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- Part I: Seismic interferometry by cross-correlation
- Part II: Seismic interferometry by multi-dimensional deconvolution
- **Part III: Beyond seismic interferometry**

Part III: Beyond seismic interferometry

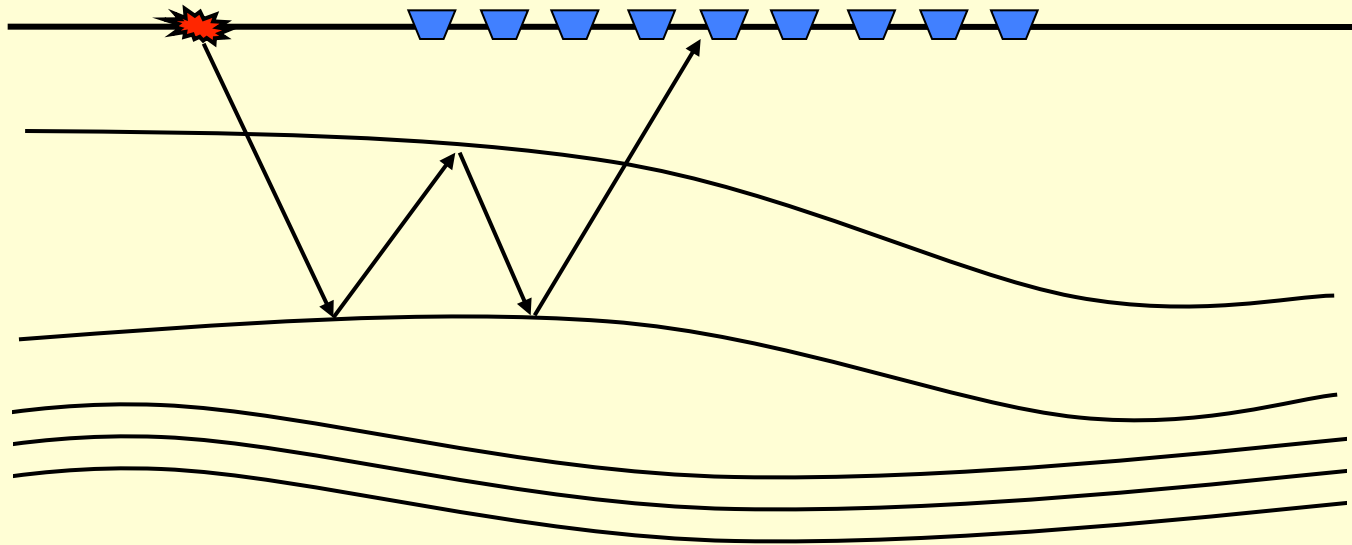
Kees Wapenaar, Jan Thorbecke, Joost van der Neut, Evert Slob,
Filippo Brogгинi, Jyoti Behura, Satyan Singh, Roel Snieder
and Ivan Vasconcelos

Also available as E-lecture:

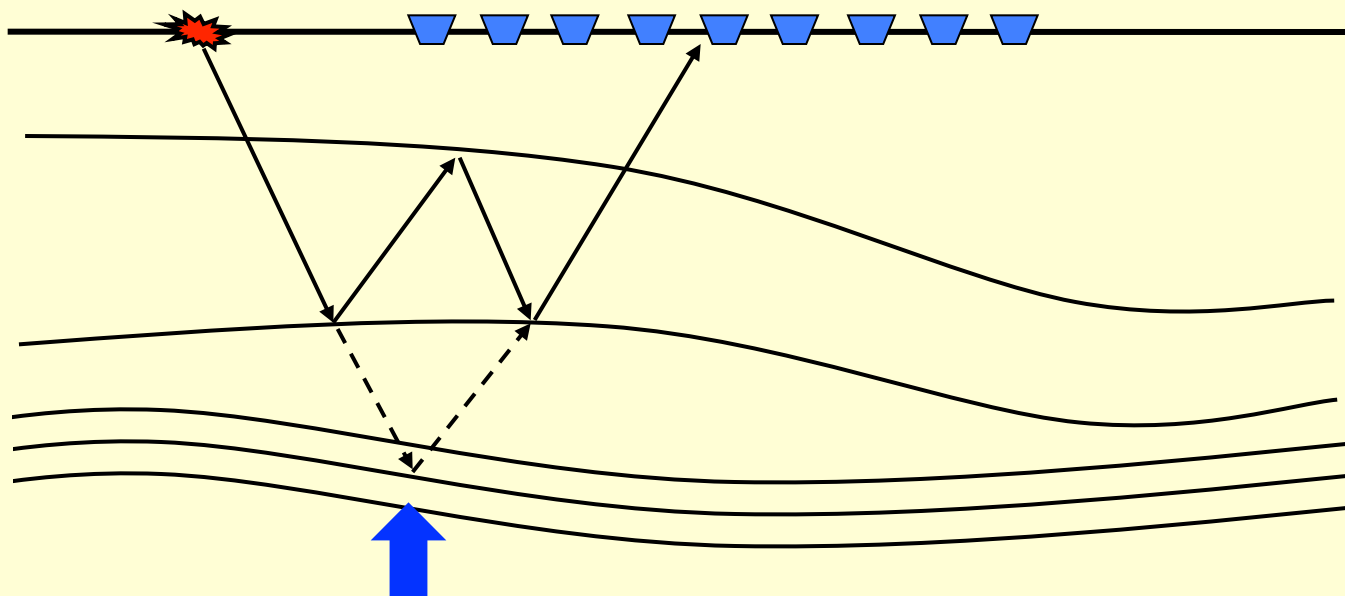
<https://youtu.be/DrpagMsK09M>

- Introduction
- Green's functions and focusing functions
- 3-D Marchenko equations
- Iterative solution
- Green's function retrieval
- Marchenko imaging
- Issues for discussion

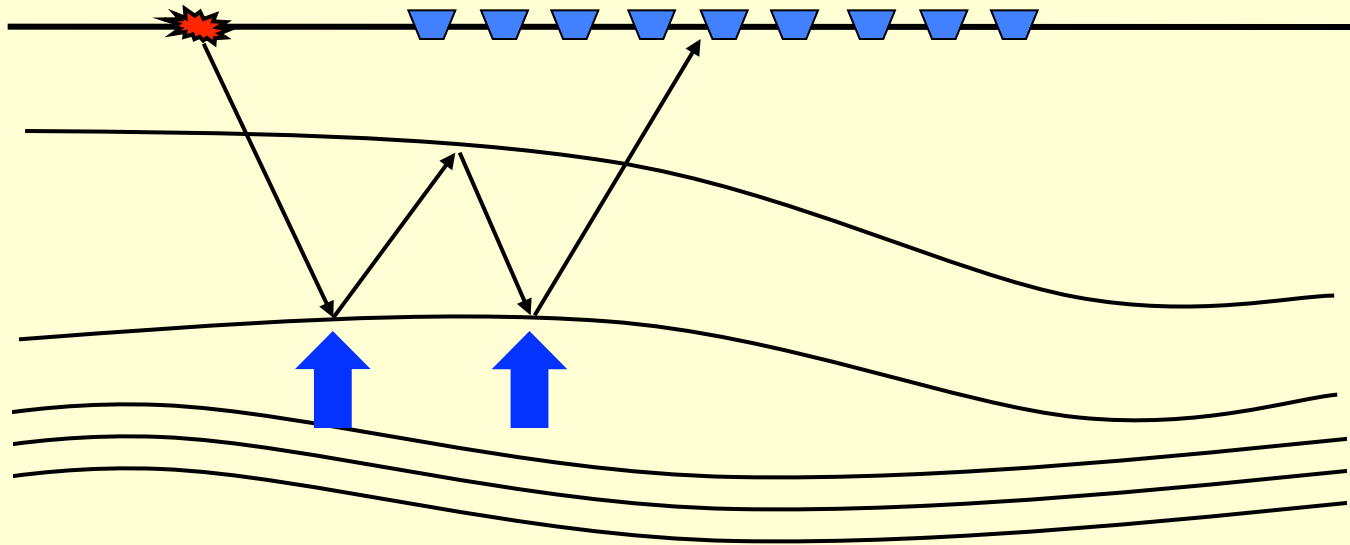
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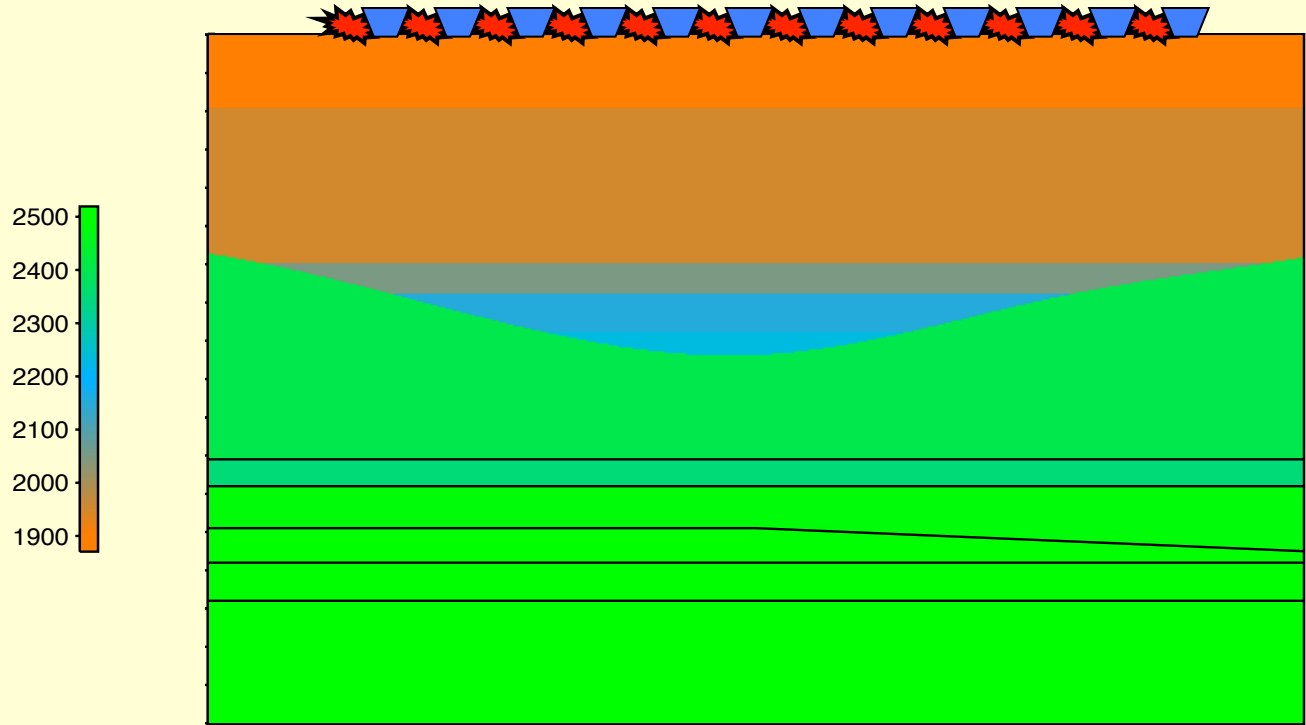
Internal multiples

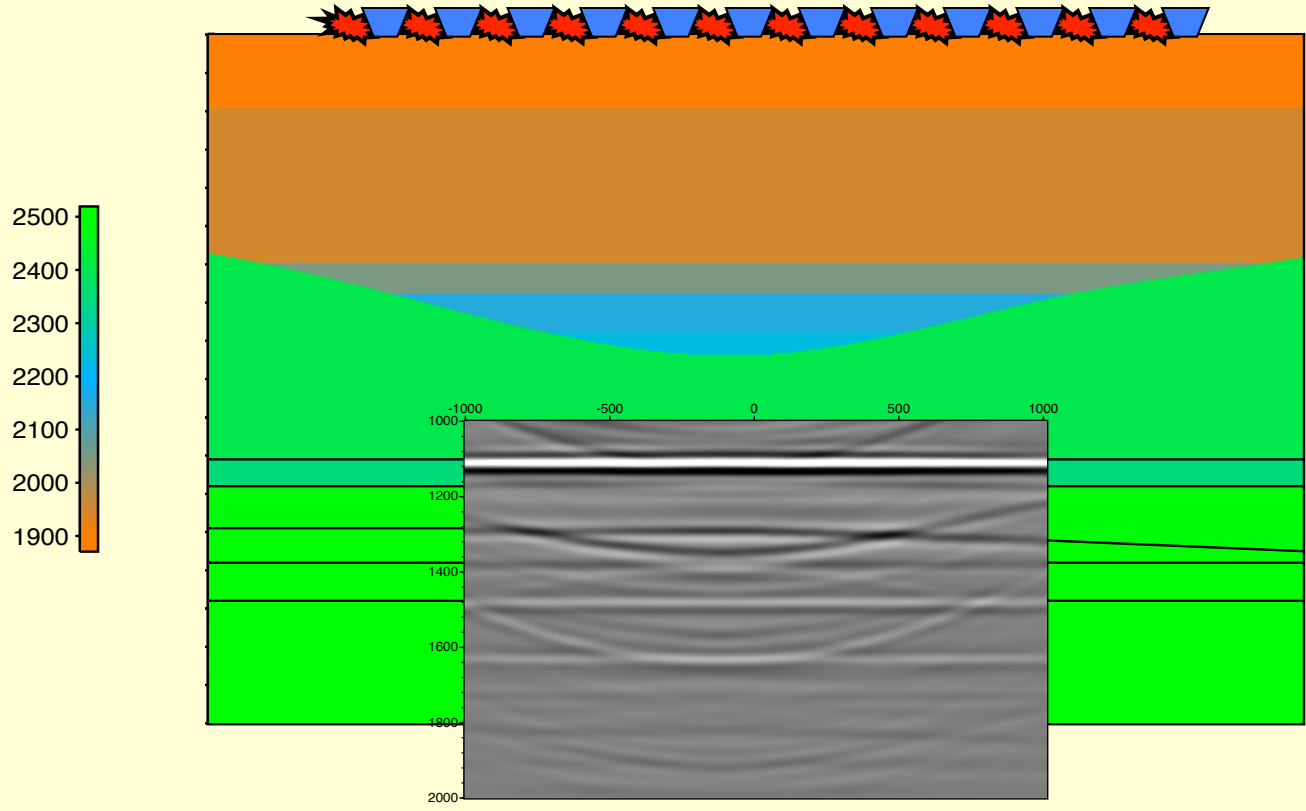


False images from internal multiples

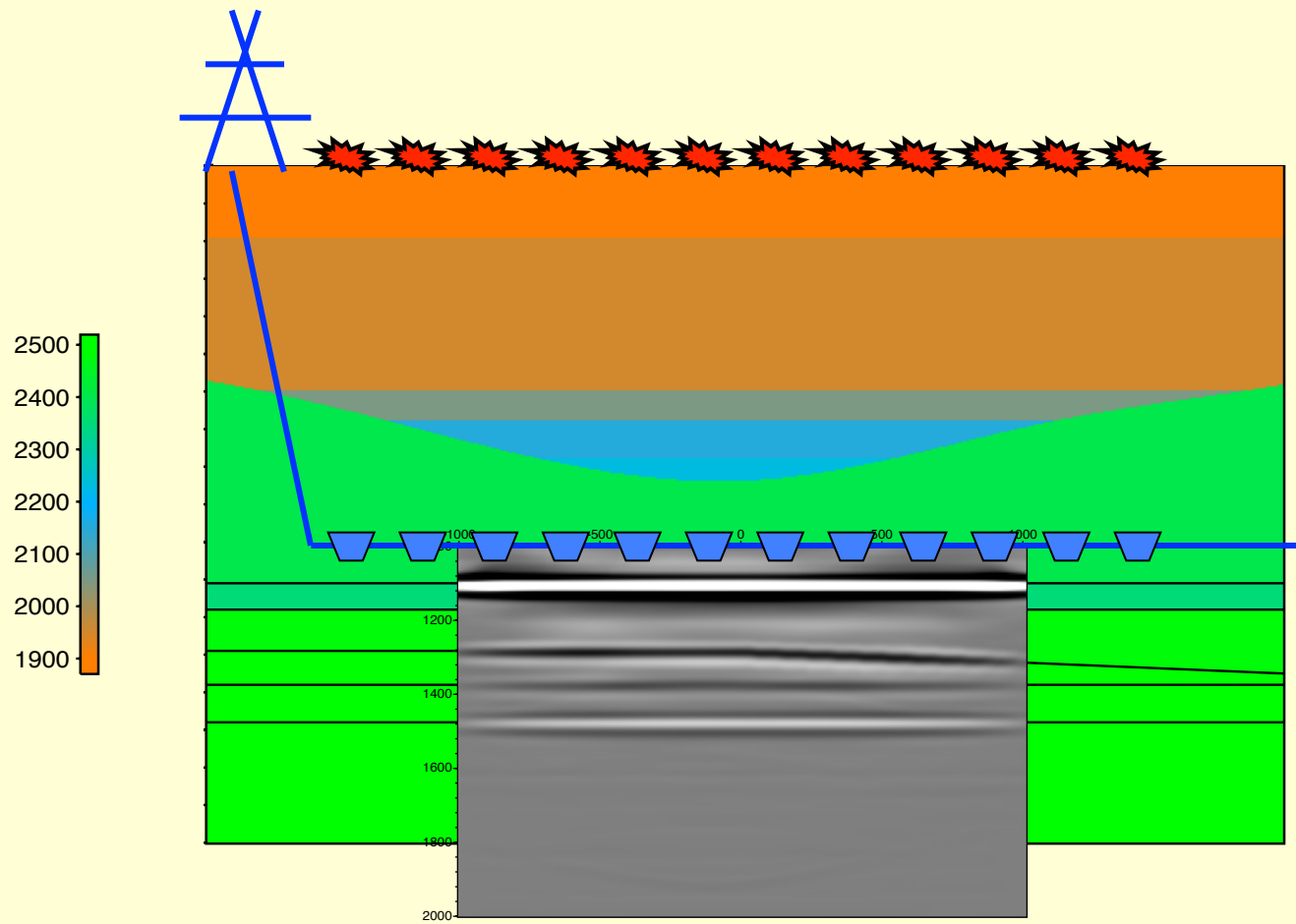


Improved illumination with internal multiples

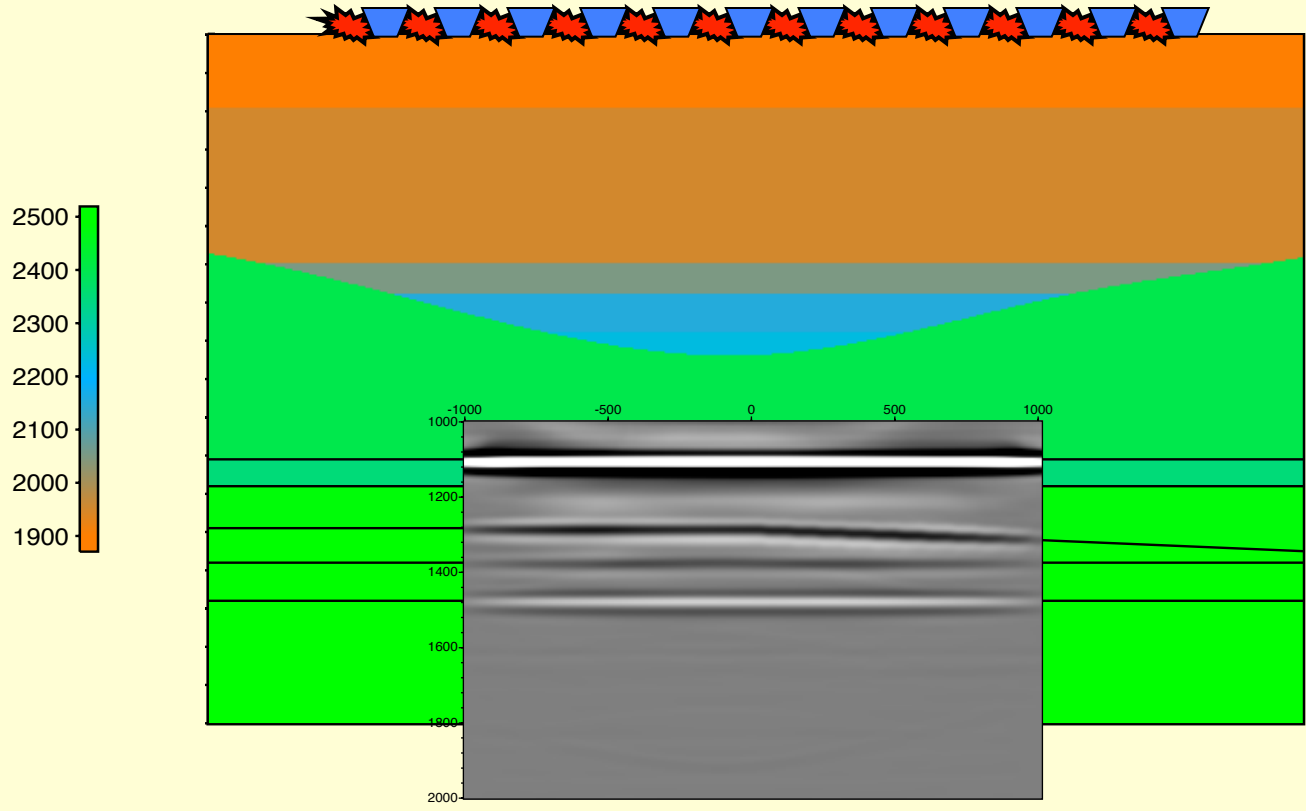




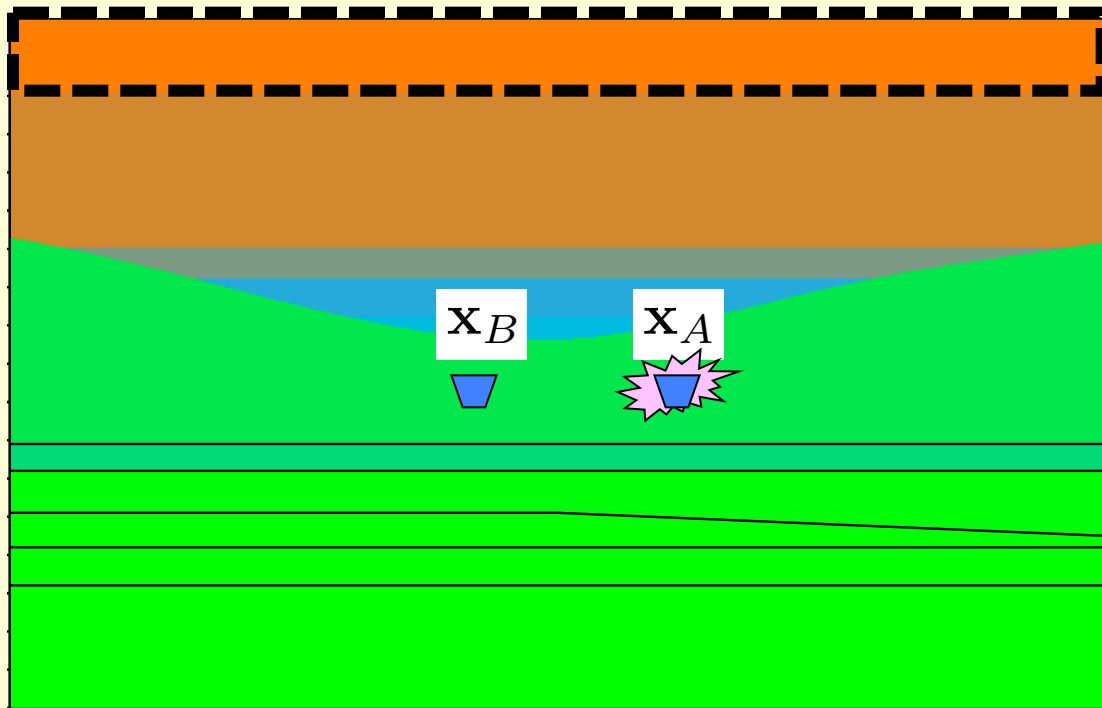
False images from internal multiples



Solution 1: receivers in a borehole



Solution 2: Marchenko imaging

$\partial\mathbb{D}$ 

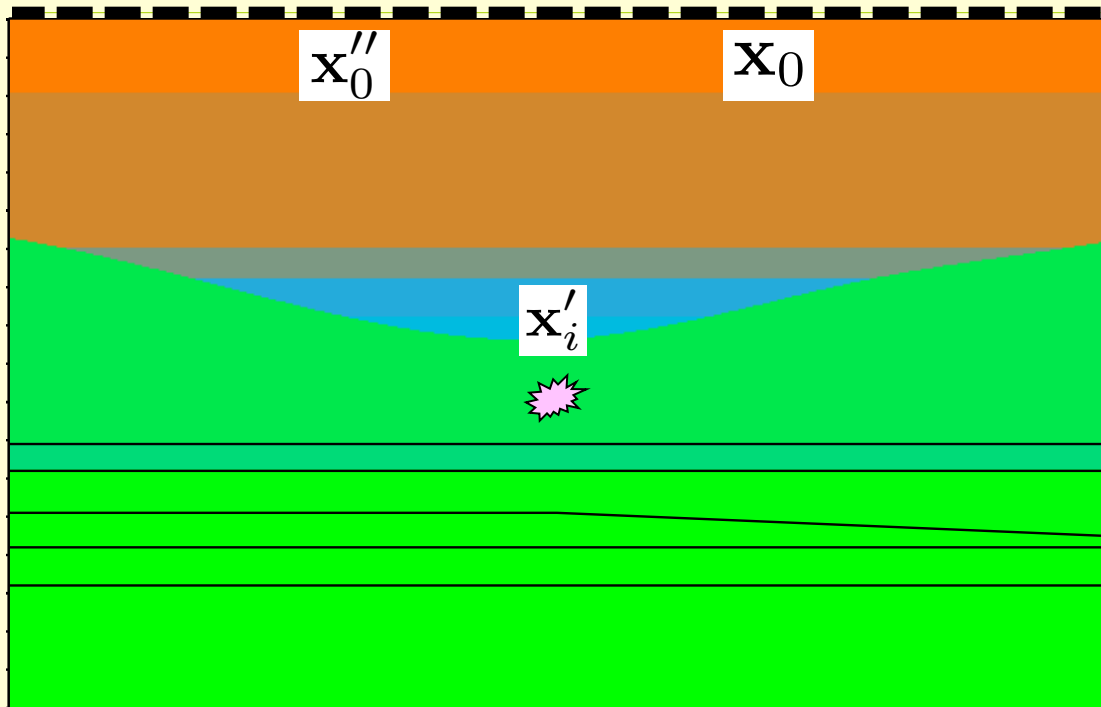
Interferometric
Green's function
representation:

Sources at $\partial\mathbb{D}$,
enclosing the medium

Receivers *inside* the
medium, at VS position

$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \approx \int_{\partial\mathbb{D}} d\mathbf{x} \int G(\mathbf{x}_B, \mathbf{x}, t + t') G(\mathbf{x}_A, \mathbf{x}, t') dt'$$

(Phys. Rev. Lett., 2004)

$\partial\mathbb{D}_0$ 

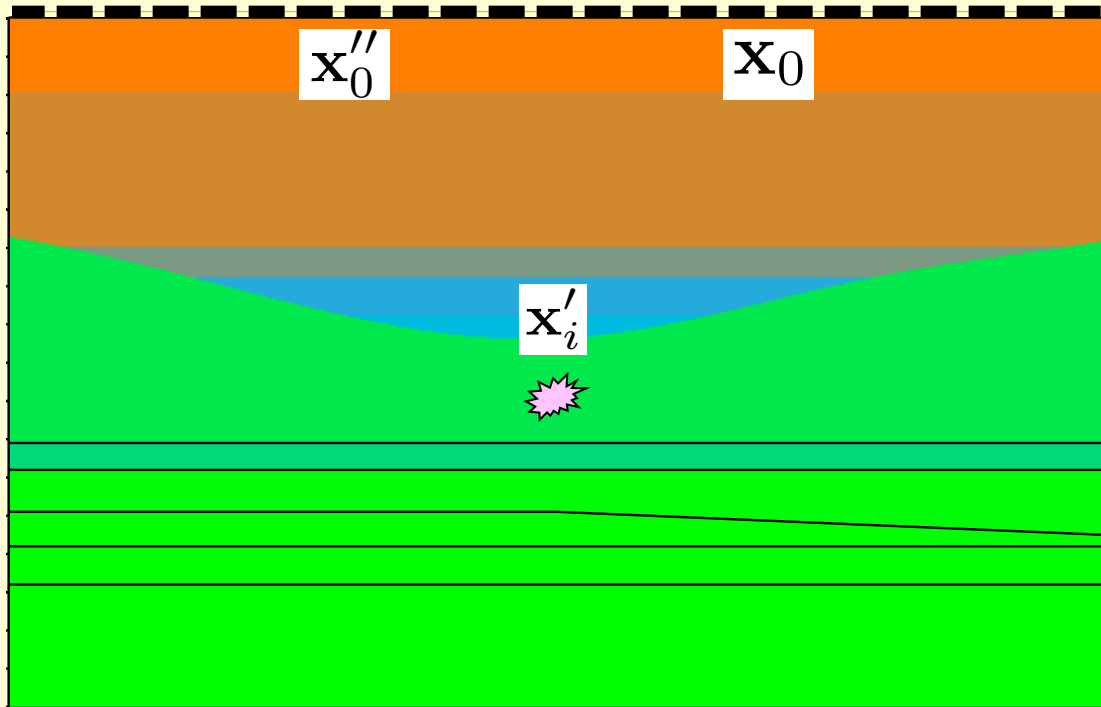
Single-sided
Green's function
representation:

Sources *and* receivers
at $\partial\mathbb{D}_0$, at *one side*
of the medium

No receiver at VS!

$$G(\mathbf{x}_0'', \mathbf{x}_i', t) = f_2(\mathbf{x}_i', \mathbf{x}_0'', -t) + \int_{\partial\mathbb{D}_0} d\mathbf{x}_0 \int_{-\infty}^t R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_2(\mathbf{x}_i', \mathbf{x}_0, t') dt'$$

(Phys. Rev. Lett., 2013)

$\partial\mathbb{D}_0$ 

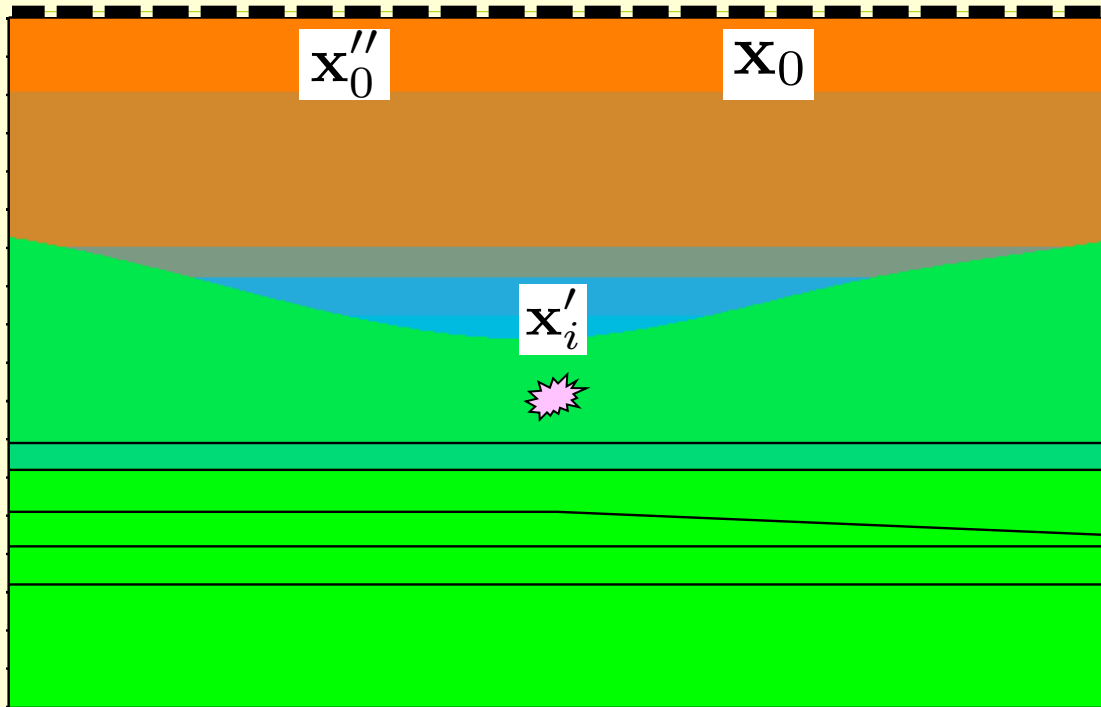
Assumption:
Lossless medium

Approximation:
Evanescent field
not included

$$G(\mathbf{x}_0'', \mathbf{x}_i', t) = f_2(\mathbf{x}_i', \mathbf{x}_0'', -t) + \int_{\partial\mathbb{D}_0} d\mathbf{x}_0 \int_{-\infty}^t R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_2(\mathbf{x}_i', \mathbf{x}_0, t') dt'$$

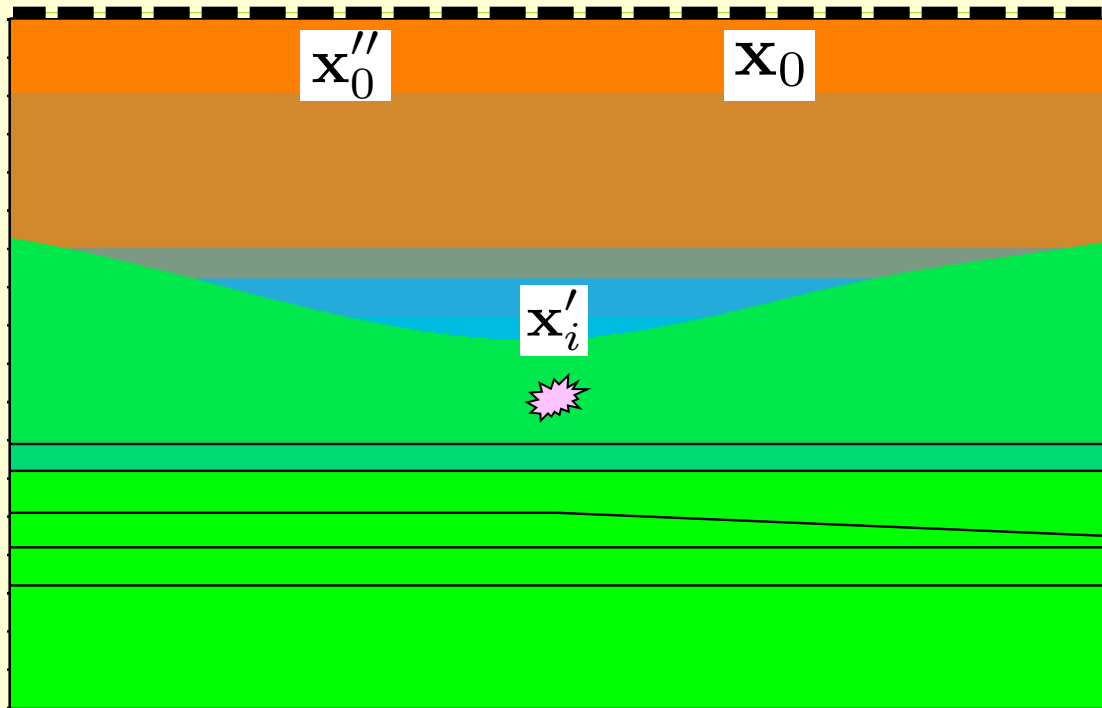
(Phys. Rev. Lett., 2013)

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$\partial\mathbb{D}_0$ 

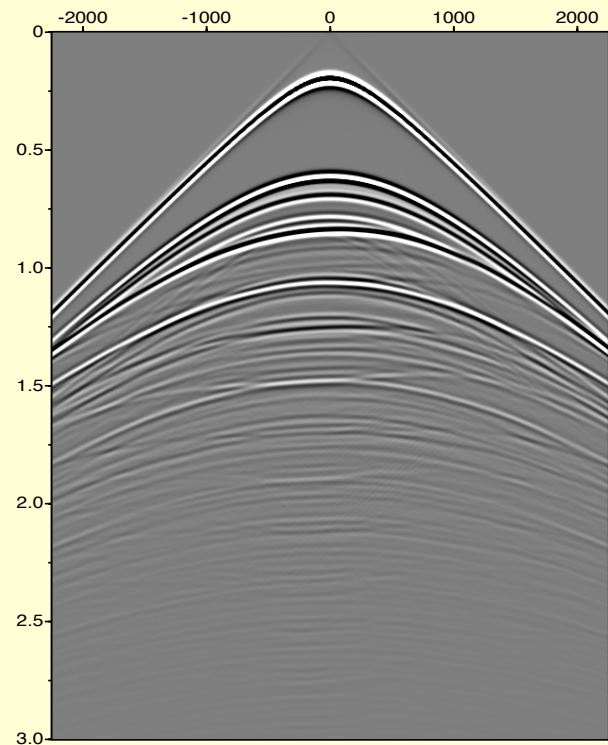
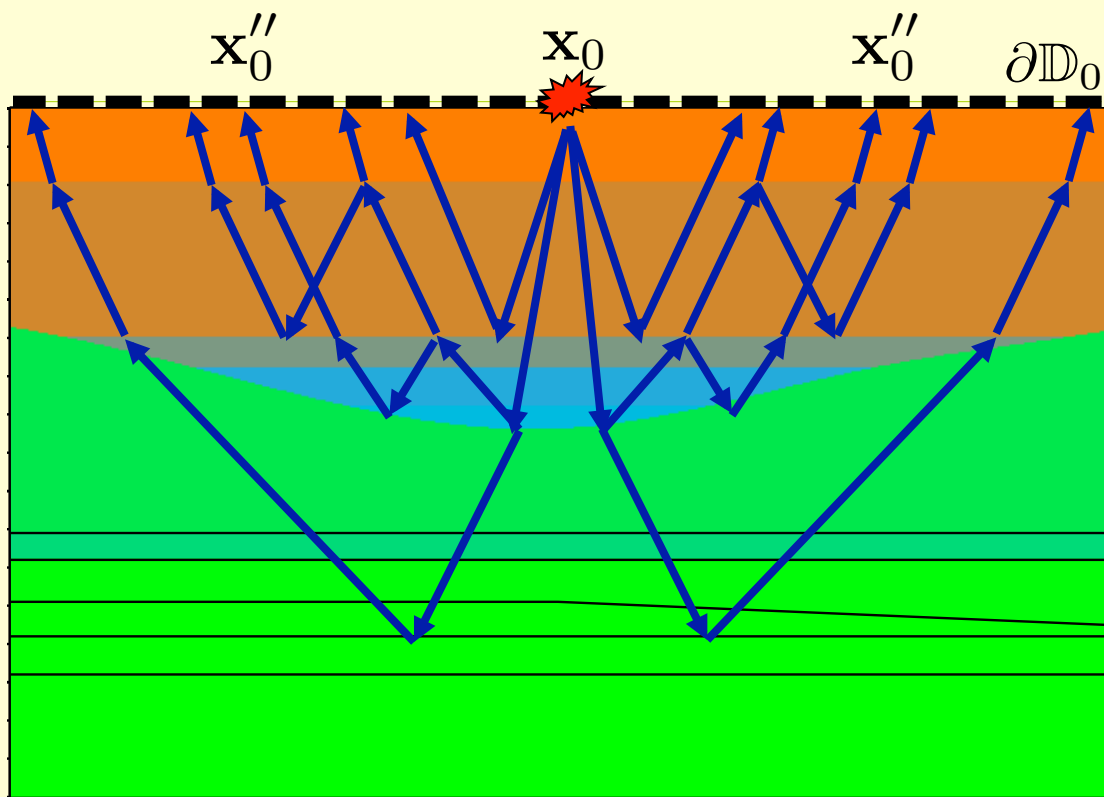
Single-sided
Green's function
representation

$$G(\mathbf{x}_0'', \mathbf{x}_i', t) - f_2(\mathbf{x}_i', \mathbf{x}_0'', -t) = \int_{\partial\mathbb{D}_0} d\mathbf{x}_0 \int_{-\infty}^t R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_2(\mathbf{x}_i', \mathbf{x}_0, t') dt'$$

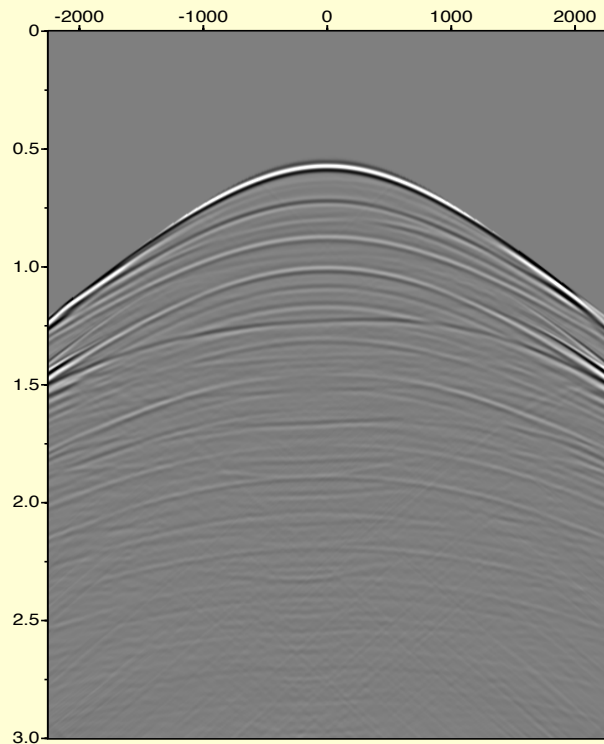
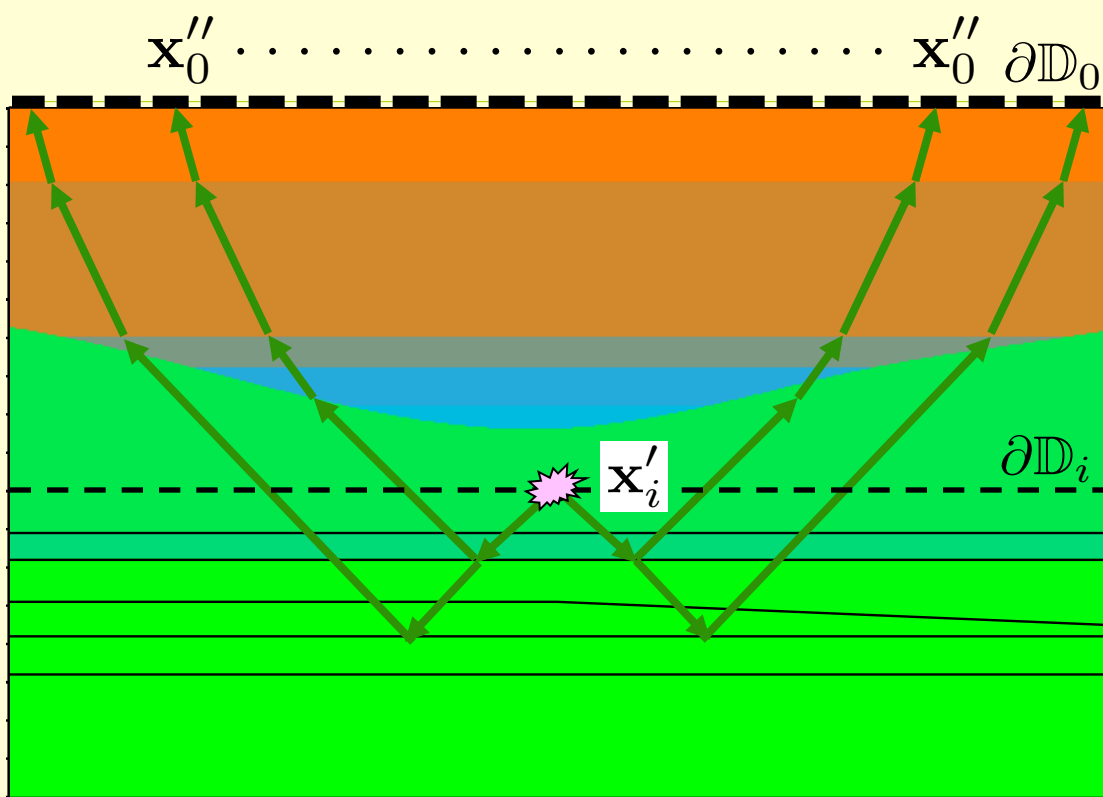
$\partial\mathbb{D}_0$ 

Decomposed
Single-sided
Green's function
representations

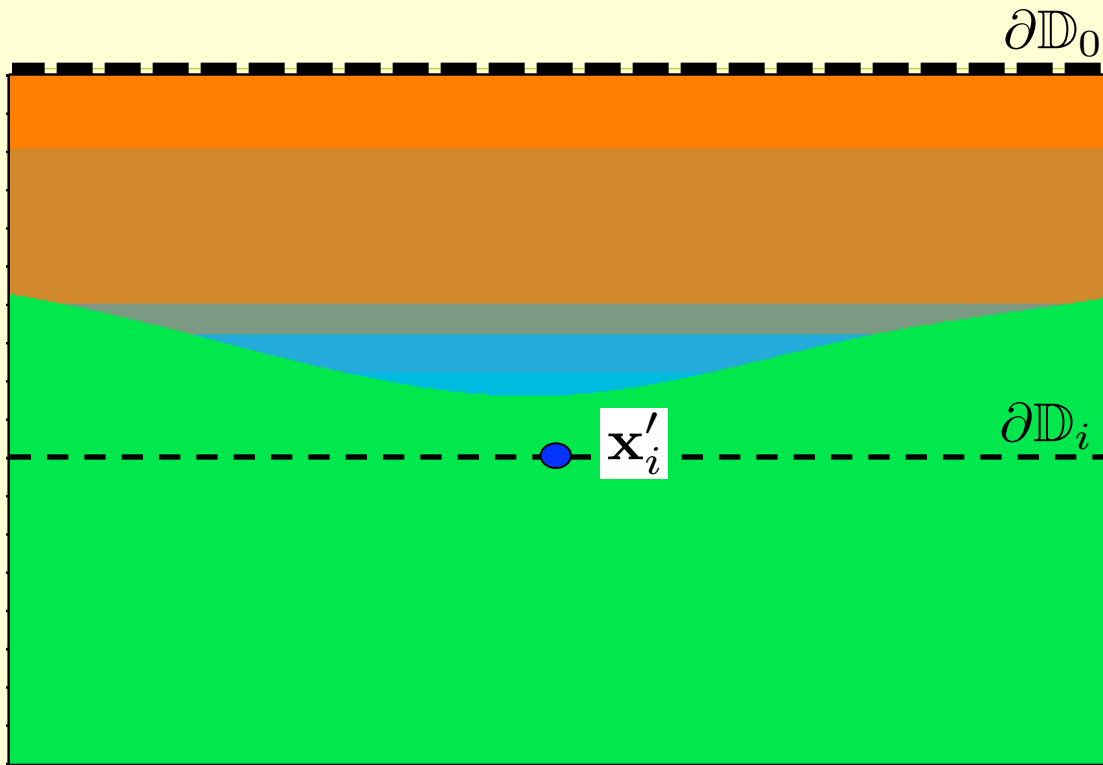
$$G^{-,+}(\mathbf{x}_0'', \mathbf{x}_i', t) + f_1^{-}(\mathbf{x}_0'', \mathbf{x}_i', t) = \int_{\partial\mathbb{D}_0} d\mathbf{x}_0 \int_{-\infty}^t R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^{+}(\mathbf{x}_0, \mathbf{x}_i', t') dt'$$

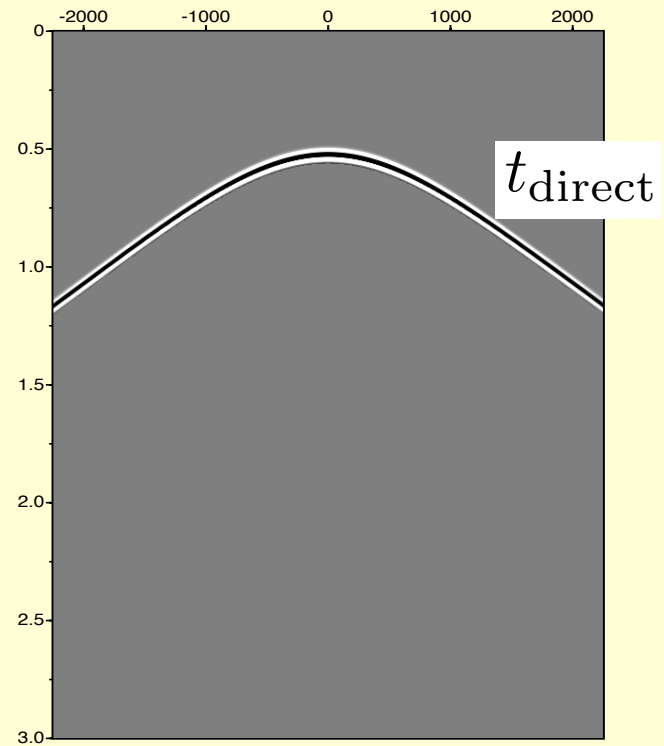
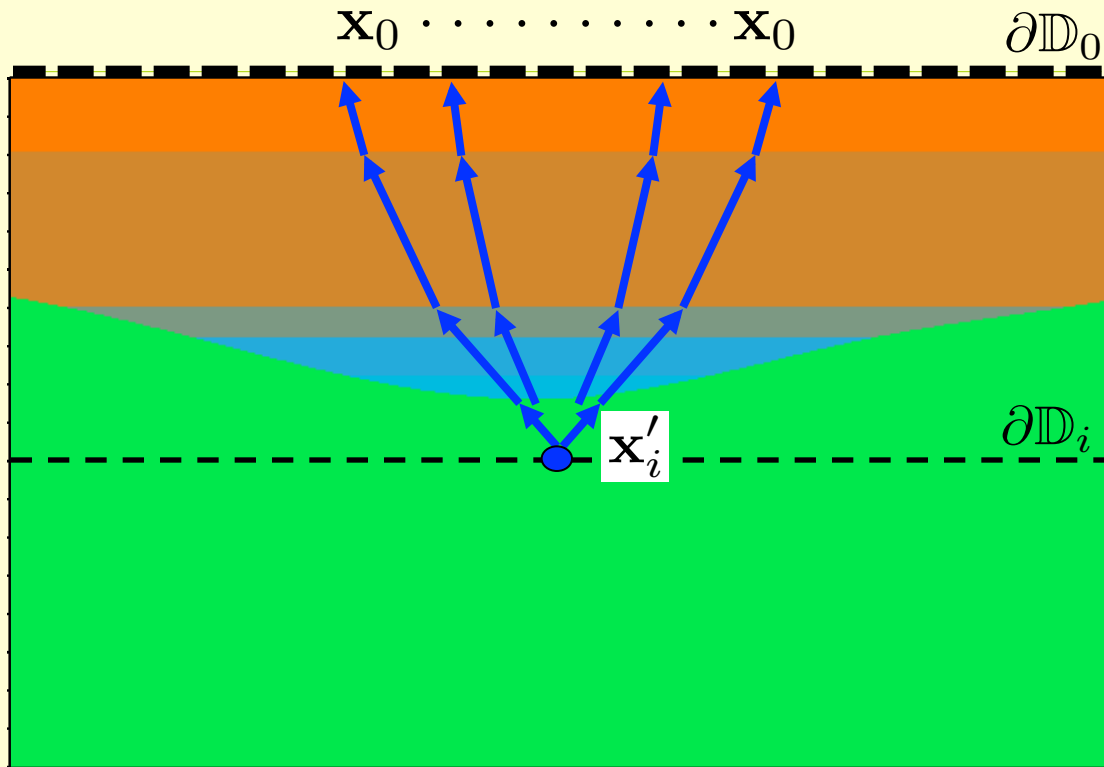


$$G^{-,+}(\mathbf{x}_0'', \mathbf{x}_i', t) + f_1^-(\mathbf{x}_0'', \mathbf{x}_i', t) = \int_{\partial\mathbb{D}_0} d\mathbf{x}_0 \int_{-\infty}^t R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^+(\mathbf{x}_0, \mathbf{x}_i', t') dt'$$

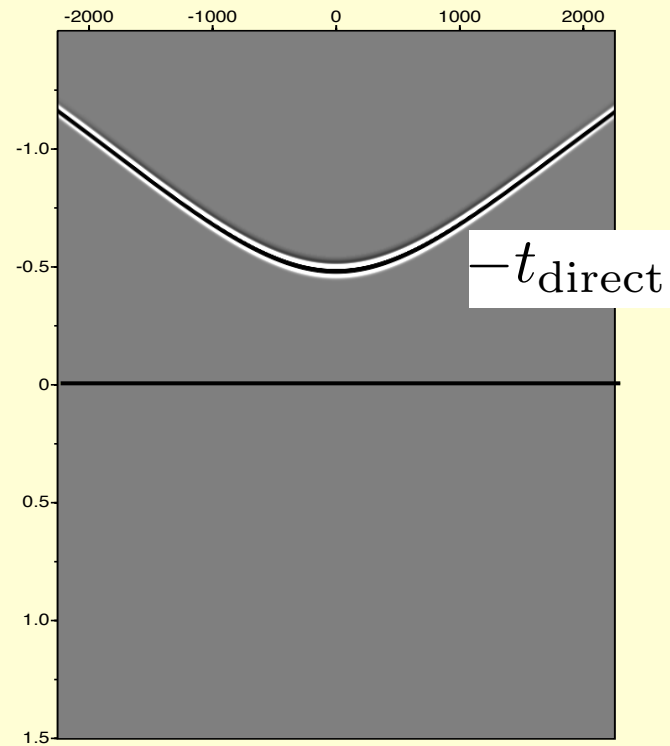
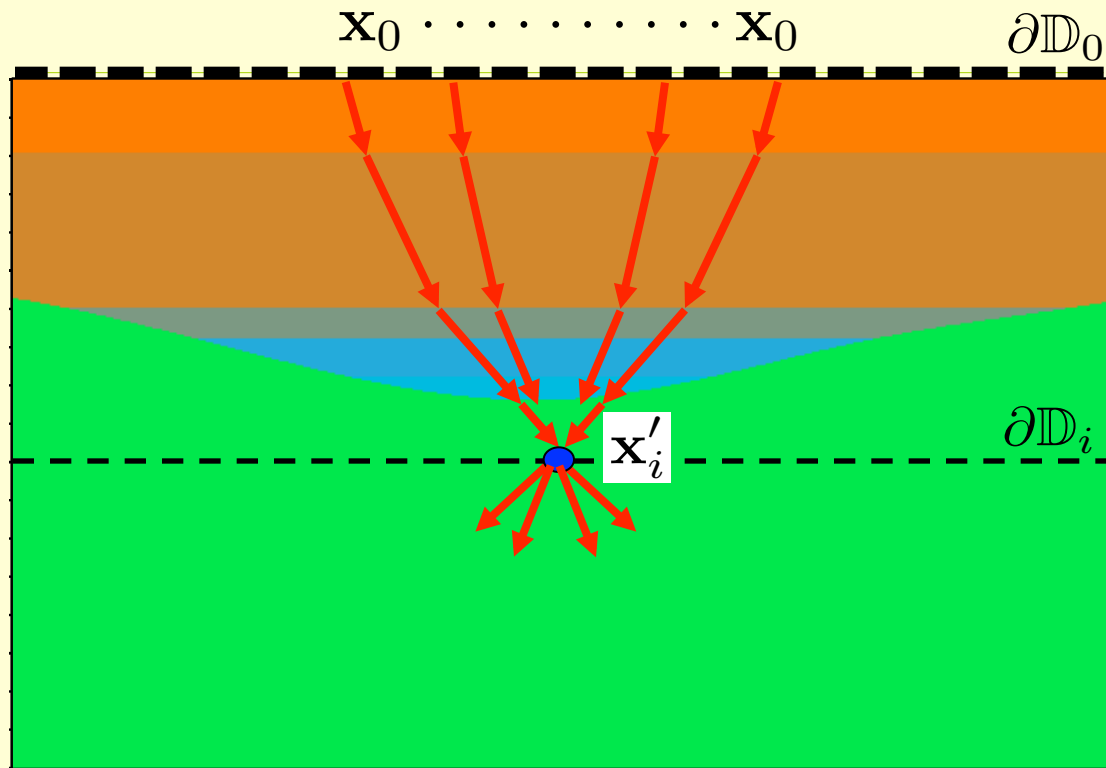


$$G^{-,+}(\mathbf{x}_0'', \mathbf{x}_i', t) + f_1^-(\mathbf{x}_0'', \mathbf{x}_i', t) = \int_{\partial \mathbb{D}_0} d\mathbf{x}_0 \int_{-\infty}^t R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^+(\mathbf{x}_0, \mathbf{x}_i', t') dt'$$

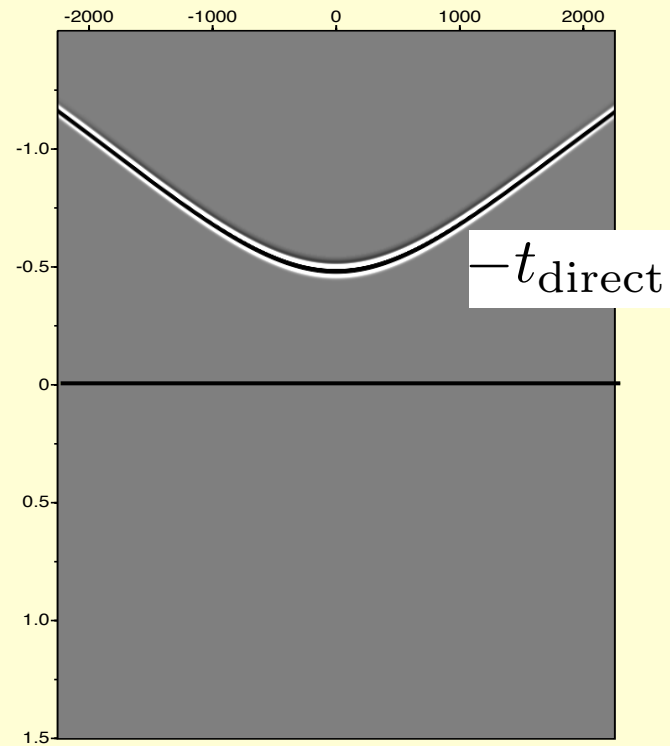
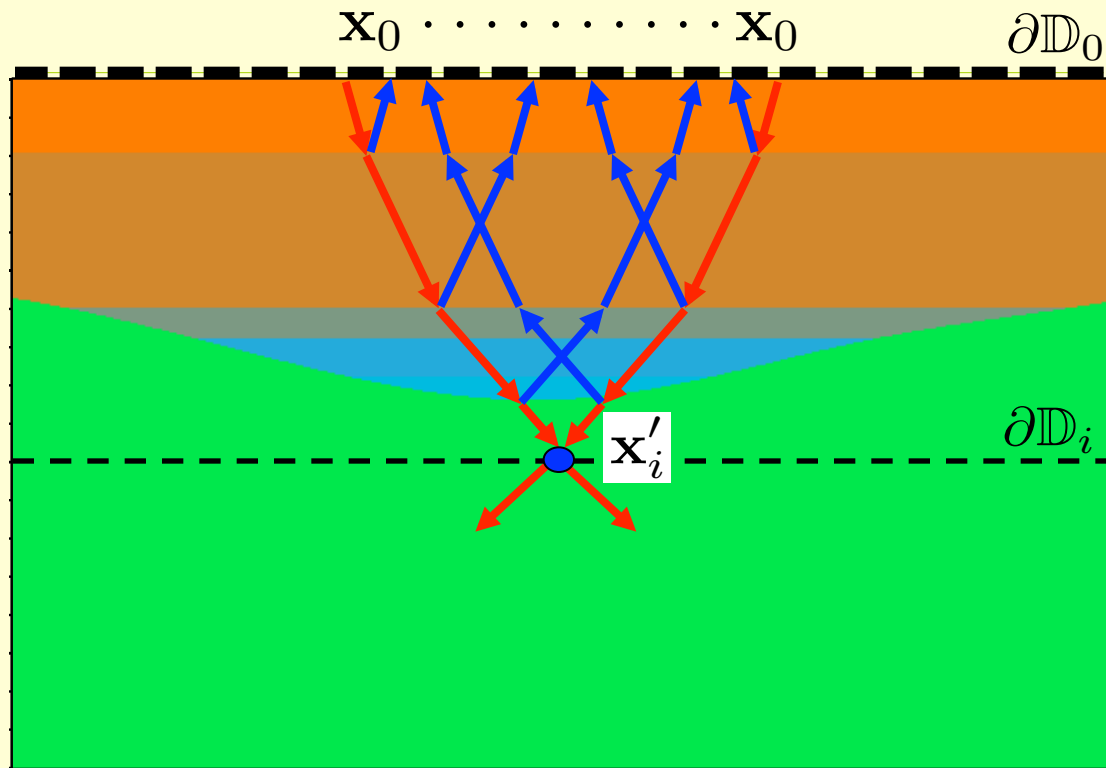




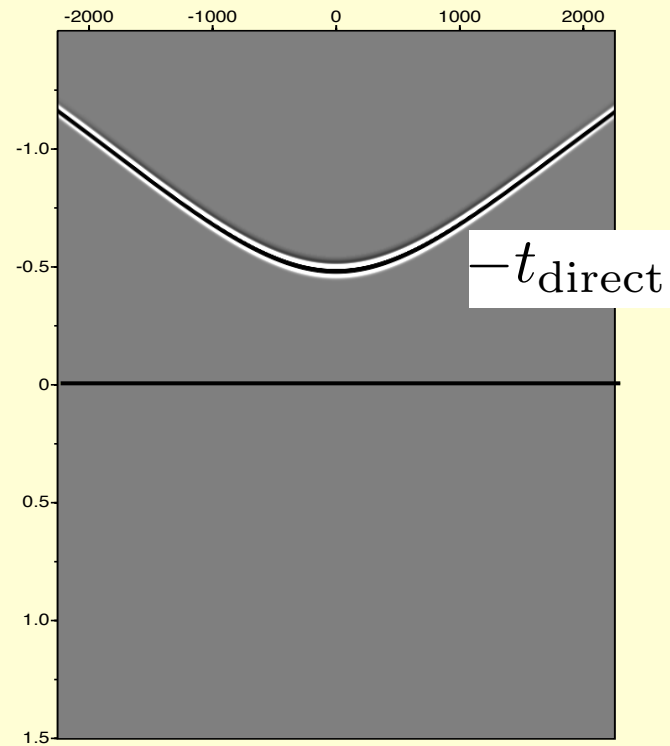
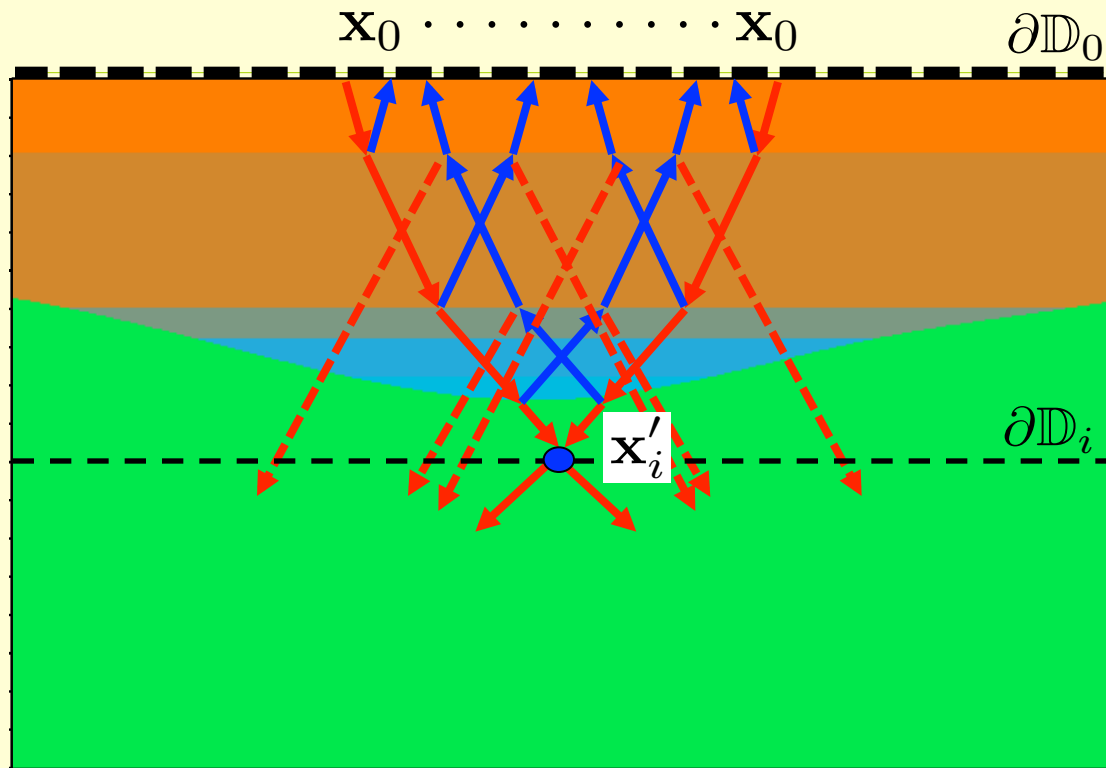
$$G_d(\mathbf{x}_0, \mathbf{x}'_i, t)$$



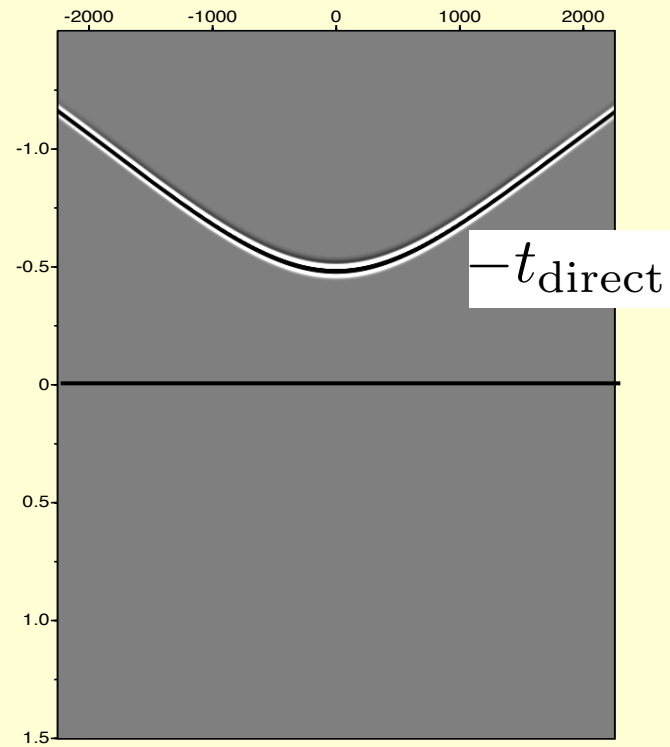
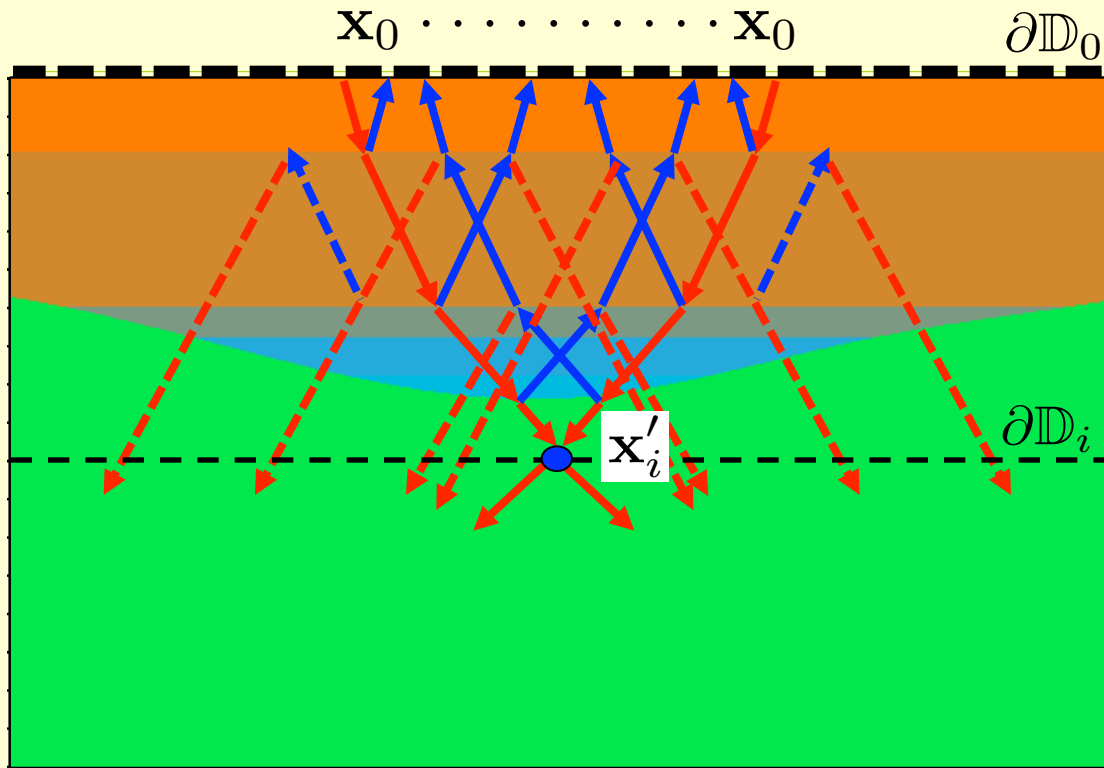
$$f_1^+(\mathbf{x}_0, \mathbf{x}'_i, t)$$



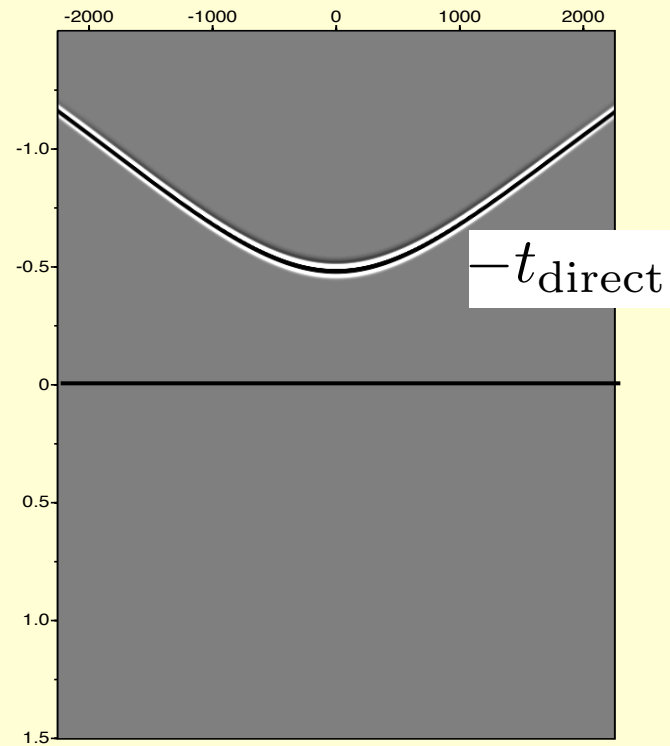
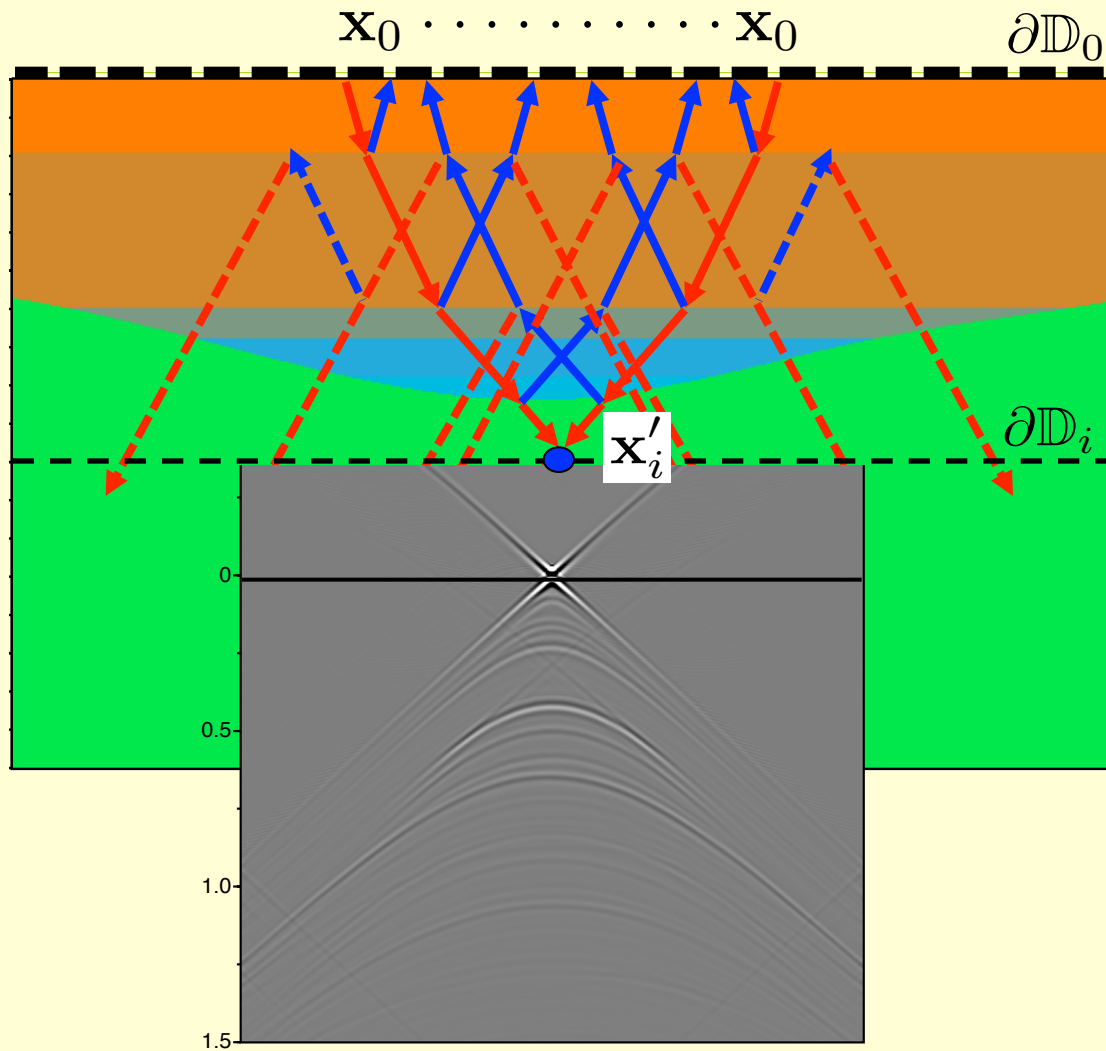
$$f_1^+(\mathbf{x}_0, \mathbf{x}'_i, t)$$



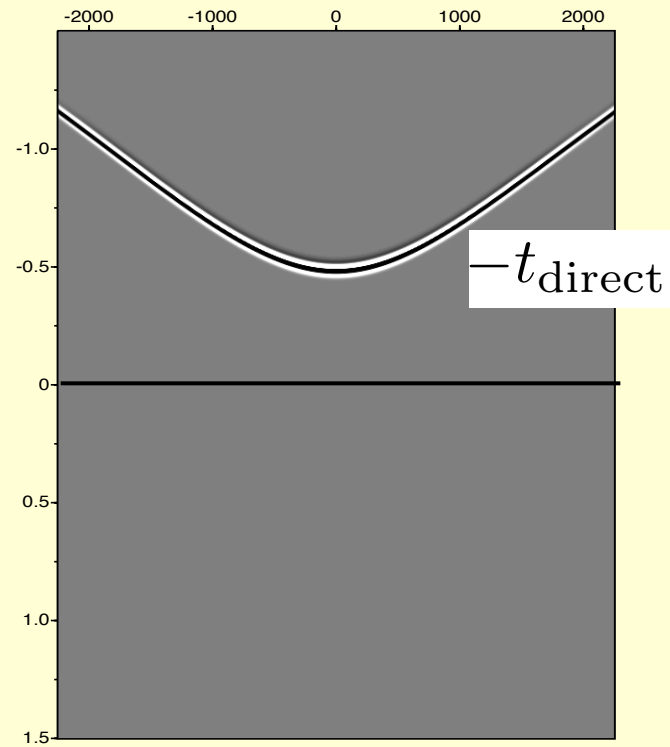
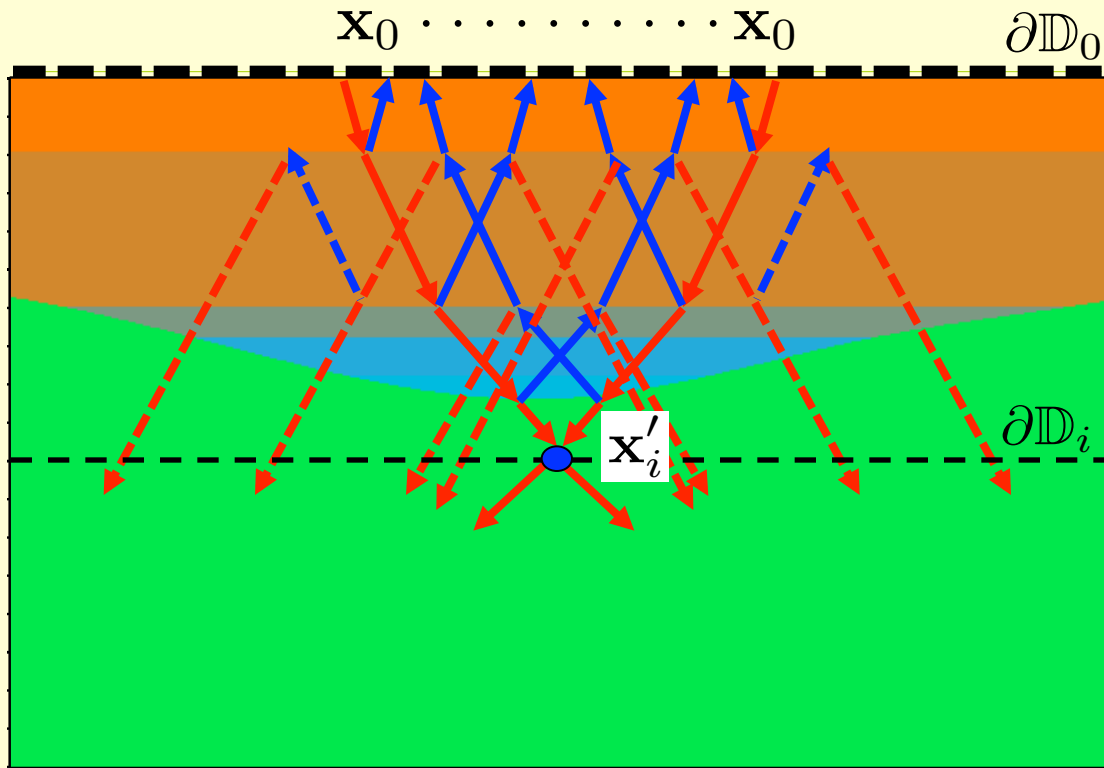
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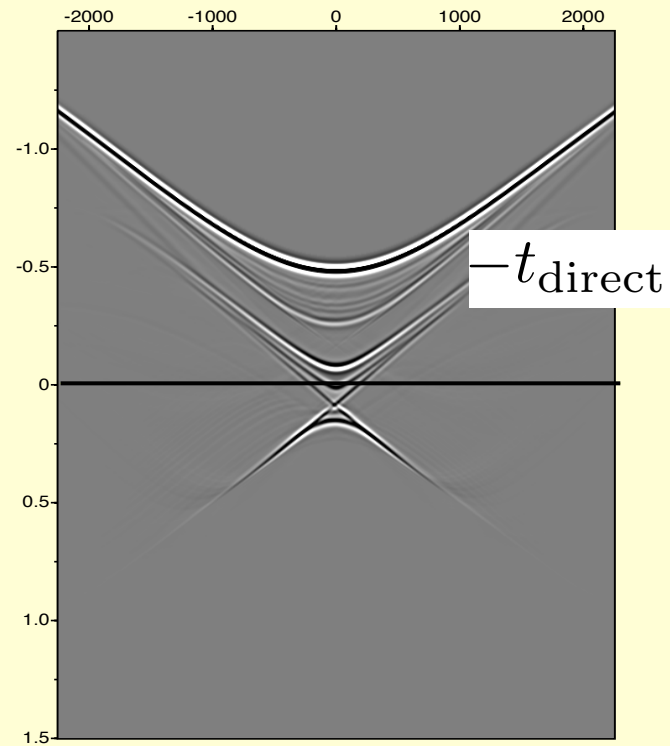
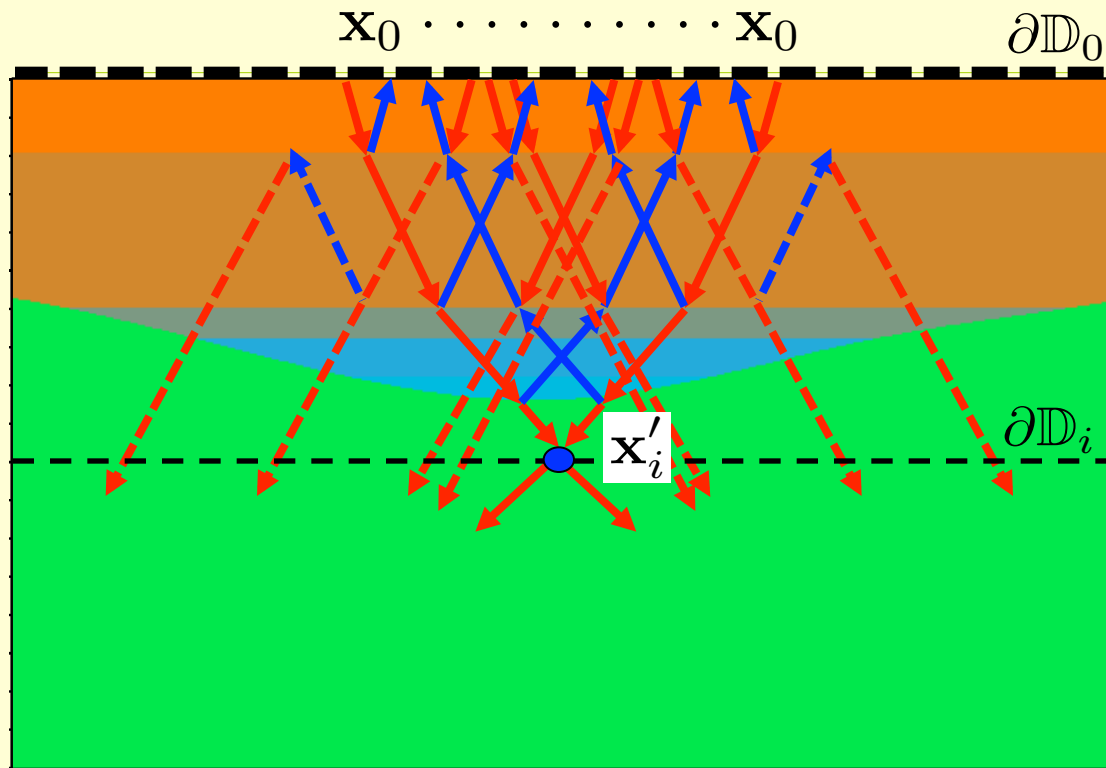
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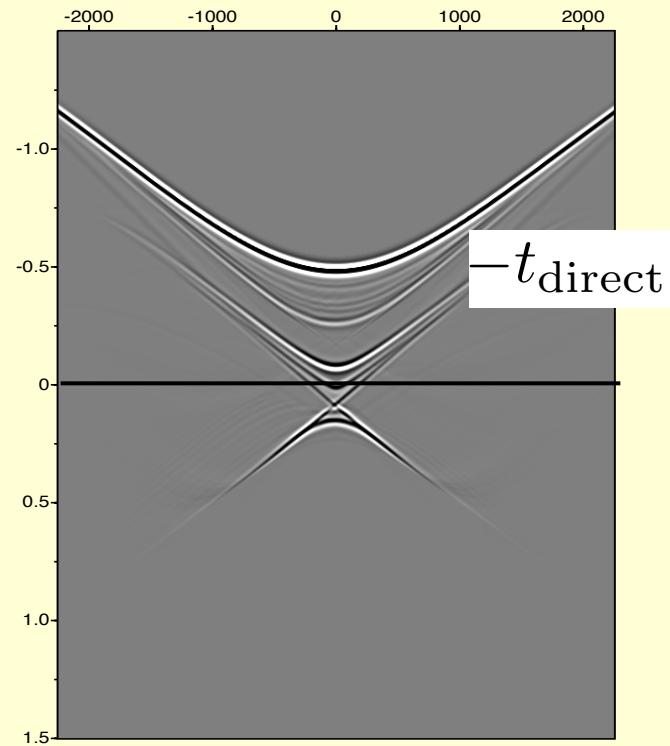
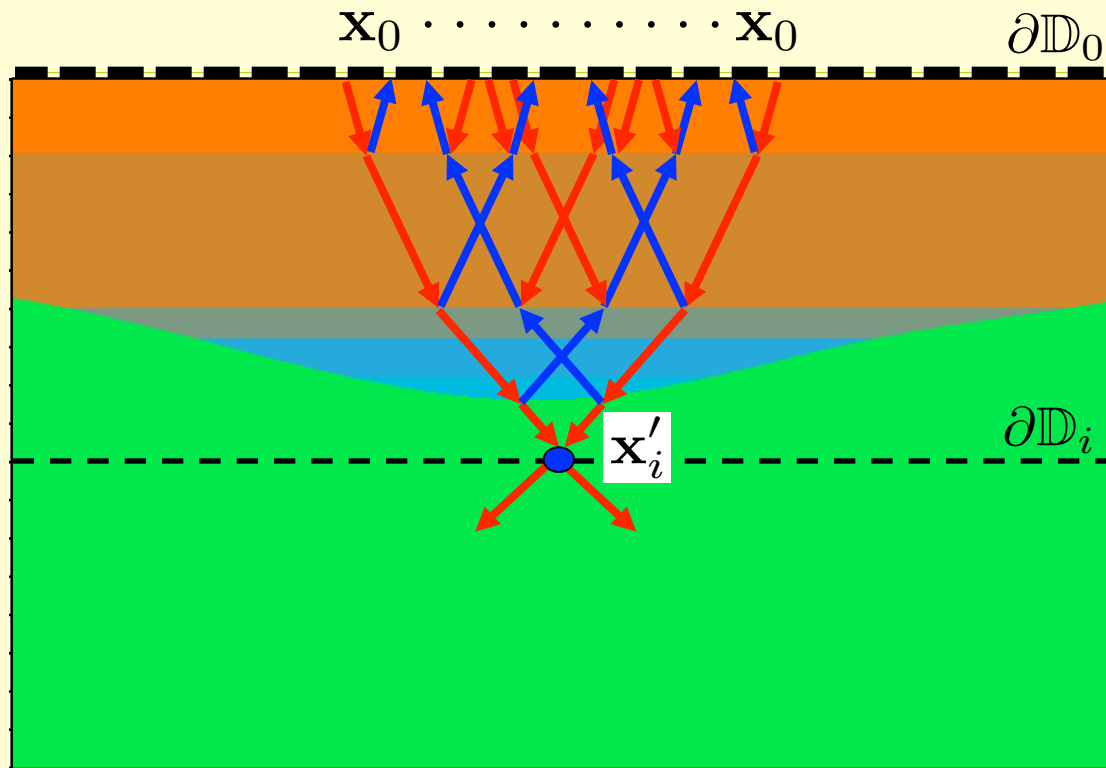
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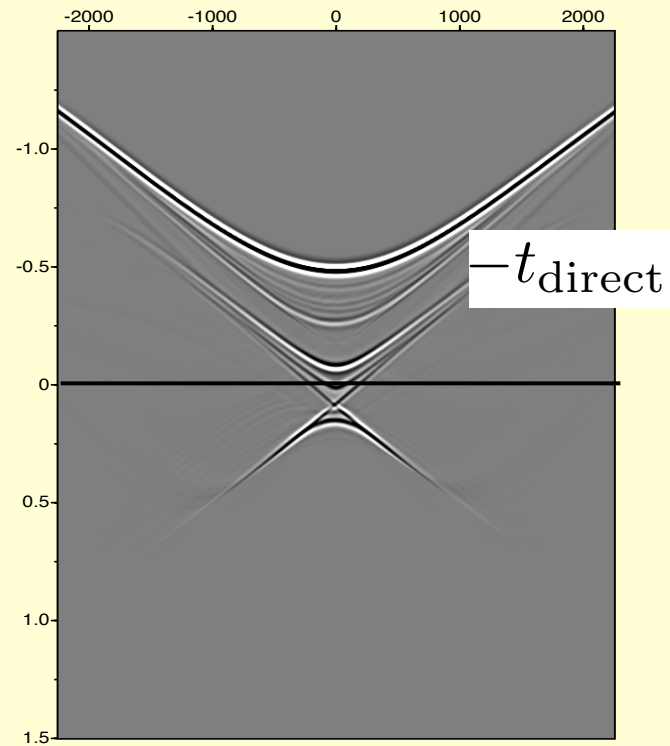
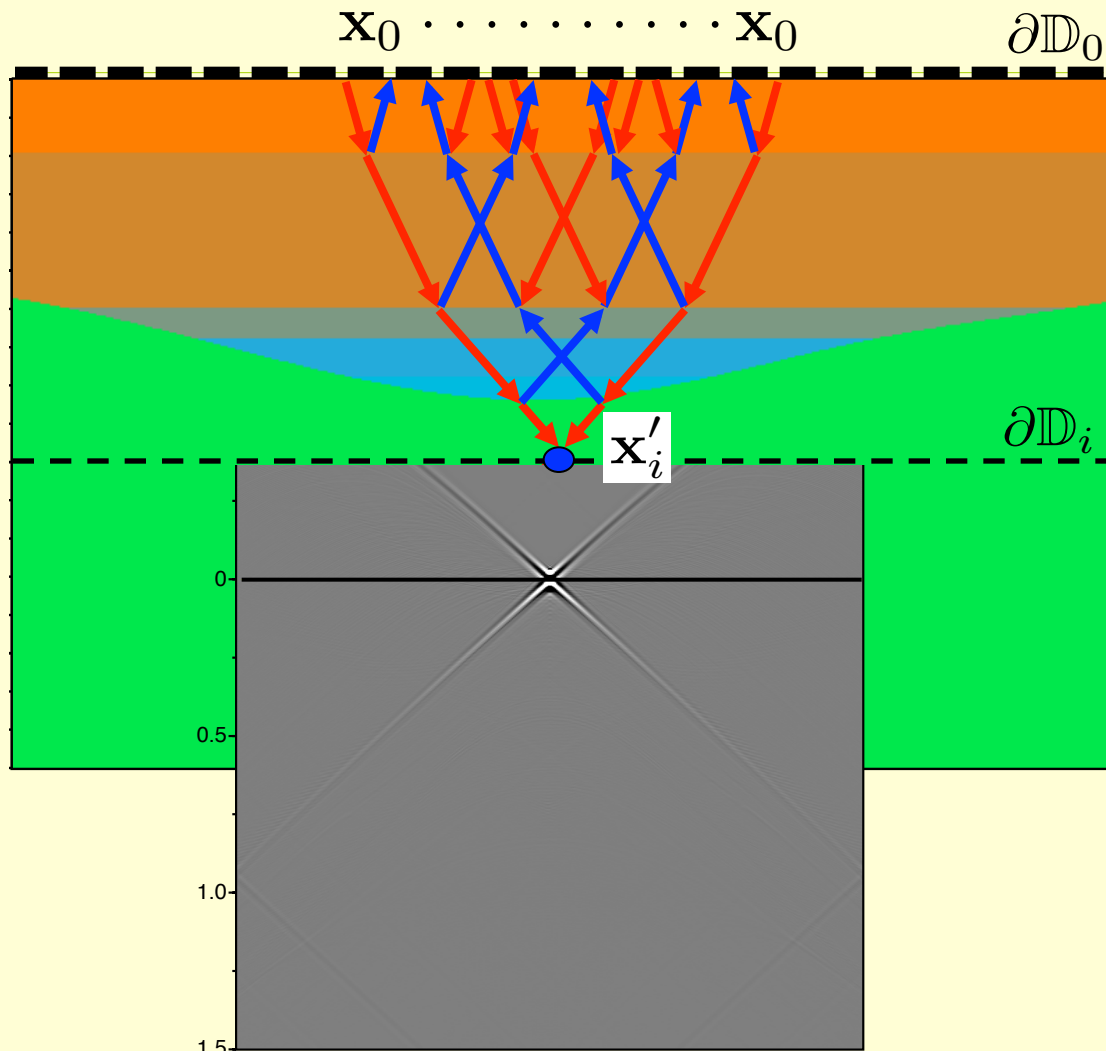
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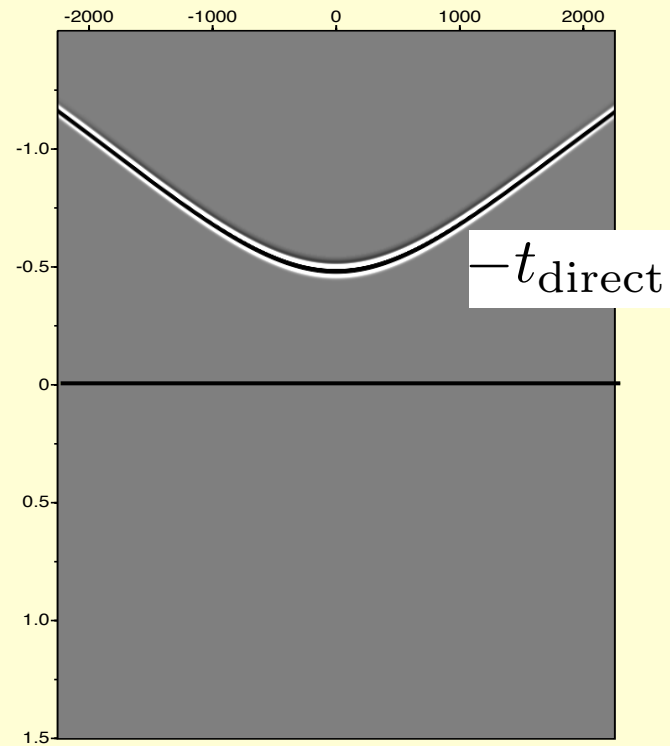
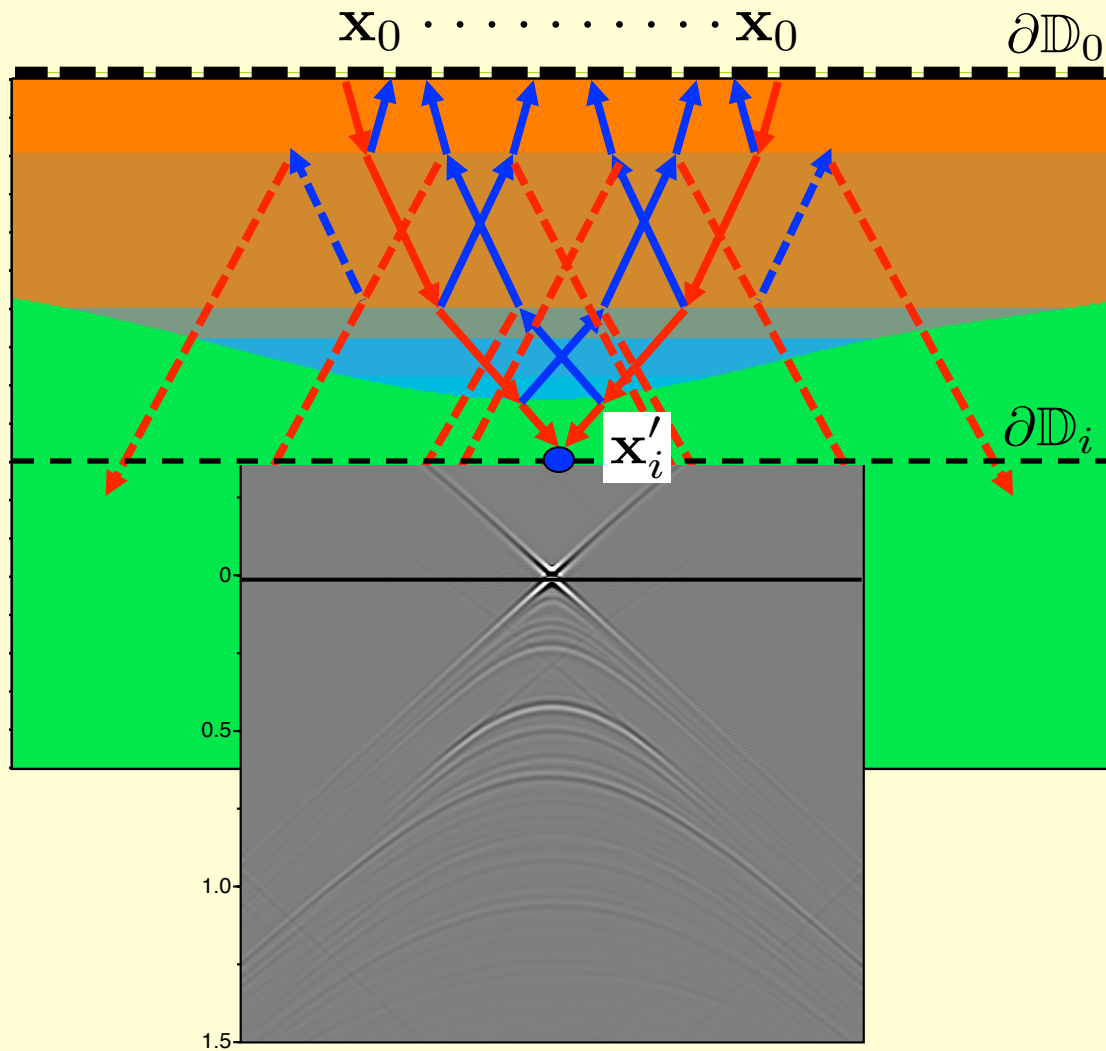
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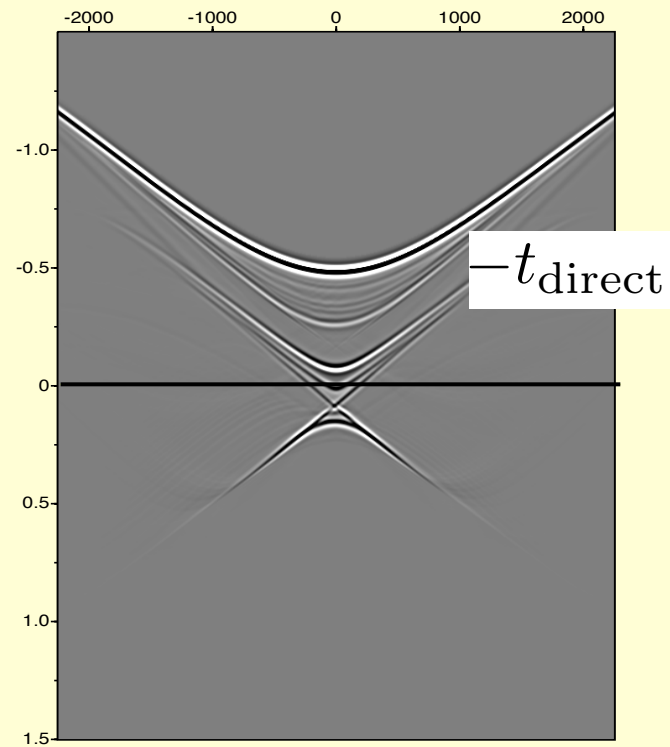
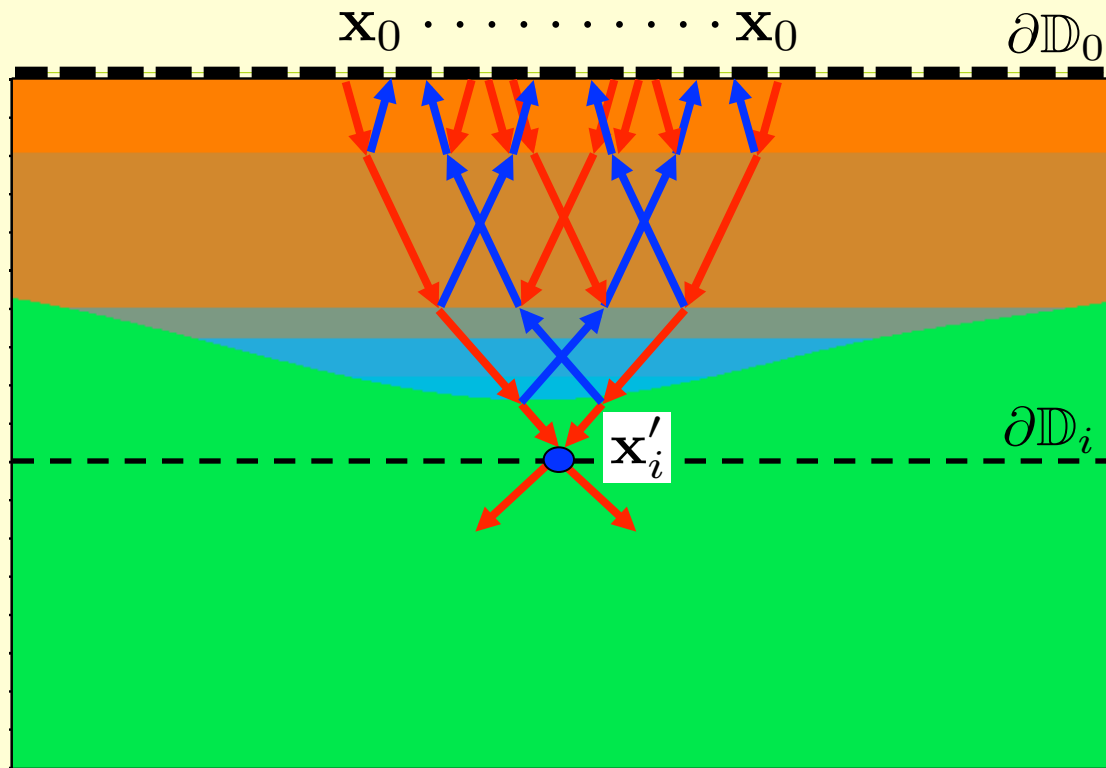
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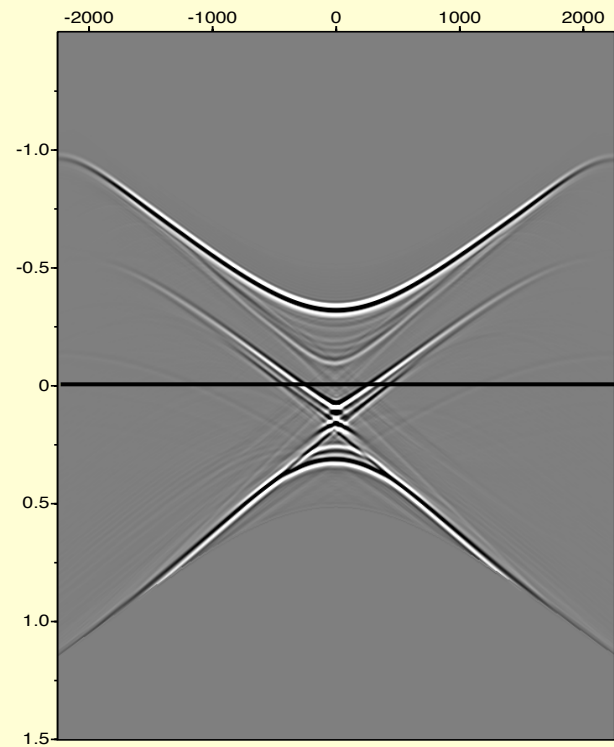
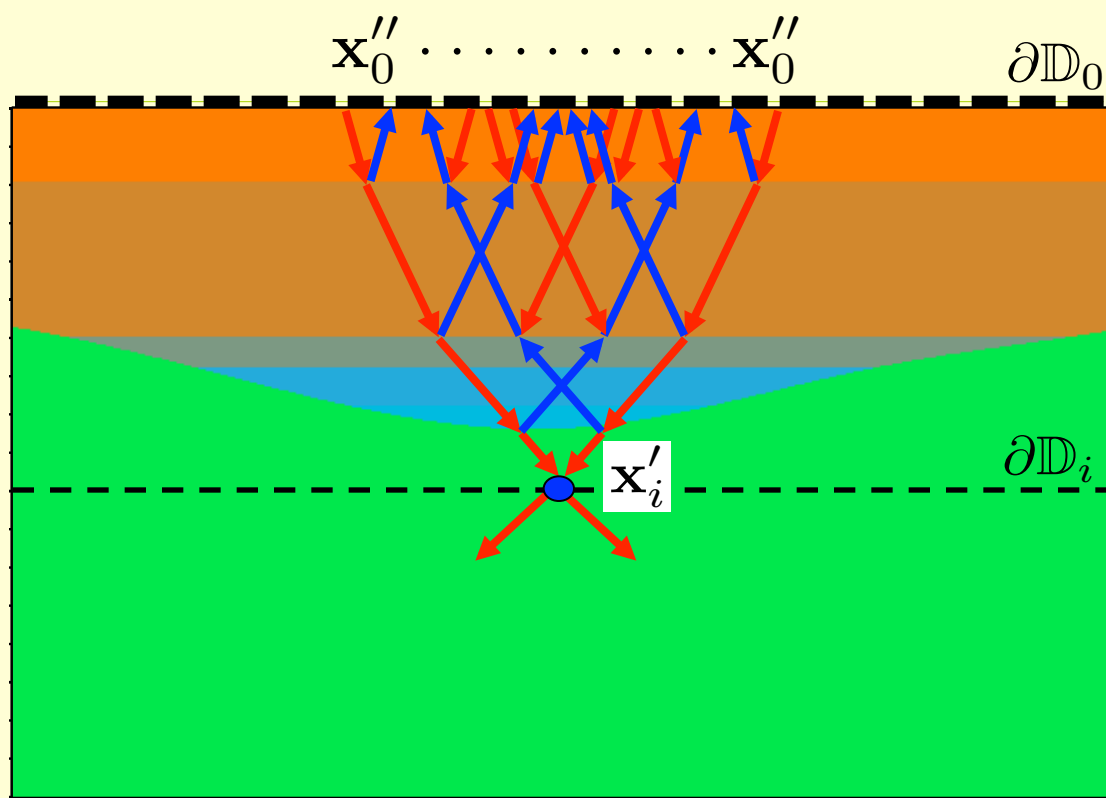
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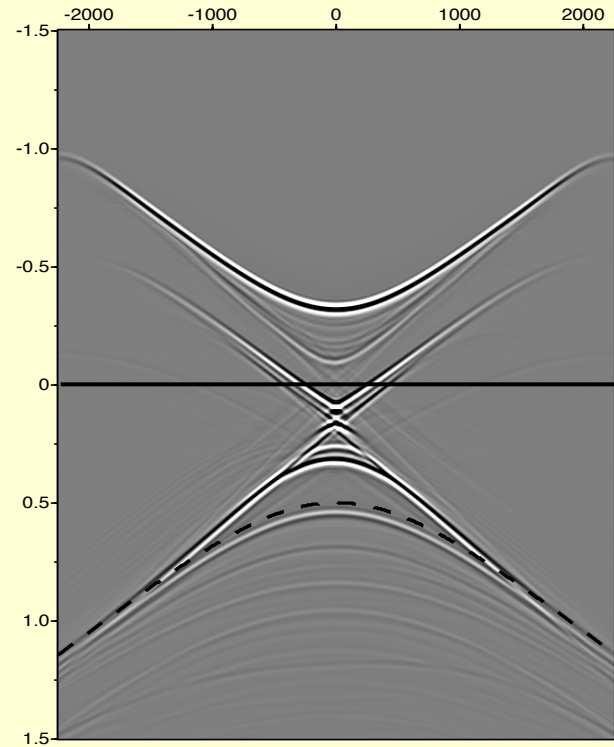
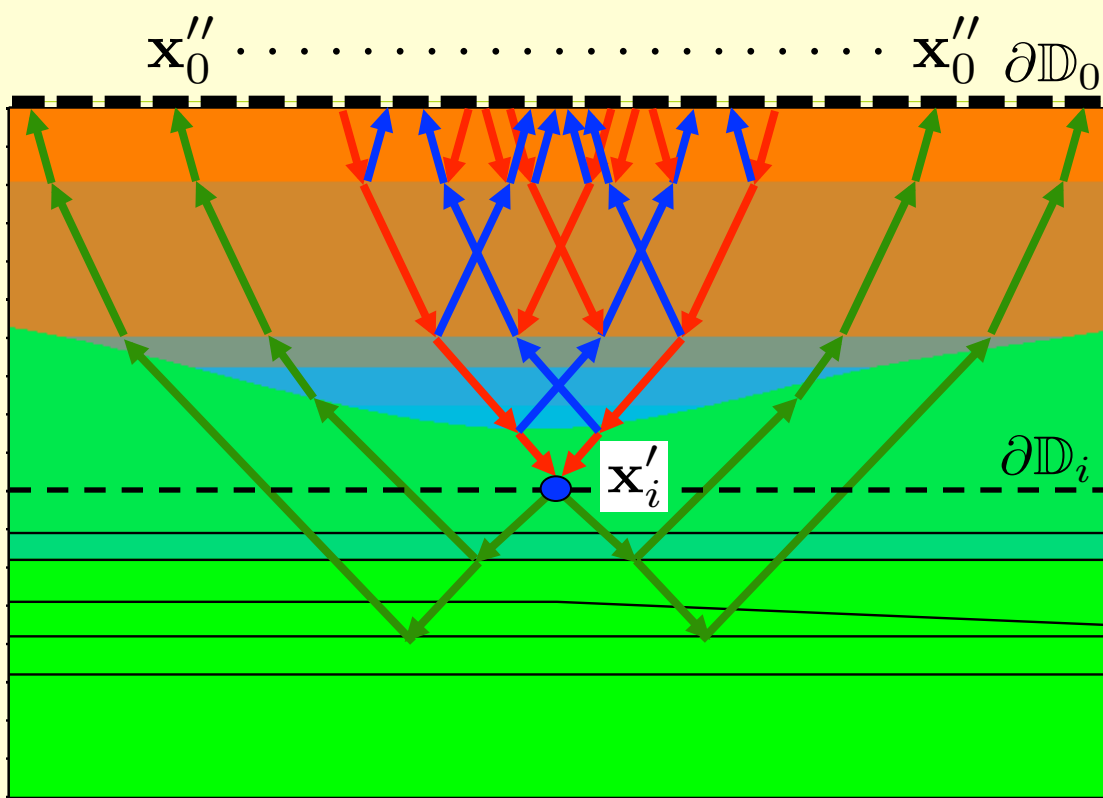
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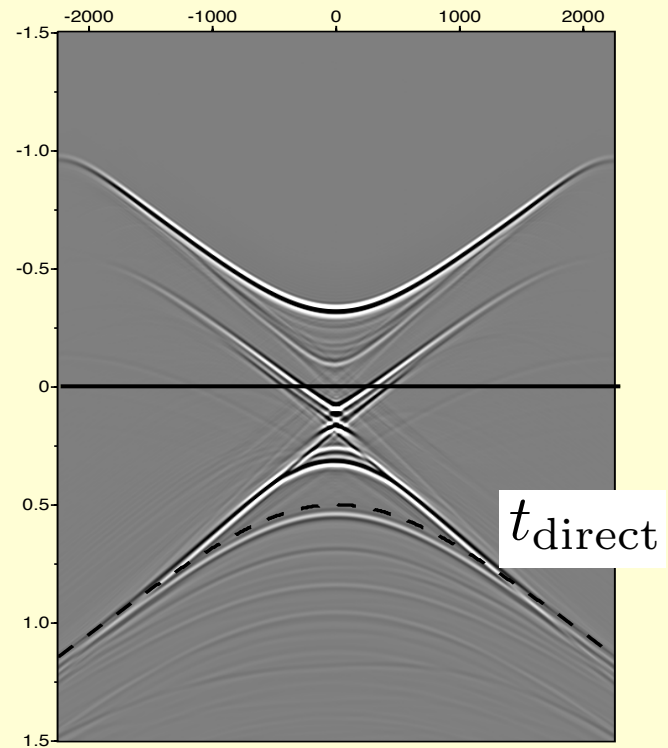
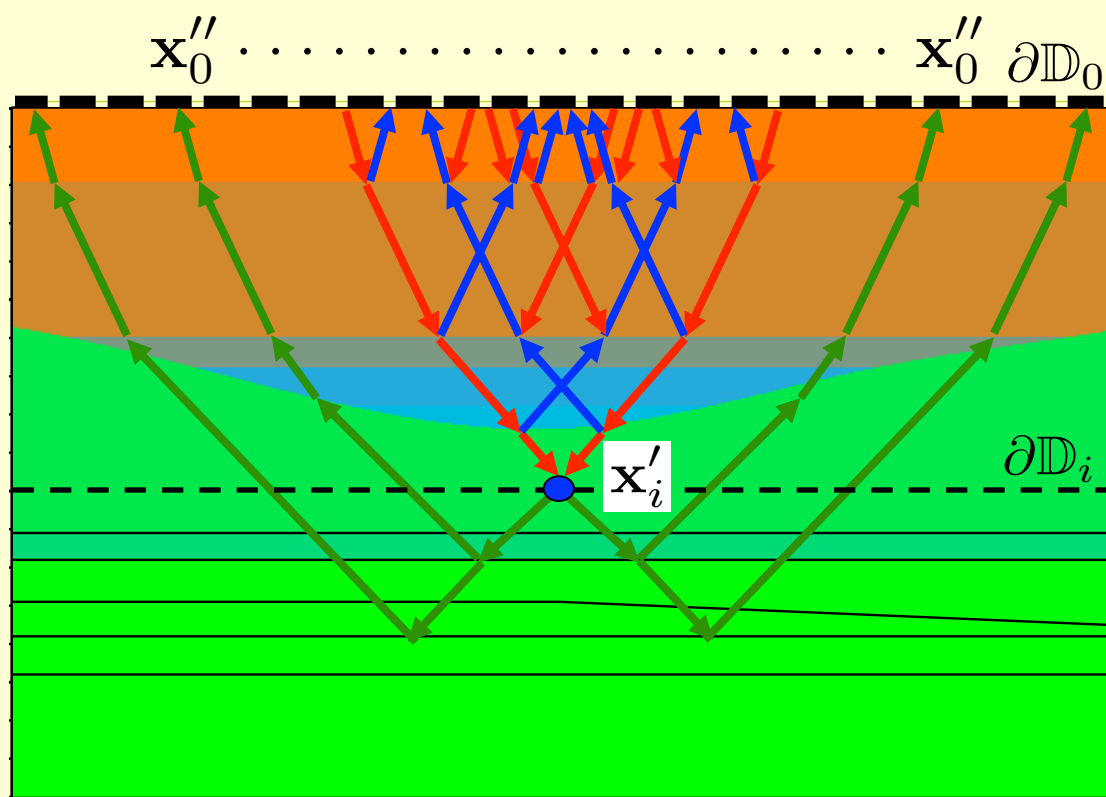
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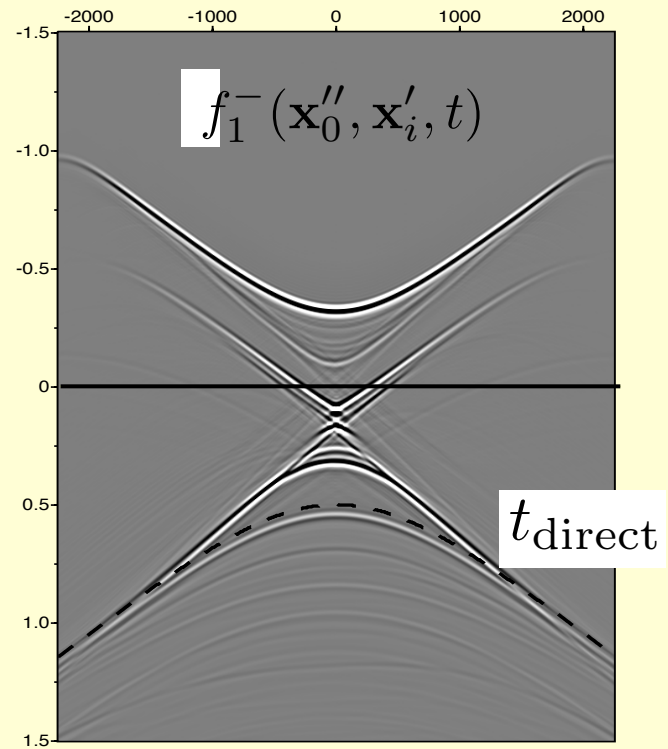
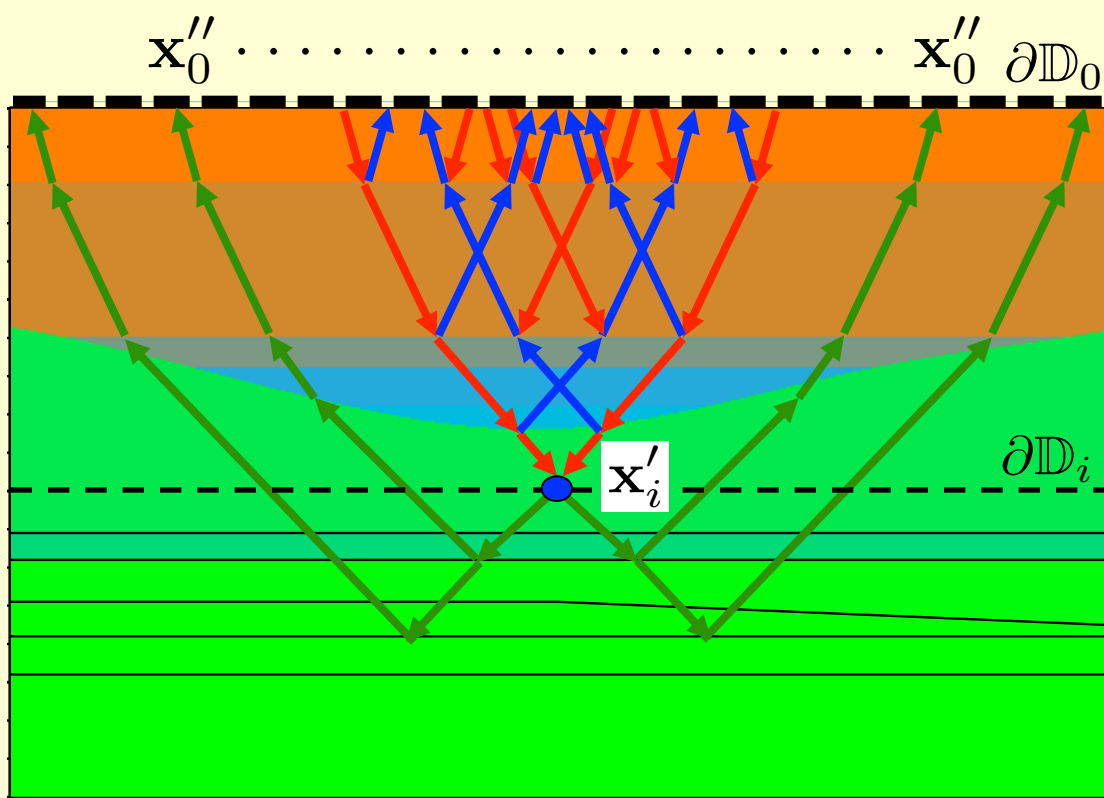
$$f_1^-(\mathbf{x}''_0, \mathbf{x}'_i, t) = \int_{\partial\mathbb{D}_0} d\mathbf{x}_0 \int_{-\infty}^t R(\mathbf{x}''_0, \mathbf{x}_0, t - t') f_1^+(\mathbf{x}_0, \mathbf{x}'_i, t') dt'$$



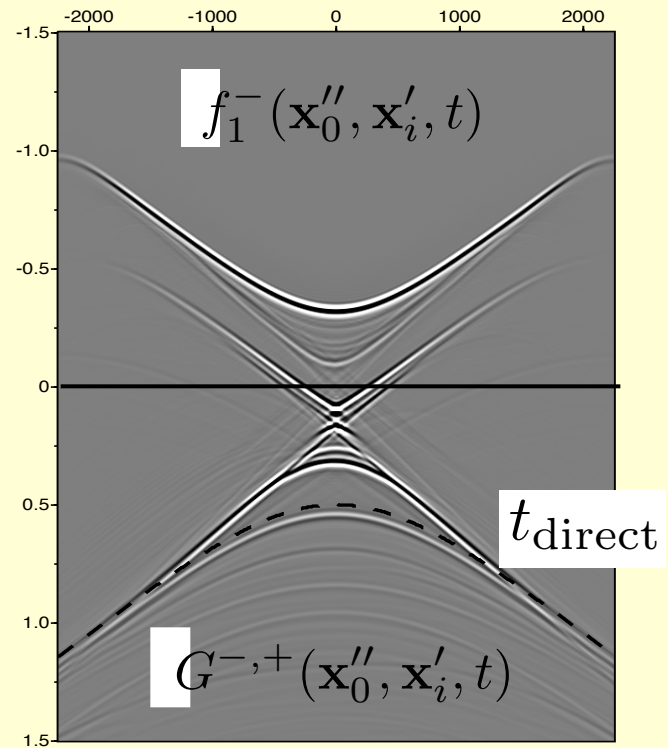
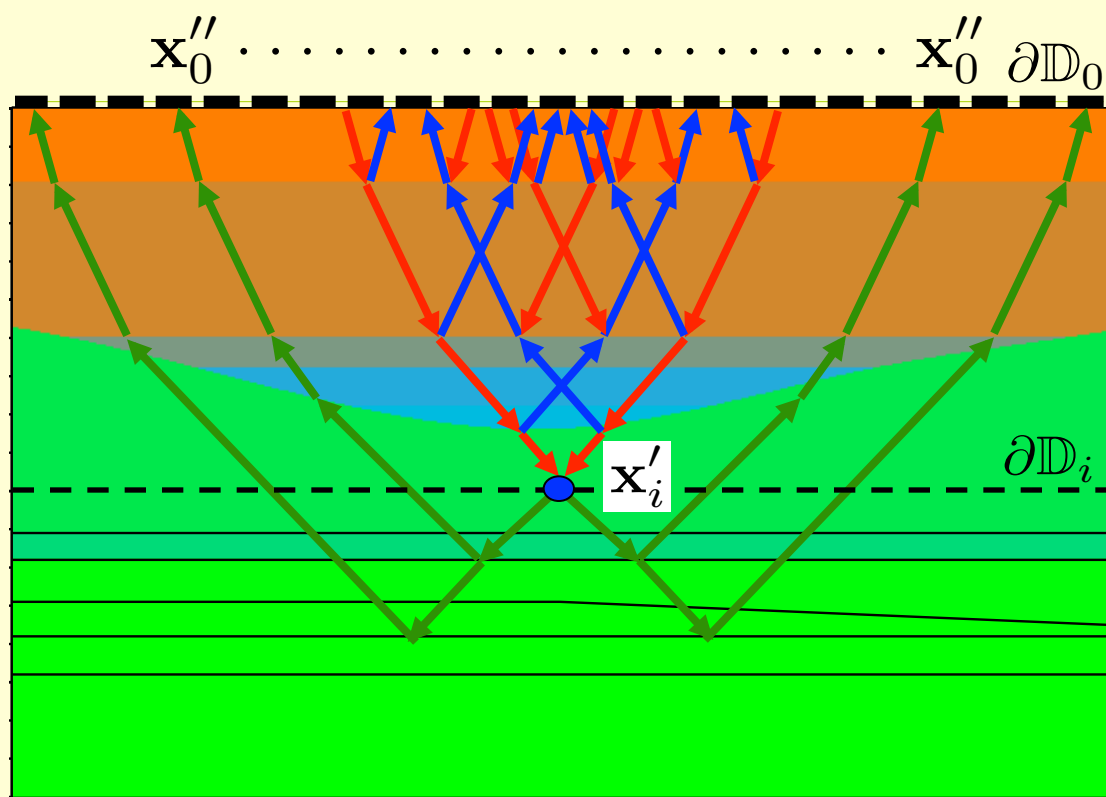
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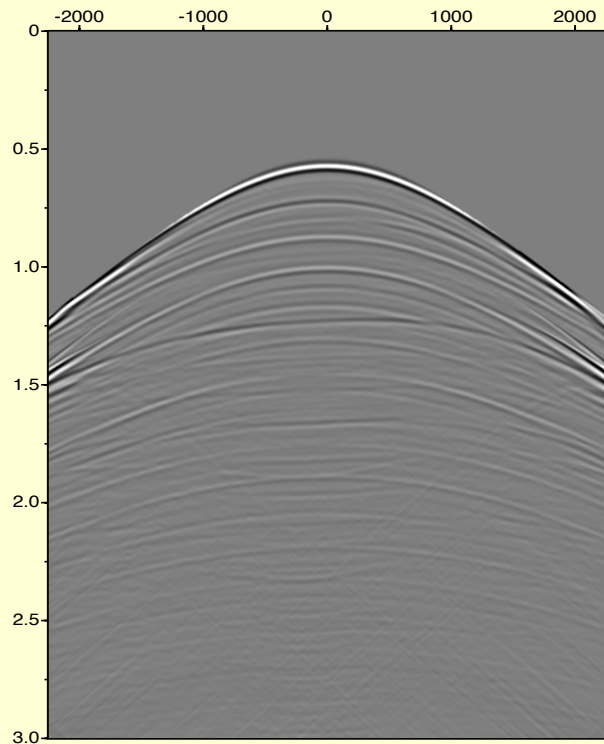
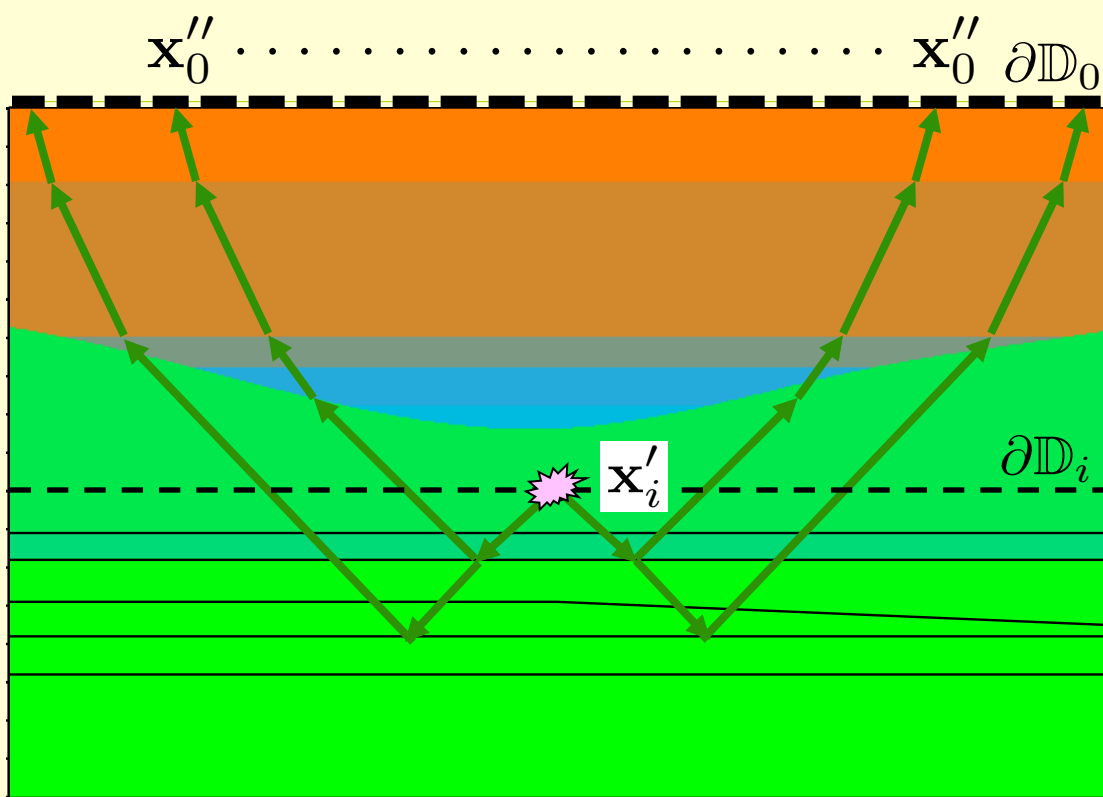
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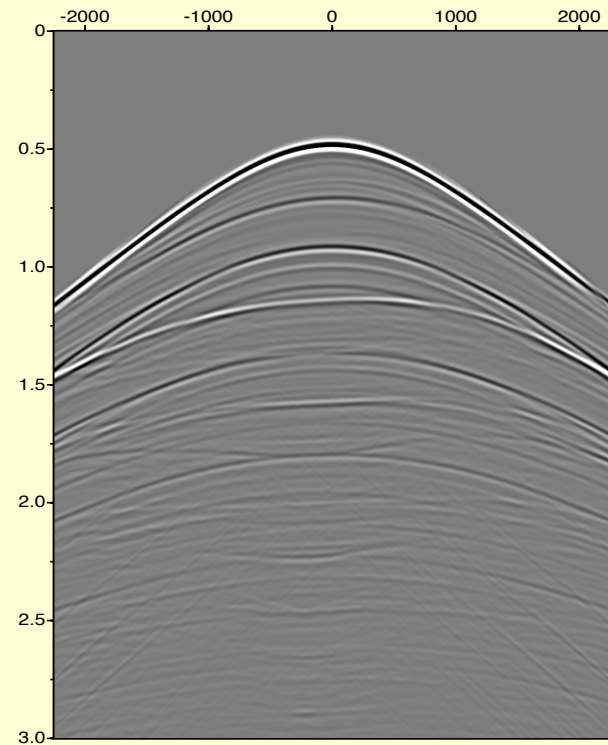
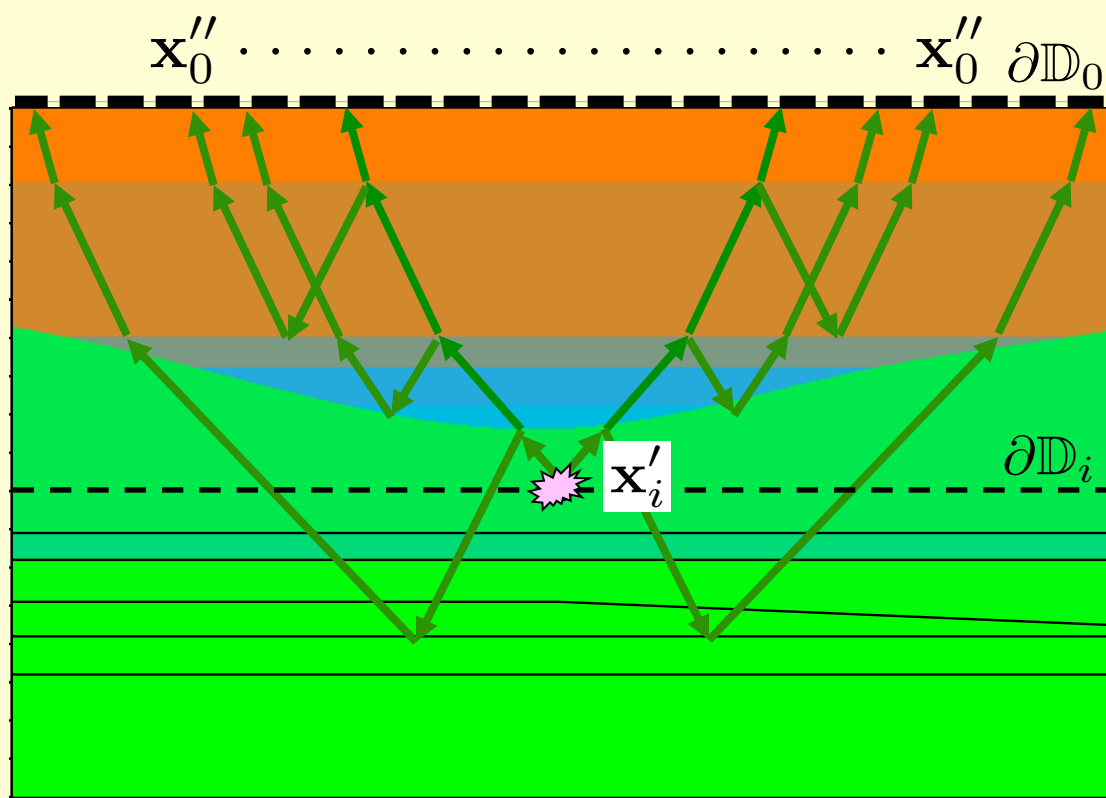
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$$G^{-,+}(\mathbf{x}_0'', \mathbf{x}_i', t) + f_1^-(\mathbf{x}_0'', \mathbf{x}_i', t) = \int_{\partial\mathbb{D}_0} d\mathbf{x}_0 \int_{-\infty}^t R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^+(\mathbf{x}_0, \mathbf{x}_i', t') dt'$$

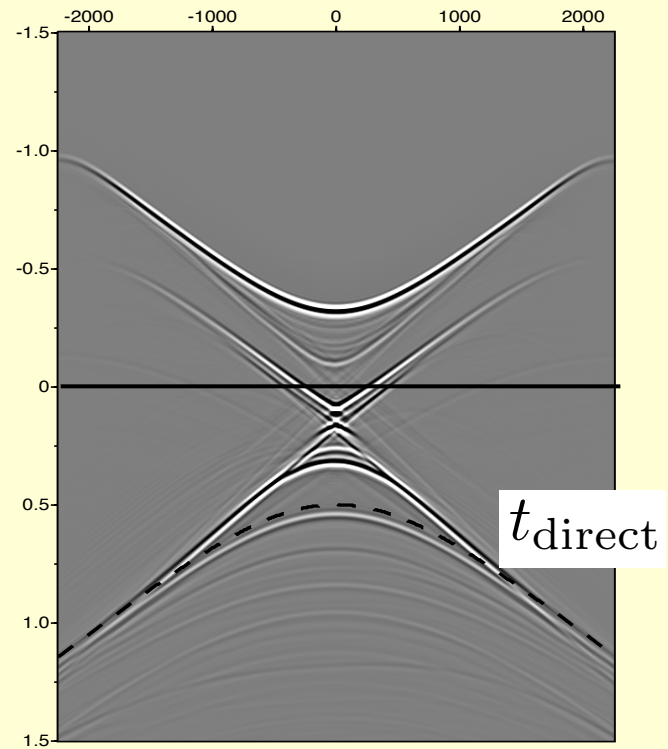
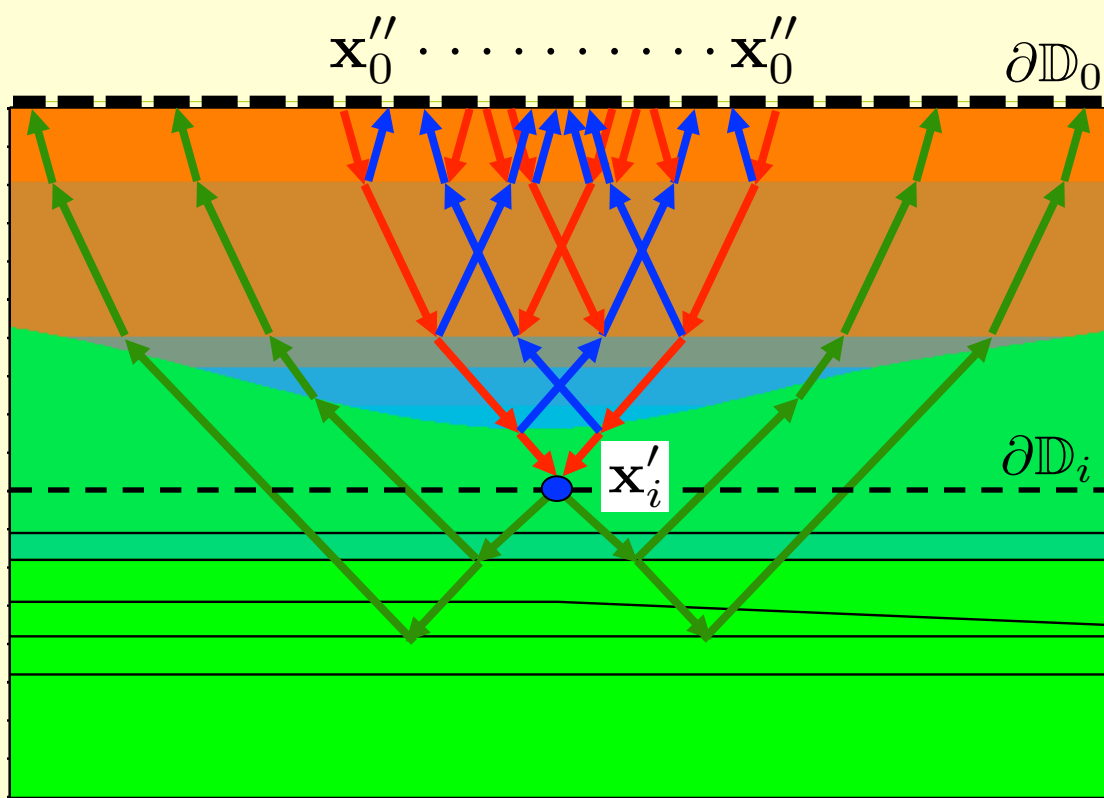


$$G^{-,+}(\mathbf{x}''_0, \mathbf{x}'_i, t) + f_1^-(\mathbf{x}''_0, \mathbf{x}'_i, t) = \int_{\partial \mathbb{D}_0} d\mathbf{x}_0 \int_{-\infty}^t R(\mathbf{x}''_0, \mathbf{x}_0, t - t') f_1^+(\mathbf{x}_0, \mathbf{x}'_i, t') dt'$$

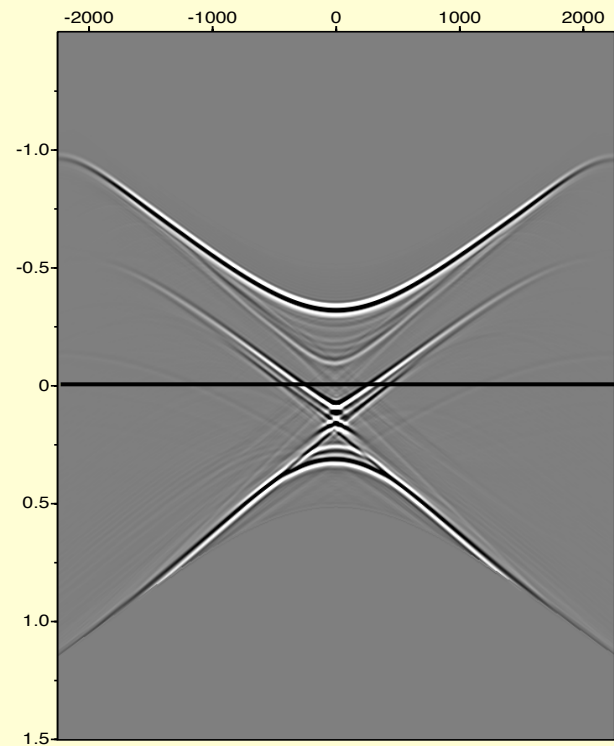
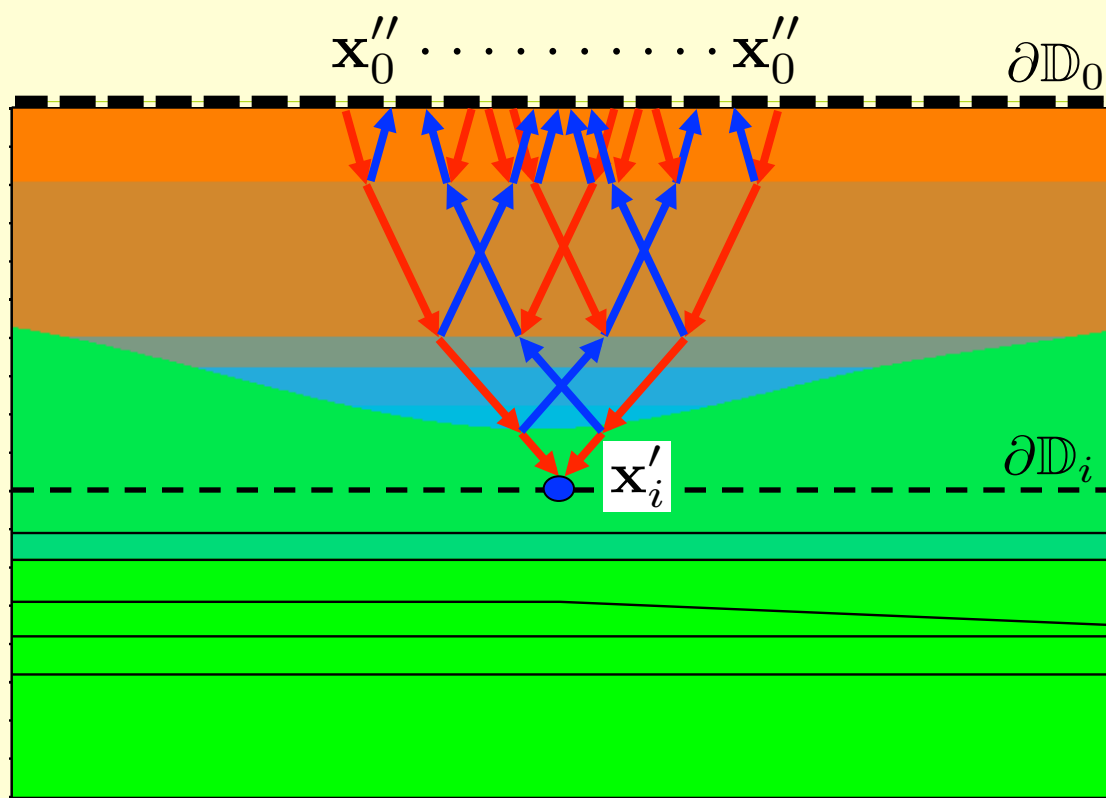


$$\textcircled{G^{-,-}(\mathbf{x}_0'', \mathbf{x}'_i, t)} + f_1^+(\mathbf{x}_0'', \mathbf{x}'_i, -t) = \int_{\partial\mathbb{D}_0} d\mathbf{x}_0 \int_{-\infty}^t R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^-(\mathbf{x}_0, \mathbf{x}'_i, -t') dt'$$

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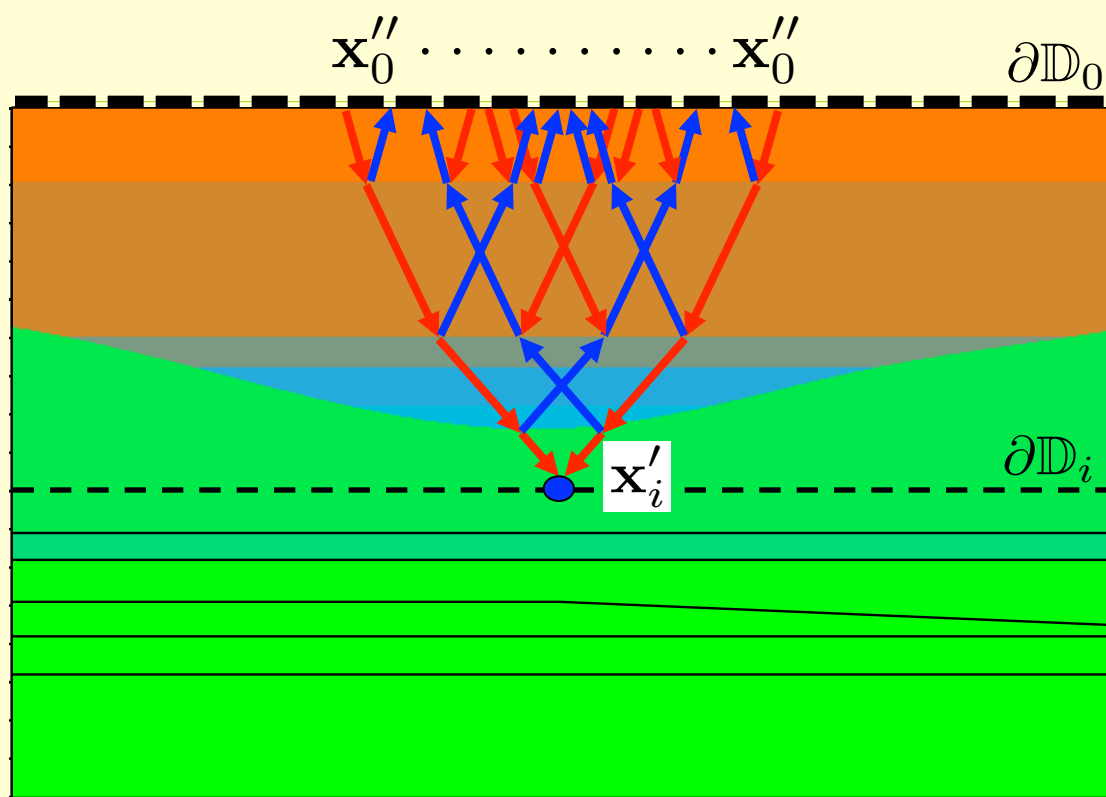


$$G^{-,+}(\mathbf{x}''_0, \mathbf{x}'_i, t) + f_1^{-}(\mathbf{x}''_0, \mathbf{x}'_i, t) = \int_{\partial\mathbb{D}_0} d\mathbf{x}_0 \int_{-\infty}^t R(\mathbf{x}''_0, \mathbf{x}_0, t - t') f_1^{+}(\mathbf{x}_0, \mathbf{x}'_i, t') dt'$$



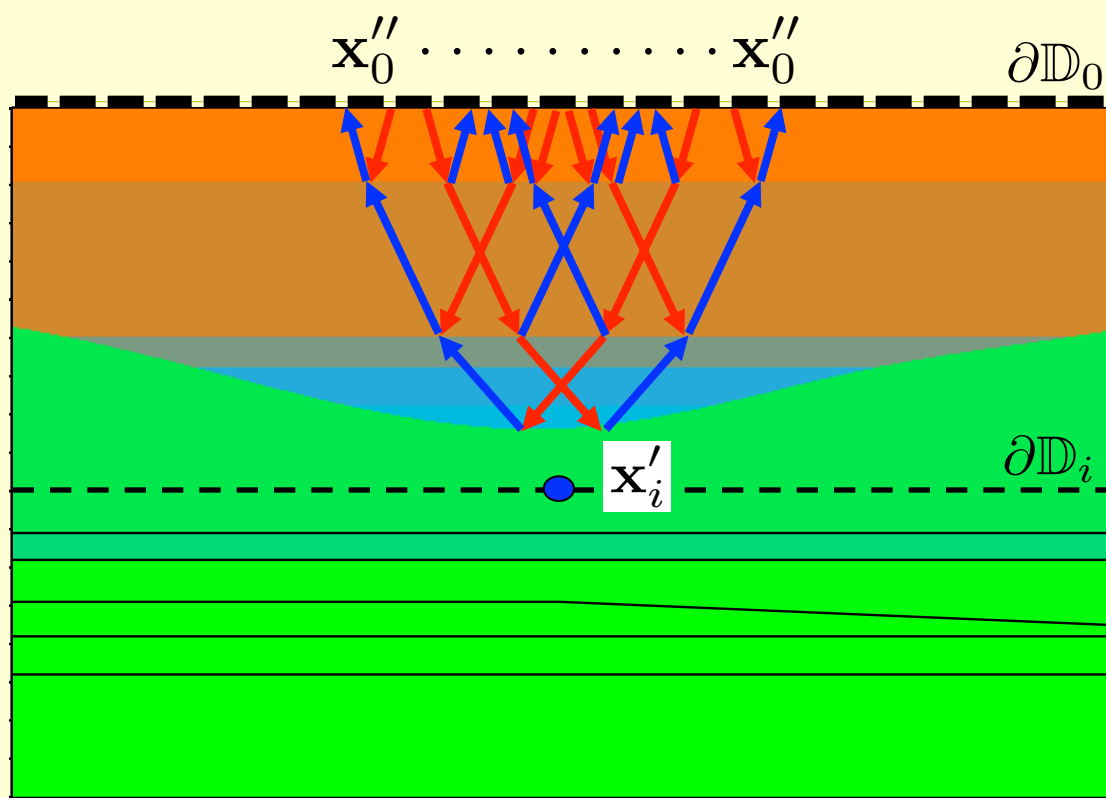
$t < t_{\text{direct}} :$

$$f_1^-(\mathbf{x}''_0, \mathbf{x}'_i, t) = \int_{\partial\mathbb{D}_0} d\mathbf{x}_0 \int_{-\infty}^t R(\mathbf{x}''_0, \mathbf{x}_0, t - t') f_1^+(\mathbf{x}_0, \mathbf{x}'_i, t') dt'$$



Measured reflection data

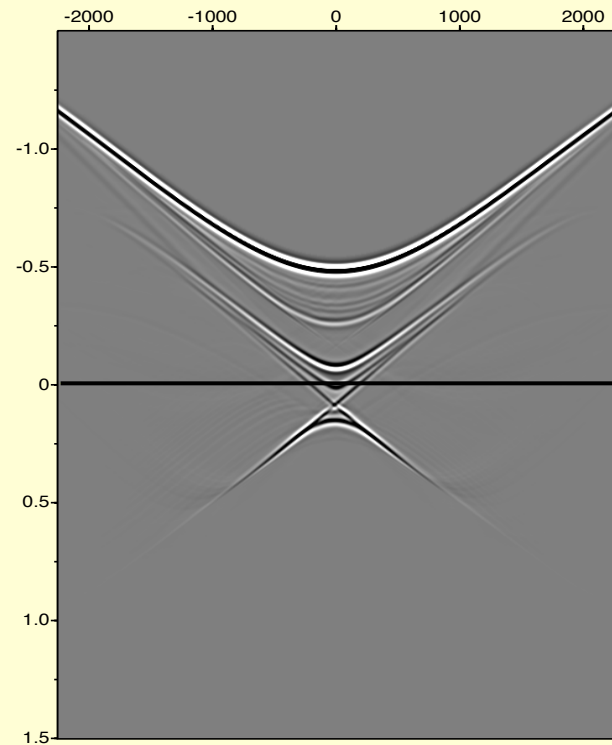
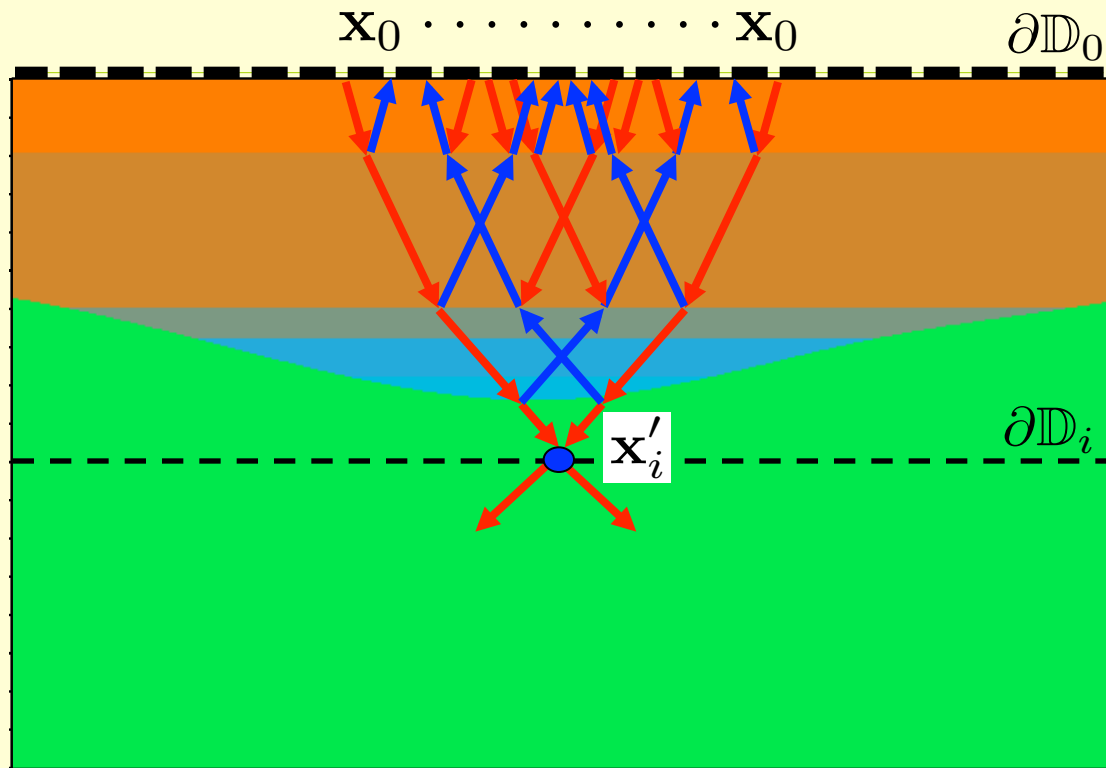
$$t < t_{\text{direct}} : \quad f_1^-(\mathbf{x}_0'', \mathbf{x}_i', t) = \int_{\partial D_0} d\mathbf{x}_0 \int_{-\infty}^t R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^+(\mathbf{x}_0, \mathbf{x}_i', t') dt'$$



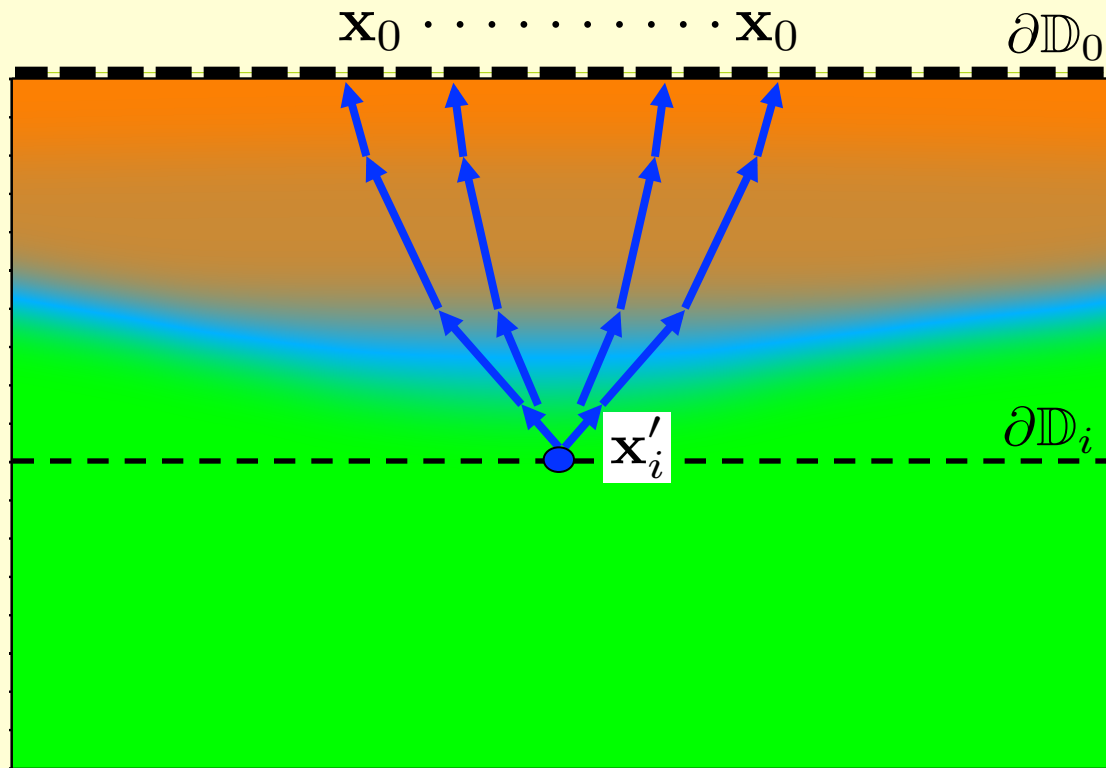
Measured reflection data

$$t < t_{\text{direct}} : \quad f_1^+(\mathbf{x}_0'', \mathbf{x}_i', -t) = \int_{\partial\mathbb{D}_0} d\mathbf{x}_0 \int_{-\infty}^t R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^-(\mathbf{x}_0, \mathbf{x}_i', -t') dt'$$

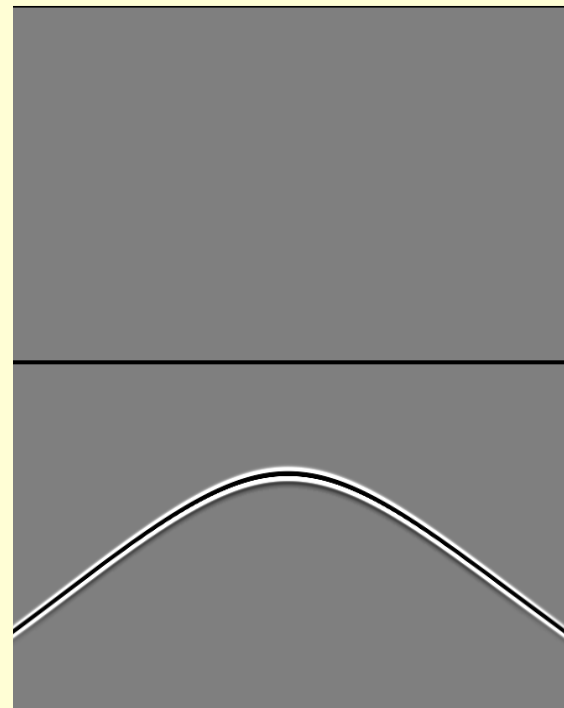
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- Issues for discussion



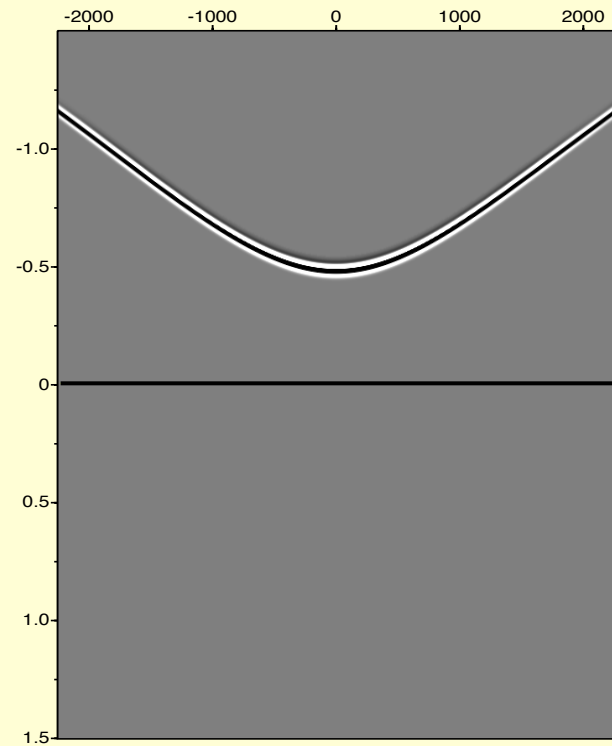
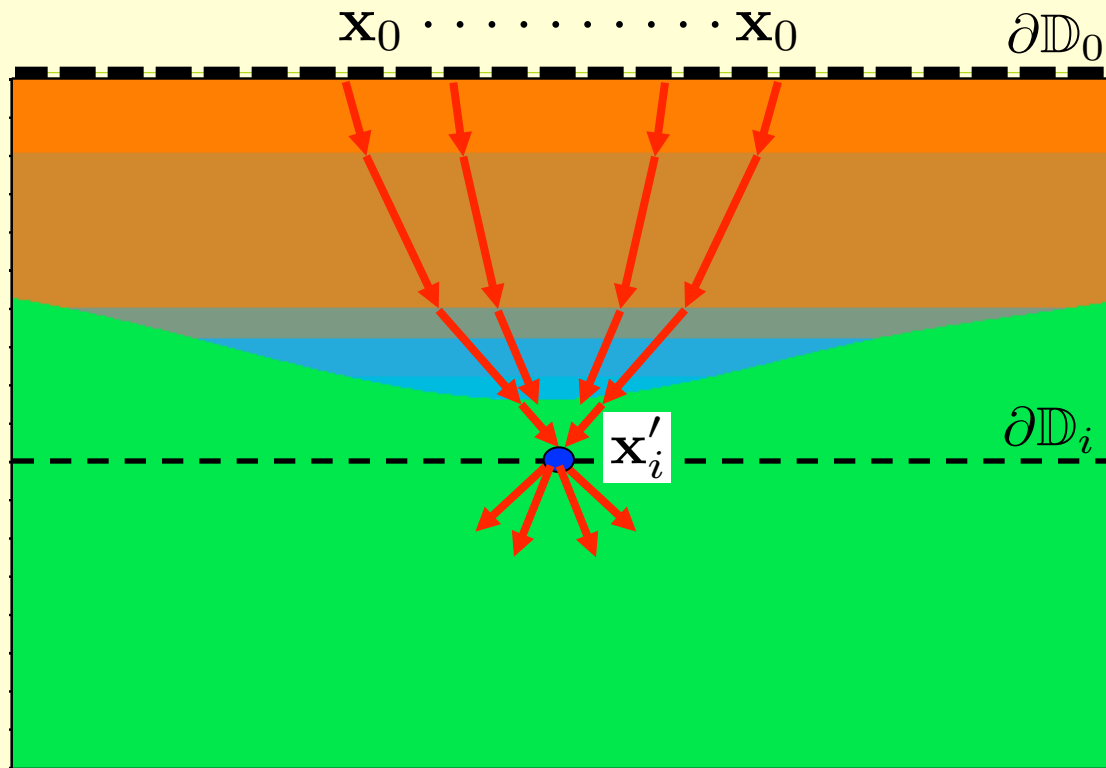
Ansatz: $f_1^+(\mathbf{x}_0, \mathbf{x}'_i, t) \approx \underbrace{G_d(\mathbf{x}_0, \mathbf{x}'_i, -t)}_{\text{Direct}} + \underbrace{M^+(\mathbf{x}_0, \mathbf{x}'_i, t)}_{\text{Coda}}$



Smooth model

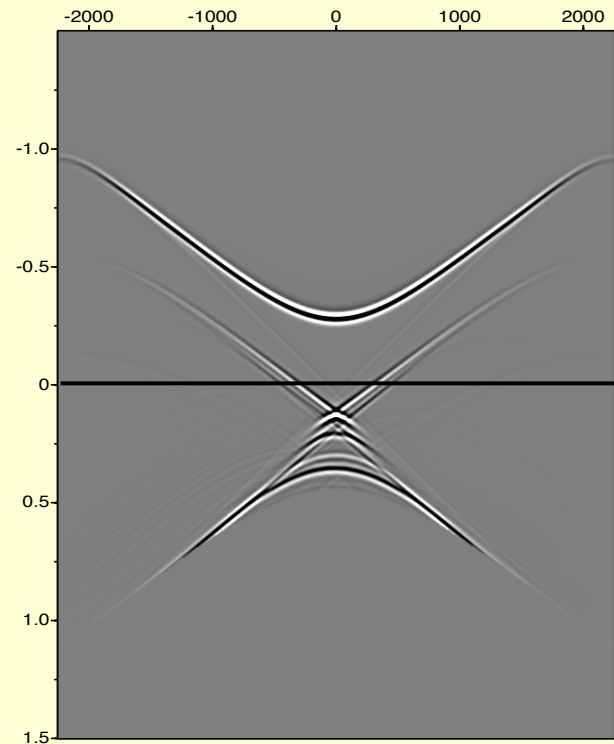
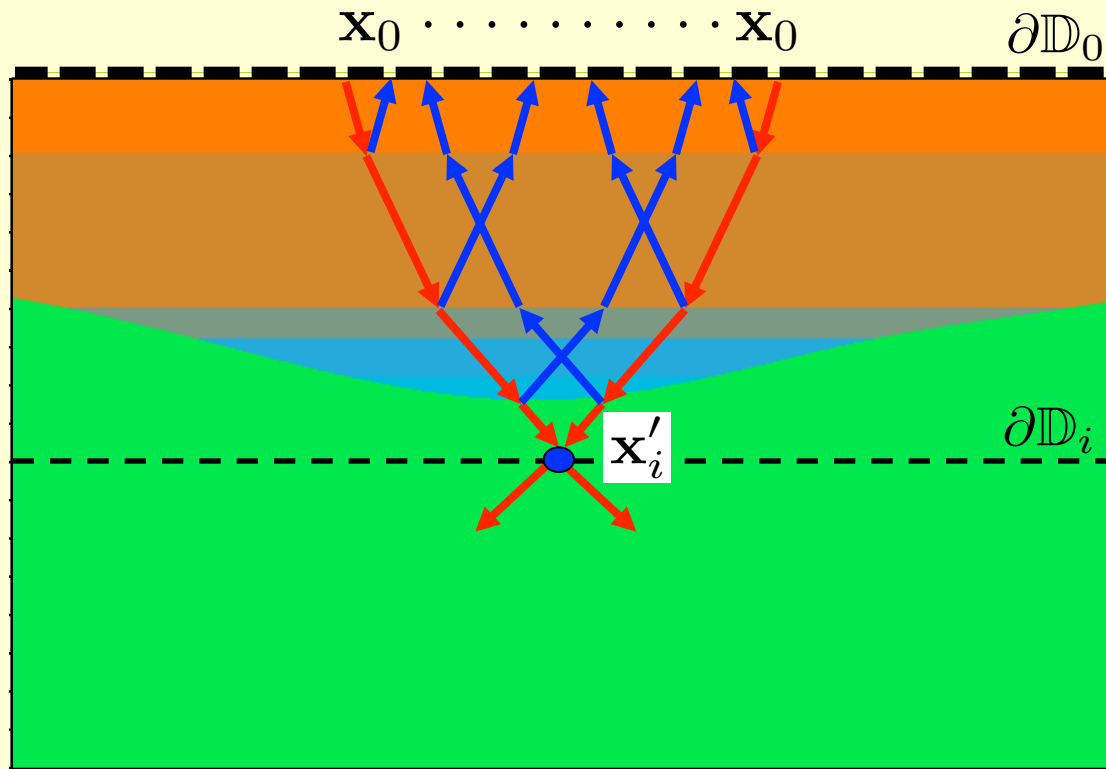


$$G_d(\mathbf{x}_0, \mathbf{x}'_i, t)$$



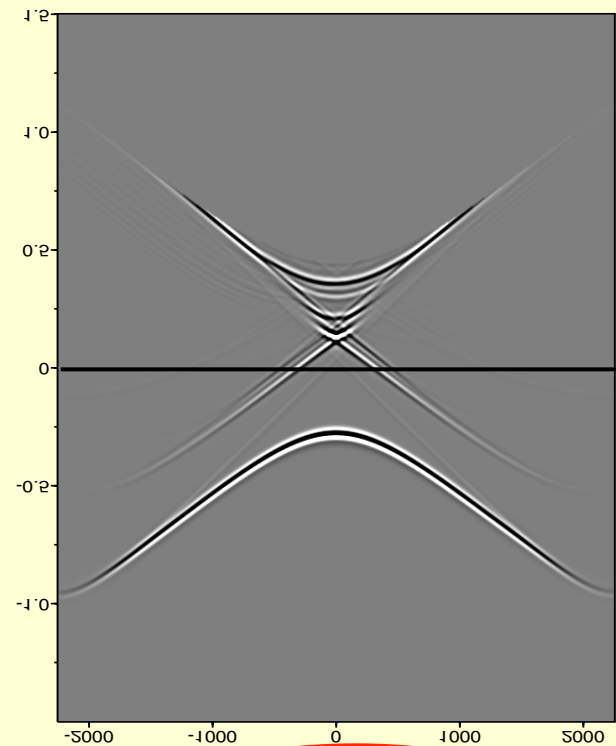
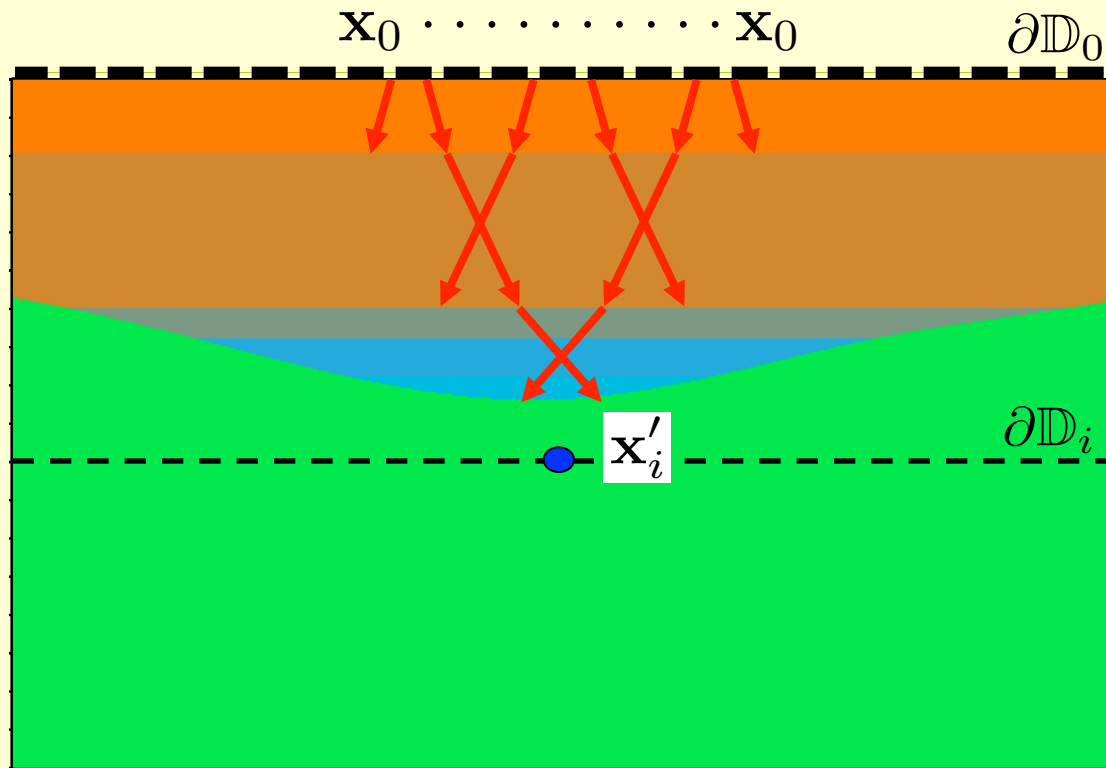
$$f_{1,0}^+(\mathbf{x}_0, \mathbf{x}'_i, t)$$

$$\approx G_d(\mathbf{x}_0, \mathbf{x}'_i, -t)$$

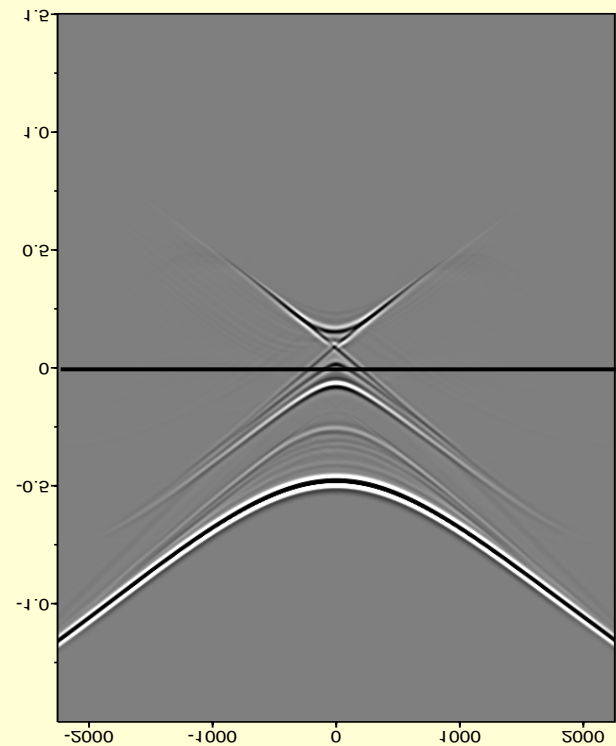
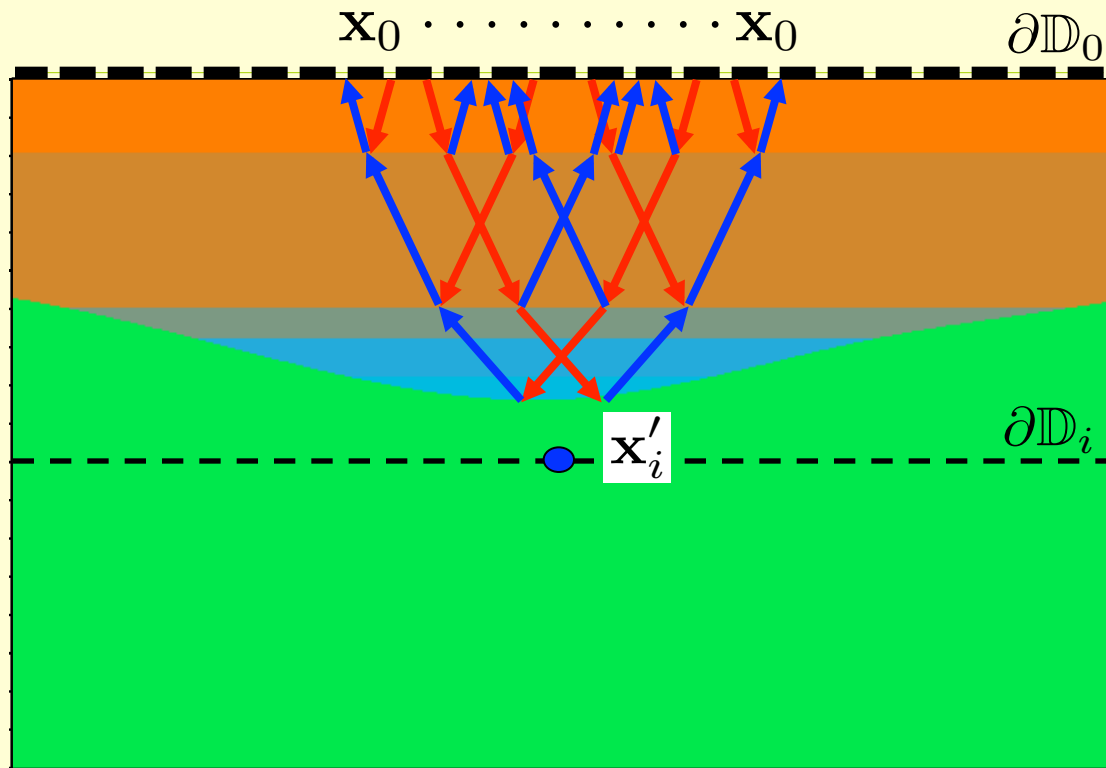


$$t < t_{\text{direct}} : f_{1,0}^-(\mathbf{x}''_0, \mathbf{x}'_i, t) = \int_{\partial D_0} d\mathbf{x}_0 \int_{-\infty}^t R(\mathbf{x}''_0, \mathbf{x}_0, t - t') f_{1,0}^+(\mathbf{x}_0, \mathbf{x}'_i, t') dt'$$

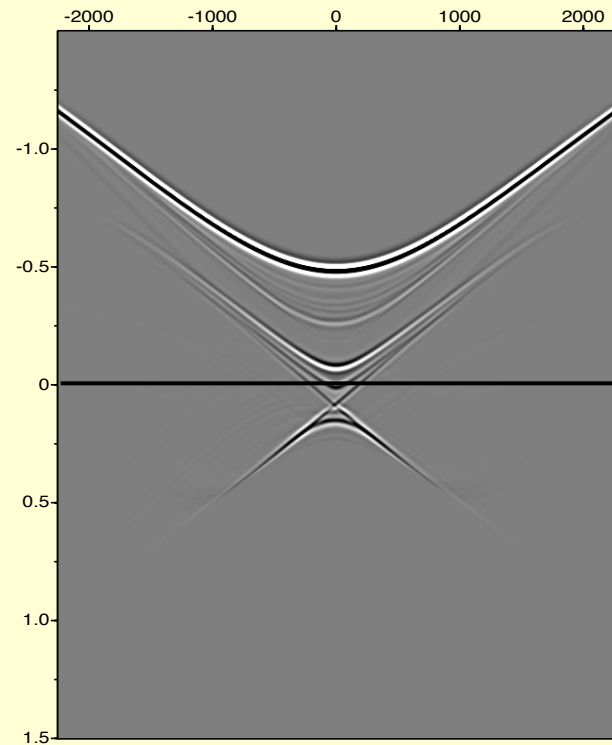
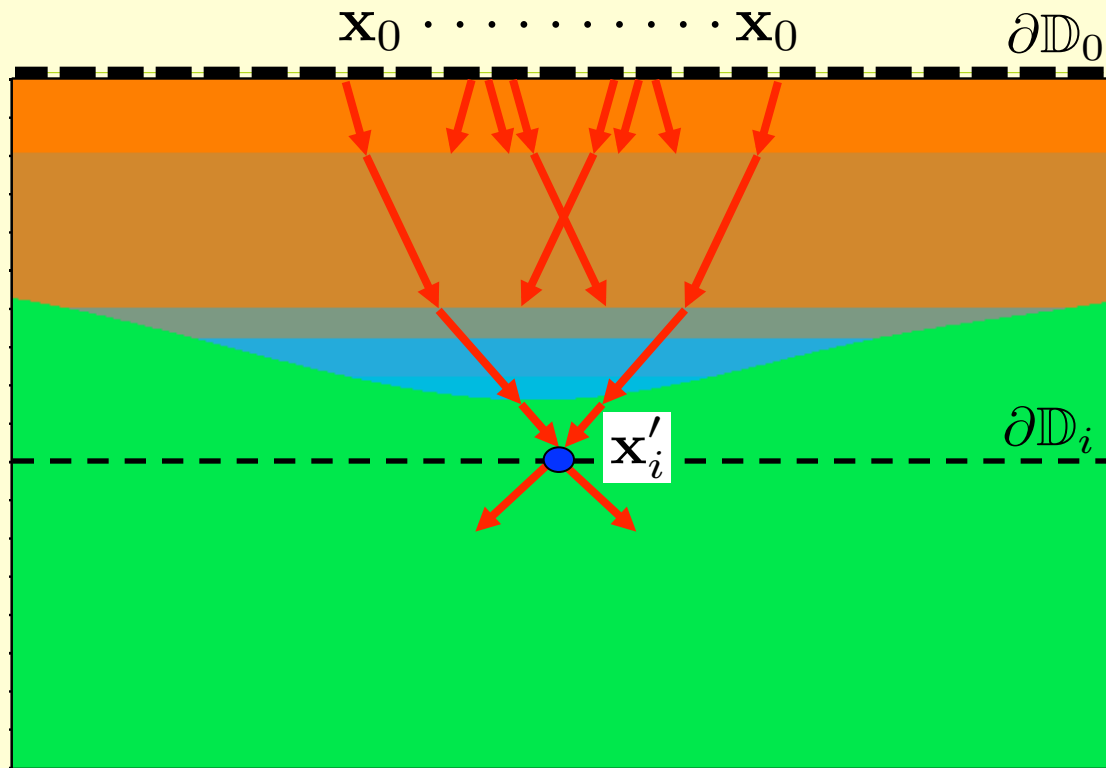
Reflection data



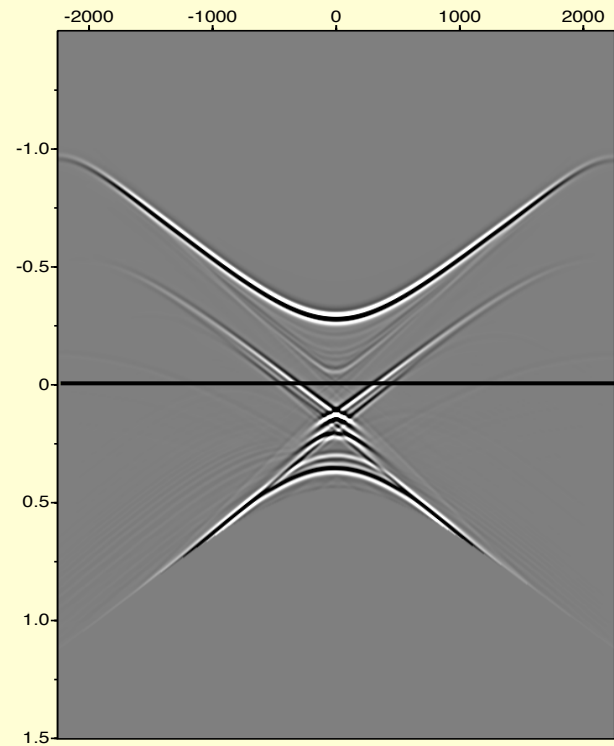
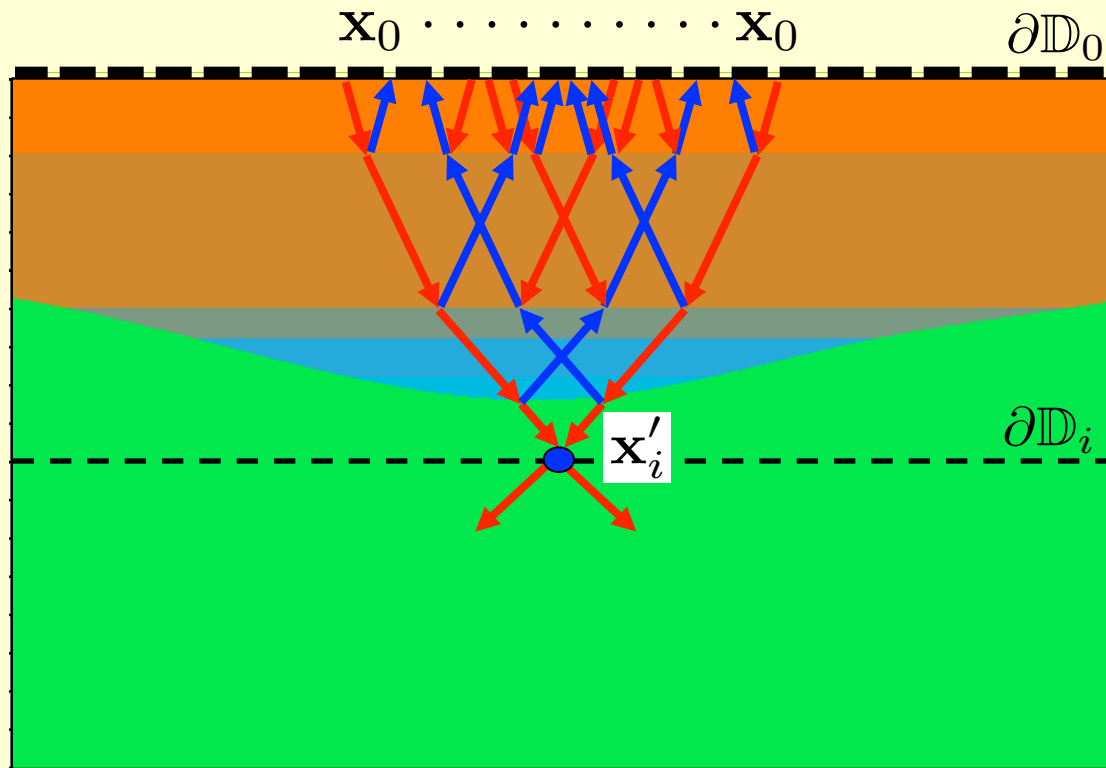
$$f_{1,0}^-(\mathbf{x}_0, \mathbf{x}'_i, -t)$$



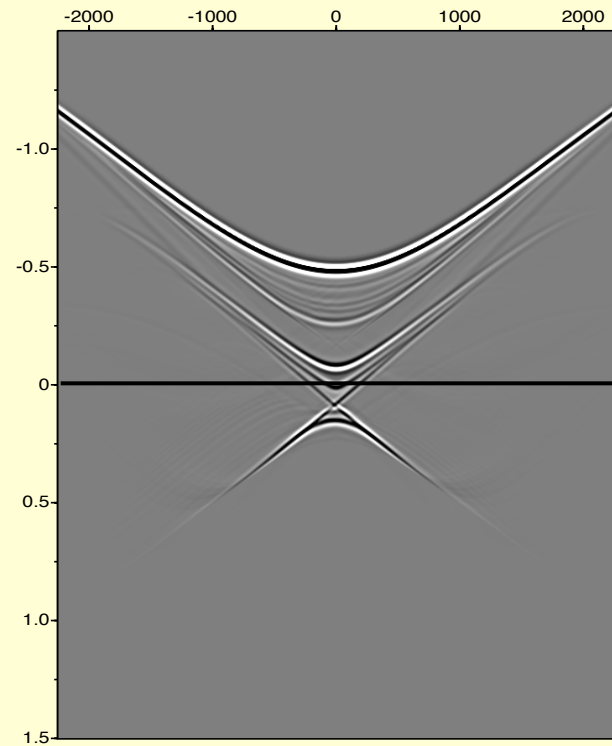
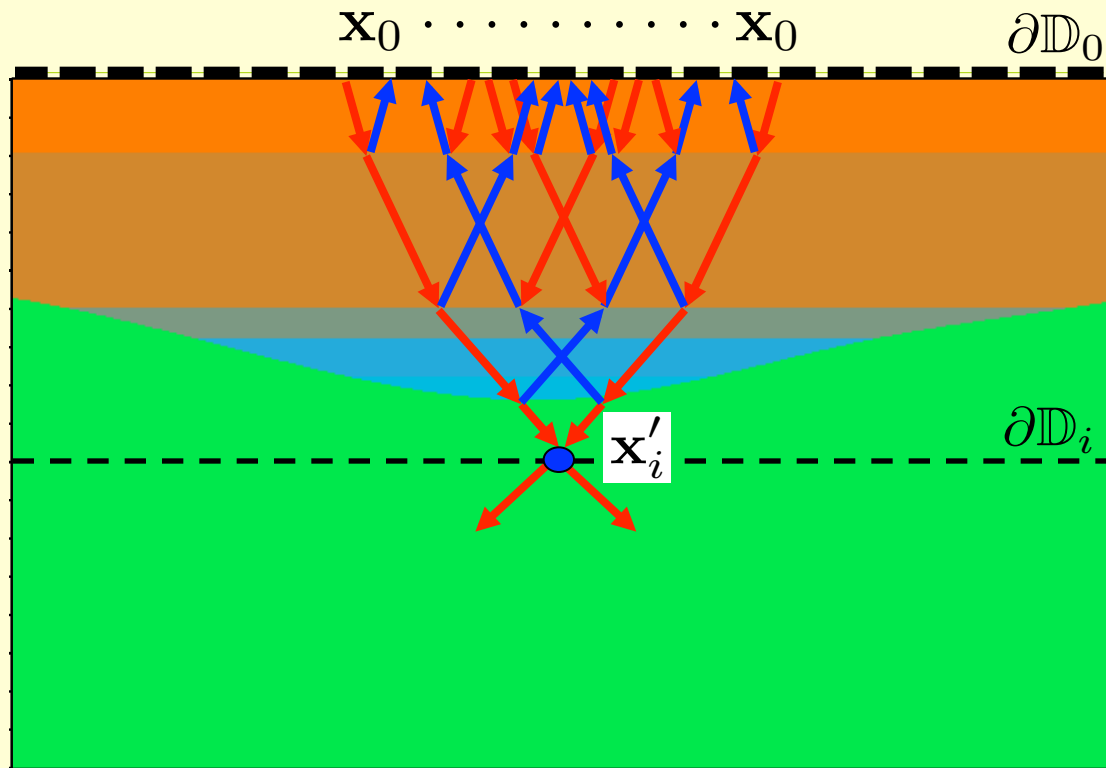
$$f_{1,1}^+(\mathbf{x}_0, \mathbf{x}'_i, -t)$$



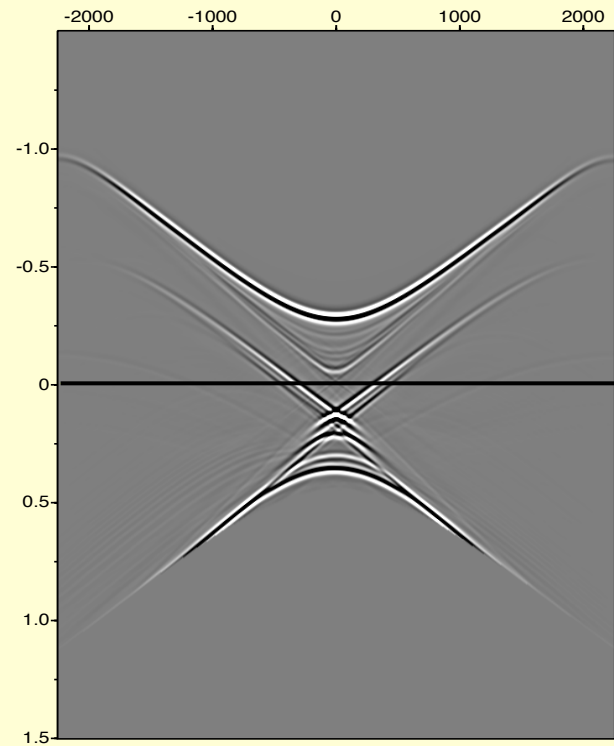
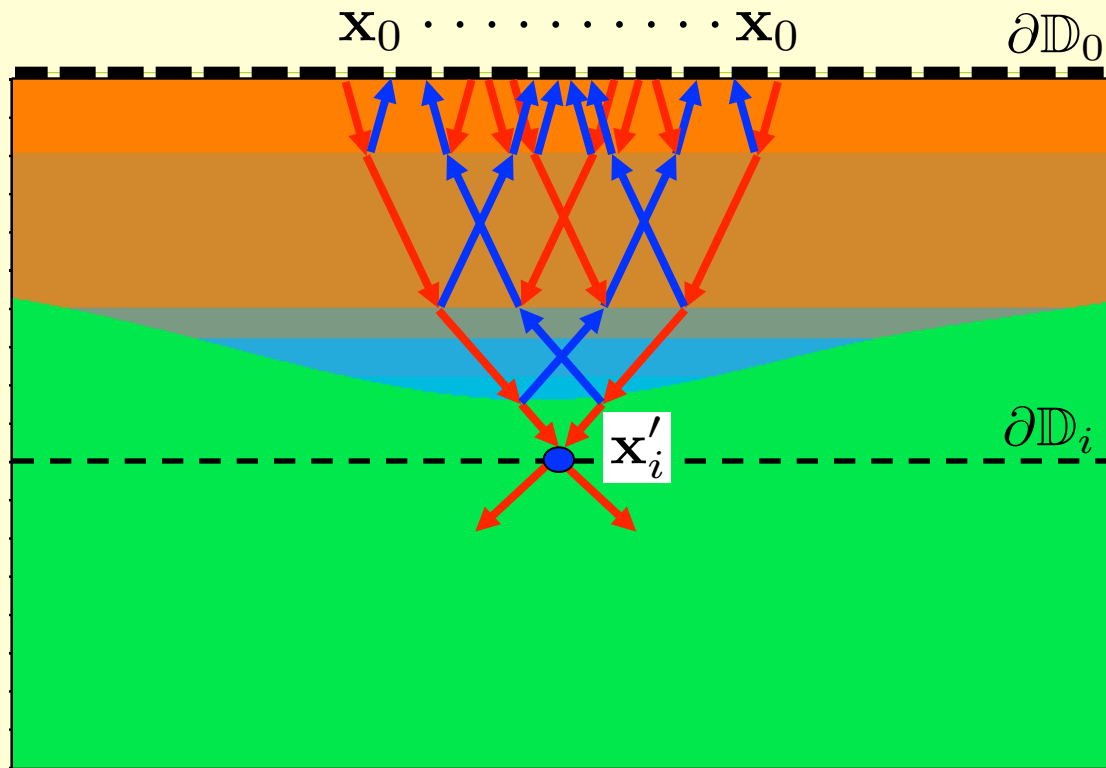
$$f_{1,1}^+(\mathbf{x}_0, \mathbf{x}'_i, t)$$



$$f_{1,1}^{-1}(\mathbf{x}_0, \mathbf{x}'_i, t)$$

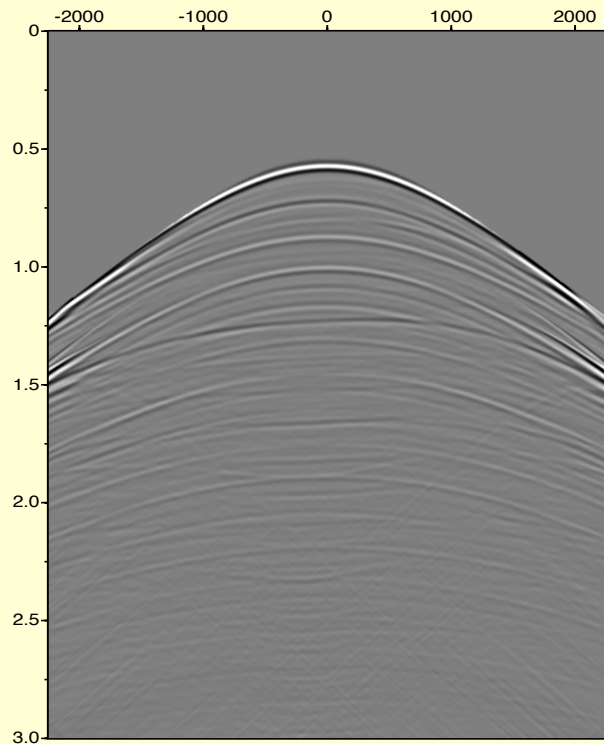
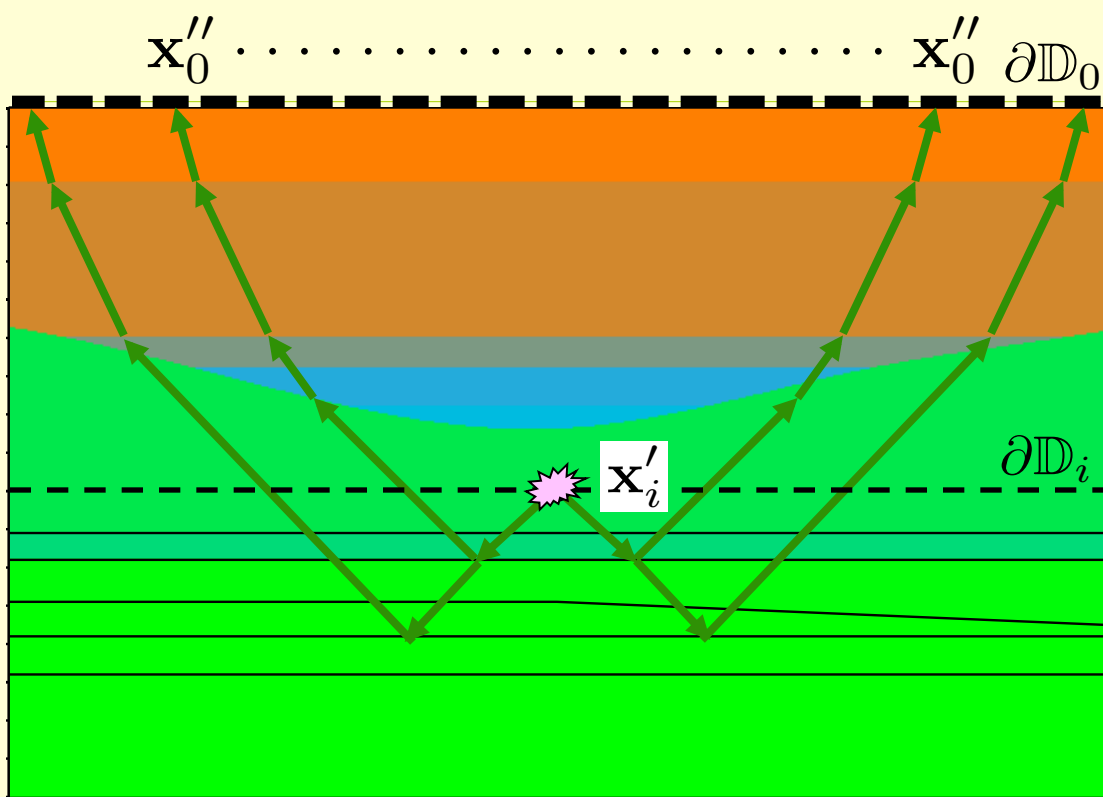


$$f_{1,4}^+(\mathbf{x}_0, \mathbf{x}'_i, t)$$

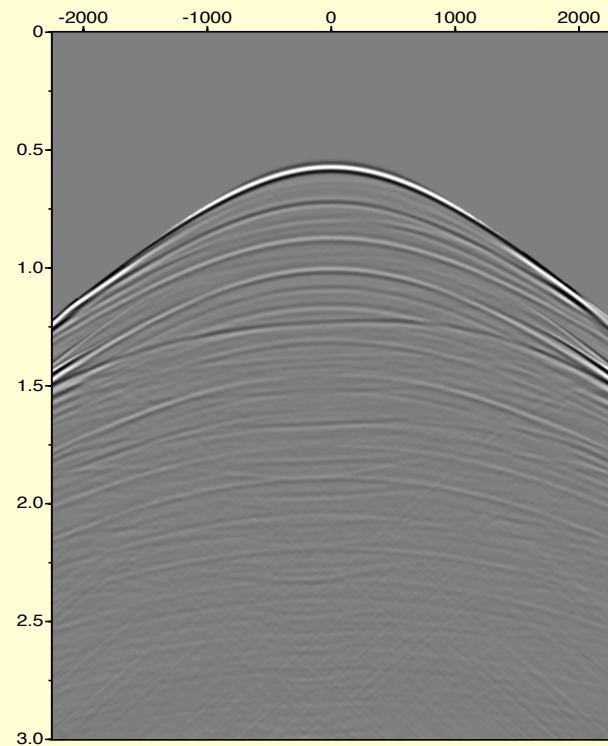
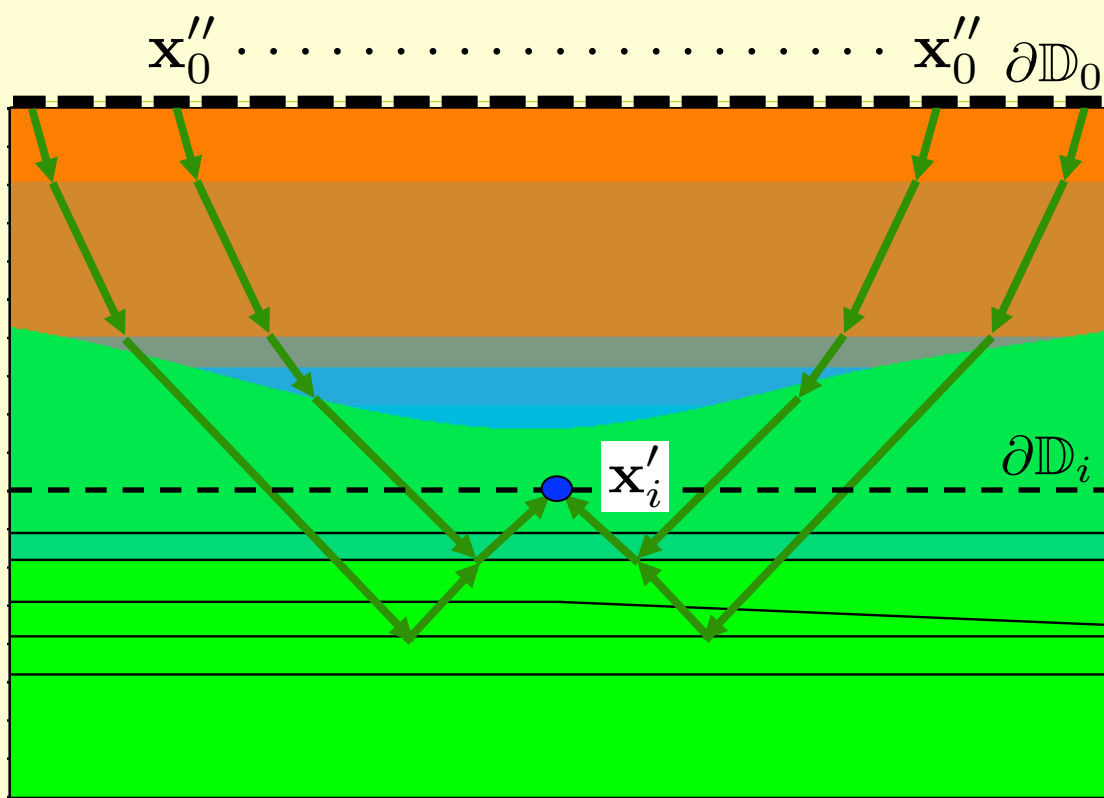


$$f_{1,4}^{-}(\mathbf{x}_0, \mathbf{x}'_i, t)$$

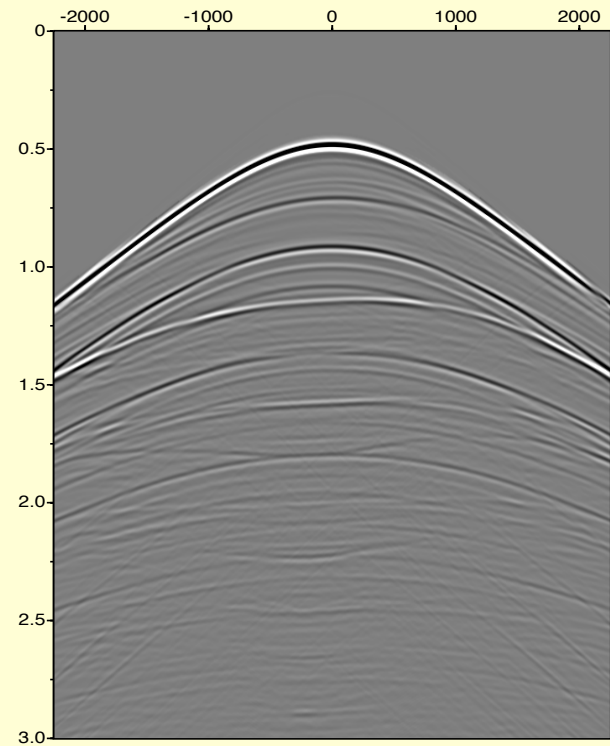
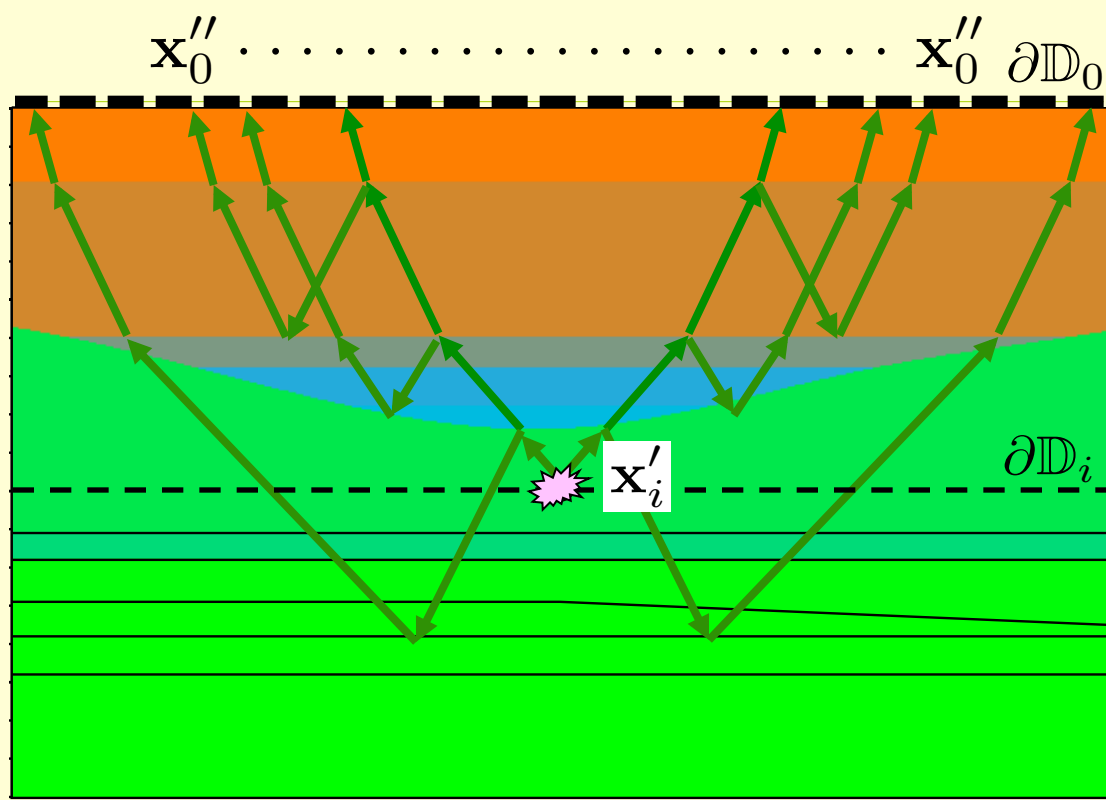
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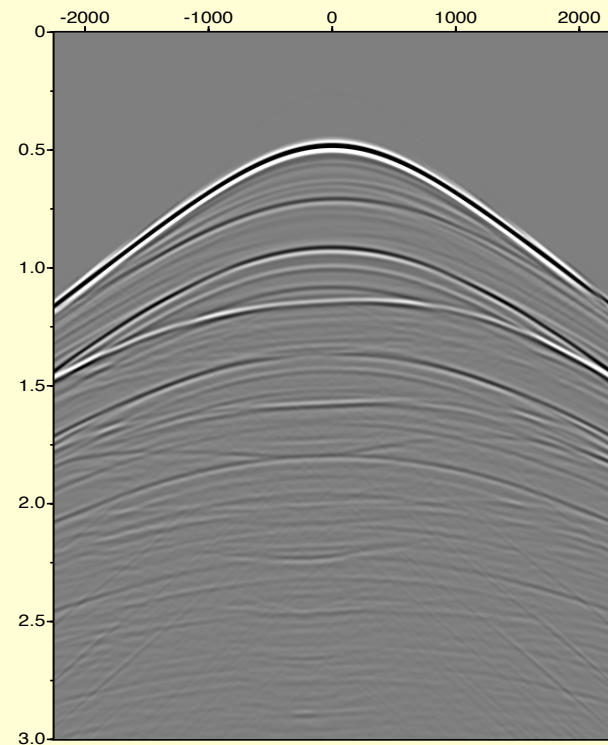
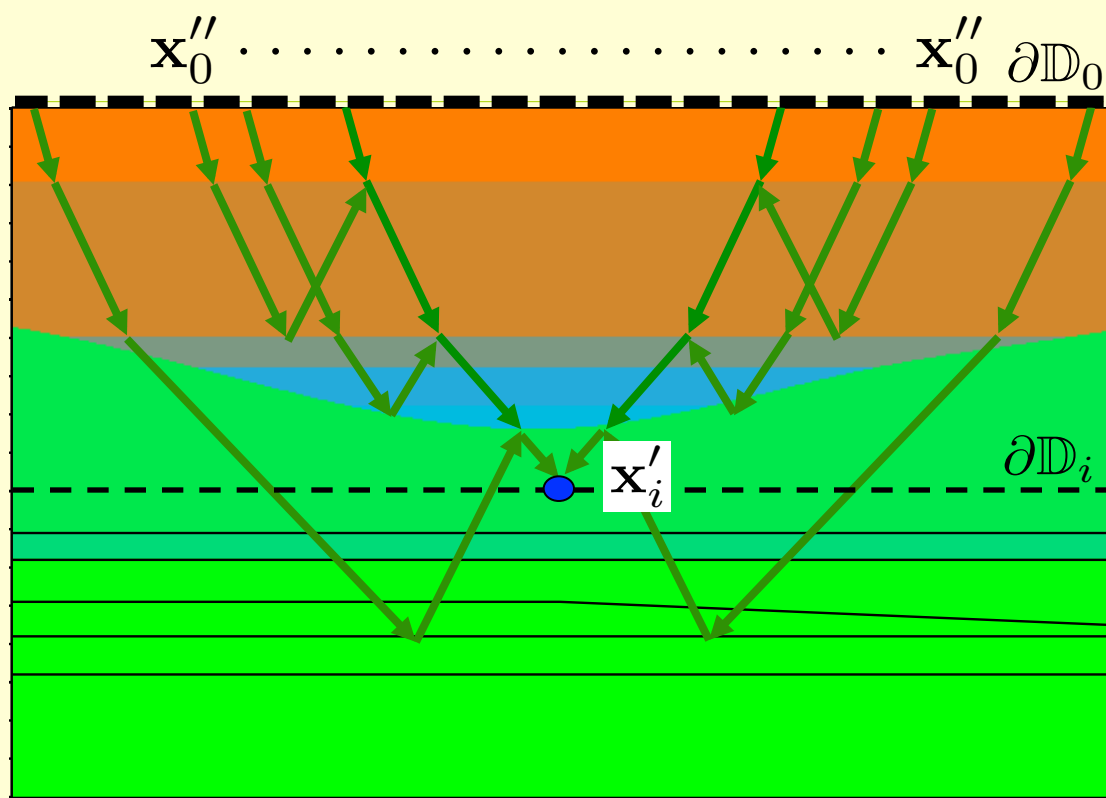
$$G^{-,+}(\mathbf{x}''_0, \mathbf{x}'_i, t) + f_1^-(\mathbf{x}''_0, \mathbf{x}'_i, t) = \int_{\partial\mathbb{D}_0} d\mathbf{x}_0 \int_{-\infty}^t R(\mathbf{x}''_0, \mathbf{x}_0, t - t') f_1^+(\mathbf{x}_0, \mathbf{x}'_i, t') dt'$$



$$G^{-,+}(\mathbf{x}_i', \mathbf{x}_0'', t) + f_1^-(\mathbf{x}_0'', \mathbf{x}_i', t) = \int_{\partial\mathbb{D}_0} d\mathbf{x}_0 \int_{-\infty}^t R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^+(\mathbf{x}_0, \mathbf{x}_i', t') dt'$$

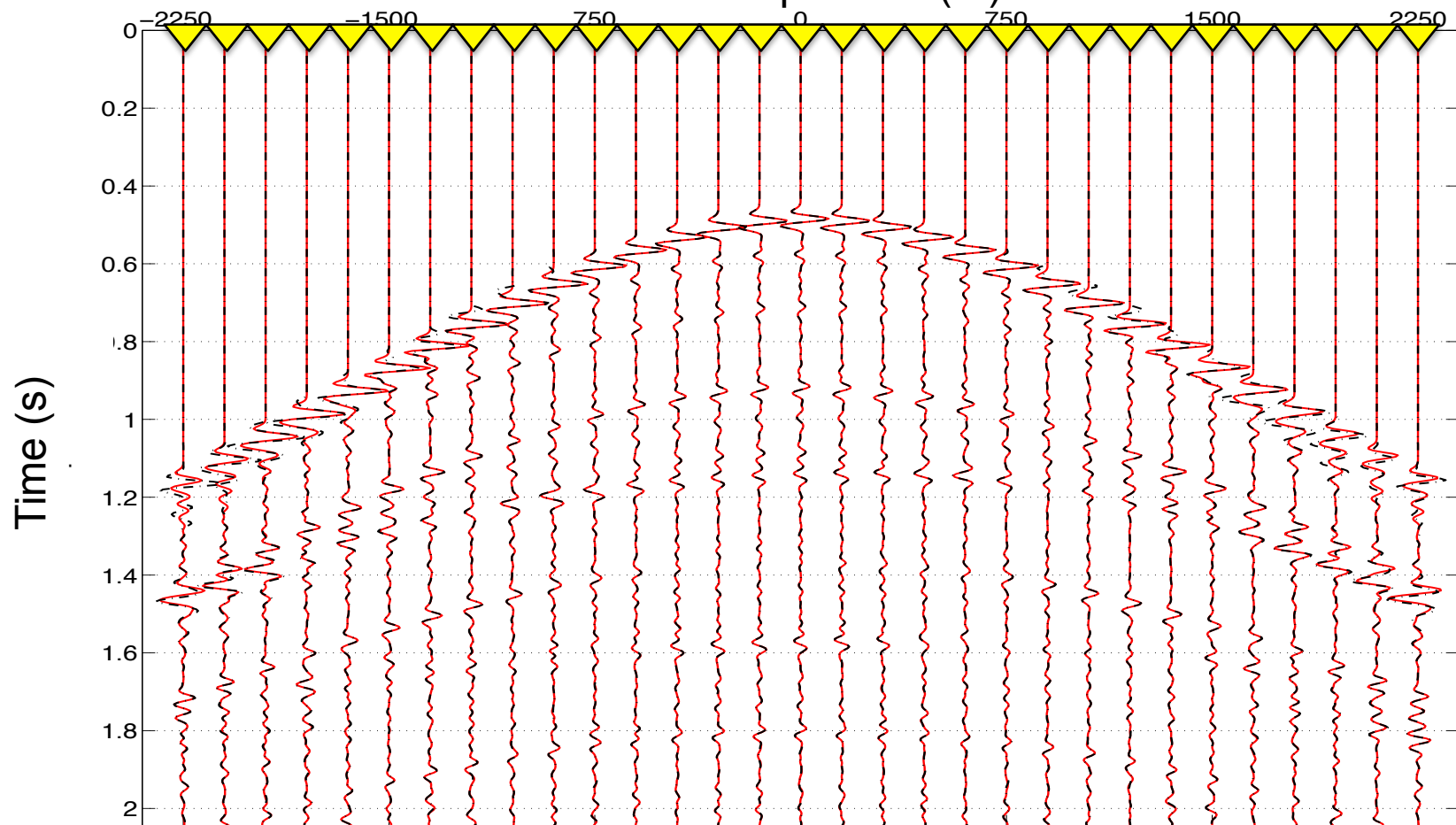


$$\textcircled{-G^{-,-}(\mathbf{x}_0'', \mathbf{x}_i', t)} + f_1^+(\mathbf{x}_0'', \mathbf{x}_i', -t) = \int_{\partial\mathbb{D}_0} d\mathbf{x}_0 \int_{-\infty}^t R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^-(\mathbf{x}_0, \mathbf{x}_i', -t') dt'$$



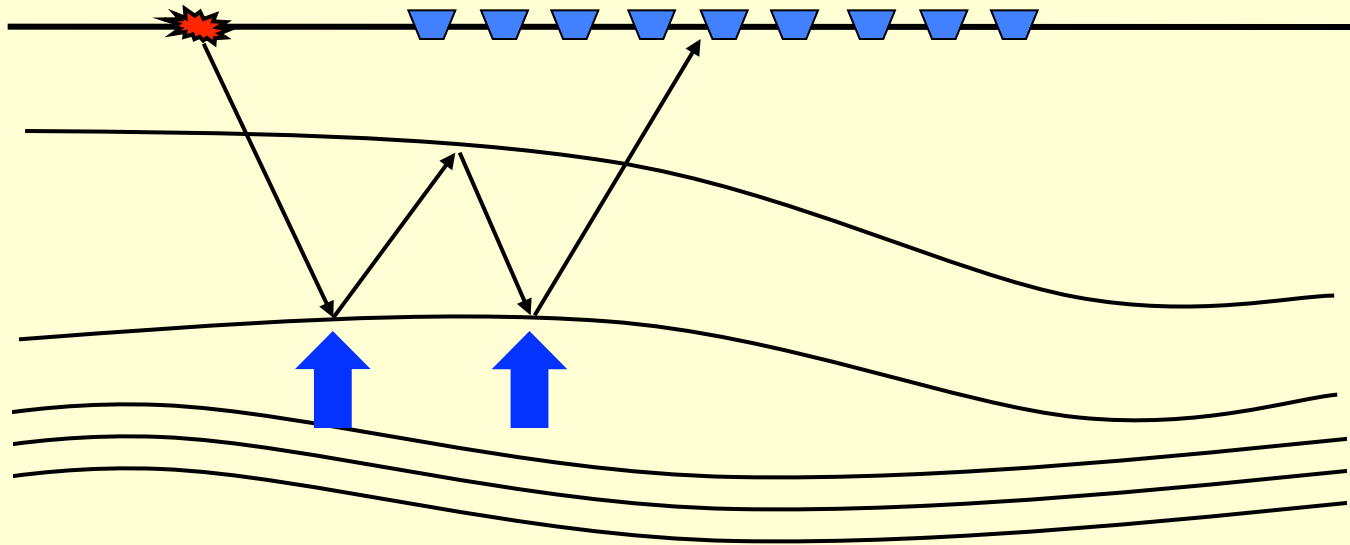
$$-G^{+,+}(\mathbf{x}_i', \mathbf{x}_0'', t) + f_1^+(\mathbf{x}_0'', \mathbf{x}_i', -t) = \int_{\partial\mathbb{D}_0} d\mathbf{x}_0 \int_{-\infty}^t R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^-(\mathbf{x}_0, \mathbf{x}_i', -t') dt'$$

Lateral position (m)

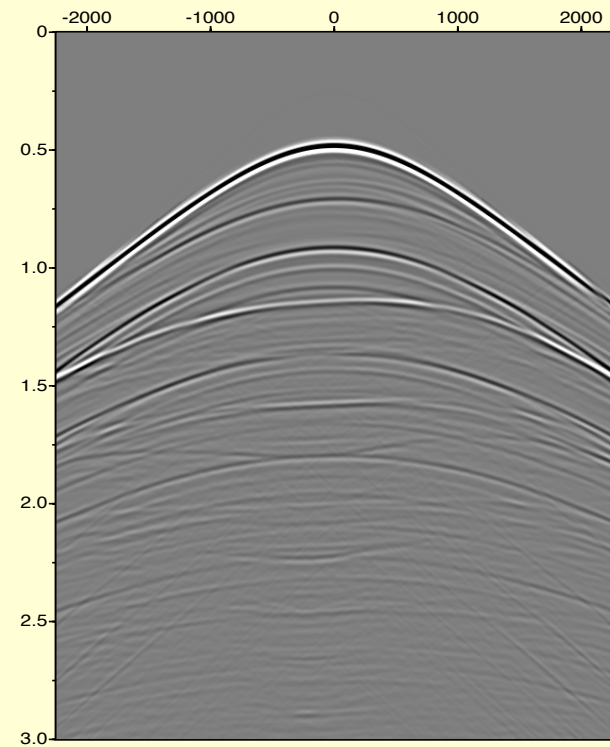
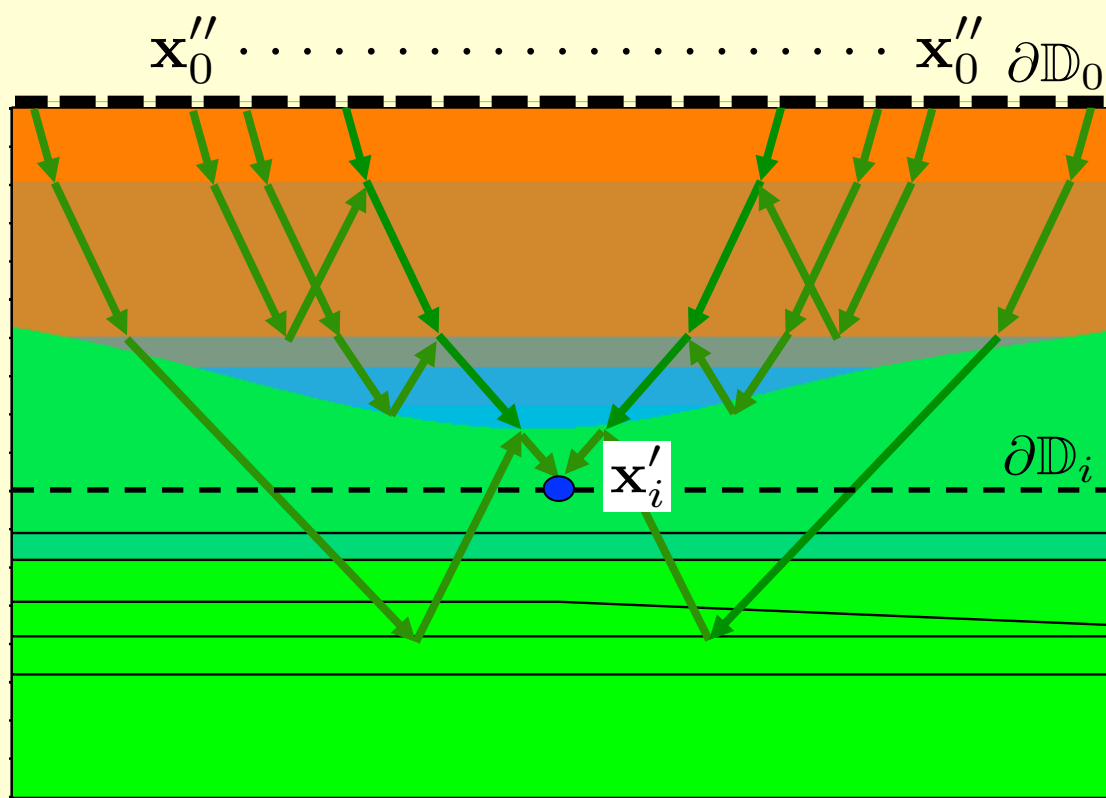


$$G^{+,+}(\mathbf{x}'_i, \mathbf{x}''_0, t) + G^{-,+}(\mathbf{x}'_i, \mathbf{x}''_0, t)$$

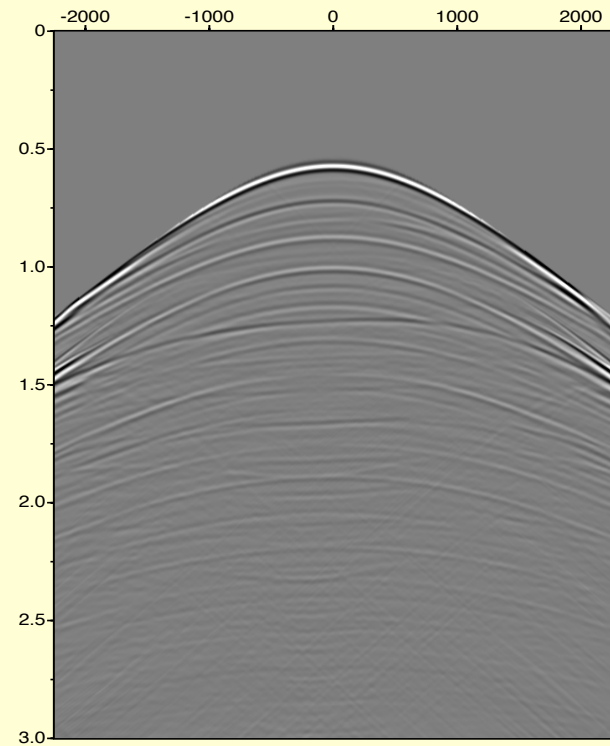
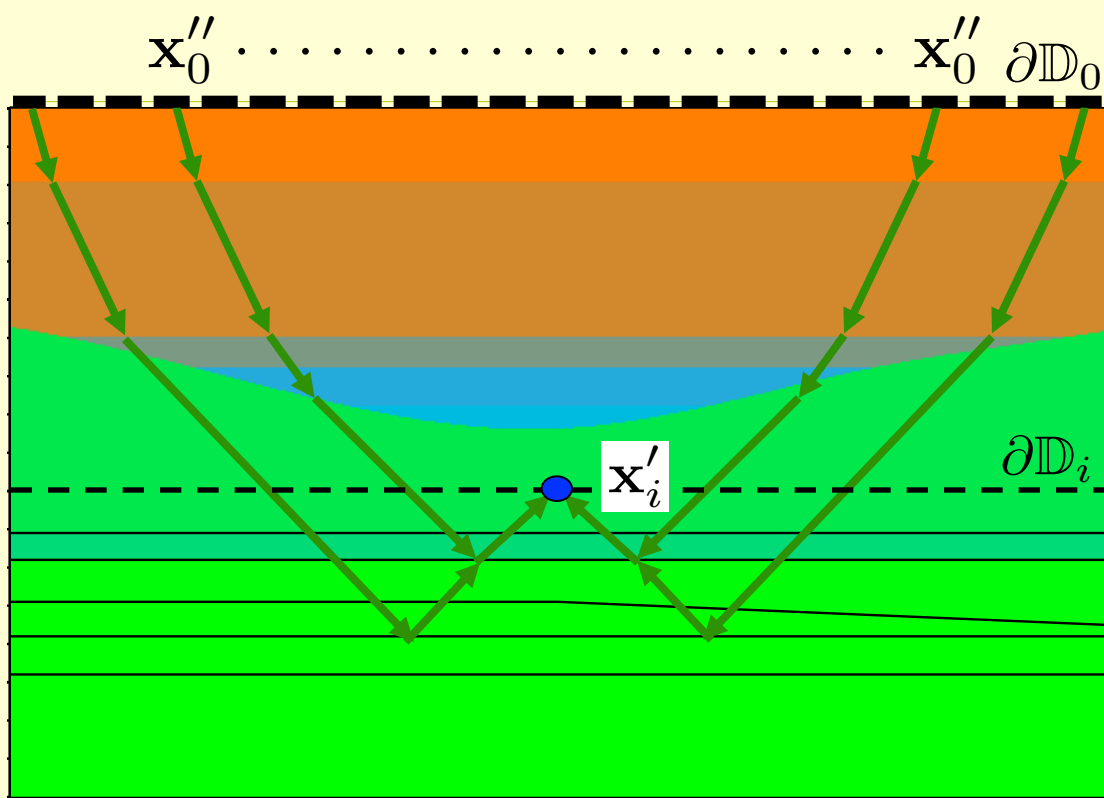
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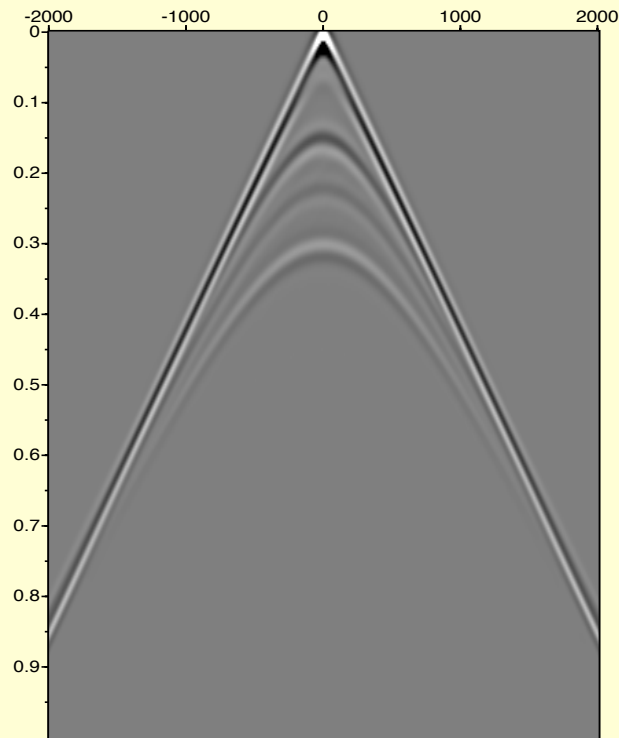
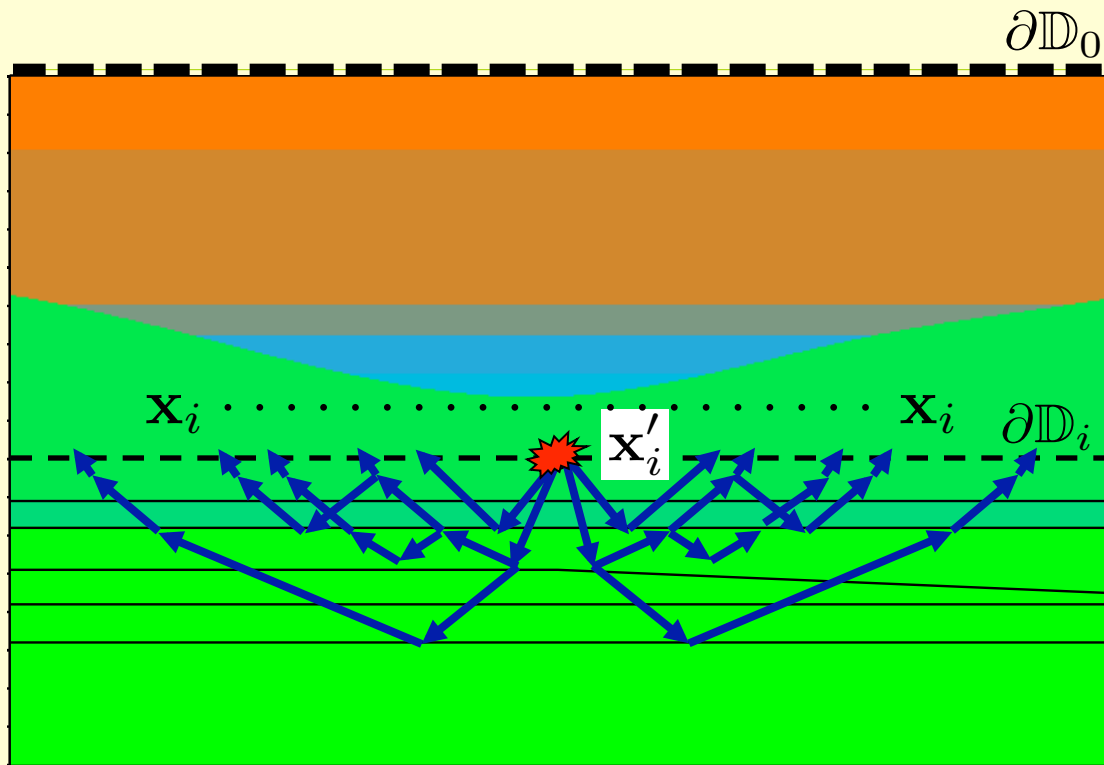
Improved illumination with internal multiples



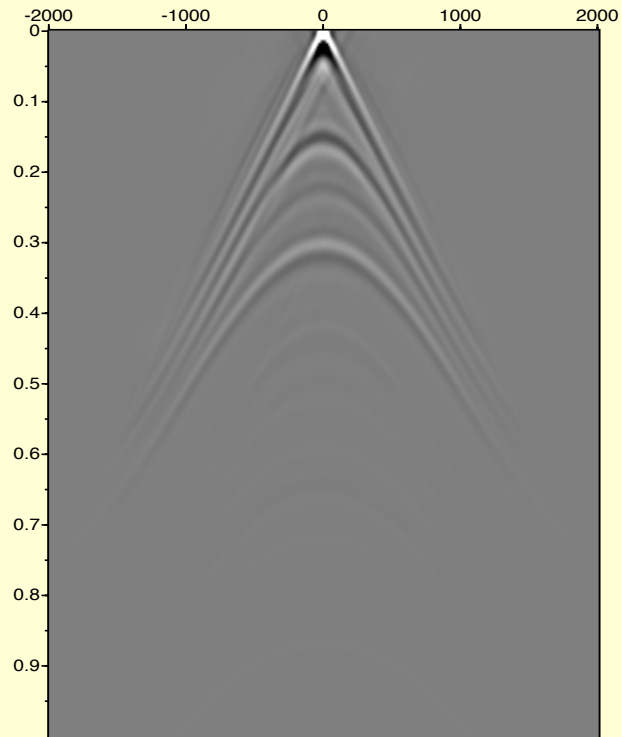
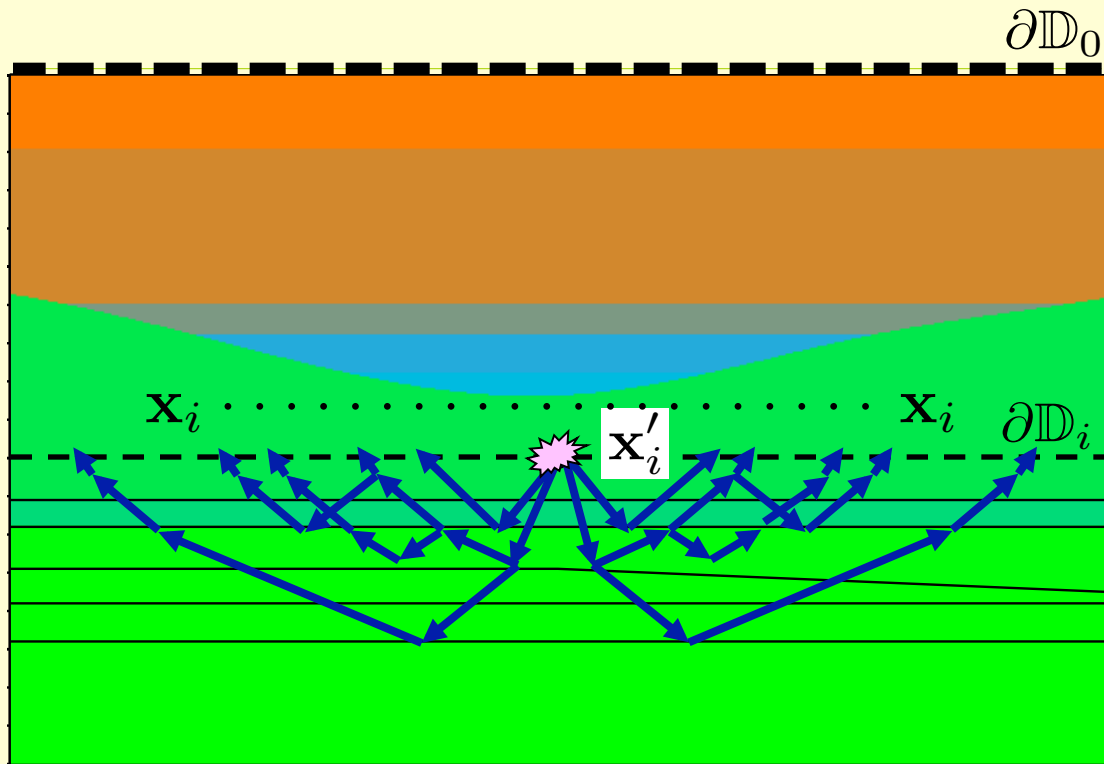
$$G^{-,+}(\mathbf{x}_i, \mathbf{x}''_0, t) = \int_{\partial\mathbb{D}_i} d\mathbf{x}'_i \int_{-\infty}^{\infty} R(\mathbf{x}_i, \mathbf{x}'_i, t') G^{+,+}(\mathbf{x}'_i, \mathbf{x}''_0, t - t') dt'$$



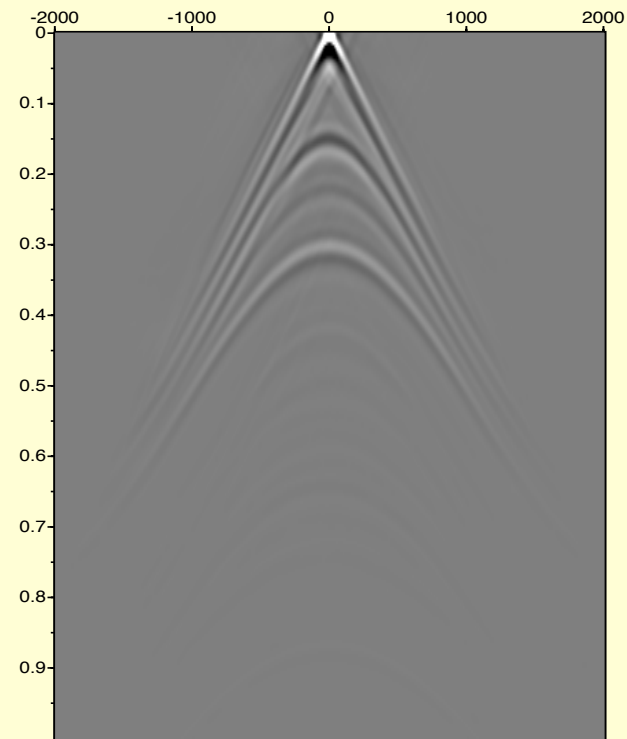
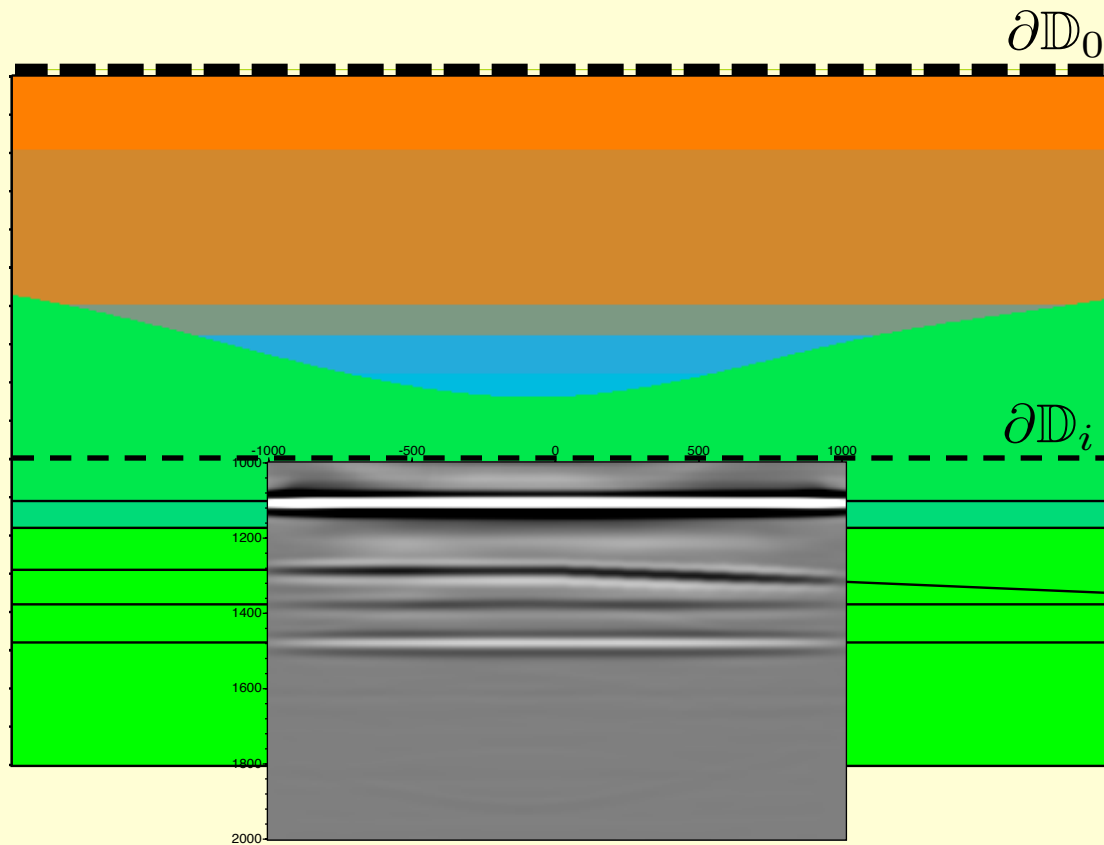
$$G^{-,+}(\mathbf{x}_i, \mathbf{x}_0'', t) = \int_{\partial\mathbb{D}_i} d\mathbf{x}'_i \int_{-\infty}^{\infty} R(\mathbf{x}_i, \mathbf{x}'_i, t') G^{+,+}(\mathbf{x}'_i, \mathbf{x}_0'', t - t') dt'$$



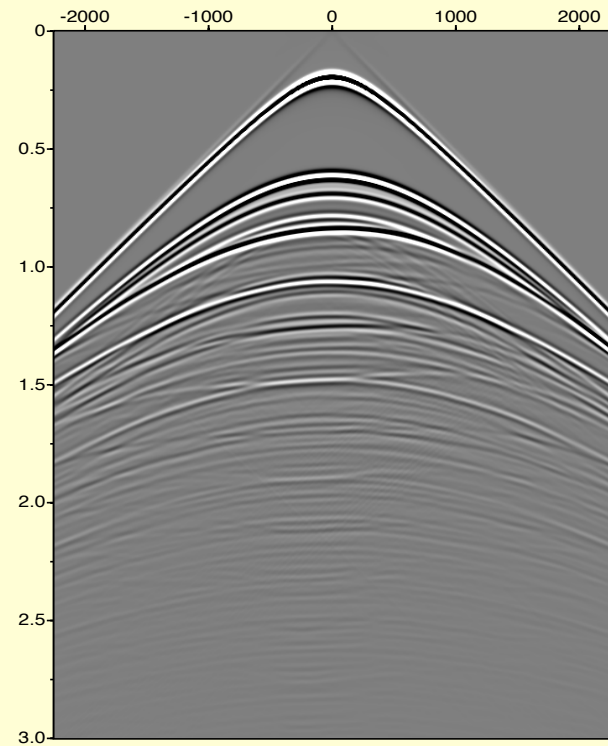
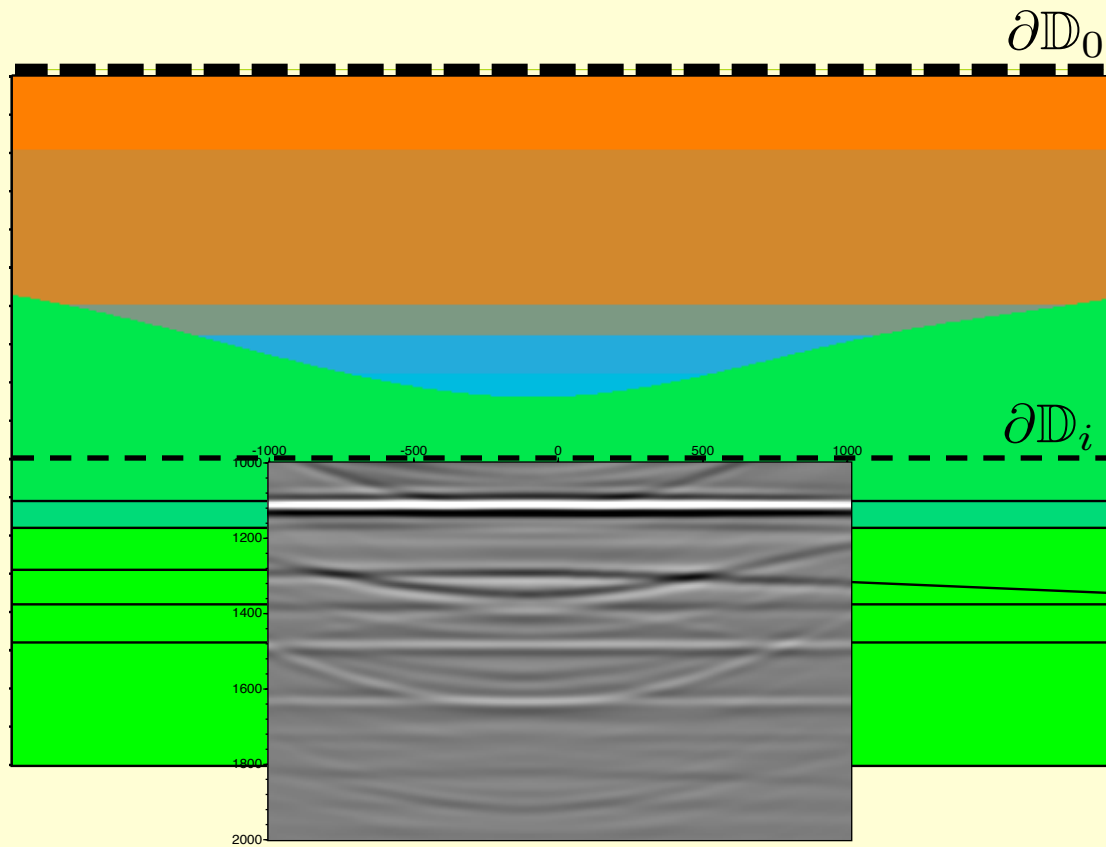
$$G^{-,+}(\mathbf{x}_i, \mathbf{x}_0'', t) = \int_{\partial\mathbb{D}_i} d\mathbf{x}'_i \int_{-\infty}^{\infty} R(\mathbf{x}_i, \mathbf{x}'_i, t') G^{+,+}(\mathbf{x}'_i, \mathbf{x}_0'', t - t') dt'$$



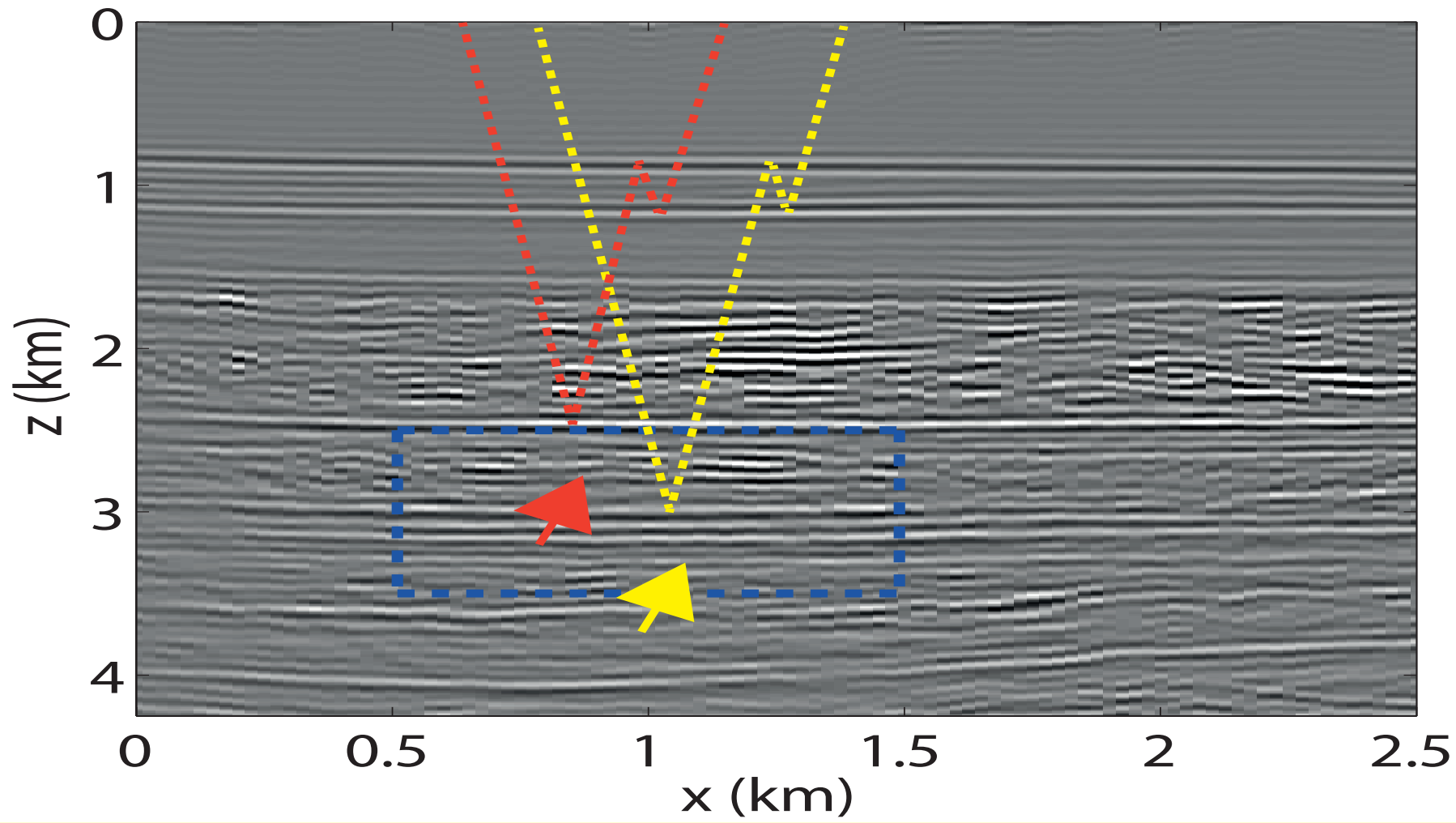
$$G^{-,+}(\mathbf{x}_i, \mathbf{x}''_0, t) = \int_{\partial\mathbb{D}_i} d\mathbf{x}'_i \int_{-\infty}^{\infty} R(\mathbf{x}_i, \mathbf{x}'_i, t') G^{+,+}(\mathbf{x}'_i, \mathbf{x}''_0, t - t') dt'$$

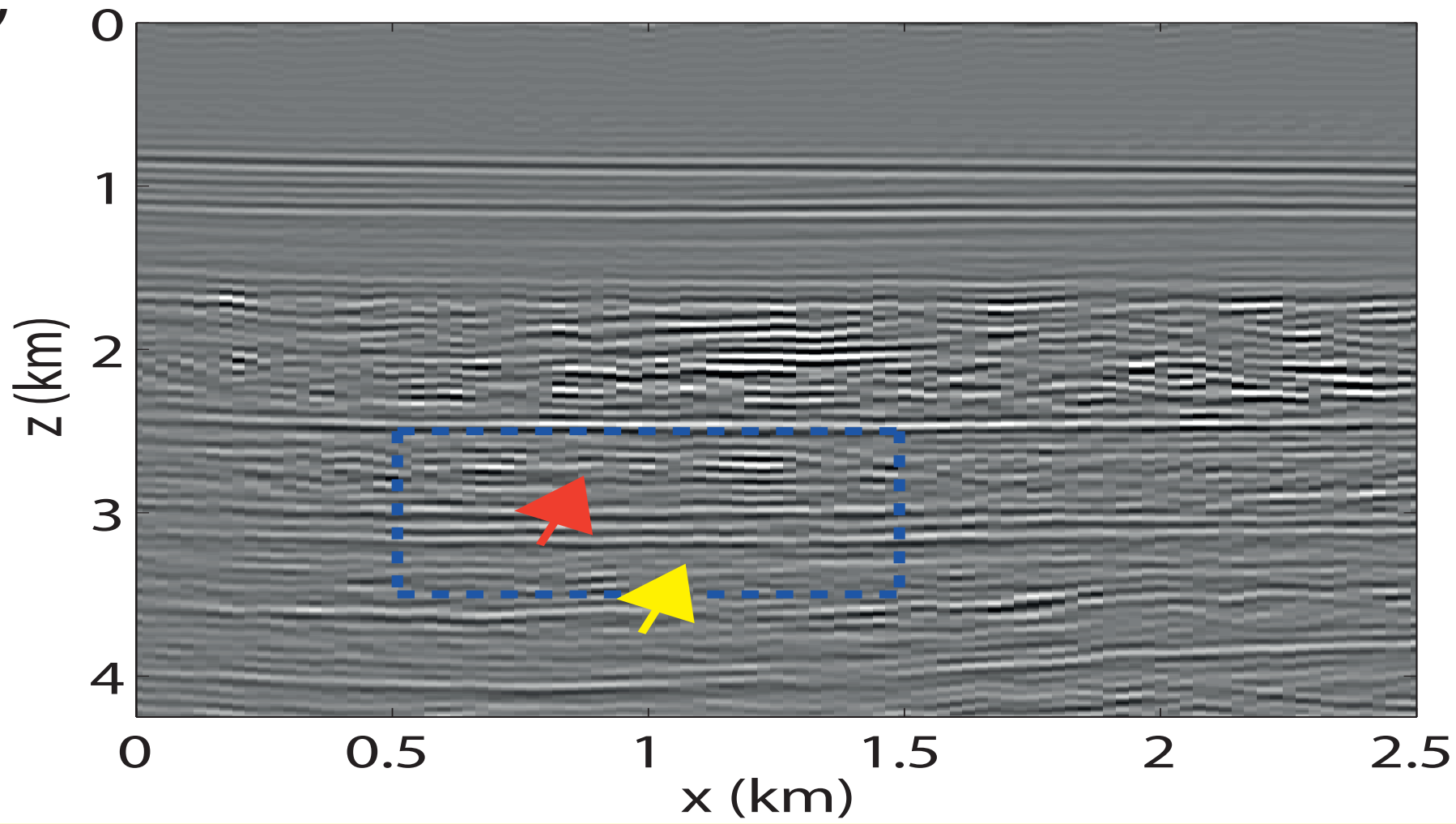


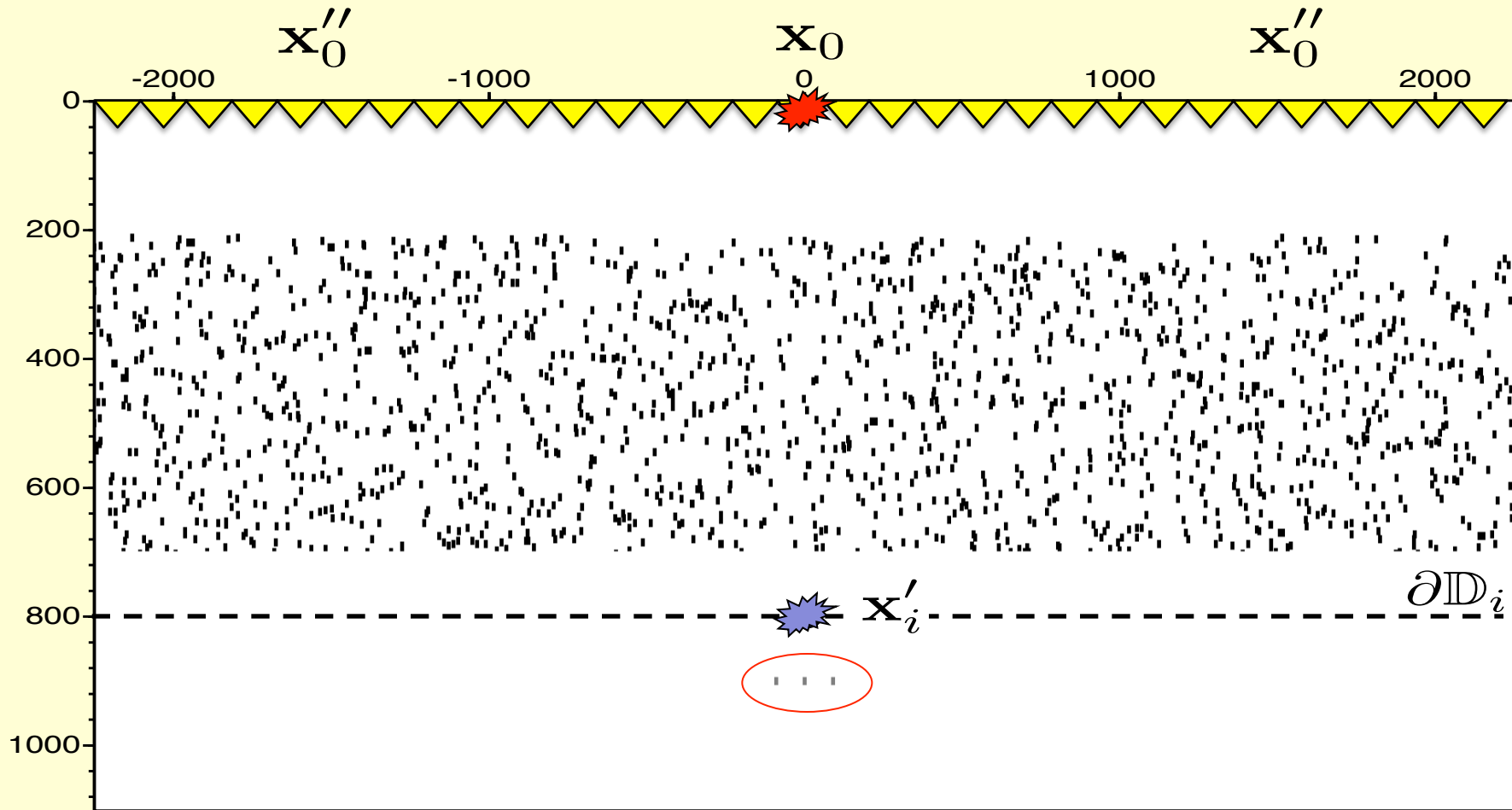
Marchenko imaging

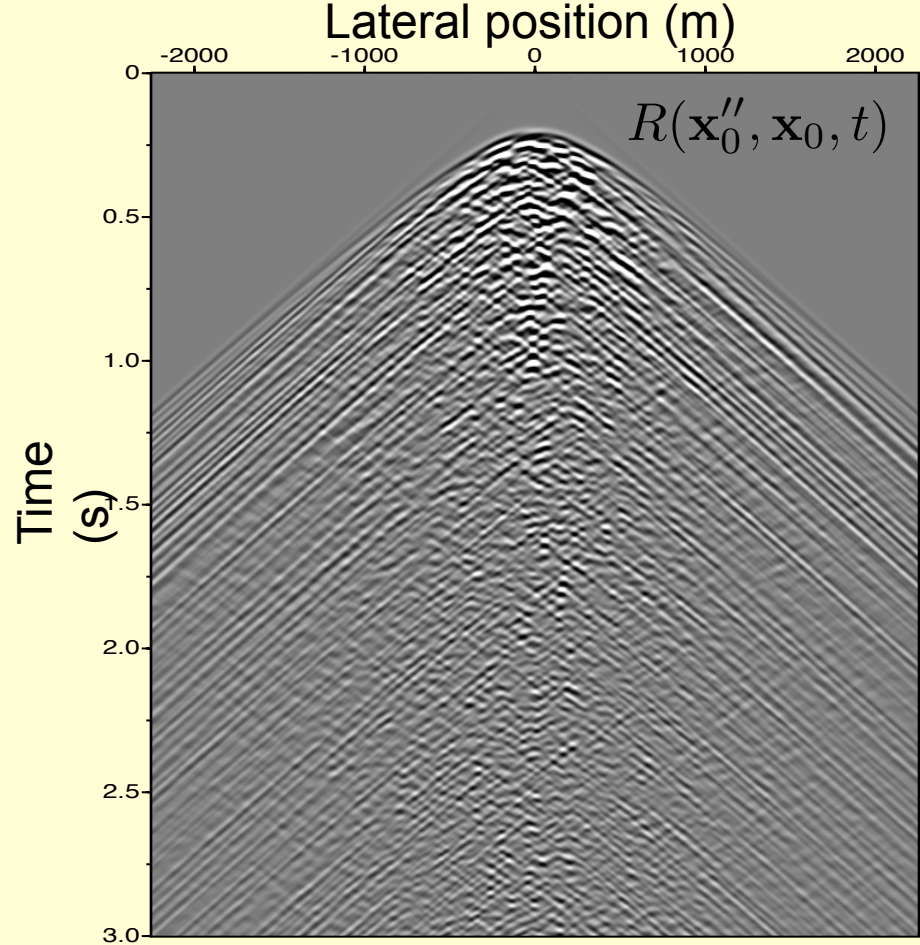


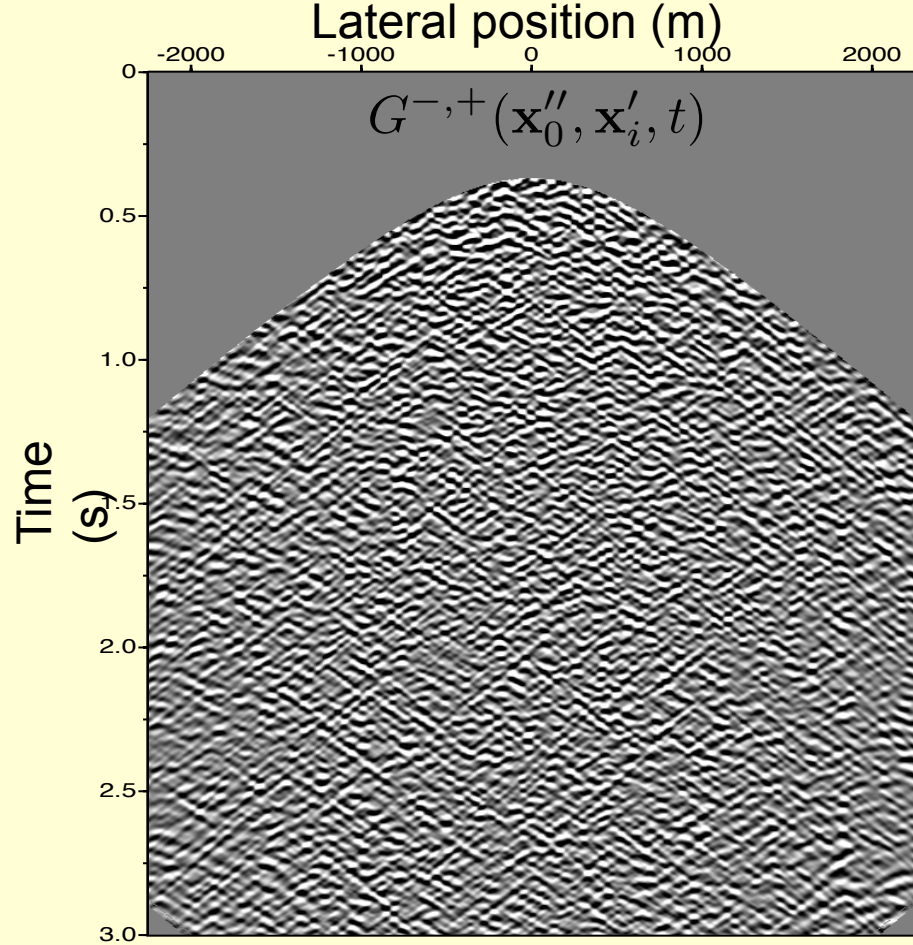
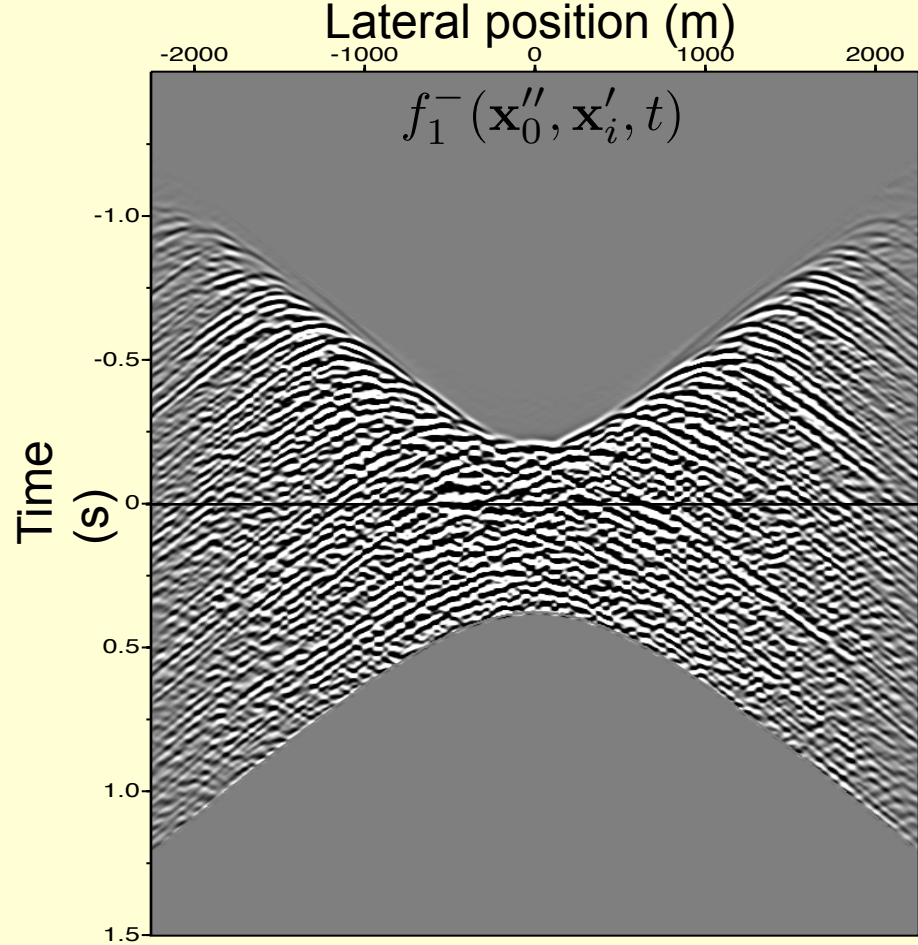
Standard imaging



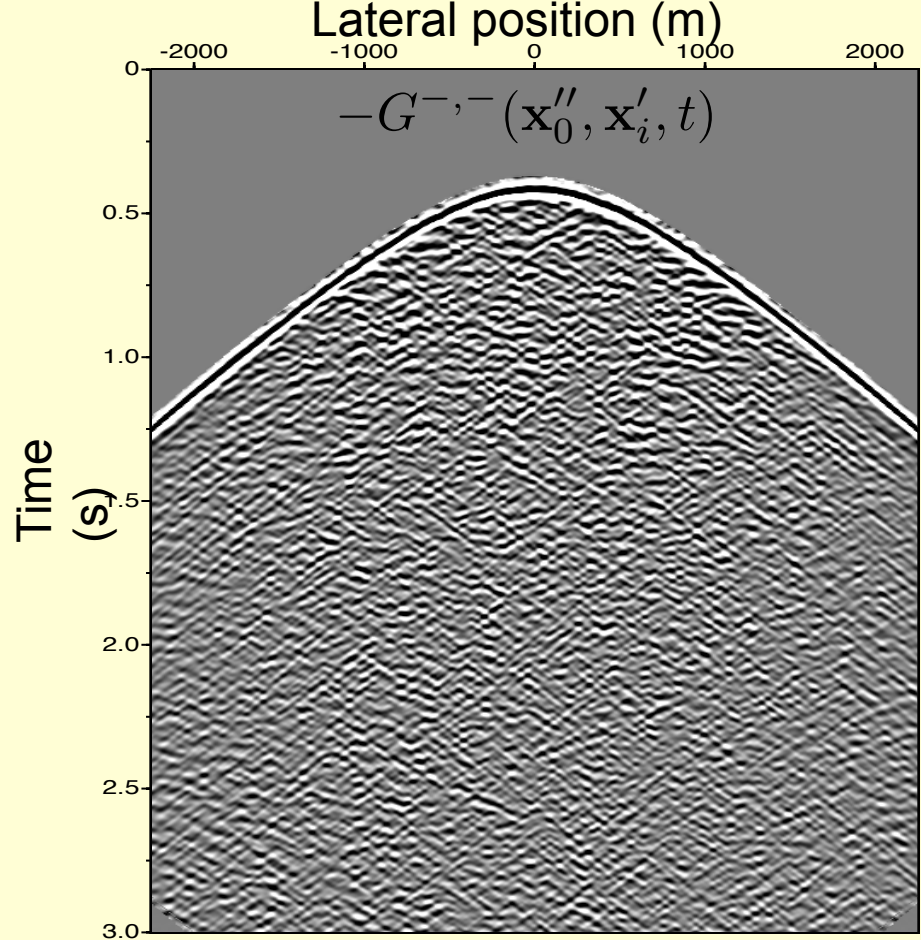
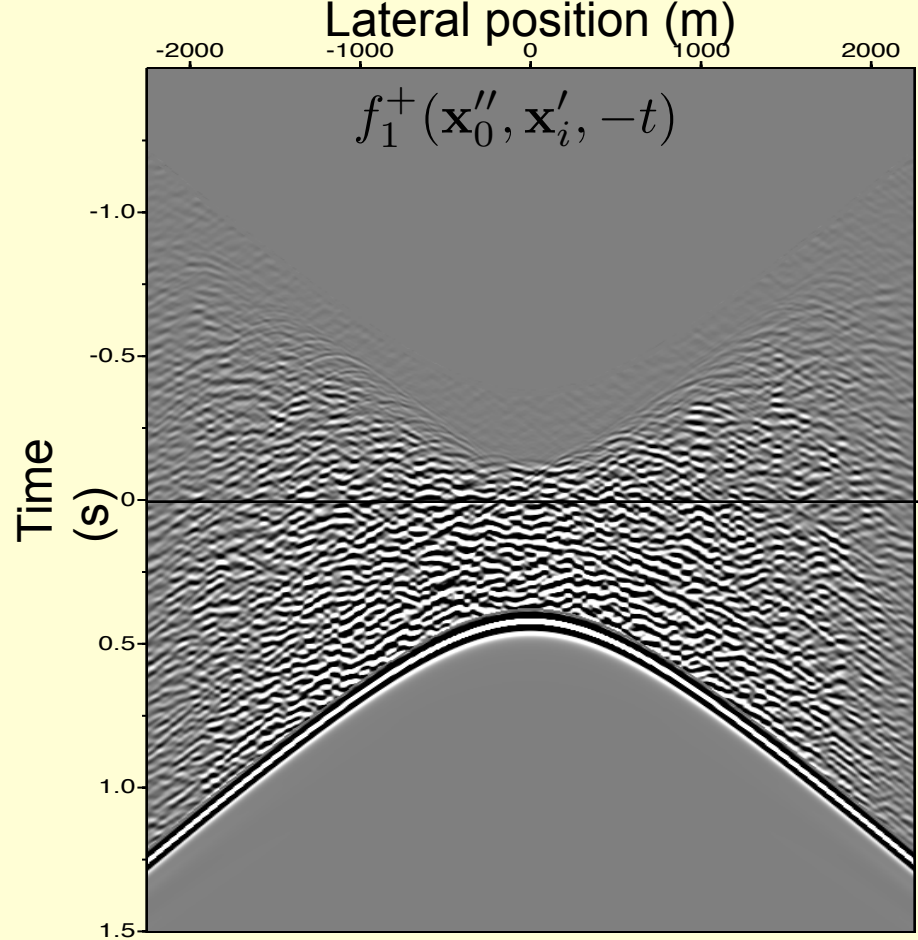




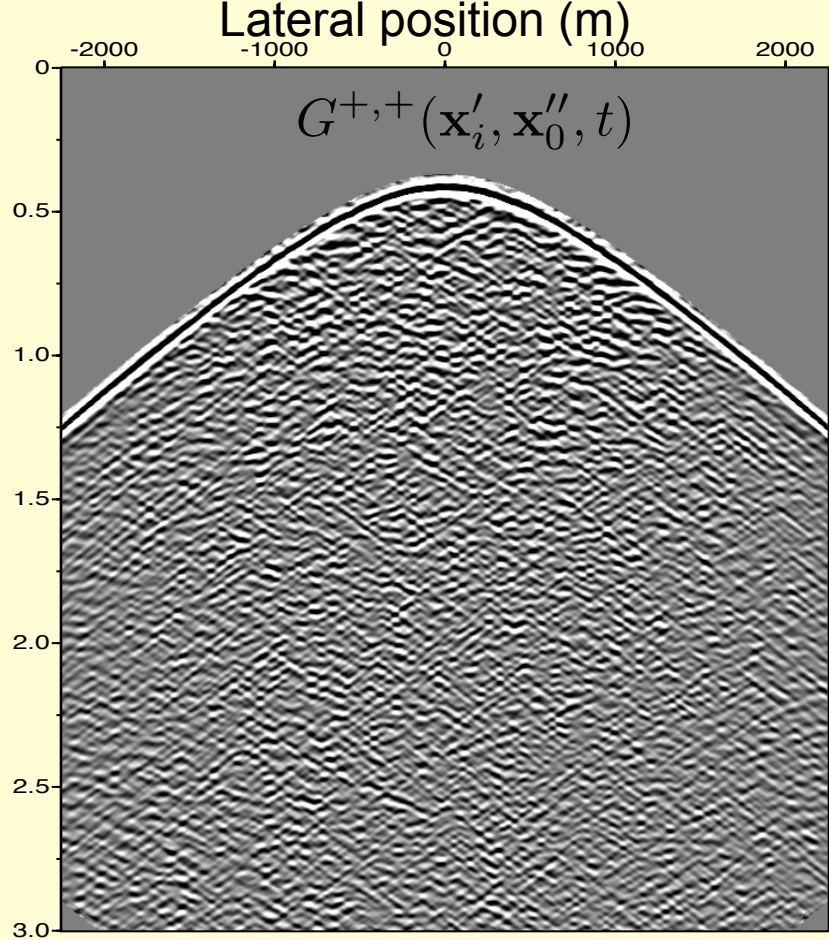
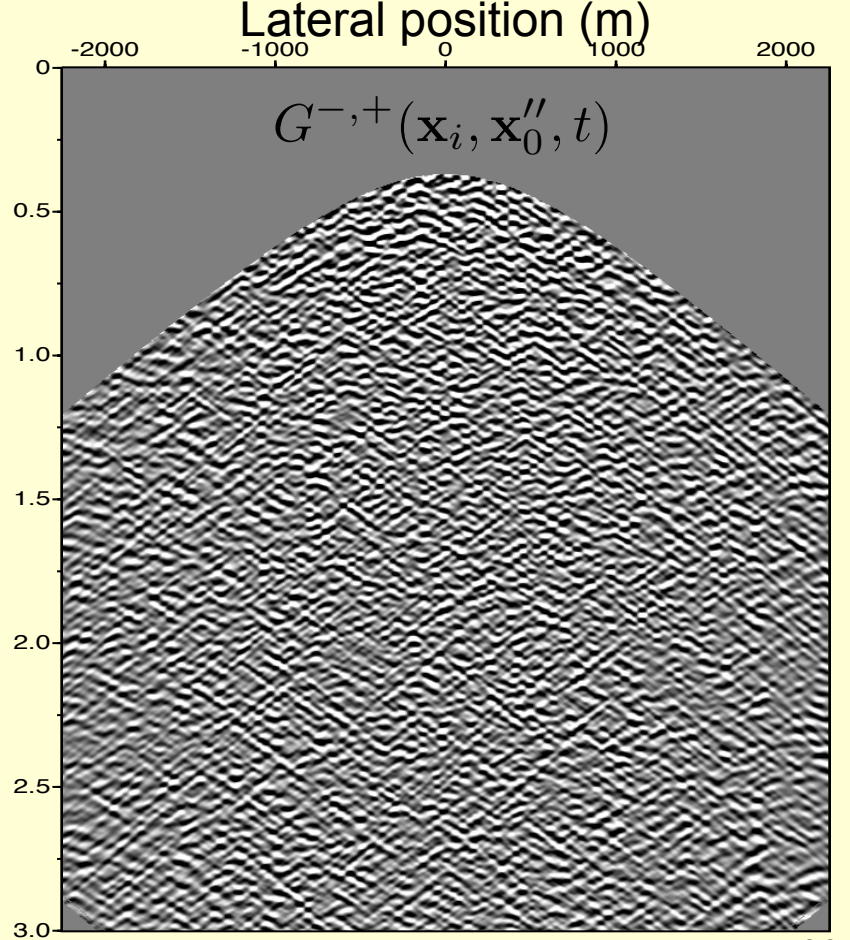




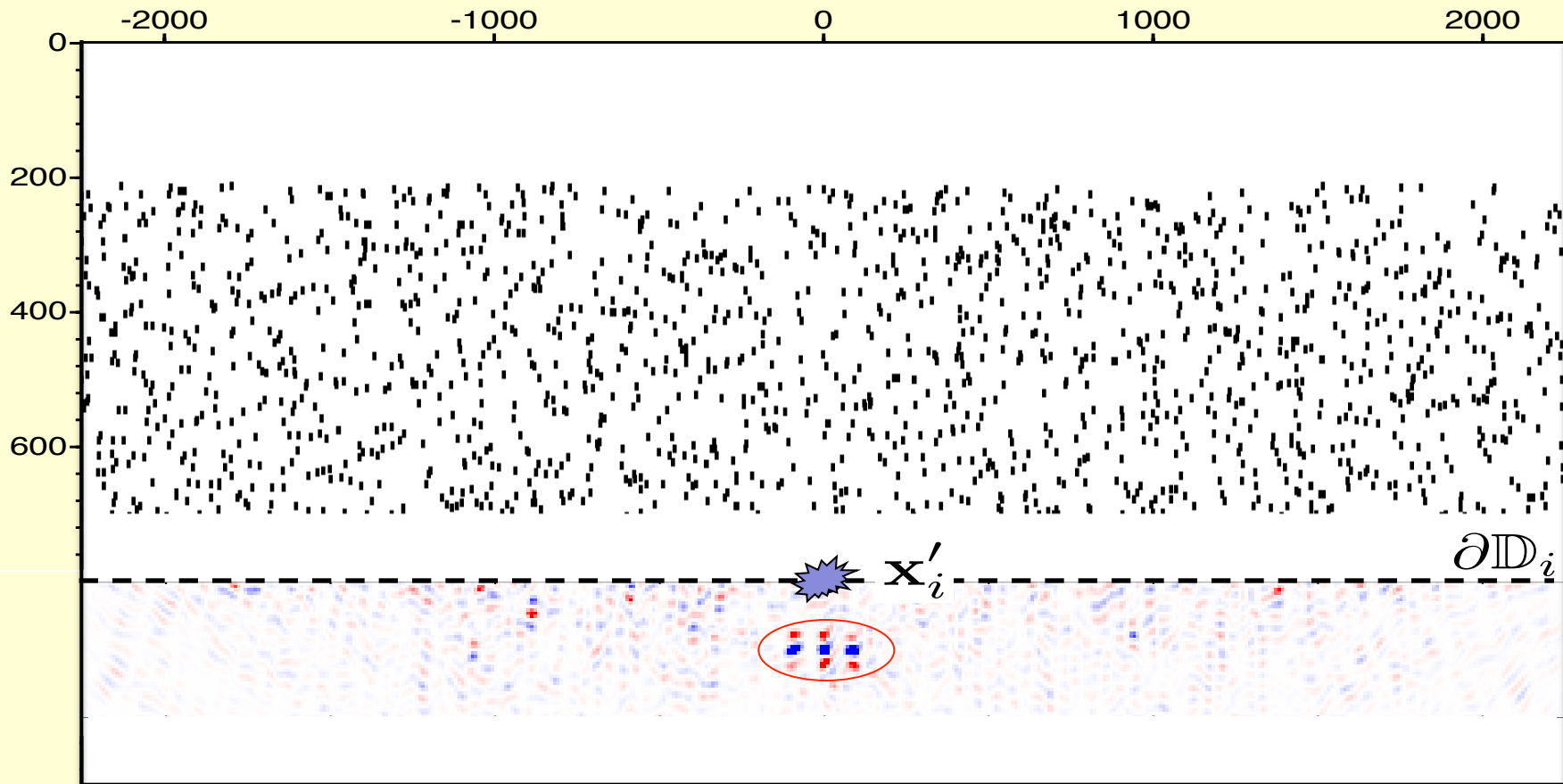
$$G^{-,+}(\mathbf{x}_0'', \mathbf{x}'_i, t) + f_1^-(\mathbf{x}_0'', \mathbf{x}'_i, t) = \int_{\partial\mathbb{D}_0} d\mathbf{x}_0 \int_{-\infty}^t R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^+(\mathbf{x}_0, \mathbf{x}'_i, t') dt'$$



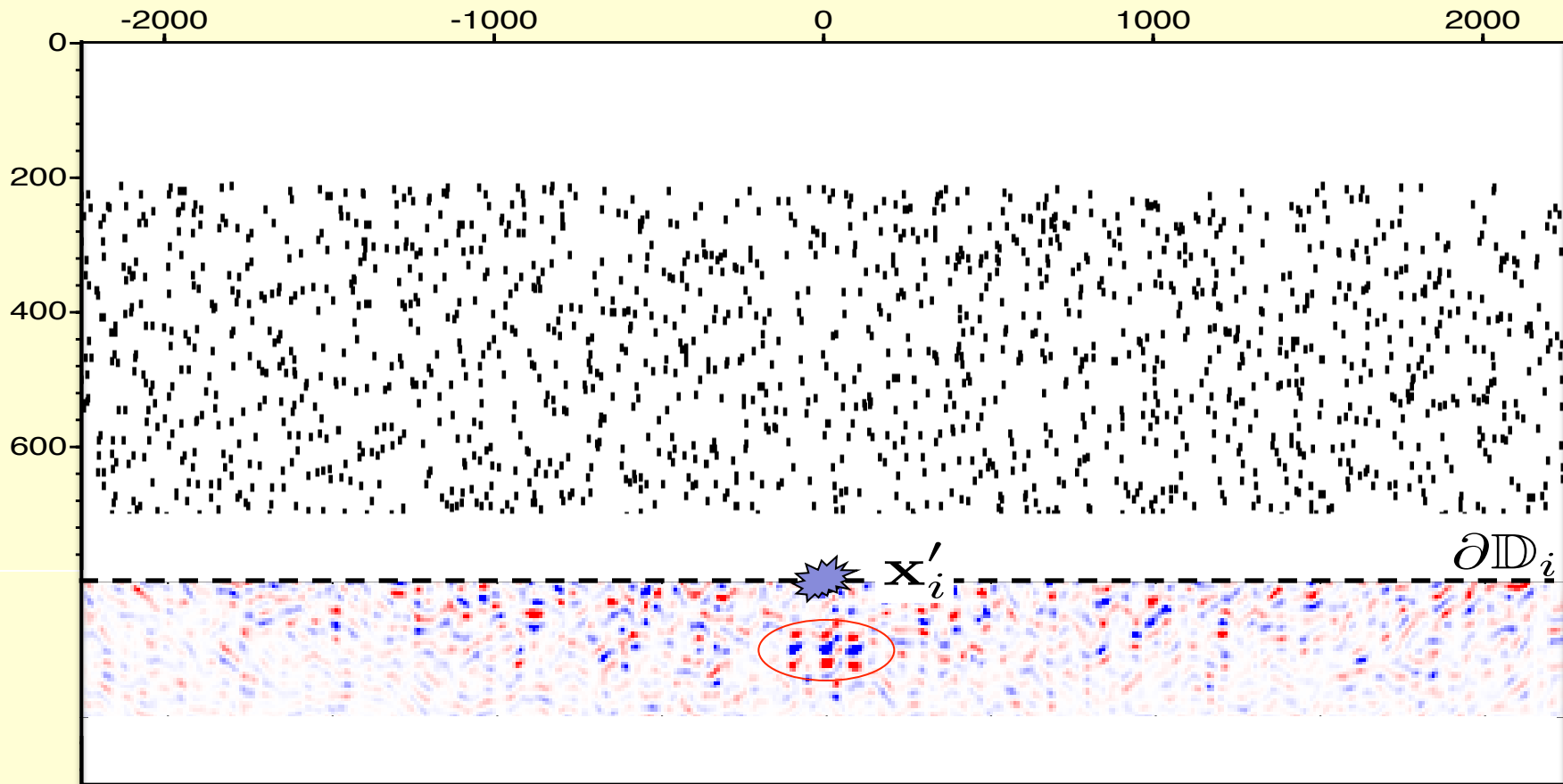
$$-G^{-,-}(\mathbf{x}_0'', \mathbf{x}'_i, t) + f_1^+(\mathbf{x}_0'', \mathbf{x}'_i, -t) = \int_{\partial\mathbb{D}_0} d\mathbf{x}_0 \int_{-\infty}^t R(\mathbf{x}_0'', \mathbf{x}_0, t - t') f_1^-(\mathbf{x}_0, \mathbf{x}'_i, -t') dt'$$



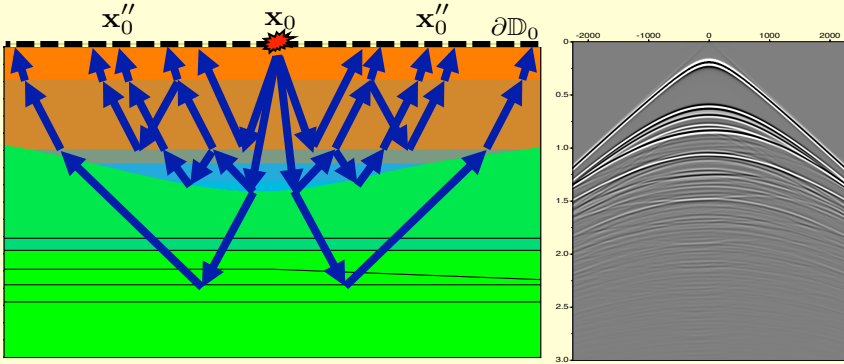
$$G^{-,+}(\mathbf{x}_i, \mathbf{x}_0'', t) = \int_{\partial\mathbb{D}_i} d\mathbf{x}'_i \int_{-\infty}^{\infty} R(\mathbf{x}_i, \mathbf{x}'_i, t') G^{+,+}(\mathbf{x}'_i, \mathbf{x}_0'', t - t') dt'$$



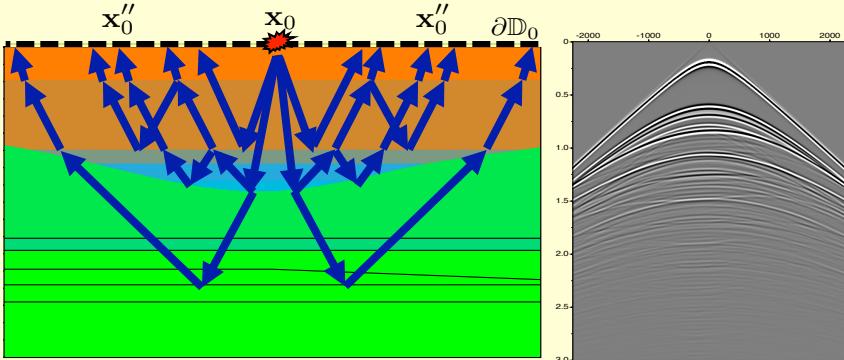
Marchenko imaging



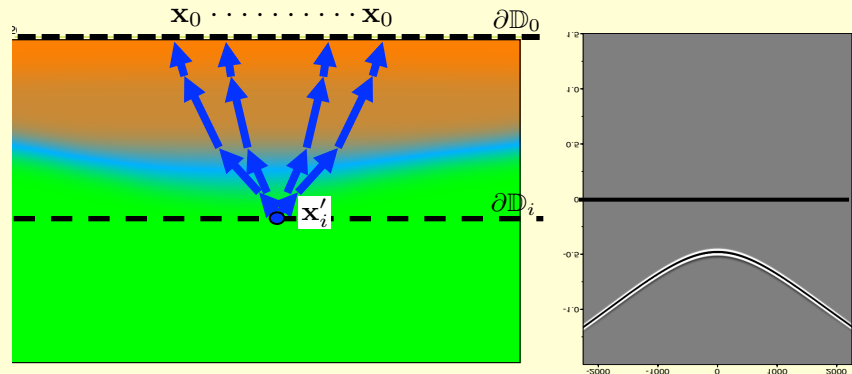
Standard imaging



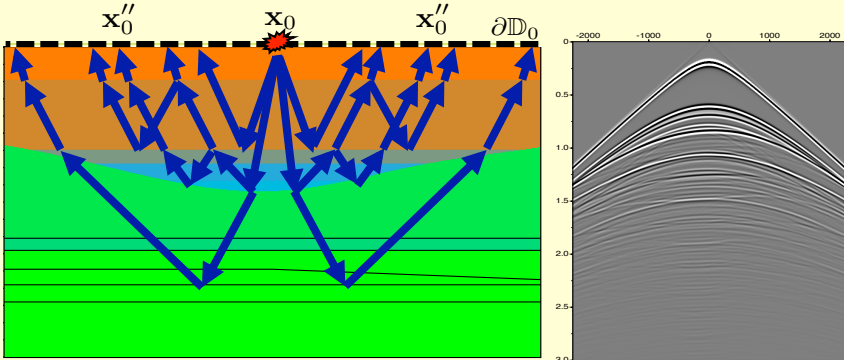
Reflection data



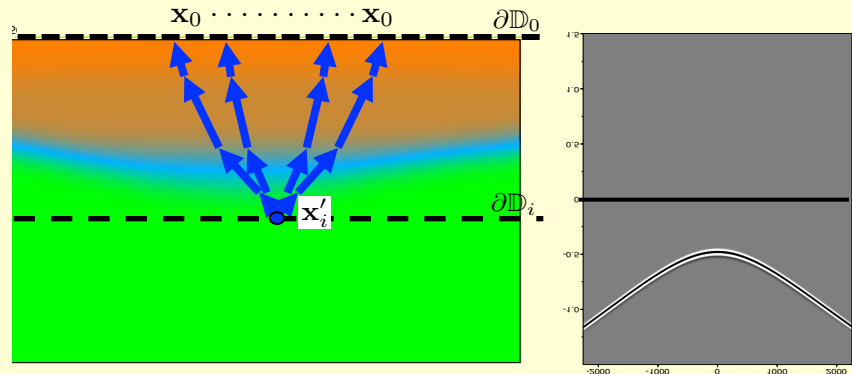
Reflection data



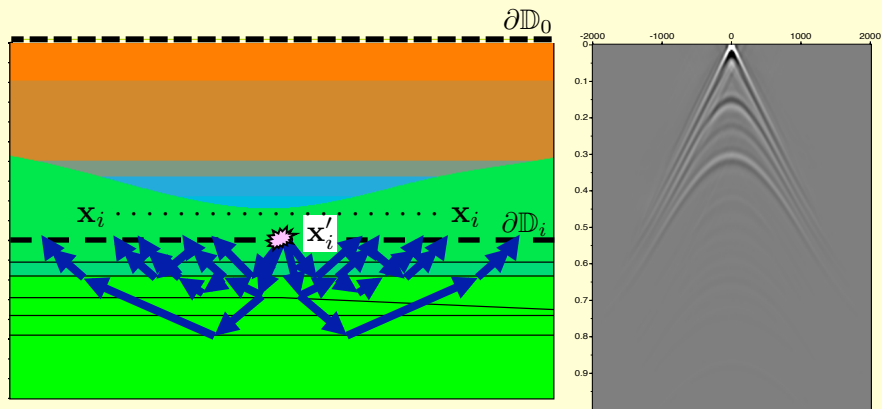
First arrivals



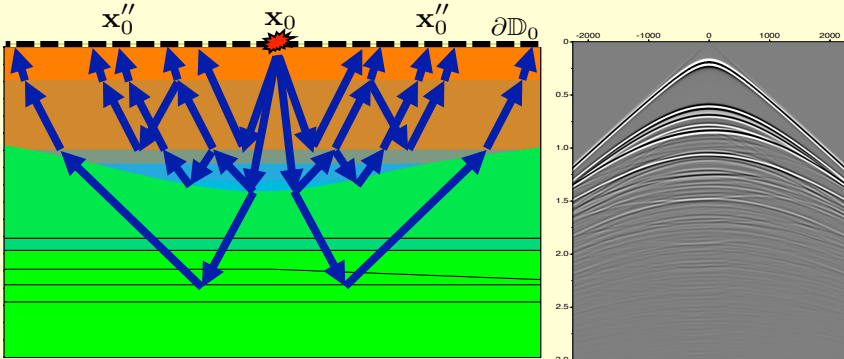
Reflection data



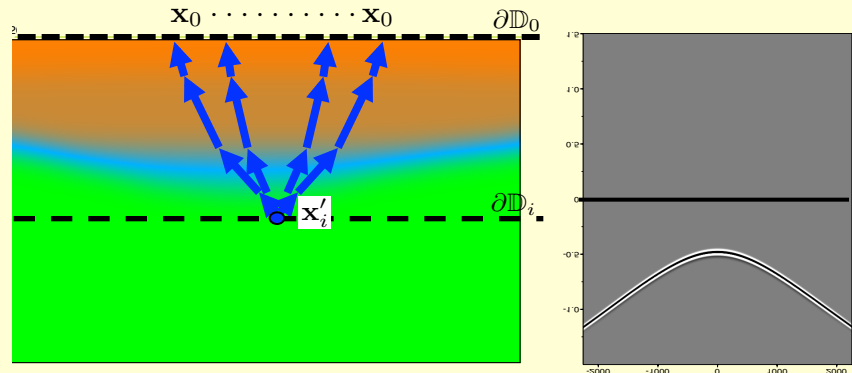
First arrivals



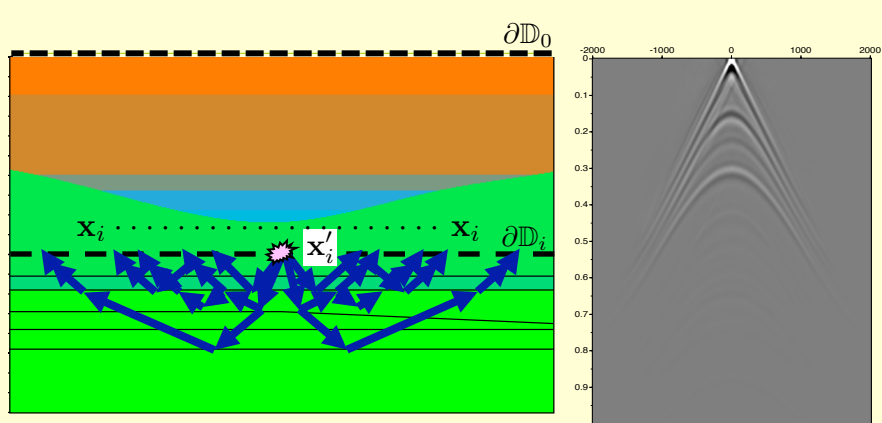
Data-driven redatuming, AVO analysis



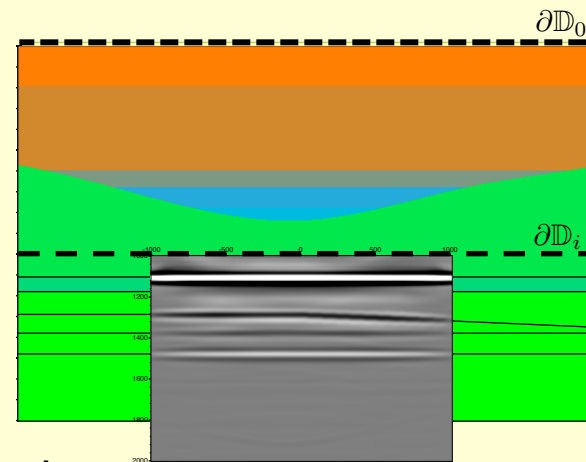
Reflection data



First arrivals



Data-driven redatuming, AVO analysis and imaging



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Issues for discussion

- Non-recursive (target oriented)

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Issues for discussion

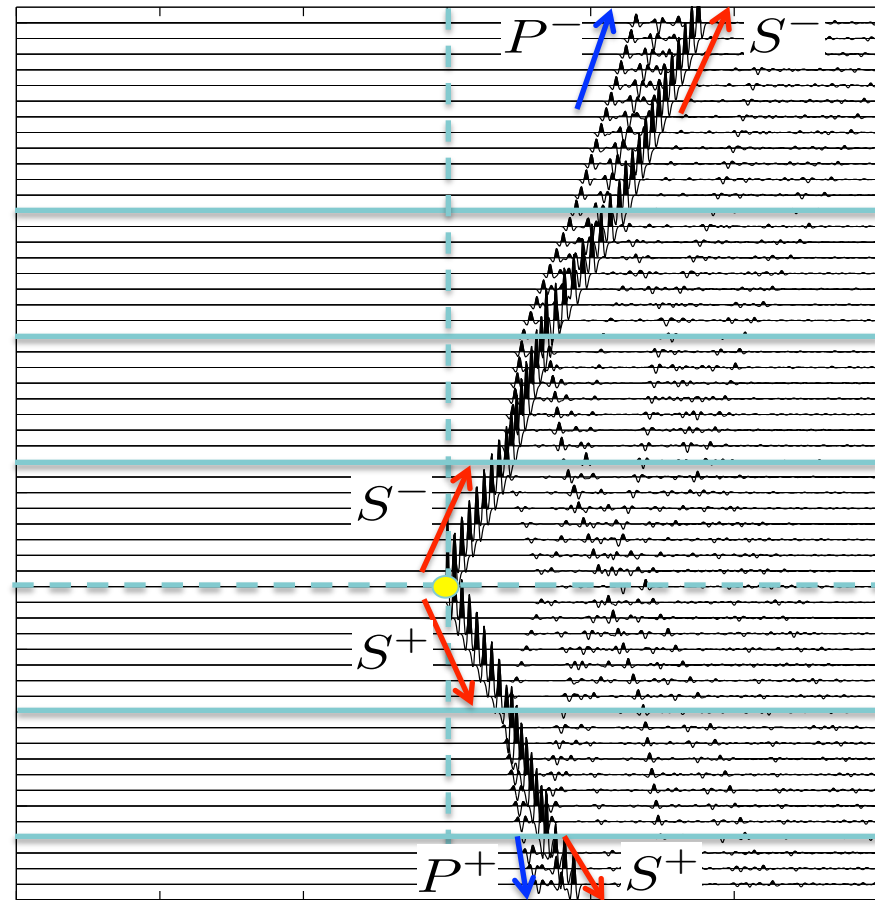
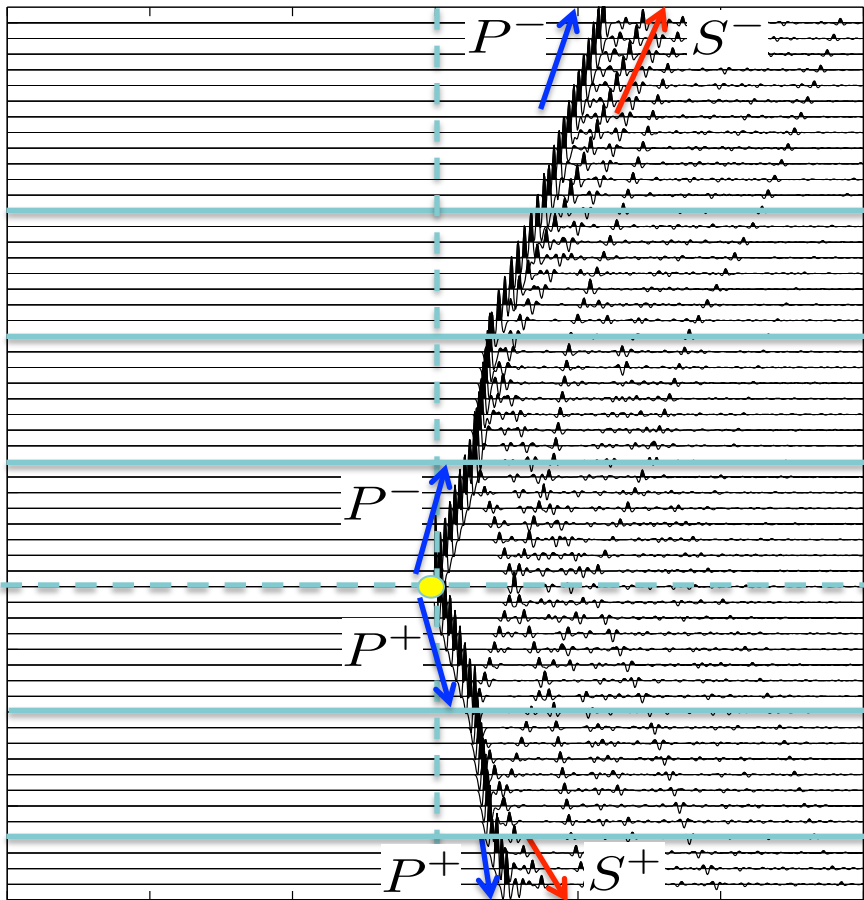
- Non-recursive (target oriented)
- Sampling issues (3D)
- Dissipation
- Adaptive prediction/subtraction

Issues for discussion

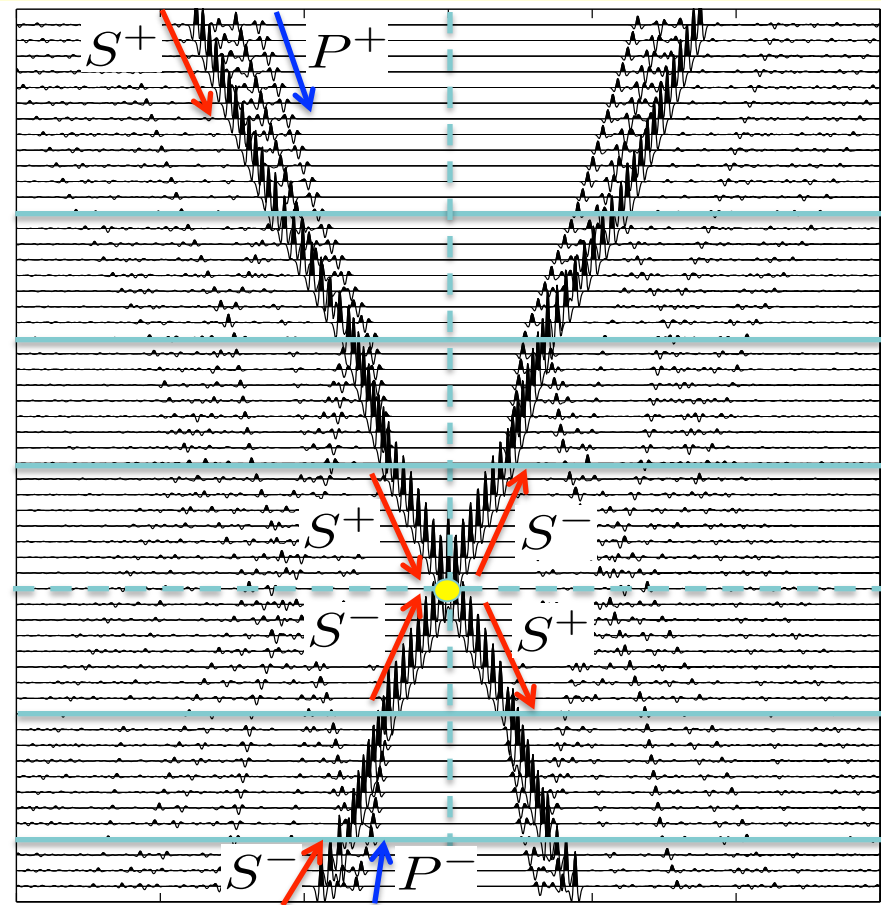
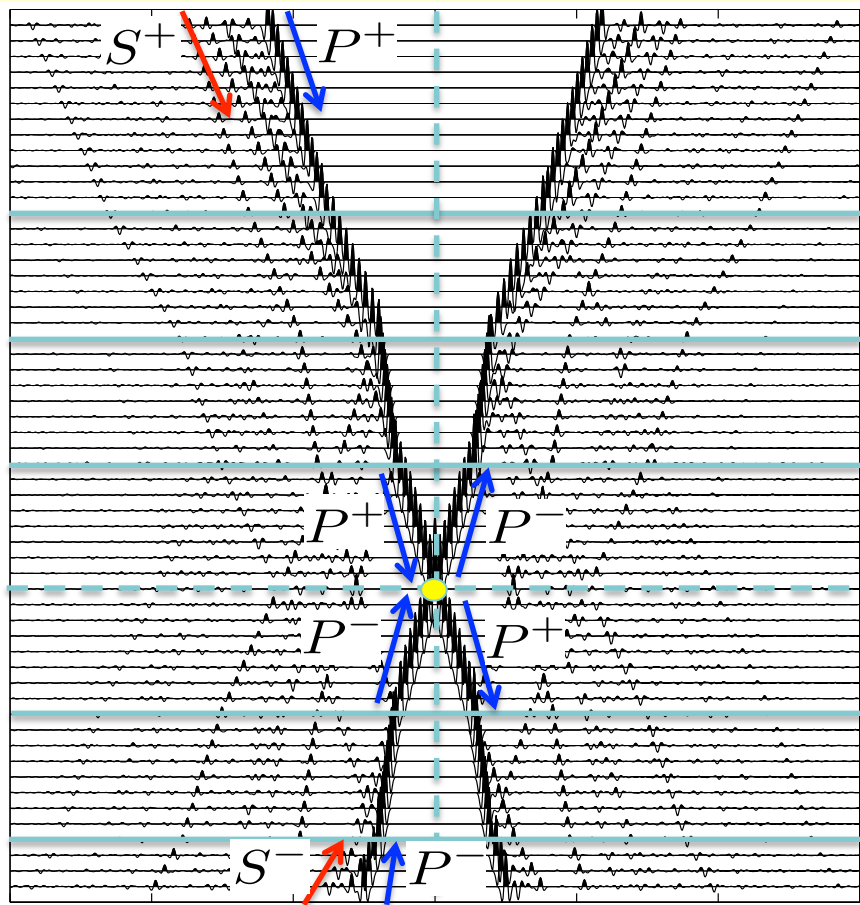
- Non-recursive (target oriented)
- Sampling issues (3D)
- Dissipation
- Adaptive prediction/subtraction
- Ambiguity direct arrival (refracted waves)

Issues for discussion

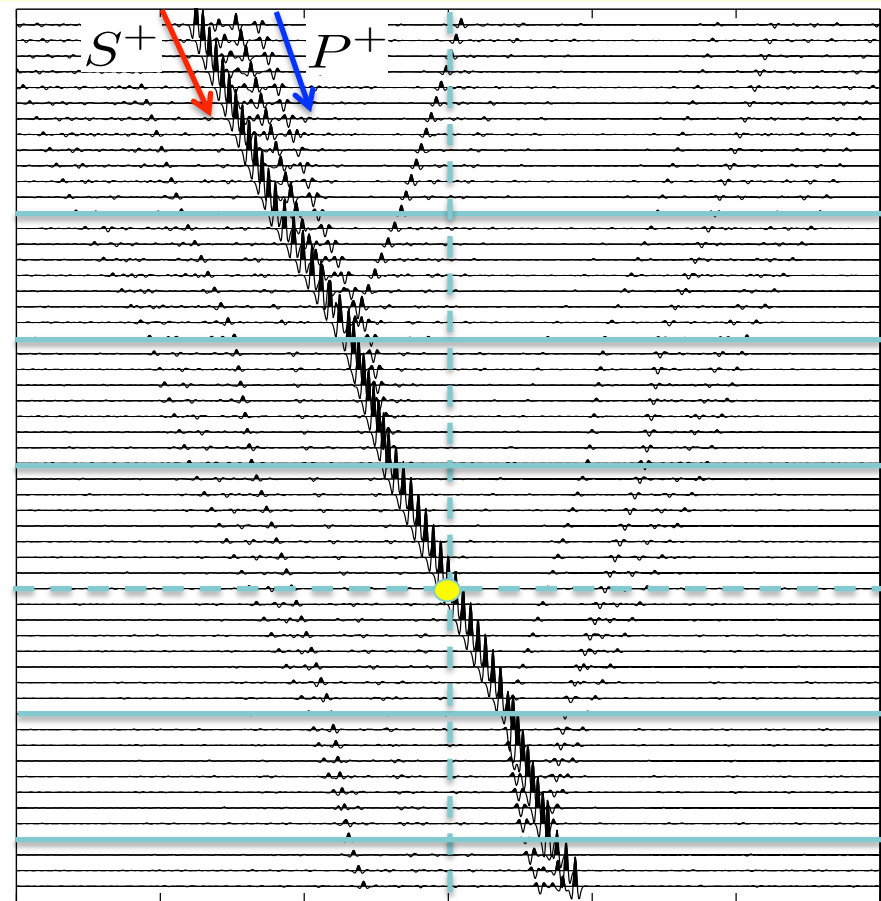
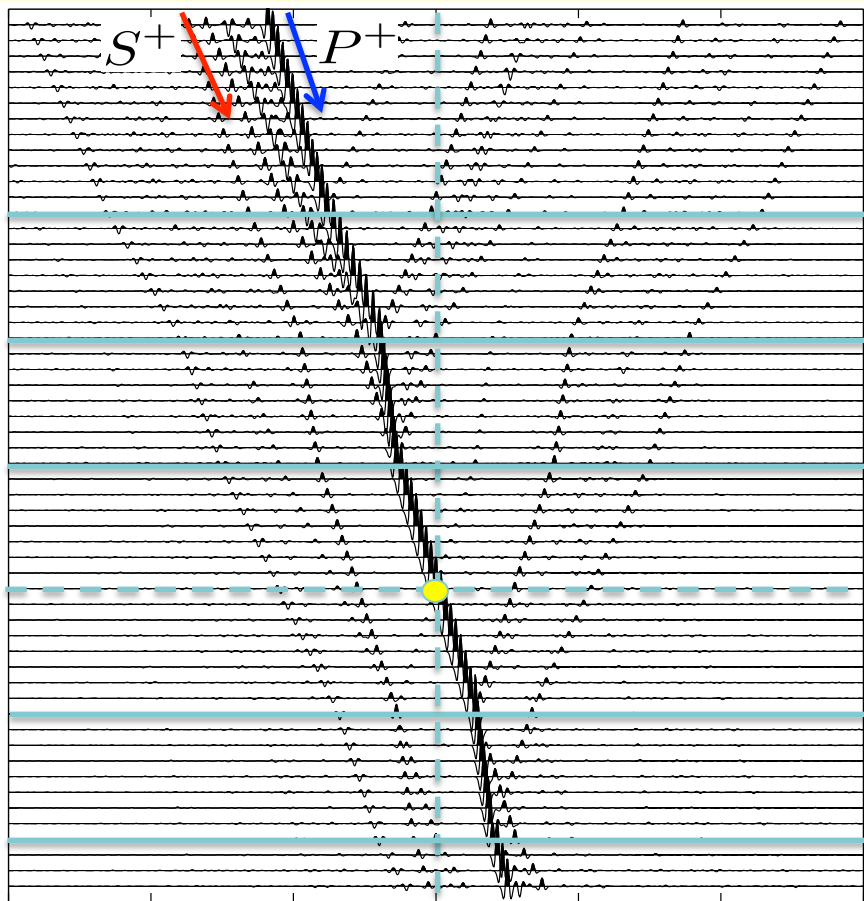
- Non-recursive (target oriented)
- Sampling issues (3D)
- Dissipation
- Adaptive prediction/subtraction
- Ambiguity direct arrival (refracted waves)
- Extension to elastodynamic waves



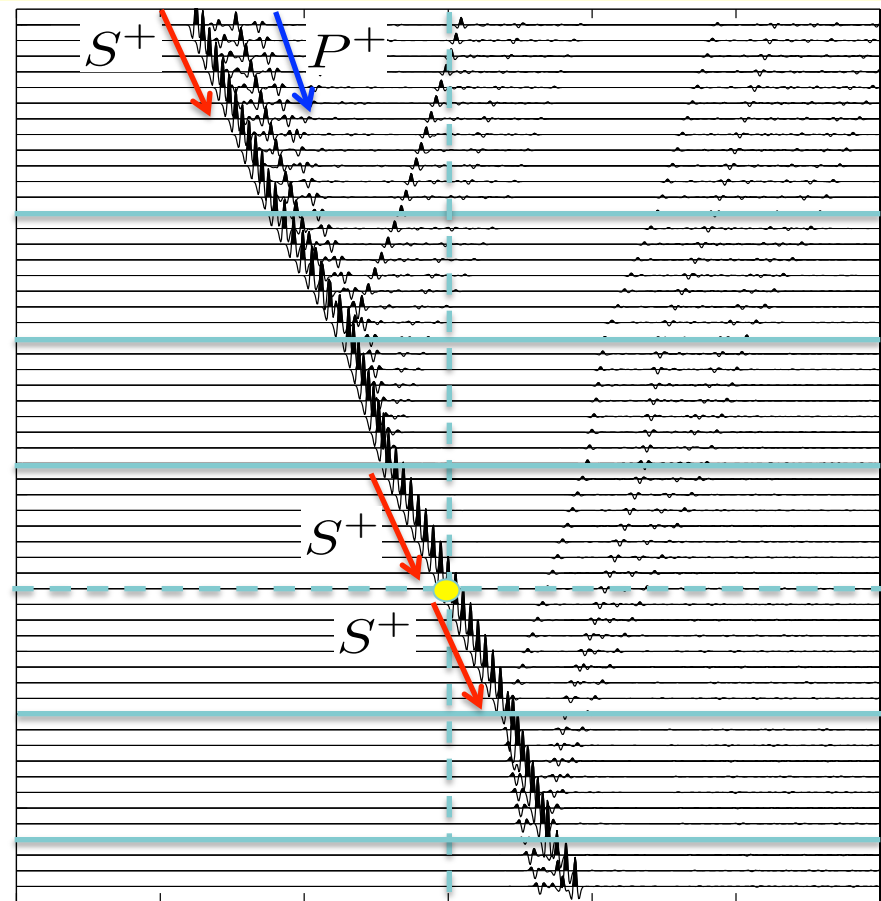
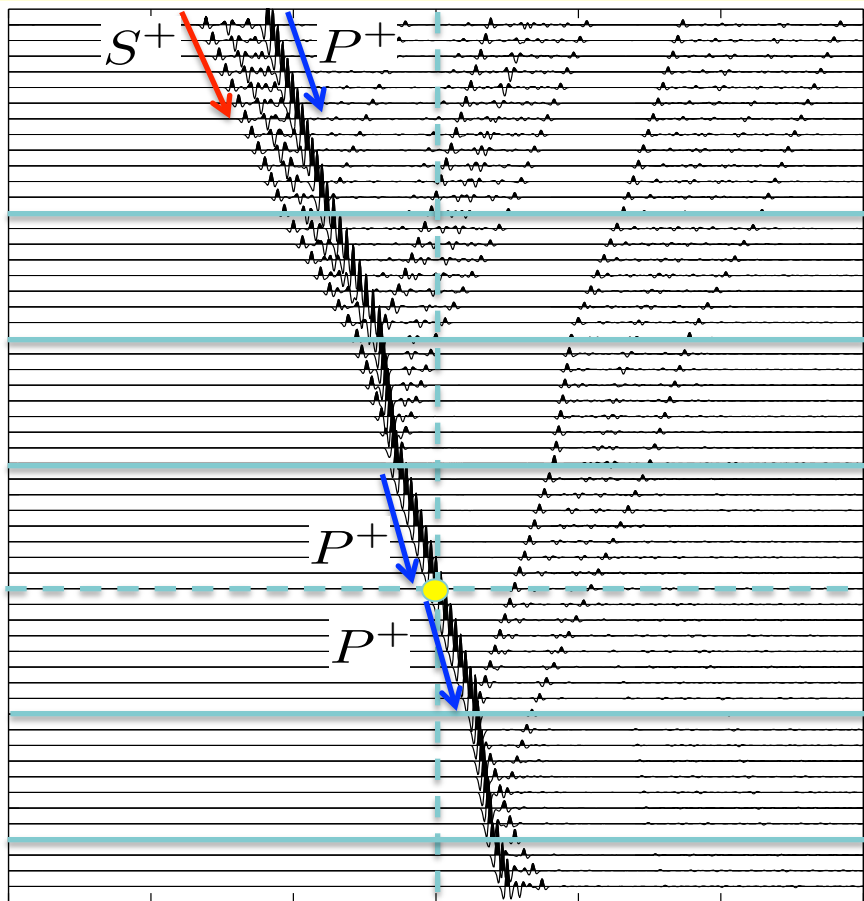
Elastodynamic Green's functions



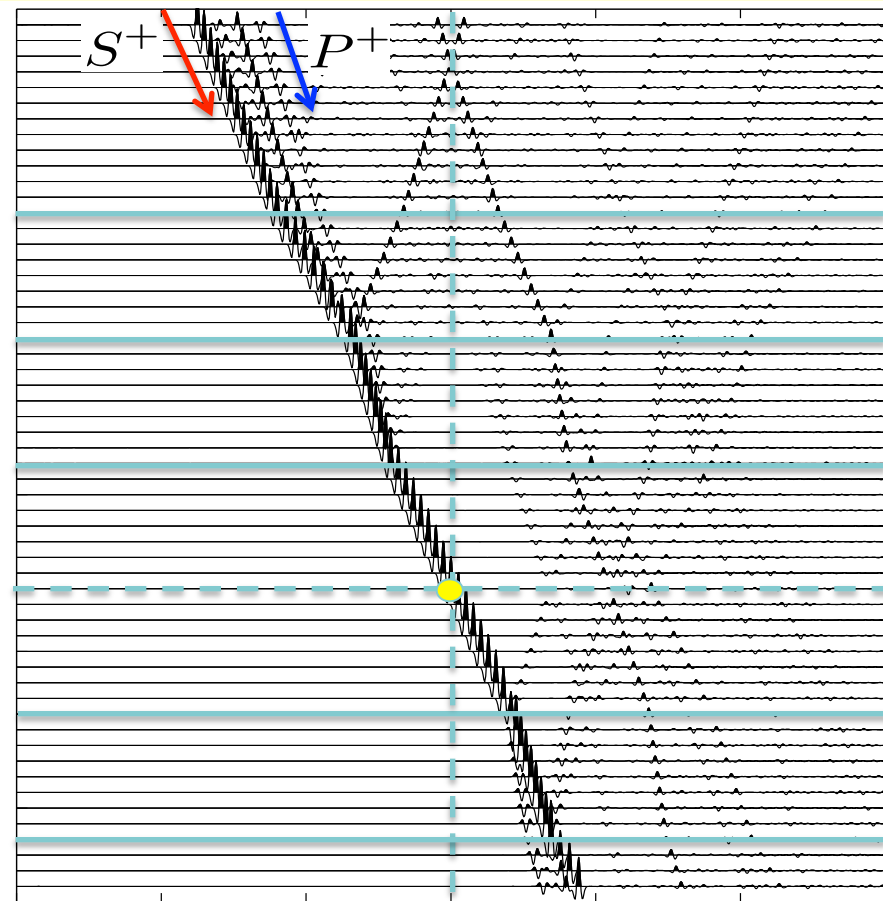
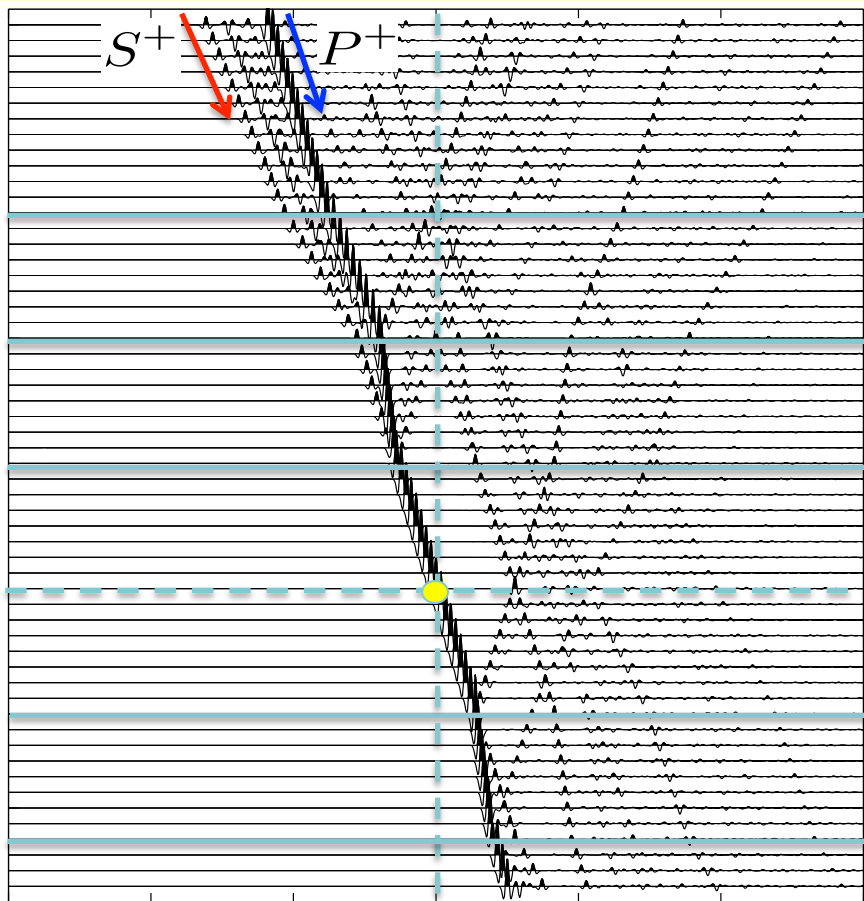
Time-reversal, two-sided illumination



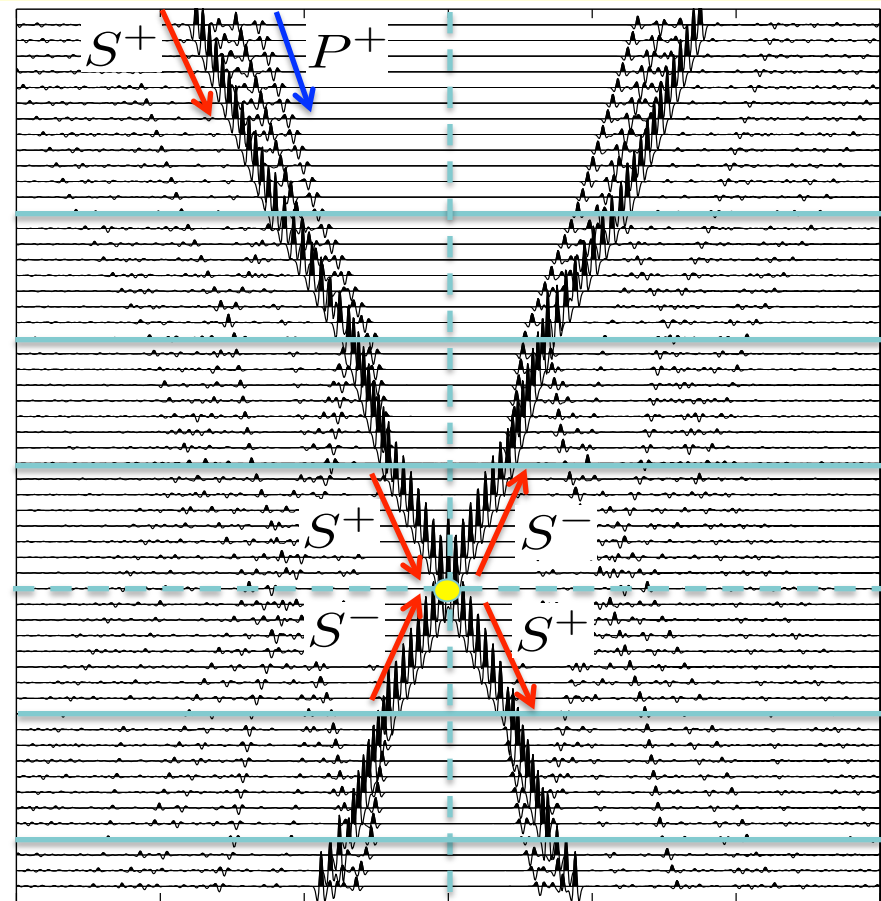
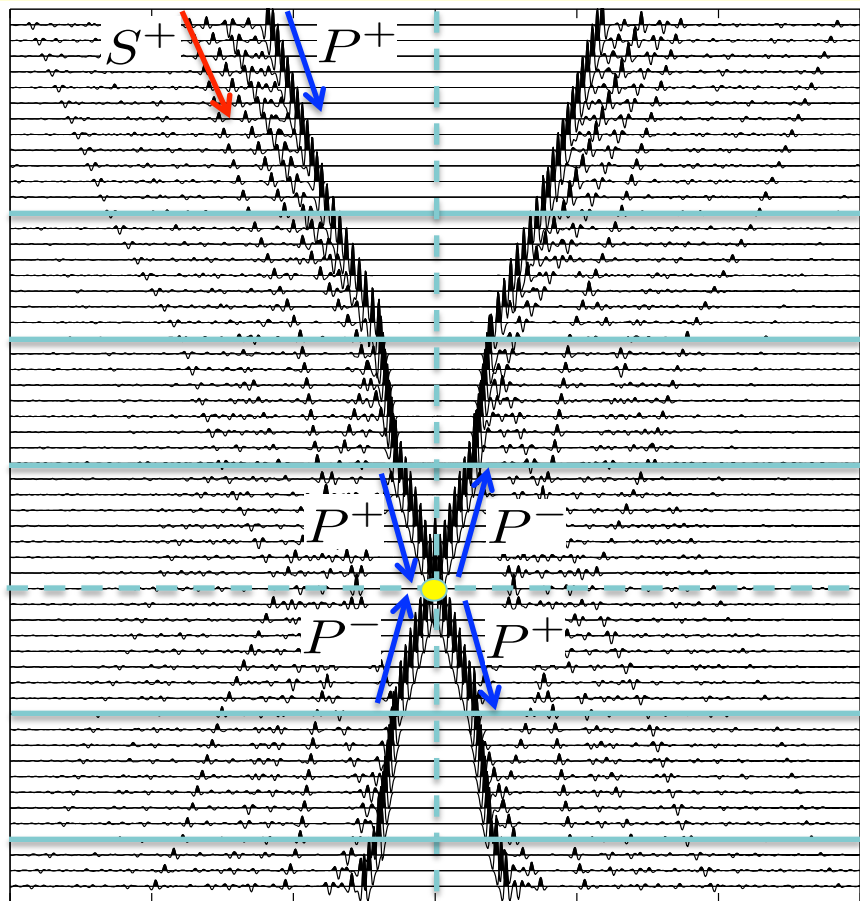
Time-reversal, one-sided illumination



Marchenko, one-sided, $\mathbf{F}_1^+(z_0, z_A, t) \cdot \mathbf{F}_1^-(z_0, z_A, -t)$



Marchenko, one-sided, $\mathbf{F}_1^+(z_0, z_A, t) - \mathbf{F}_1^-(z_0, z_A, -t)$



Marchenko, one-sided, plus its time-reversal