shape: Journal of the Optical Society of America, 60, 1051–1059.

Oristaglio, M. L., 1989, An inverse scattering formula that uses all the data: Inverse Problems, 5, 1097–1105.

$$G_0(\mathbf{x}_B, \mathbf{x}_A, t) - G_0(\mathbf{x}_B, \mathbf{x}_A, -t) \approx -\frac{2}{\rho c} \frac{\partial}{\partial t} \int_S G_0(\mathbf{x}_B, \mathbf{x}, t) * G_0(\mathbf{x}_A, \mathbf{x}, -t) d^2 \mathbf{x}$$



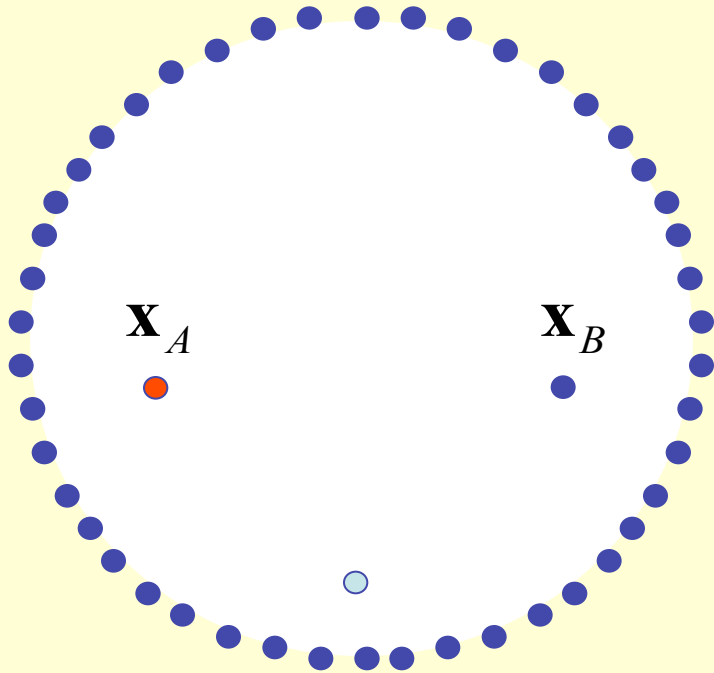
Representation proposed in the context of Green's function retrieval from ambient noise:

Wapenaar, K., 2003, Synthesis of an Inhomogeneous medium from its acoustic transmission response: *Geophysics*, 68, 1756-1759

Weaver, R. L., and O. I. Lobkis, 2004, Diffuse fields in open systems and the emergence of the Green's function L: *Journal of the Acoustical Society of America*, 116, 2731–2734

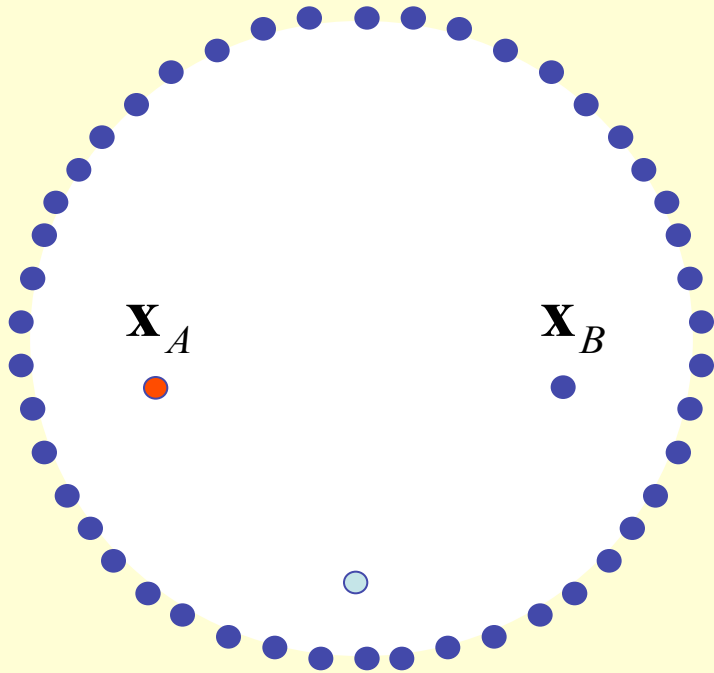
Wapenaar, K., 2004, Retrieving the elastodynamic Green's function of an arbitrary inhomogeneous medium by cross correlation: *Physical Review Letters*, 93, 254301

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Sum or difference of G 's?

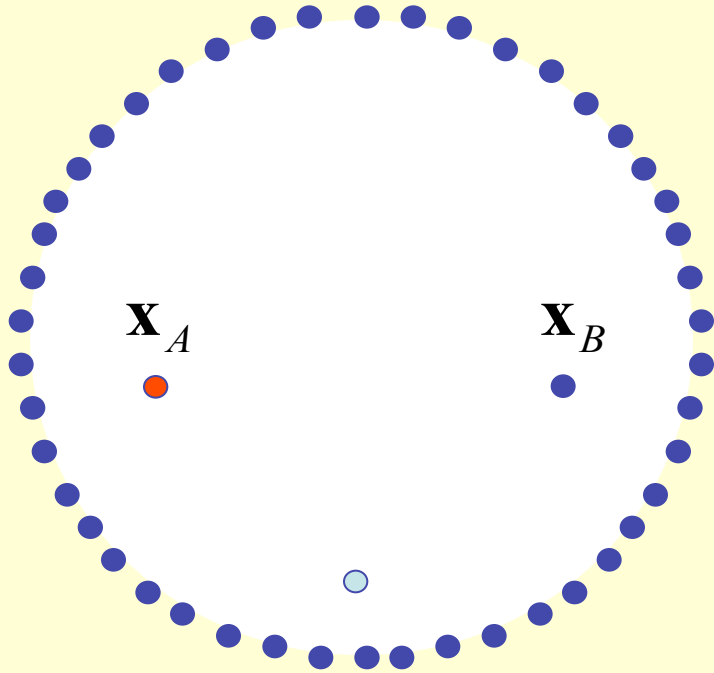
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Sum or difference of G 's?

$$G(\mathbf{x}_B, \mathbf{x}_A, t) = \frac{\partial}{\partial t} G_0(\mathbf{x}_B, \mathbf{x}_A, t)$$

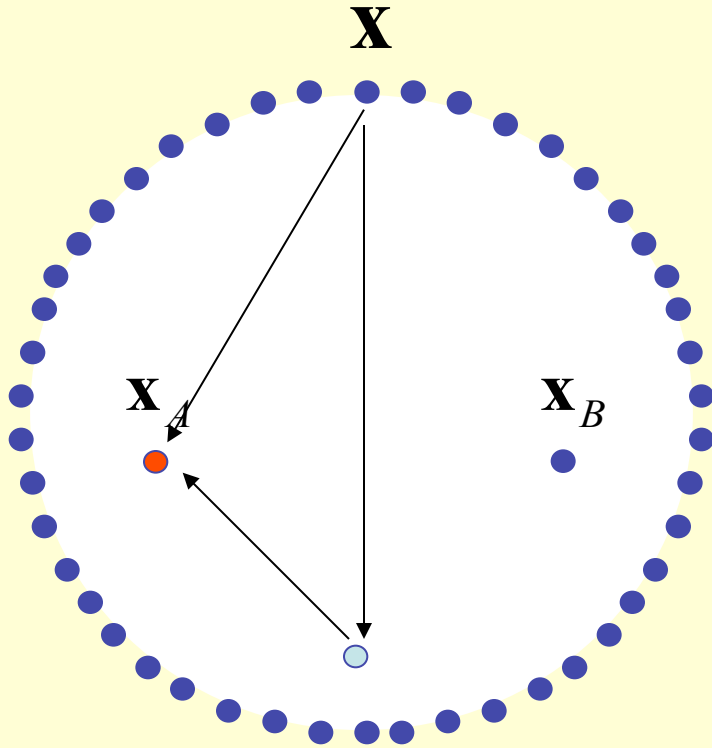
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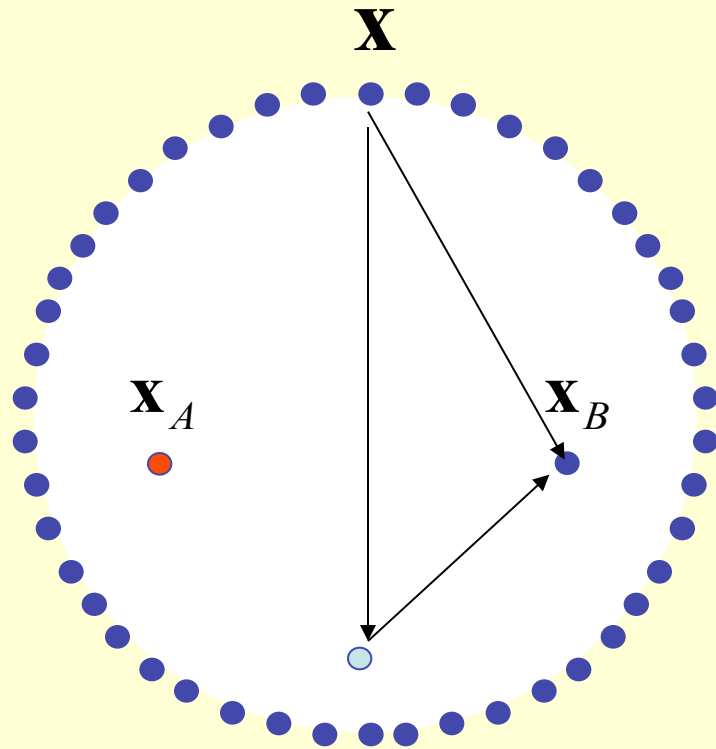
$$G(\mathbf{x}_B, \mathbf{x}_A, t) = \frac{\partial}{\partial t} G_0(\mathbf{x}_B, \mathbf{x}_A, t)$$

$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \approx \frac{2}{\rho c} \int_S G(\mathbf{x}_B, \mathbf{x}, t) * G(\mathbf{x}_A, \mathbf{x}, -t) d^2 \mathbf{x}$$



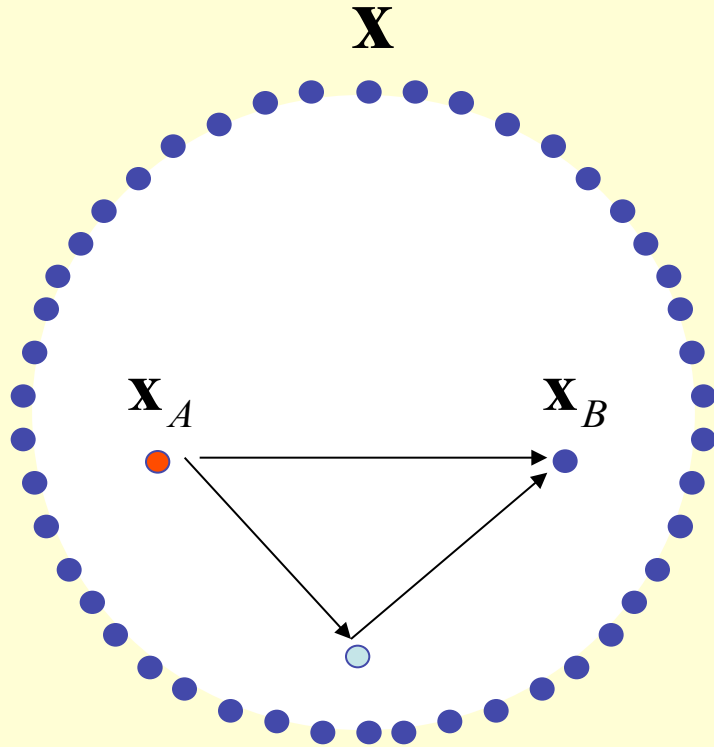
Source at \mathbf{x} ,
Receiver at \mathbf{x}_A

$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \approx \frac{2}{\rho c} \int_S G(\mathbf{x}_B, \mathbf{x}, t) * G(\mathbf{x}_A, \mathbf{x}, -t) d^2 \mathbf{x}$$



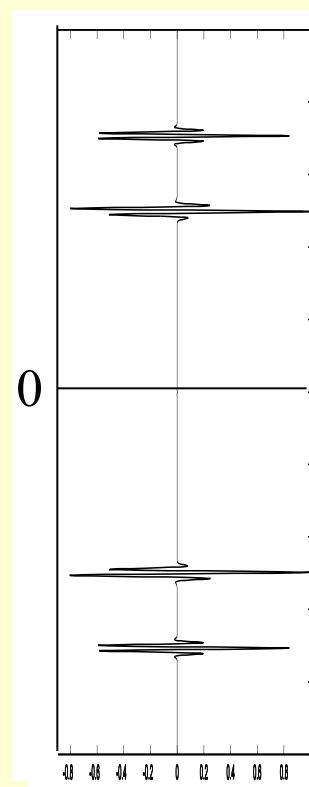
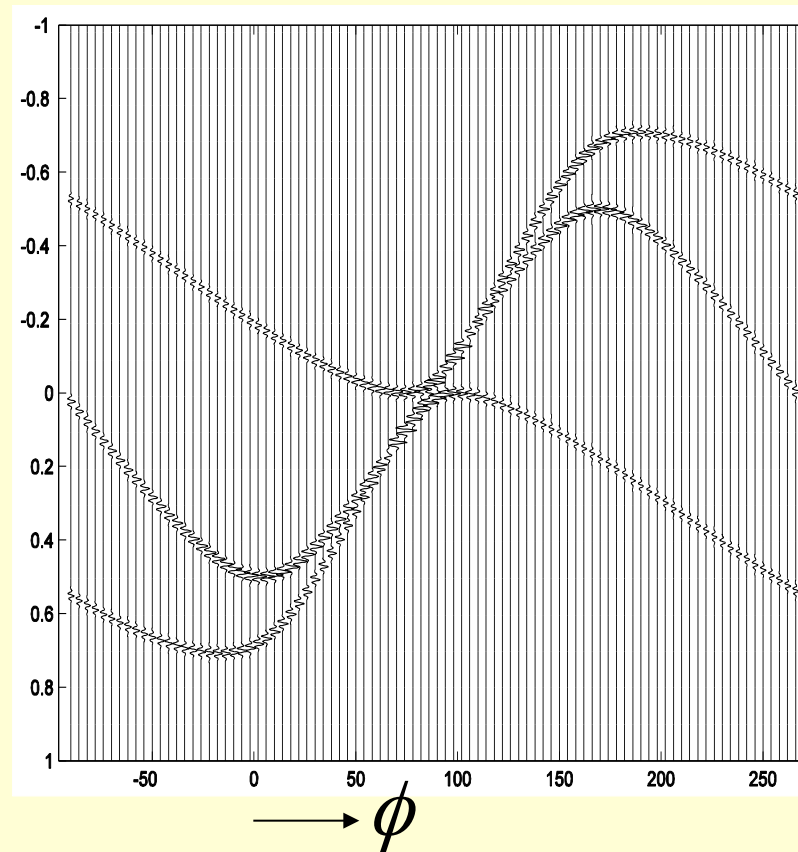
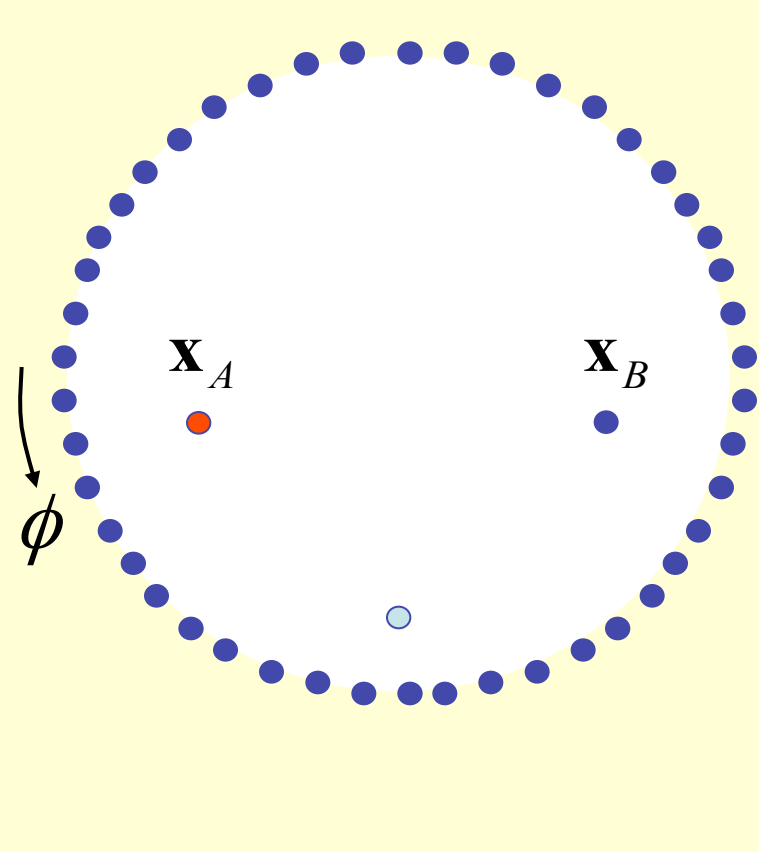
Source at \mathbf{x} ,
Receiver at \mathbf{x}_B

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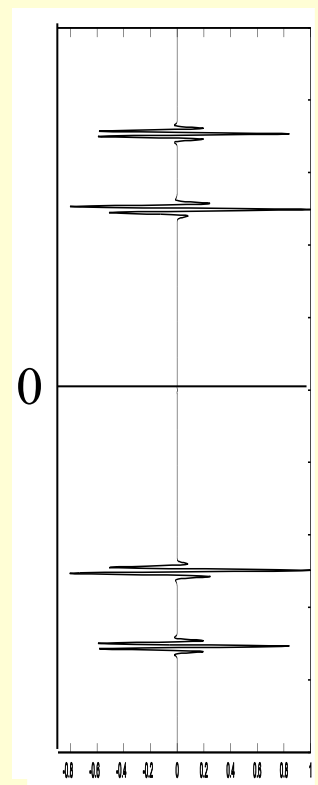
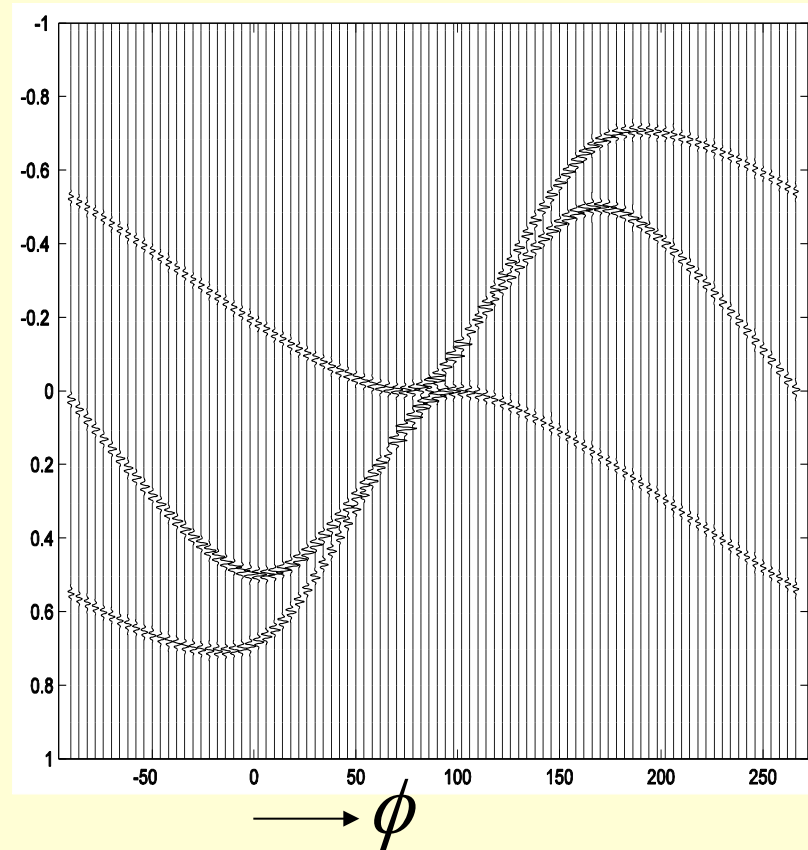
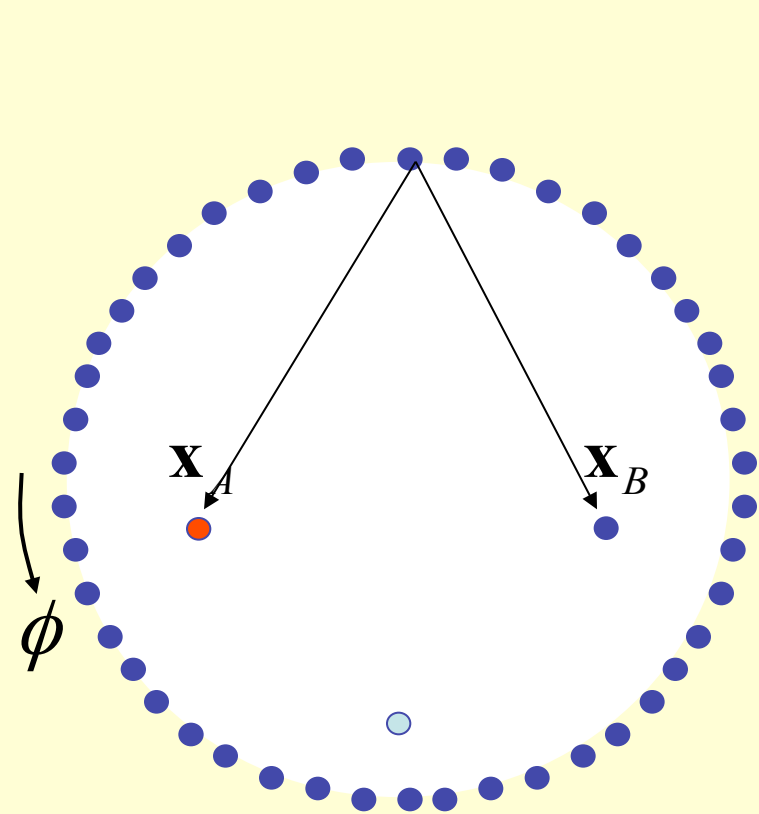


Source at \mathbf{x}_A ,
Receiver at \mathbf{x}_B

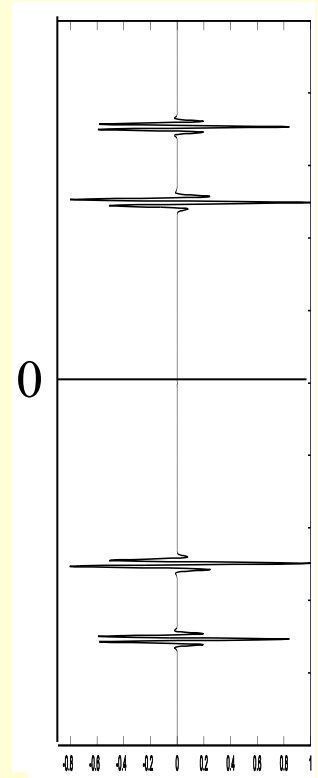
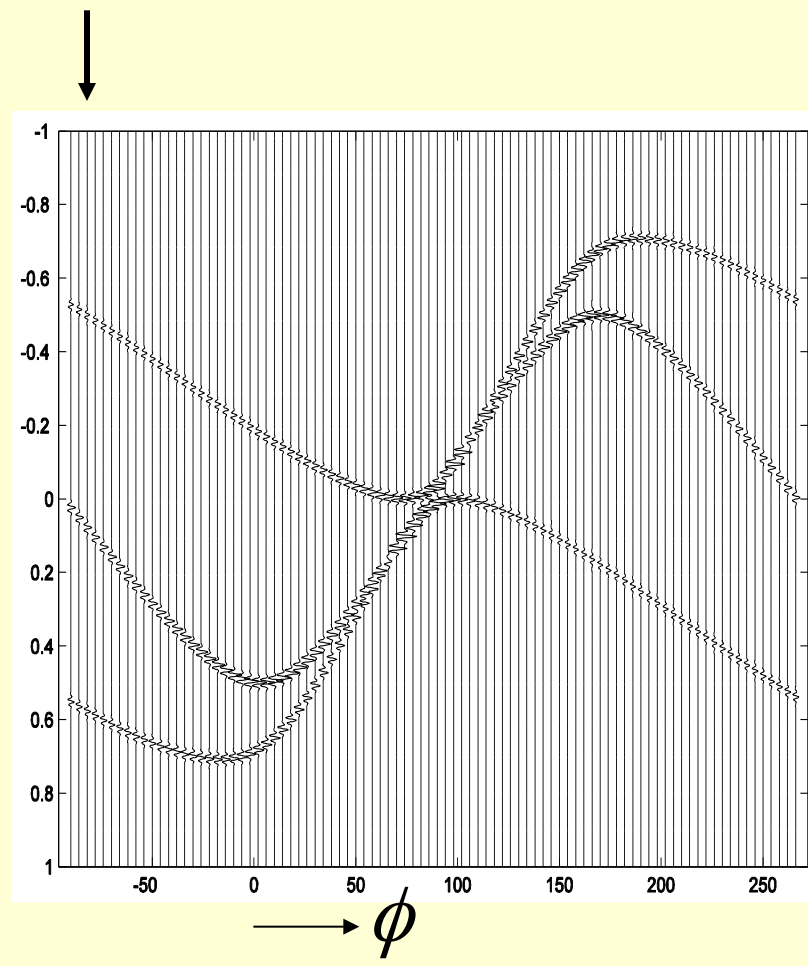
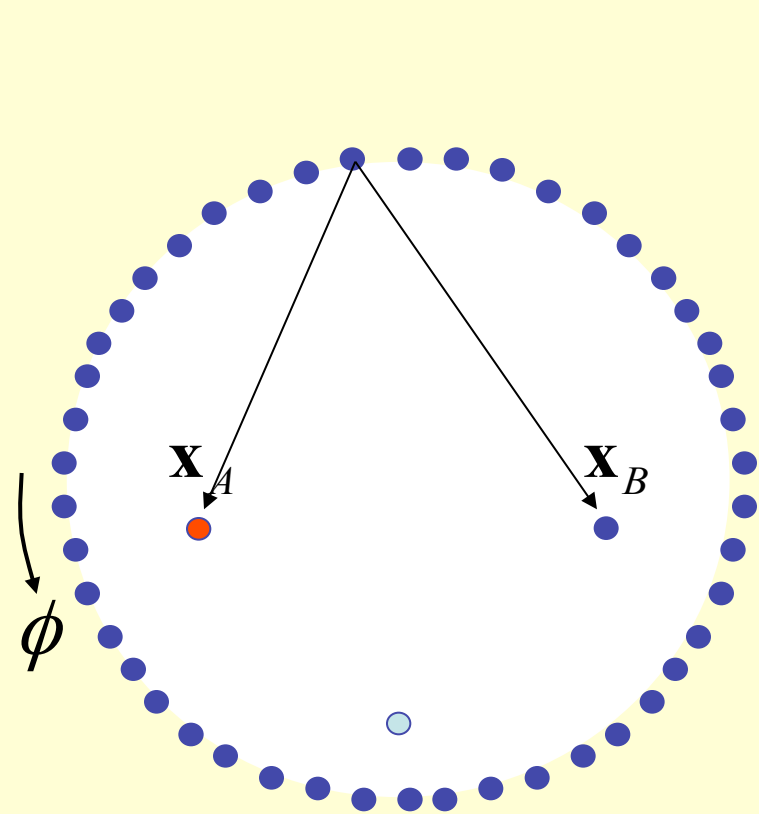
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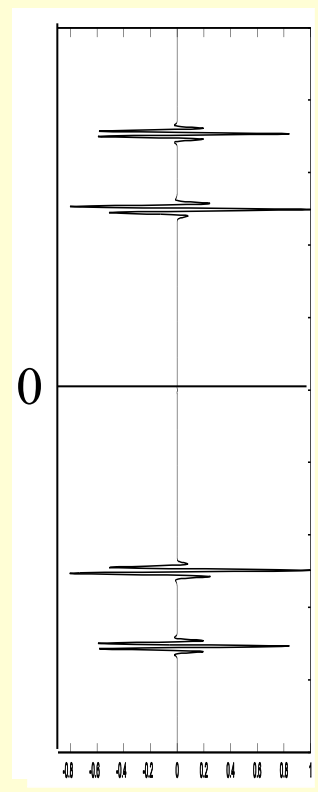
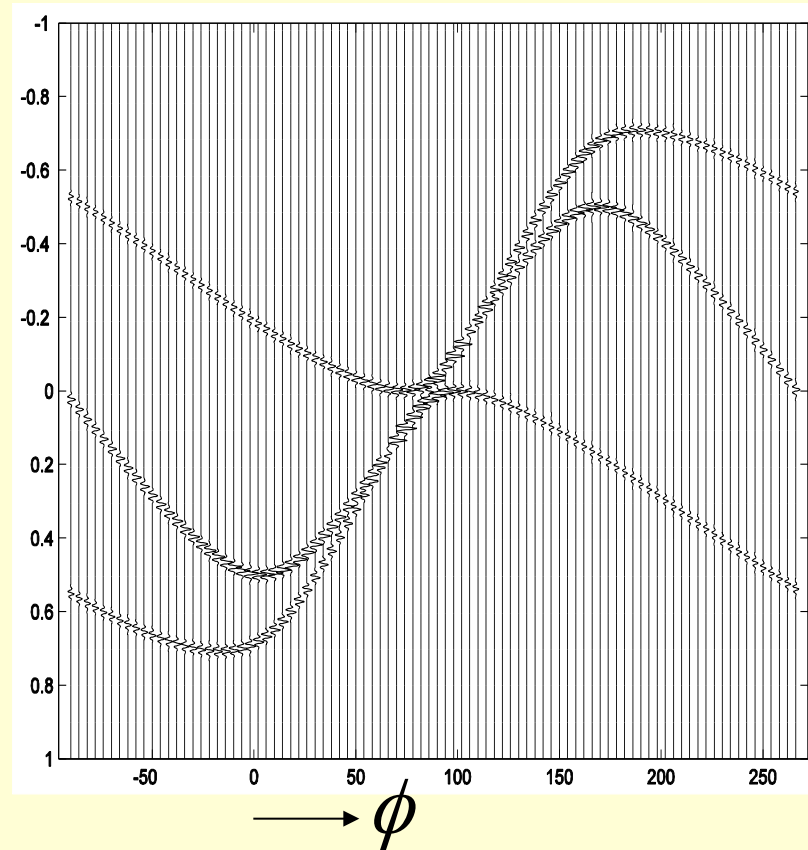
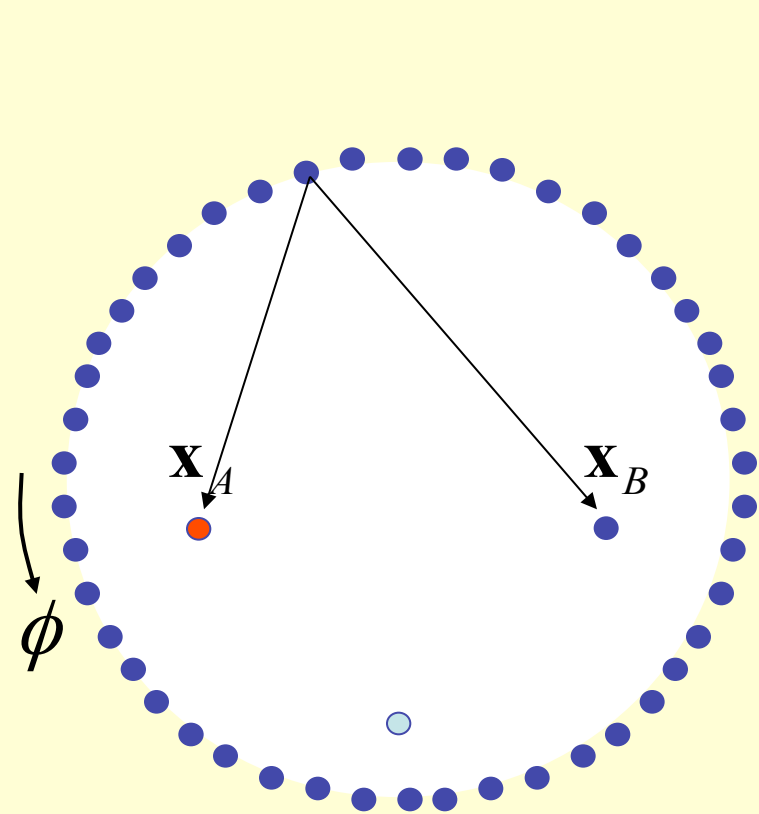
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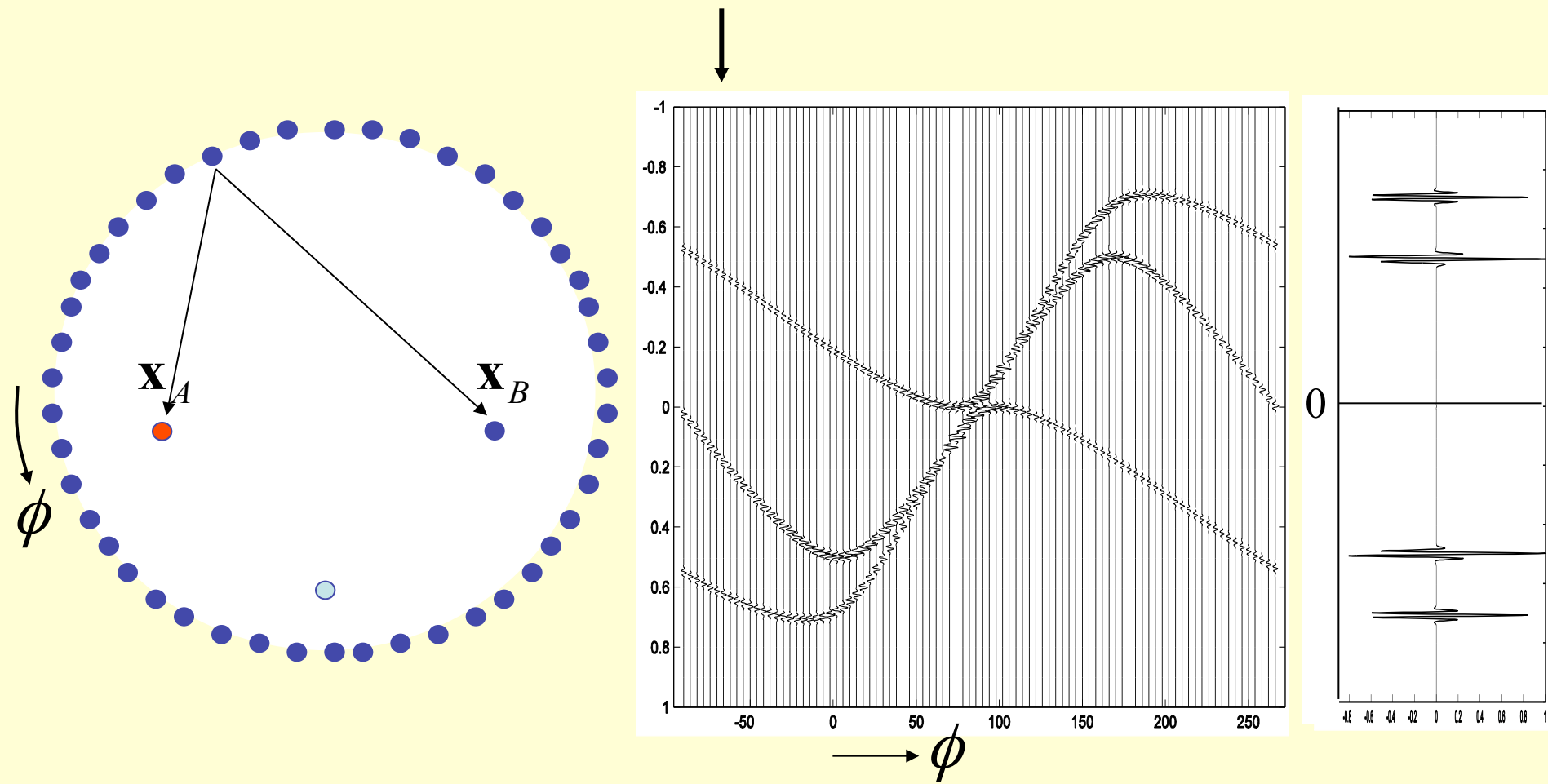
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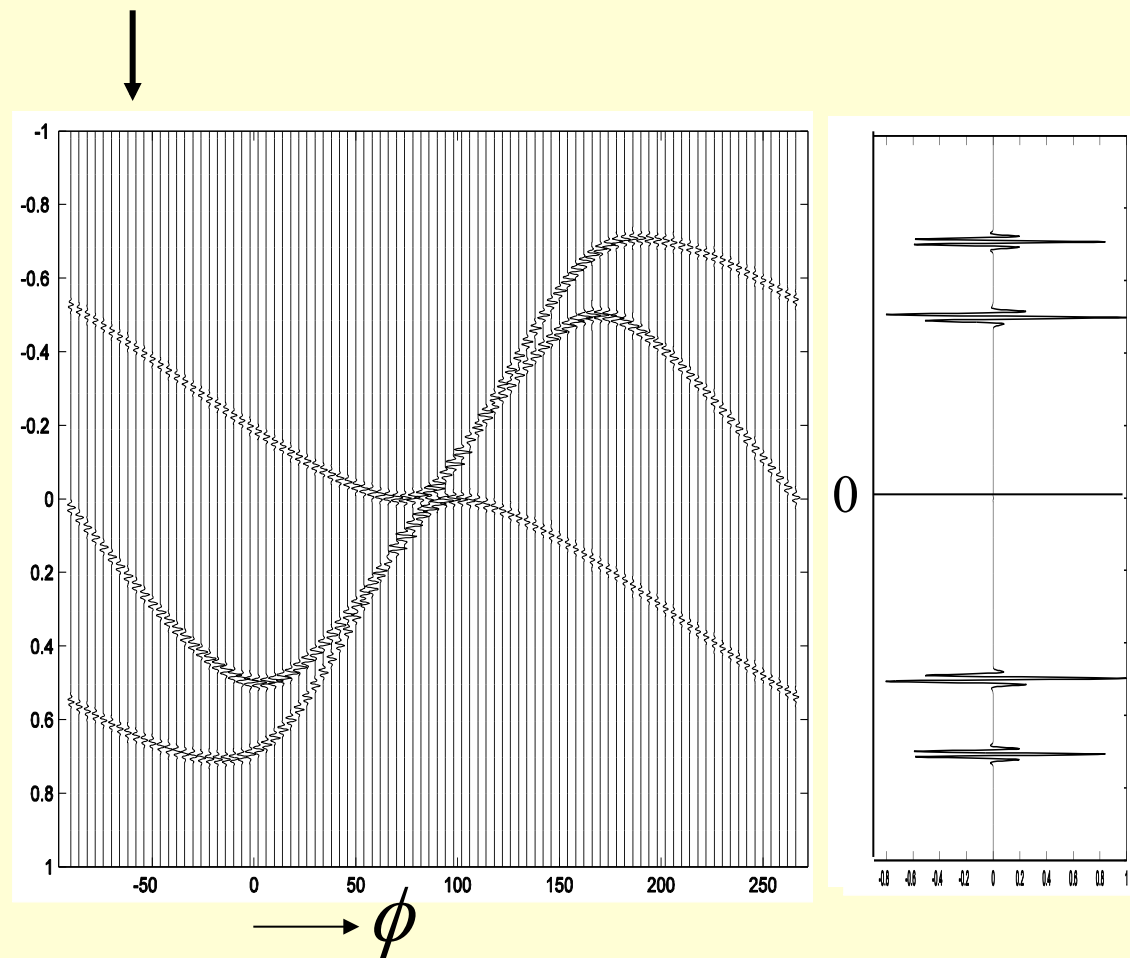
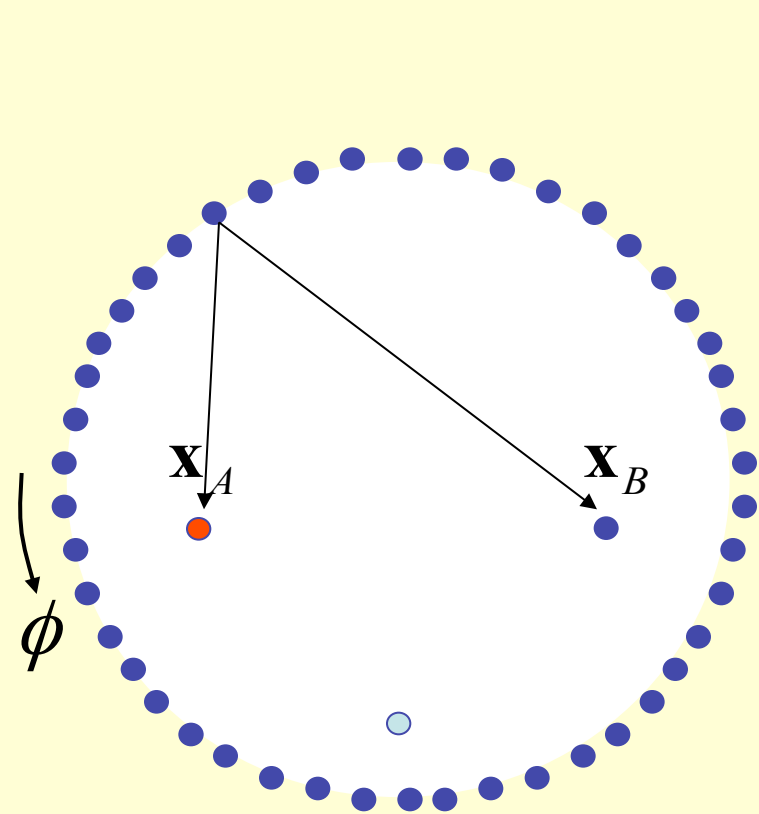
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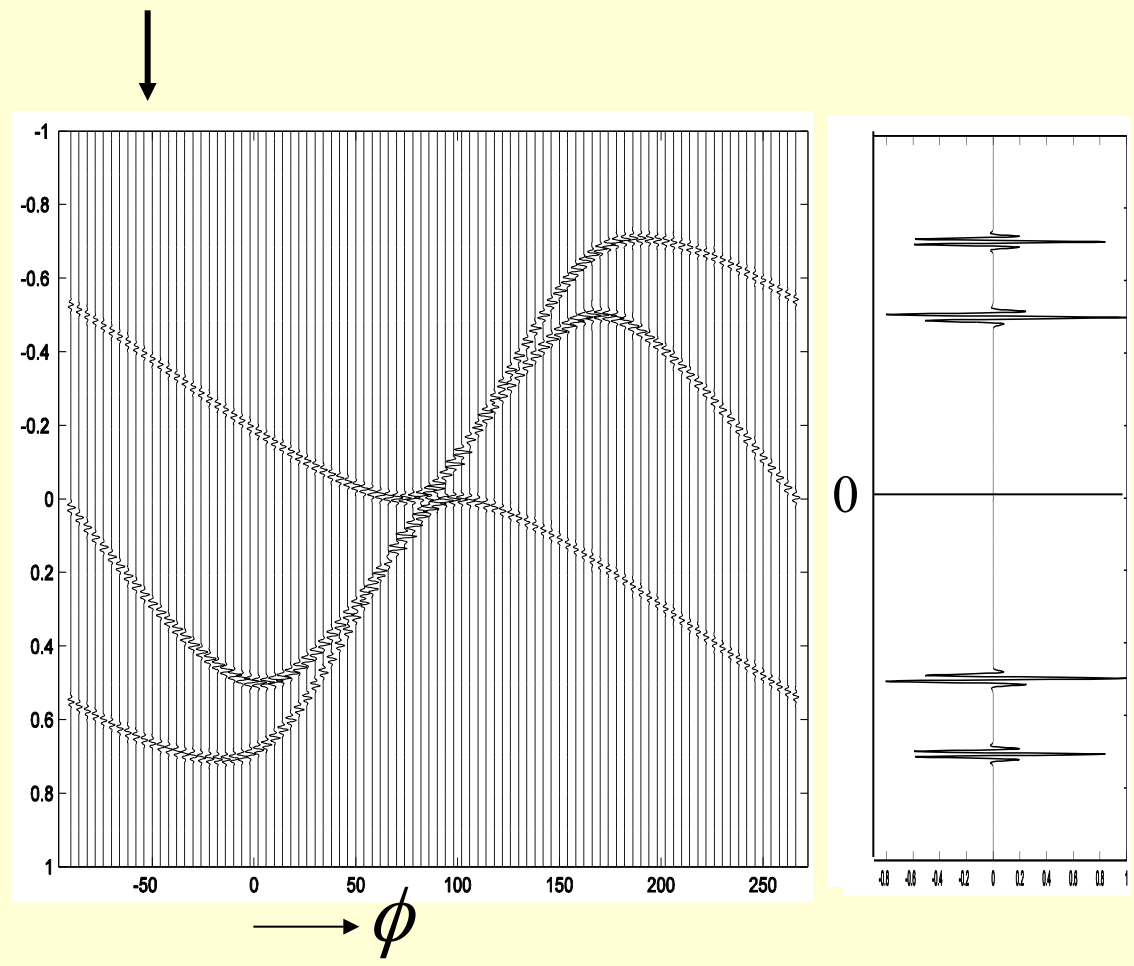
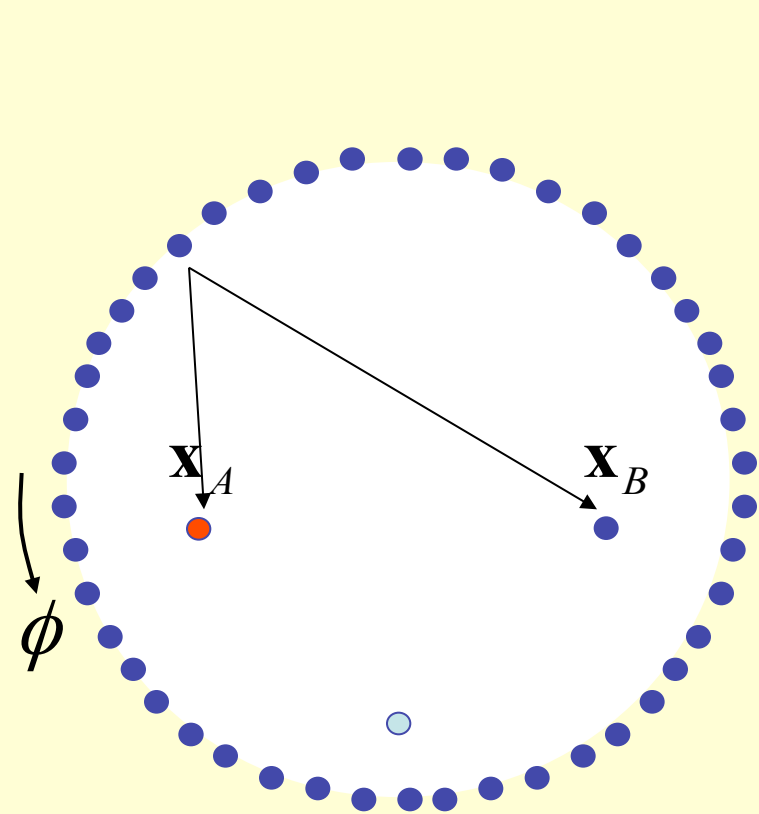
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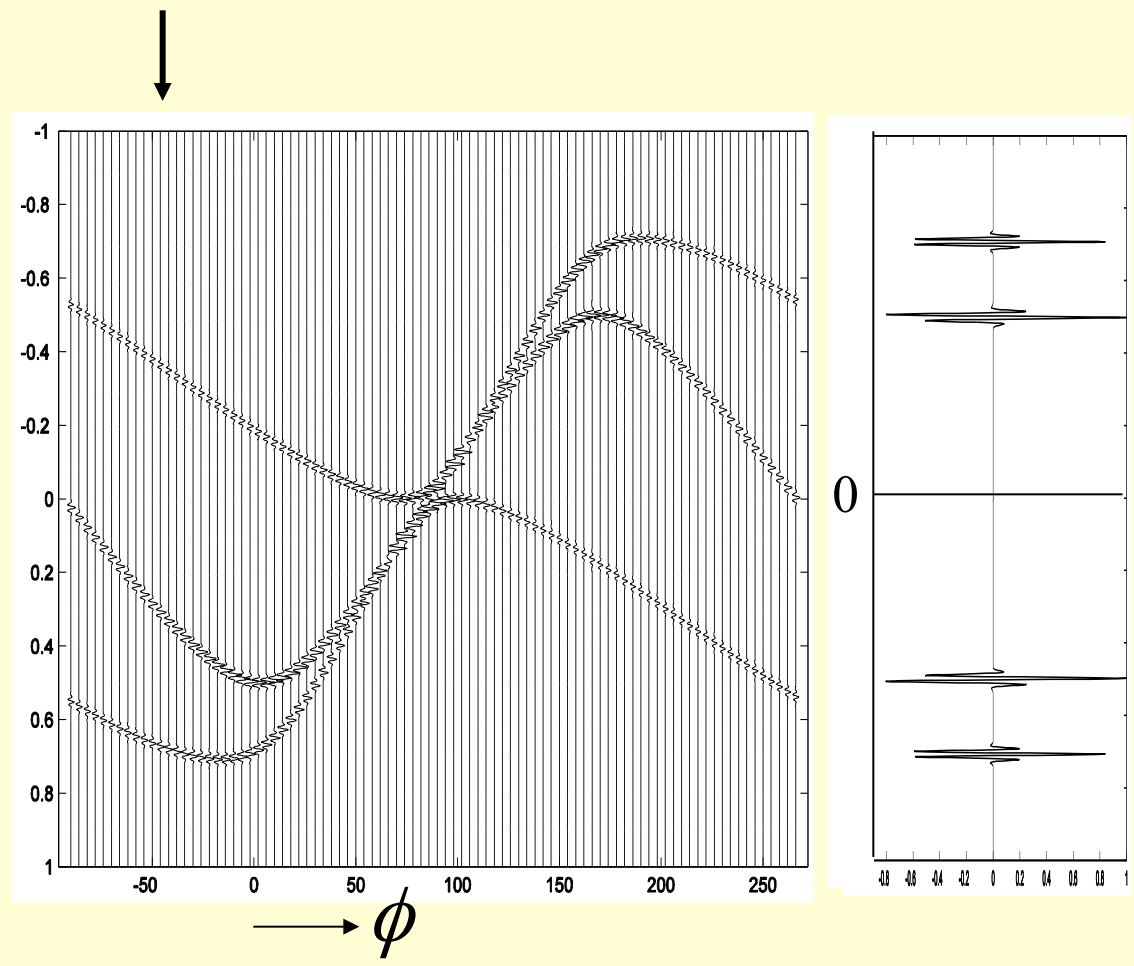
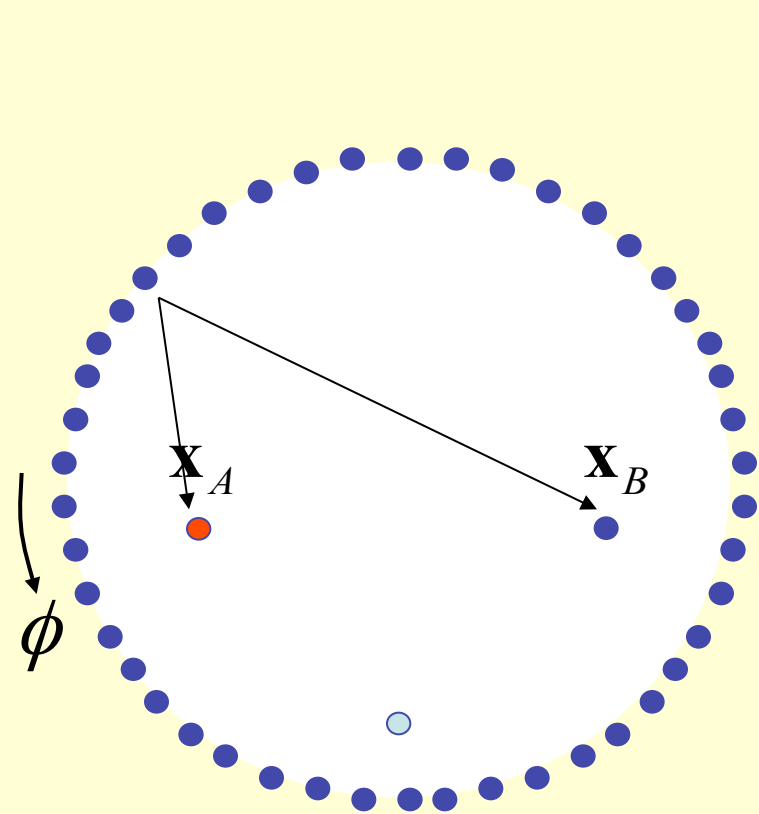
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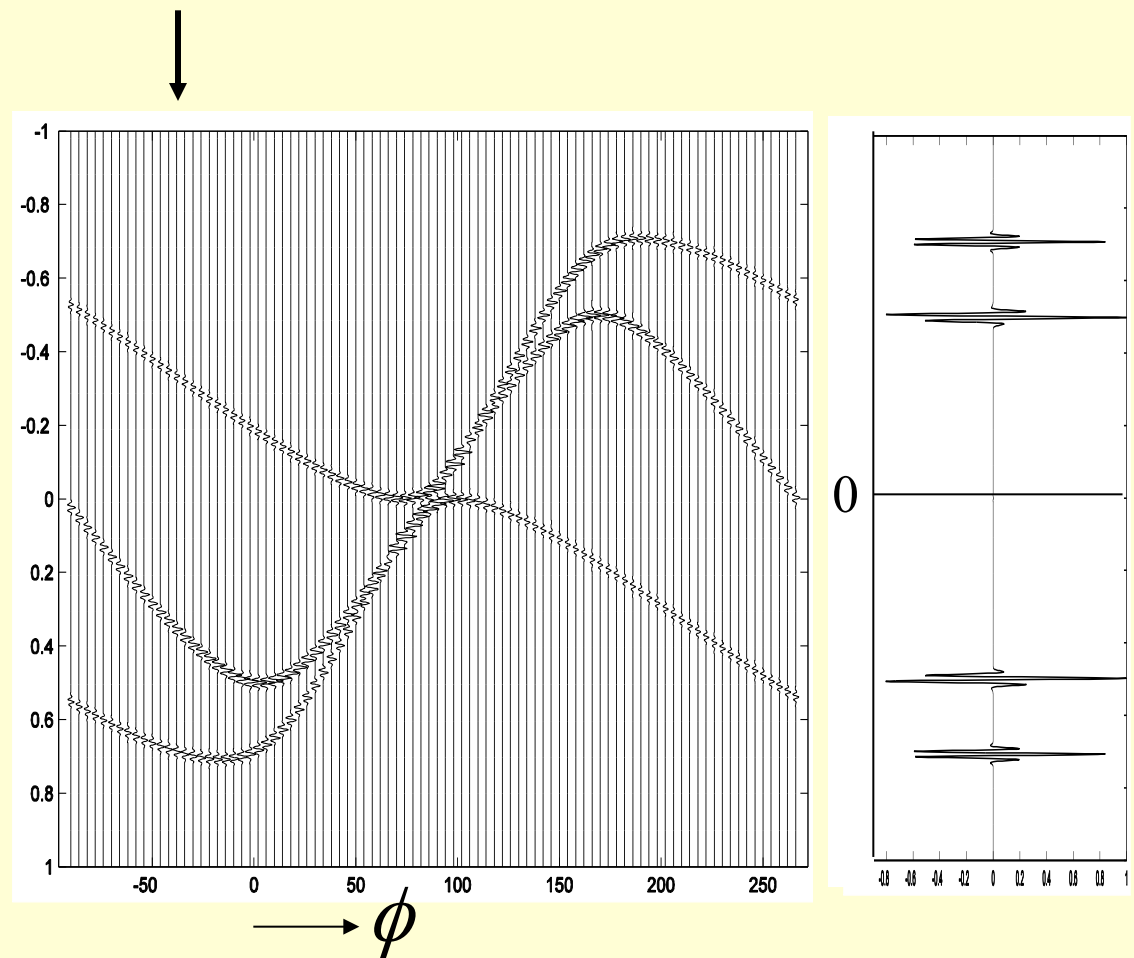
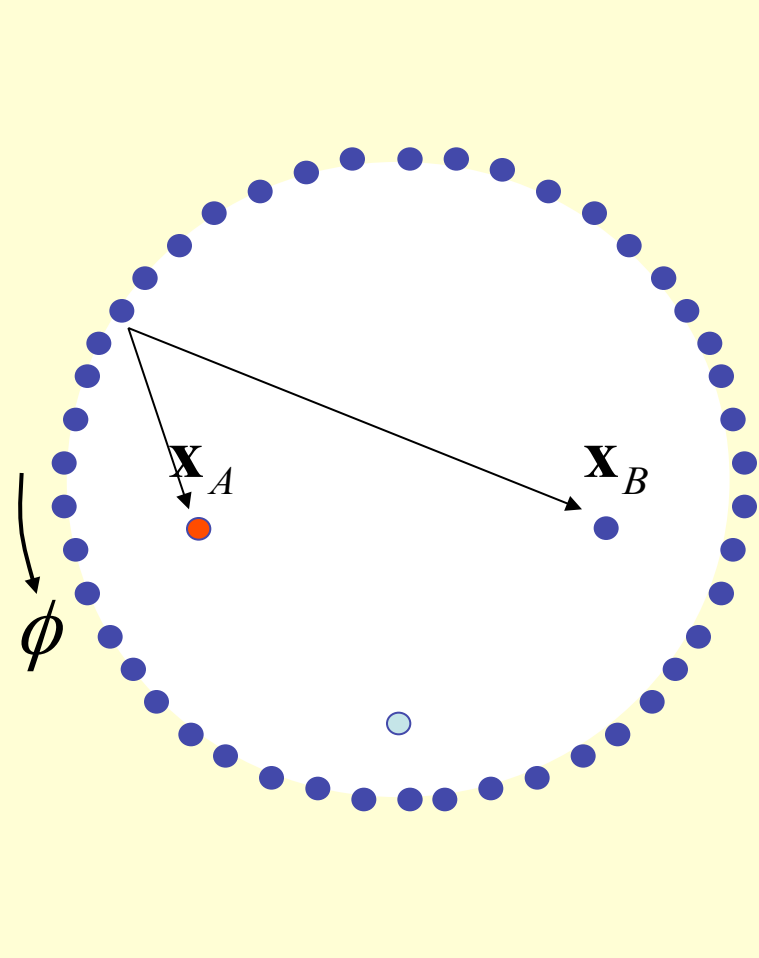
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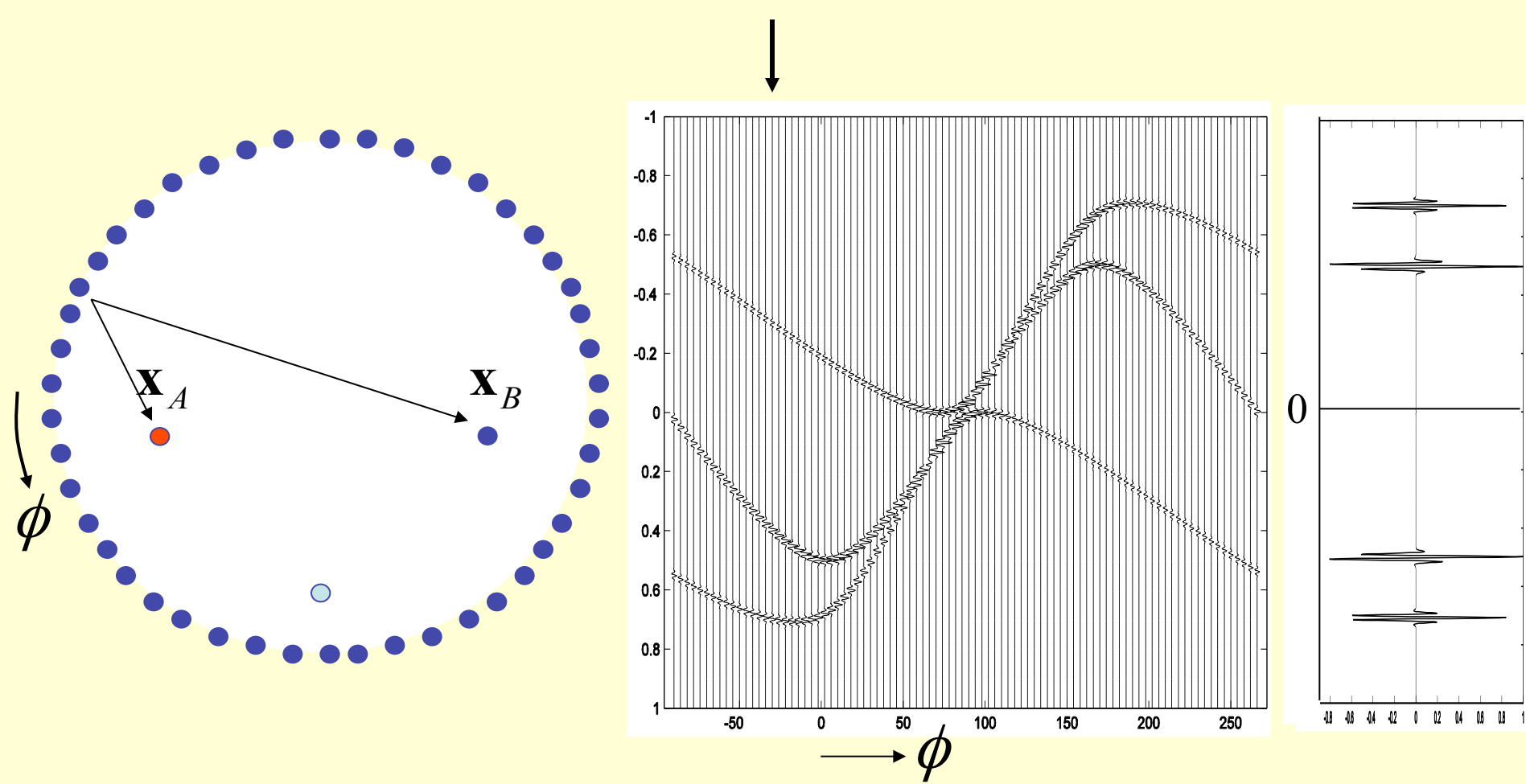
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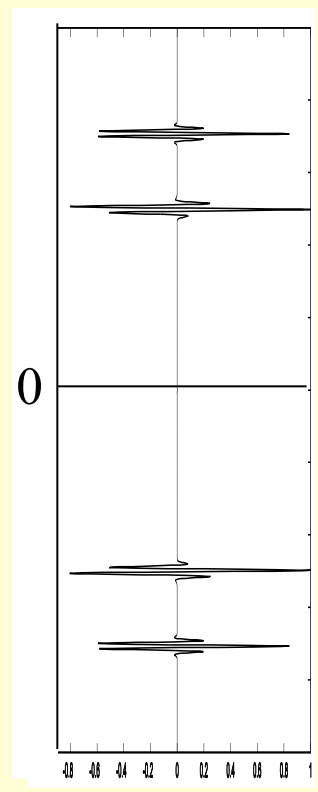
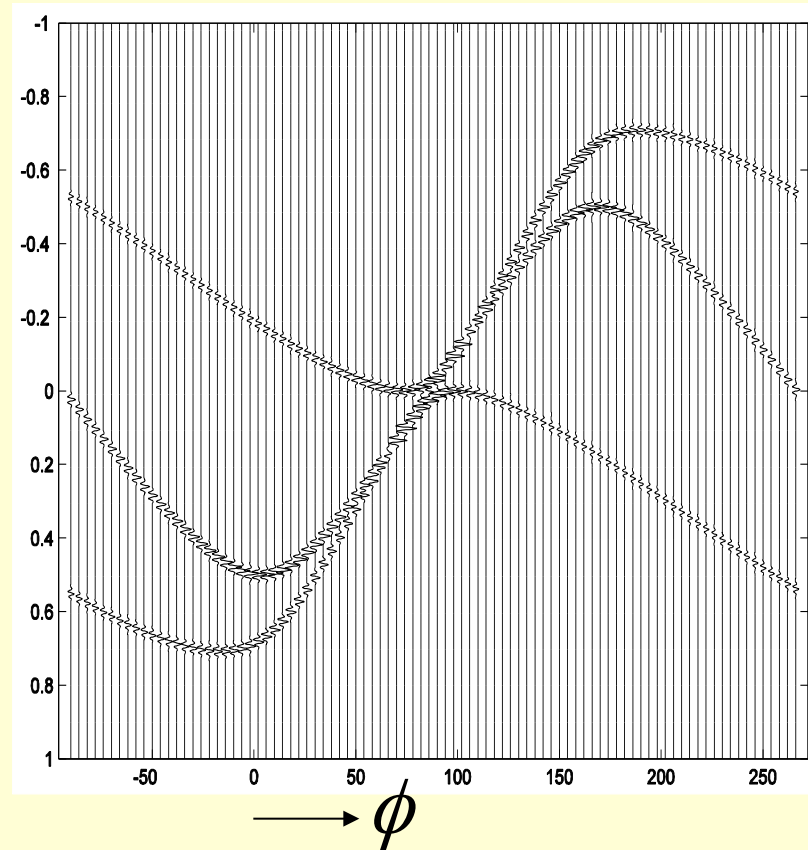
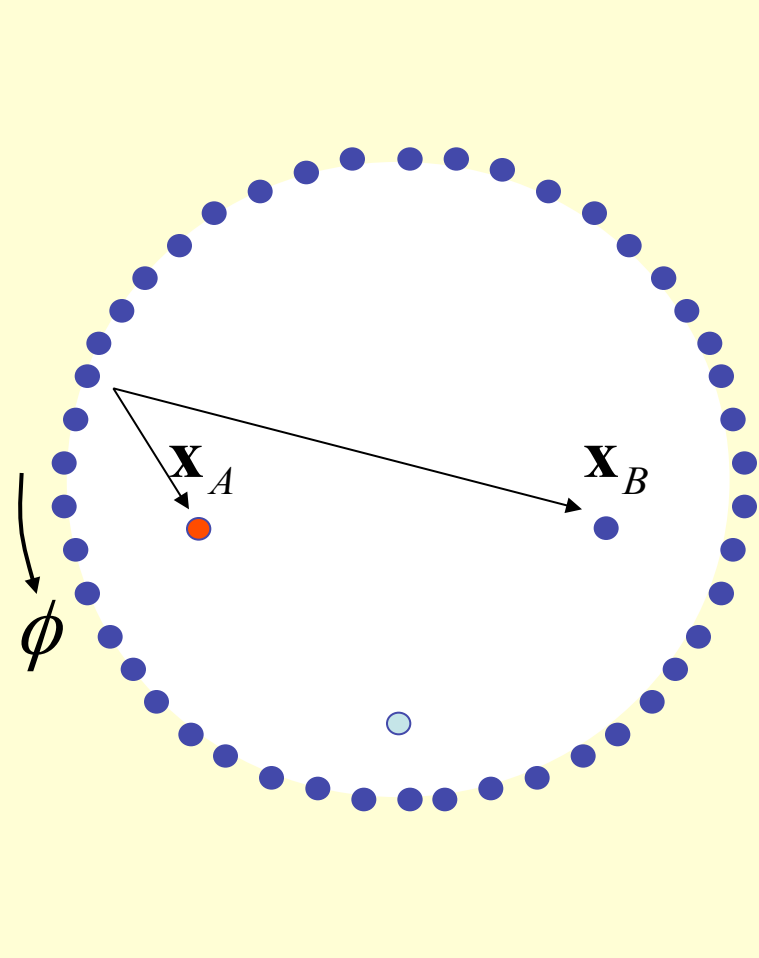
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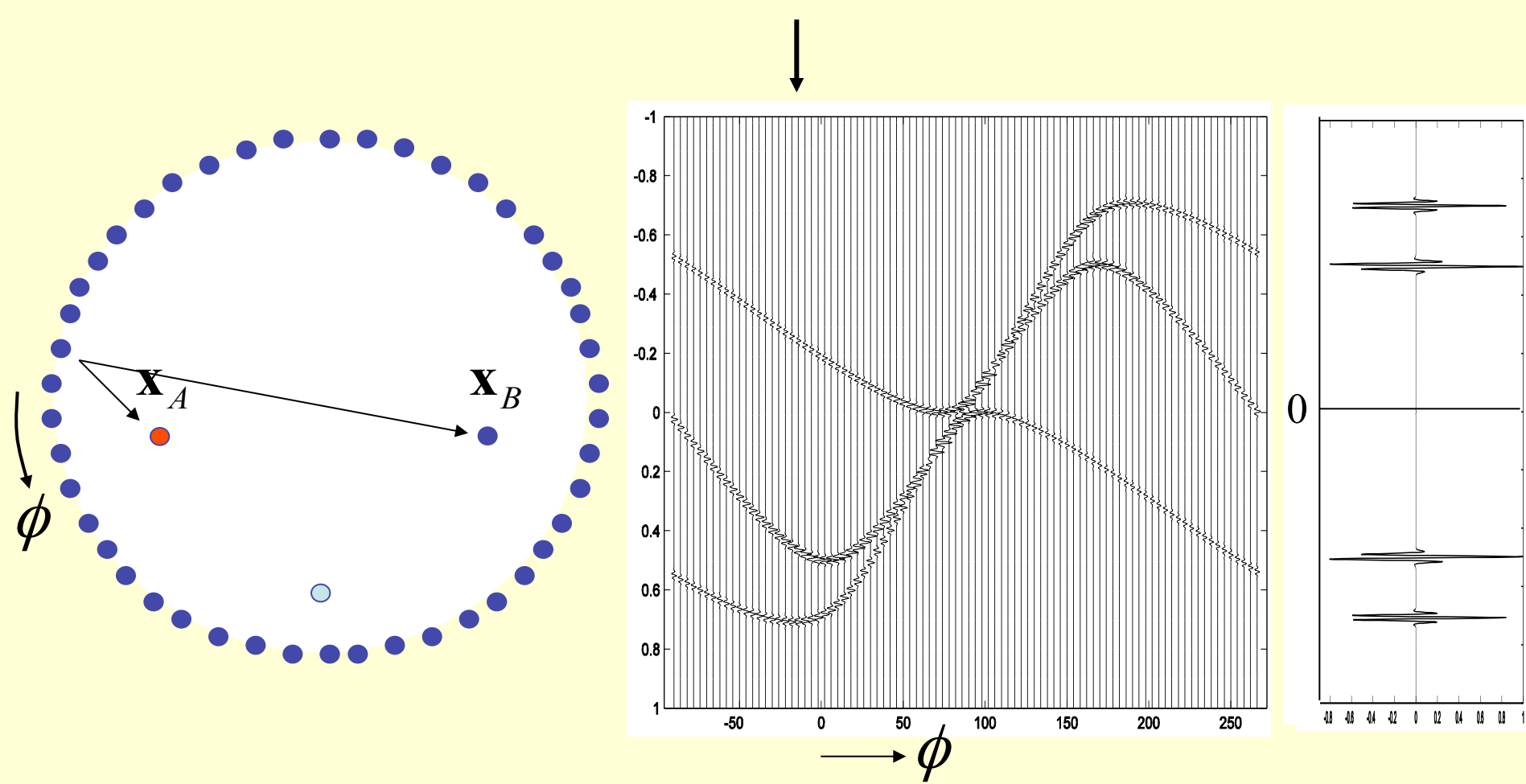
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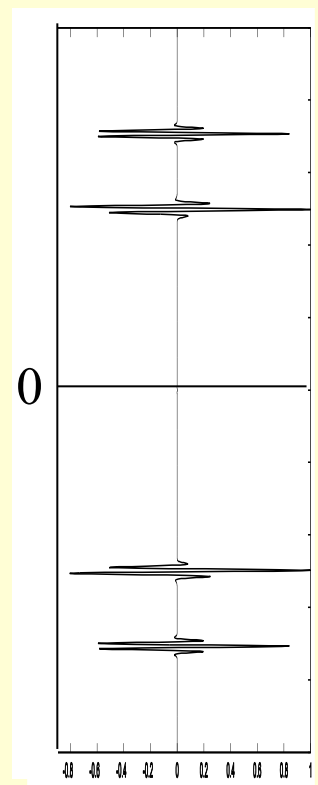
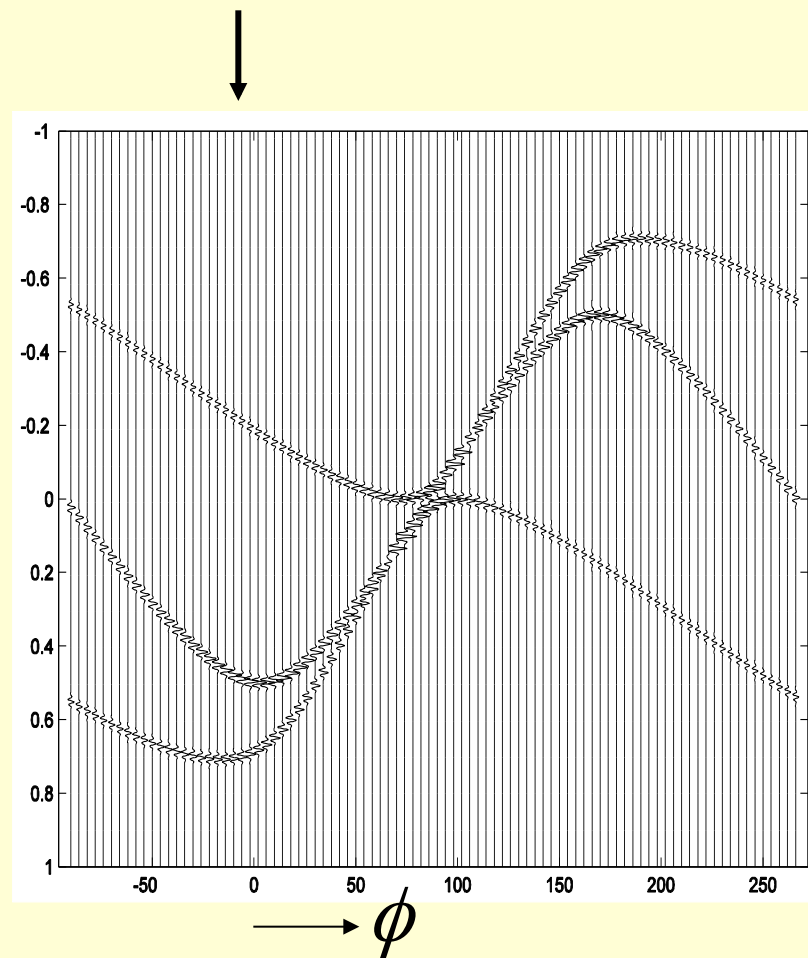
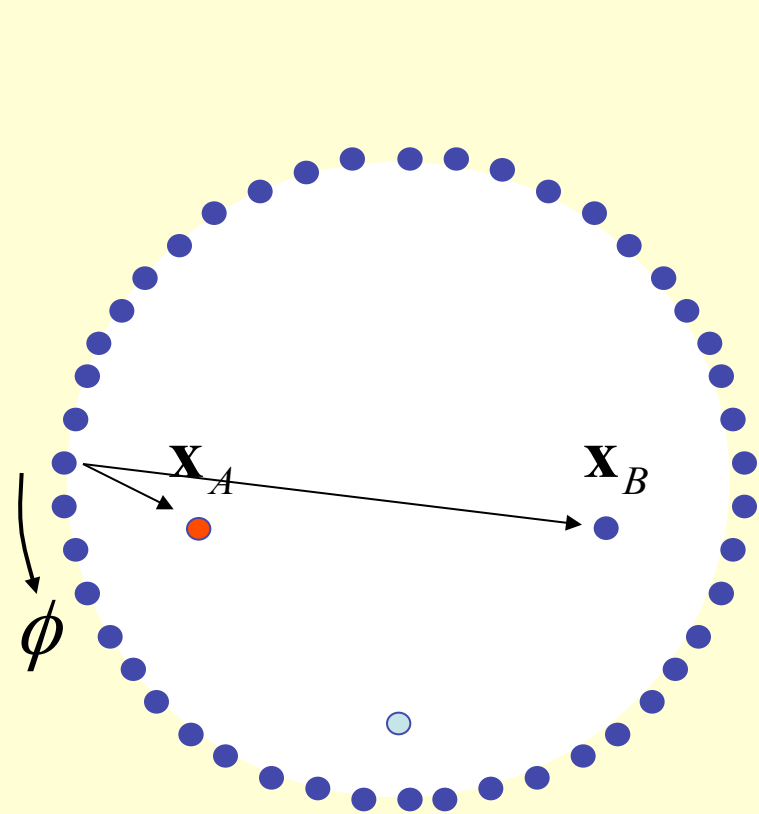
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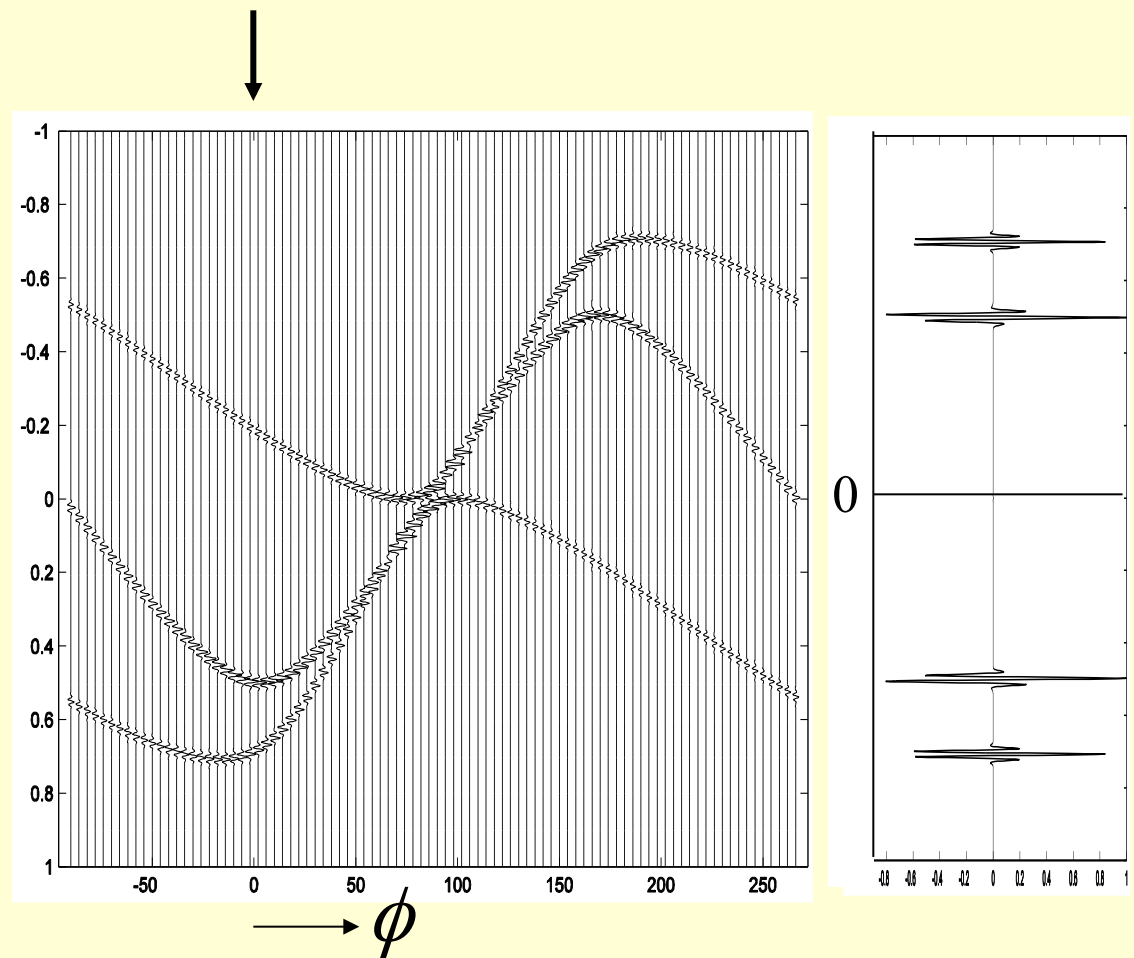
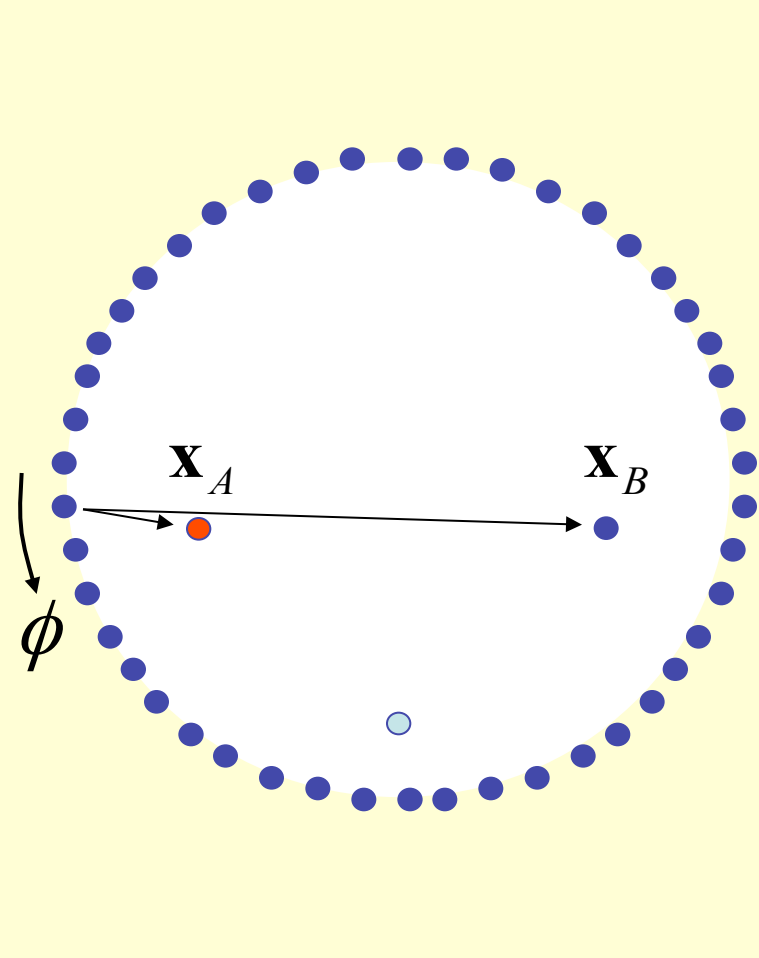
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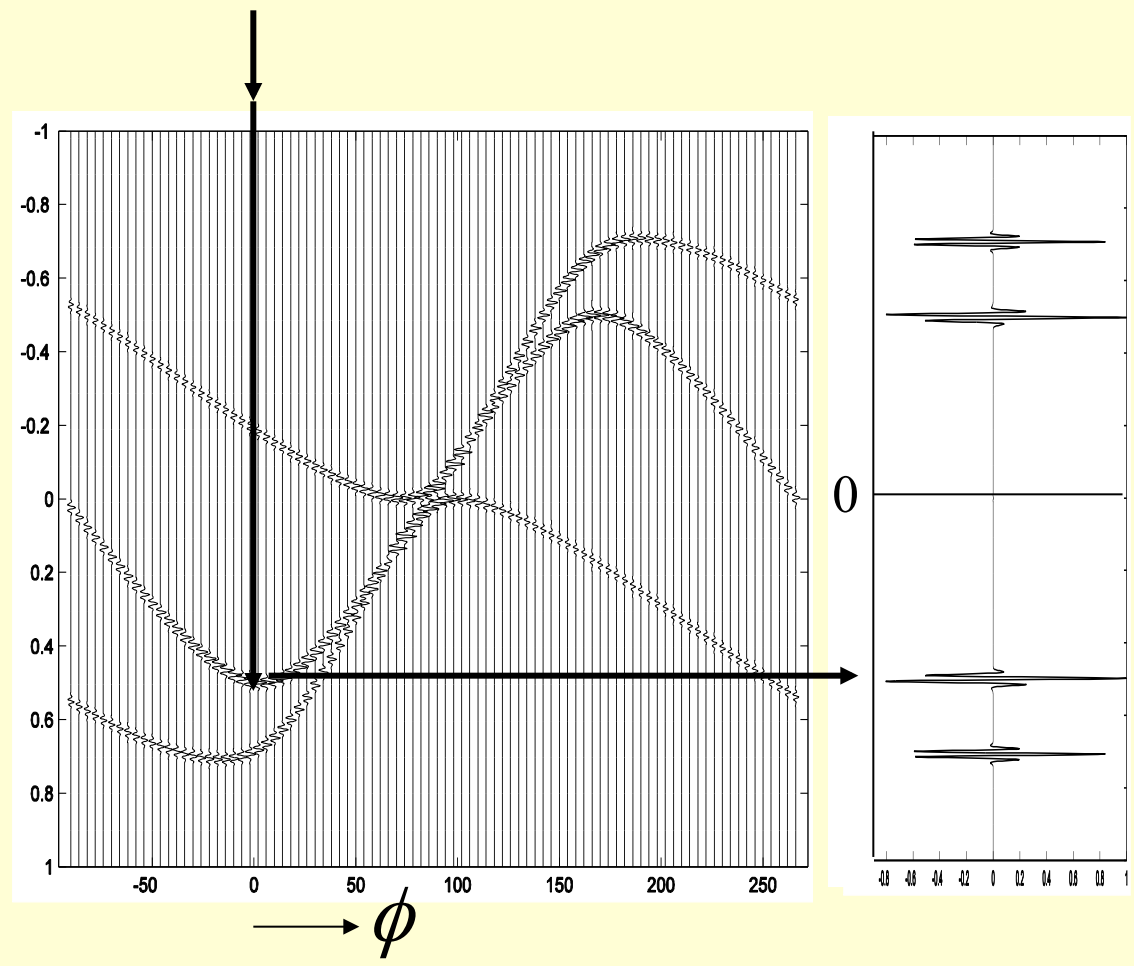
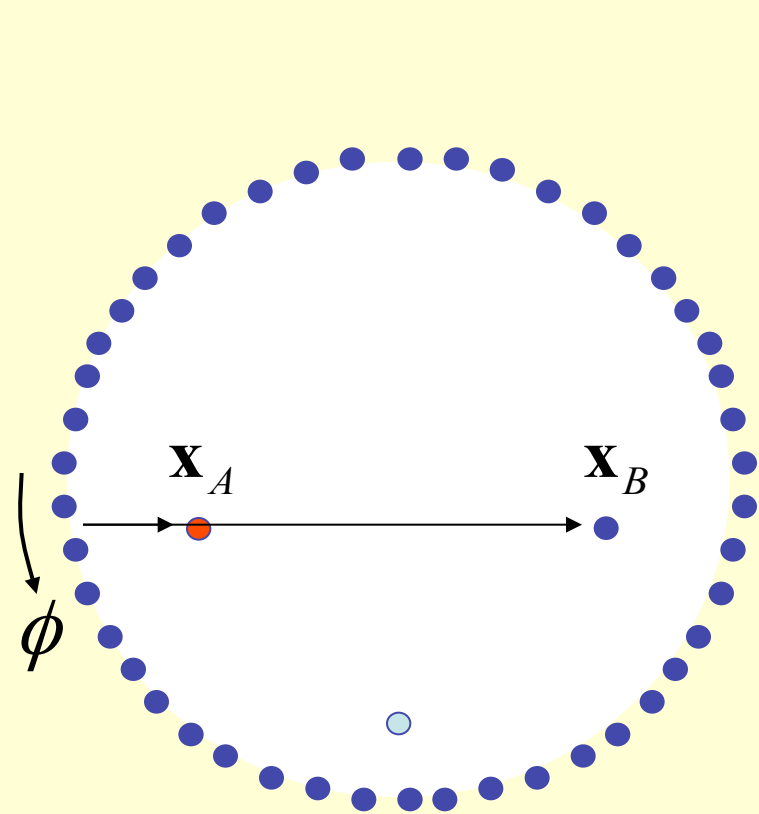
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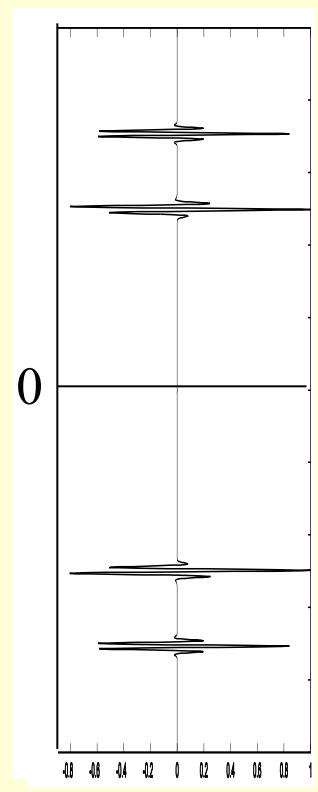
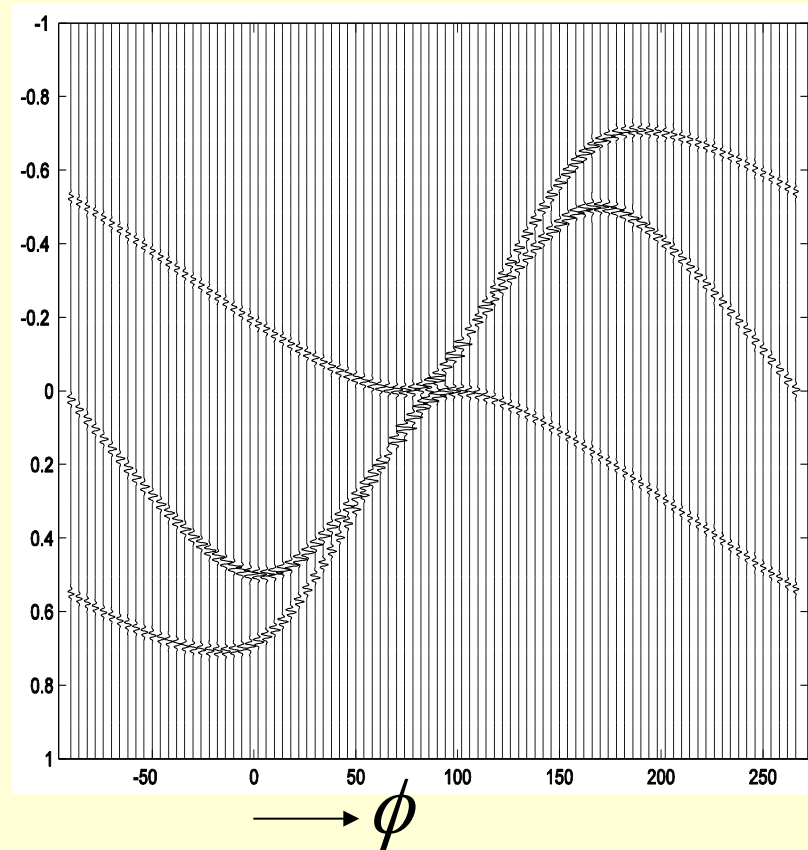
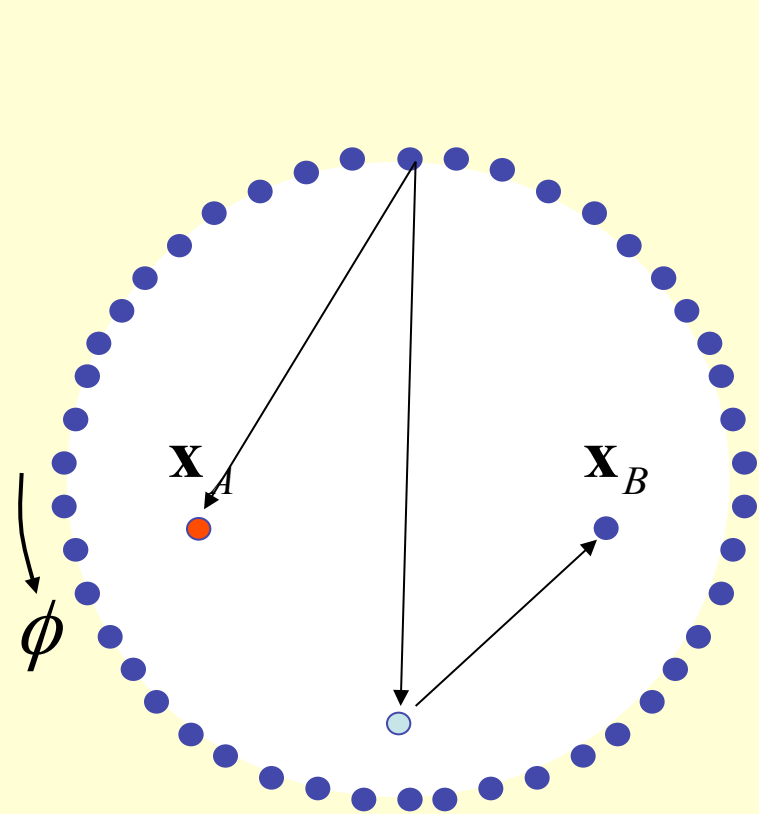
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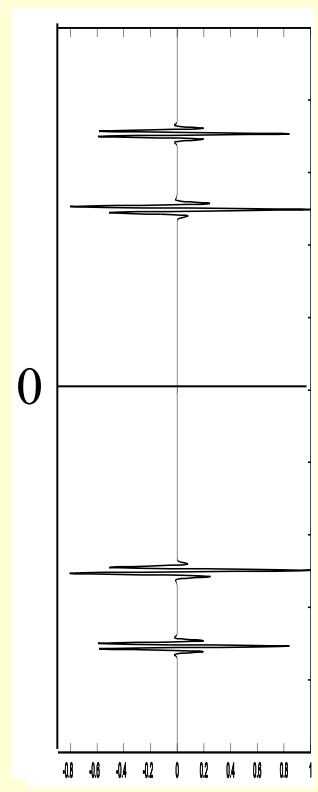
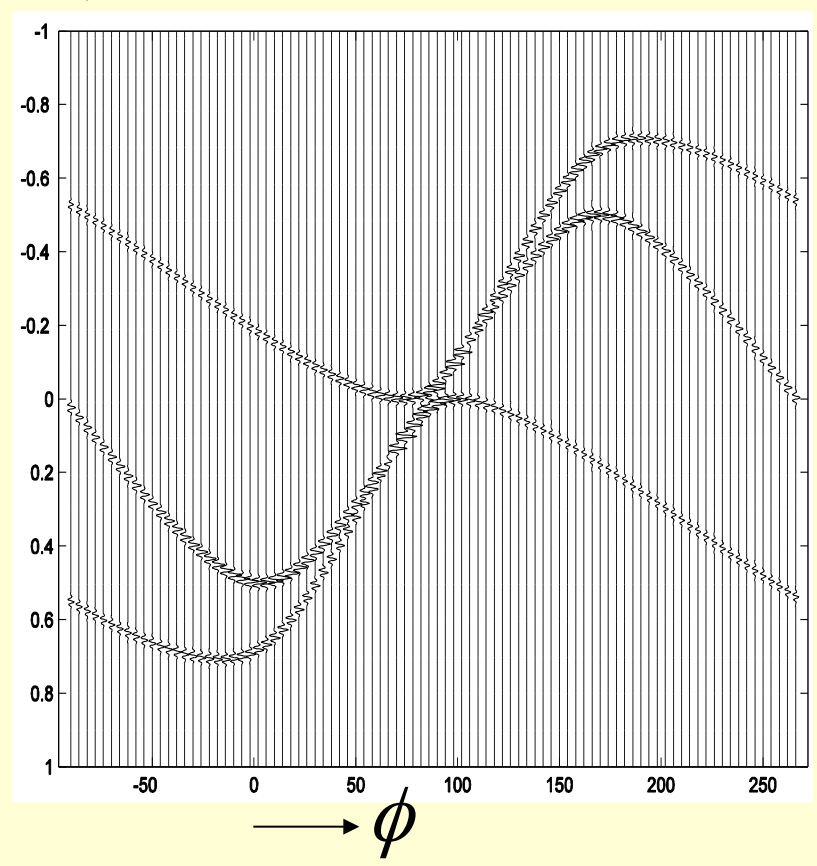
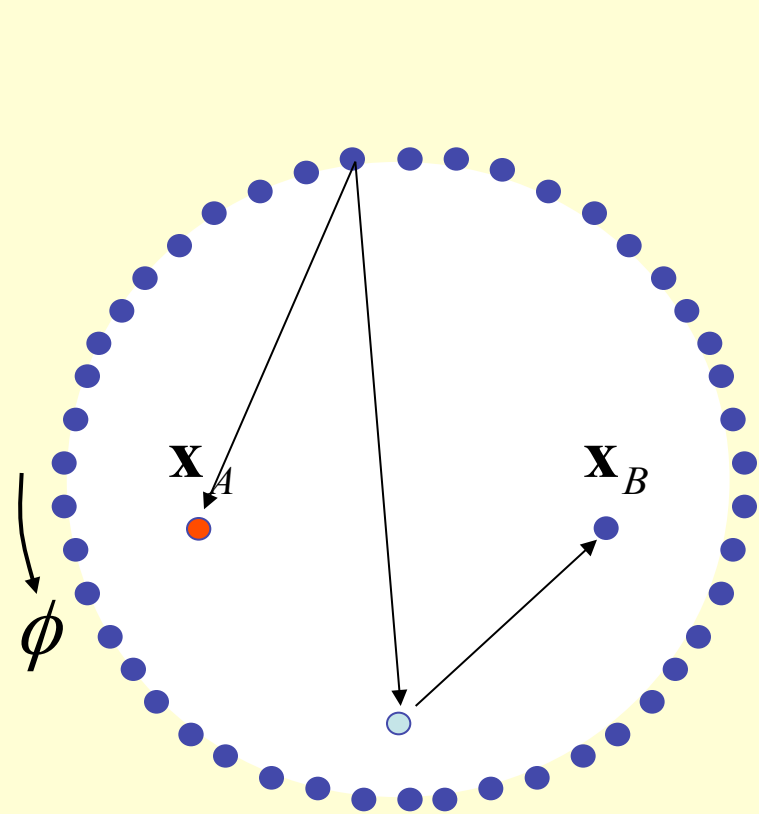
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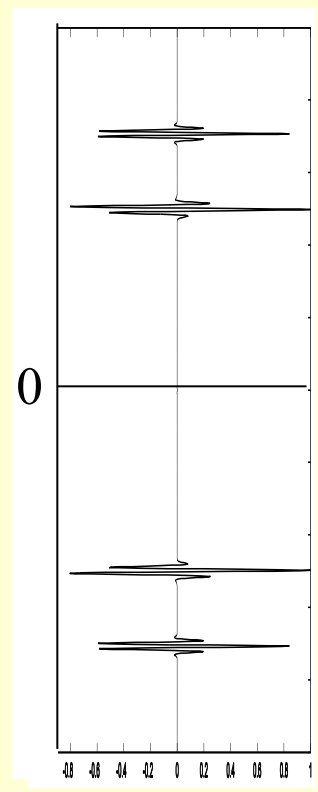
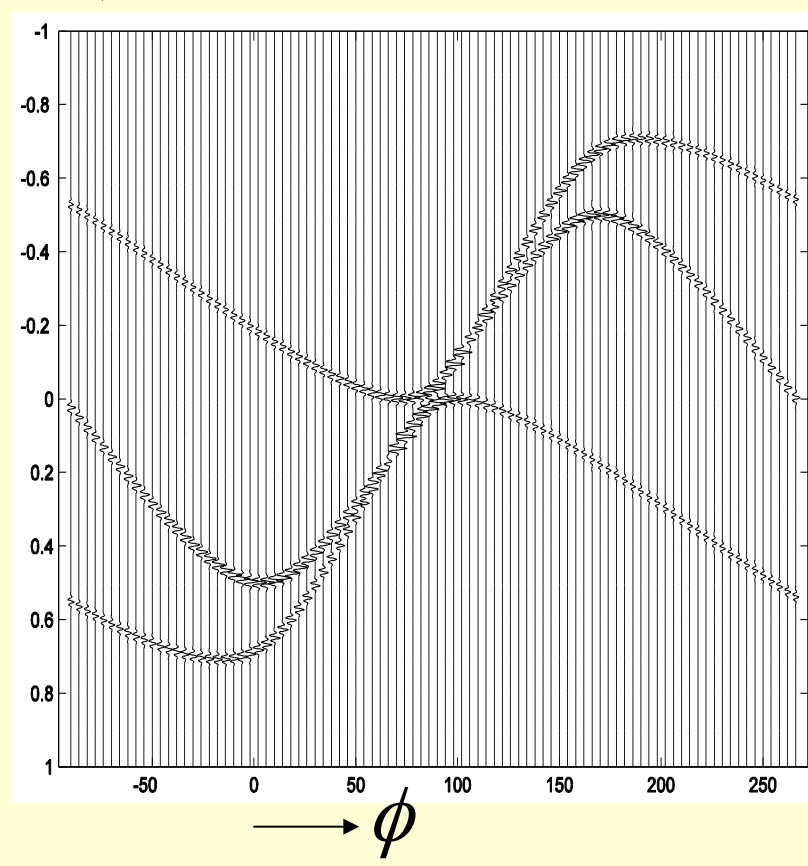
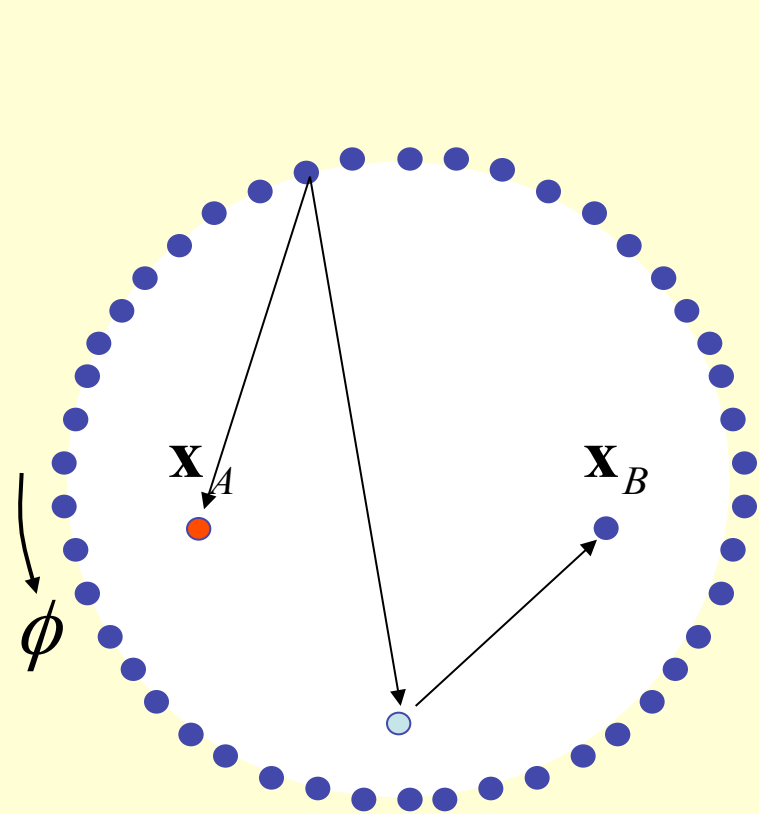
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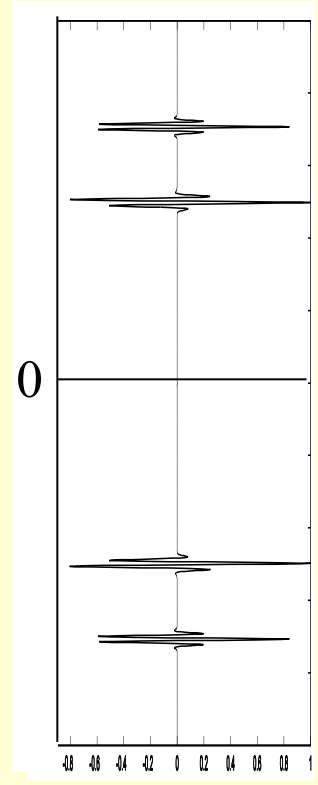
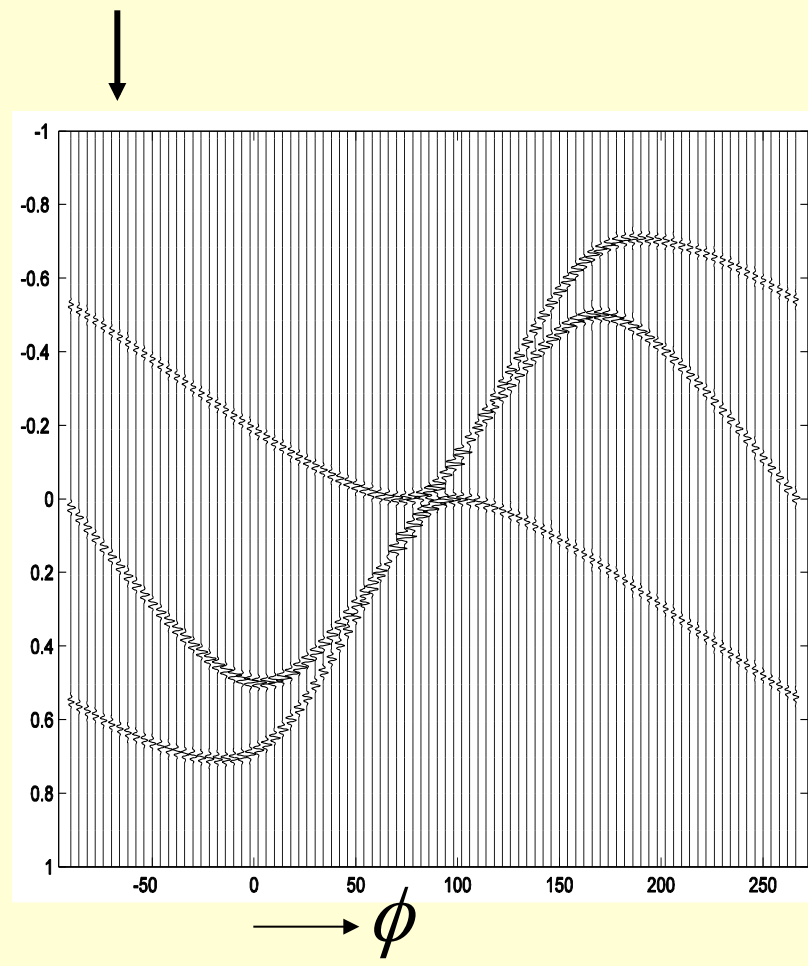
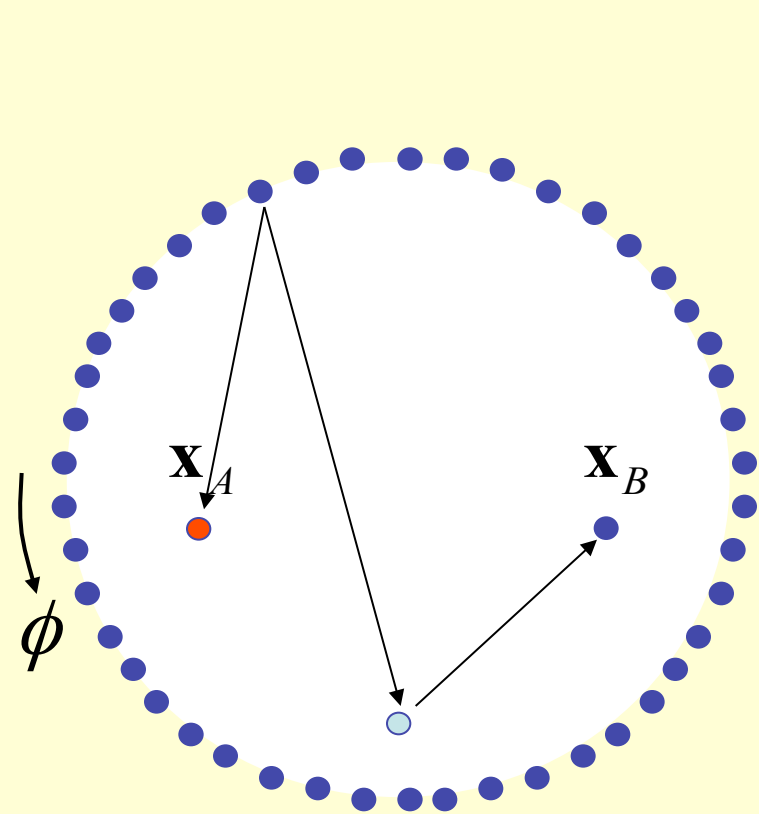
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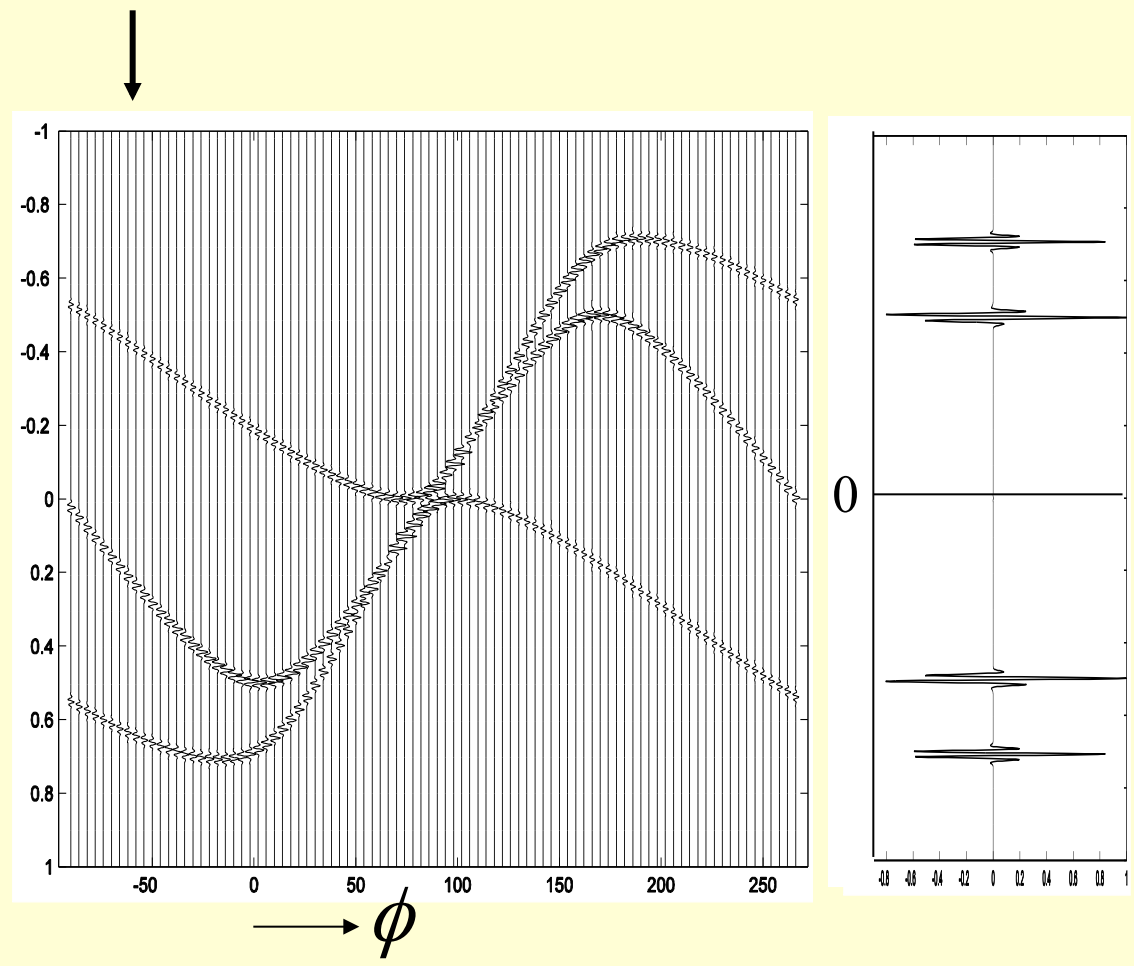
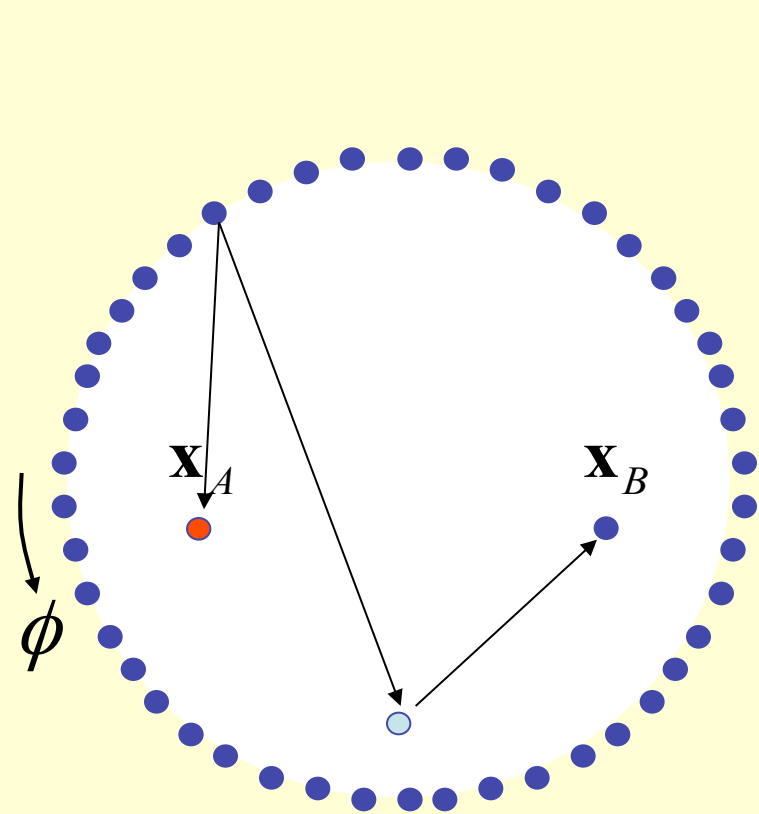
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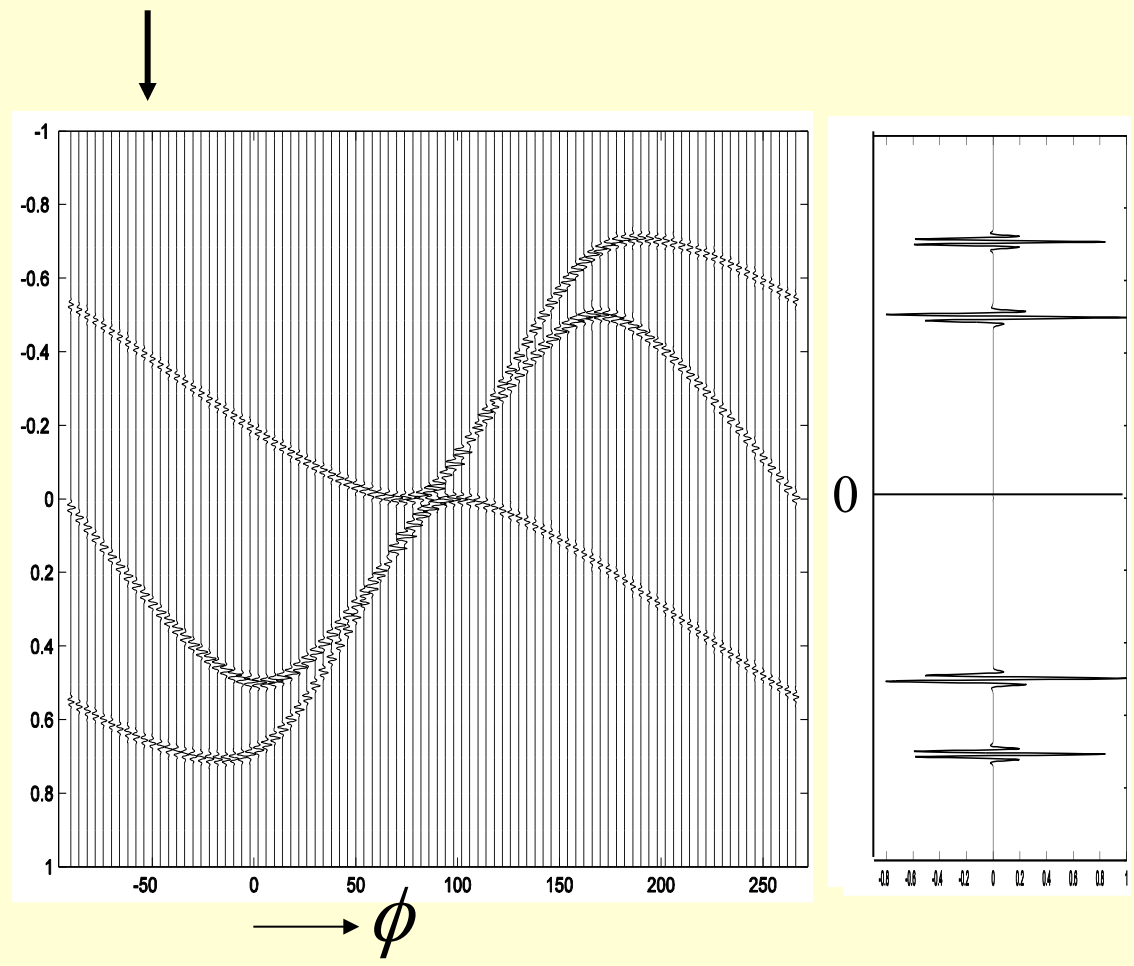
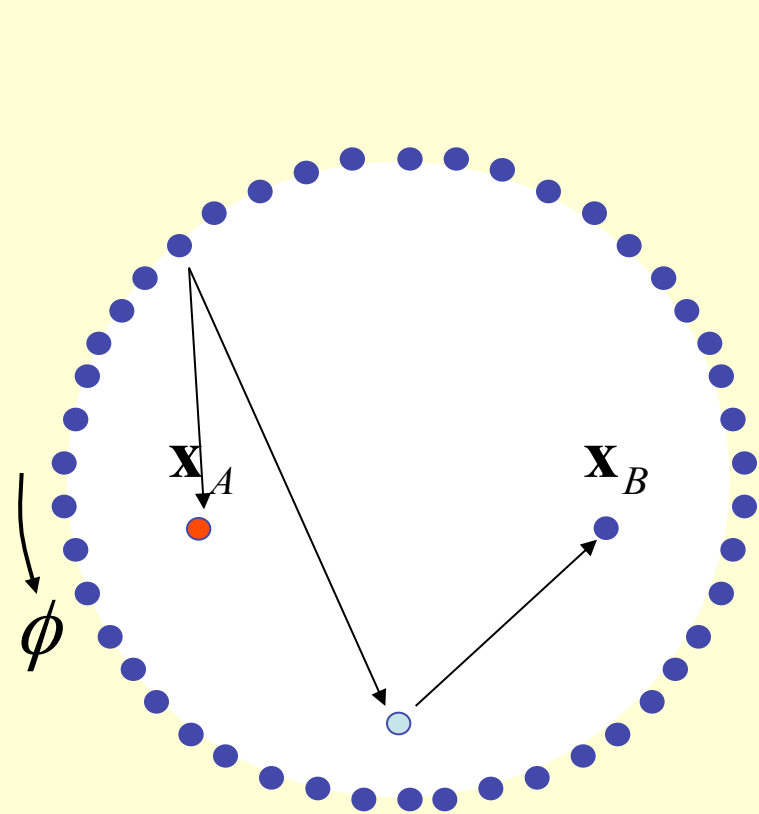
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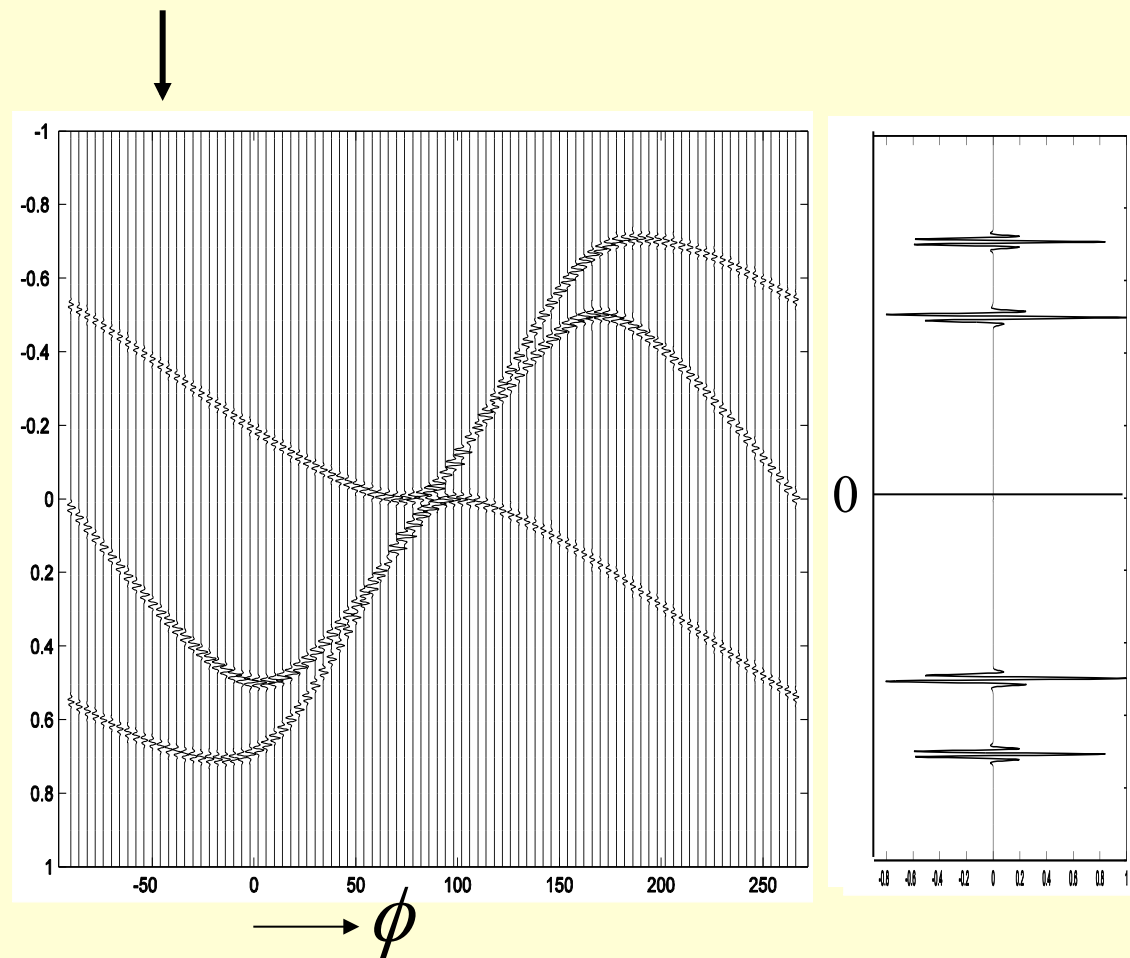
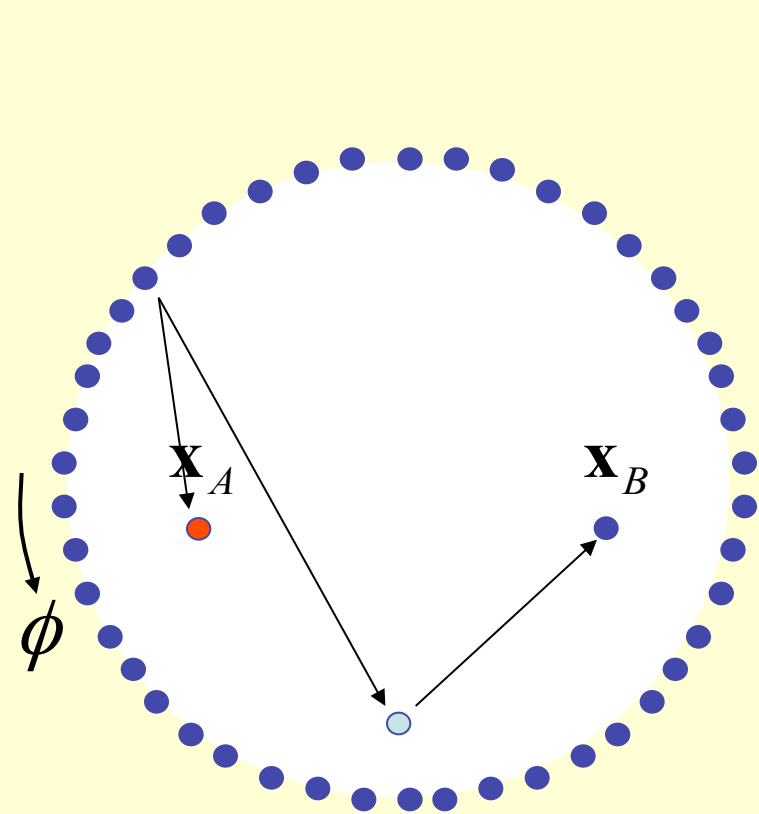
$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \approx \frac{2}{\rho c} \int_S G(\mathbf{x}_B, \mathbf{x}, t) * G(\mathbf{x}_A, \mathbf{x}, -t) d^2 \mathbf{x}$$



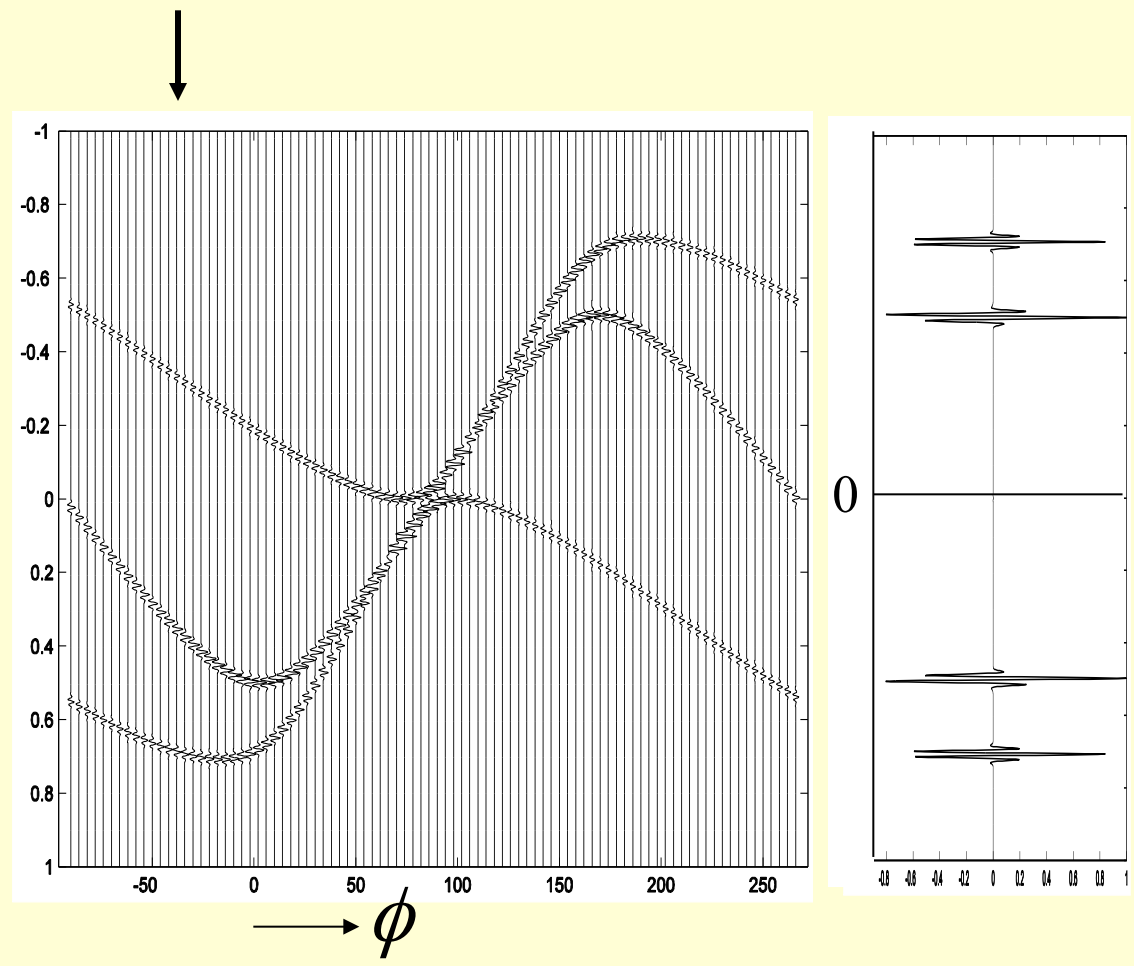
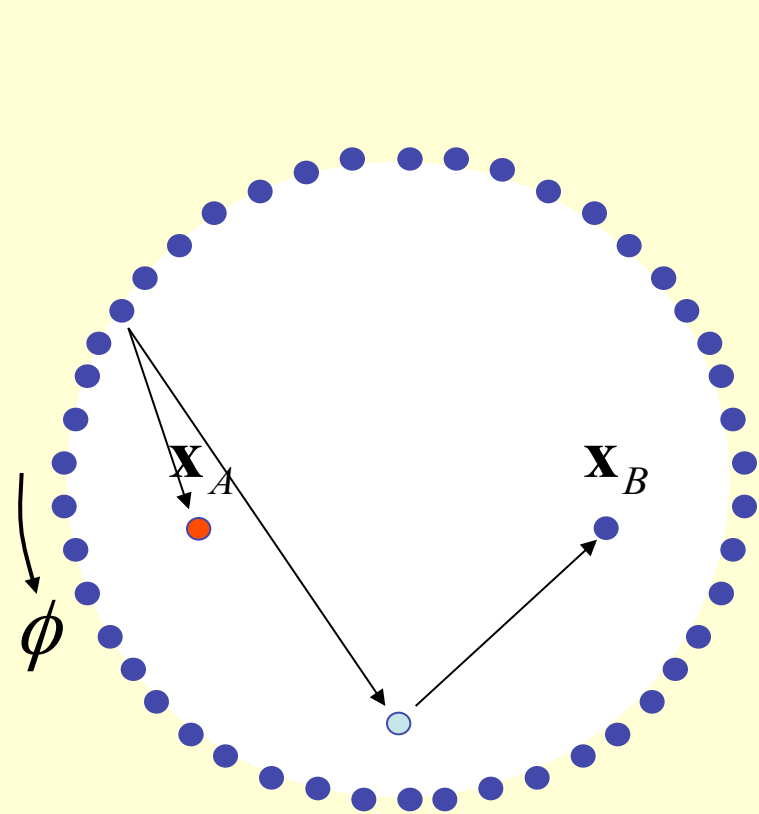
$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \approx \frac{2}{\rho c} \int_S G(\mathbf{x}_B, \mathbf{x}, t) * G(\mathbf{x}_A, \mathbf{x}, -t) d^2 \mathbf{x}$$



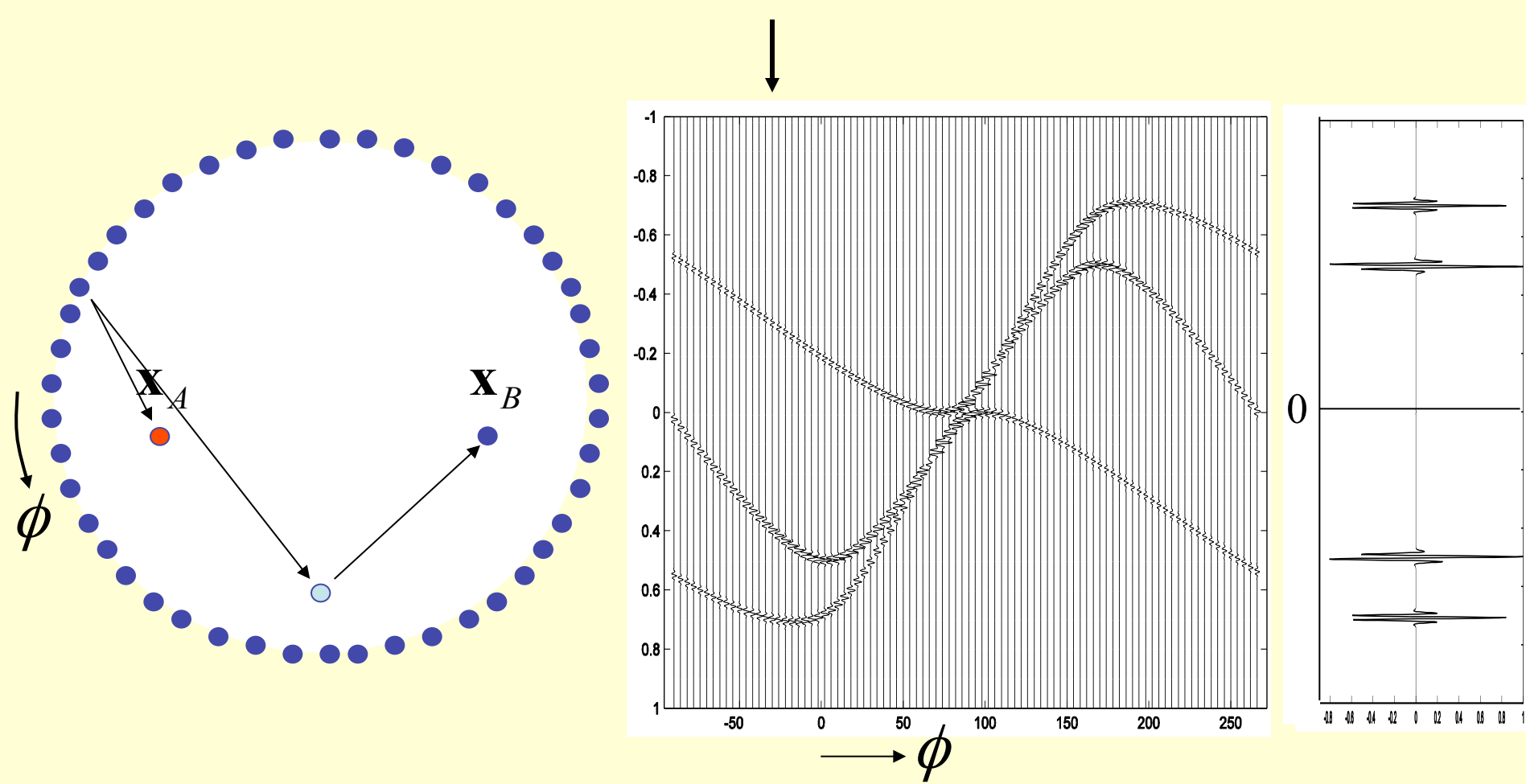
$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \approx \frac{2}{\rho c} \int_S G(\mathbf{x}_B, \mathbf{x}, t) * G(\mathbf{x}_A, \mathbf{x}, -t) d^2 \mathbf{x}$$



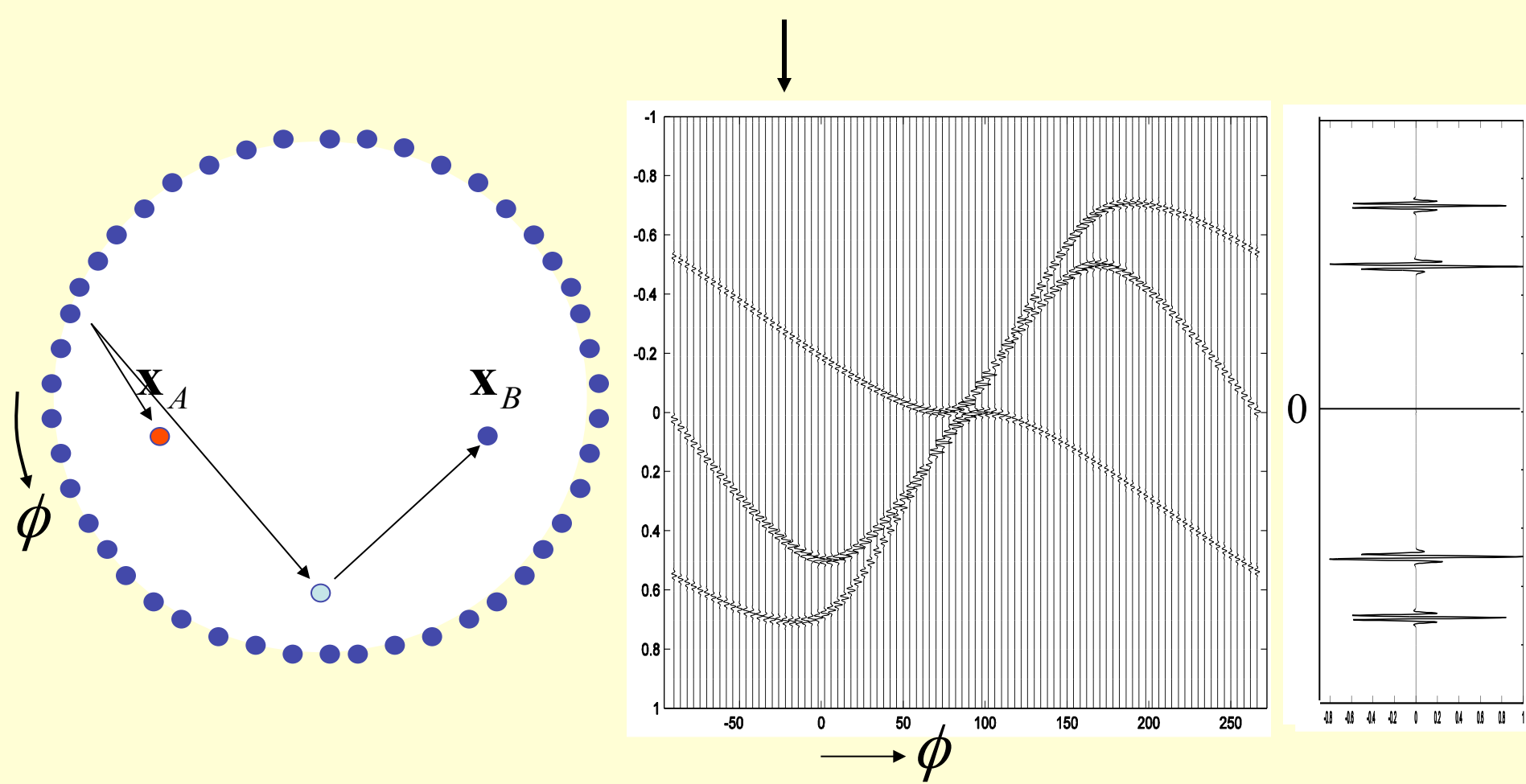
$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \approx \frac{2}{\rho c} \int_S G(\mathbf{x}_B, \mathbf{x}, t) * G(\mathbf{x}_A, \mathbf{x}, -t) d^2 \mathbf{x}$$



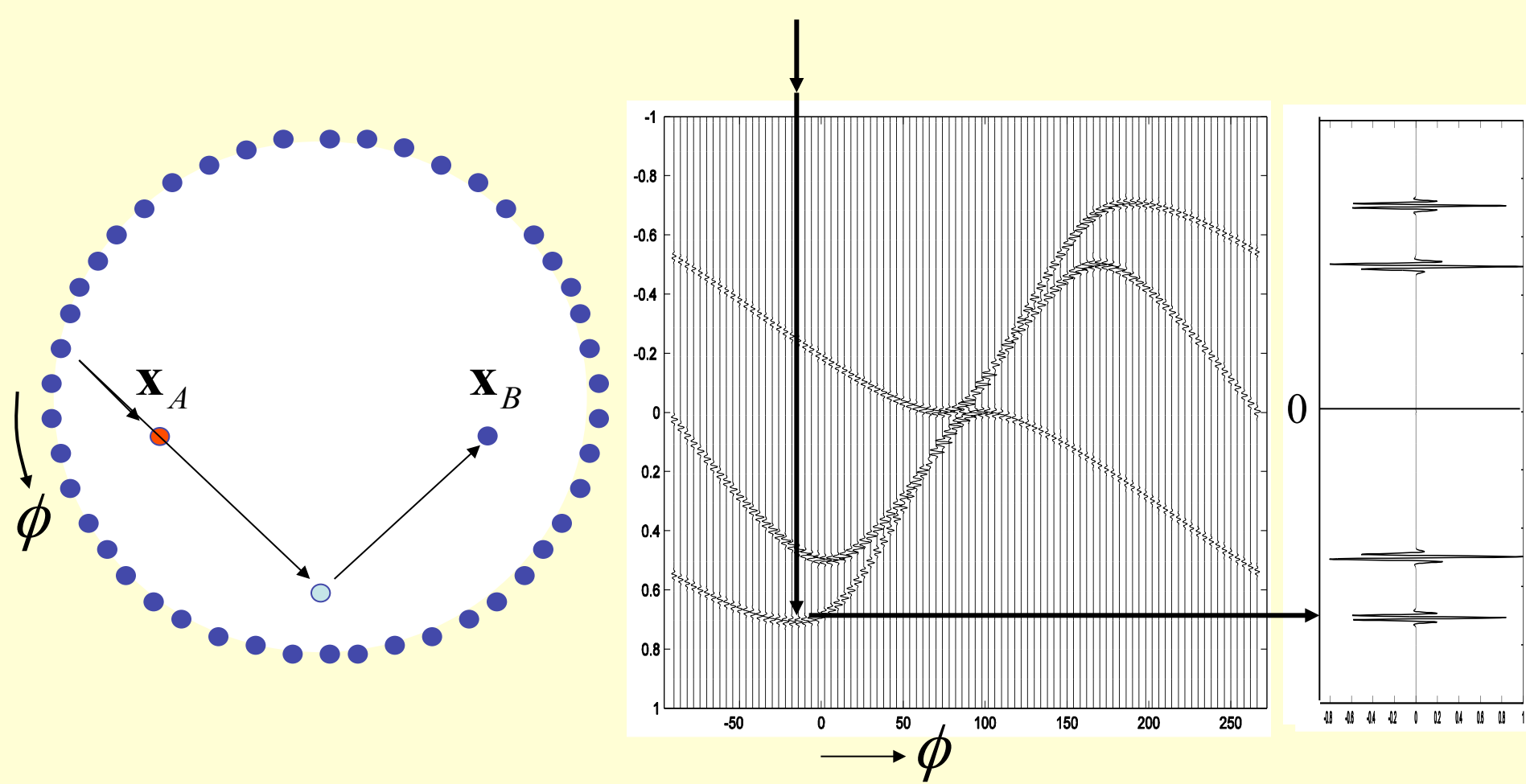
$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \approx \frac{2}{\rho c} \int_S G(\mathbf{x}_B, \mathbf{x}, t) * G(\mathbf{x}_A, \mathbf{x}, -t) d^2 \mathbf{x}$$



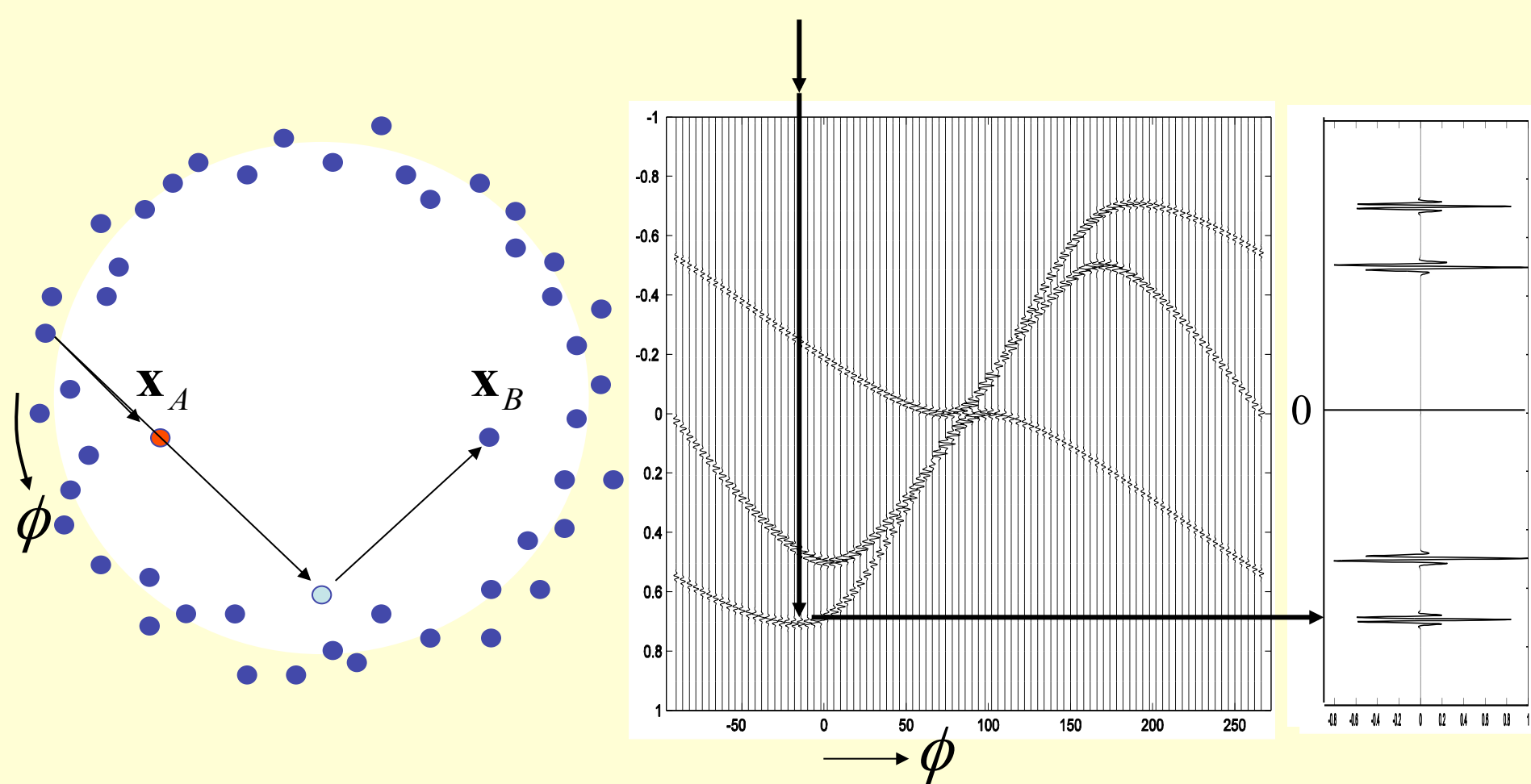
$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \approx \frac{2}{\rho c} \int_S G(\mathbf{x}_B, \mathbf{x}, t) * G(\mathbf{x}_A, \mathbf{x}, -t) d^2 \mathbf{x}$$



$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \approx \frac{2}{\rho c} \int_S G(\mathbf{x}_B, \mathbf{x}, t) * G(\mathbf{x}_A, \mathbf{x}, -t) d^2 \mathbf{x}$$

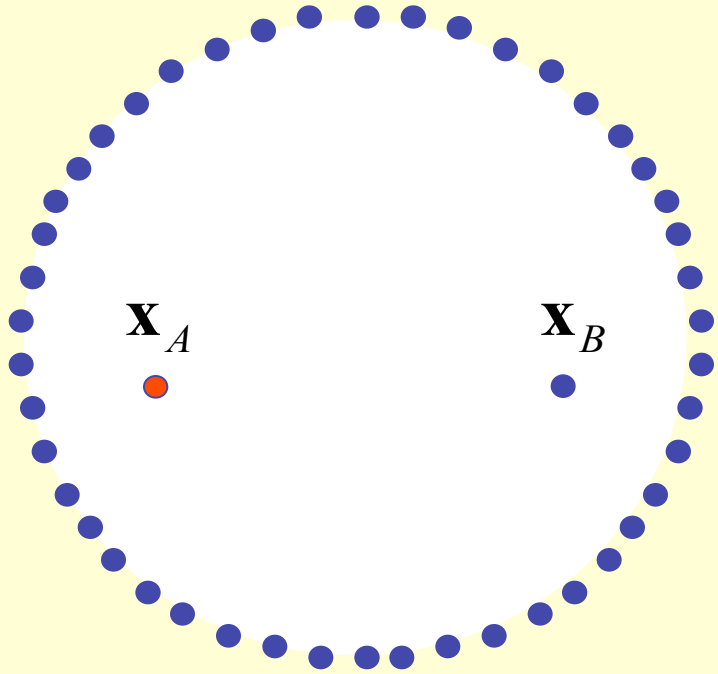


$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \approx \frac{2}{\rho c} \int_S G(\mathbf{x}_B, \mathbf{x}, t) * G(\mathbf{x}_A, \mathbf{x}, -t) d^2 \mathbf{x}$$



$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \approx \frac{2}{\rho c} \int_S G(\mathbf{x}_B, \mathbf{x}, t) * G(\mathbf{x}_A, \mathbf{x}, -t) d^2 \mathbf{x}$$

Uncorrelated noise sources:



$$p(\mathbf{x}_A, t) =$$

$$\int_S G(\mathbf{x}_A, \mathbf{x}, t) * N(\mathbf{x}, t) d^2 \mathbf{x}$$

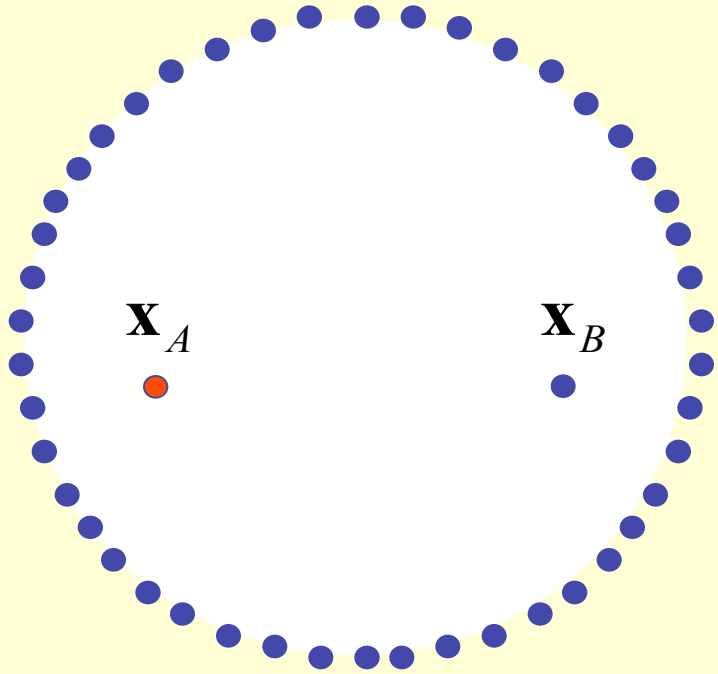
$$p(\mathbf{x}_B, t) =$$

$$\int_S G(\mathbf{x}_B, \mathbf{x}', t) * N(\mathbf{x}', t) d^2 \mathbf{x}'$$

$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \approx$$

$$\frac{2}{\rho c} \int_S G(\mathbf{x}_B, \mathbf{x}, t) * G(\mathbf{x}_A, \mathbf{x}, -t) d^2 \mathbf{x}$$

Uncorrelated noise sources:



$$p(\mathbf{x}_A, t) =$$

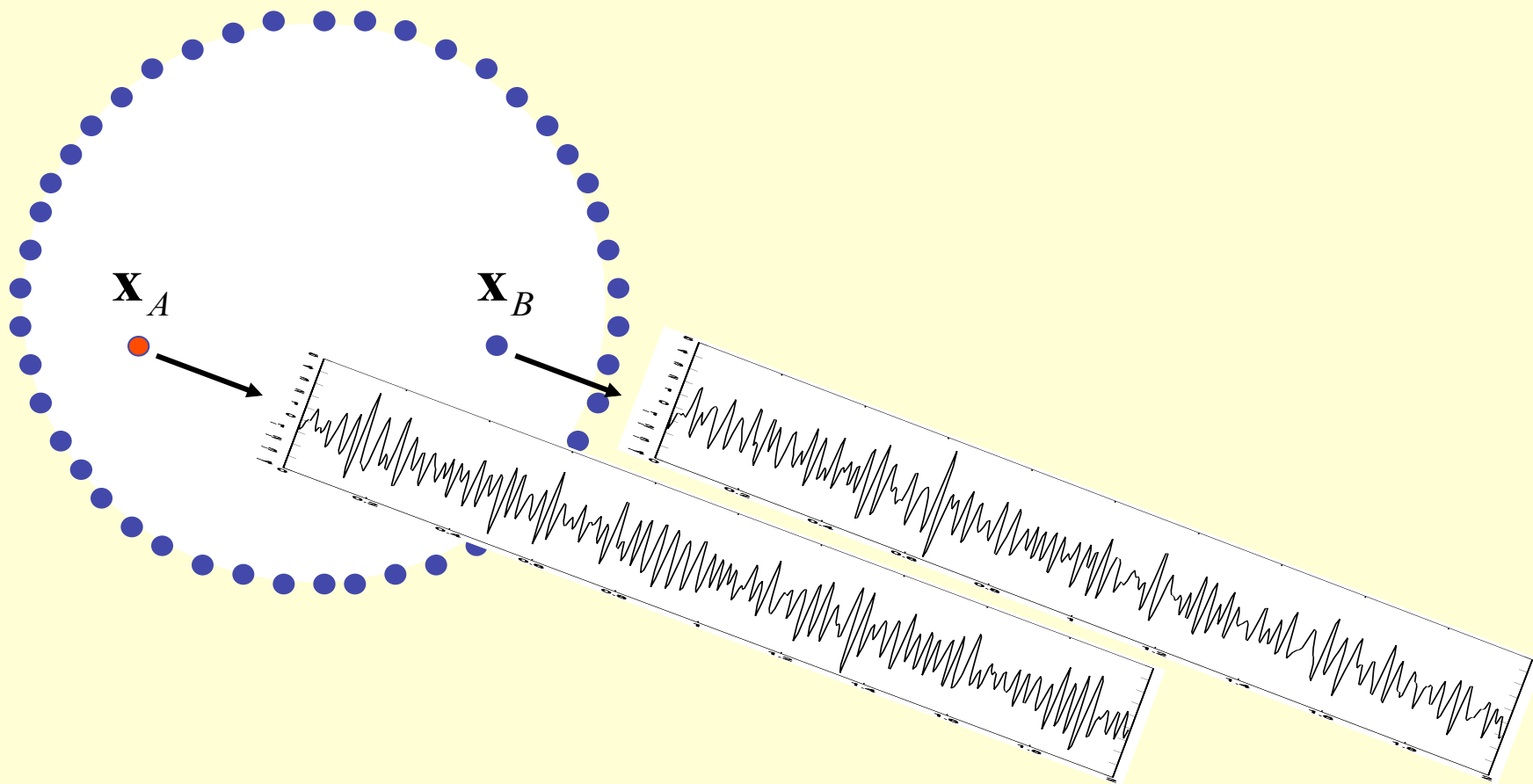
$$\int_S G(\mathbf{x}_A, \mathbf{x}, t) * N(\mathbf{x}, t) d^2 \mathbf{x}$$

$$p(\mathbf{x}_B, t) =$$

$$\int_S G(\mathbf{x}_B, \mathbf{x}', t) * N(\mathbf{x}', t) d^2 \mathbf{x}'$$

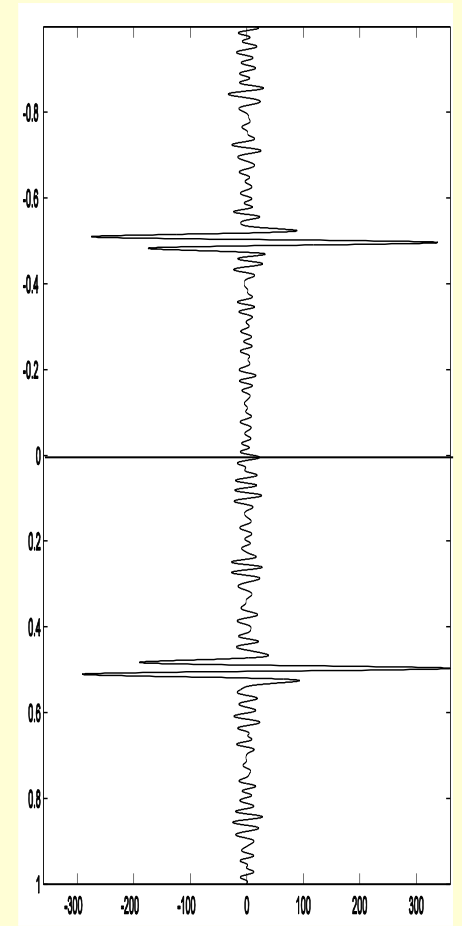
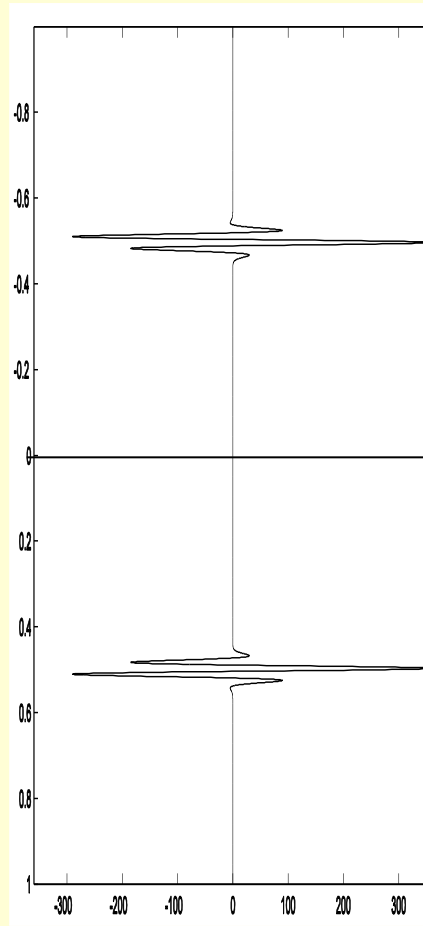
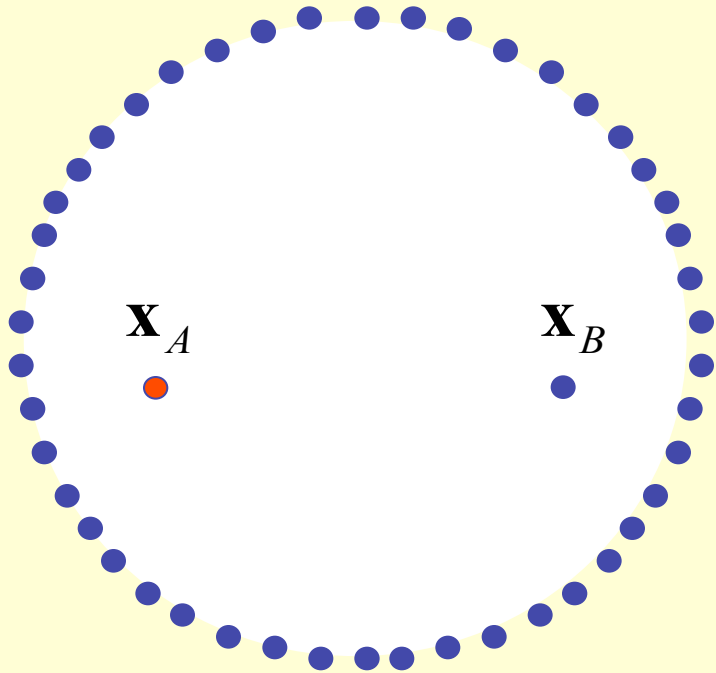
$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \approx$$

$$\langle p(\mathbf{x}_B, t) * p(\mathbf{x}_A, -t) \rangle$$



$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \approx$$

$$\langle p(\mathbf{x}_B, t) * p(\mathbf{x}_A, -t) \rangle$$



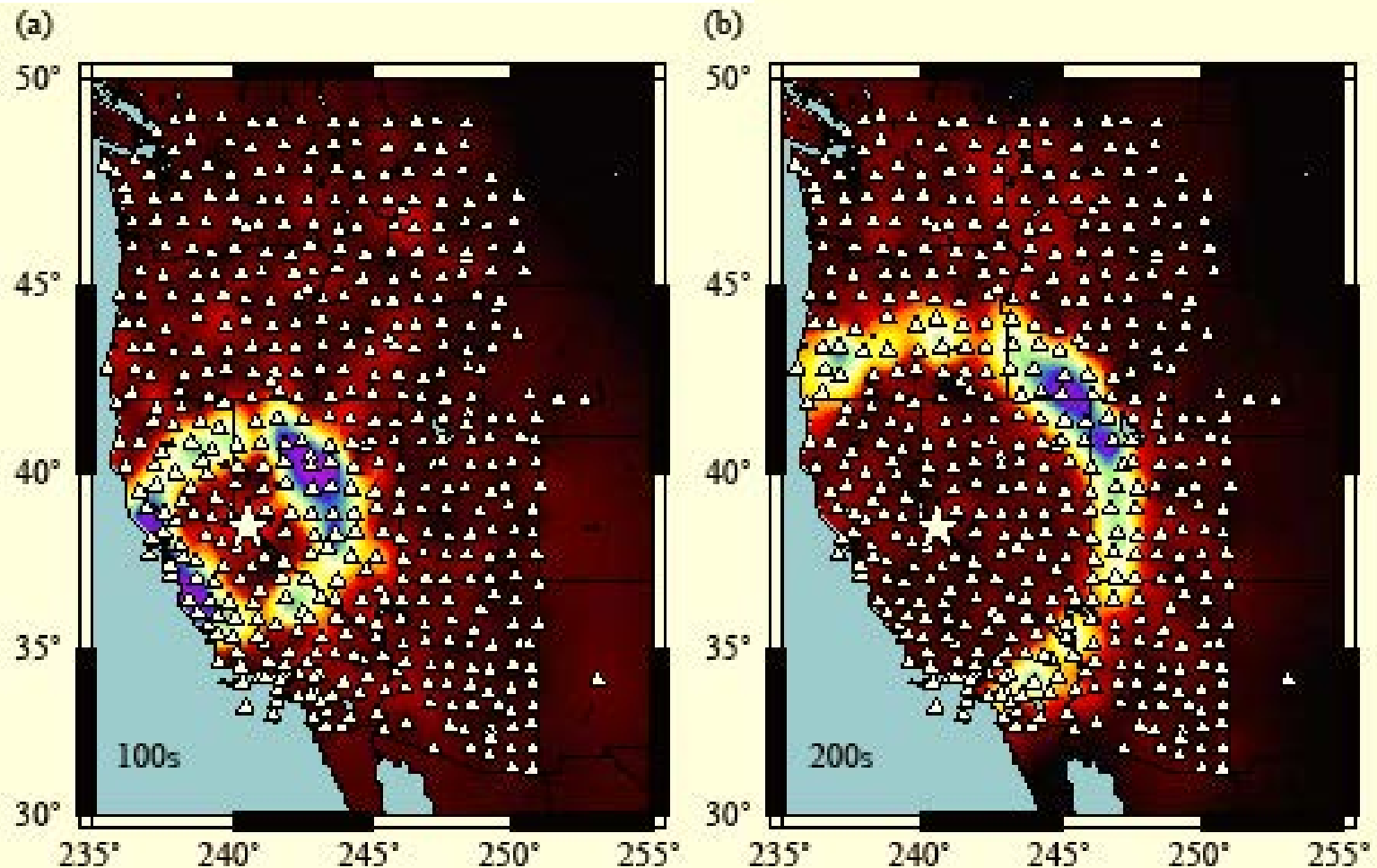
$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \approx$$

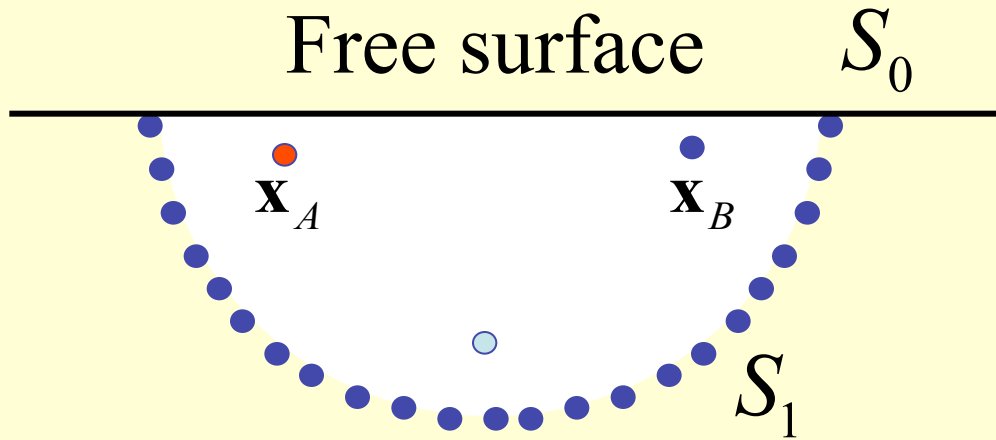
$$\langle p(\mathbf{x}_B, t) * p(\mathbf{x}_A, -t) \rangle$$

Eikonal tomography: surface wave tomography by phase front tracking across a regional broad-band seismic array

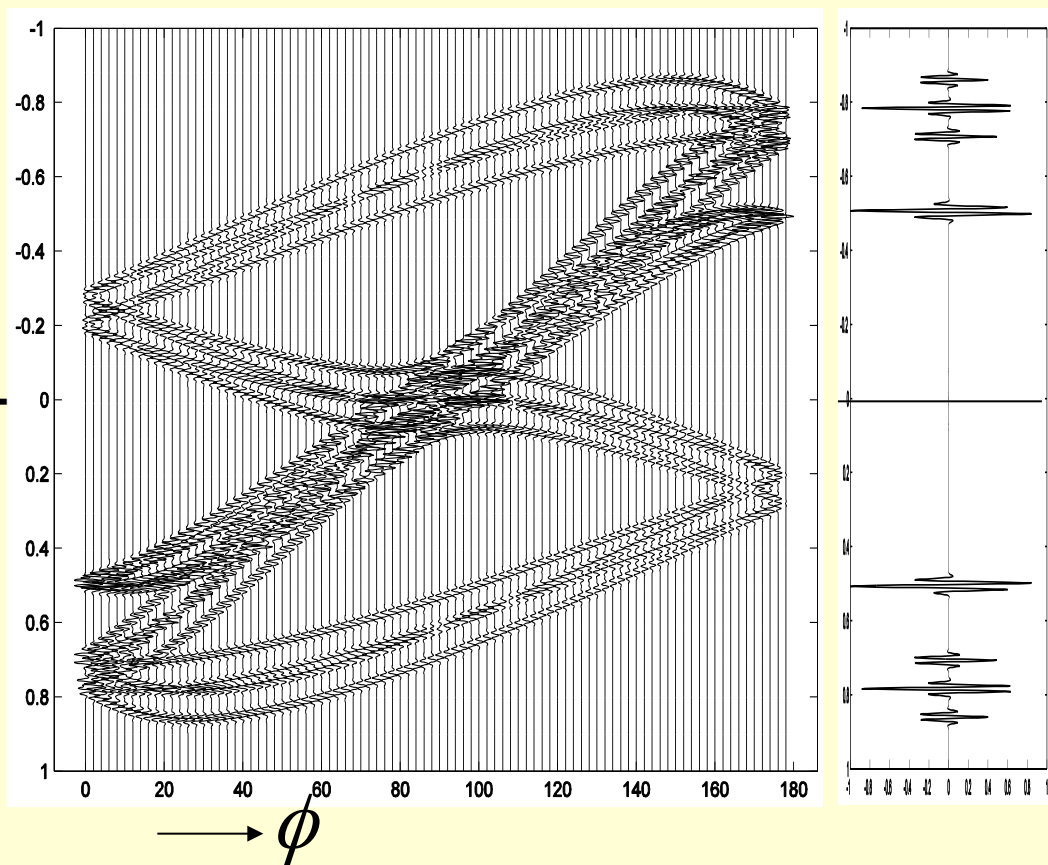
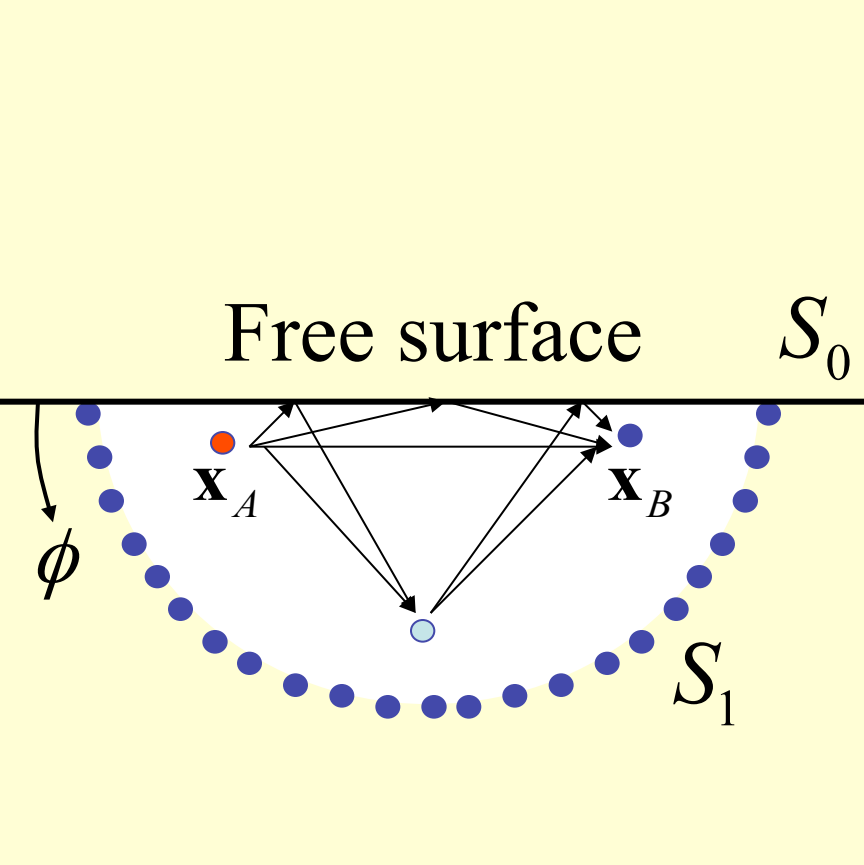
Fan-Chi Lin,¹ Michael H. Ritzwoller¹ and Roel Snieder²

Geophys. J. Int. (2009) **177**, 1091–1110

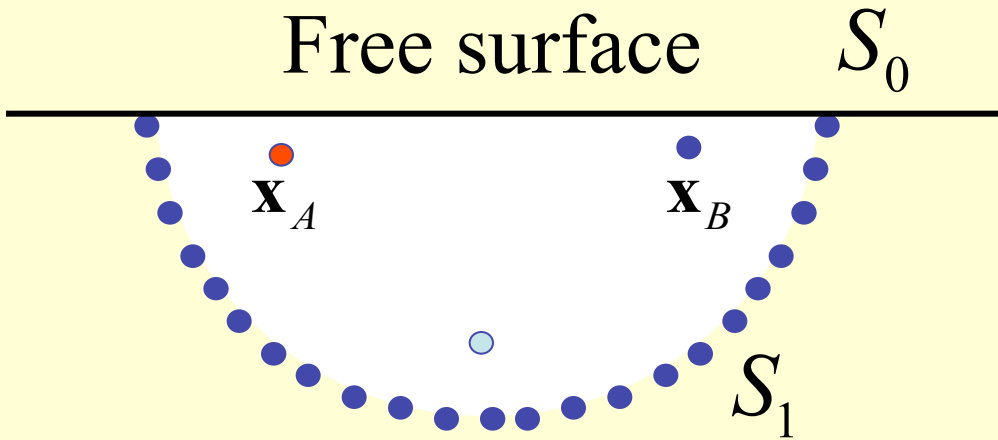




$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \approx \frac{2}{\rho c} \int_{S_1} G(\mathbf{x}_B, \mathbf{x}, t) * G(\mathbf{x}_A, \mathbf{x}, -t) d^2 \mathbf{x}$$



$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \approx \frac{2}{\rho c} \int_{S_1} G(\mathbf{x}_B, \mathbf{x}, t) * G(\mathbf{x}_A, \mathbf{x}, -t) d^2 \mathbf{x}$$



Uncorrelated
noise sources

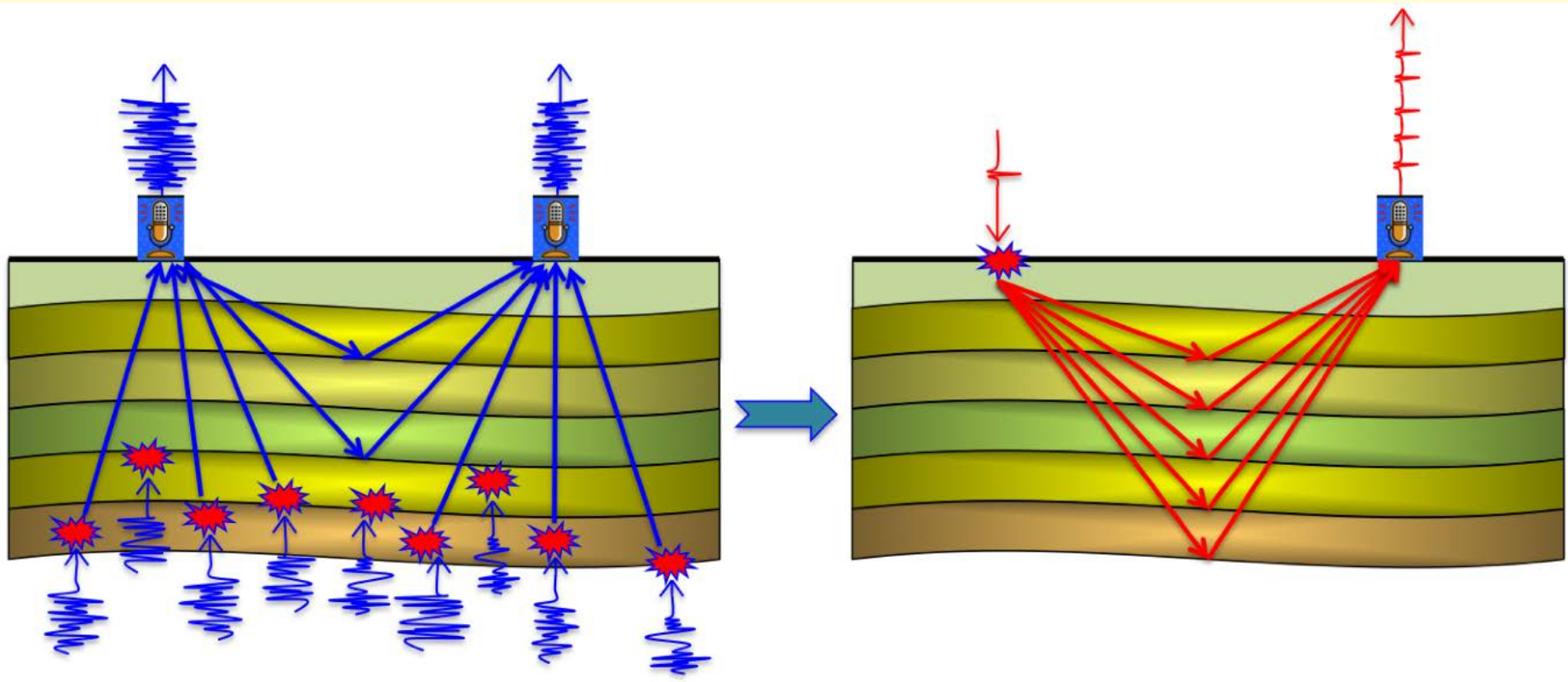
$$p(\mathbf{x}_A, t) = \int_{S_1} G(\mathbf{x}_A, \mathbf{x}, t) * N(\mathbf{x}, t) d^2 \mathbf{x}$$

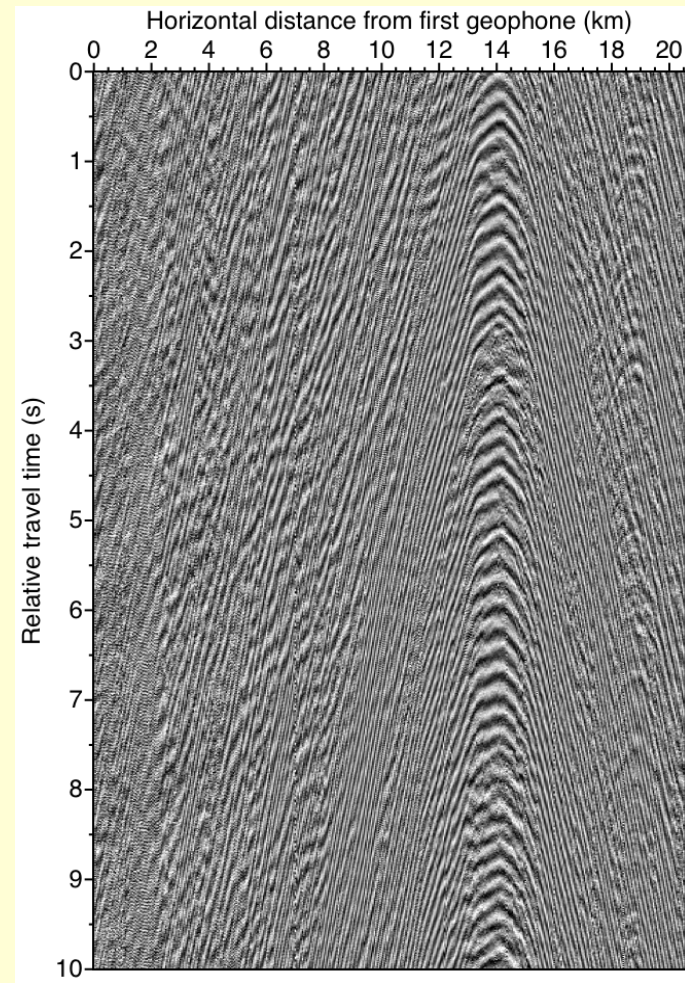
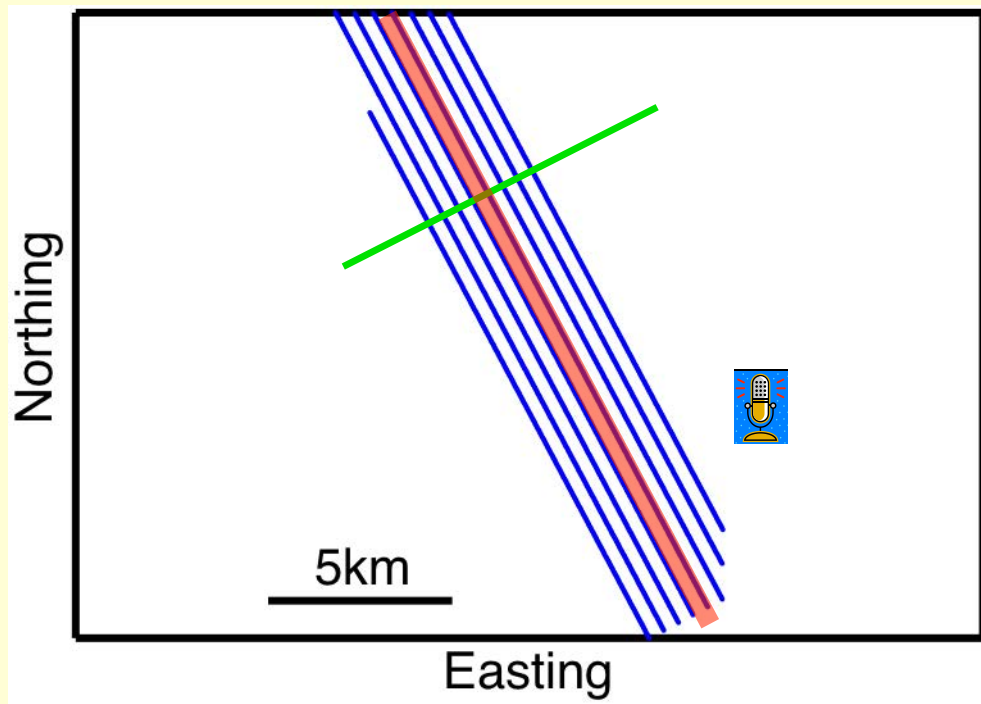
$$p(\mathbf{x}_B, t) = \int_{S_1} G(\mathbf{x}_B, \mathbf{x}, t) * N(\mathbf{x}, t) d^2 \mathbf{x}$$

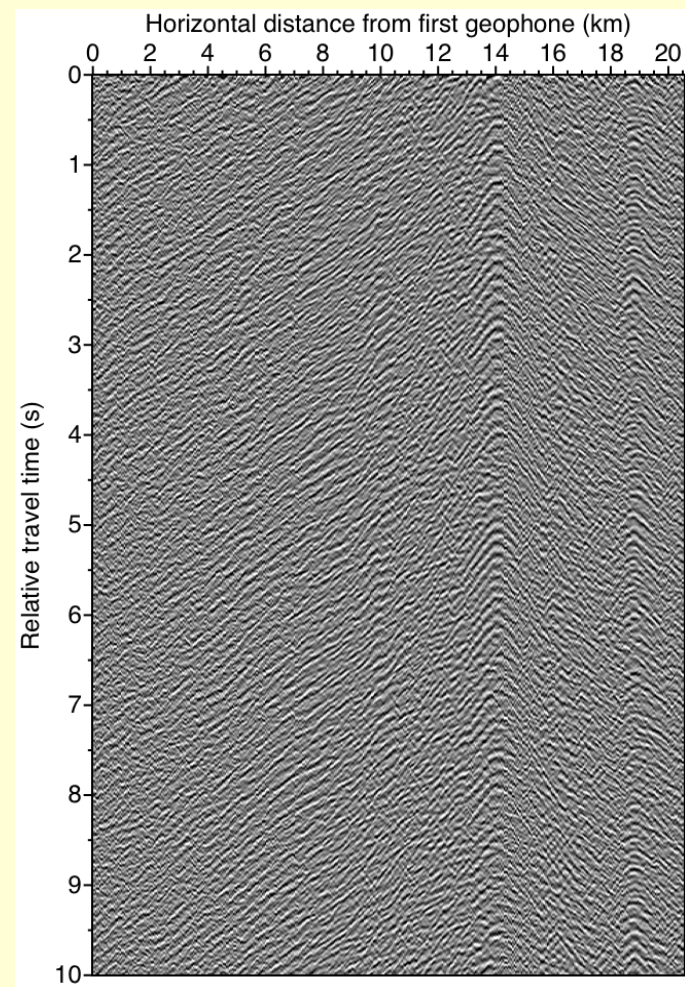
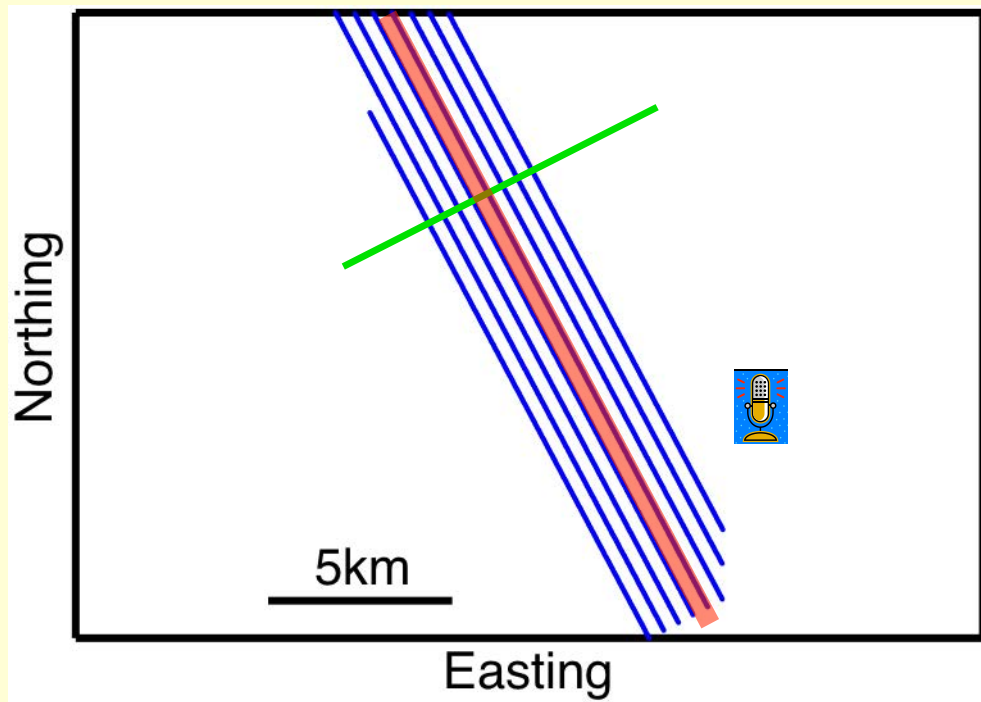
$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \approx \langle p(\mathbf{x}_B, t) * p(\mathbf{x}_A, -t) \rangle$$

Reflection images from ambient seismic noise

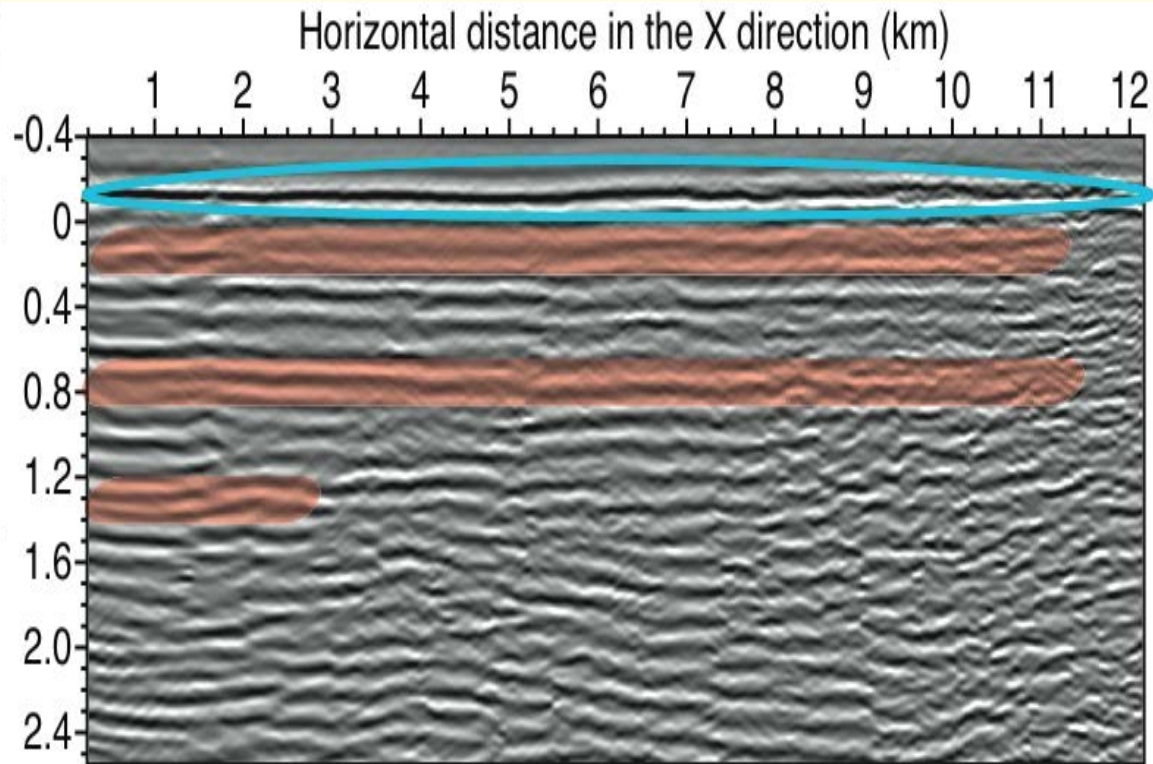
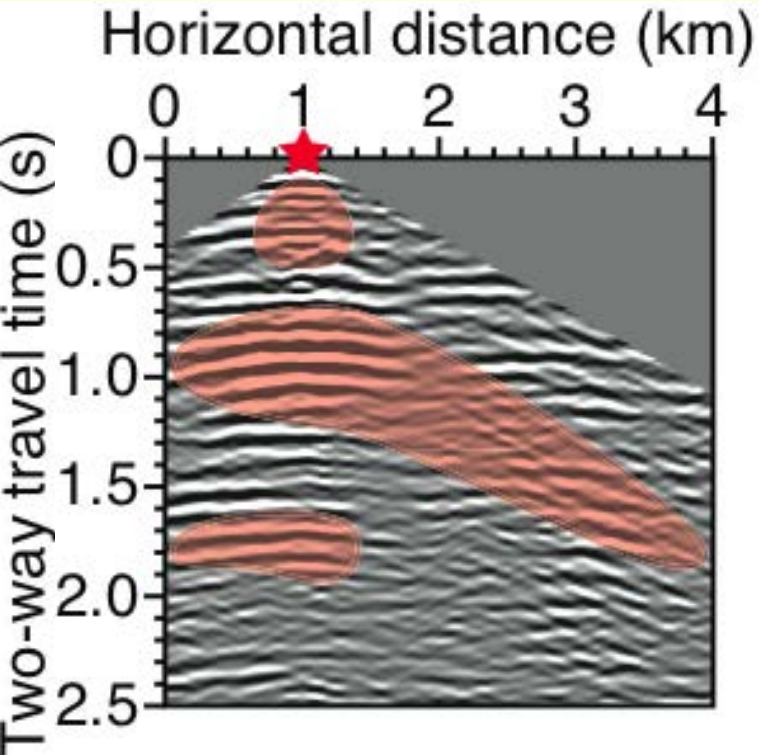
Deyan Draganov¹, Xander Campman², Jan Thorbecke¹, Arie Verdel², and Kees Wapenaar¹



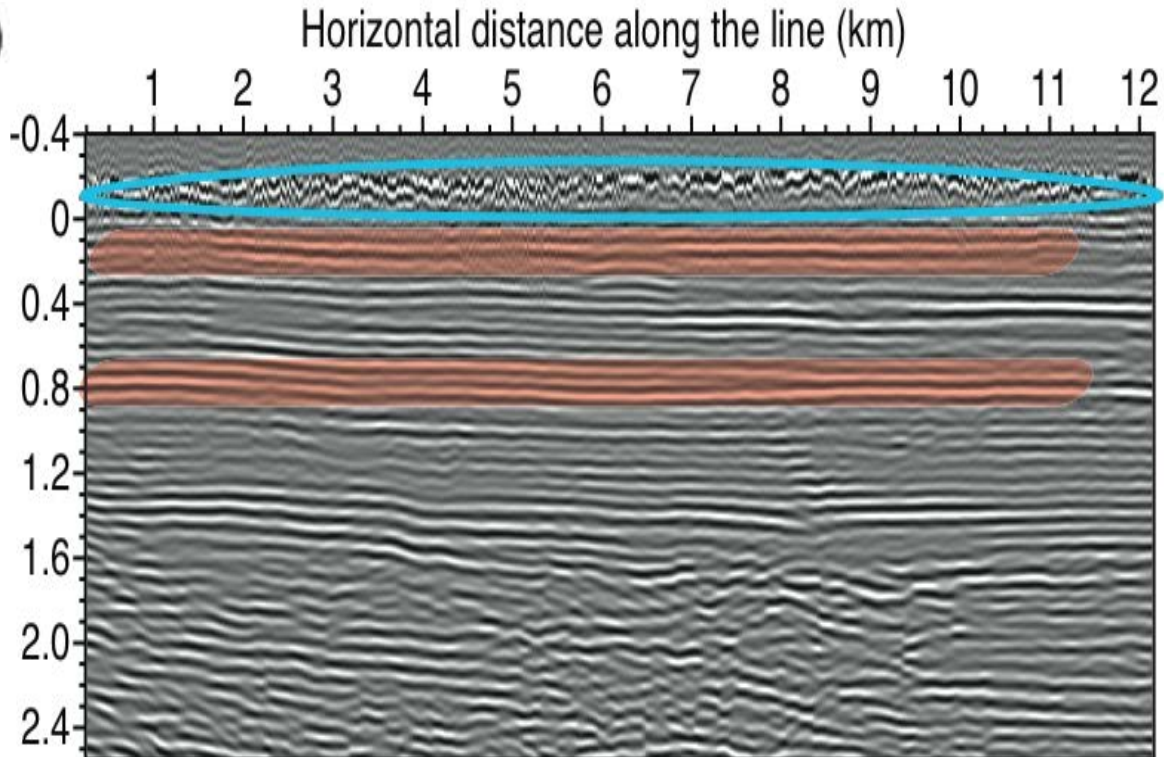
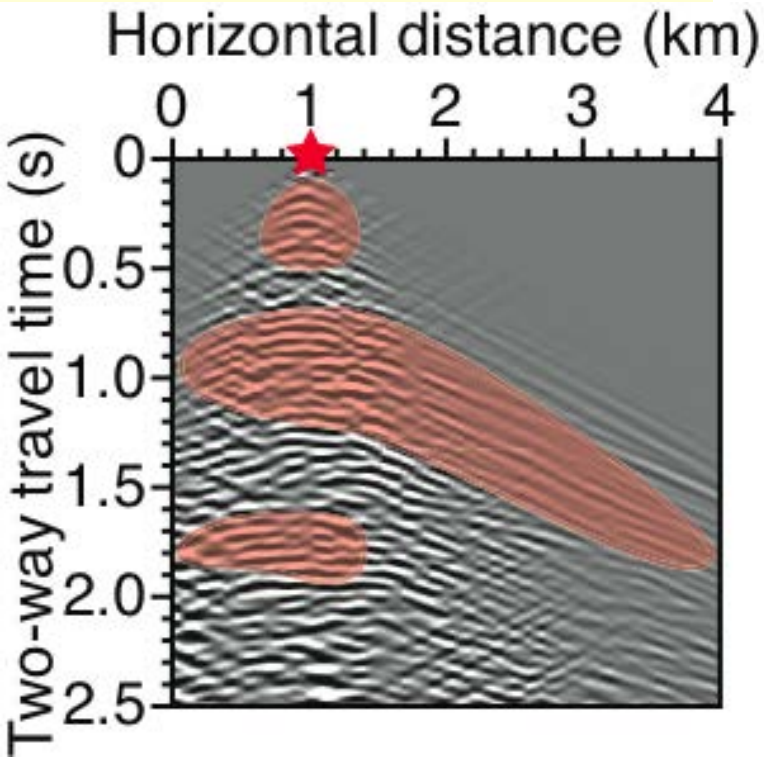


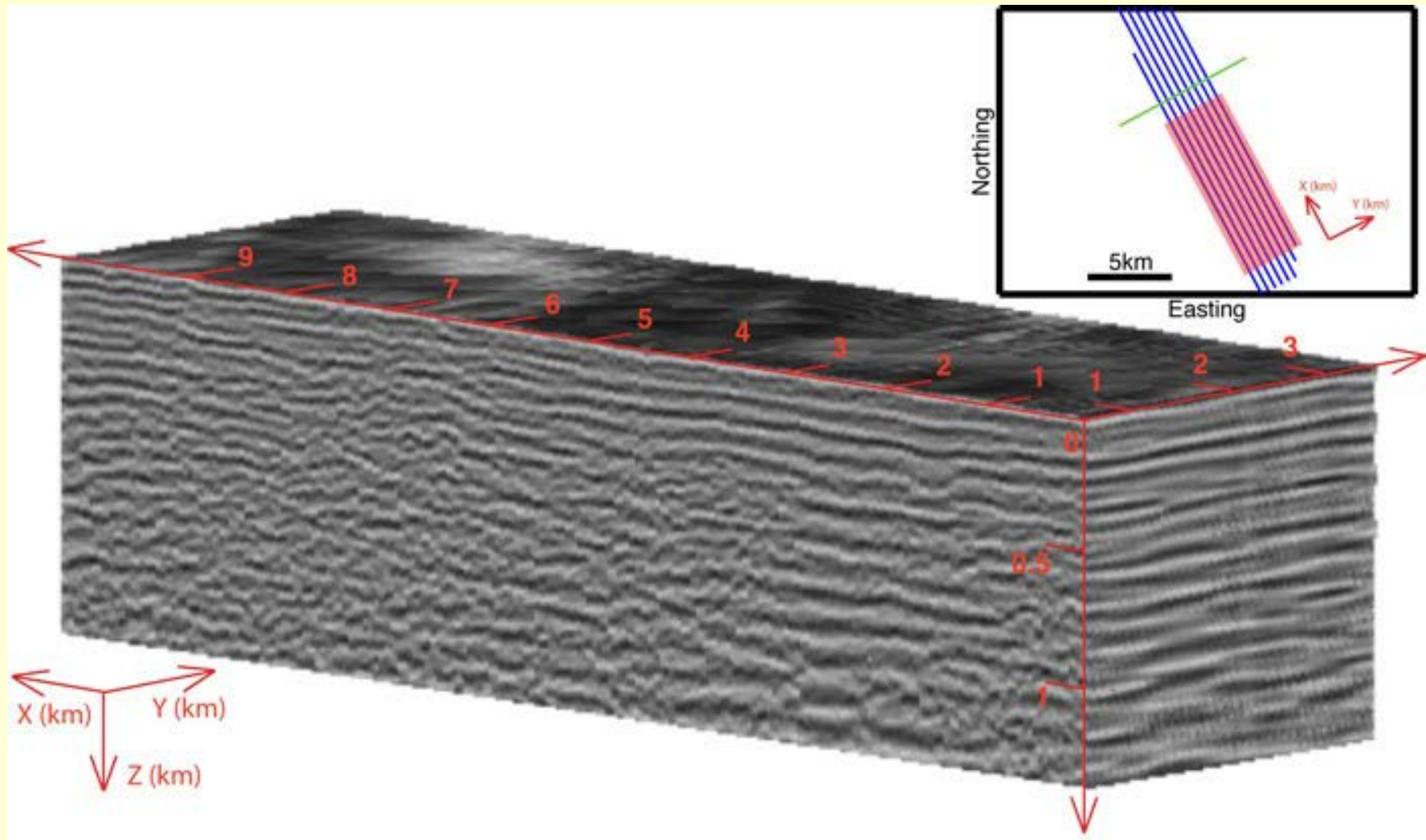


Passive data



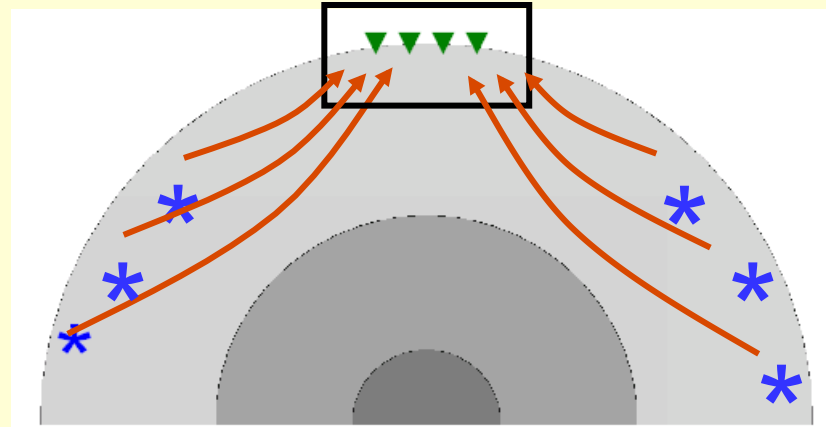
Active data





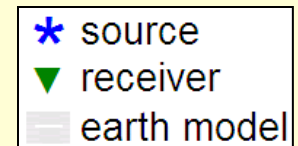
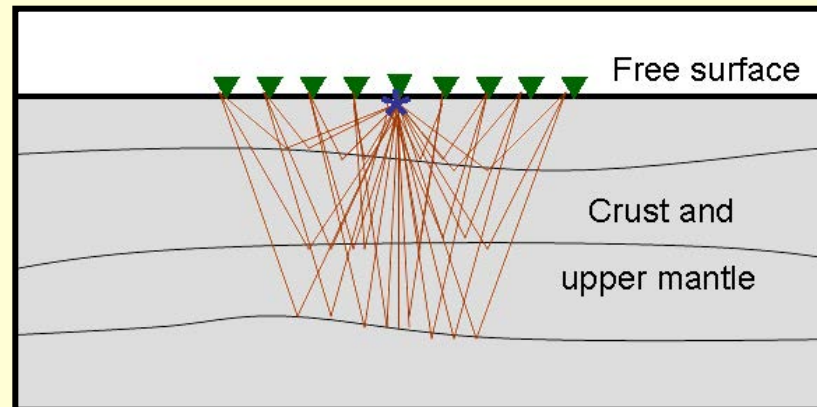
Lithospheric imaging with seismic interferometry

Ruigrok et al., 2010, GJI, 183, 339-357

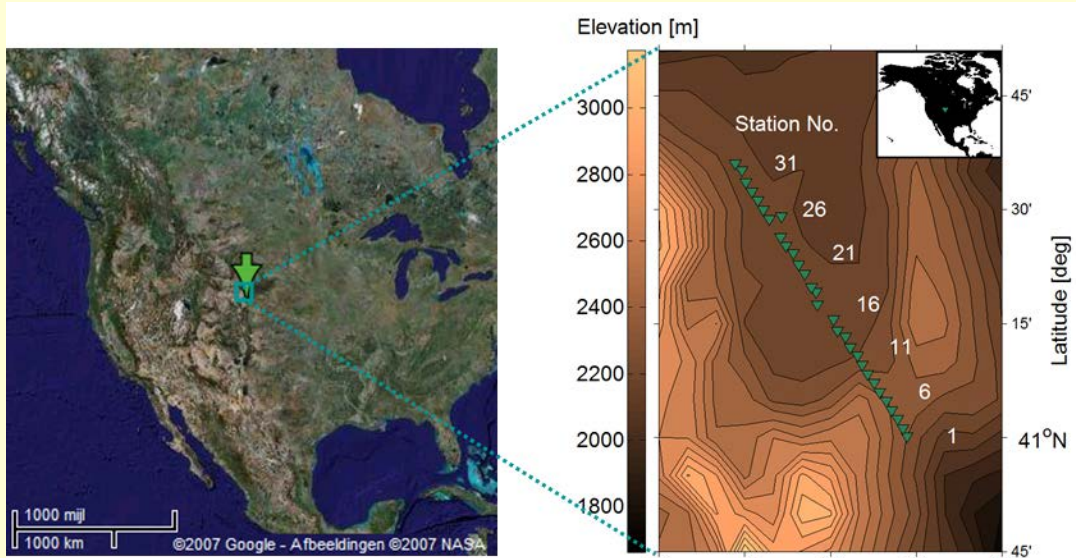


Seismic Interferometry (SI)

(by cross-correlation)



Laramie array



Receiver
configuration

Source
configuration

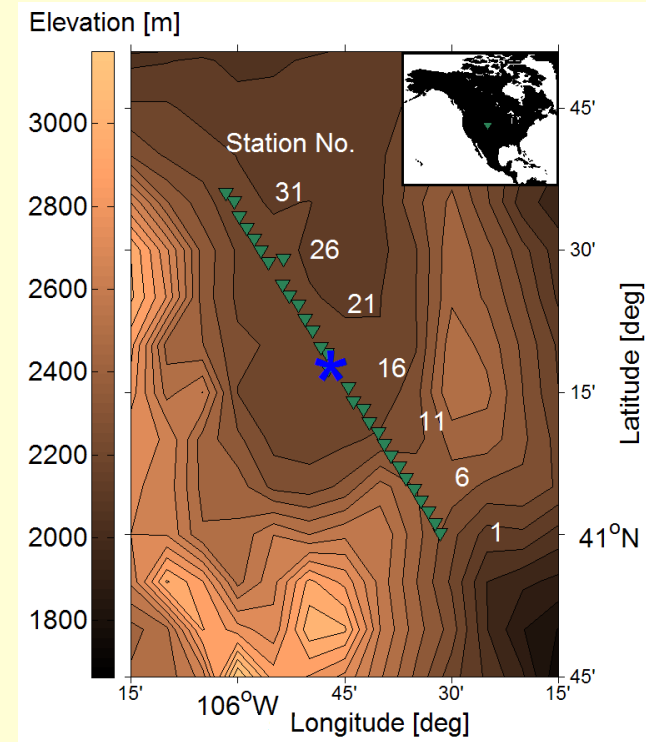
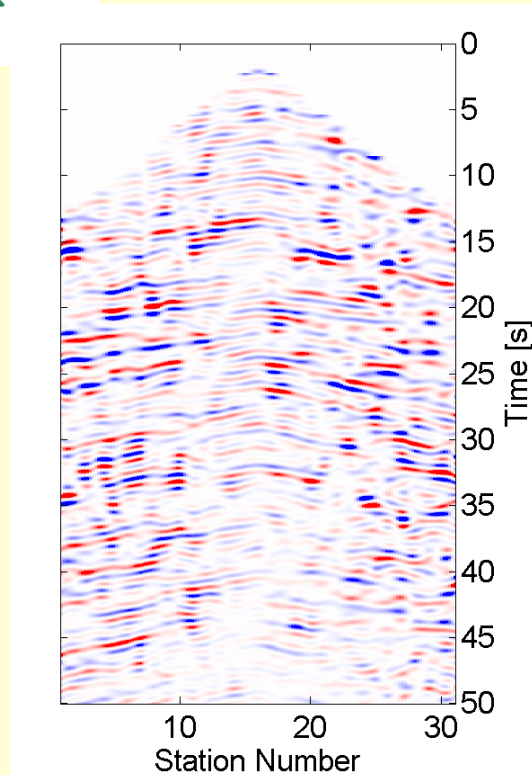
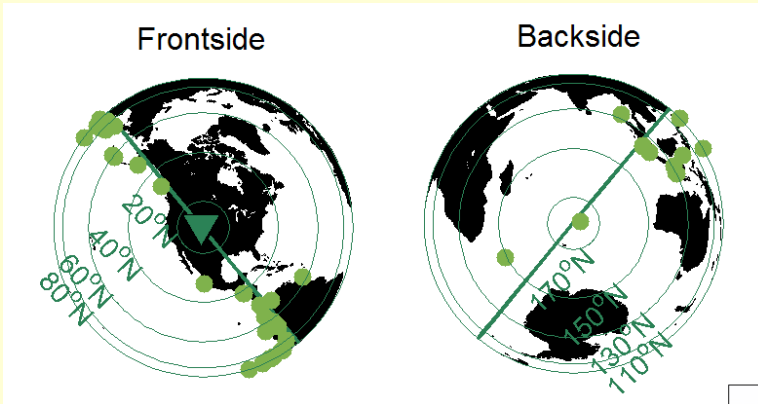
Frontside



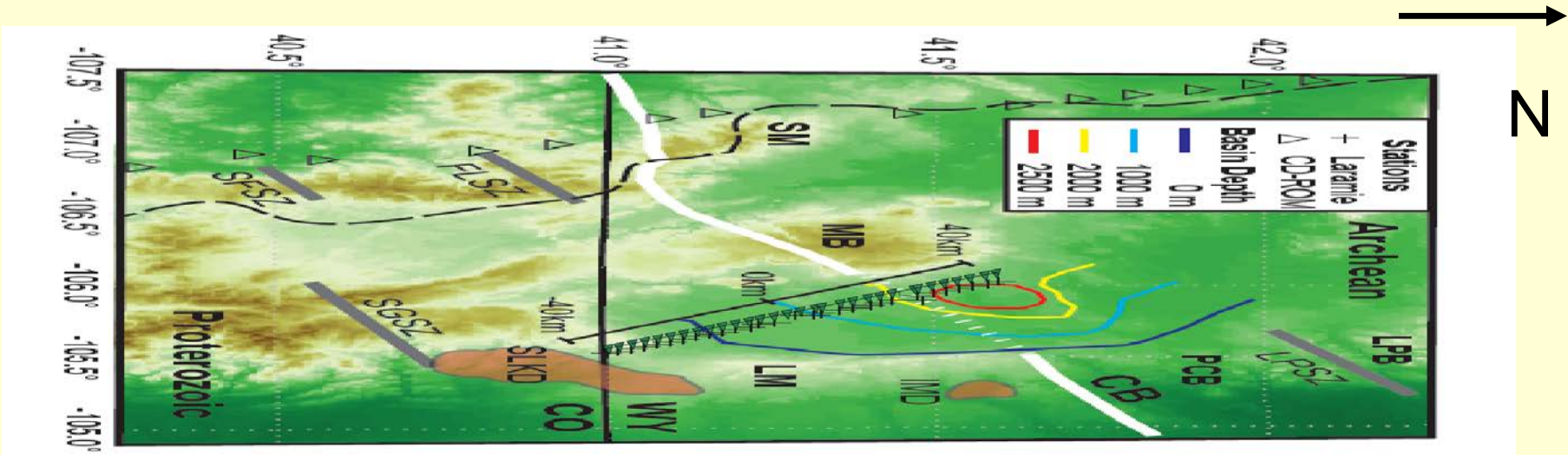
Backside



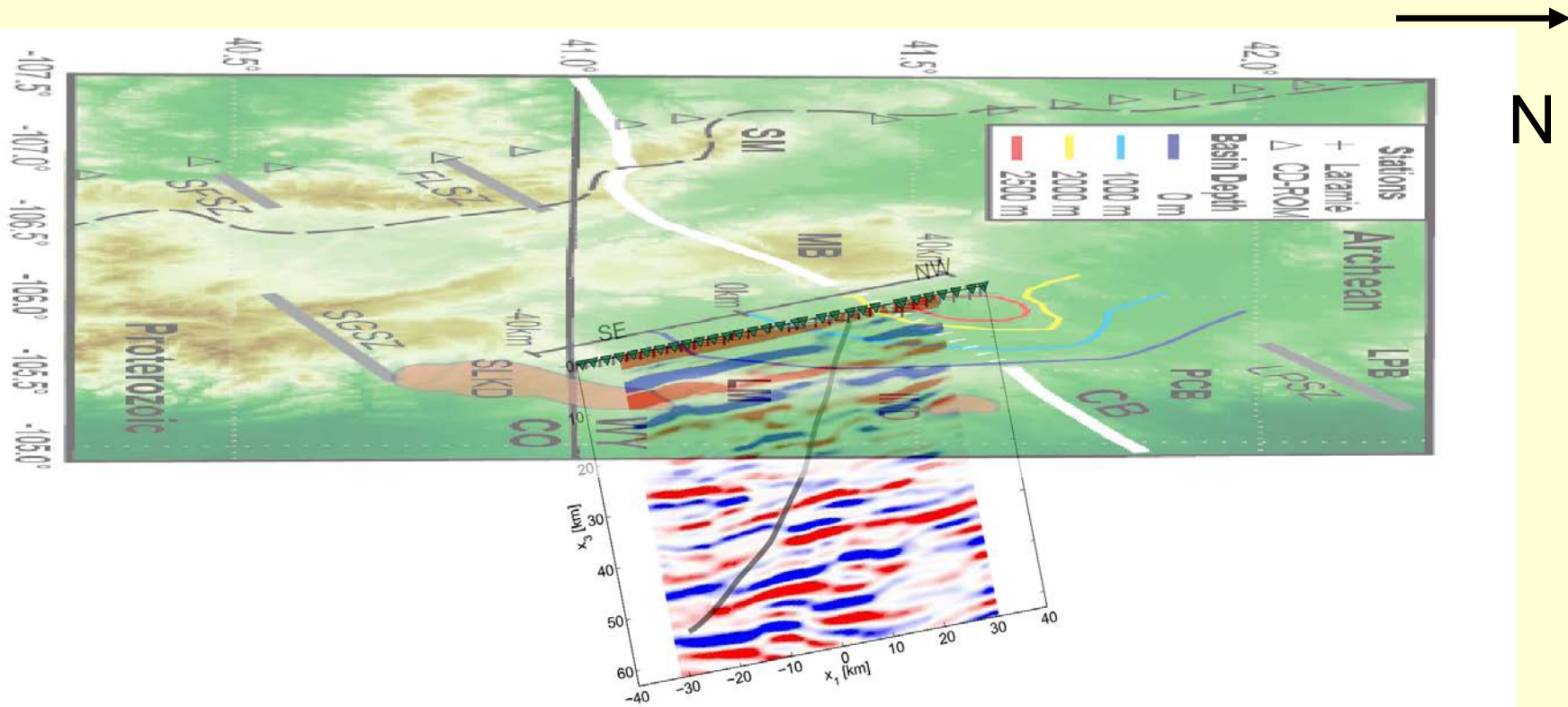
Retrieved reflection response



Final image

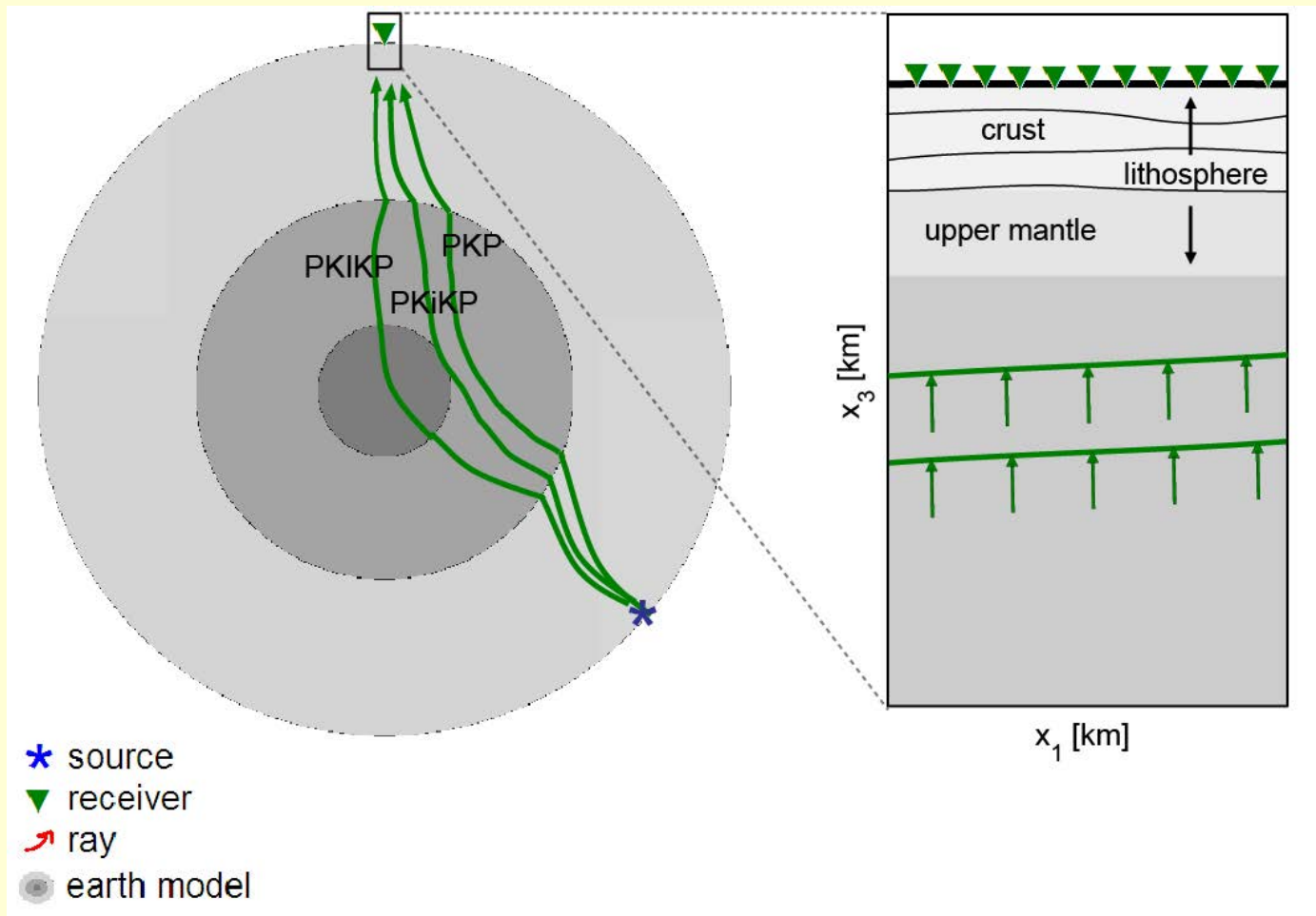


Final image

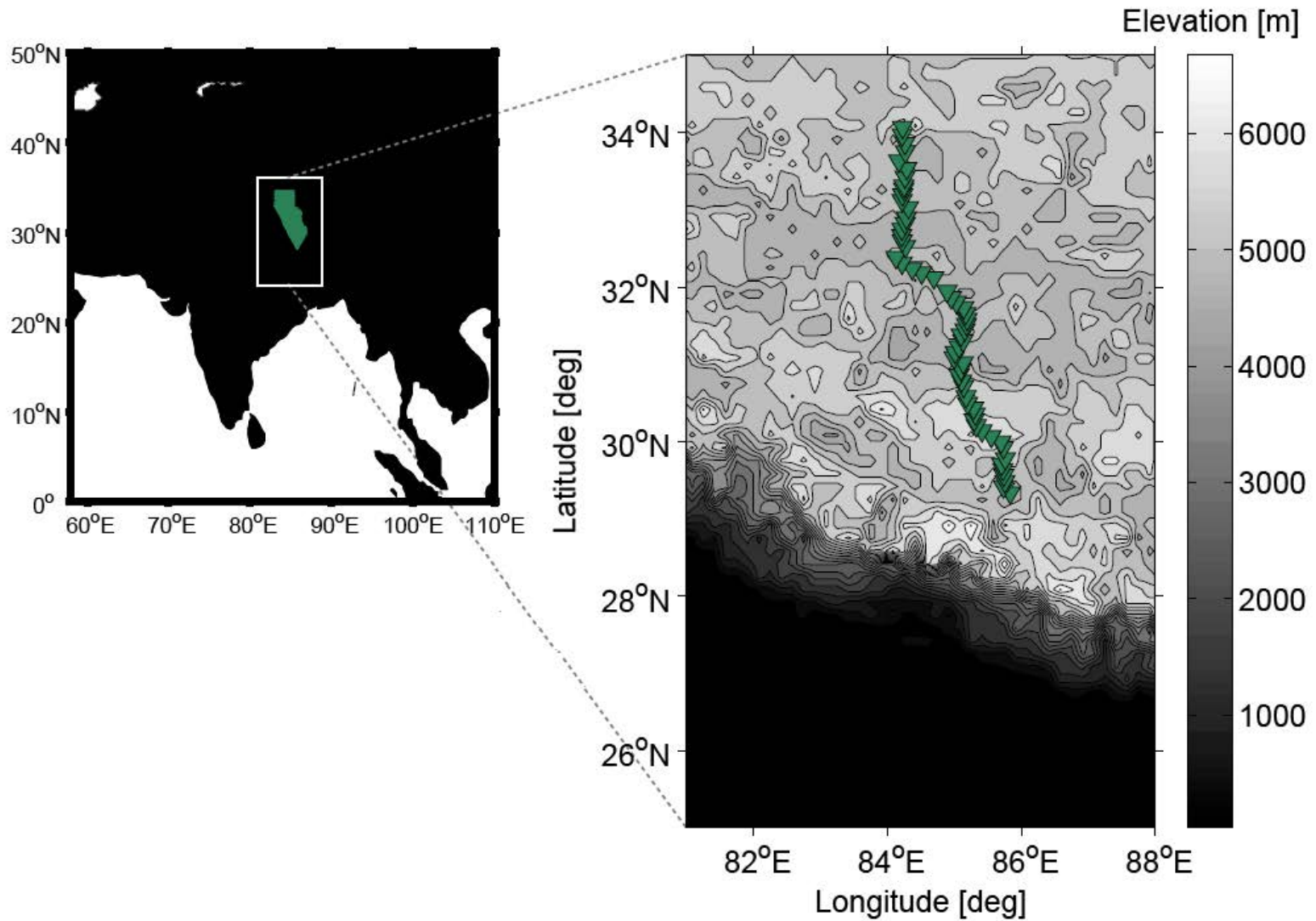


Global-phase seismic interferometry

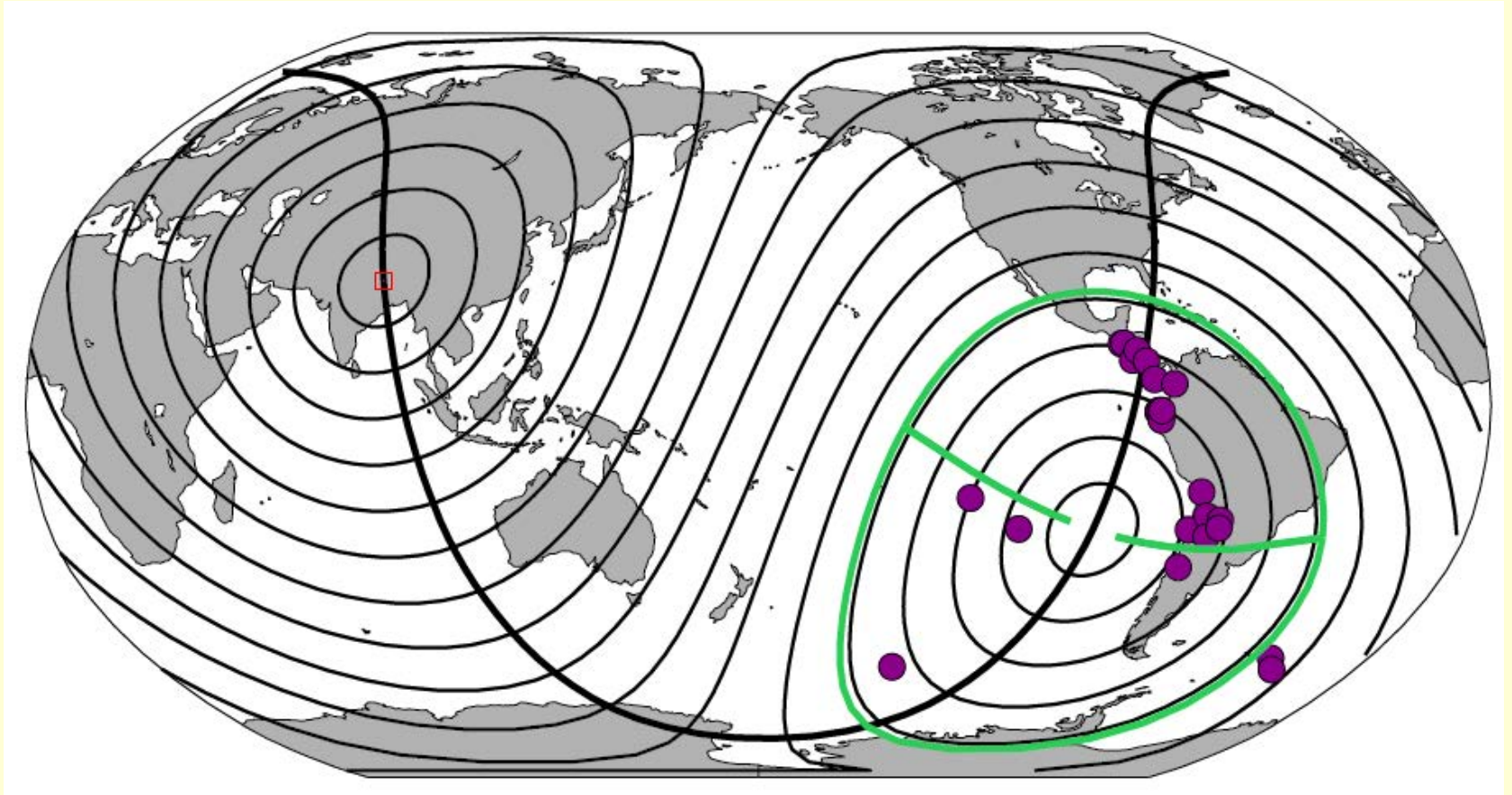
Ruigrok and Wapenaar, 2012, GRL, 39, L11303



Application to HiCLIMB array

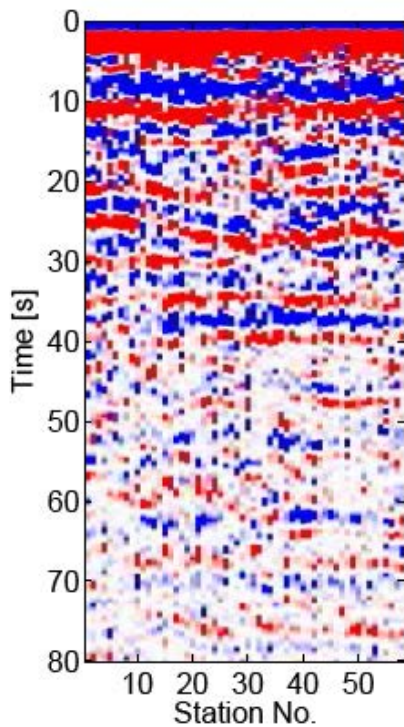


Input: global phases (PKP/PKIKP)

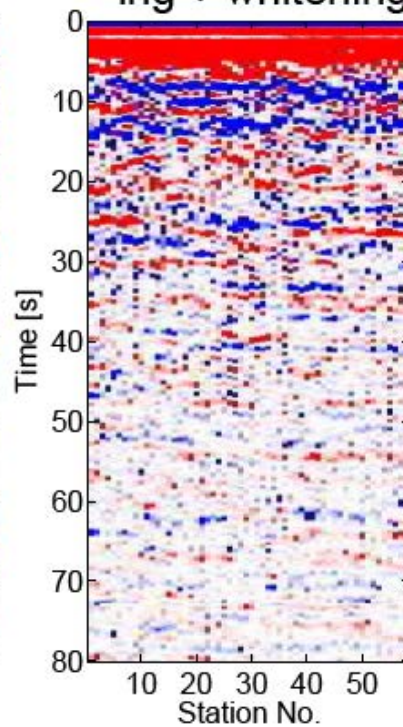


From raw to processed interferometry result

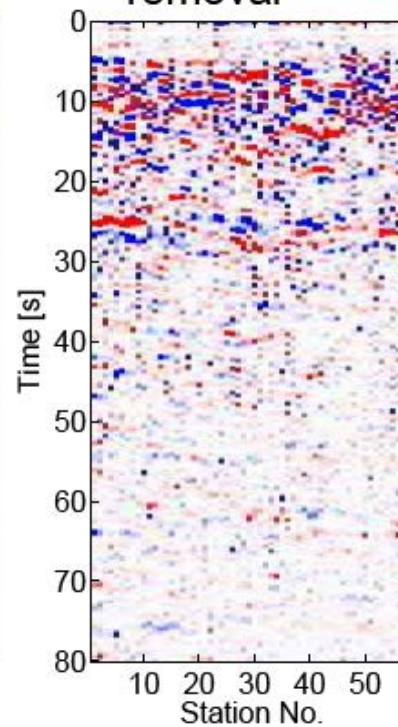
(a): 'raw'



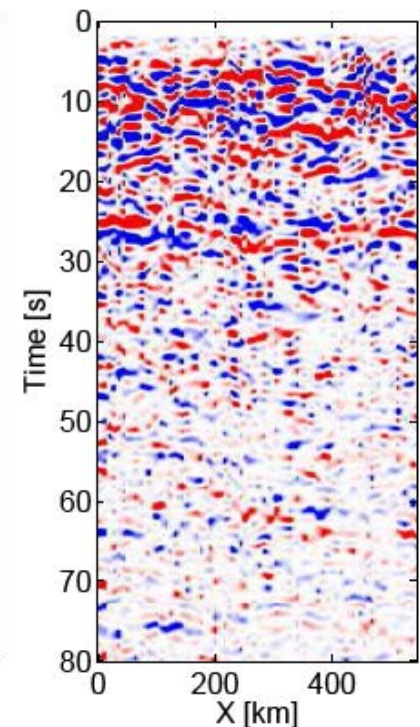
(b): bandpass filtering + whitening



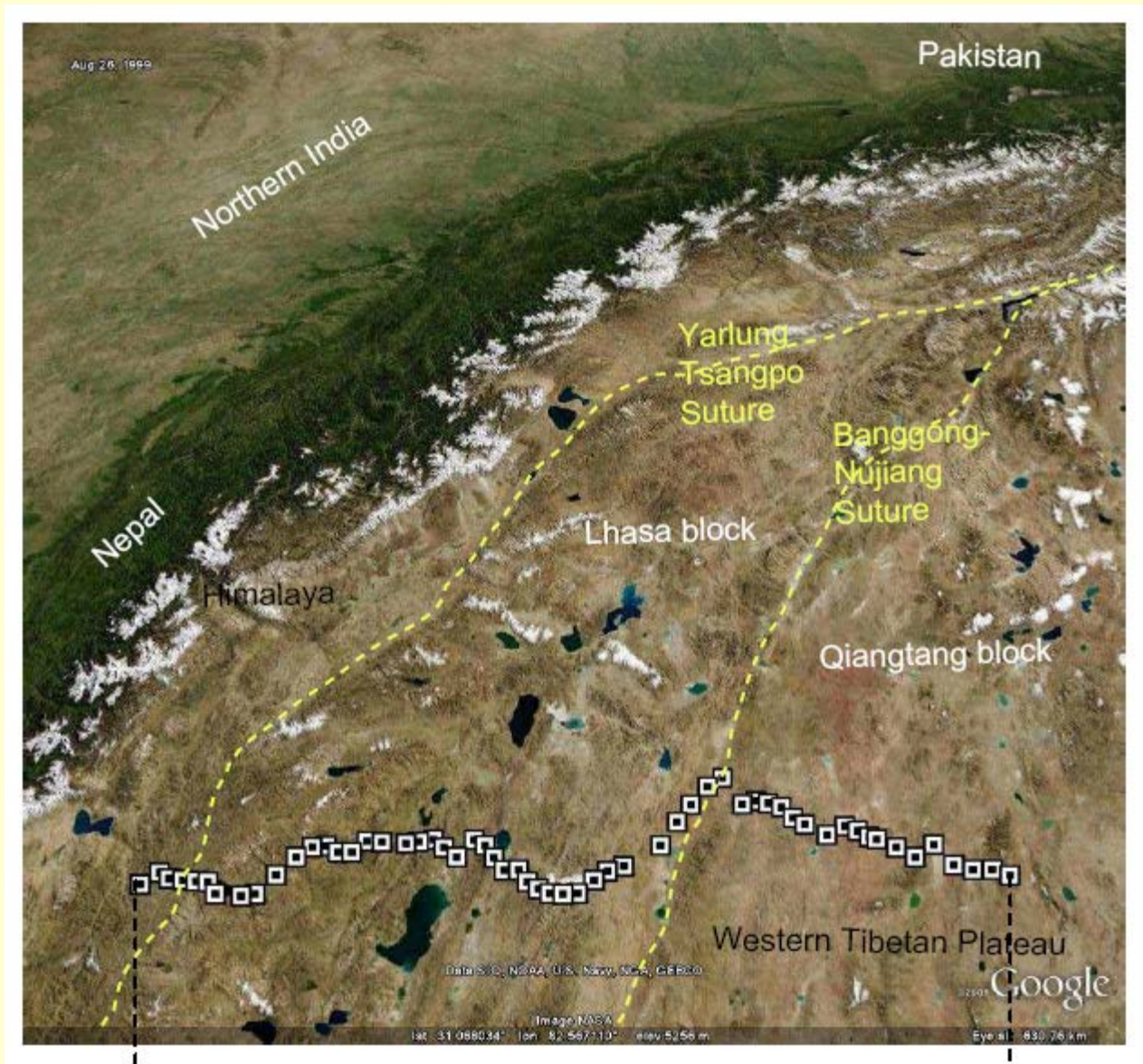
(c): + direct-wave removal



(d): + regularization



Array + tectonic overview



Final Image

