Advances and Challenges in understanding the amplitude in noise correlations

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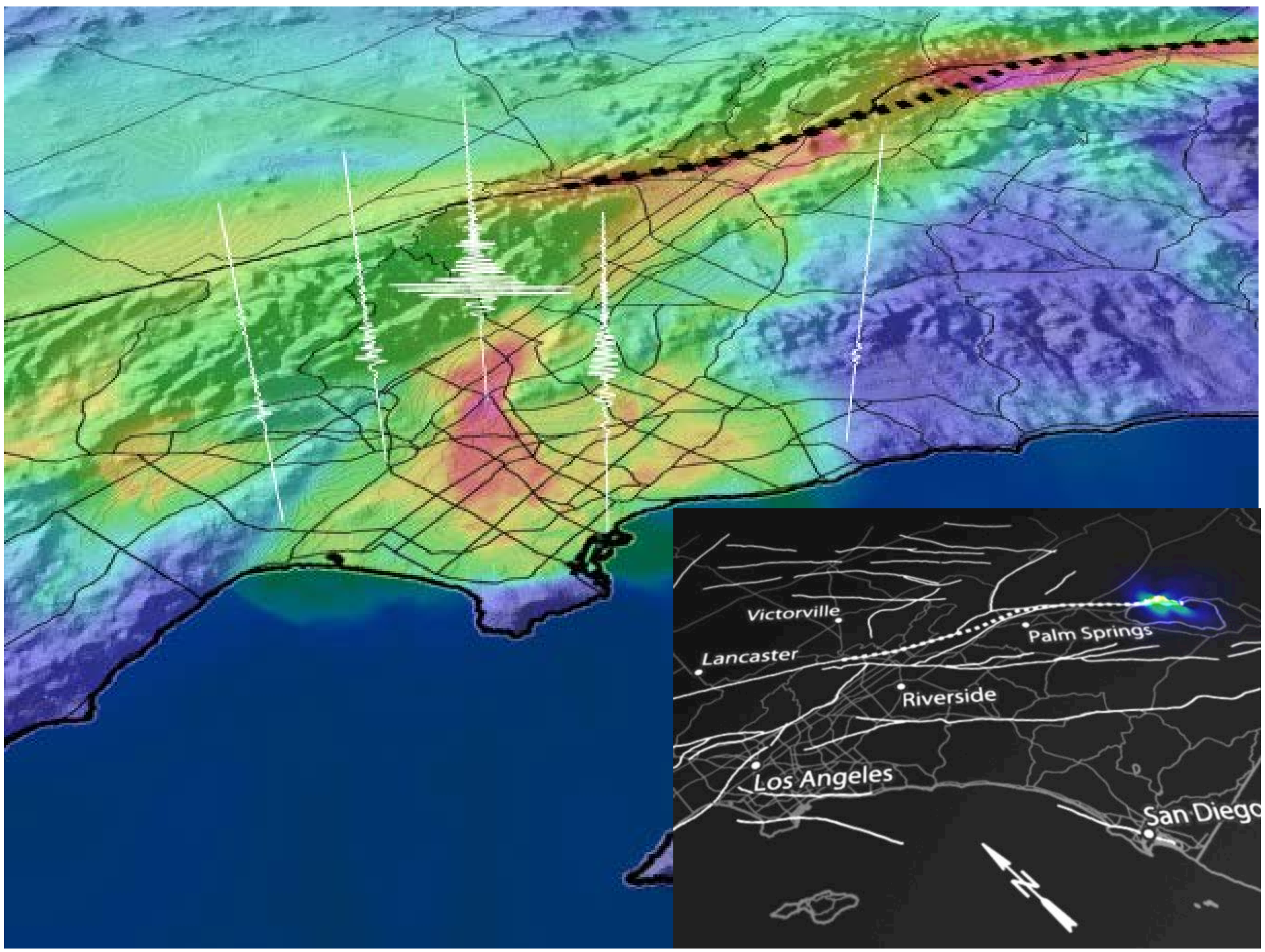
Earth, Atmospheric, and Planetary Sciences, MIT

Physics Department - Universidad de los Andes



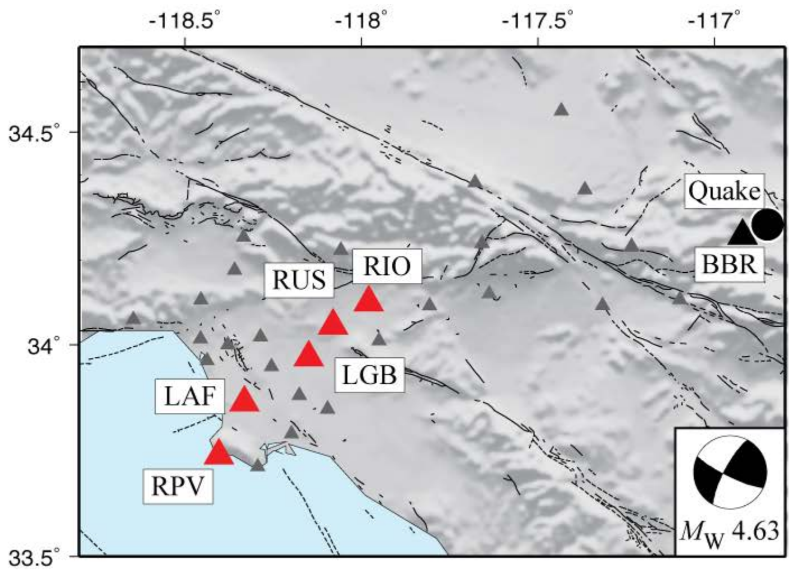
April 22, 2013

Cargese Summer School

Corsica, France

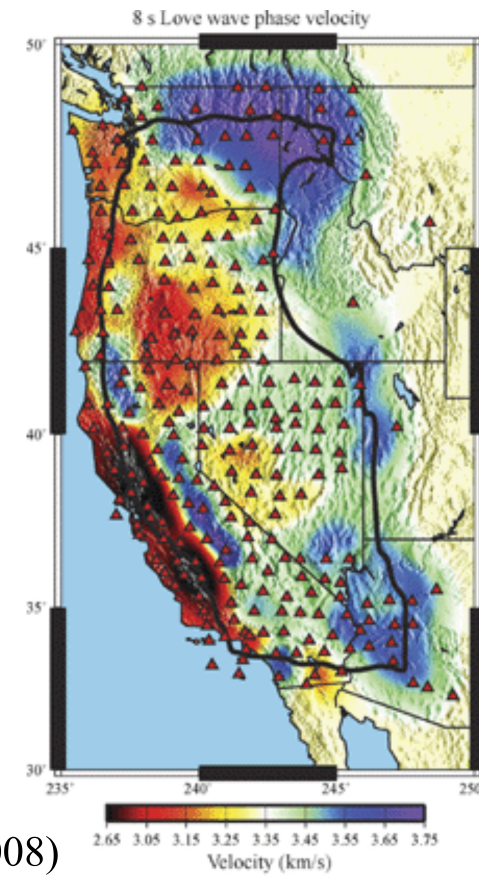
Scale of our problem

Local Scale
Building
10's meters



Local - Regional
Sedimentary Basins
10-100's km

Regional - Global
Western US
100-1000's km



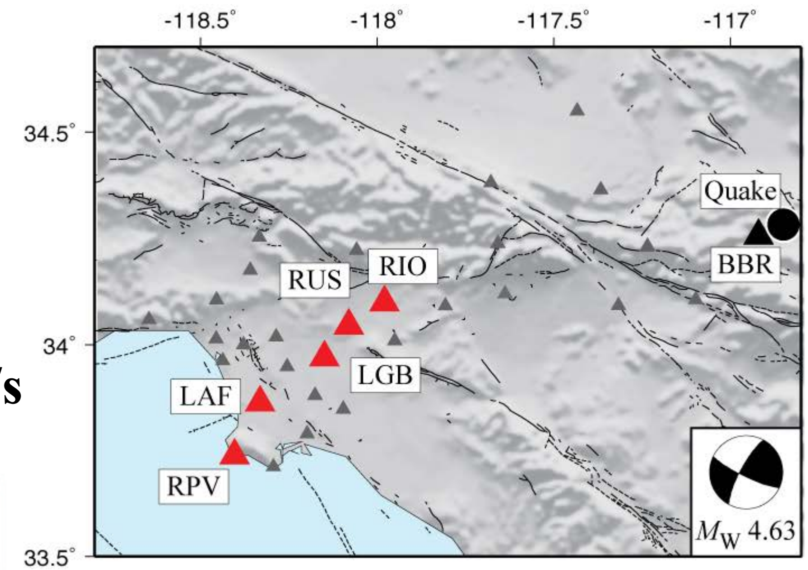
Lin et al. GJI (2008)

Scale of our problem

Local Scale

Freq: 0.5–5 Hz

Prop Vel 100 – 200 m/s



Local – Regional

Freq: 0.1 – 0.5 Hz

T: 2 – 10 sec

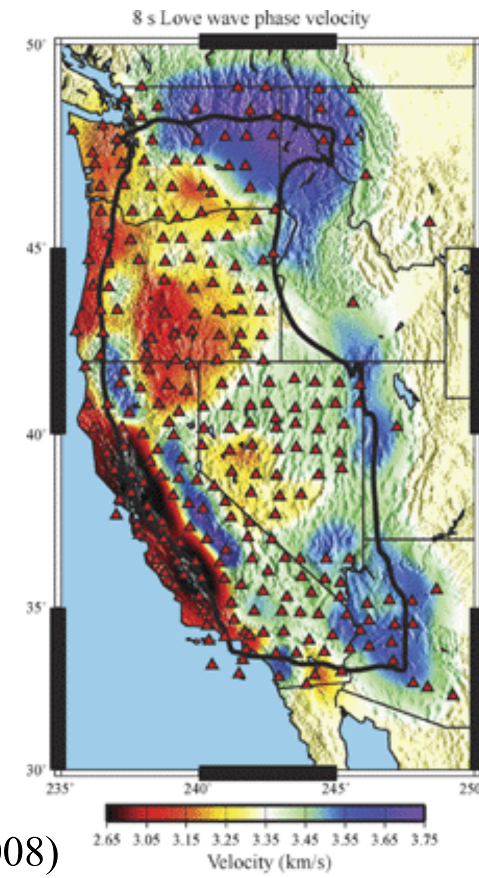
Prop Vel: ~ 3 km/s

Regional - Global

Freq: 0.125–0.025 Hz

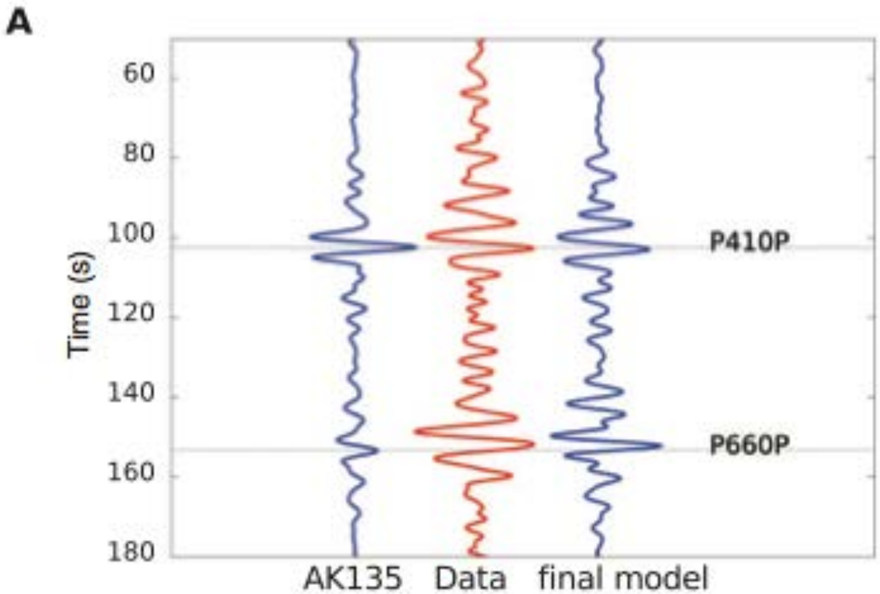
T: 8 – 40 sec

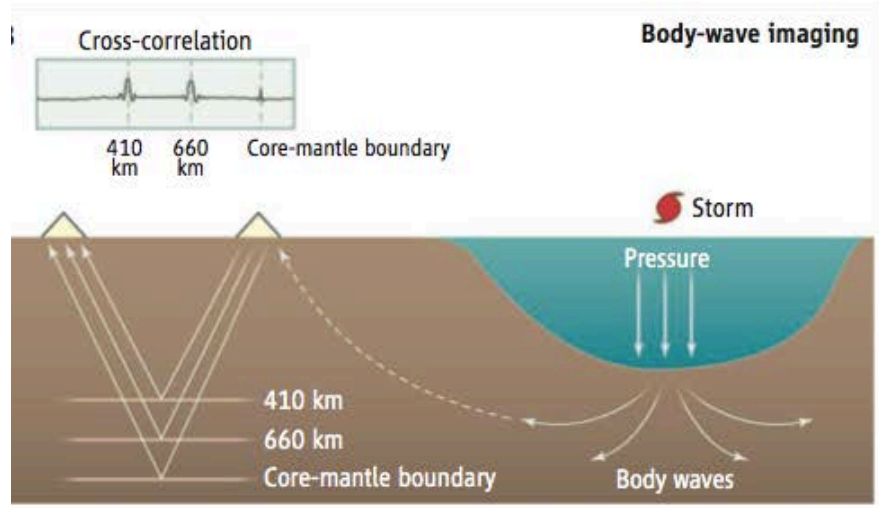
Prop Vel: 3 – 4 km/s



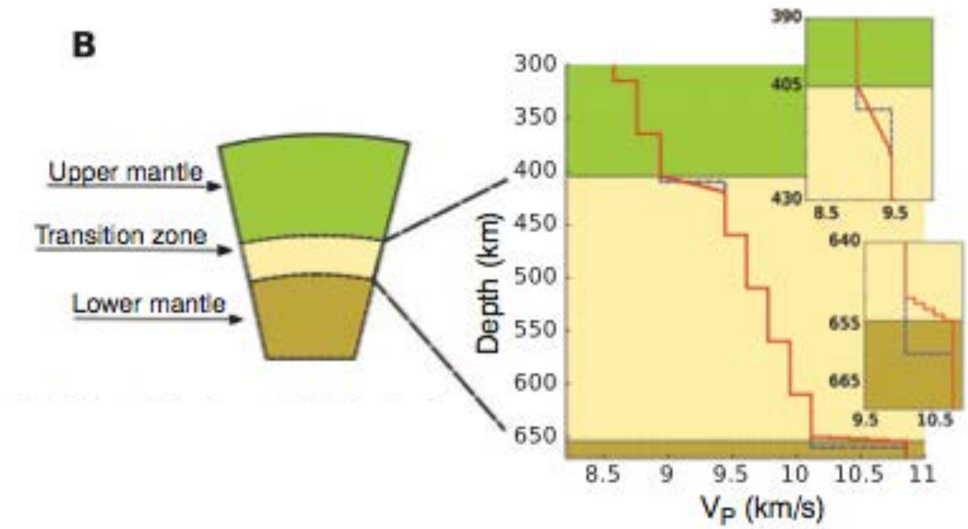
Lin et al. GJI (2008)

Of course, noise has gone global





Prieto, Science (2012)



Poli, et al., Science (2012)

Introduction

Amplification and Attenuation

Tomographic Imaging of Elastic Structure

Higher resolution with increase data coverage

Seismic attenuation tomography has lagged behind

More difficult.

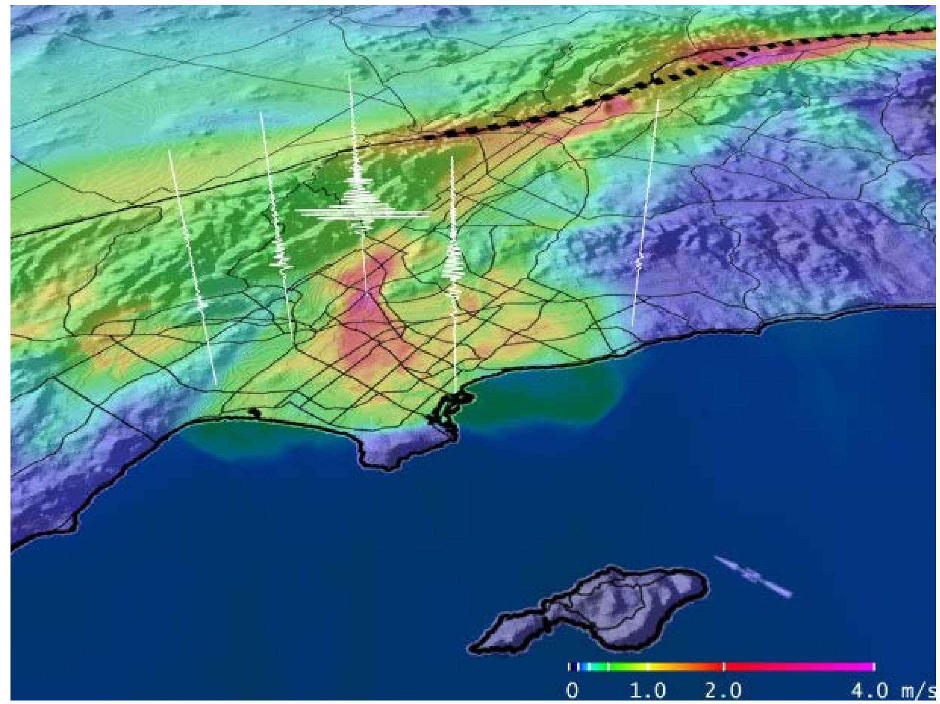
Amplitude of seismic waves affected by 3D velocity, multi-pathing, scattering, and source.

Why is this important?

Amplification and Attenuation

Recent ground motion simulations suggest wave guide by sedimentary basins and large amplitudes in the LA Basin.

Basin for major cities (Tokyo, LA, Mexico City, Bogotá, Colombia)



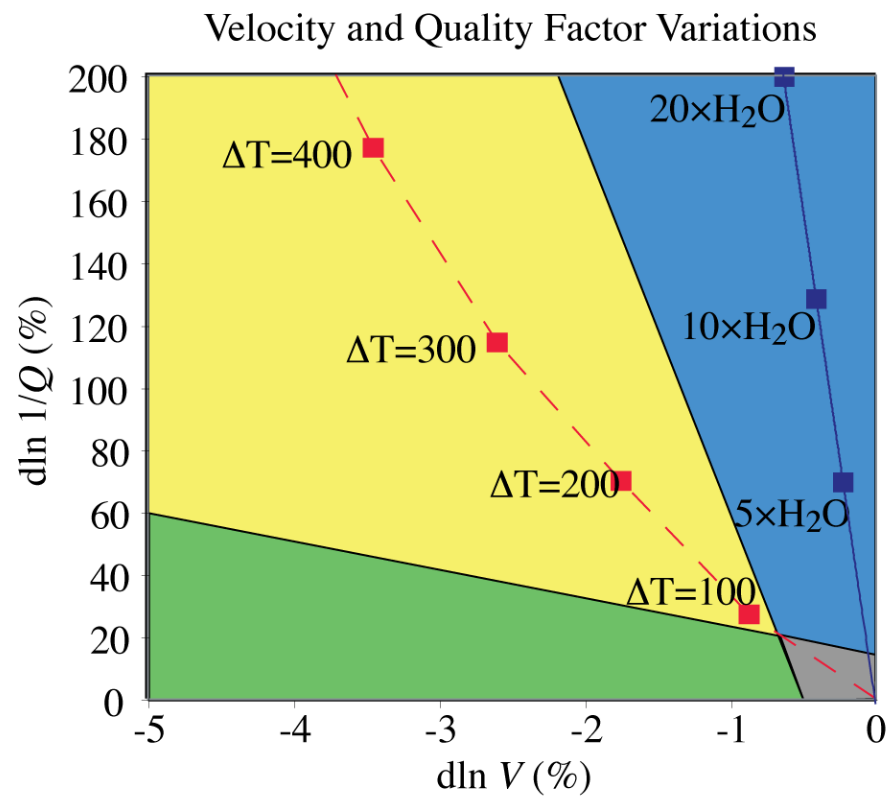
How can we validate these simulations?

Why is this important?

Amplification and Attenuation

Attenuation provides relevant physical properties of Earth.

Very sensitive to temperature
Presence of fluids (melt or water)
Tectonic activity (E,W US)



[*Lawrence & Wysession*., 2005: AGU Monograph; *Karato*, 2003: AGU Monograph]

How can we validate these simulations?
Can we use amplitudes to monitor changes in the media?

Amplitude information

Is there reliable amplitude information in noise correlations?

Geometrical Spreading
Basin Amplification
Attenuation
…

YES

(under some conditions)
STILL DEBATED HOW

… substantial and competing amplitude anomalies due to elastic and anelastic variations … (Savage et al., 2010)

For today …

Is there reliable amplitude information in noise correlations?

1. First suggestions on amplitude information
2. Some theoretical questions
3. A numerical example
4. Two recent success stories
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… substantial and competing amplitude anomalies due to elastic and anelastic variations … (Savage et al., 2010)

Ambient noise Amplitudes and Attenuation

results of previous studies are encouraging

Weaver and Lobkis, 2001

Larose et al., 2007

Prieto and Beroza, 2008

Matzel, 2008

Prieto et al., 2009

Taylor et al., 2009

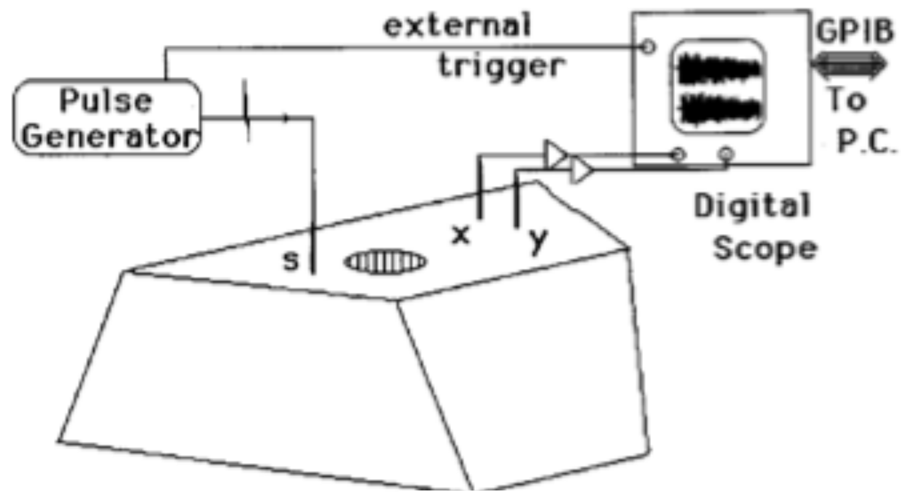
Cupillard and Capdeville, 2010

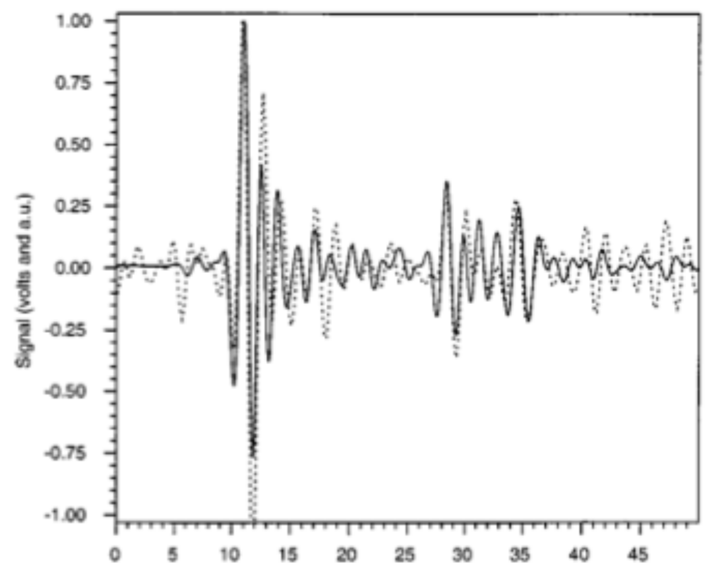
Lawrence and Prieto, 2011

Lin et al., 2011; 2012

Weaver, 2011a; 2011b; 2013

Prieto et al., 2011





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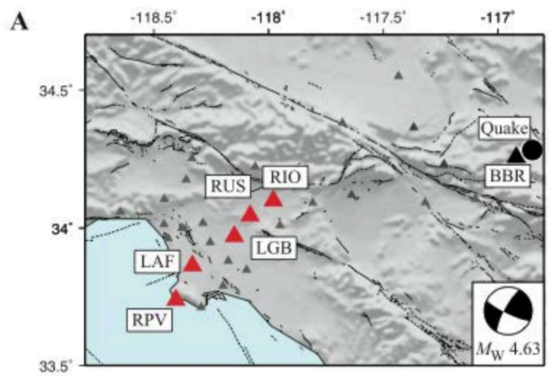

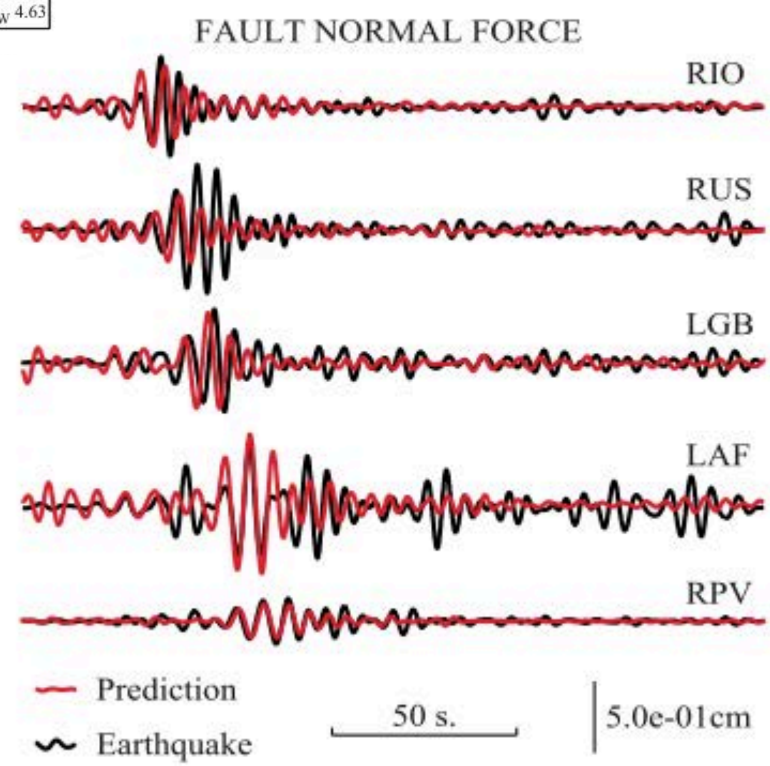

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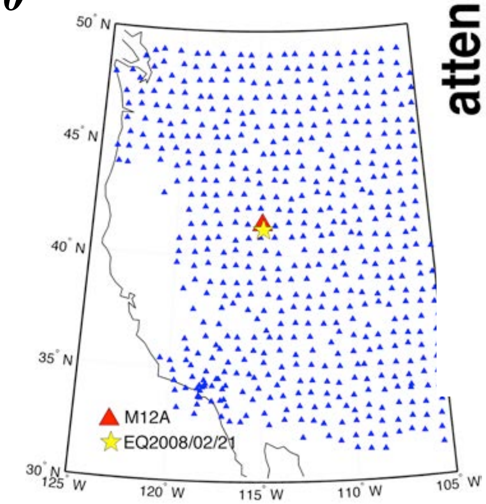

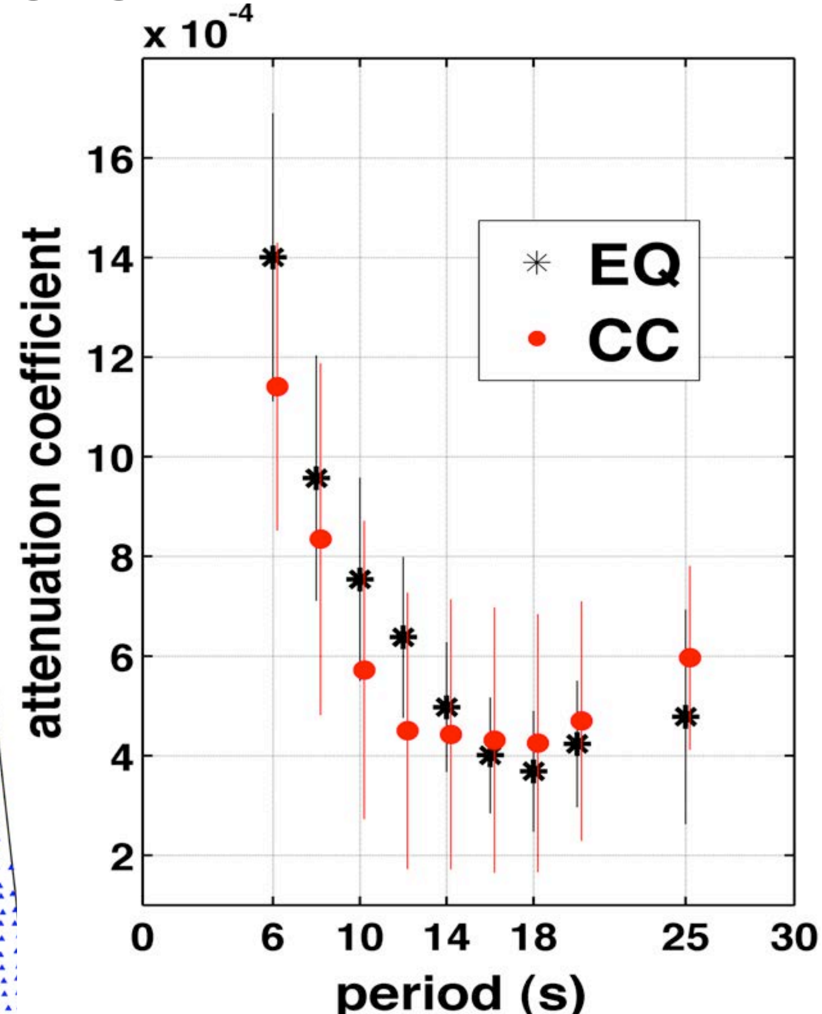

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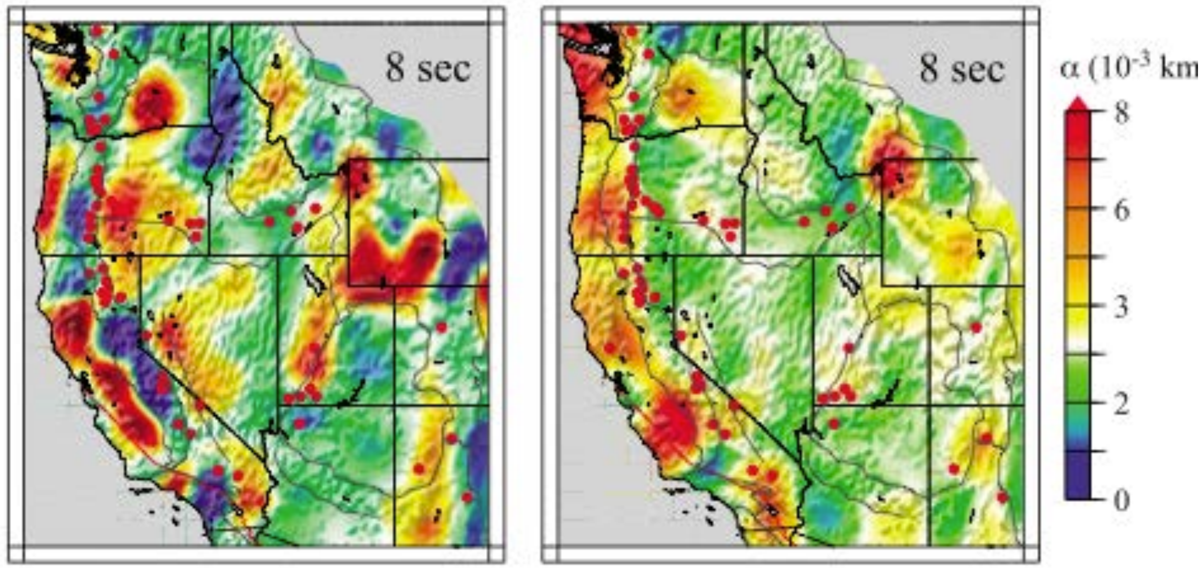

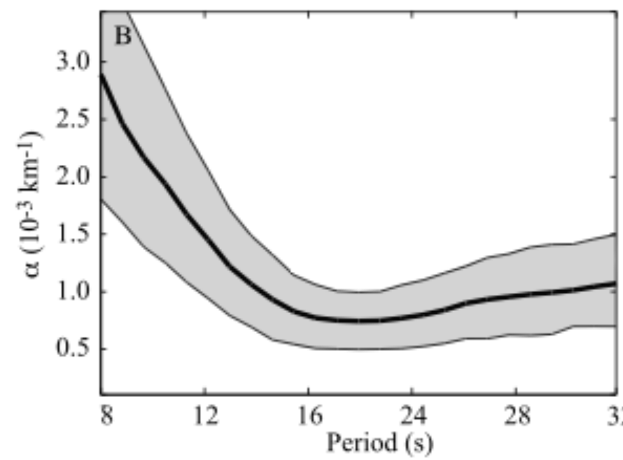

For today …

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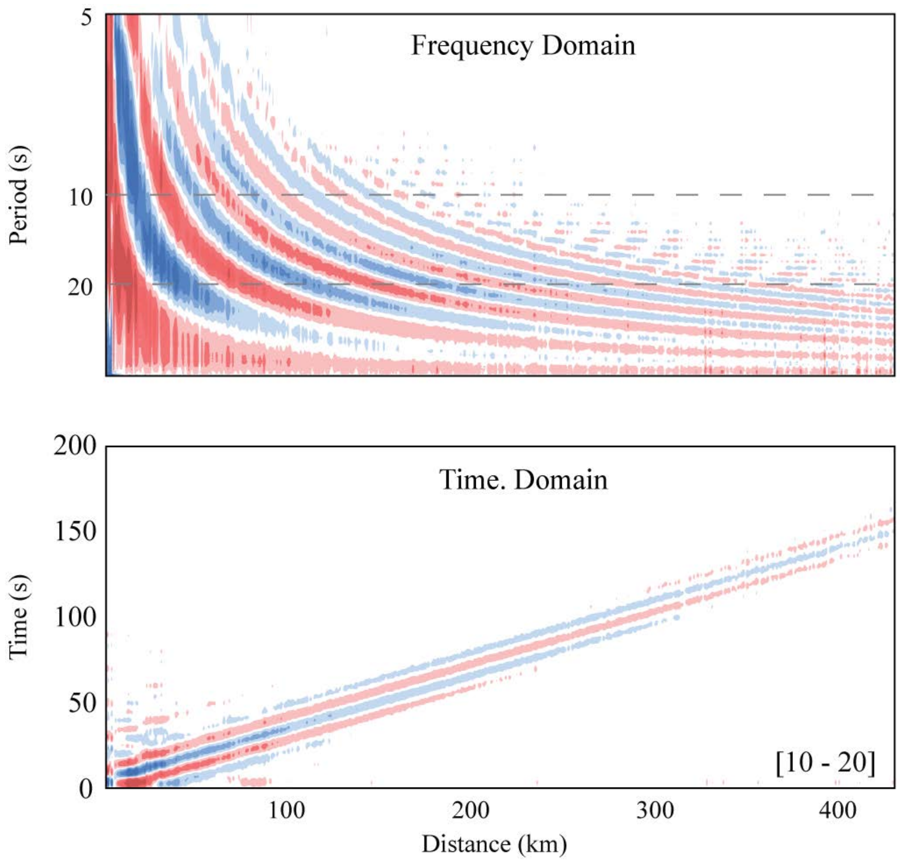
… substantial and competing amplitude anomalies due to elastic and anelastic variations … (Savage et al., 2010)

Correlations - $e^{\alpha r}$

Based on seismic observations, we suggested



Prieto et al. CRG (2011)

Aki (1957) SPAC Method

Freq: $\mathrm{Re}\left[\gamma_{AB}(\omega,r)\right] = \left\langle \frac{u_A \, u_B^*}{\langle |u_A| \rangle \langle |u_B| \rangle} \right\rangle = J_0\left(\frac{\omega r}{C}\right)$

C = Phase Velocity

Colin de Verdière (2006)

Time: $\frac{\partial}{\partial \tau}\langle C_{AB}(\tau)\rangle = \frac{-\sigma^2}{4a}\left(G_{AB}(\tau) - G_{AB}(-\tau)\right)$

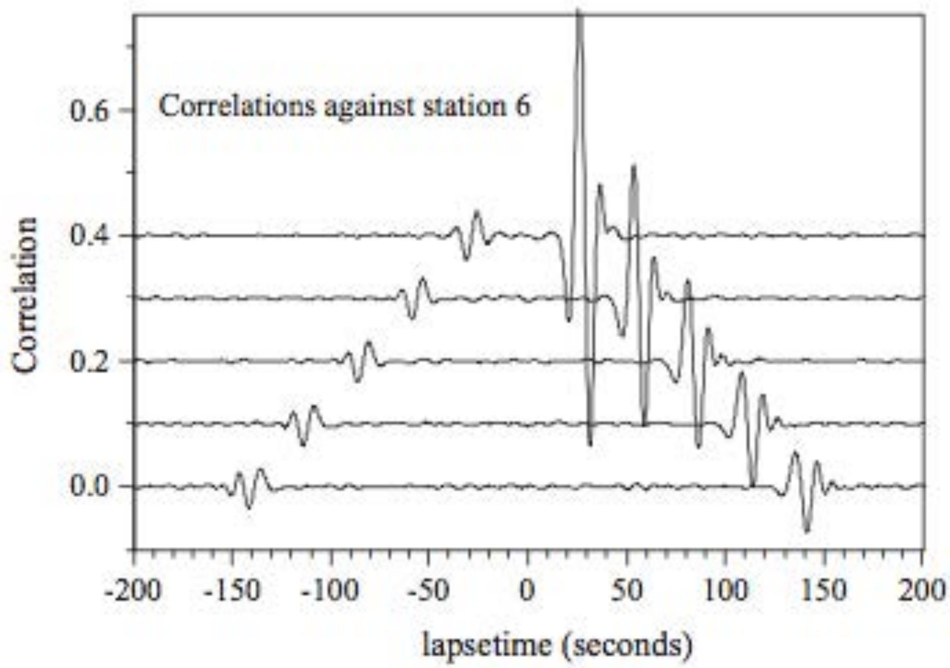
$$\mathrm{Re}\left[\gamma_{AB}(\omega,r)\right] = J_0\left(\frac{\omega r}{C}\right)\exp(-\alpha r)$$

Theory – Weaver et al.

Amplitude information present, but not simple $e^{\alpha r}$

Inference of attenuation from comparison of empirical coherencies to Jo(ωr/c) exp(-α r) is problematic (Weaver, CRG).

Site factors, noise intensity (directivity) and attenuation all have an effect on amplitude measurements



$$X_{i\to j} = s_i s_j B_i(\hat{\boldsymbol{n}}_{i\to j})\sqrt{2\pi c/\omega\ |\vec{r}_i - \vec{r}_j|}\ \exp\left(-\ \left|\ \int_{\vec{r}_j}^{\vec{r}_i} \alpha dx\ \right|\right)$$

Theory – Tsai

Behavior of $e^{-\alpha r}$ only in special cases

$$\hat{C}^{E}_{xy} = \frac{1}{I_0(\alpha r_{xy})} J_0\left(\frac{\omega r_{xy}}{c}\right).$$

Uniform distribution of far-field sources

$$\hat{C}^{E}_{xy} = \frac{A_\theta^2}{\overline{A}^2} e^{-\alpha r_{xy}} J_0\left(\frac{\omega r_{xy}}{c}\right).$$

Including far and near-field source

Theory – Nakahara

SPAC method in disipative media (Nakahara, GJI, 2013)

Analysis of the SPAC expressions for 2-D cases shows that the conjecture of *Prieto et al*. (2009) is not strict but approximately good for small attenuation.

$$C_{1,2}(\mathbf{r}, \omega) = J_0(k_0 r) \quad \text{for} \quad \text{2-D},$$

$$C_{1,2}(\mathbf{r}, \omega) = \frac{\sin(k_0 r)}{k_0 r} \quad \text{for} \quad \text{3-D}.$$

Non-dissipative media

$$C_{1,2}(\mathbf{r}, \omega) \approx \exp(-\kappa r)\left[J_0(k_0 r)\left(1 + \frac{2\kappa}{\pi k_0}\right) + \frac{\kappa}{2k_0} Y_0(k_0 r)\right].$$

$$C_{1,2}(\mathbf{r}, \omega) = \frac{\sin k_0 r}{k_0 r} \exp(-\kappa r).$$

Dissipative, assuming large separations and small attenuation

Theory – Nakahara

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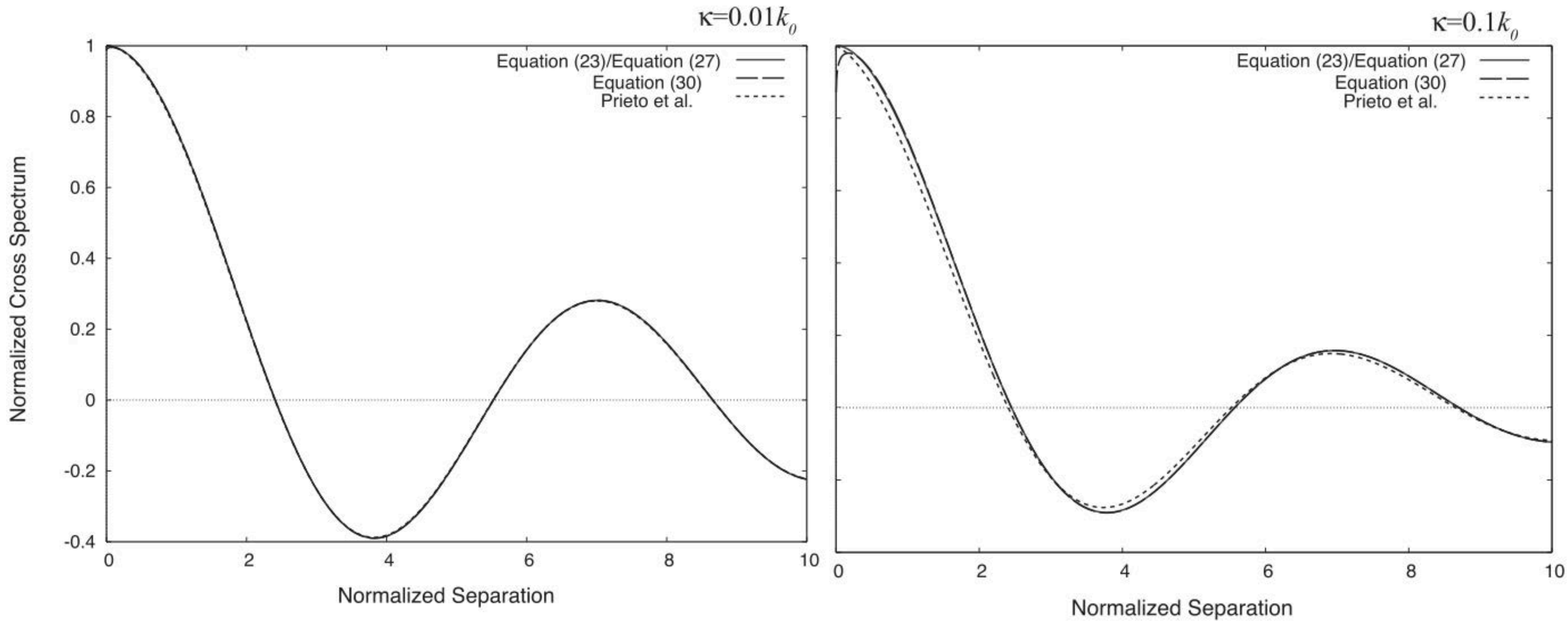

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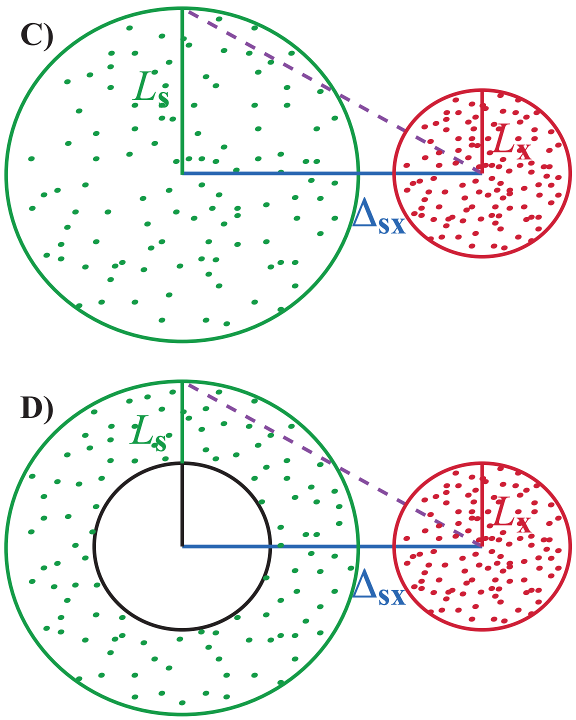
Numerical Simulation

Lawrence et al. In prep.



Do simulations support our claim?

We generate

- random array of sensors (red)
- random location of "noise" sources
- 3 months of synthetic data

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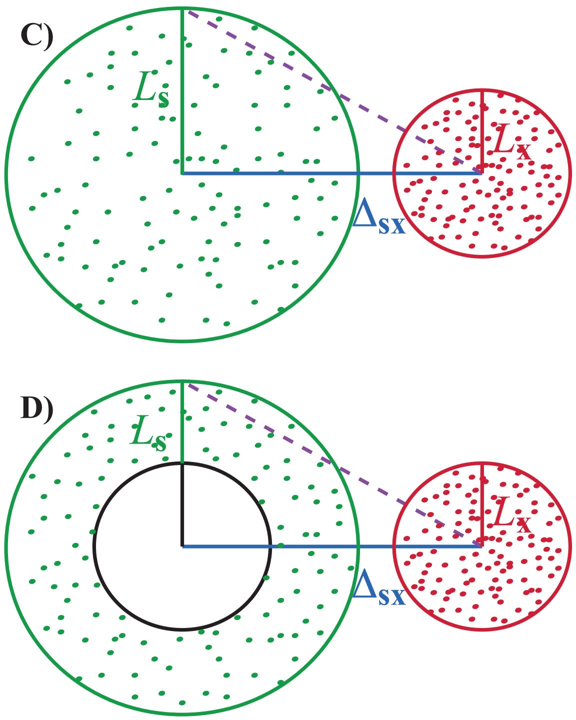

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$$u(\mathbf{x},\mathbf{s}) = \int_A F(\mathbf{s},\omega) e^{i\omega t} e^{-ikr_{sx}} e^{-\alpha(\omega) r_{sx}} r^{-0.5} dA$$

$F(\mathbf{s},\omega)$ – source at location $\boldsymbol{s}$

α - attenuation coefficient

r_{sx} – source receiver distance

A – integration over sources

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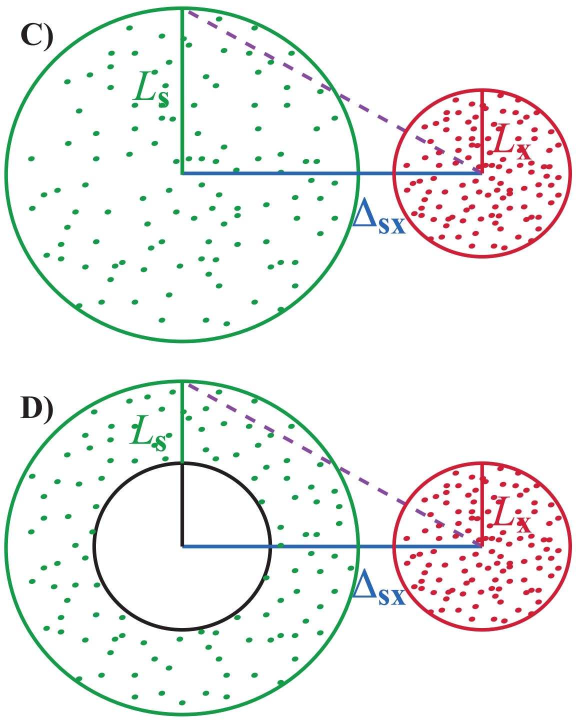

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$$\bar{\gamma}_{\mathbf{xy}}(\omega) = \left\langle \frac{\sum_{j=1}^{N_s} F(\mathbf{s}_j,\omega) e^{i\omega t} e^{-ikr_{\mathbf{s}_j\mathbf{x}}} e^{-\alpha(\omega) r_{\mathbf{s}_j\mathbf{x}}} r^{-0.5} \sum_{j=1}^{N_s} F(\mathbf{s}_j,\omega) e^{i\omega t} e^{-ikr_{\mathbf{s}_j\mathbf{y}}} e^{-\alpha(\omega) r_{\mathbf{s}_j\mathbf{y}}} r^{-0.5}}{\left\{\left|\sum_{j=1}^{N_s} F(\mathbf{s}_j,\omega) e^{i\omega t} e^{-ikr_{\mathbf{s}_j\mathbf{x}}} e^{-\alpha(\omega) r_{\mathbf{s}_j\mathbf{x}}} r^{-0.5}\right|\right\}\left\{\left|\sum_{j=1}^{N_s} F(\mathbf{s}_j,\omega) e^{i\omega t} e^{-ikr_{\mathbf{s}_j\mathbf{y}}} e^{-\alpha(\omega) r_{\mathbf{s}_j\mathbf{y}}} r^{-0.5}\right|\right\}} \right\rangle$$

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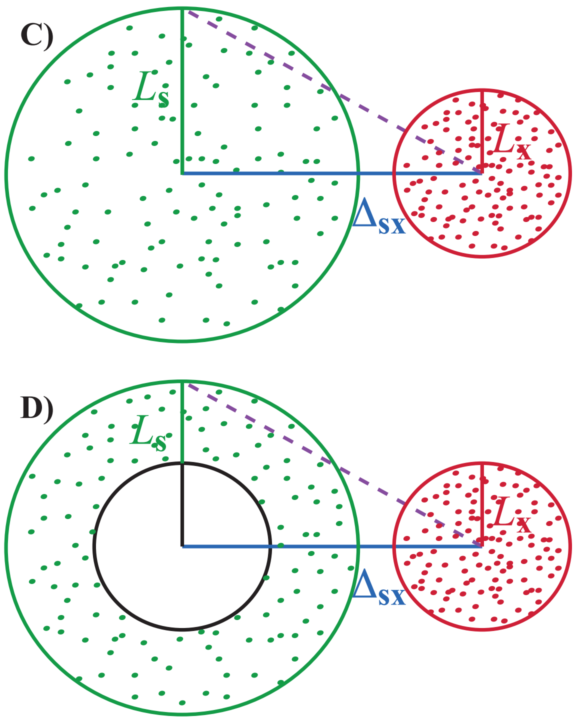
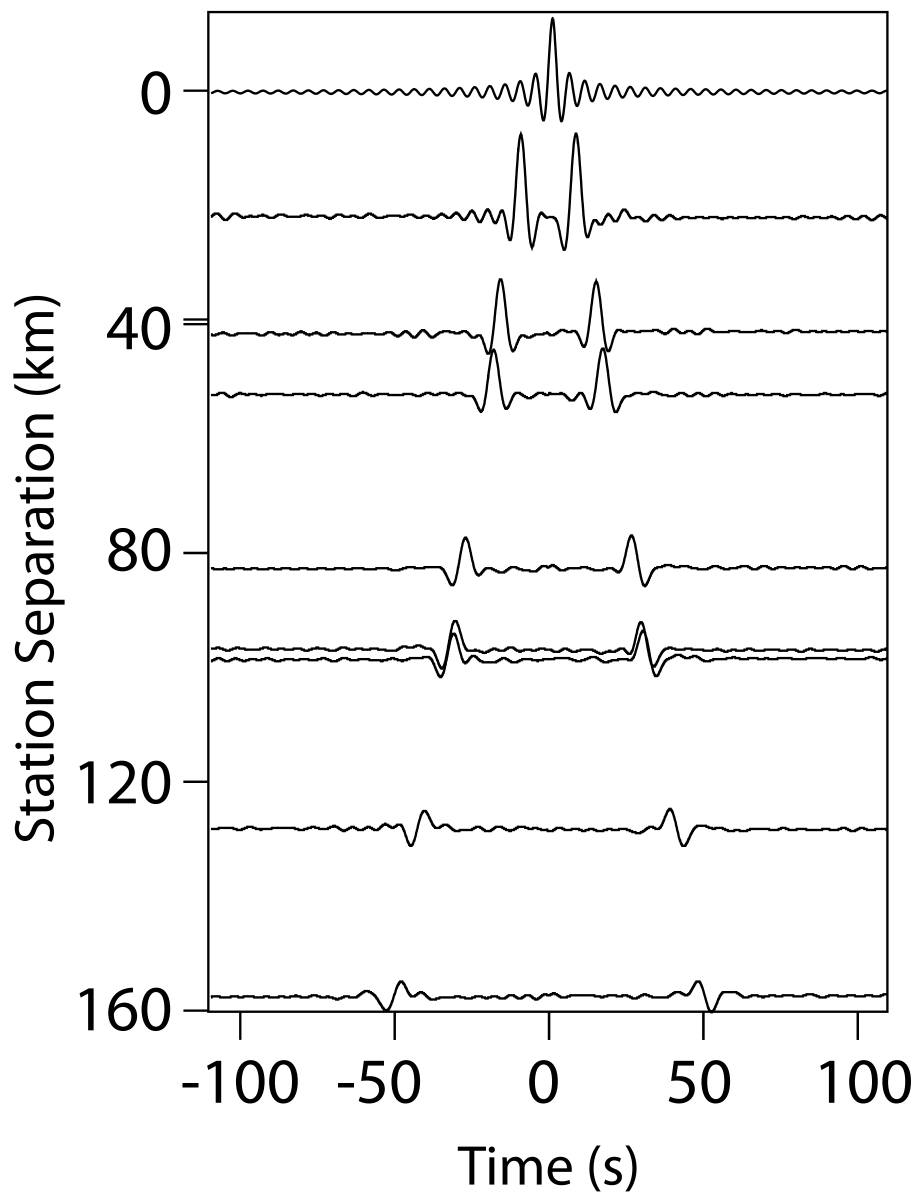
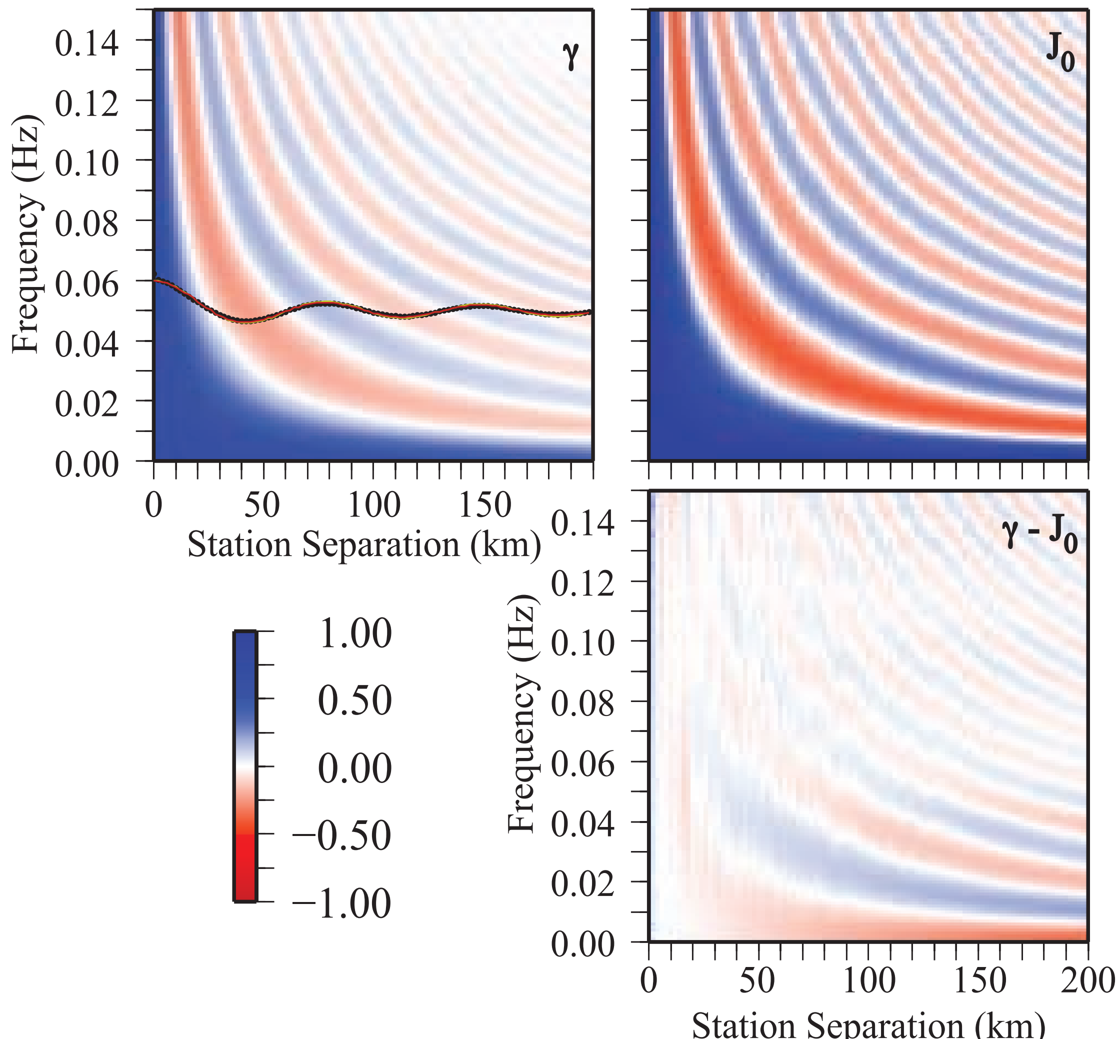




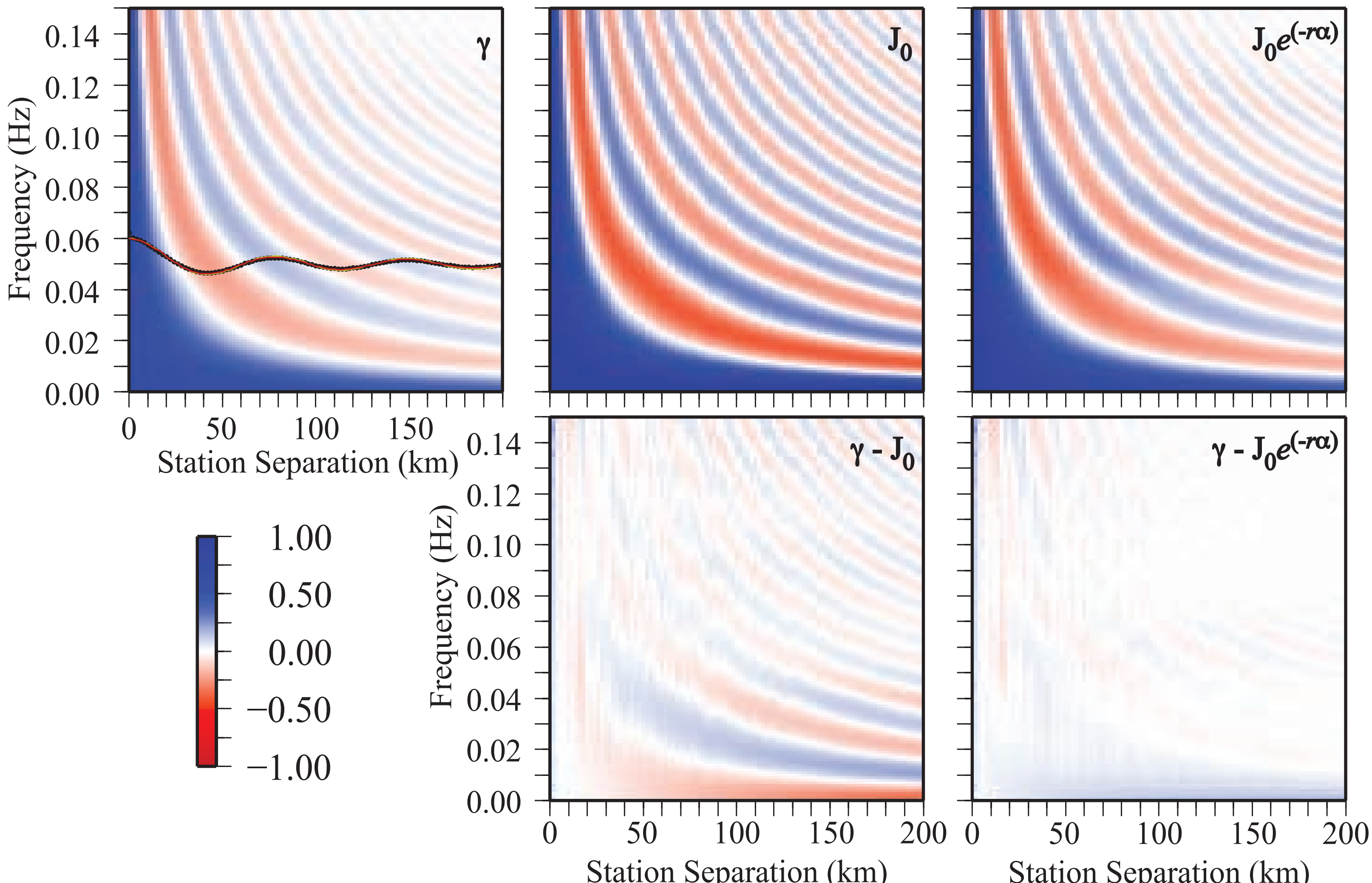
Accurate phase velocity and amplitude decay observed in simulated cross-correlations.

Numerical Simulation



Simulated coherencies match Bessel function for elastic case

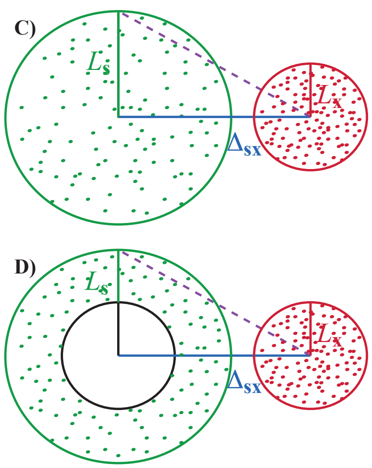
Numerical Simulation

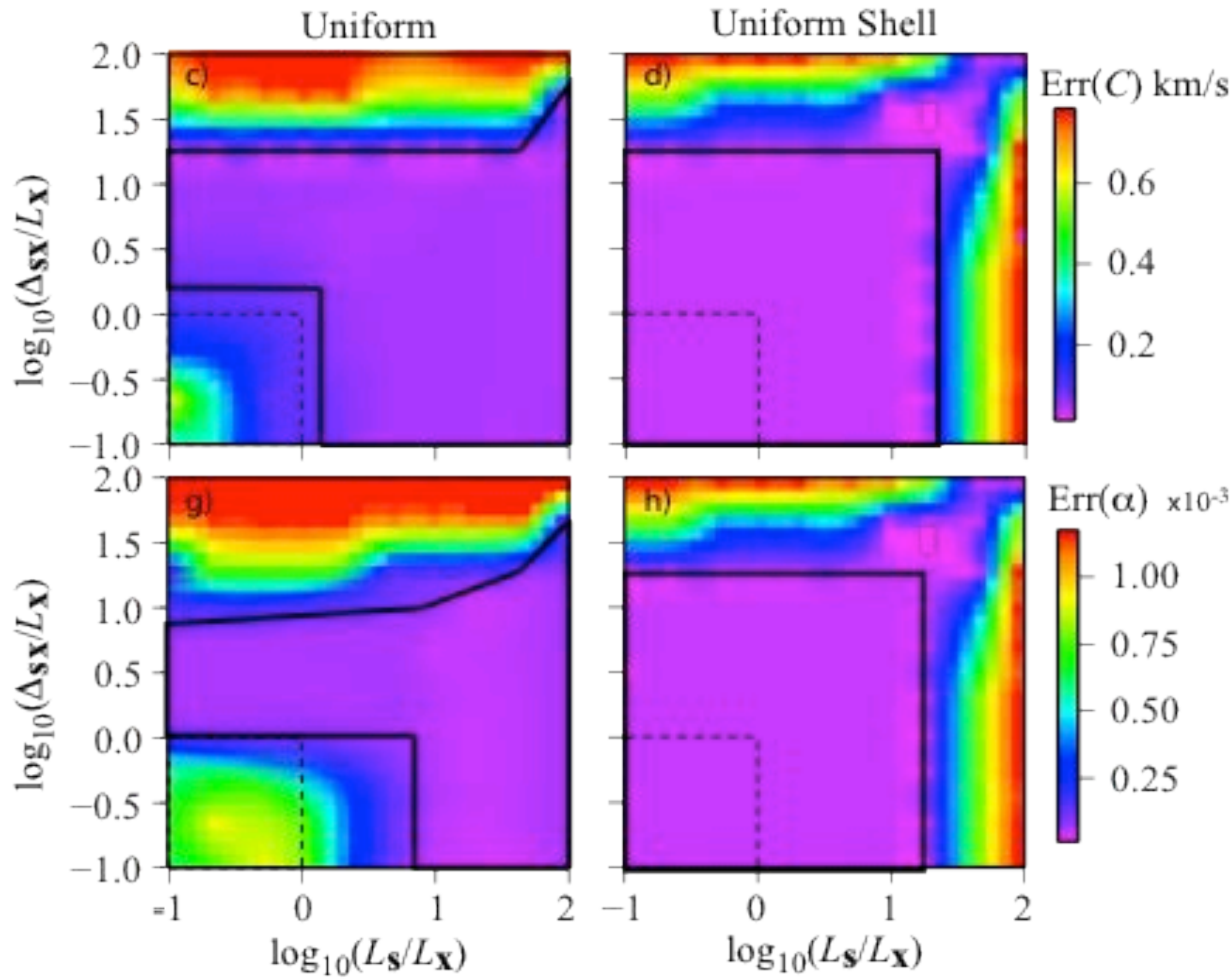


Simulated coherencies match attenuated Bessel function better

Numerical Simulation

Lawrence et al. In prep.

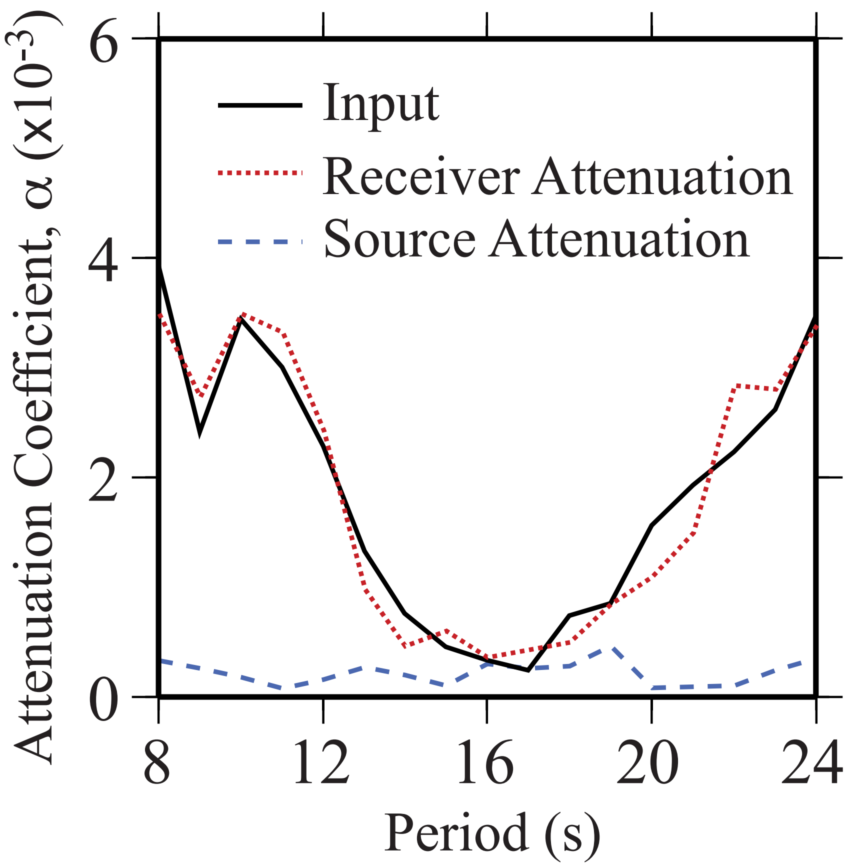


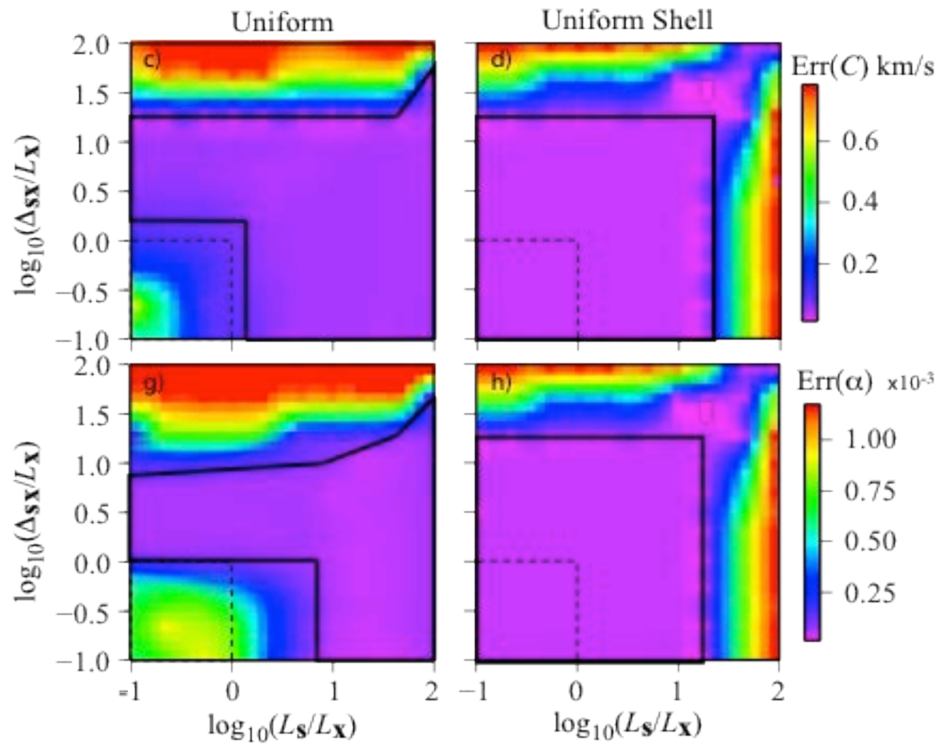


Does not always work, but does on realistic source distributions

Numerical Simulation

Lawrence et al. In prep.





Attenuation measured, is the interstation attenuation

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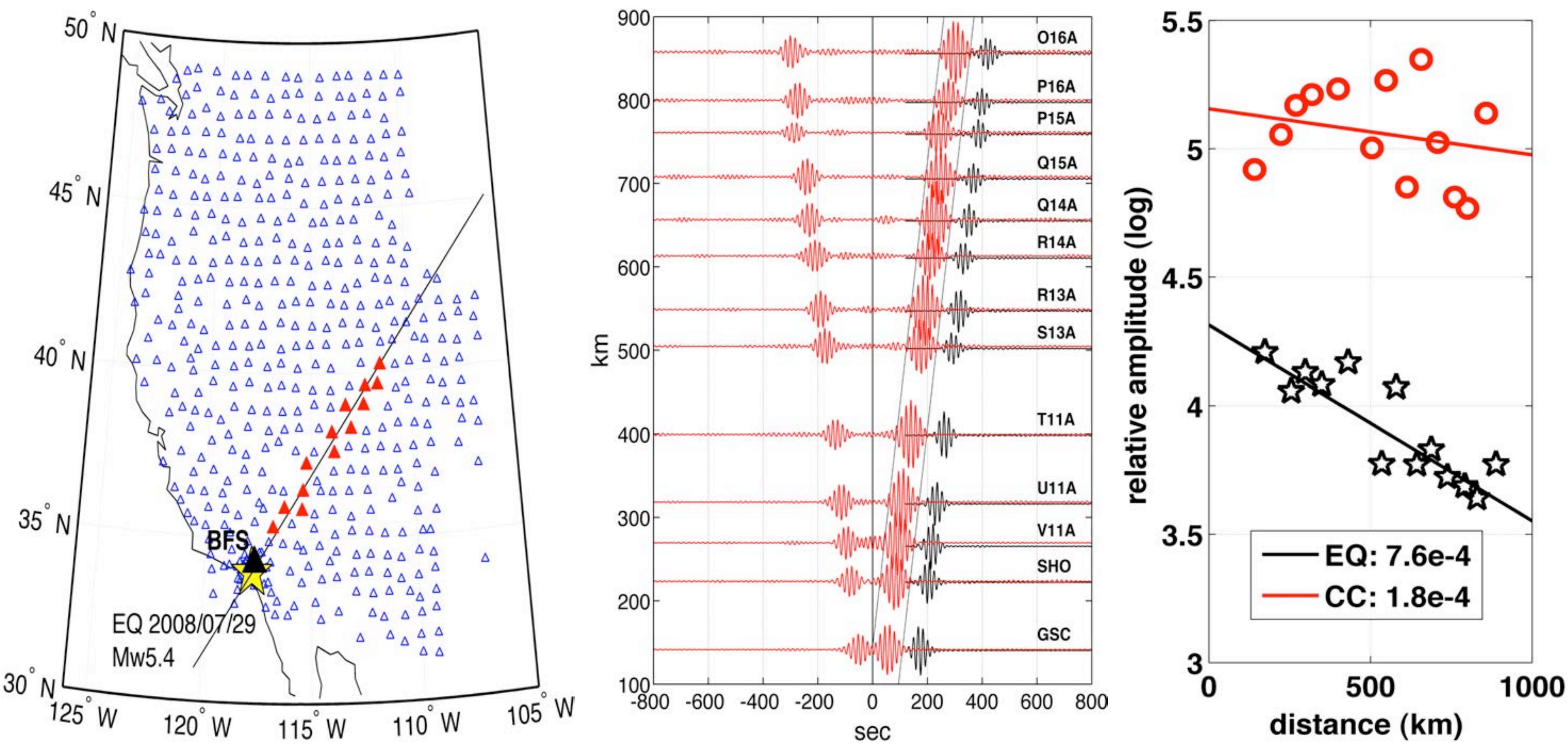
… substantial and competing amplitude anomalies due to elastic and anelastic variations … (Savage et al., 2010)

Attenuation from noise – C^3

Amplitude decay – line-array examples

18 sec
geometric spreading corrected
clear EGF, yet **biased** attenuation

(Zhang and Yang., 2013)



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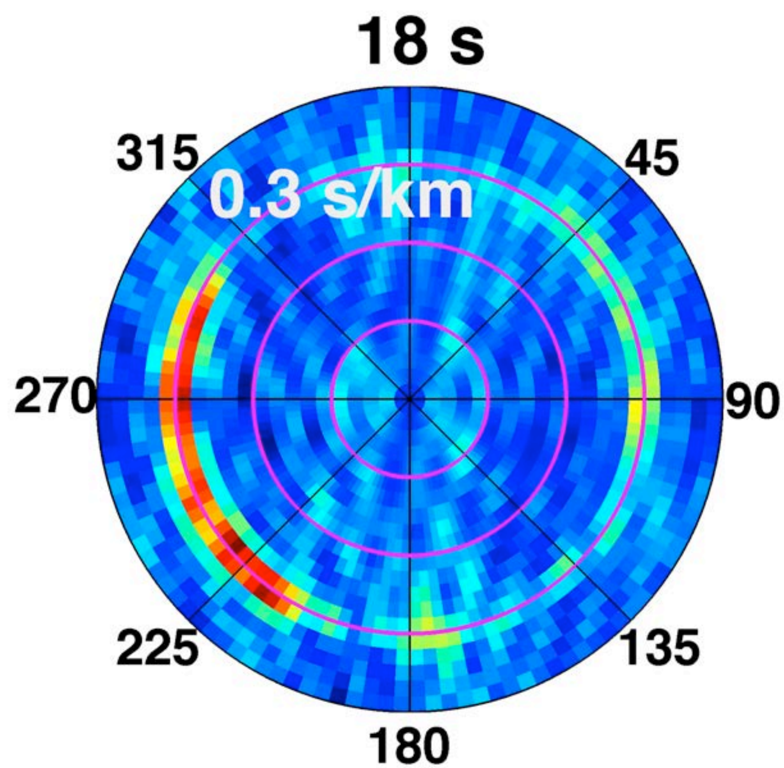
May this bias be due to
noise intensity?
noise directivity?



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Amplitude decay – line-array examples

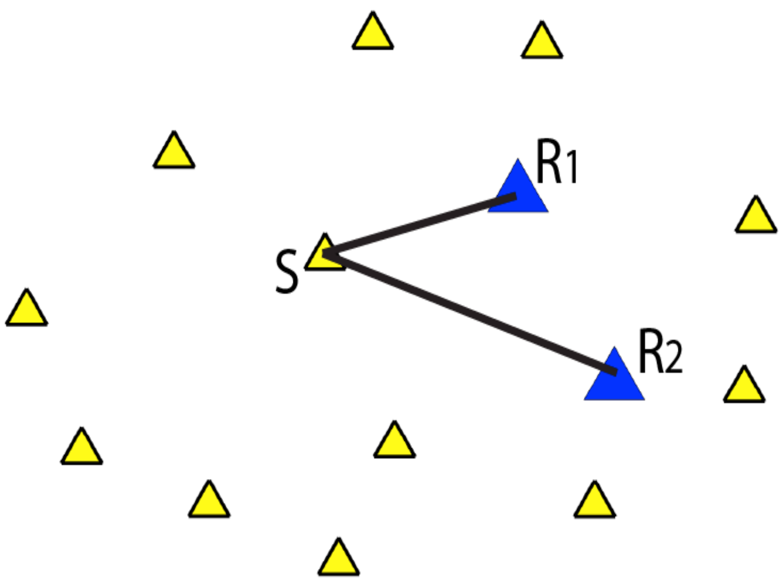
(Zhang and Yang., 2013)

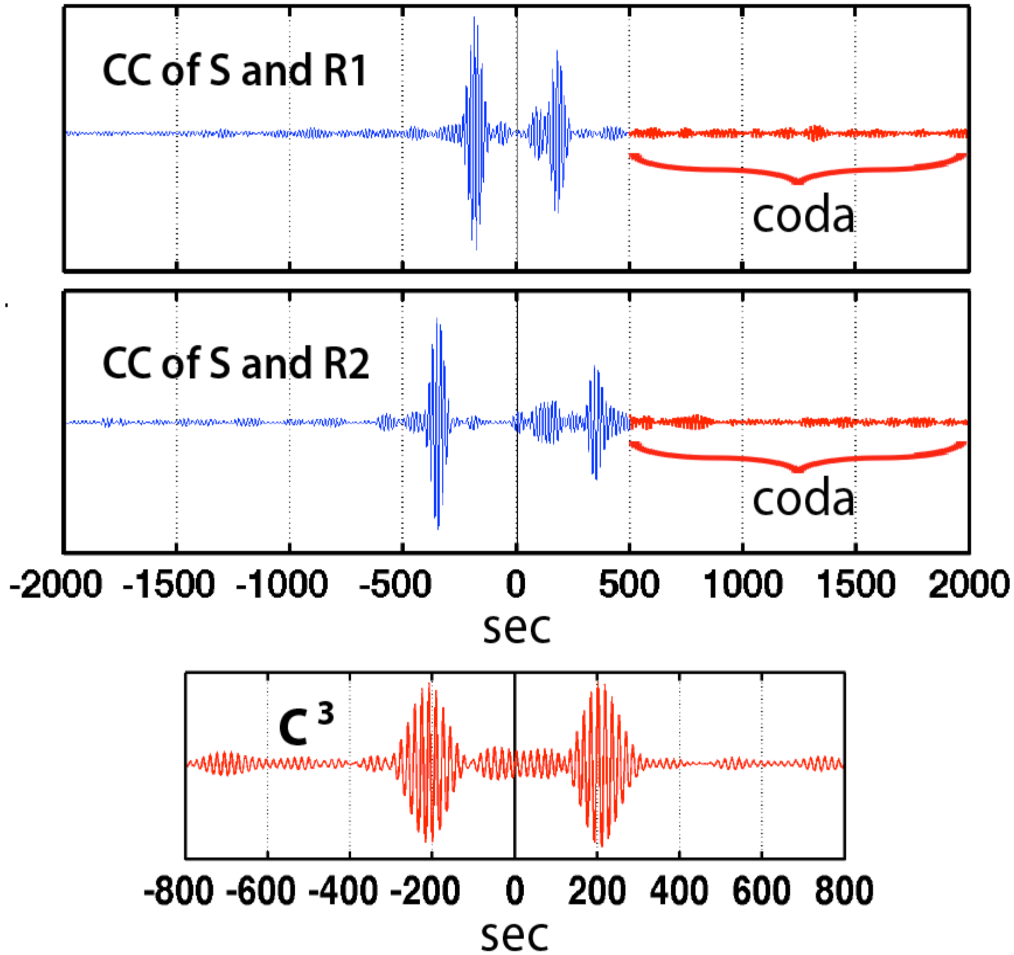
Stehly et al., 2008

Garnier & Papanicolaou, 2009

de Ridder et al., 2009

Froment et al., 2011



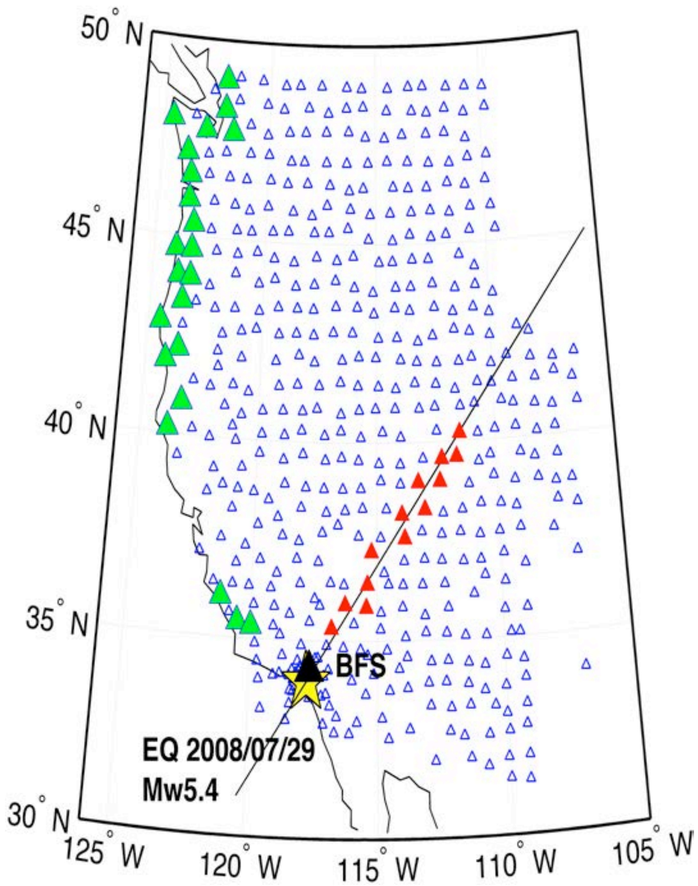


Attenuation from noise – C³

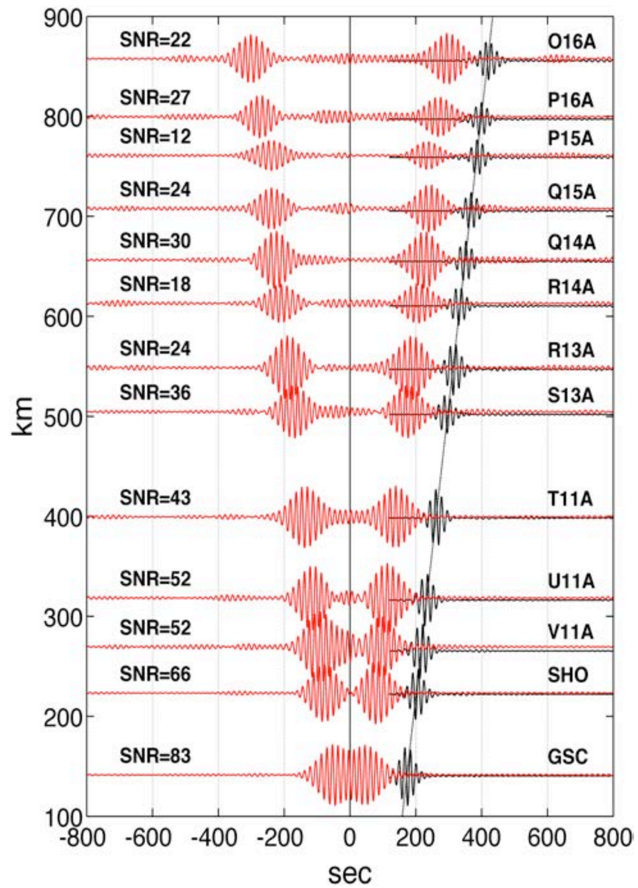
Amplitude decay – line-array examples

(Zhang and Yang., 2013)

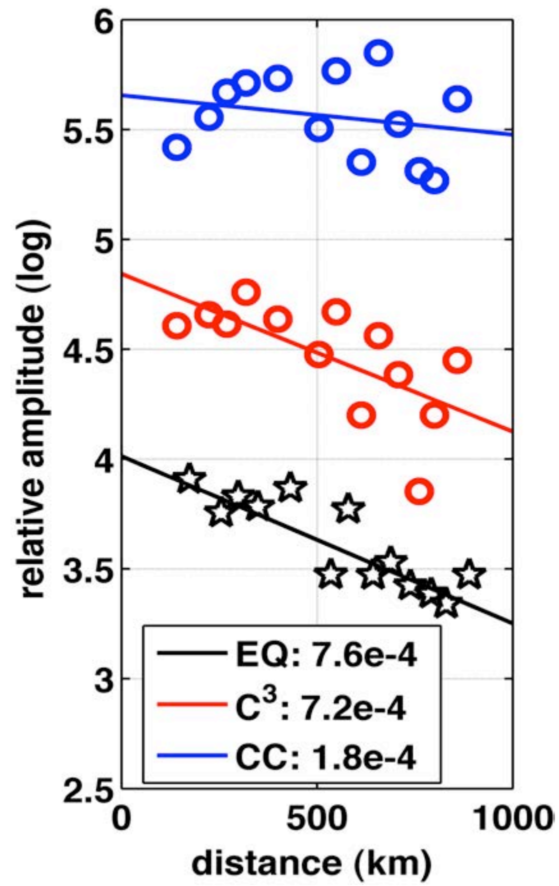
coastal coda-stations



symmetric EGF with high SNR



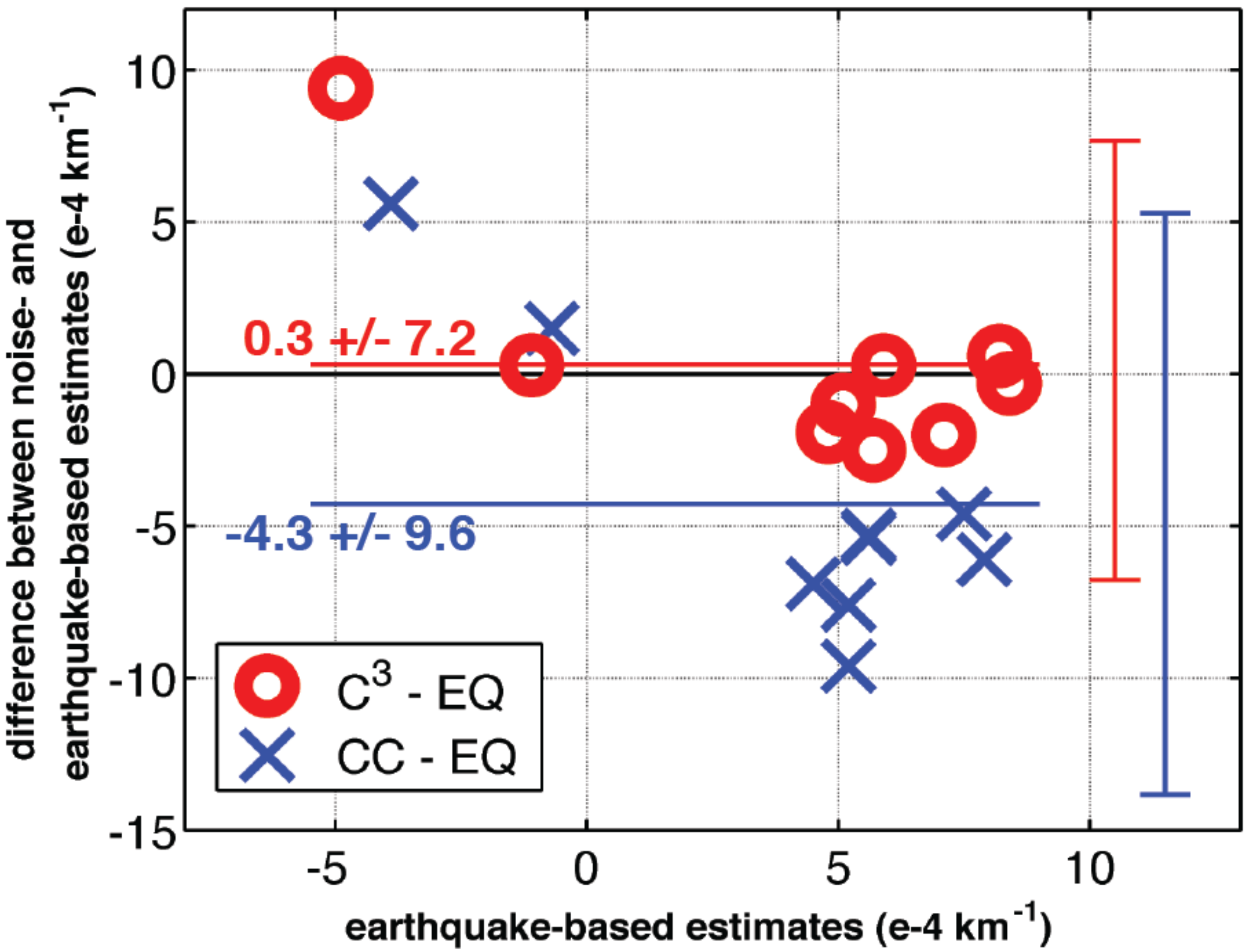
CC vs. C³



Attenuation from noise – C^3

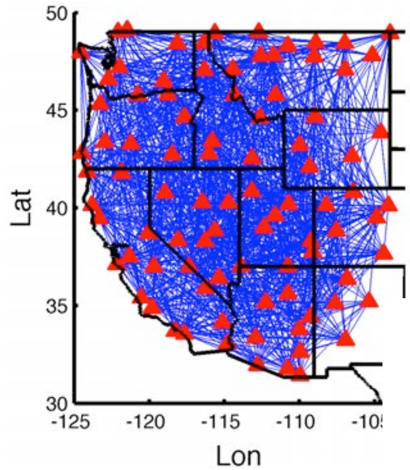
Amplitude decay – line-array examples

(Zhang and Yang., 2013)



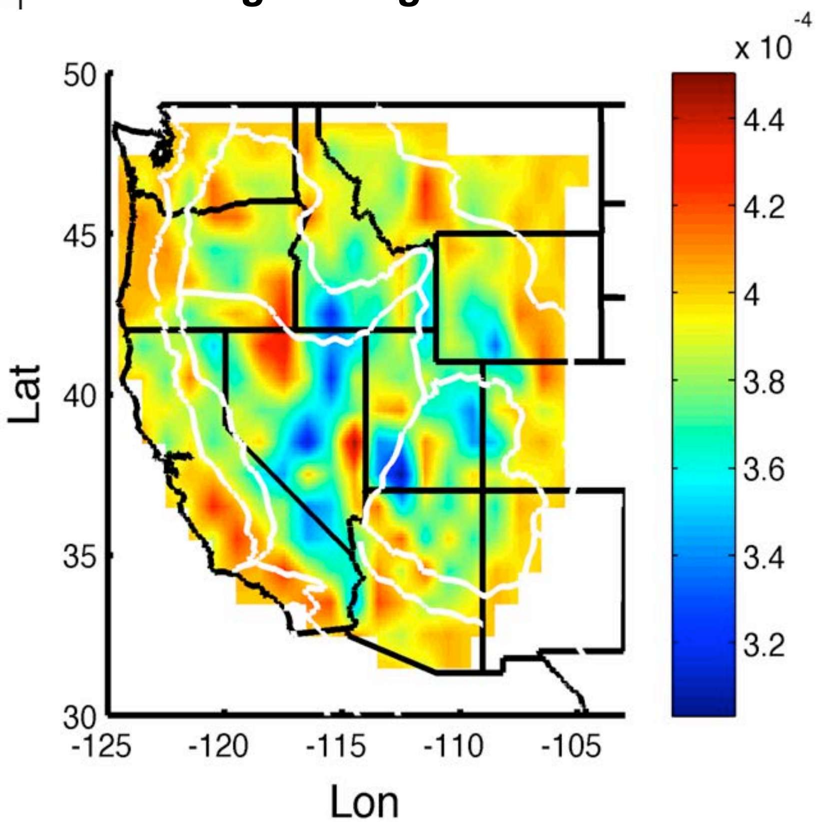
Attenuation from noise – C^3

Amplitude decay – line-array examples

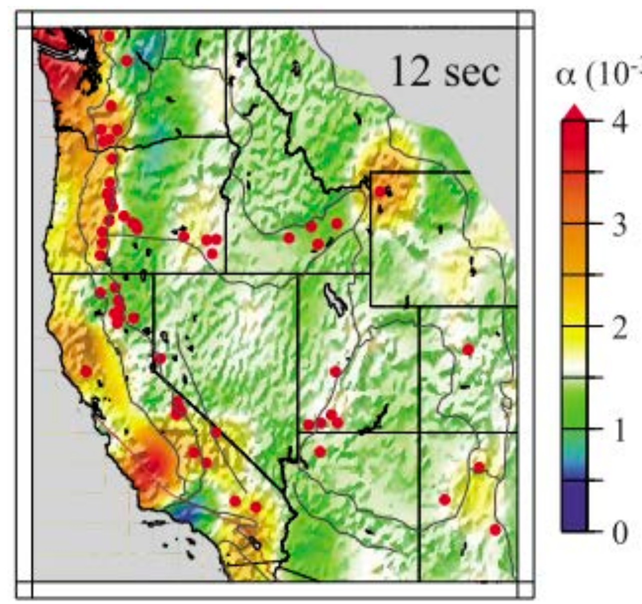


a preliminary tomography using C^3

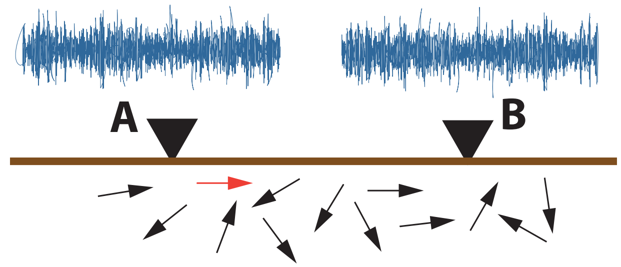
Zhang & Yang SSA 2013

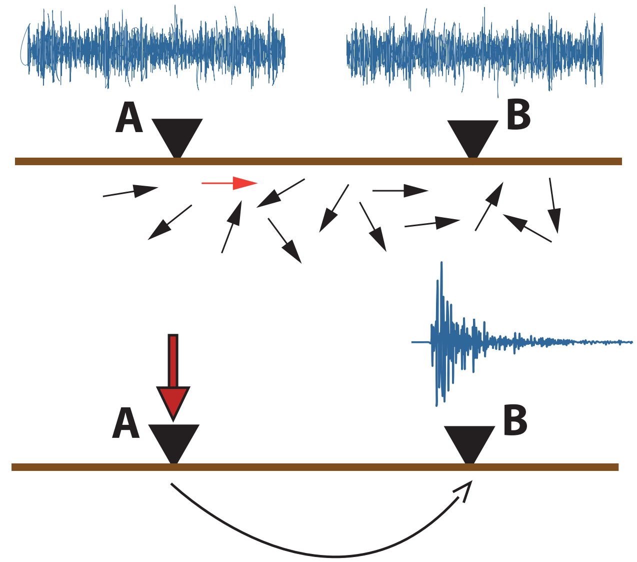


Lawrence & Prieto, JGR 2011



Weak coherent
ambient
seismic field
recorded at stations





➔ **Path Effects**

STANFORD UNIVERSITY

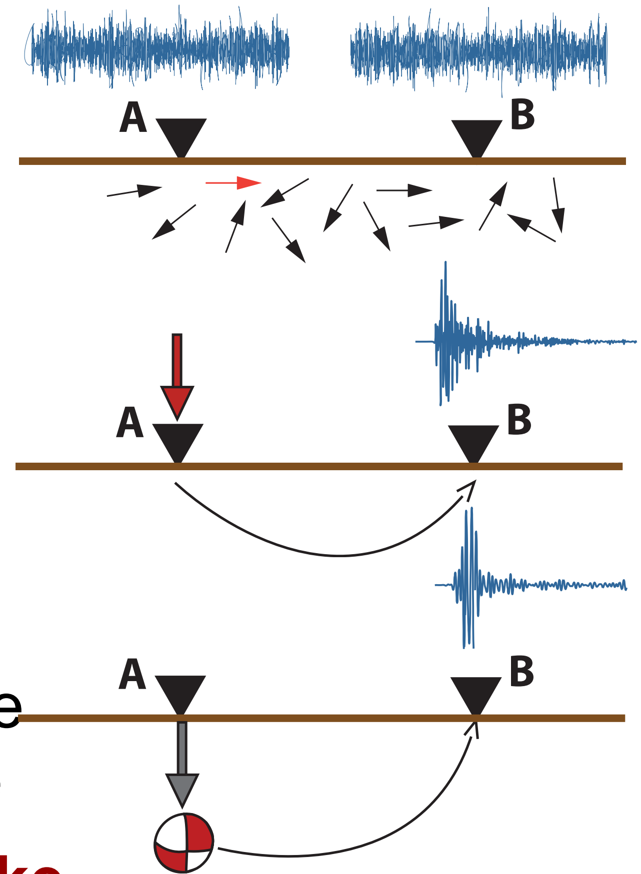
Weak coherent
ambient
seismic field
recorded at stations

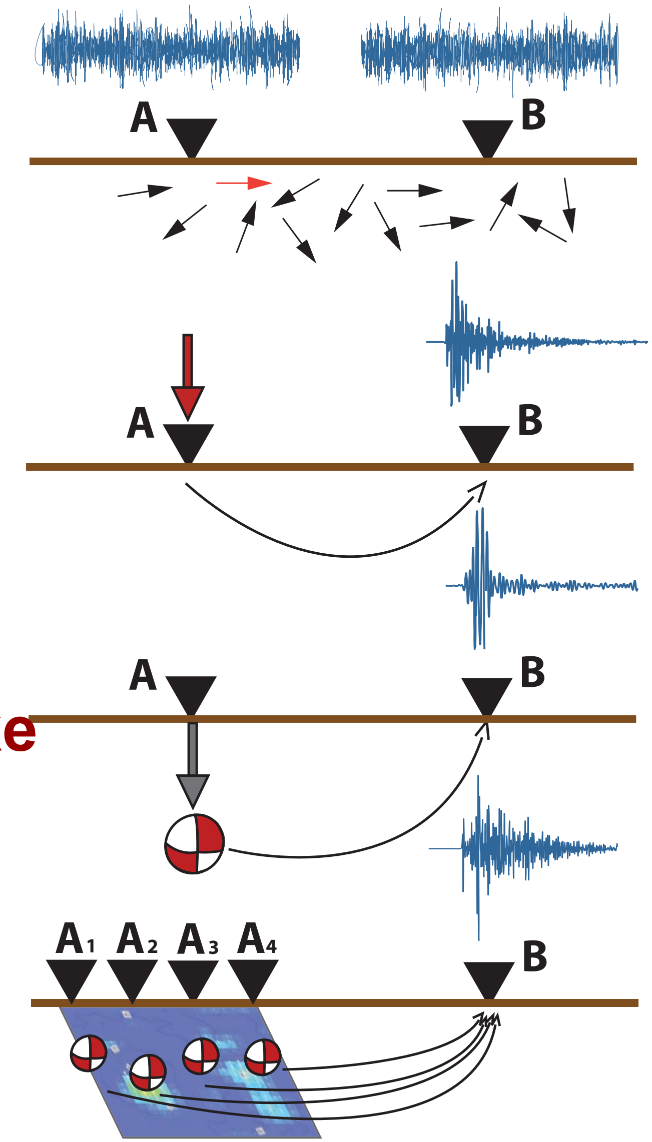
Extract impulse response

➔ **Path Effects**

Convert surface impulse response
to buried double-couple response

➔ **Point Source Virtual Earthquake**





➔ **Path Effects**

➔ **Point Source Virtual Earthquake**

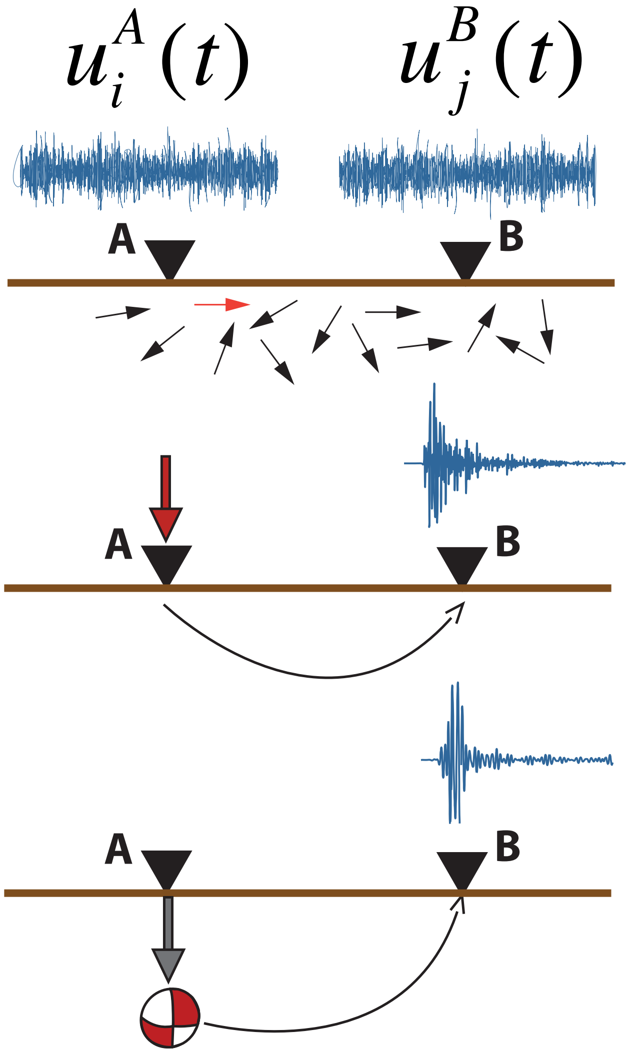
Model extended-source response using the representation theorem

➔ **Extended Virtual Earthquake**

VIRTUAL EARTHQUAKE APPROACH

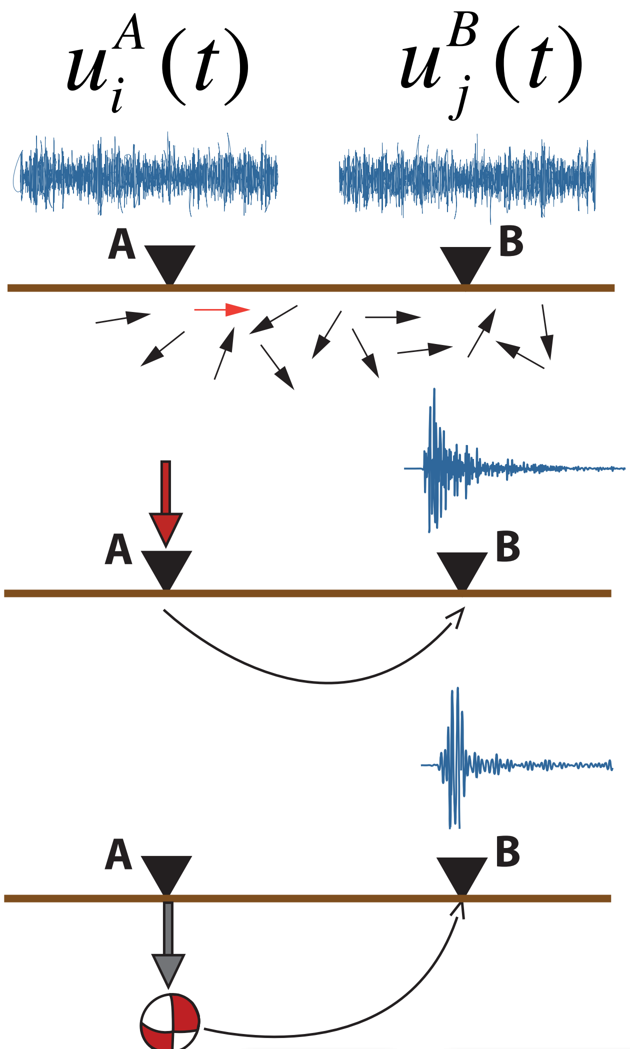
STANFORD UNIVERSITY

Depth and mechanism correction



Isotropic radiation from surface source

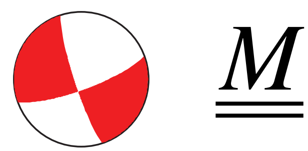
Azimuth-dependent radiation from a deep source

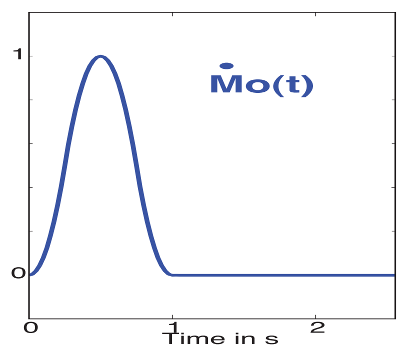


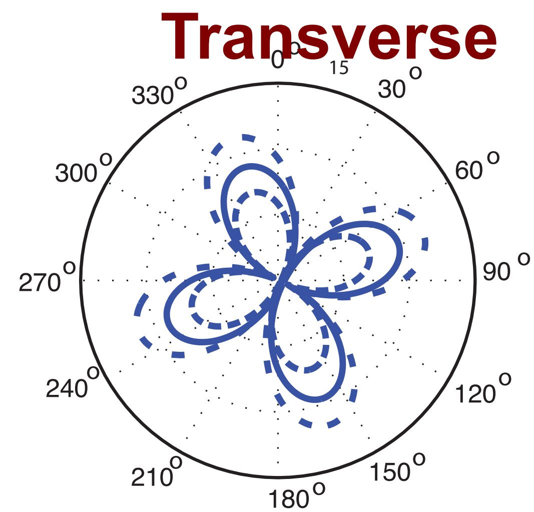
Depth and mechanism correction

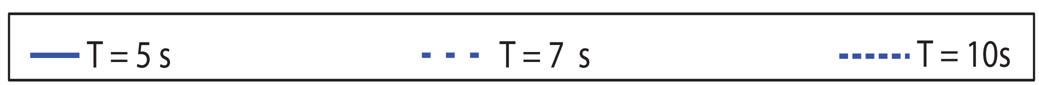
$l_1(0,\omega)$ = disp eigenfunction, surface

$l_1(h,\omega)$ = disp eigenfunction at depth



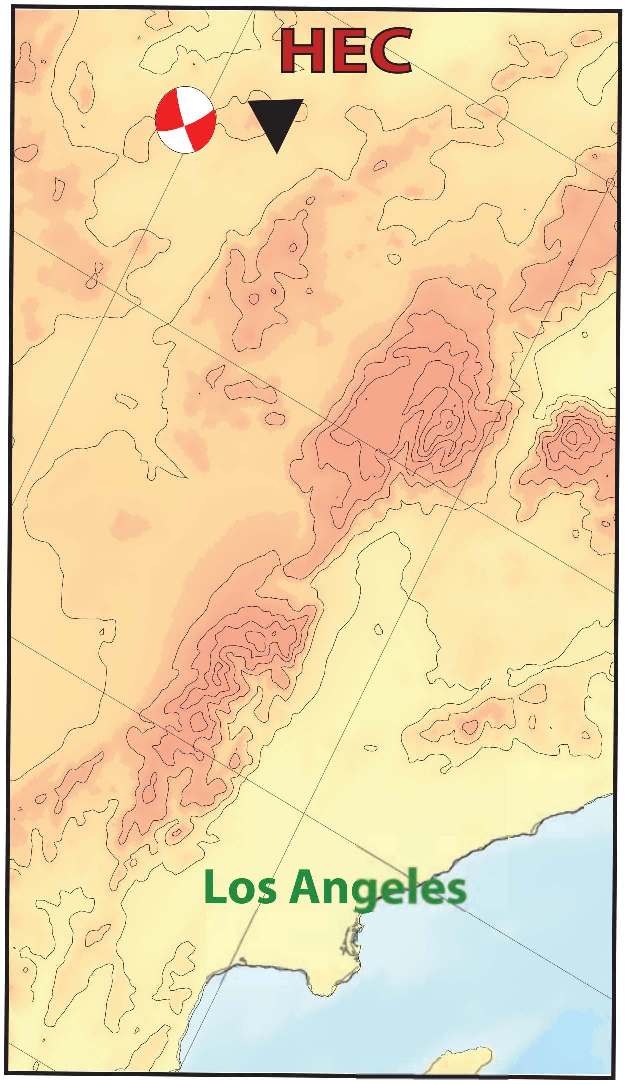






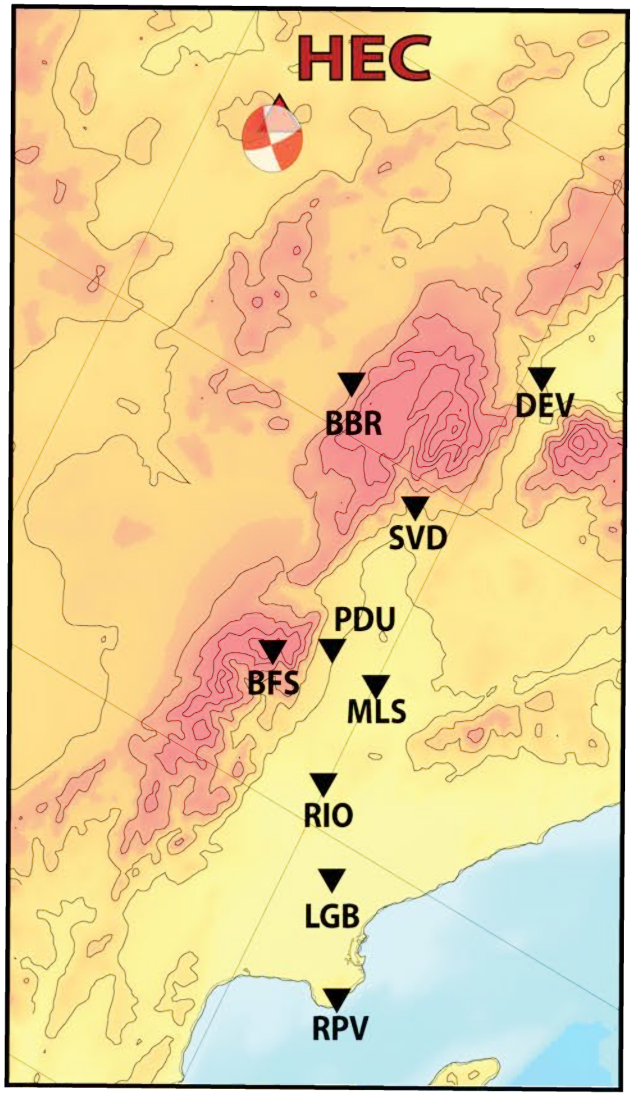
$$\hat{u}_T(h,\omega) = \frac{1}{l_1(0,\omega)}\left[-ik_L \hat{M}_{TR}(\omega) l_1(h,\omega) + l_1'(h,\omega)\hat{M}_{TD}(\omega)\right]\hat{T}_{TT}^{AN}(\omega)$$

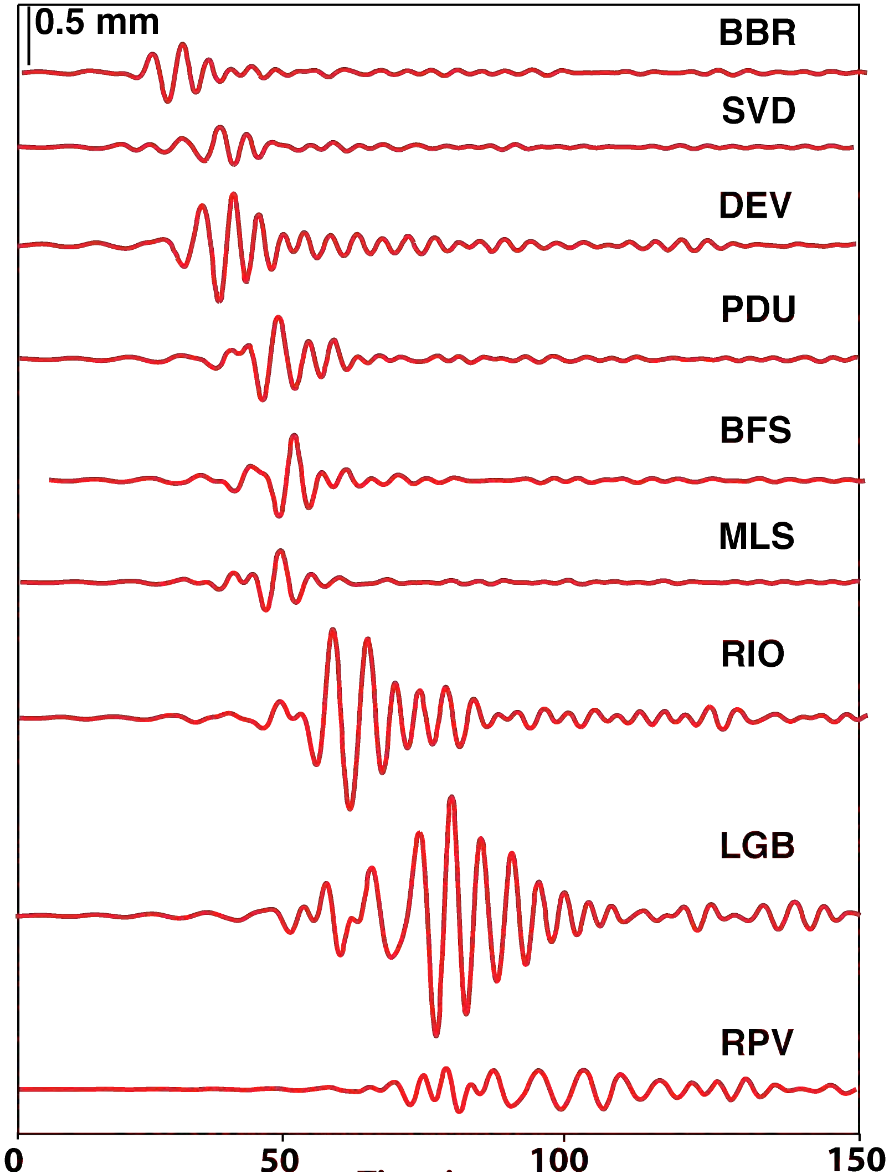
12/06/08 M5.1



Validation against real data

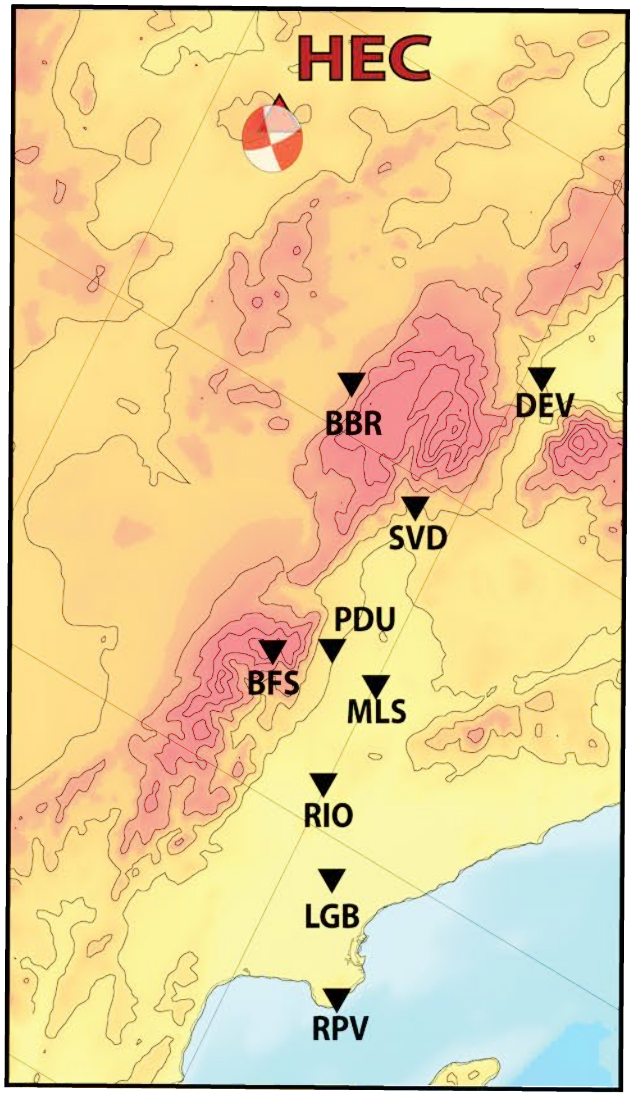
Love Waves



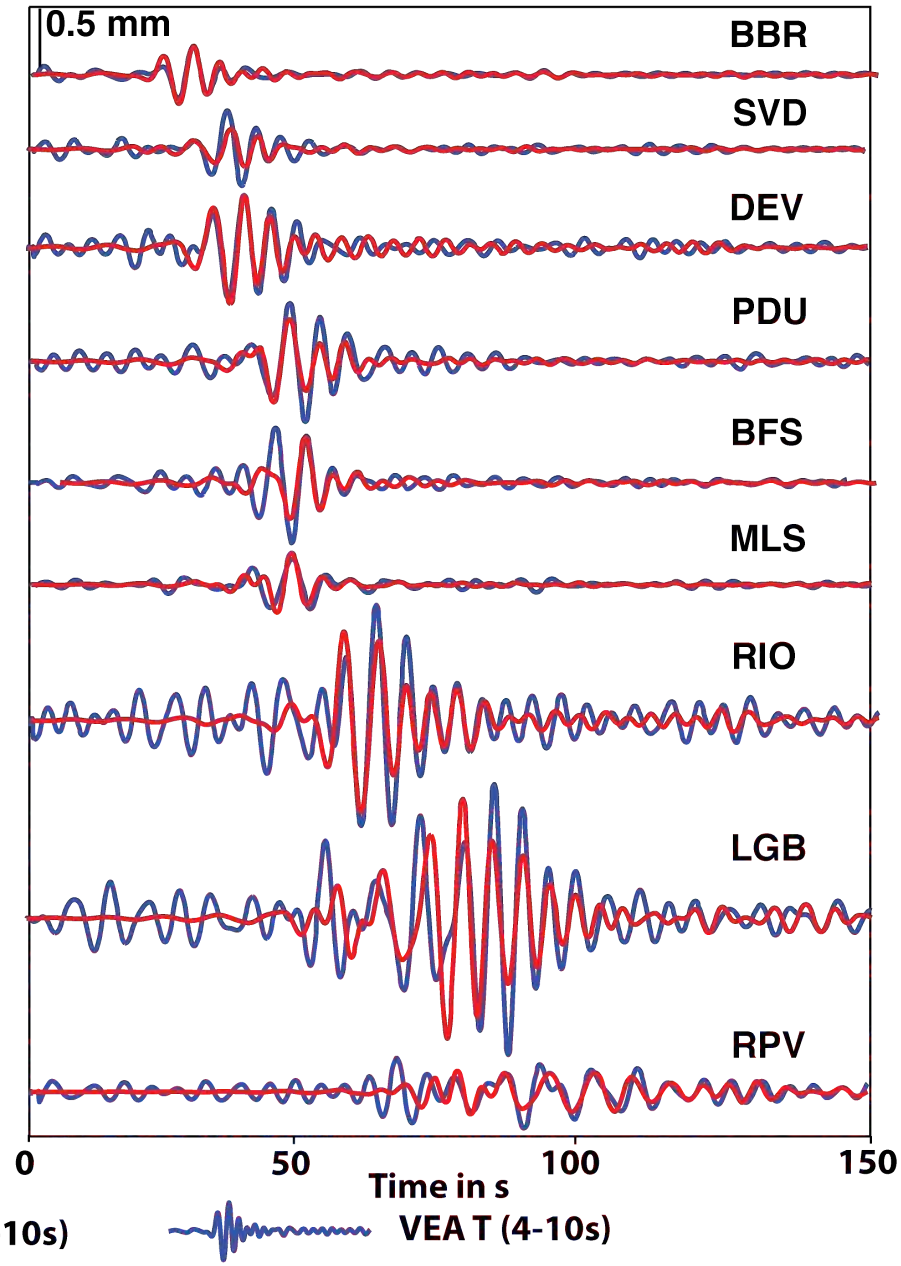


Displacement T (4-10s)

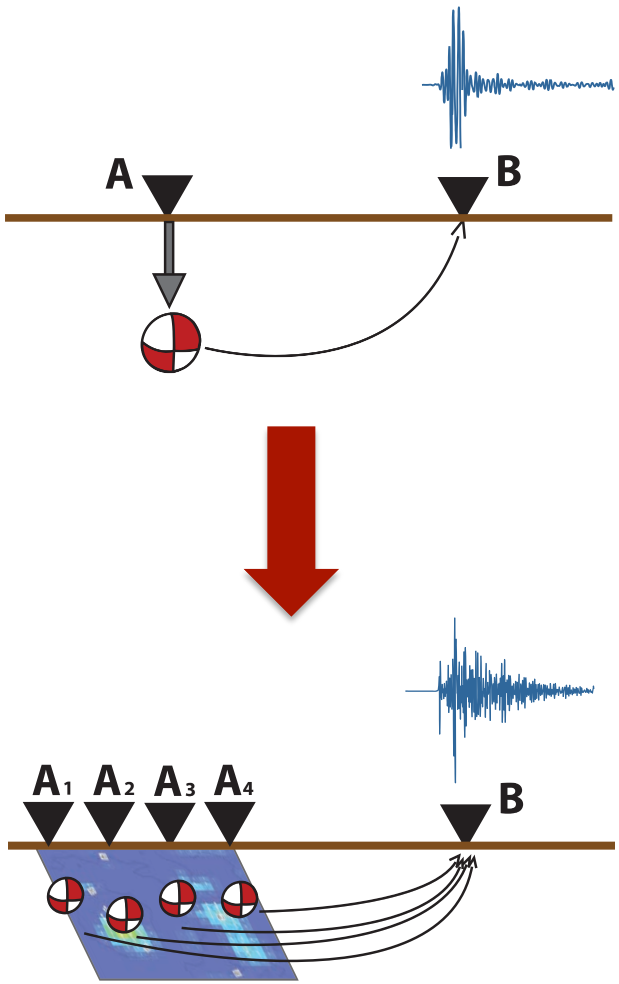
Love Waves



HEC
BBR
DEV
SVD
PDU
BFS
MLS
RIO
LGB
RPV



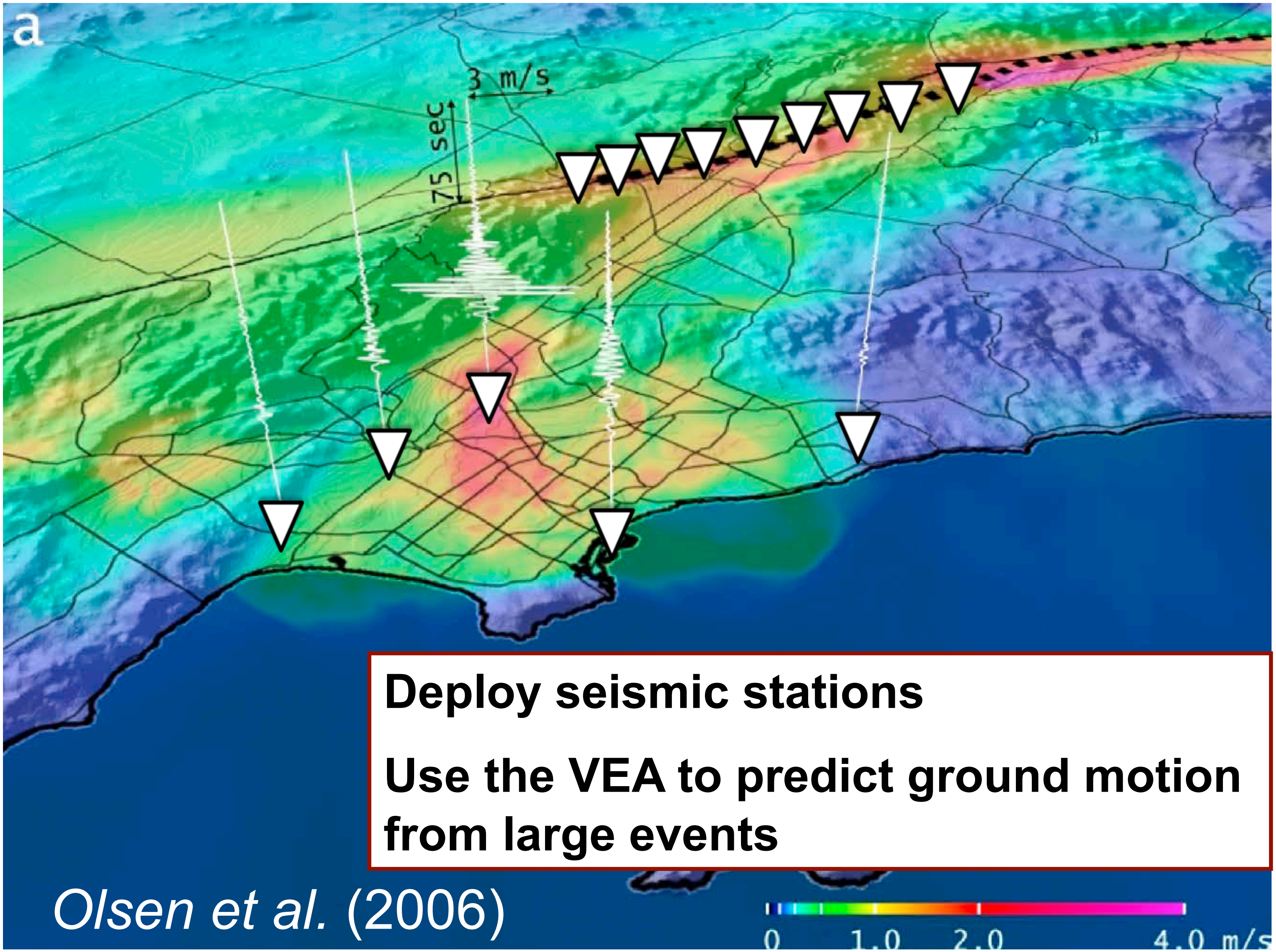
0.5 mm
BBR
SVD
DEV
PDU
BFS
MLS
RIO
LGB
RPV
0
50
100
150
Time in s
Displacement T (4-10s)
VEA T (4-10s)



Point Source $u_j(\underline{x}, t - t_j)$

Representation theorem to extend the source

$$U_i(\underline{x}, t) = \sum_{j=1}^{N} \int_0^h u_j((x_j, z'), t - t_j)\,dz'$$

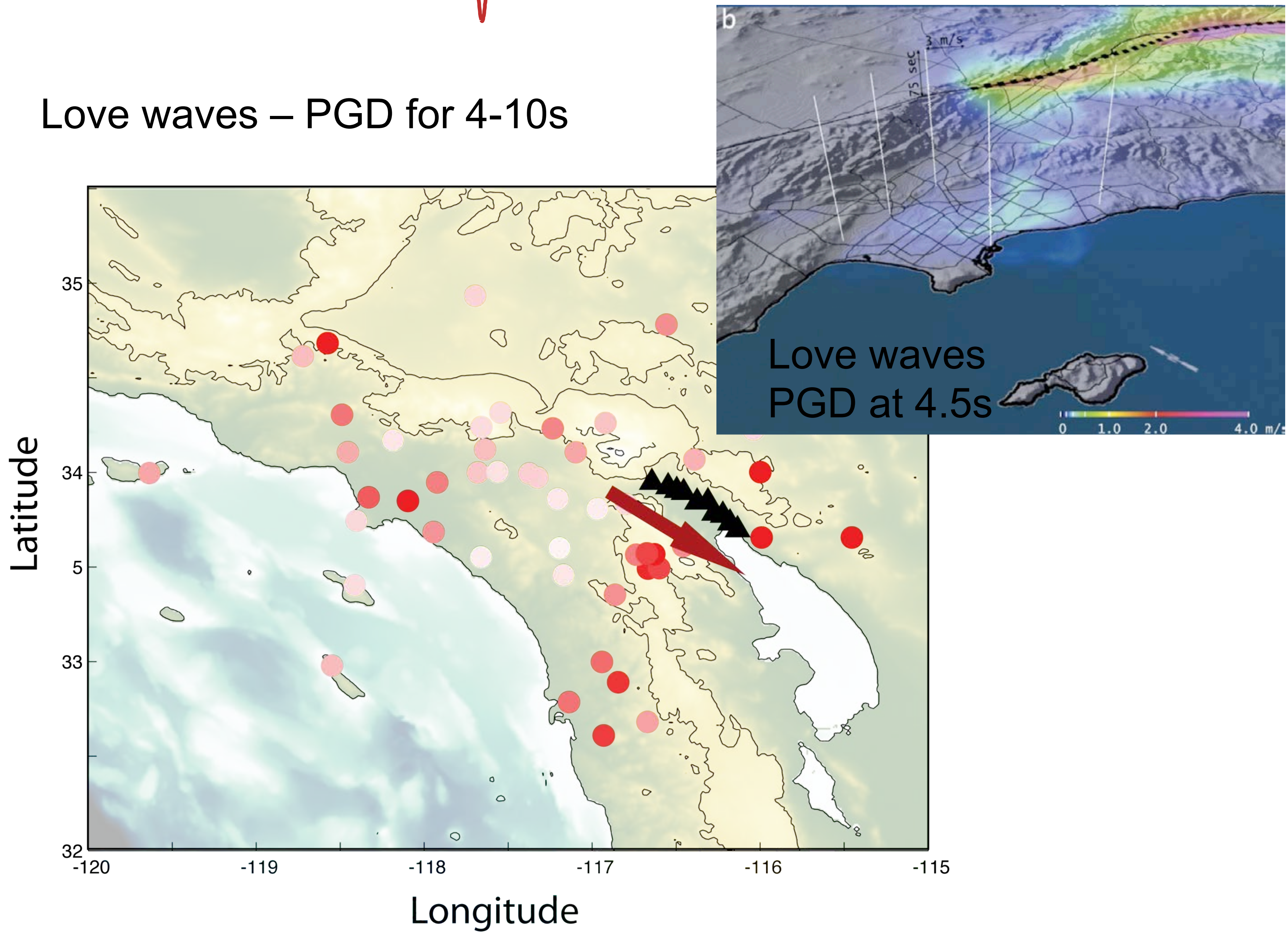


Deploy seismic stations

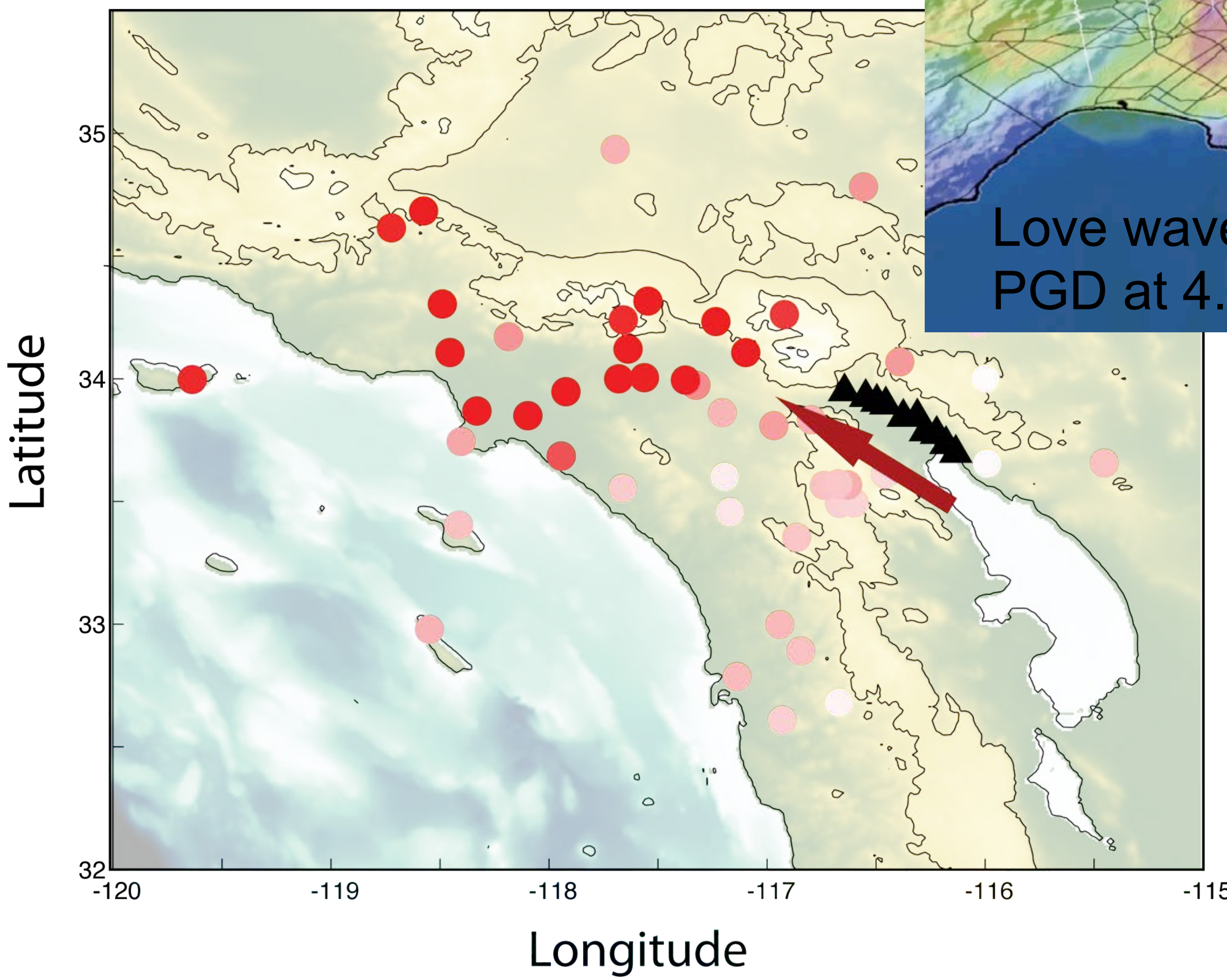
Use the VEA to predict ground motion from large events

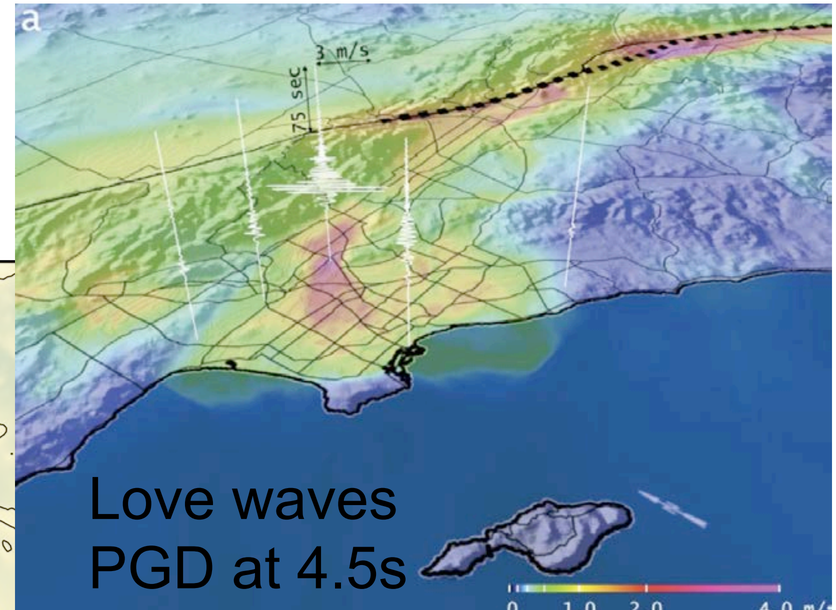
Olsen et al. (2006)

Love waves – PGD for 4-10s

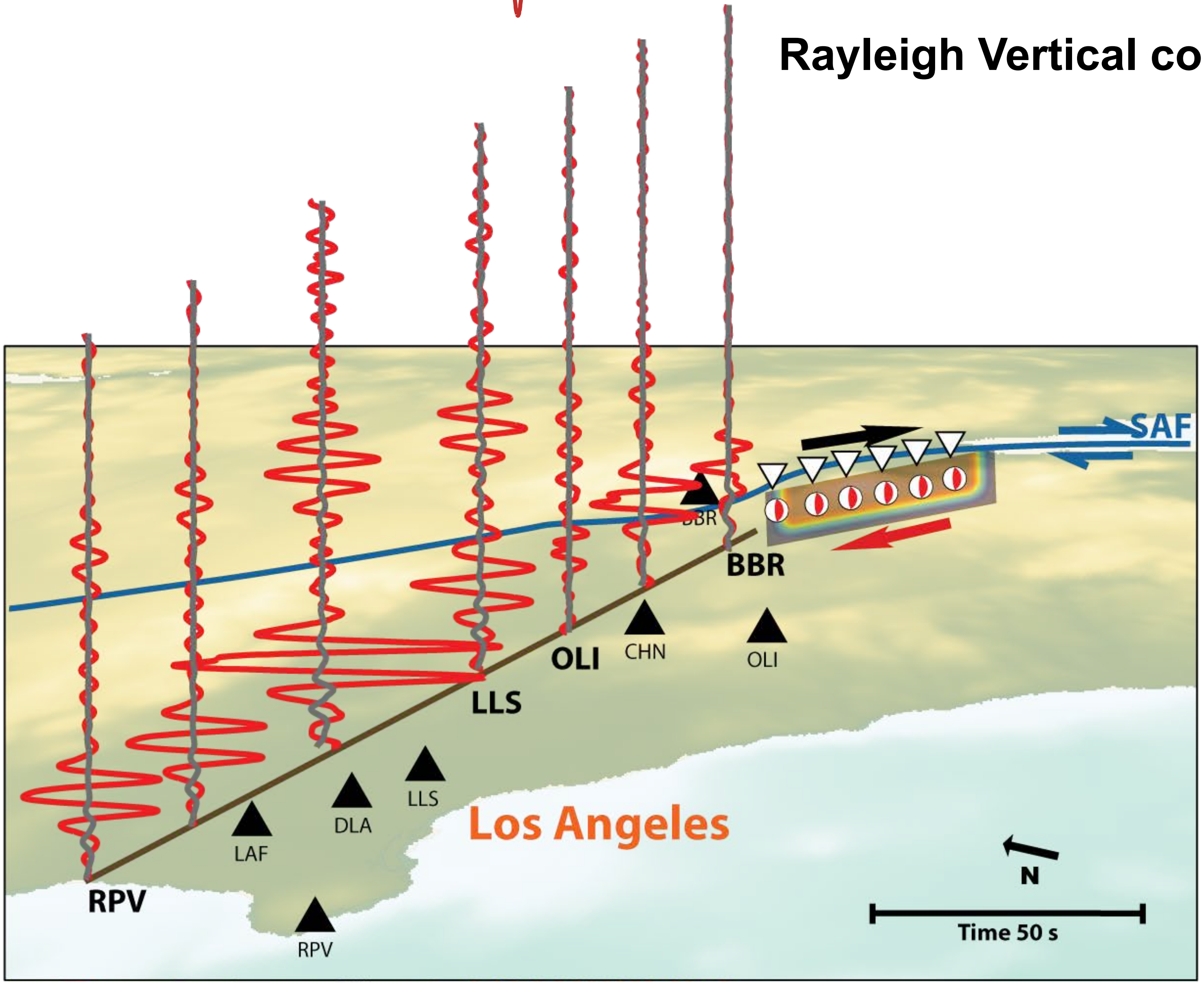


Love waves – PGD for 4-10s





Rayleigh Vertical components



For today …

Is there reliable amplitude information in noise correlations?

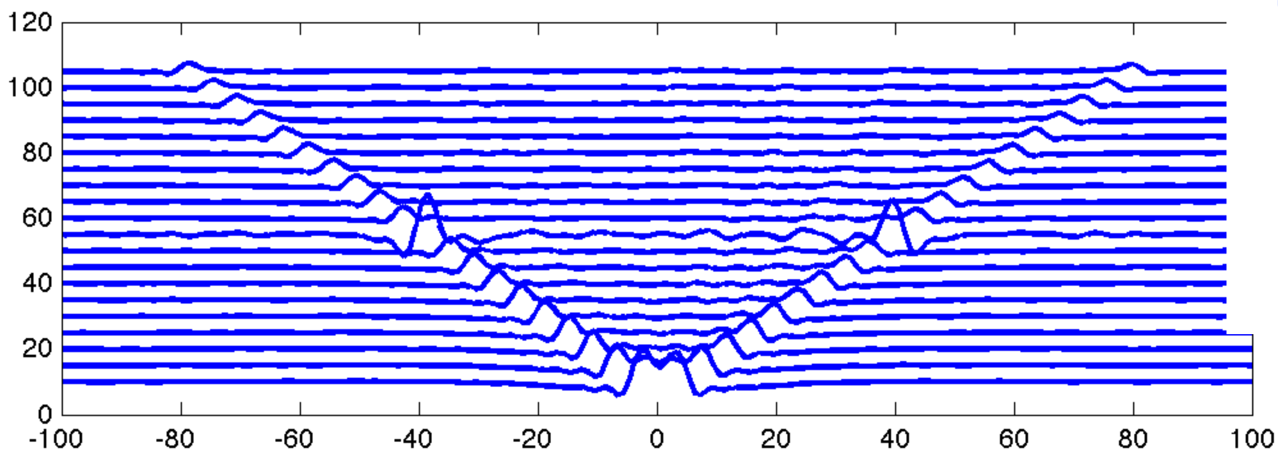
1. First suggestions on amplitude information
2. Some theoretical explanations
3. A numerical example
4. Two recent success stories
5. Always improving, how can we do better?

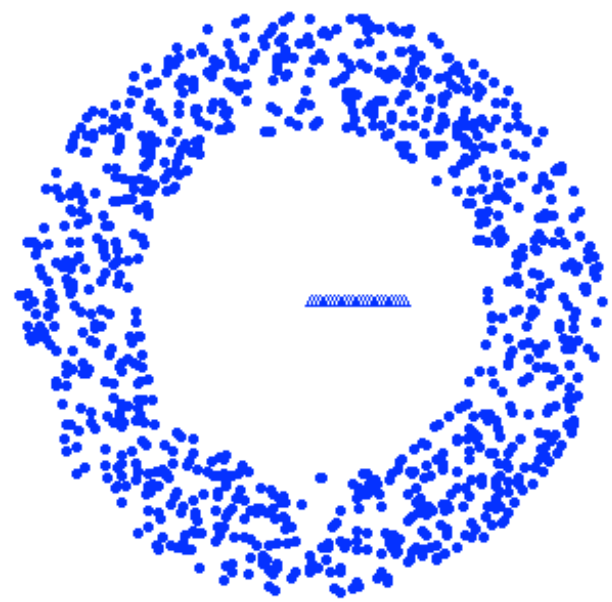
… substantial and competing amplitude anomalies due to elastic and anelastic variations … (Savage et al., 2010)

Can we do better?

Standard practice suggests that we can

$$H1 = \frac{Gx}{Gy} \qquad\qquad H1 = \frac{G_x \bullet conj(G_y)}{G_{xx}}$$





Can we do better?

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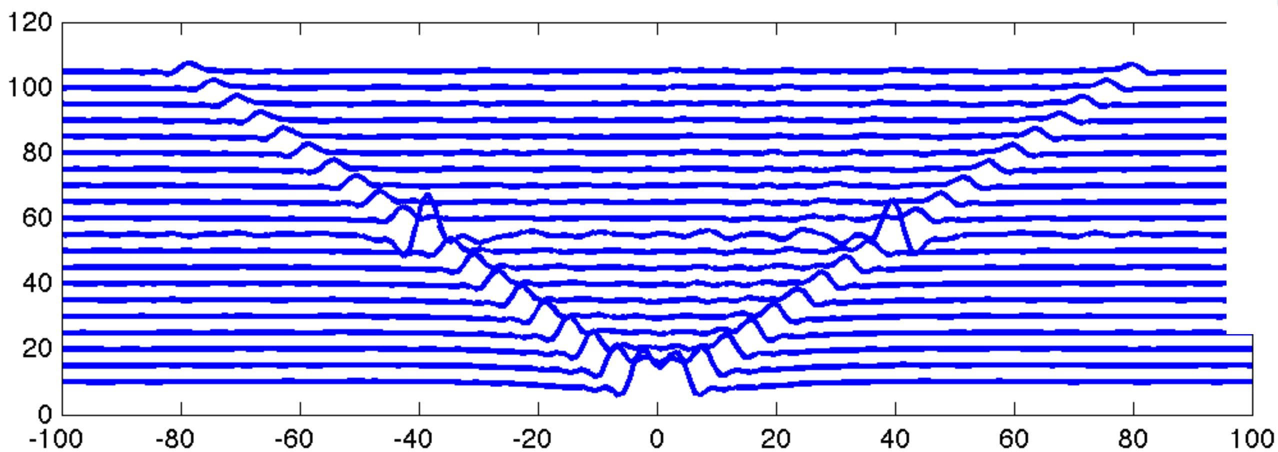
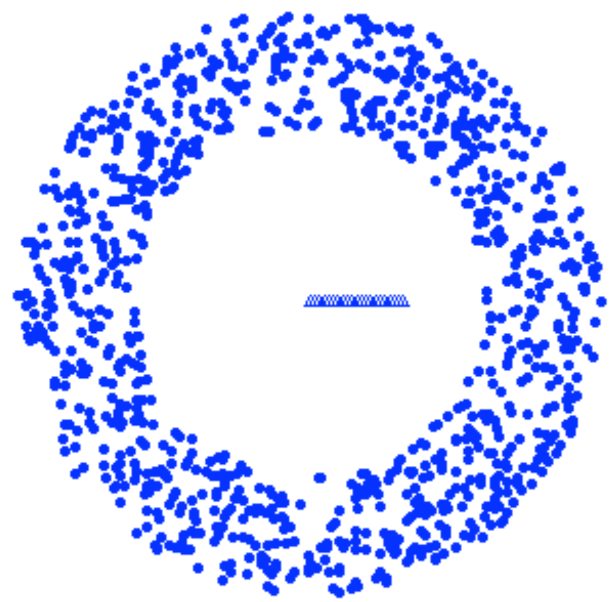
But, every H1 function is
based on two-station correlations.

System is a multiple input/output



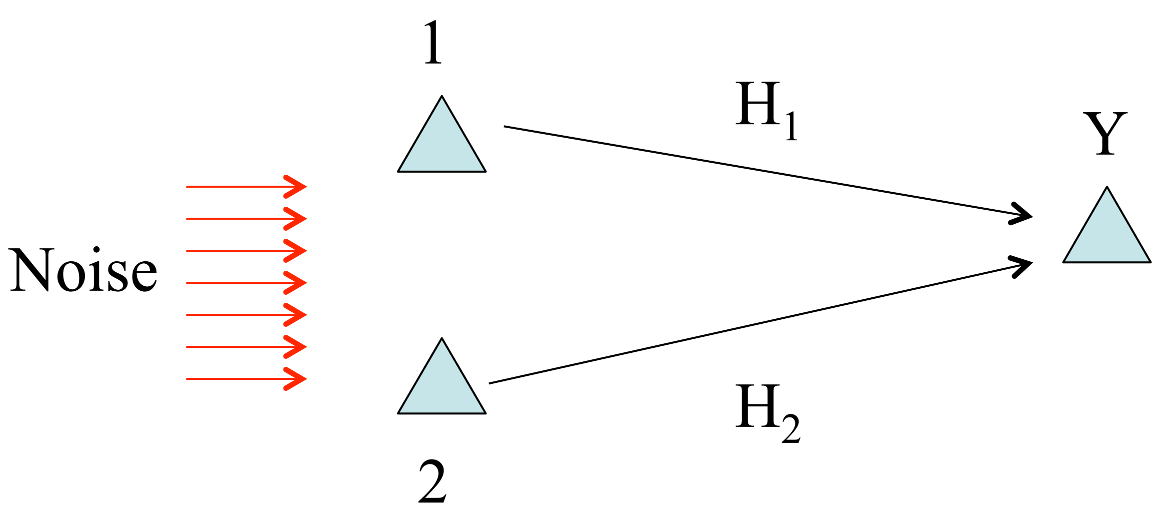


Can we do better?

In a multiple Input/Output system (Bendat and Piersol, 2010)

$$G_{1y}(f) = H_1(f)G_{11} + H_2G_{12}$$

$$G_{2y}(f) = H_1(f)G_{21} + H_2G_{22}$$



G_{xy} – X-Y Cross-Spec
G_{yy} – Power Spectrum
H_1 – Transfer function

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Transfer Function is based on coherent signal

$$H_1(f) = \frac{G_{1y}(f)\left[1 - \frac{G_{12}G_{2y}}{G_{22}G_{1y}}\right]}{G_{11}\left[1 - \gamma_{12}^2(f)\right]}$$

γ_{12} – Coherency between signal at stations 1 and 2

G_{xy} – X-Y Cross-Spec
G_{yy} – Power Spectrum
H_1 – Transfer function

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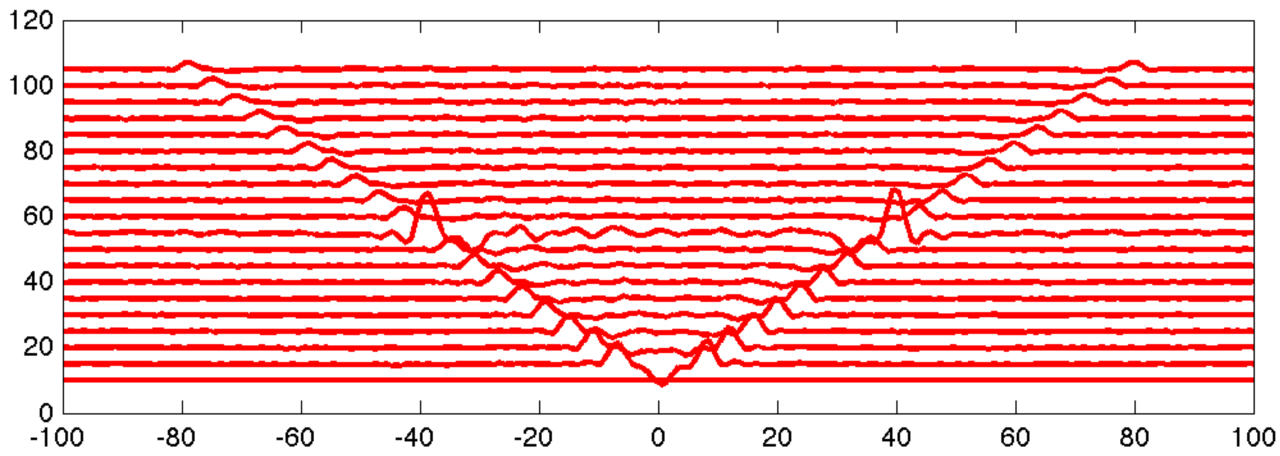

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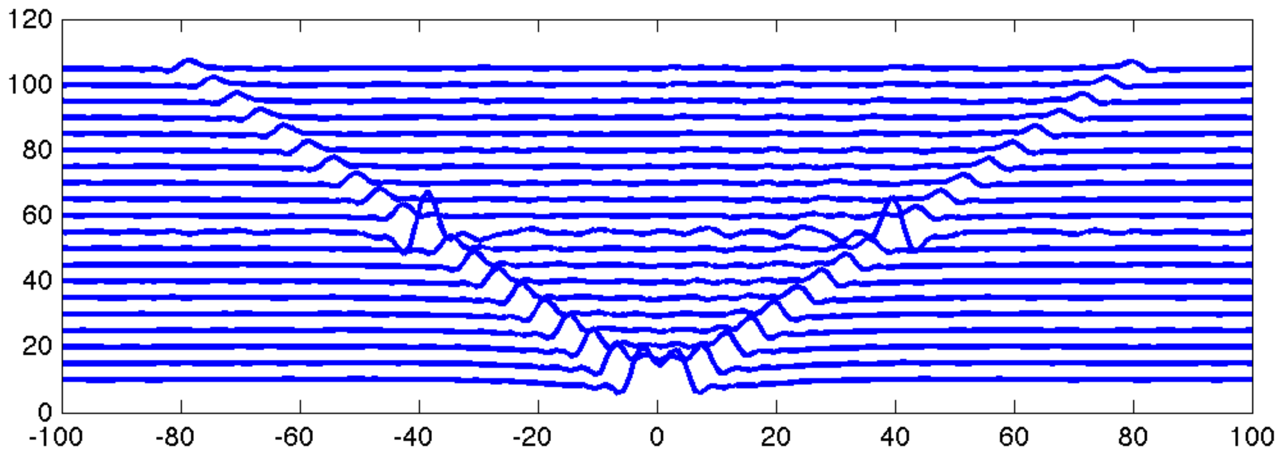

Conclusions

Ambient-noise Green's functions can be used for

Attenuation Tomography

Long-Period Ground Motion Predictions.

But, we can do better using improved signal processing

- Correct directivity and source intensity
- Apply C3 methods
- Impulse response functions, with coherency constraints.

THANK YOU !