

# Ambient noise Cross-Correlations : convergence rate & monitoring...

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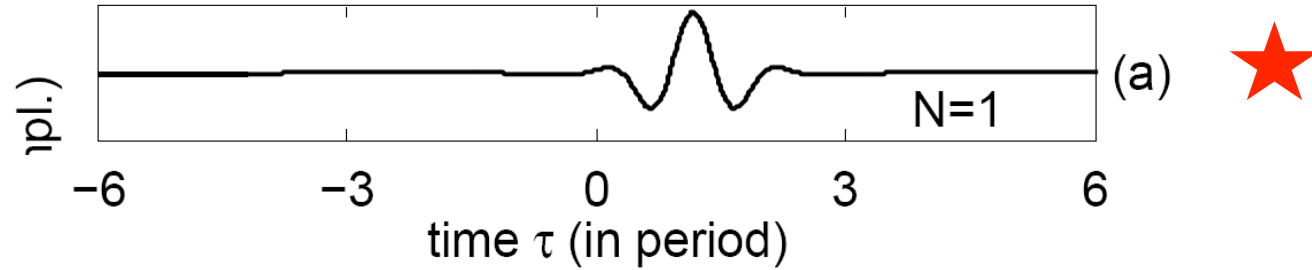
1. Convergence rate
2. Coda Wave Interferometry
3. Coda Wave Decorrelation

A ■

■ B



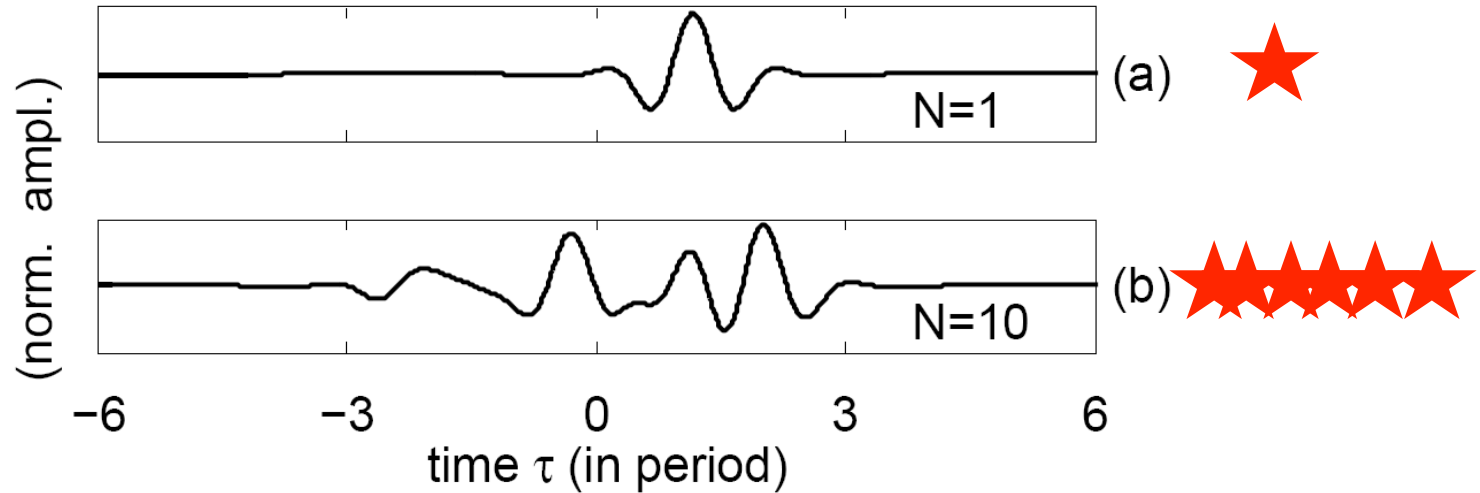
# Convergence toward the Green function



$$D = 3 \lambda$$



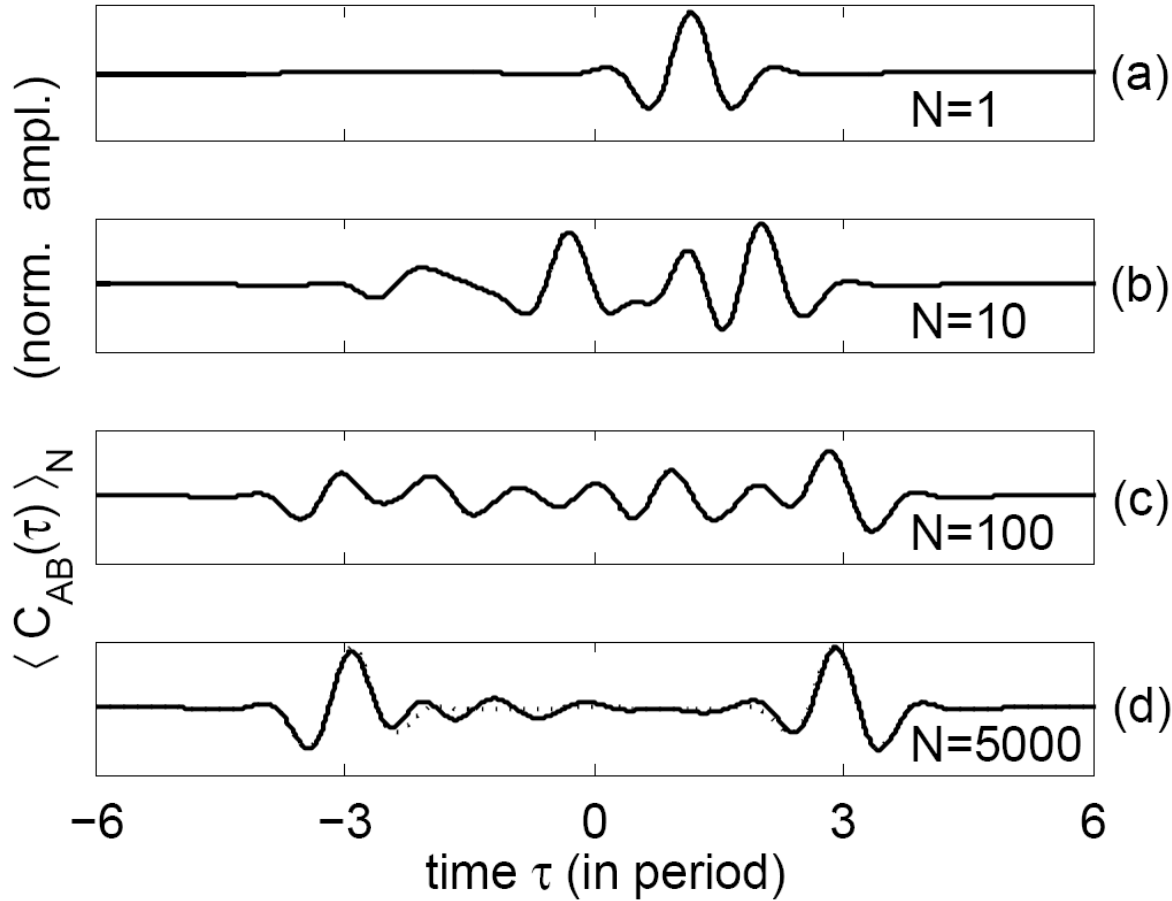
# Convergence toward the Green function

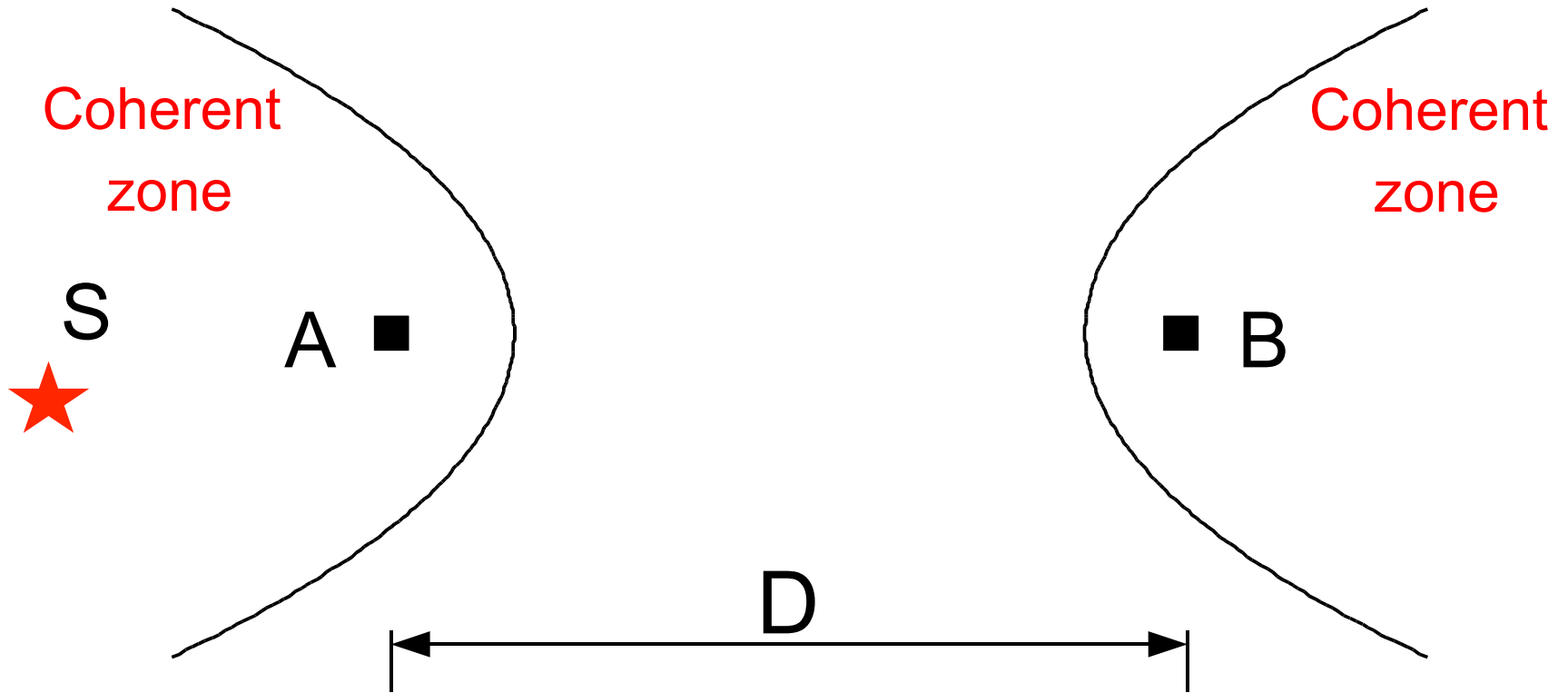


$$D = 3 \lambda$$

# Convergence toward the Green function

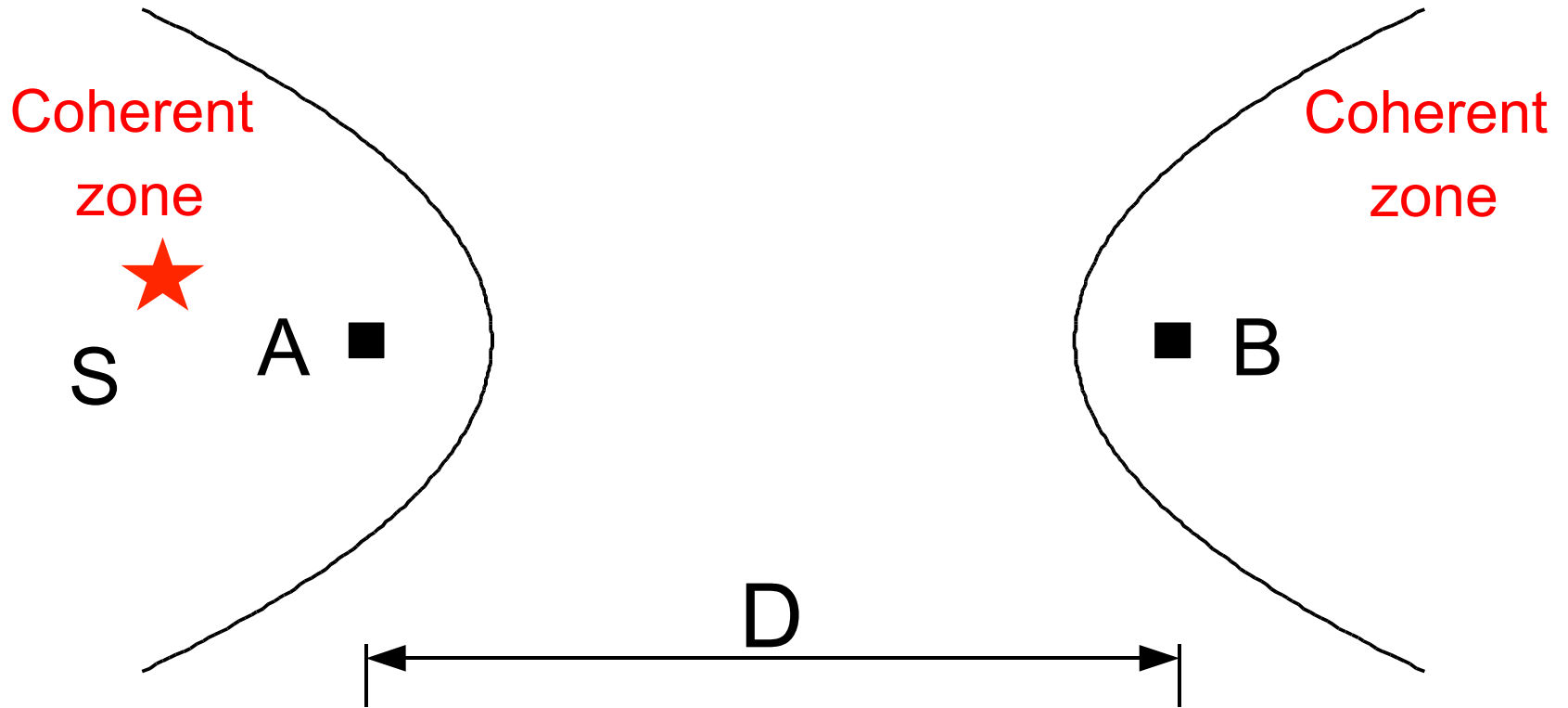
$D = 3 \lambda$





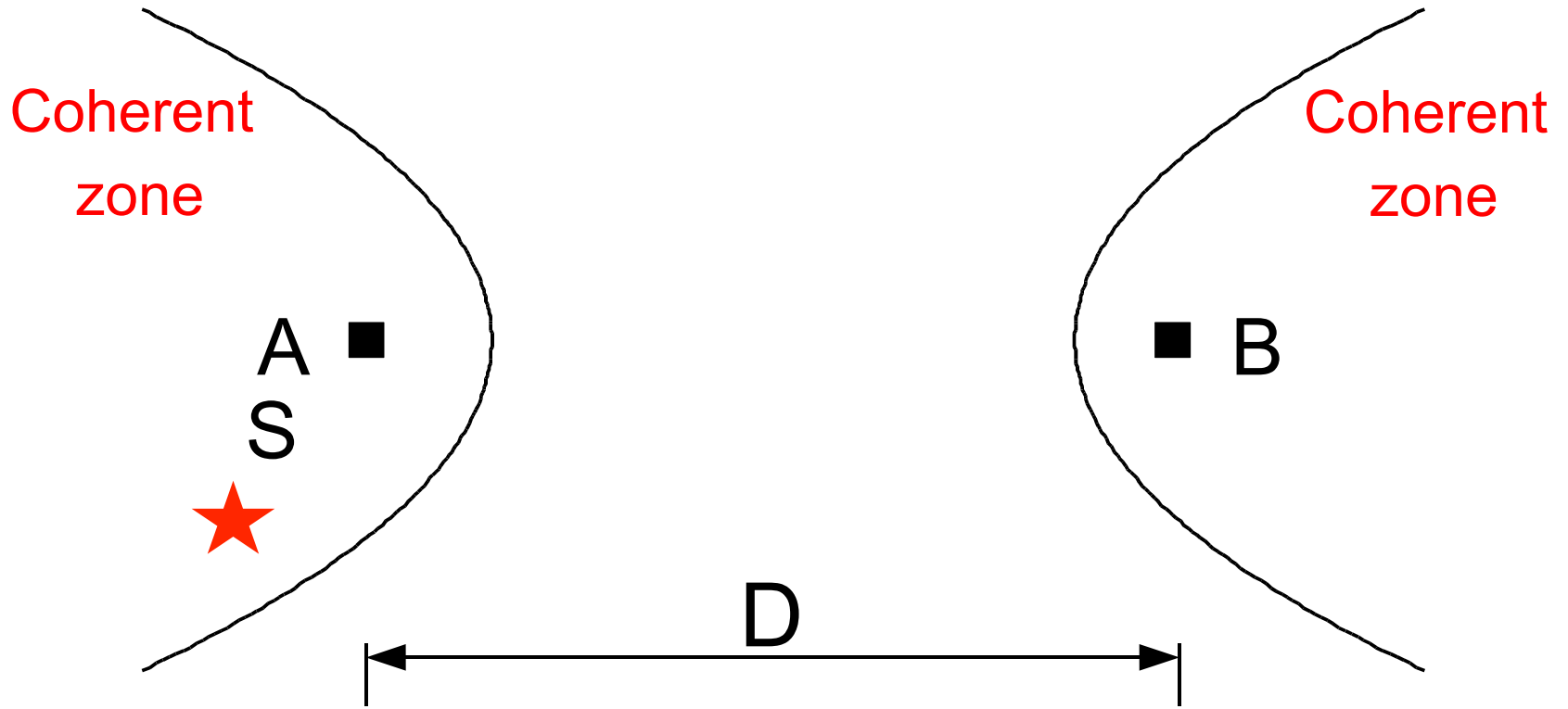
$$\Sigma \sim N$$

Roux *et al*, J. Acoust. Soc. Am. (2004)  
 Snieder, Phys. Rev. E (2004)  
 Larose, Ann. Phys. Fr (2006)



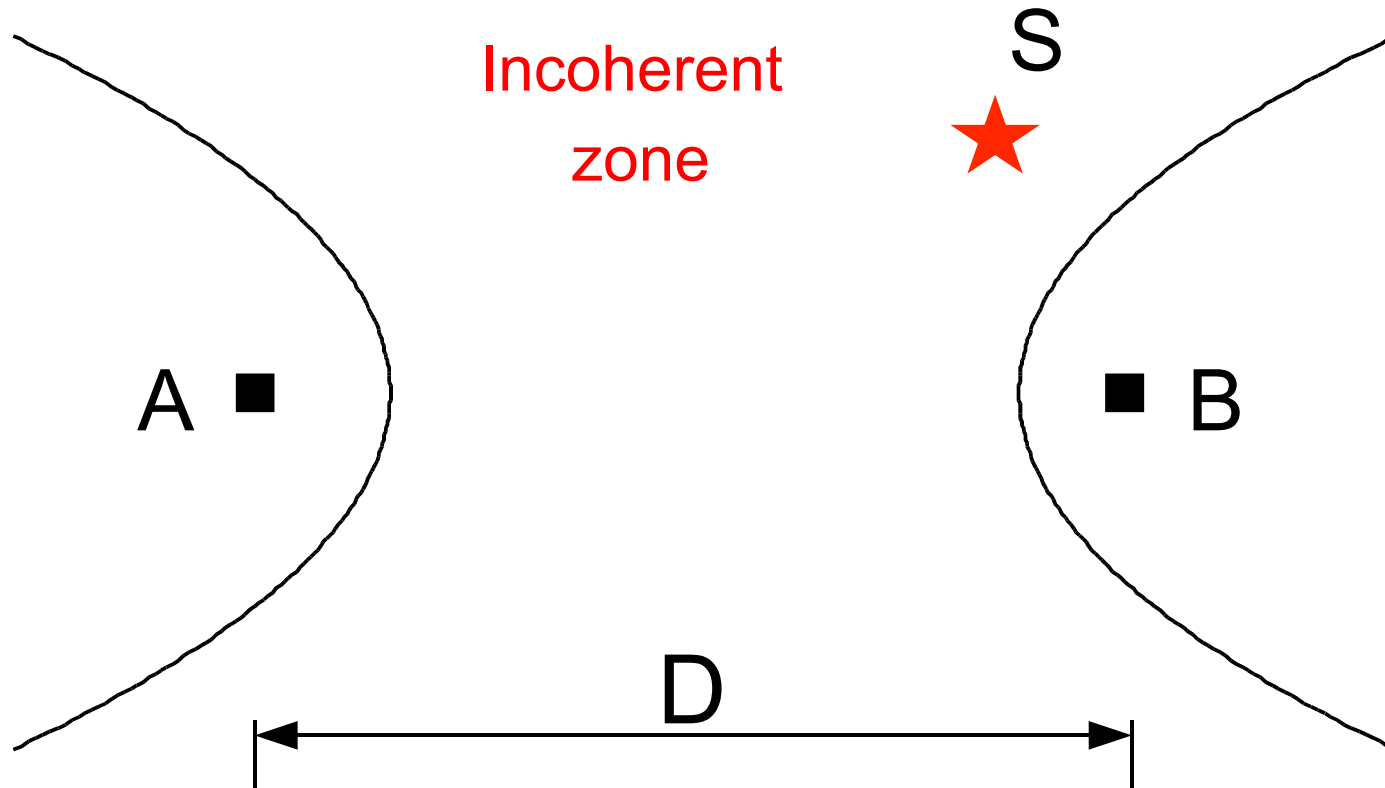
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Roux *et al*, J. Acoust. Soc. Am. (2004)  
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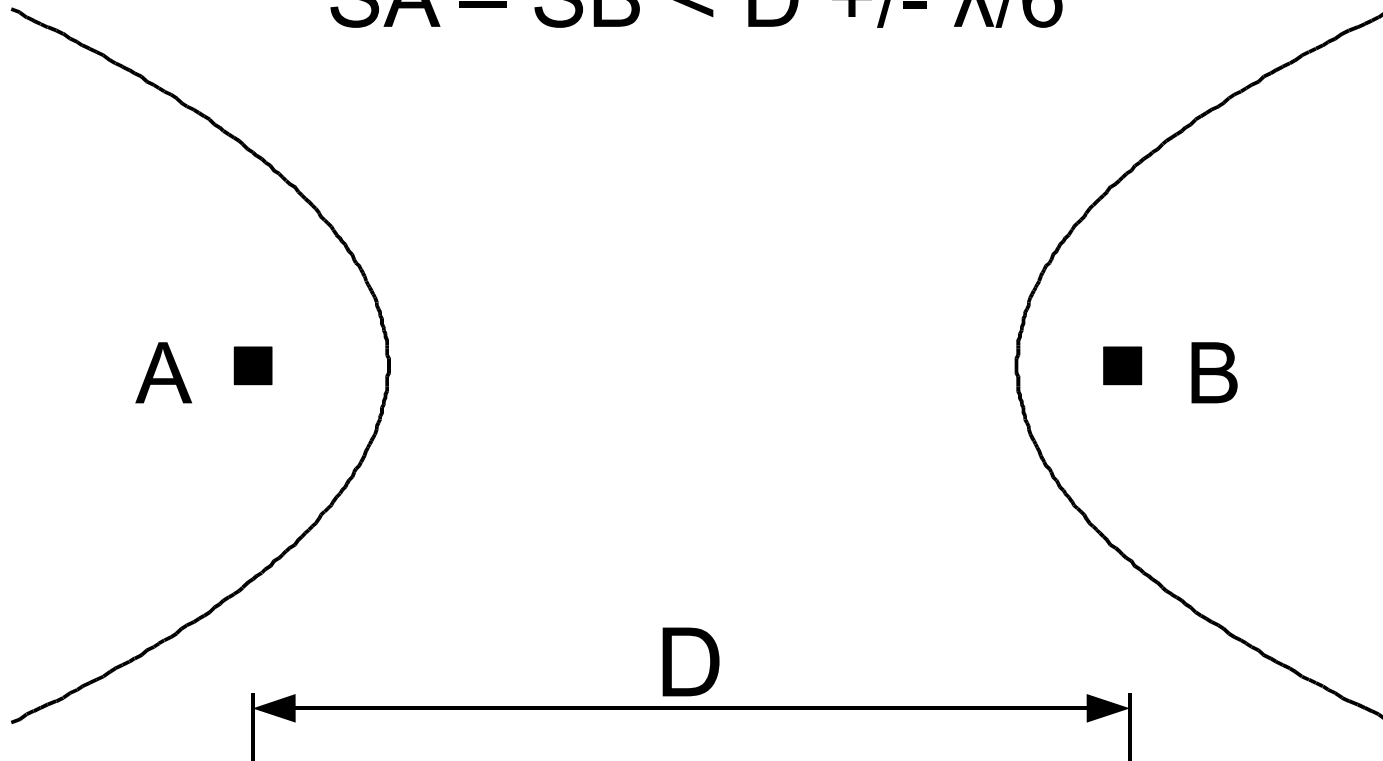
Roux *et al*, J. Acoust. Soc. Am. (2004)  
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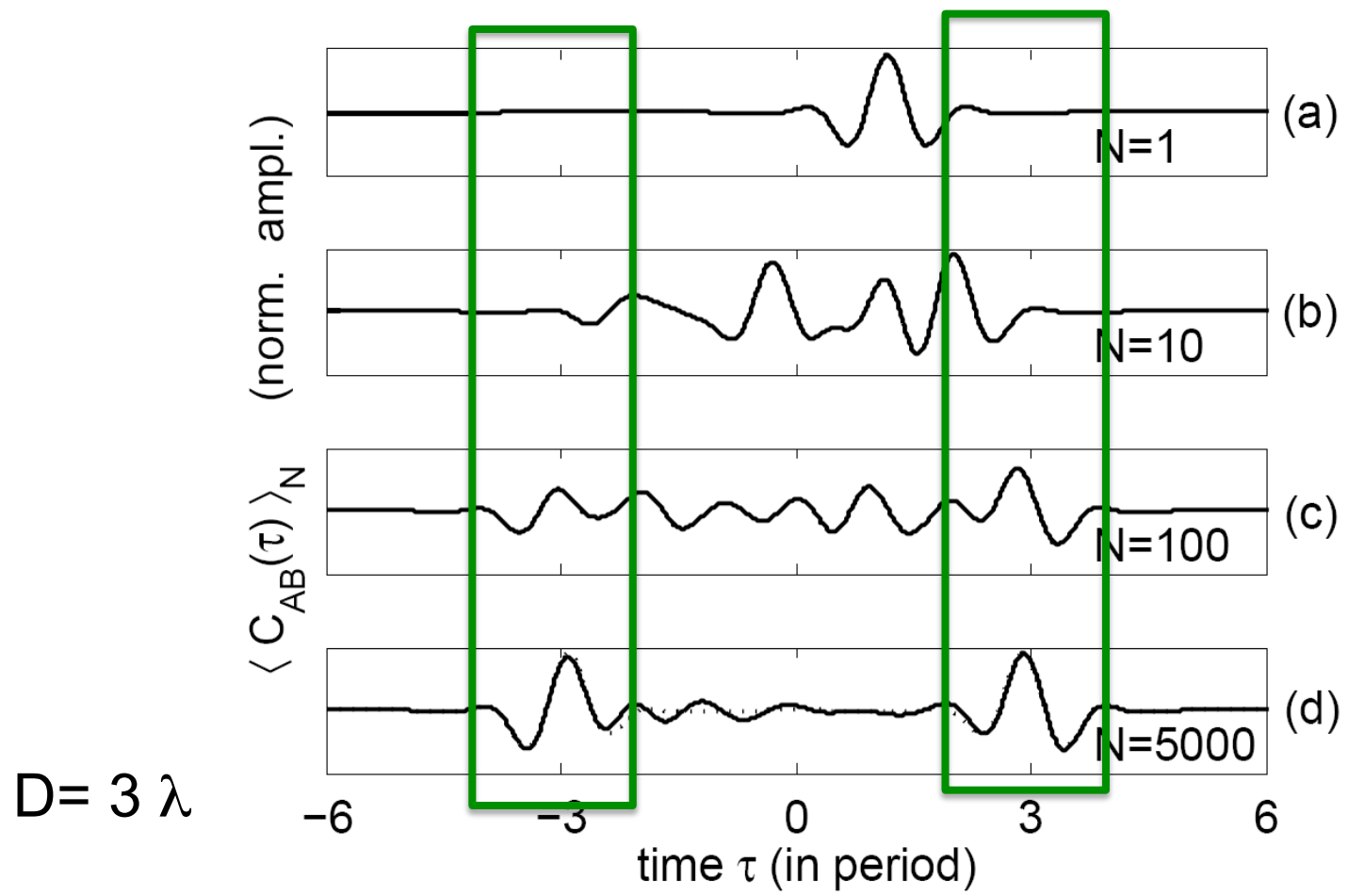
$$\Sigma \sim \sqrt{N}$$

Roux *et al*, J. Acoust. Soc. Am. (2004)  
 Snieder, Phys. Rev. E (2004)  
 Larose, Ann. Phys. Fr (2006)

Condition of coherence:  
 $SA - SB < D \pm \lambda/6$



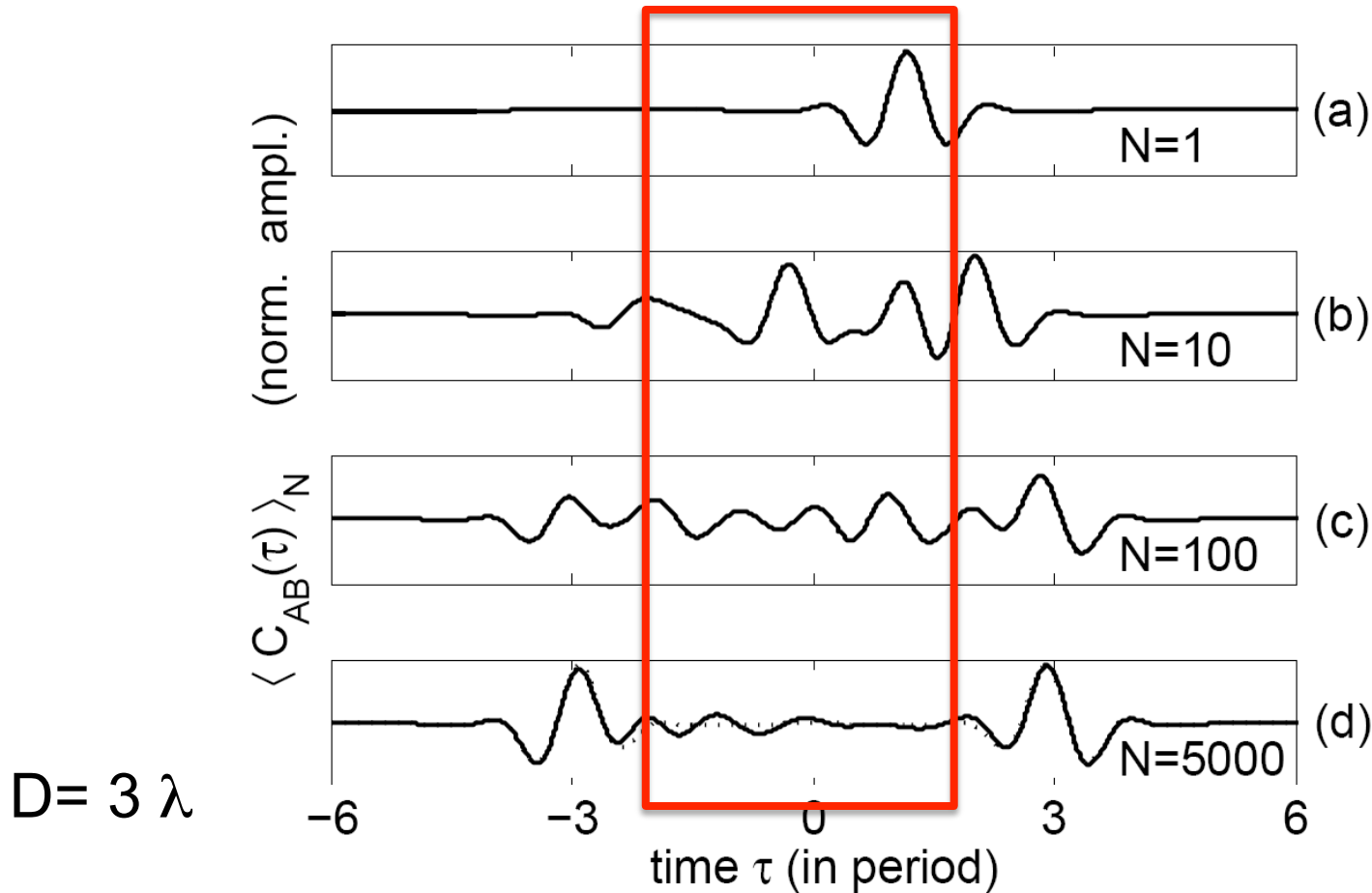
# Definition of coherent zone and SNR



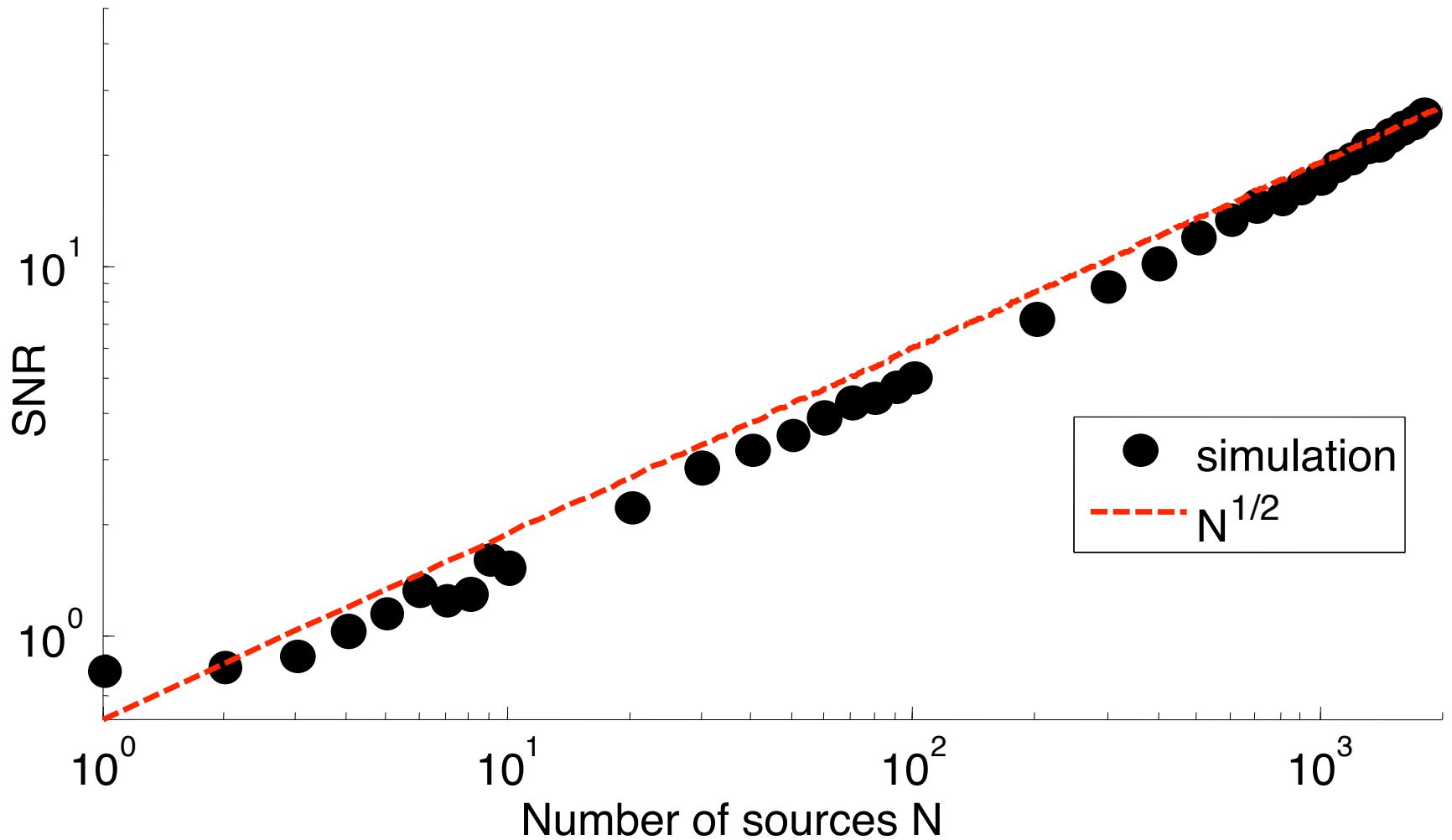
Amplitude of the signal



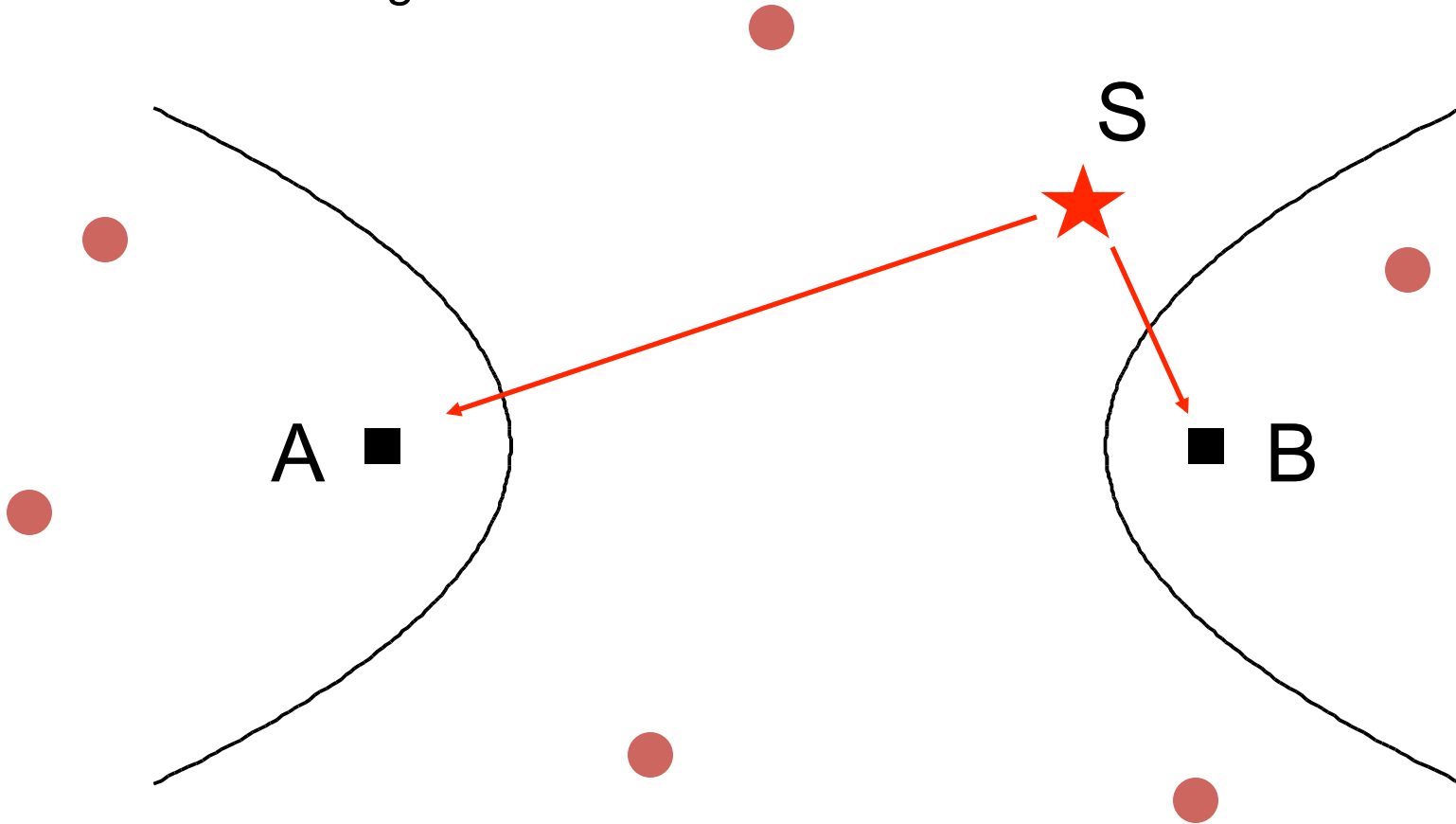
# Definition of coherent zone and SNR



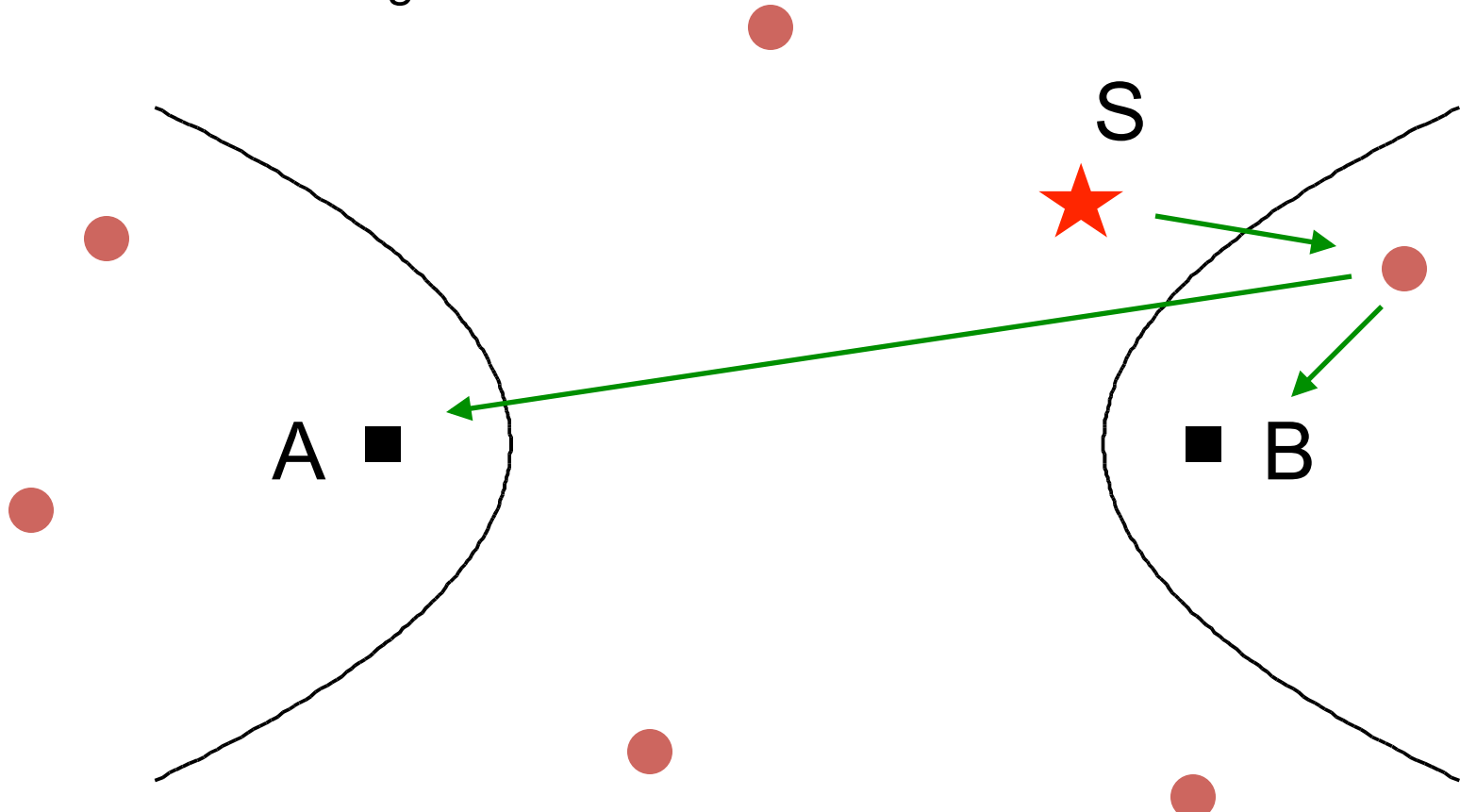
Amplitude of the noise



# Role of scattering ?

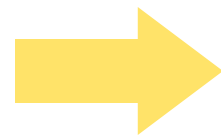
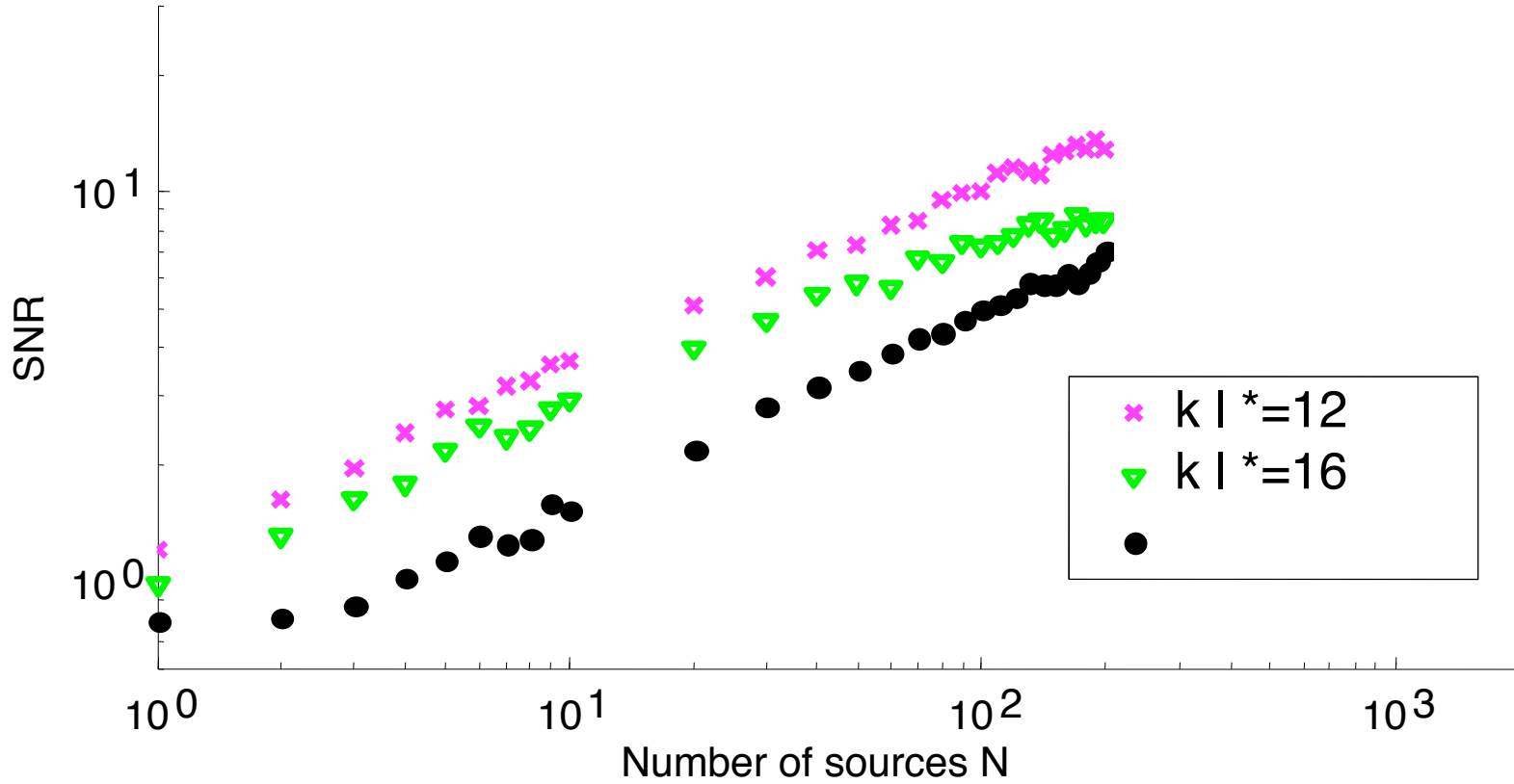


# Role of scattering ?



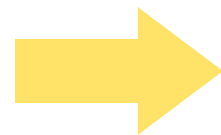
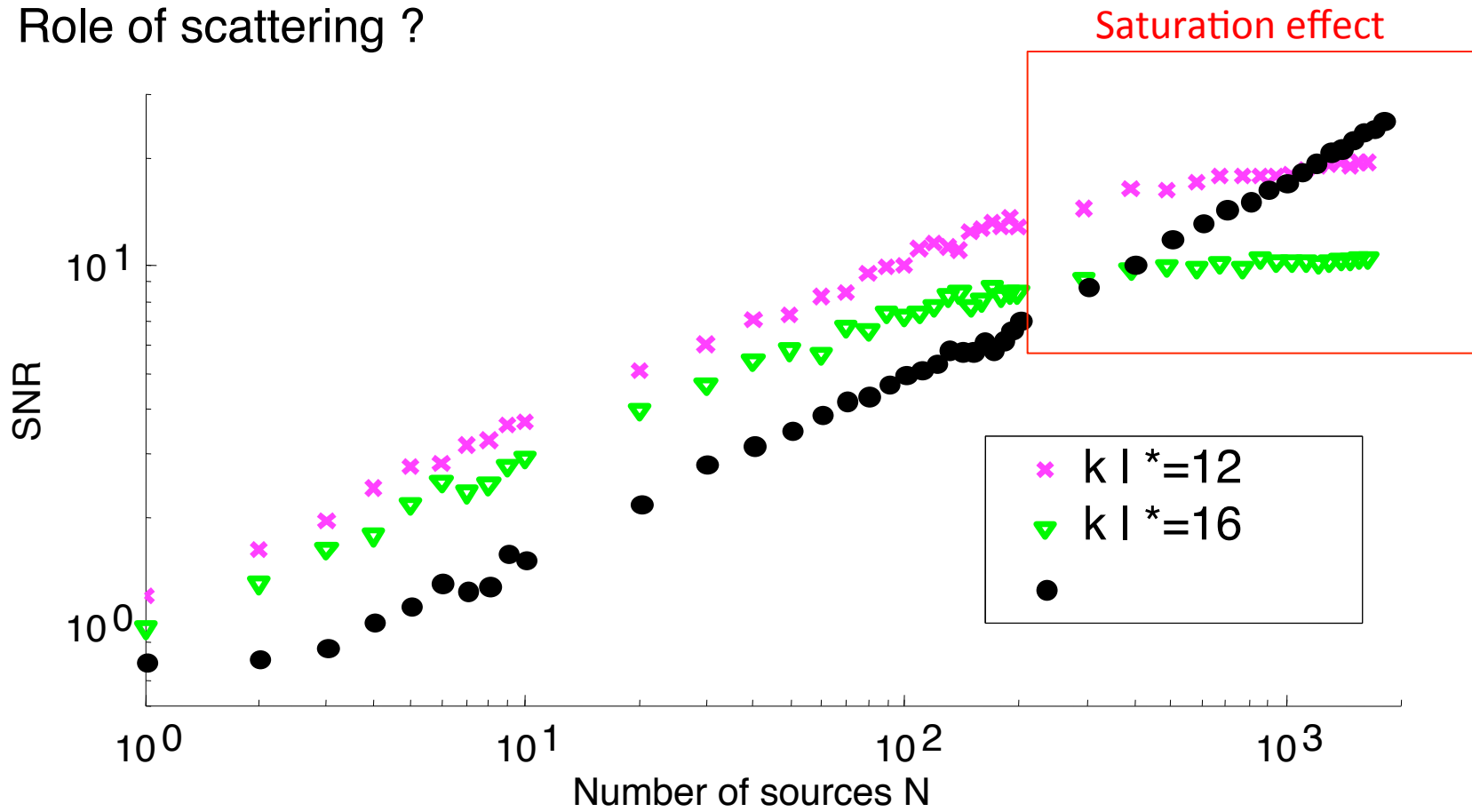
Scattering HELPS the reconstruction

# Role of scattering ?



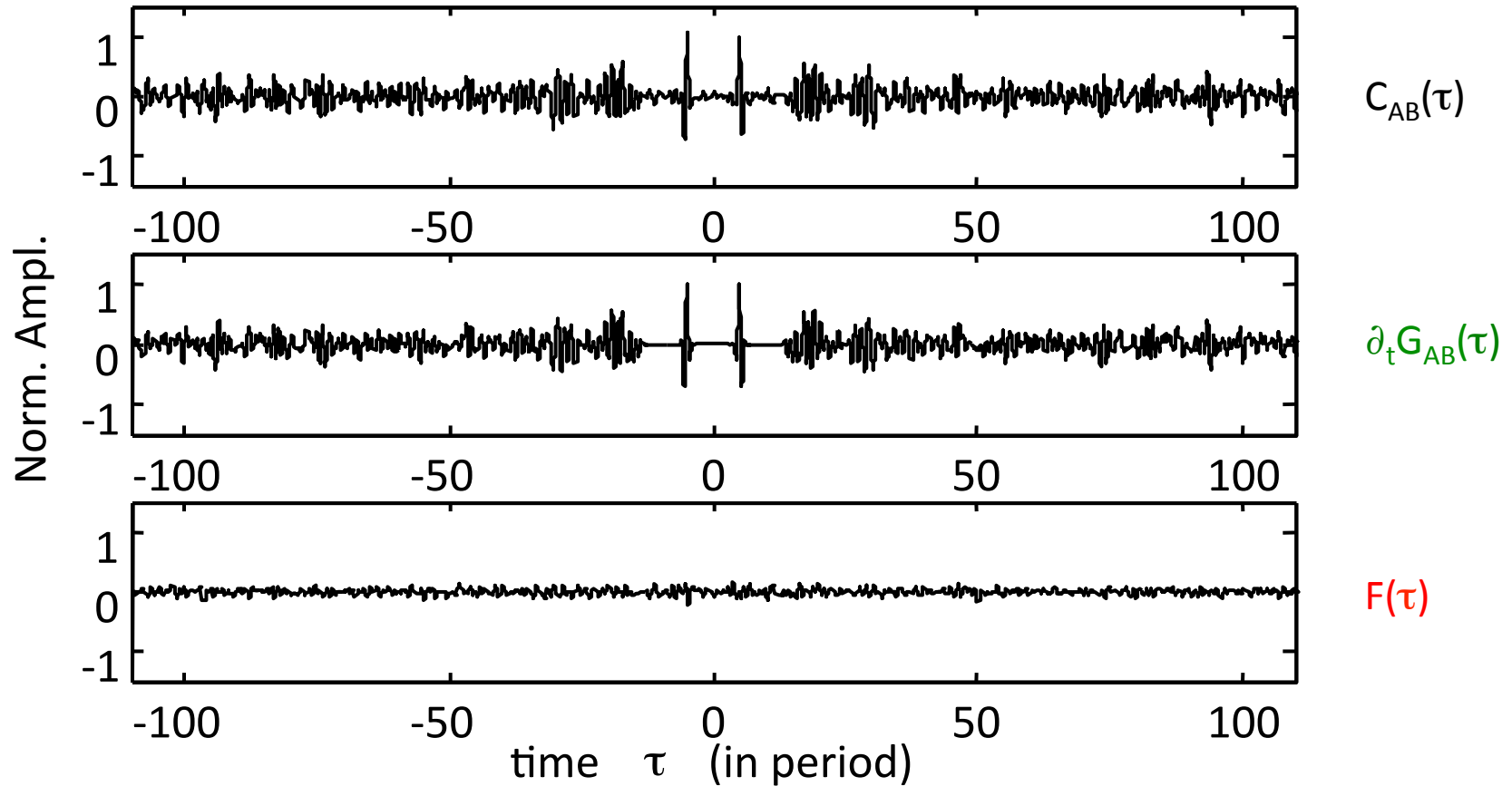
Scattering HELPS the reconstruction

### Role of scattering ?



Scattering HELPS the reconstruction

$$C_{AB}(\tau) = \partial_t G_{AB}(\tau) + F(\tau)$$



Establish a model for the AMPLITUDE of

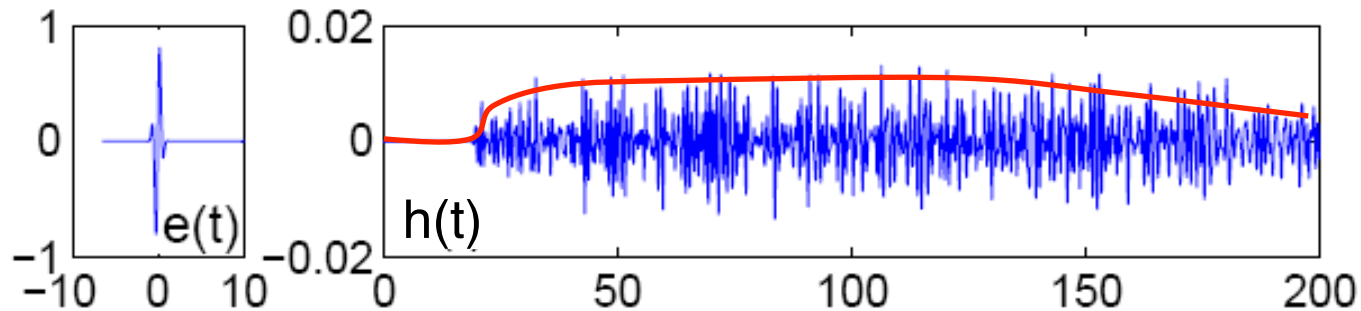
- 1) the « fluctuations »  $F(\tau)$
- 2) the « signal »  $\partial_t G_{AB}(\tau)$

« refocusing » energy in A at  $\tau=0$   $\Leftrightarrow$  autocorrelation

$$C_{AA}(\tau = 0) = \int_0^T h_A^2(t) dt$$

$$= \int_0^T \sigma^2(t) dt \quad e(t) \otimes e(-t)$$

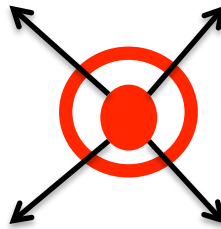
Envelope :  $\sigma(t)$





Propagation of the energy from A at  $\tau=0$   $\Leftrightarrow$  correlation in B at  $\tau$

$$\begin{aligned}
 C_{AB}(\tau) &= \int_0^T h_A(t) h_B(t + \tau) dt \\
 &= \int_0^T \sigma^2(t) dt G_{AB}(\tau) \otimes e(t) \otimes e(-t)
 \end{aligned}$$



Propagation of the energy from A at  $\tau=0$   $\Leftrightarrow$  correlation in B at  $\tau$

$$\begin{aligned}
 C_{AB}(\tau) &= \int_0^T h_A(t) h_B(t + \tau) dt \\
 &= \int_0^T \underbrace{\sigma(t) \sigma(t + \tau)}_{\text{Amplitude at the focus}} dt \underbrace{G_{AB}(\tau)}_{\text{propagation}} \otimes \underbrace{e(t) \otimes e(-t)}_{\text{Source spectrum}}
 \end{aligned}$$

Propagation of the energy from A at  $\tau=0$   $\Leftrightarrow$  correlation in B at  $\tau$

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 \end{aligned}$$

} Amplitude of the signal

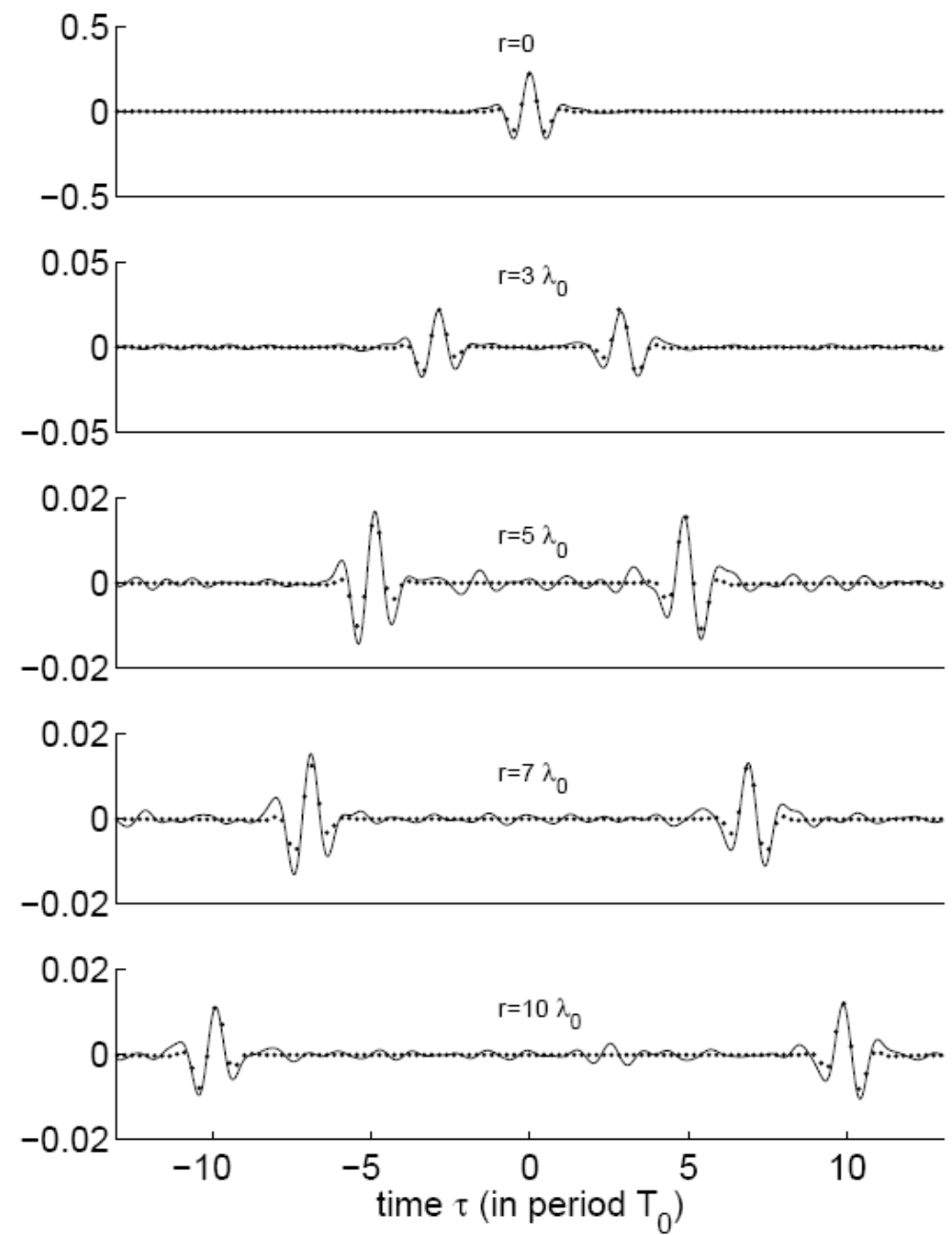
# Numerical validation

Amplitude of the signal

$$\int_0^T \sigma(t)\sigma(t + \tau)dt$$

$$\times G_{AB}(\tau) \otimes e(t) \otimes e(-t)$$

- ➔ Geometrical spreading
- ➔
- ➔



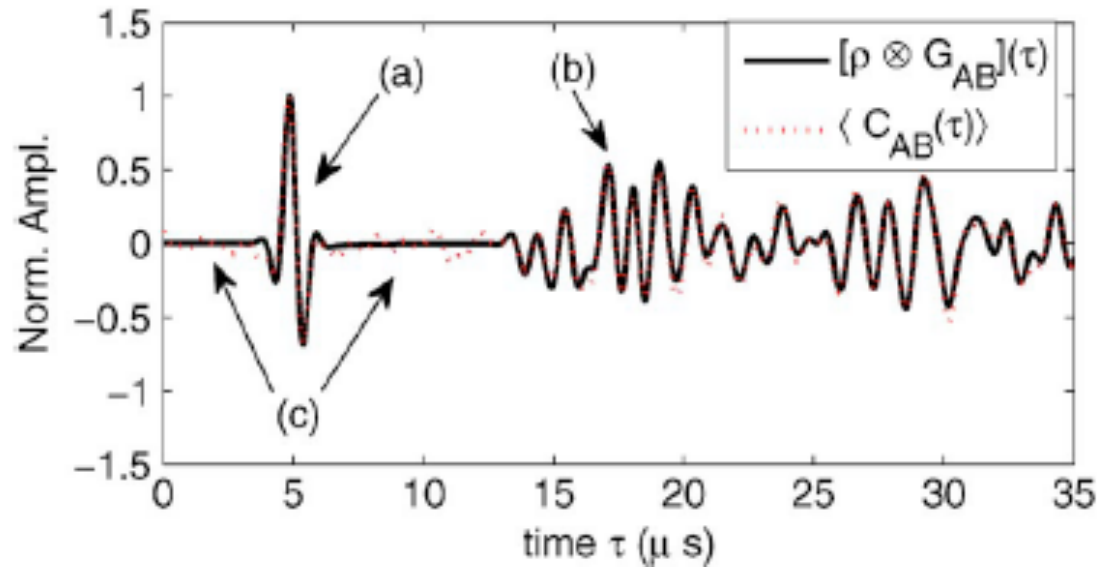
# Numerical validation

## Amplitude of the signal

$$\int_0^T \sigma(t)\sigma(t + \tau)dt$$

$$\times G_{AB}(\tau) \otimes e(t) \otimes e(-t)$$

- ➔ Geometrical spreading
- ➔ Scattered waves
- ➔



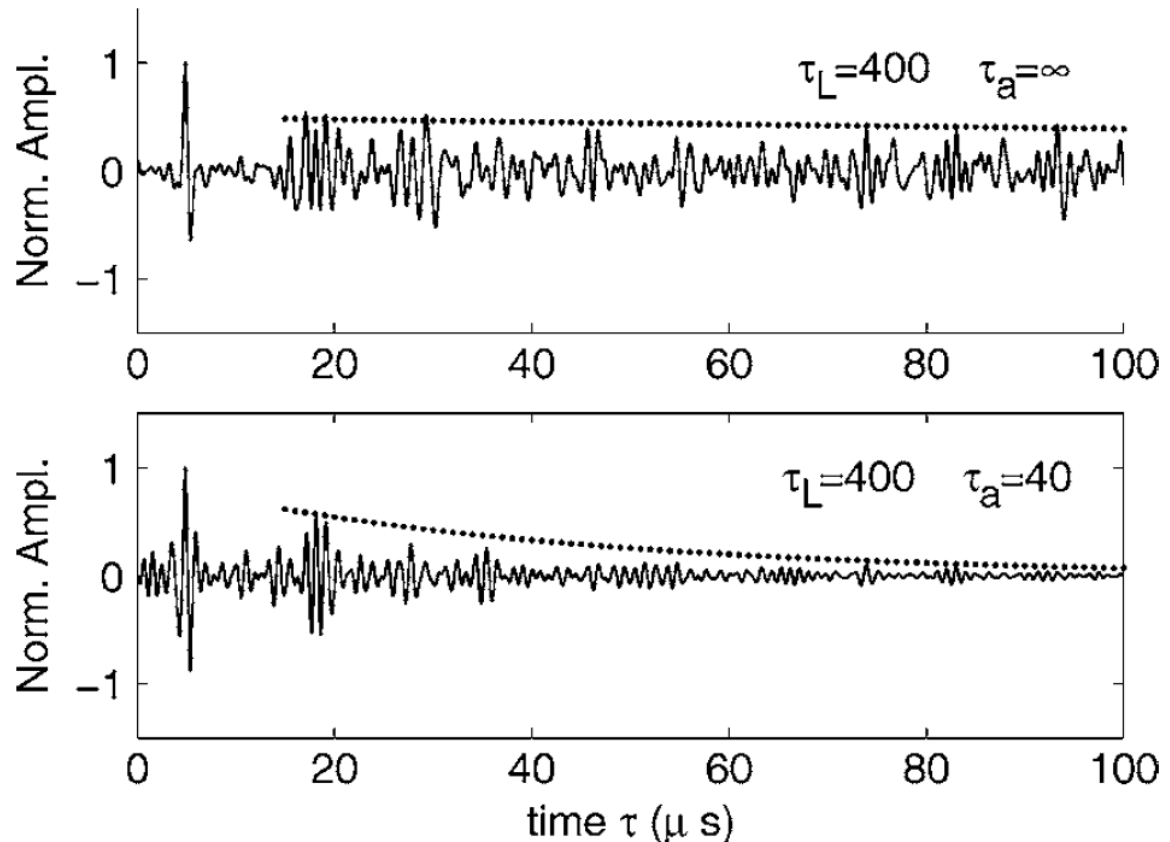
# Numerical validation

## Amplitude of the signal

$$\int_0^T \sigma(t)\sigma(t + \tau)dt$$

$$\times G_{AB}(\tau) \otimes e(t) \otimes e(-t)$$

- ➔ Geometrical spreading
- ➔ Scattered waves
- ➔ Attenuation



# Fluctuations of correlations:

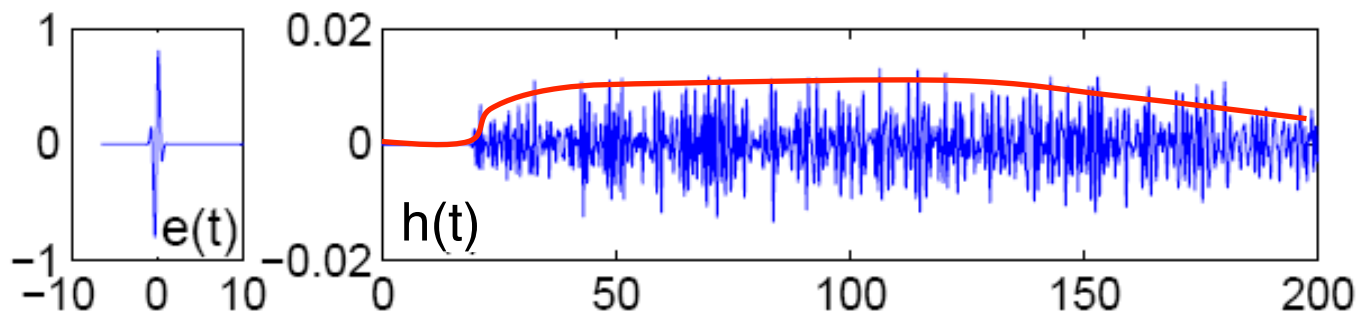
$$\text{var} \{C_{AB}\} = \langle C_{AB}^2(\tau) \rangle - \langle C_{AB}(\tau) \rangle^2$$

We assume:

Coda = succession of independent information grain = « shot noise » model  
 AND independent sources

$$\text{var}_{theo} \approx \int_0^T \sigma^2(\theta) \sigma^2(\theta + \tau) d\theta \int \rho^2(q) dq$$

Envelope :  $\sigma(t)$



## Fluctuations of correlations:

$$\text{var} \{C_{AB}\} = \langle C_{AB}^2(\tau) \rangle - \langle C_{AB}(\tau) \rangle^2$$

We assume:

Coda = succession of independent information grain = « shot noise » model  
 AND independent sources

$$\text{var}_{theo} \approx \int_0^T \sigma^2(\theta) \sigma^2(\theta + \tau) d\theta \int \rho^2(q) dq$$

$$SNR_{theo} = \frac{S_{theo}}{\sqrt{\text{var}_{theo}}} \quad SNR_{num} = \frac{S_{num}}{\sqrt{\text{var}_{num}}}$$



$$SNR_{theo}(\tau, d) = \frac{[\rho \otimes G_{AB}](\tau)}{\sqrt{\int \rho^2(t) dt}} \times \frac{\int_0^T \sigma(t)\sigma(t + \tau) dt}{\sqrt{\int_0^T \sigma^2(t)\sigma^2(t + \tau) dt}}$$

$$SNR_{theo}(\tau, d) = \frac{[\rho \otimes G_{AB}](\tau)}{\sqrt{\int \rho^2(t) dt}} \times \frac{\int_0^T \sigma(t)\sigma(t + \tau) dt}{\sqrt{\int_0^T \sigma^2(t)\sigma^2(t + \tau) dt}}$$

SNR depends on:

→ The Green function (geometrical spreading, attenuation...)

$$\sqrt{\frac{1}{(kr)^{d-1}}}$$

$$SNR_{theo}(\tau, d) = \frac{[\rho \otimes G_{AB}](\tau)}{\sqrt{\int \rho^2(t) dt}} \times \frac{\int_0^T \sigma(t)\sigma(t + \tau) dt}{\sqrt{\int_0^T \sigma^2(t)\sigma^2(t + \tau) dt}}$$

SNR depends on:

- ➔ The Green function (geometrical spreading, attenuation...)
- ➔ Source Bandwidth

$$\sqrt{\int \rho^2(t) dt} = \sqrt{\frac{1}{\Delta\omega}}$$

$$SNR_{theo}(\tau, d) = \frac{[\rho \otimes G_{AB}](\tau)}{\sqrt{\int \rho^2(t) dt}} \times \frac{\int_0^T \sigma(t)\sigma(t + \tau) dt}{\sqrt{\int_0^T \sigma^2(t)\sigma^2(t + \tau) dt}}$$

SNR depends on:

- ➔ The Green function (geometrical spreading, attenuation...)
- ➔ Source Bandwidth
- ➔ Duration and envelope of the record

In the case of stable noise...

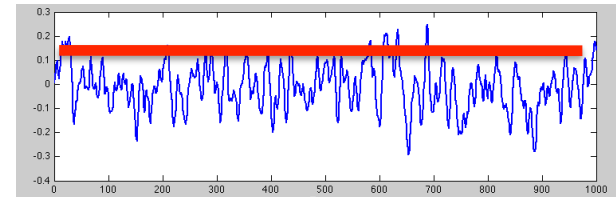
$$\frac{\int_0^T \sigma(t)\sigma(t + \tau) dt}{\sqrt{\int_0^T \sigma^2(t)\sigma^2(t + \tau) dt}} \approx \frac{T}{\sqrt{T}}$$

$$SNR_{theo}(\tau, d) = \frac{[\rho \otimes G_{AB}](\tau)}{\sqrt{\int \rho^2(t) dt}} \times \frac{\int_0^T \sigma(t)\sigma(t + \tau) dt}{\sqrt{\int_0^T \sigma^2(t)\sigma^2(t + \tau) dt}}$$

SNR depends on:

- ➔ The Green function (geometrical spreading, attenuation...)
- ➔ Source Bandwidth
- ➔ Duration and envelope of the record

In the case of stable noise...



$$SNR_{theo} \propto \sqrt{T \cdot \Delta\omega} \sqrt{\frac{1}{(kr)^{d-1}}}$$

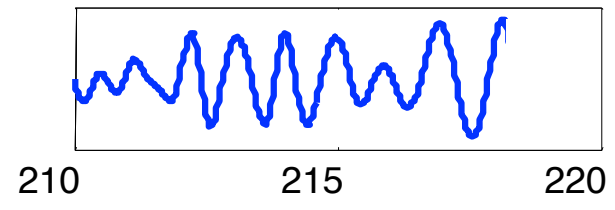
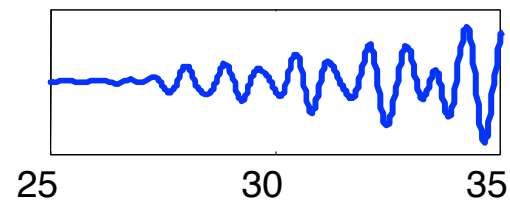
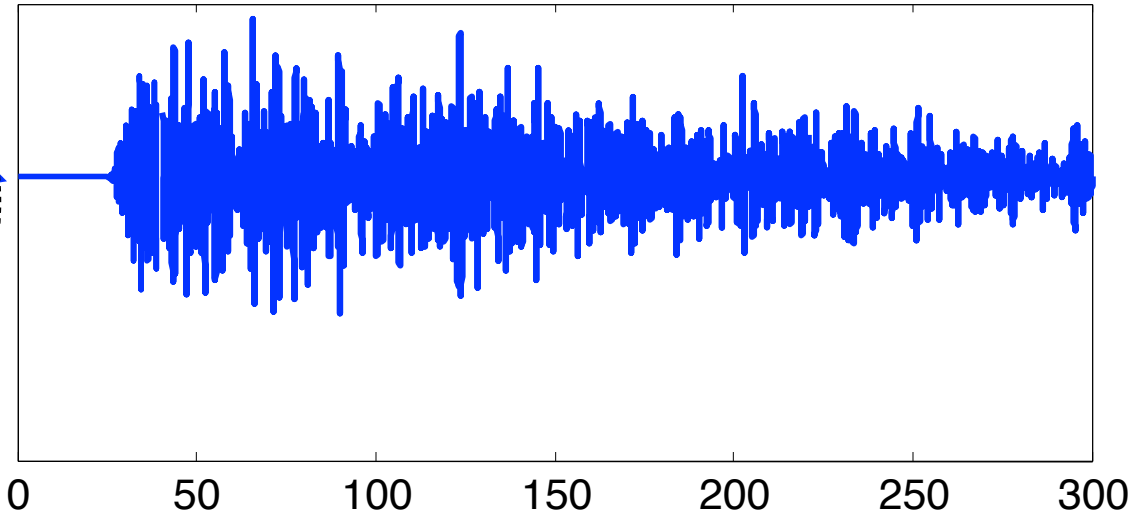
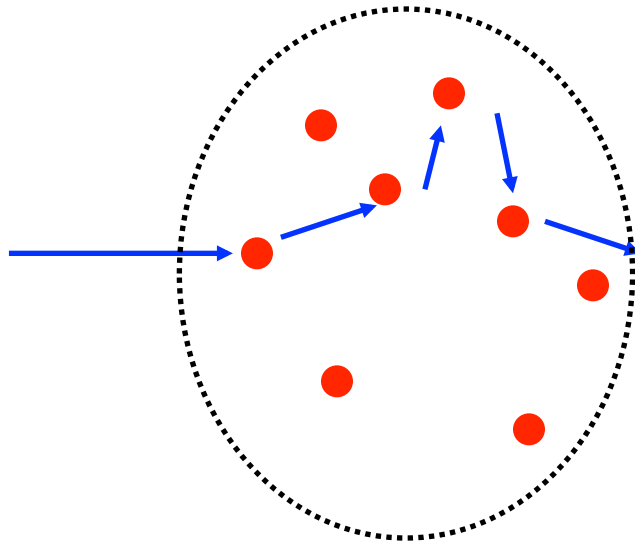
Larose *et al*, J. Appl. Phys (2008)

Cf also: Sabra *et al*. JASA 2005  
& Richard Weaver's papers

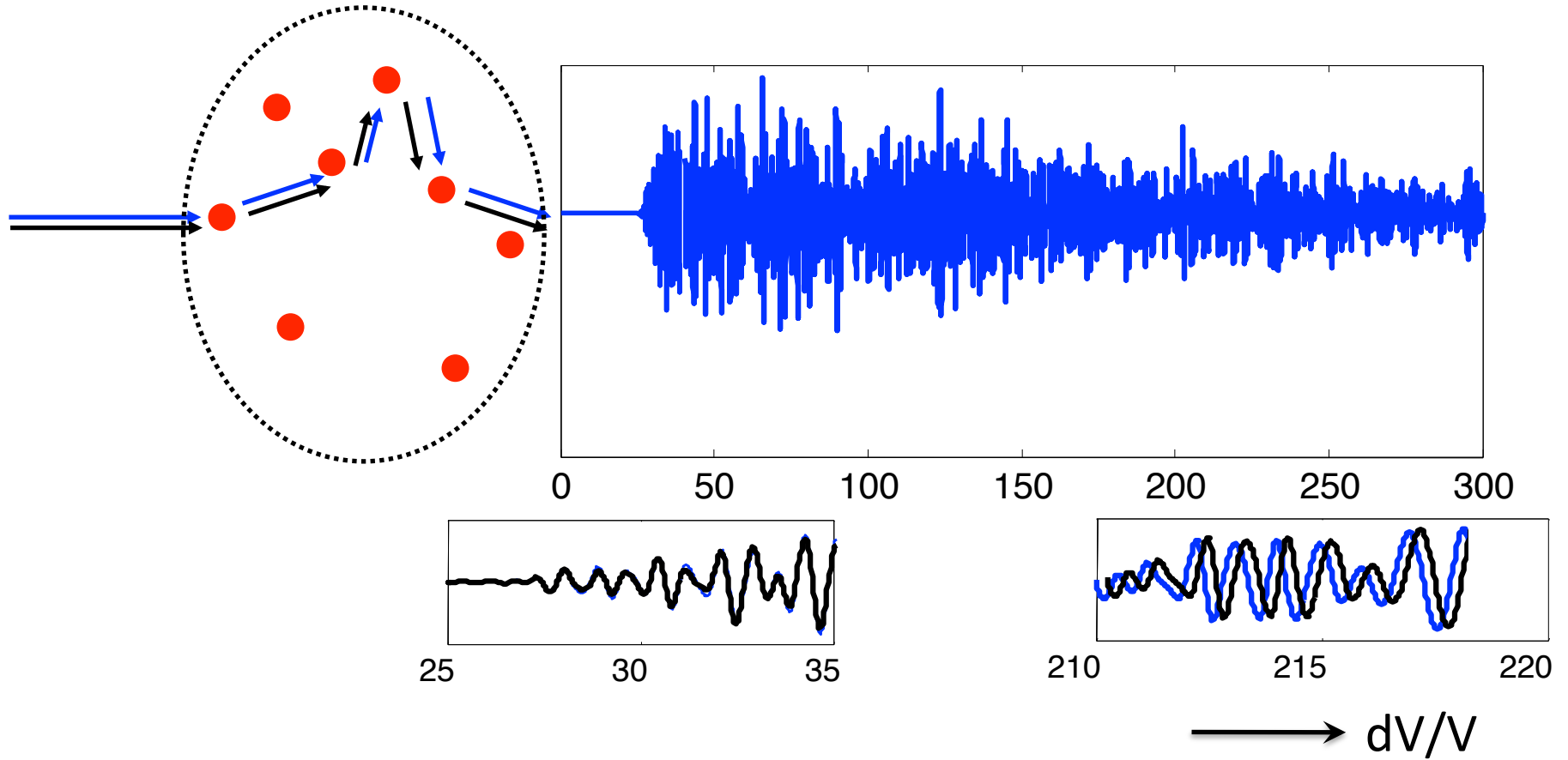
1. Convergence rate
2. Coda Wave Interferometry
3. Coda Wave Decorrelation

## Coda Wave Interferometry

- Poupinet et al 1984
- Snieder et al 2002
- ...







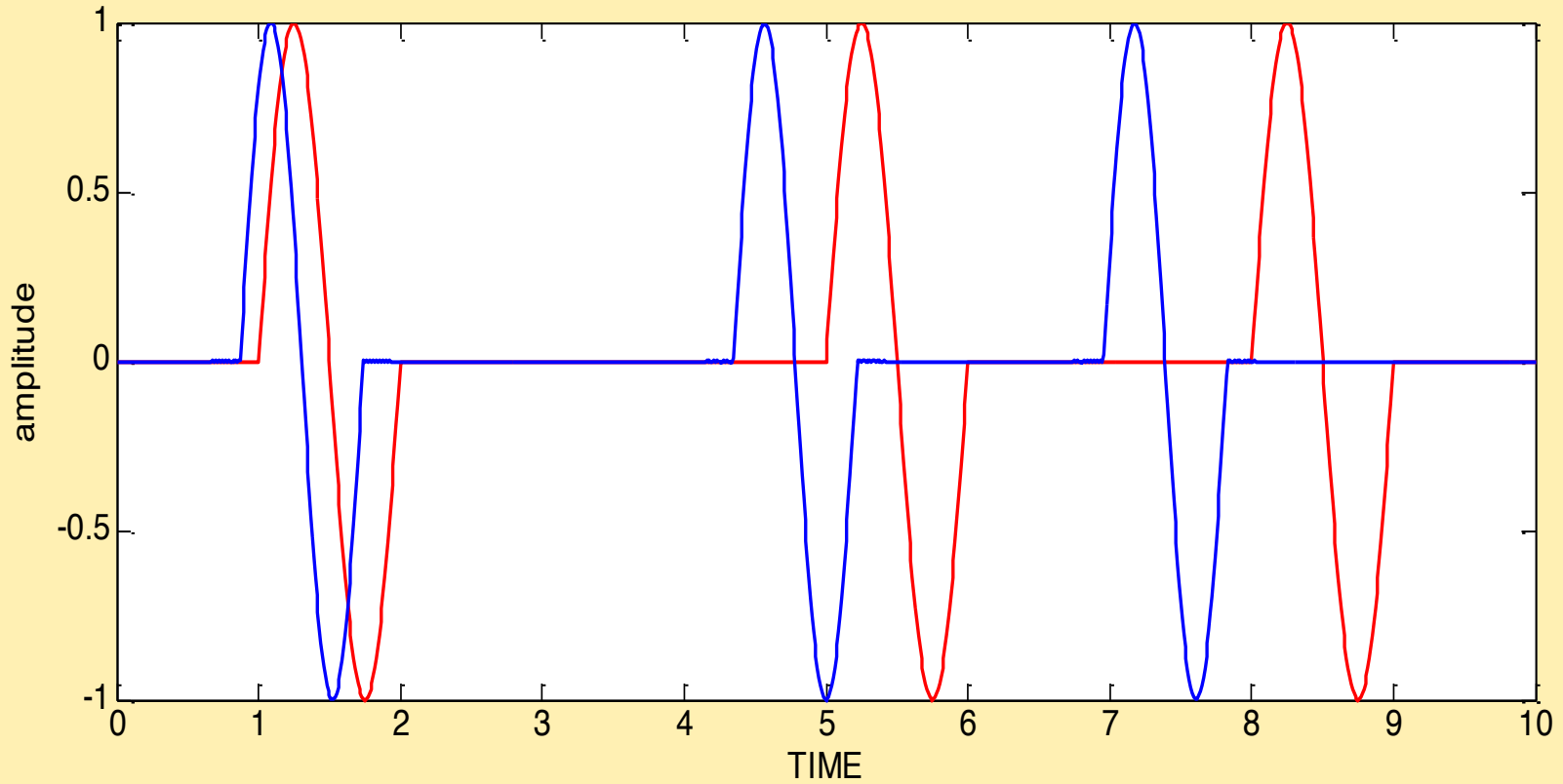
simulation

Date #1

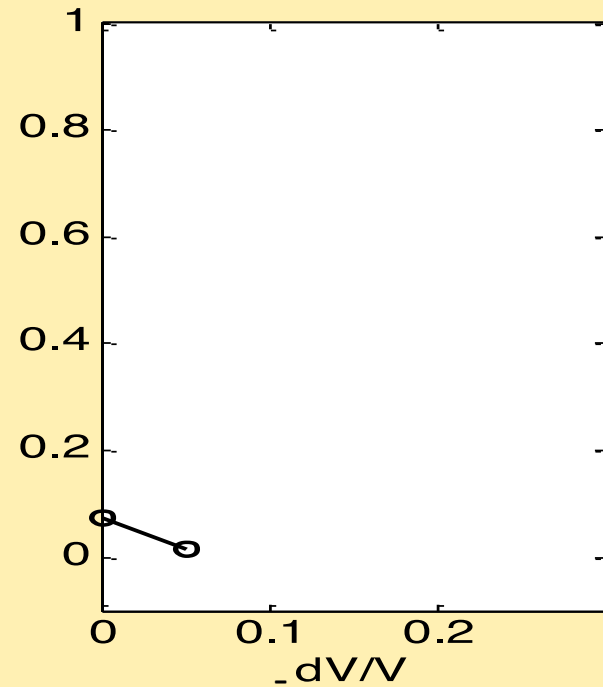
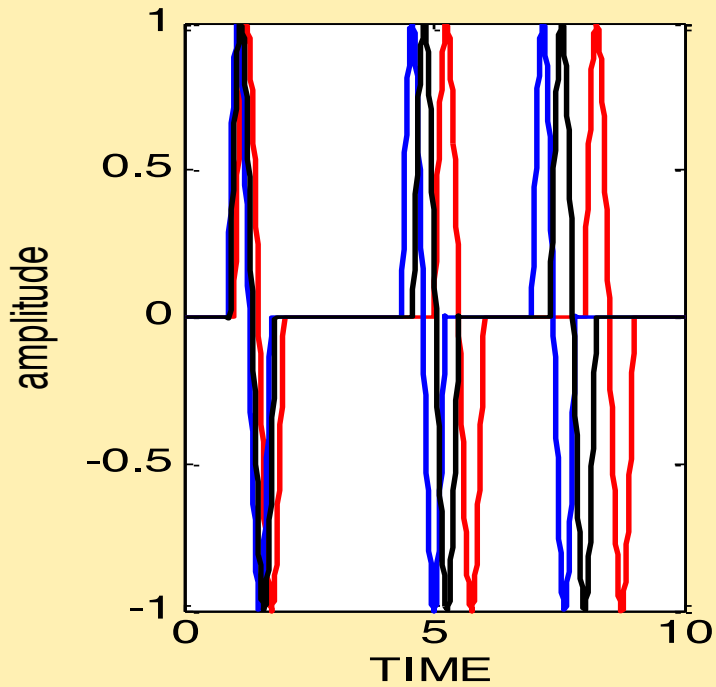
Froid = RAPIDE

Date #2

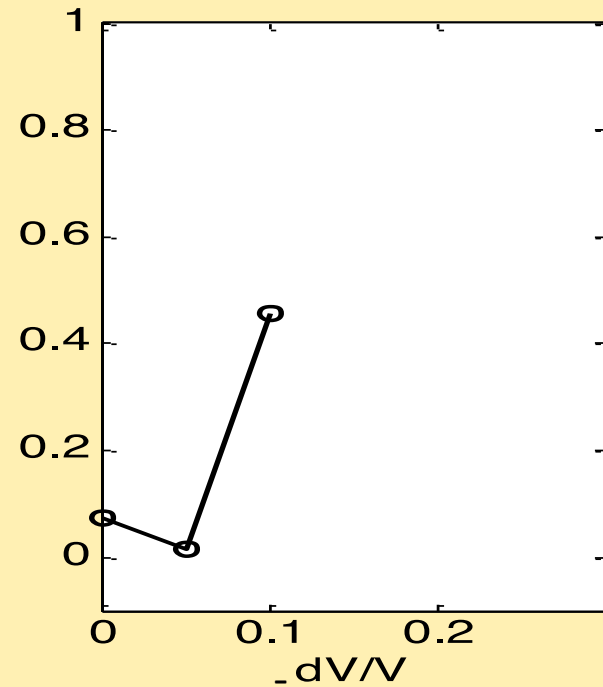
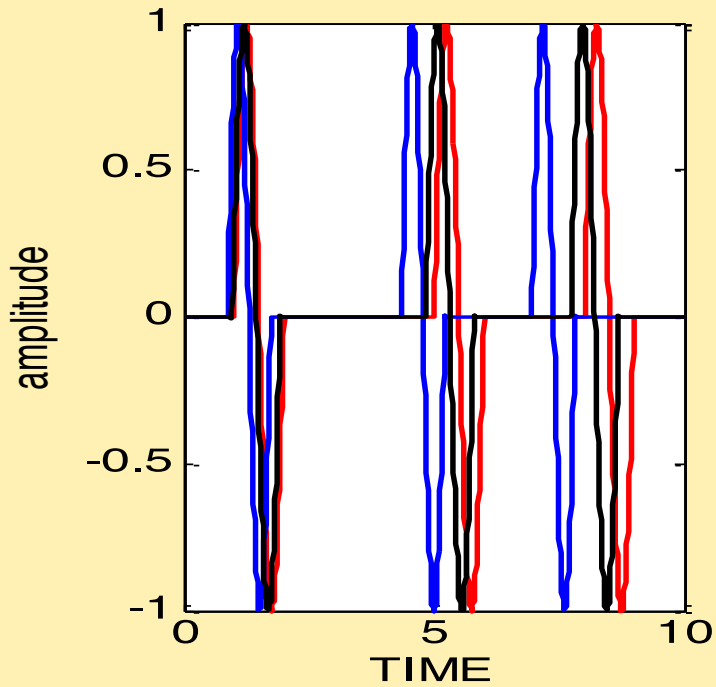
Chaud = LENT



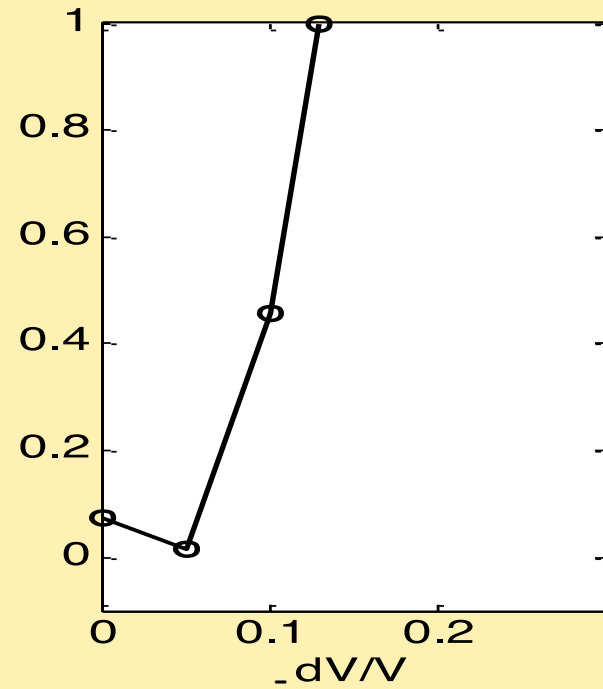
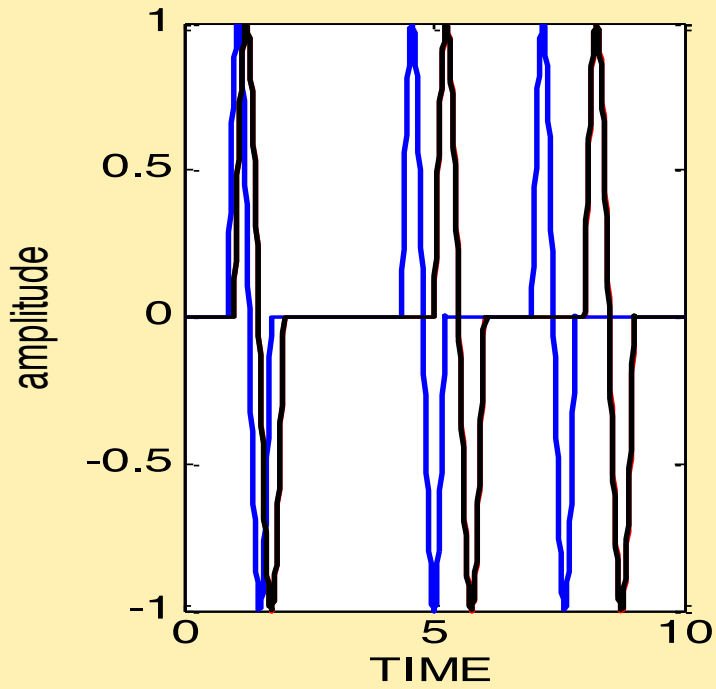
simulation



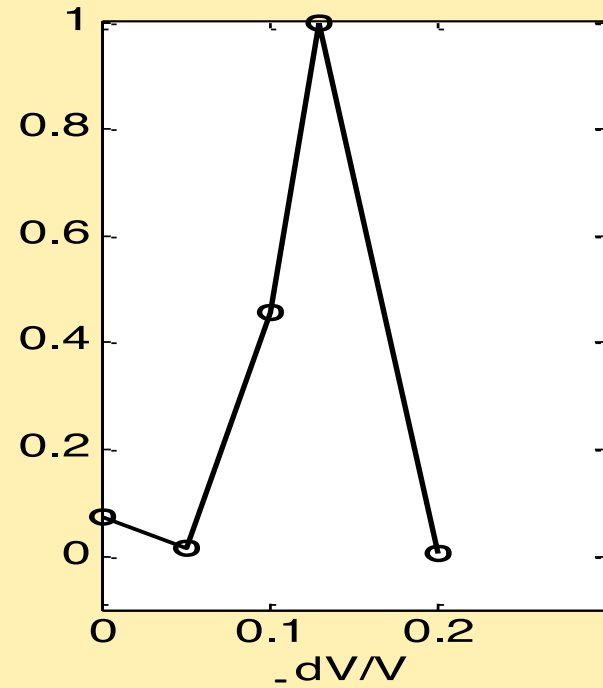
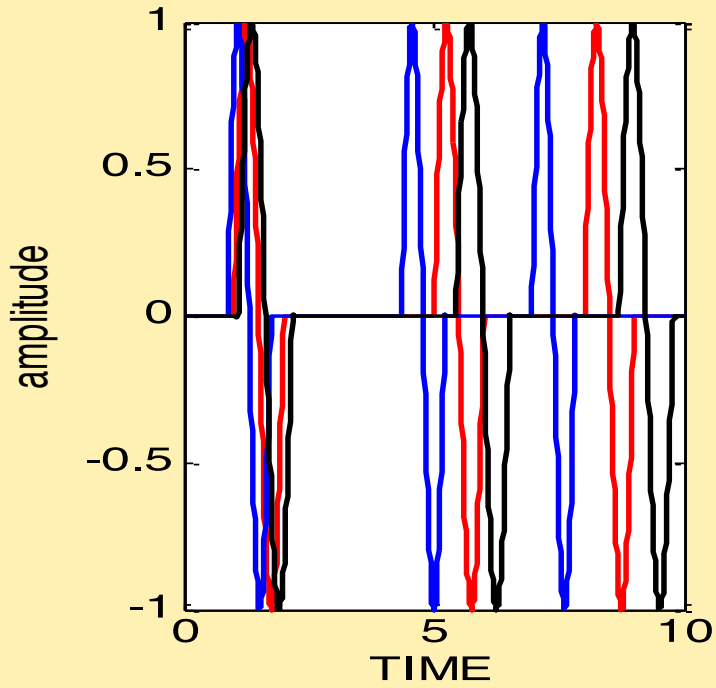
simulation



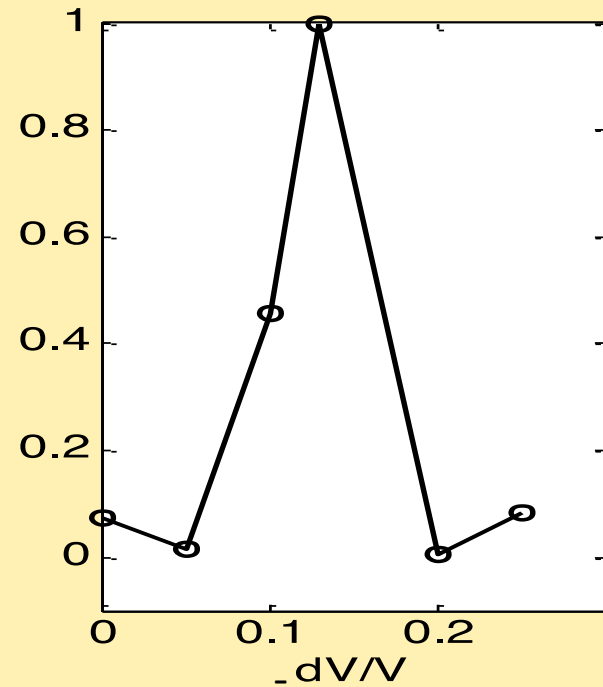
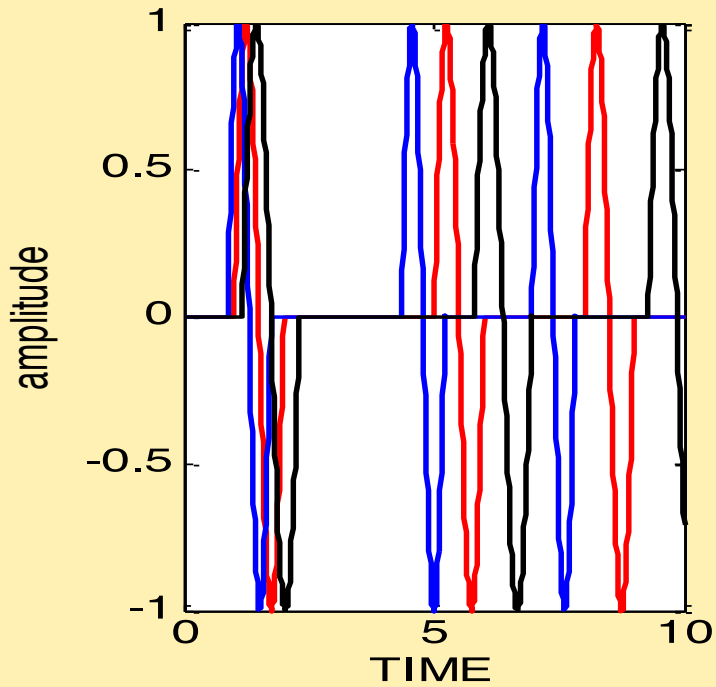
simulation



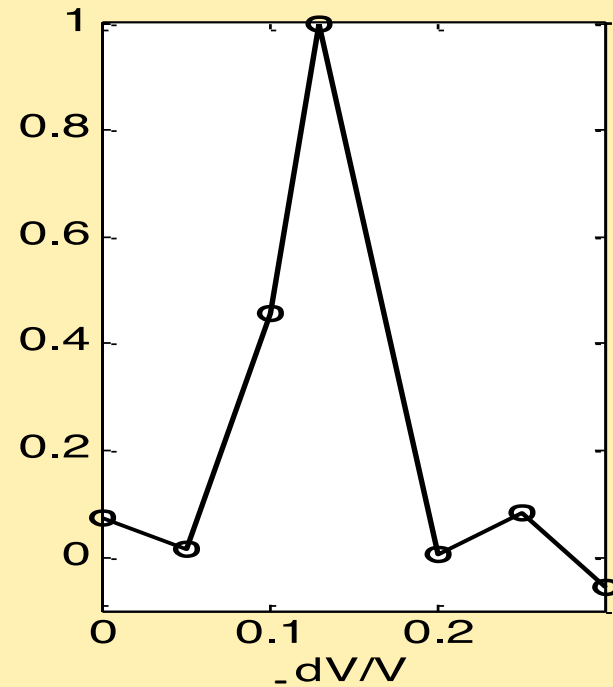
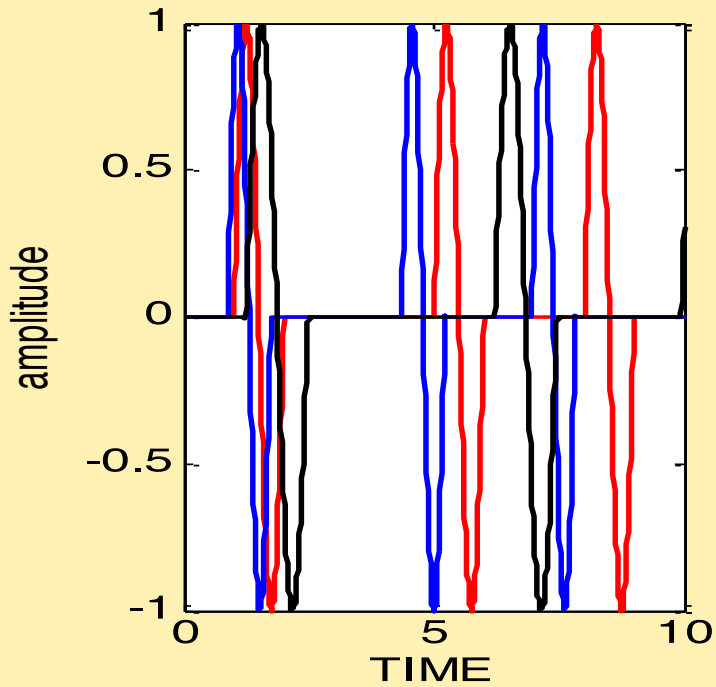
simulation



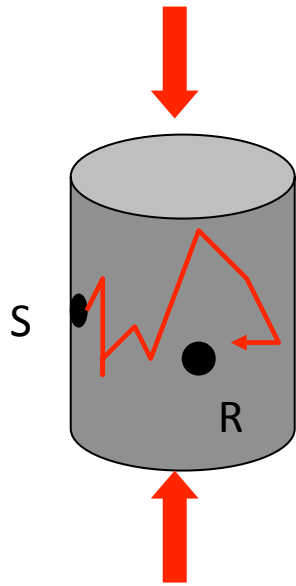
simulation



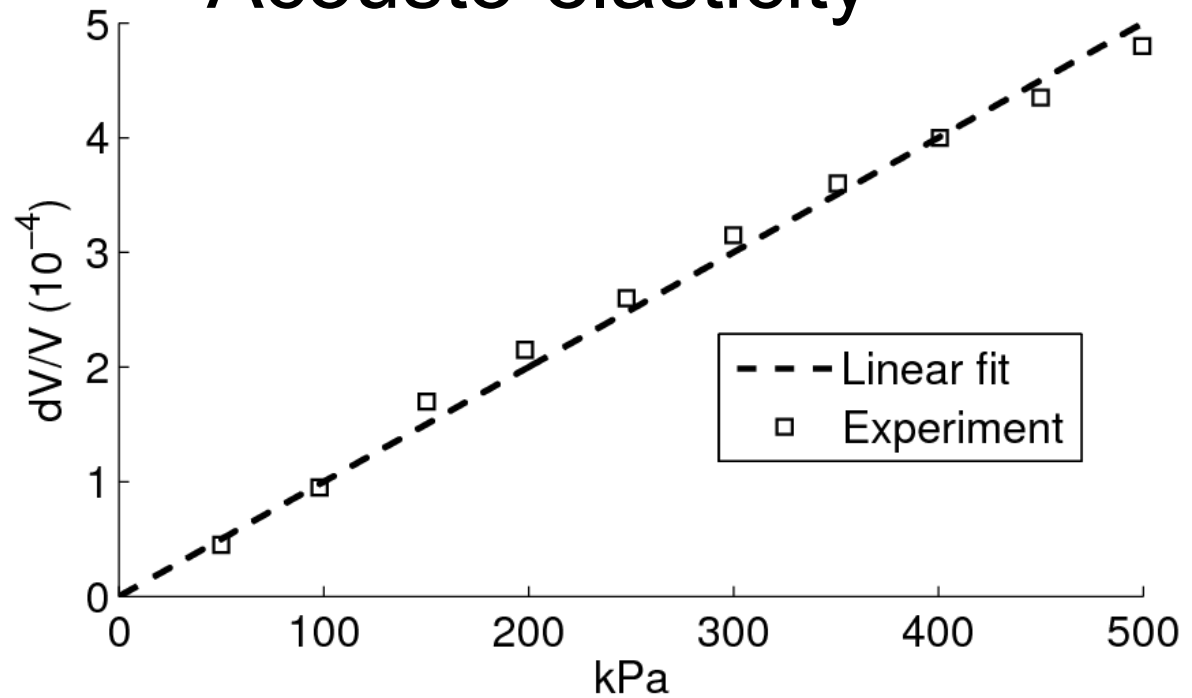
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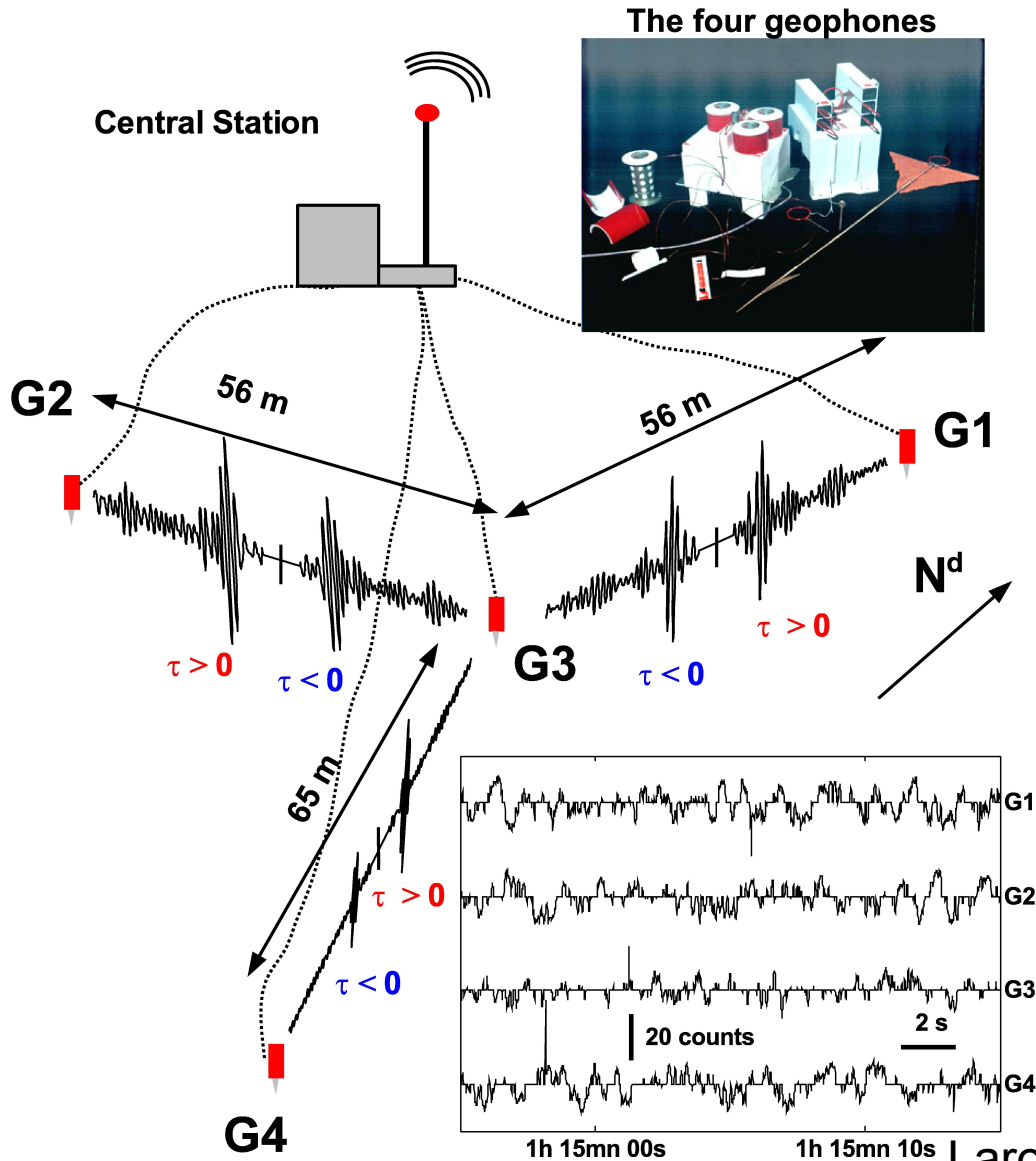




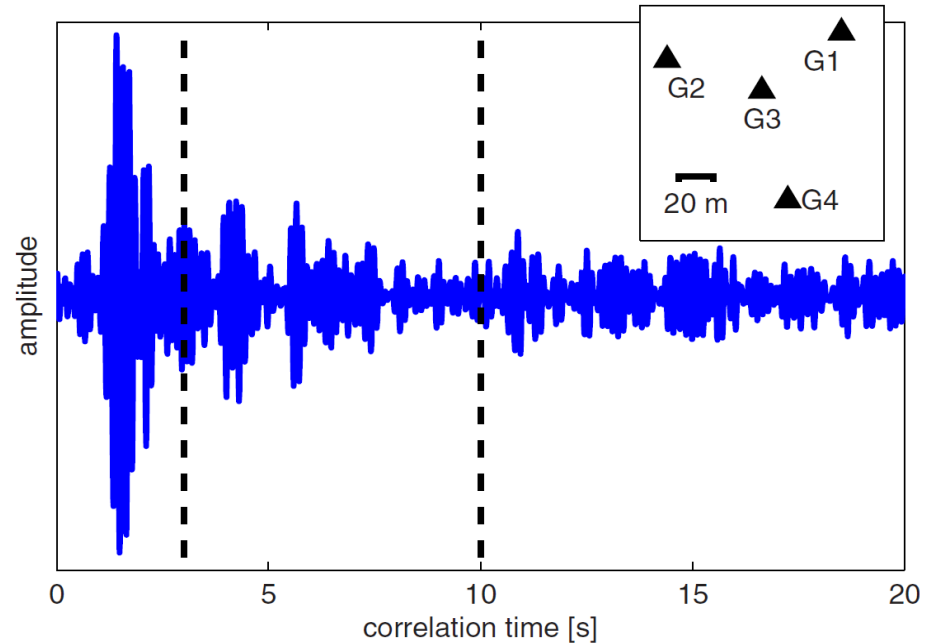
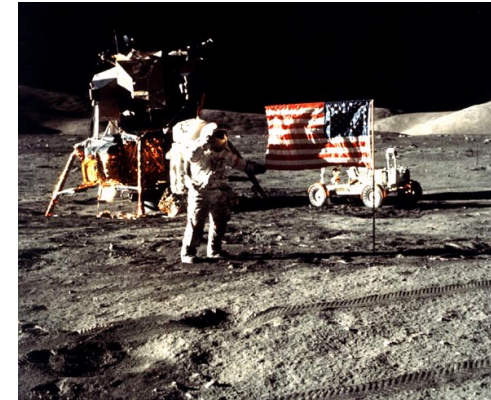
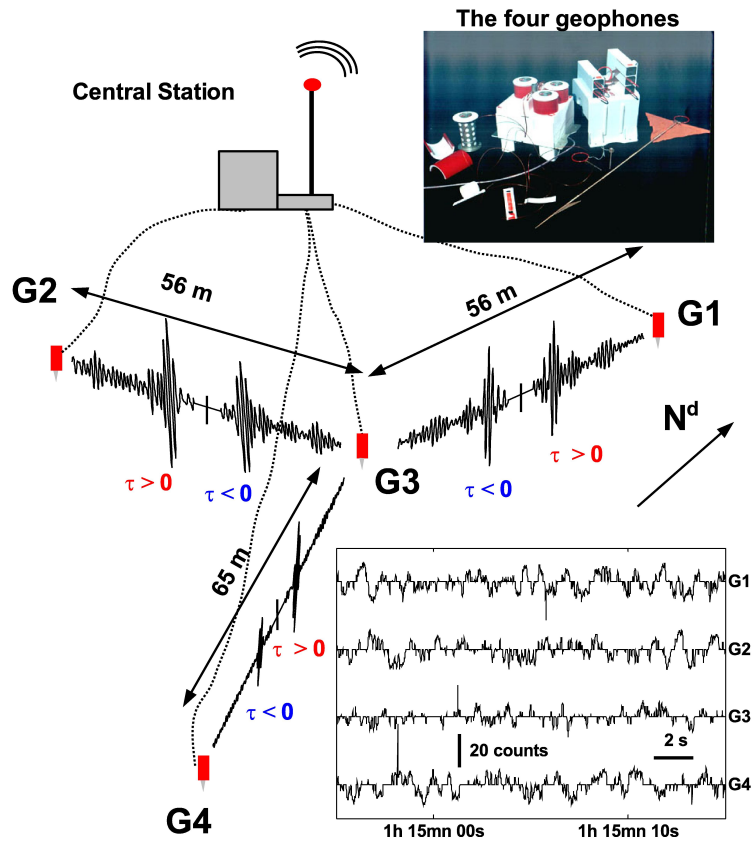
# Acousto-elasticity



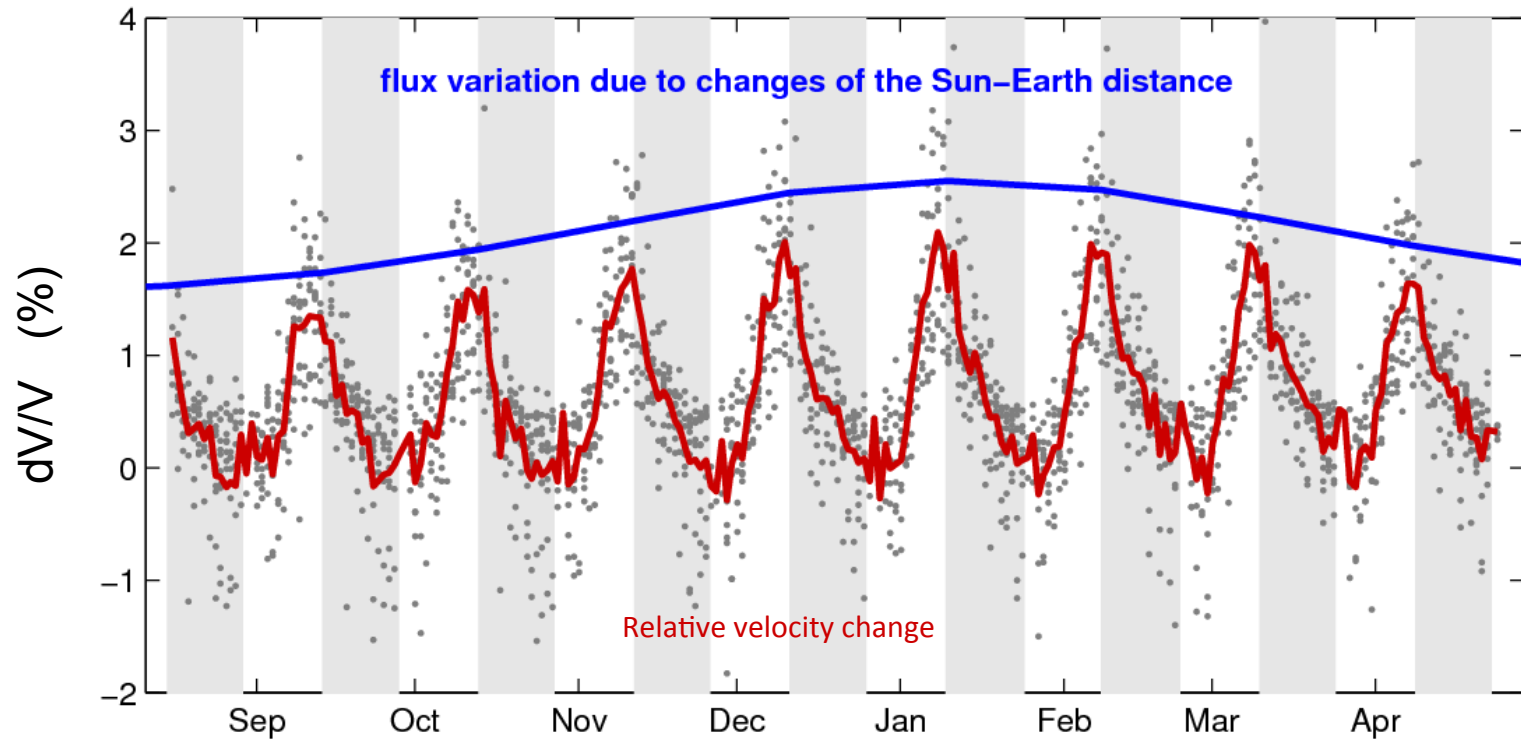








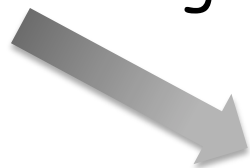
Larose *et al.*, Geophys. Res. Lett. (2005)



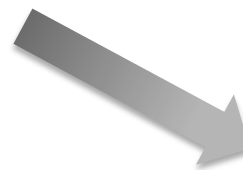
Sens-schönfelder & Larose, Phys. Rev. E (2008)

# The role of Moisture

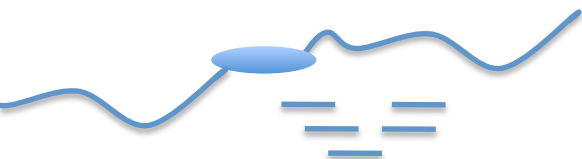
Water content change



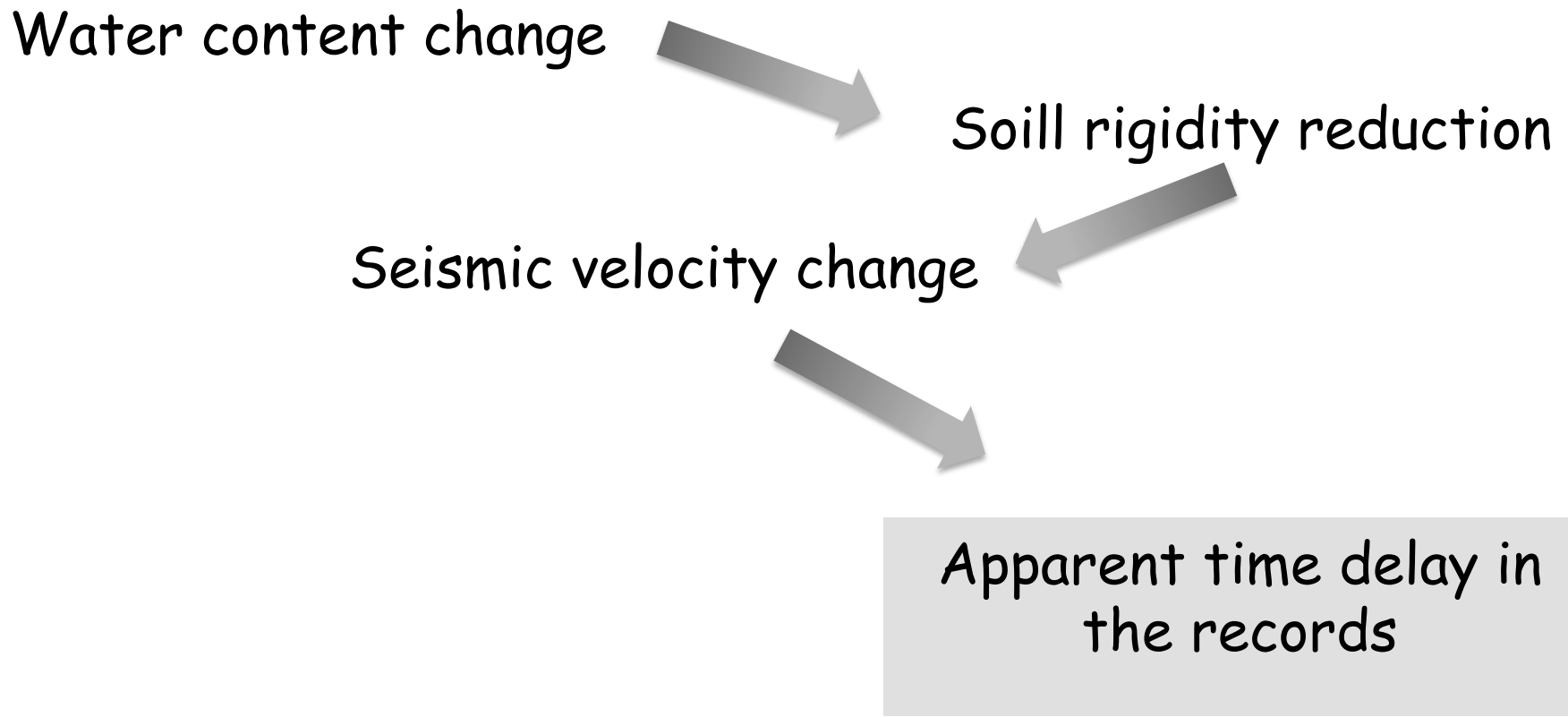
Seismic velocity change



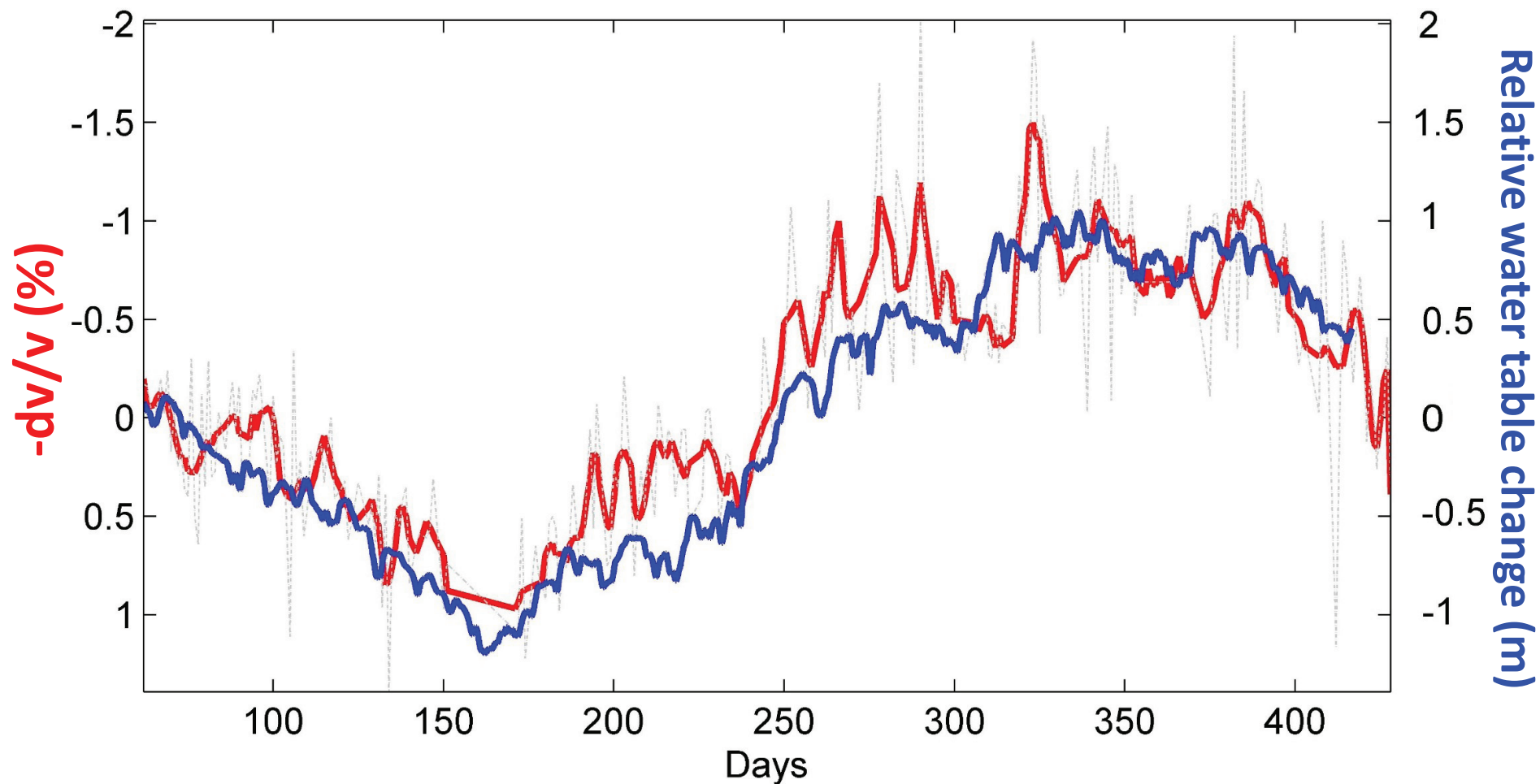
Apparent time delay in the records



# The role of Moisture



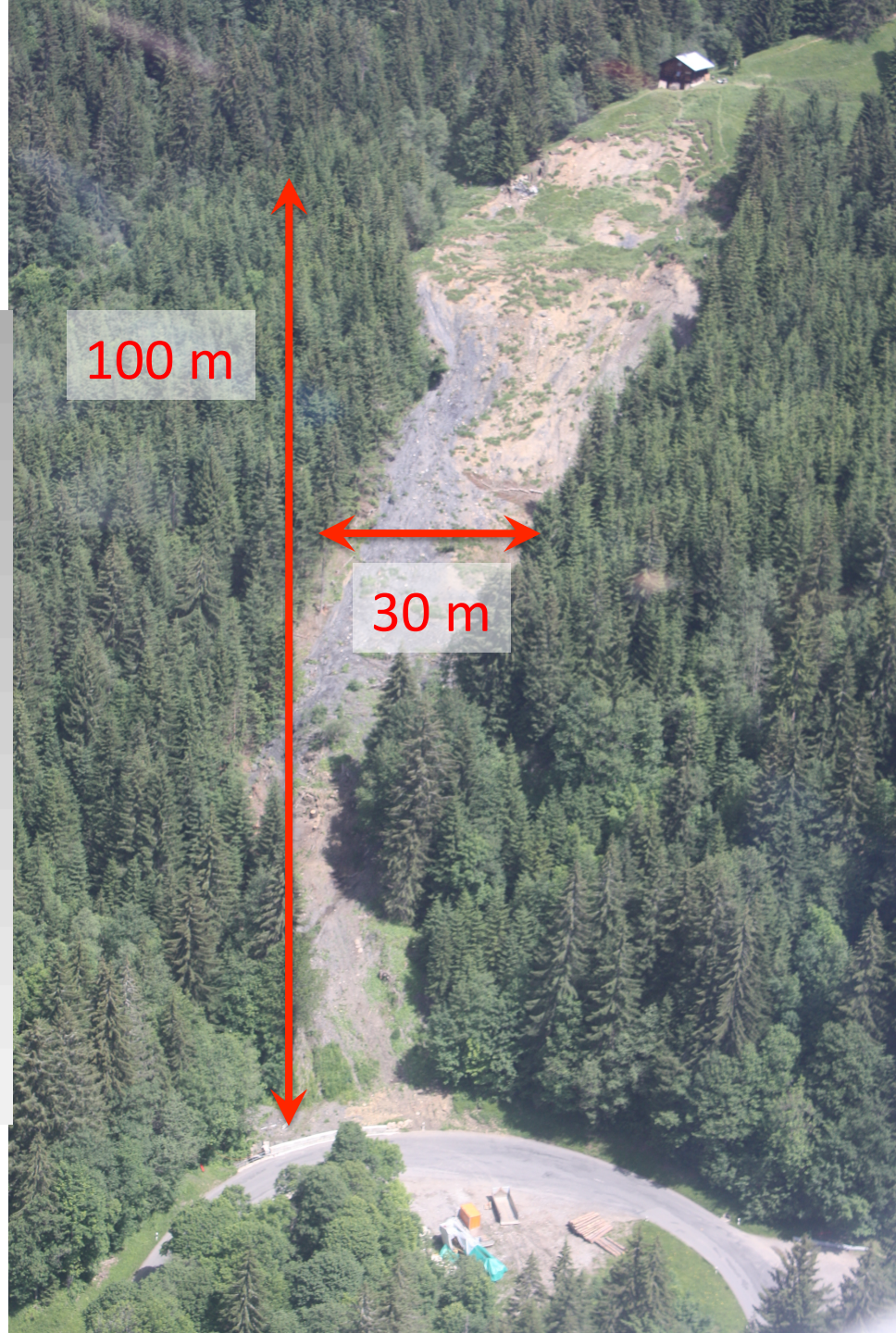
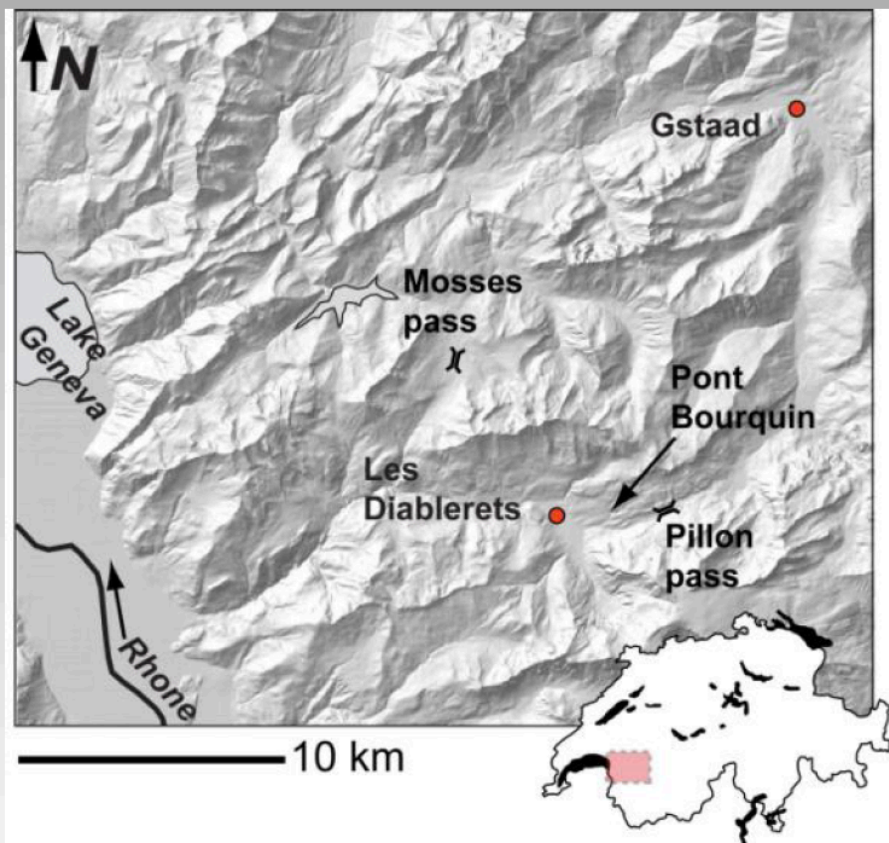
# Utiku - NZ



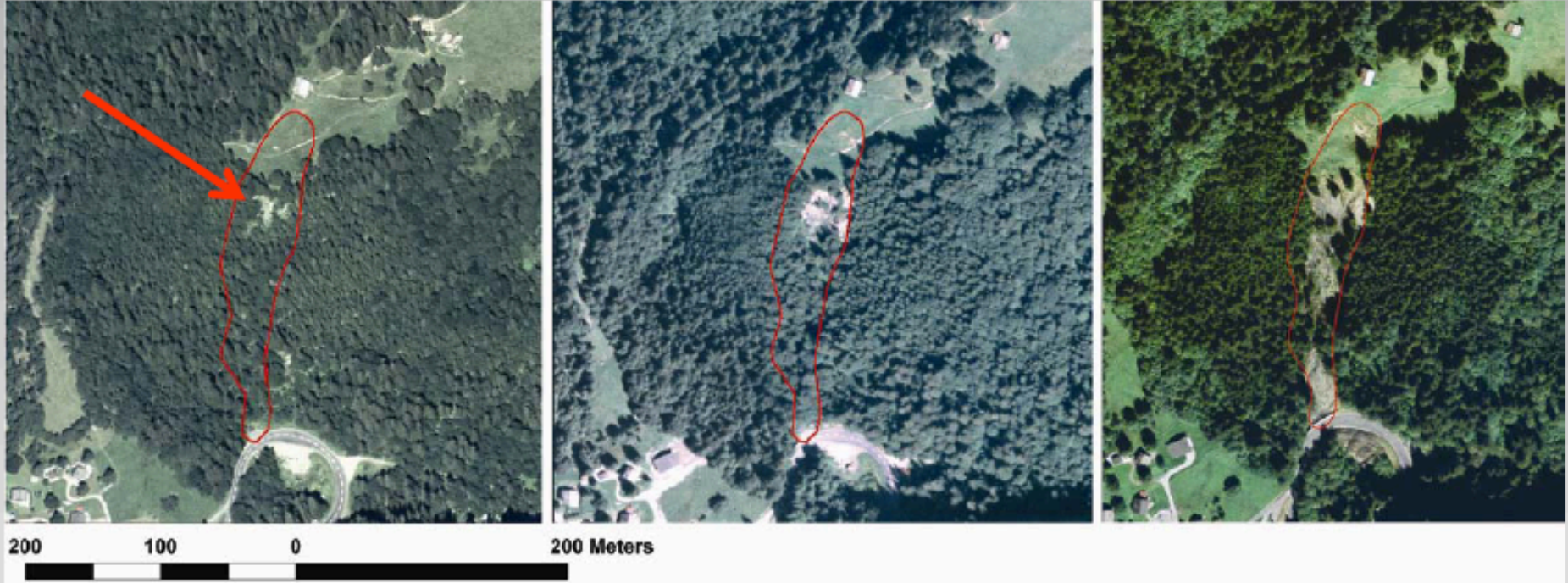
Courtesy of Voisin & Garambois



# Les Diablerets (Suisse)



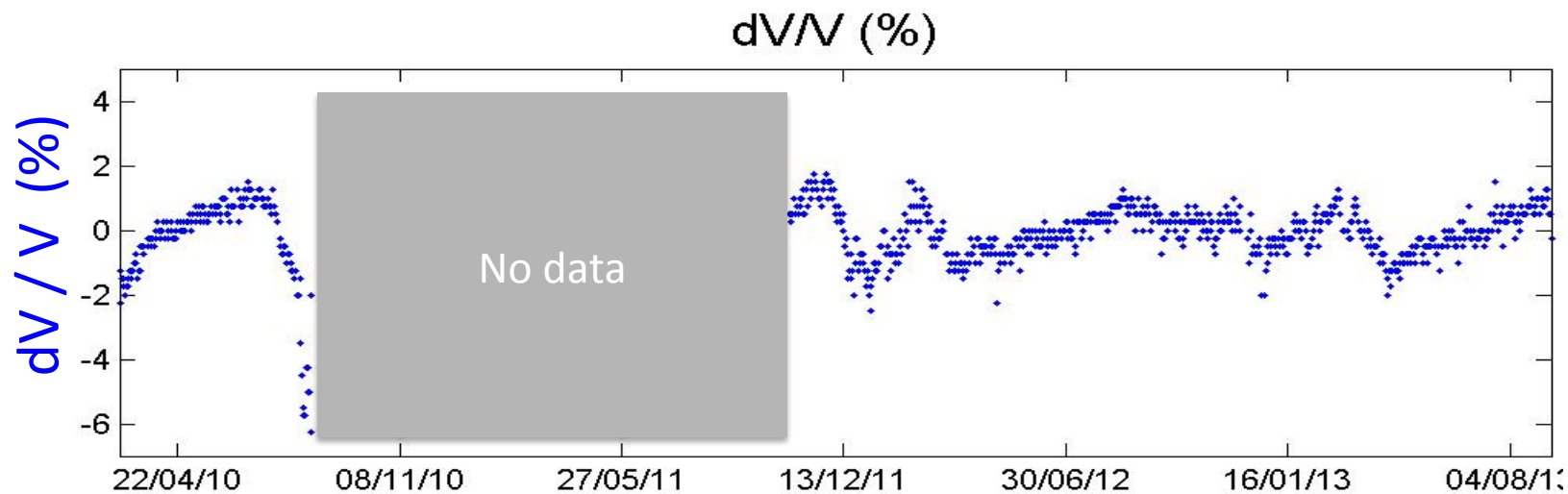


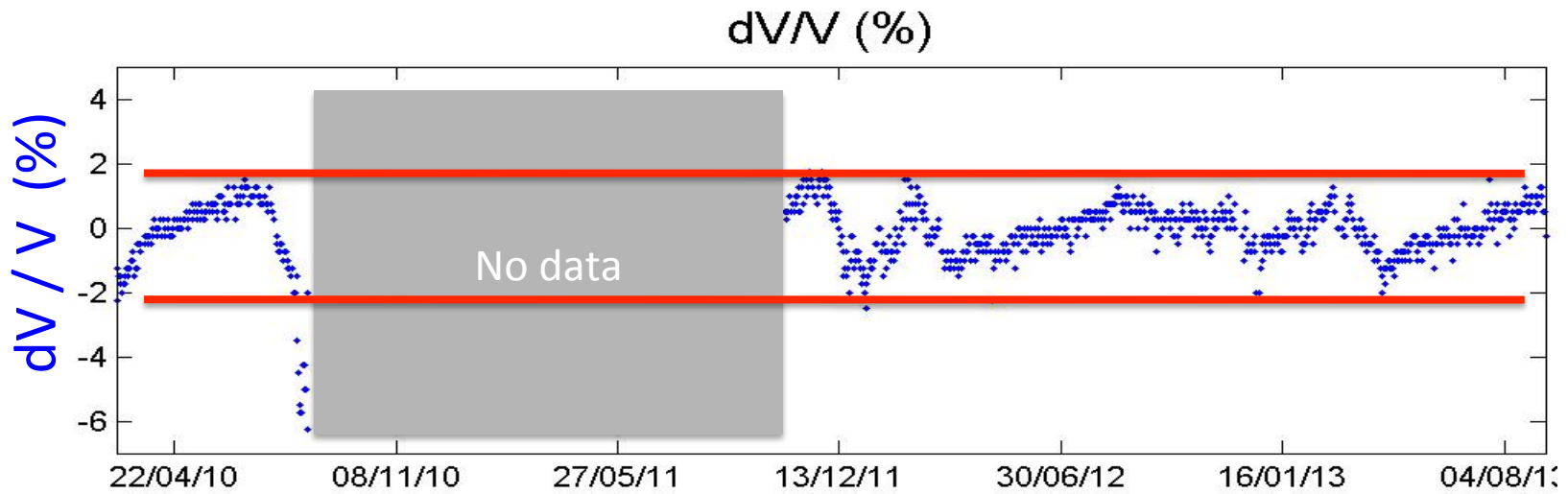


1998

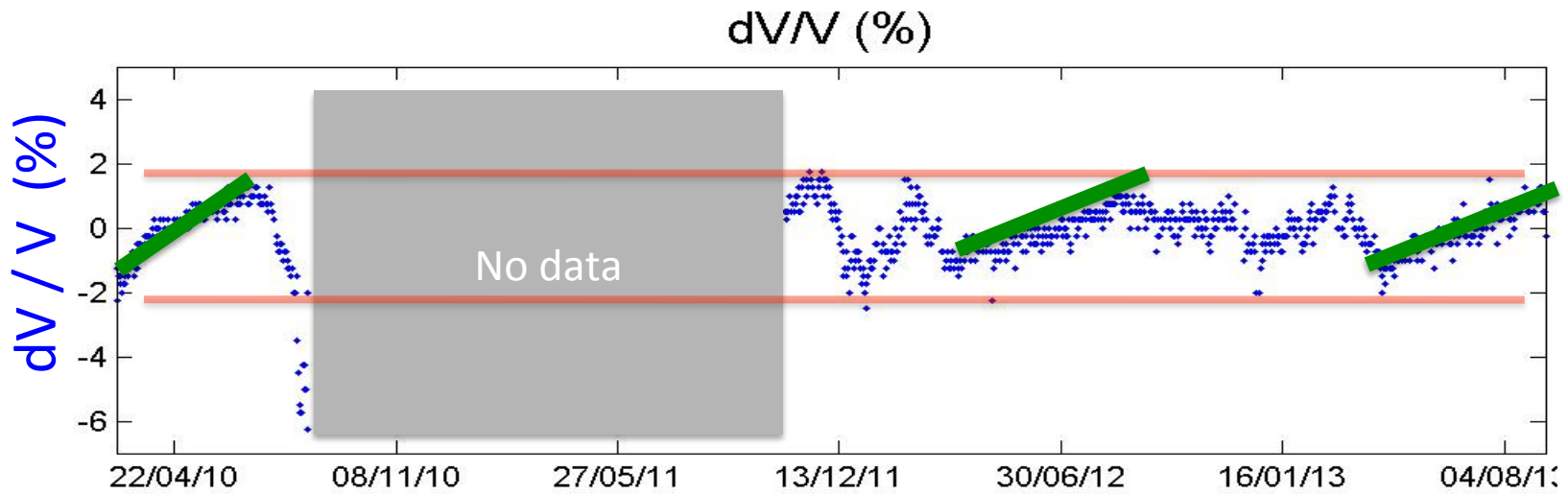
2004

2007

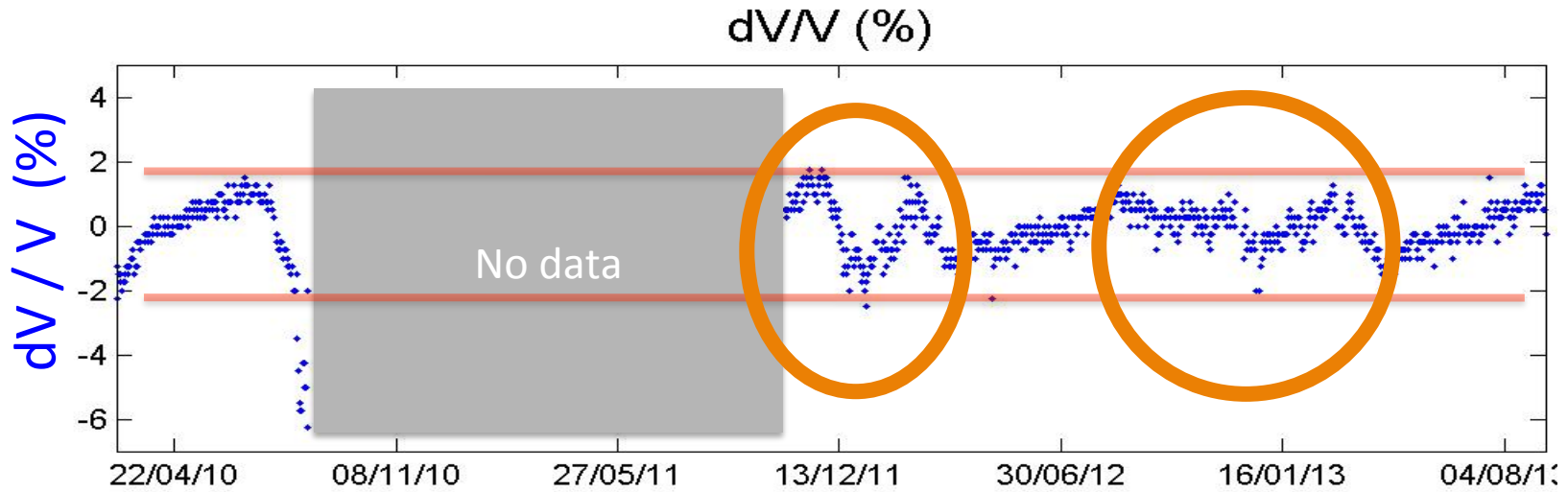




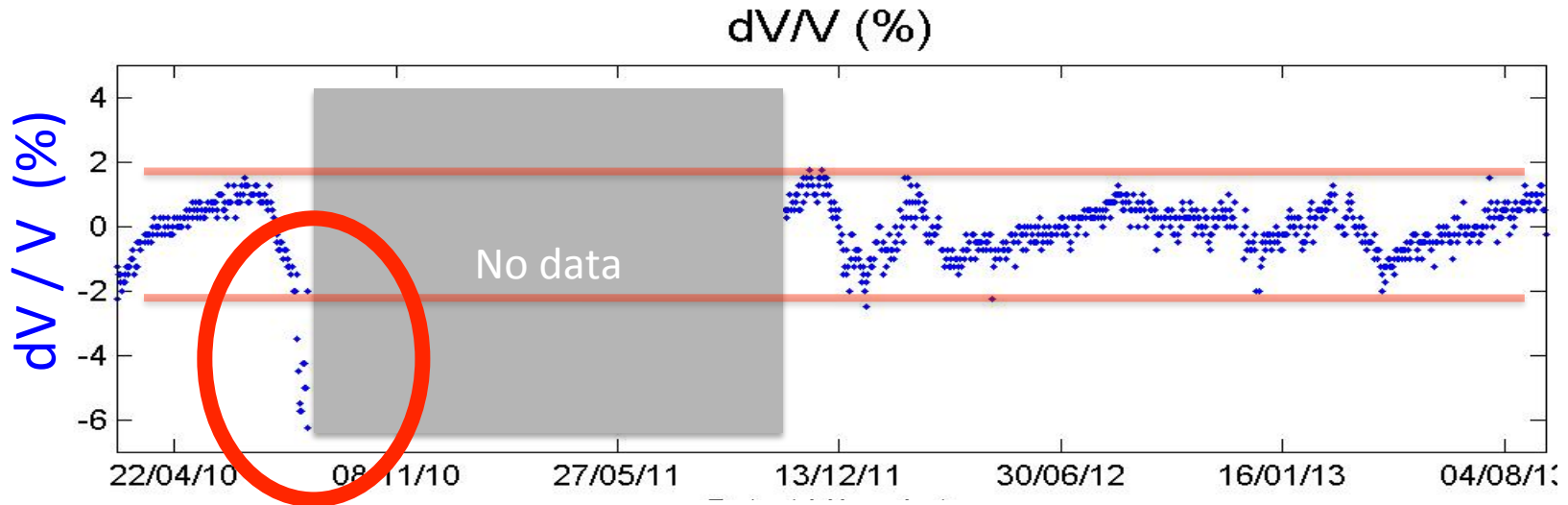
Fluctuations +/- 2%



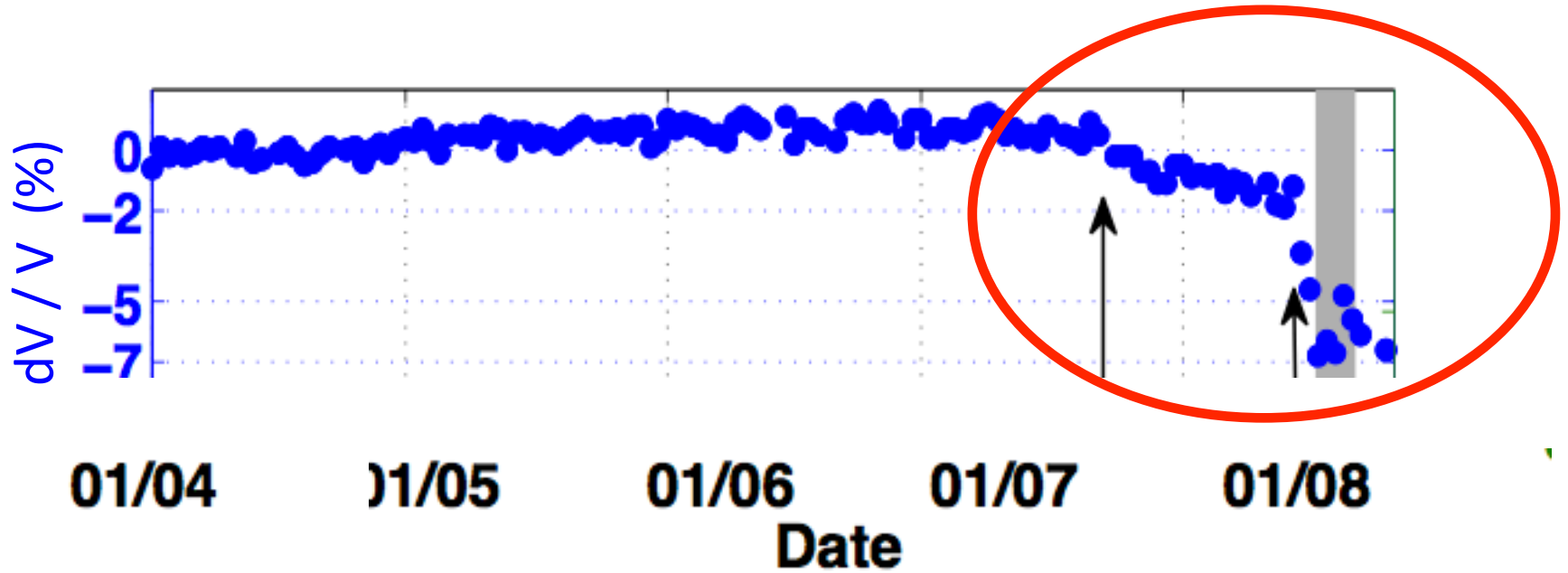
Drying during summer



Winter : Moisture / freezing / snow...

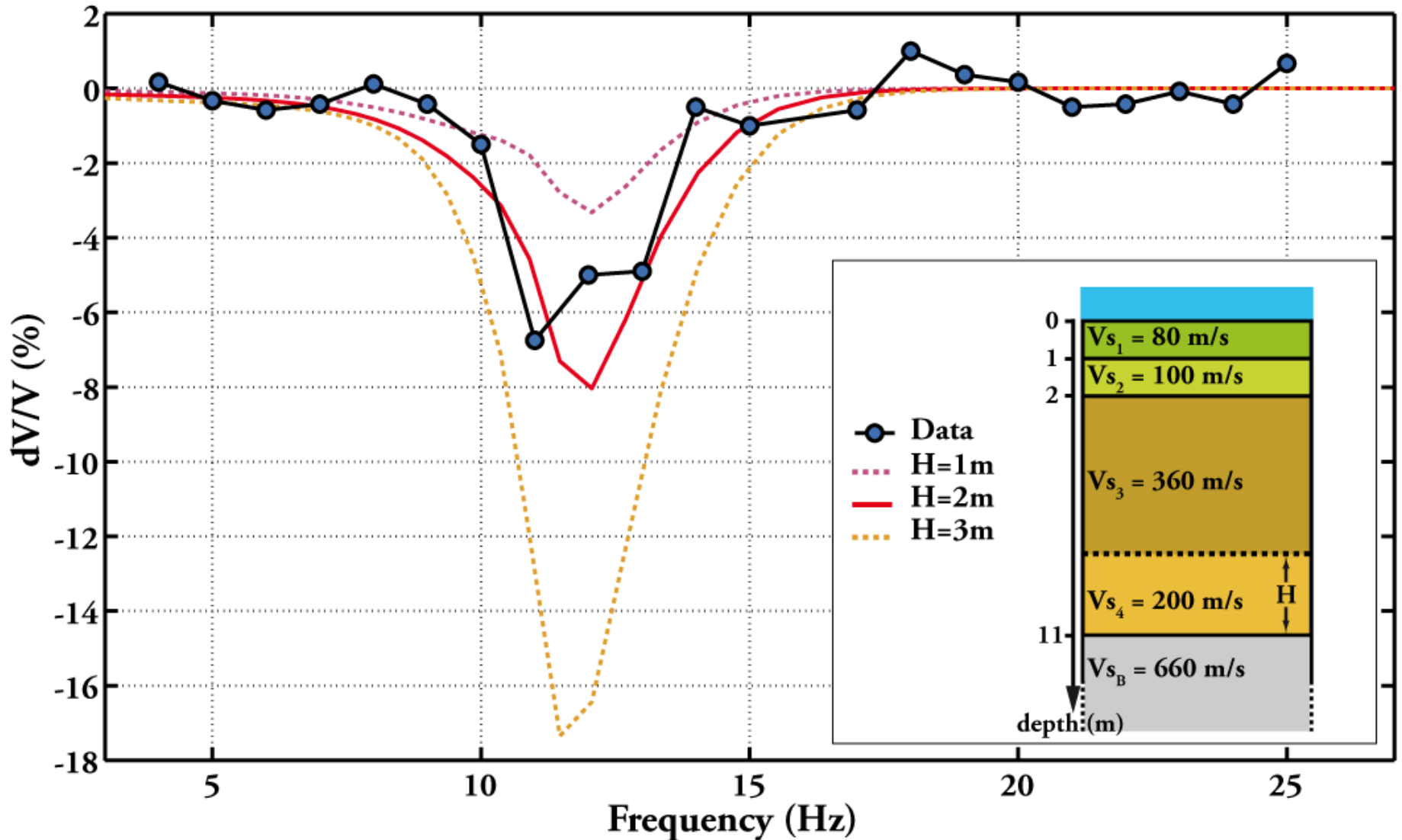


Large decrease => Liquefaction?



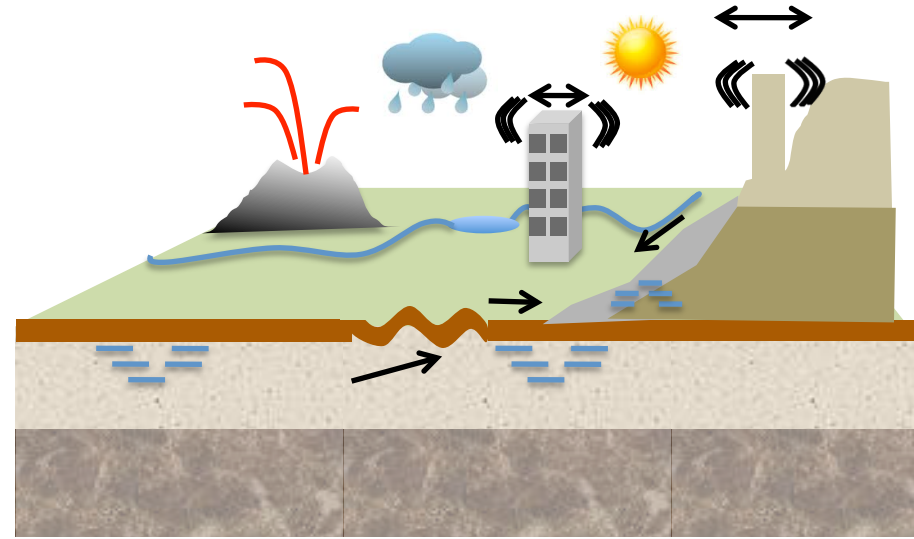
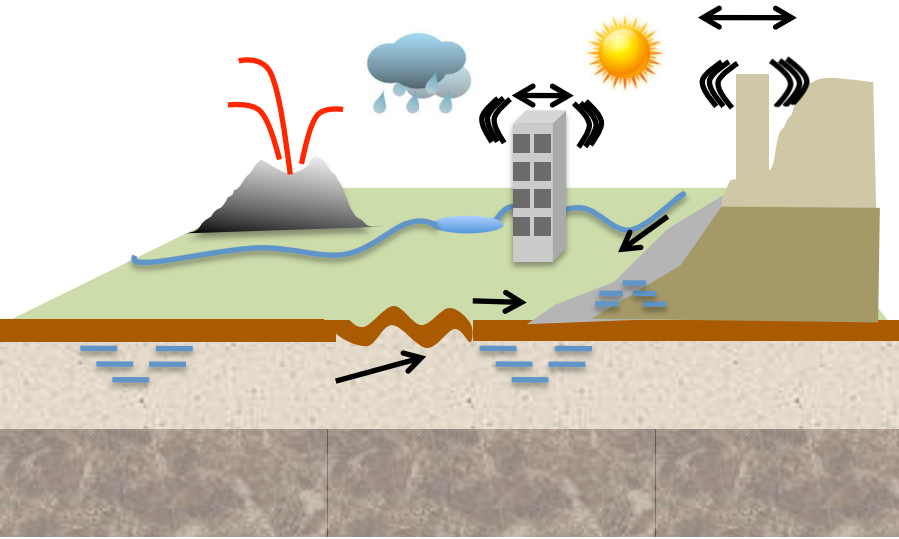
5 days precursory signal





# Environmental SEISMOLOGY

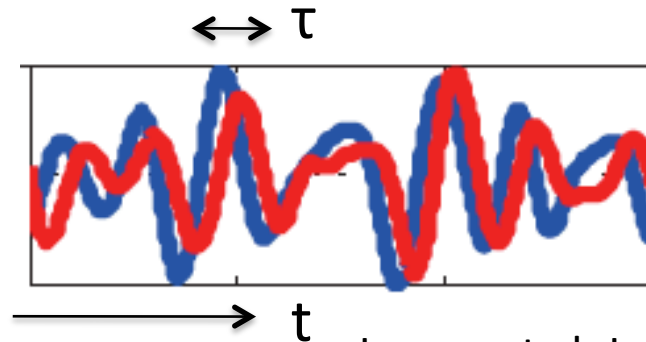




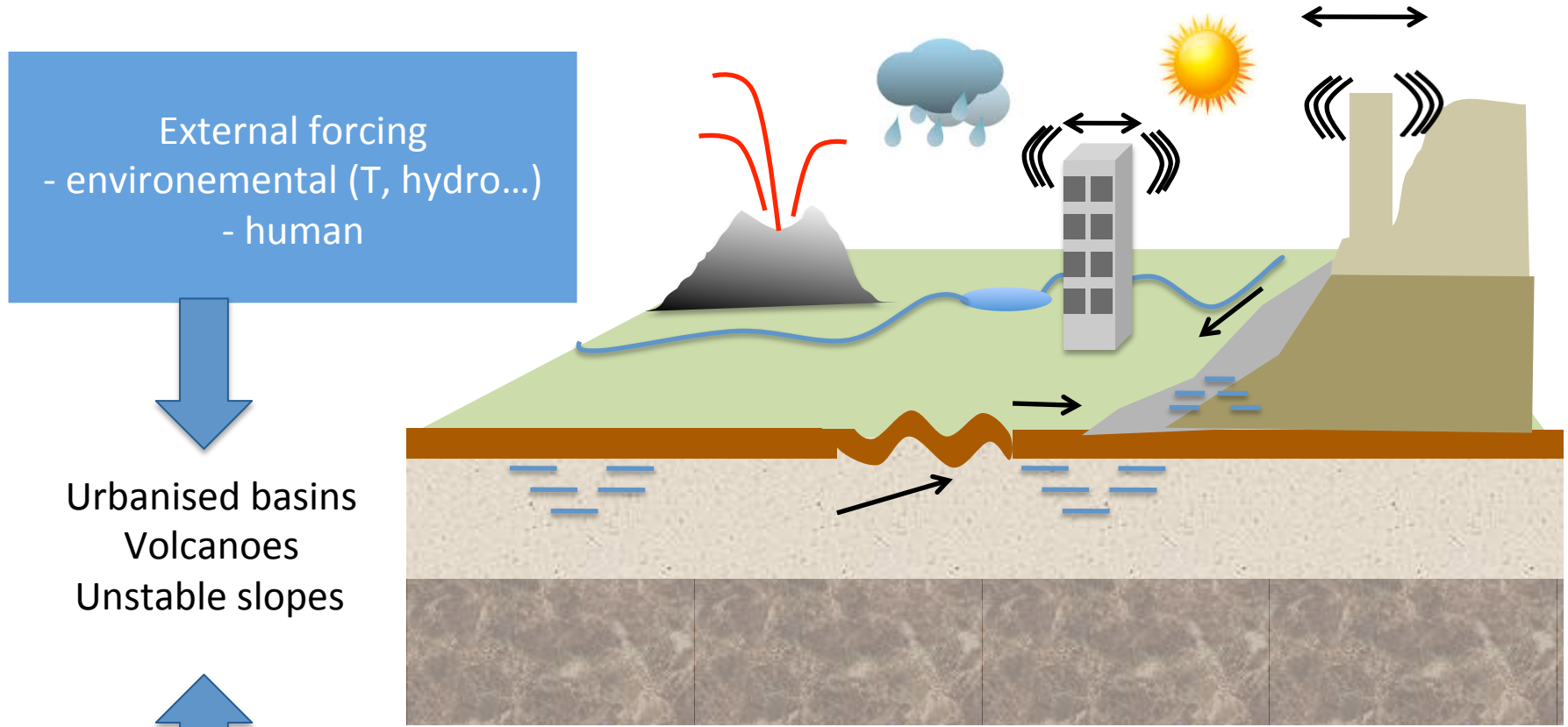
State 1



State 2







External forcing  
- environmental (T, hydro...)  
- human



Urbanised basins  
Volcanoes  
Unstable slopes



Gravity  
Damage  
Tectonics  
volcanology

Change of seismic waveforms

Learn on the environment

Discriminate internal/external forcing

Learn on the susceptibility

Discriminate Reversible/irreversible changes

Hydrology

...

Active fault, landslide, volcanology

To humidity, temperature...

Active fault, landslide, volcanology



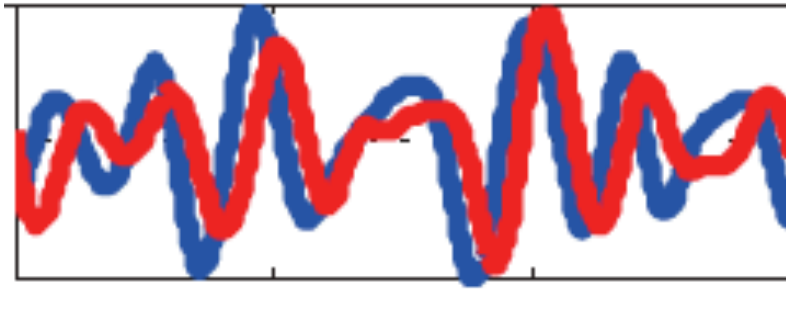
Water ressources

Natural Hazards

Non-linearities, Damage

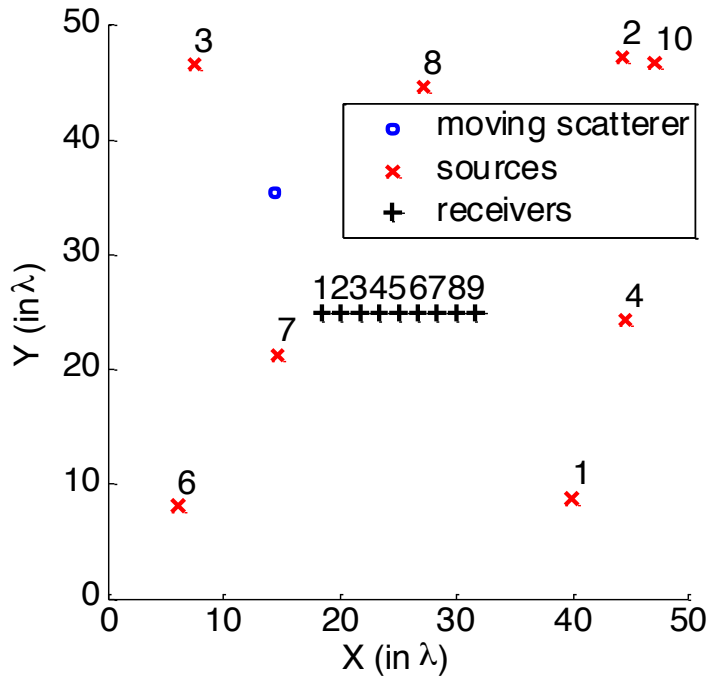
Natural Hazards

1. Convergence rate
2. Coda Wave Interferometry
3. **Coda Wave Decorrelation**

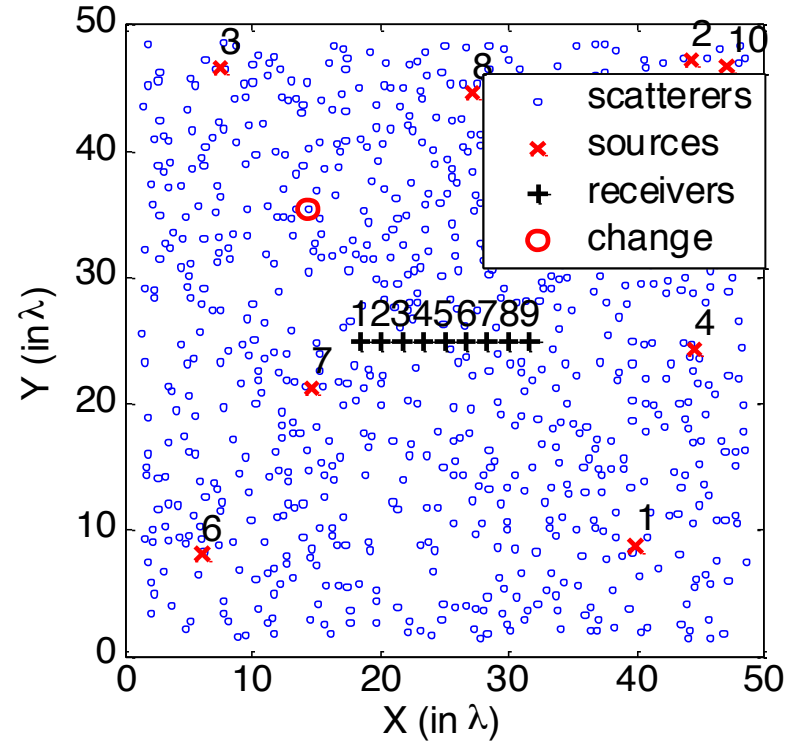


3D CARTOGRAPHIES

- Damage & cracks [m<sup>2</sup>/m<sup>3</sup>]
- Relative velocity changes [%]



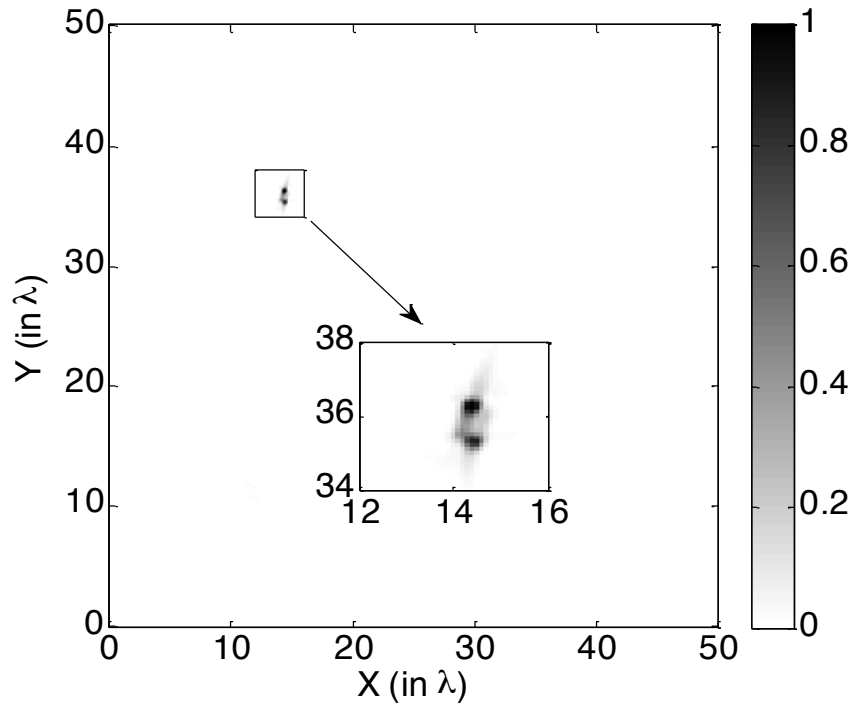
**Simple scattering**  
1 change



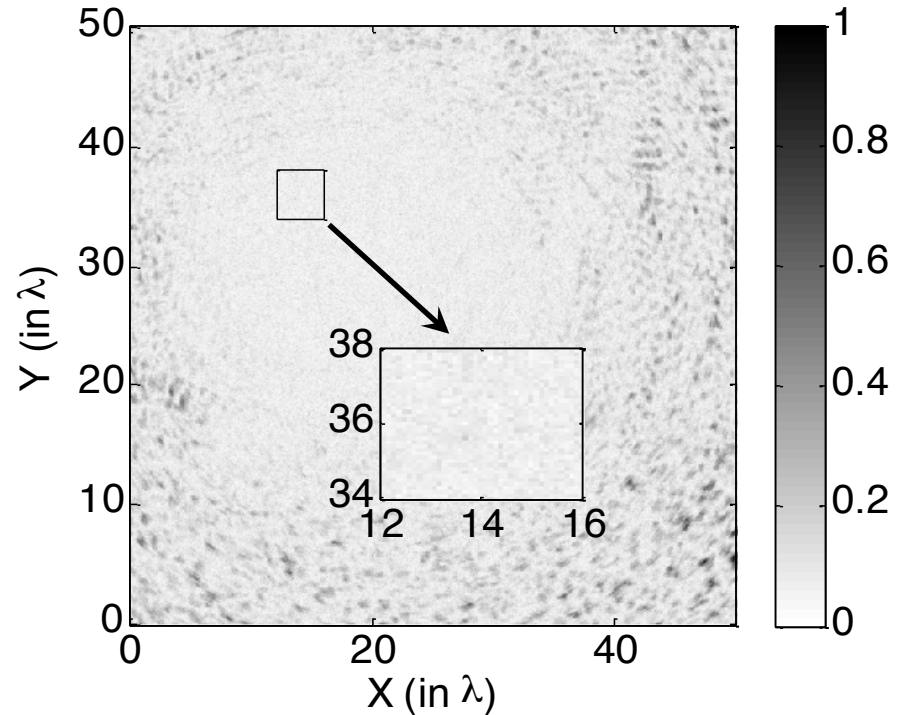
**Multiple scattering**  
800 scatterers+ 1 change



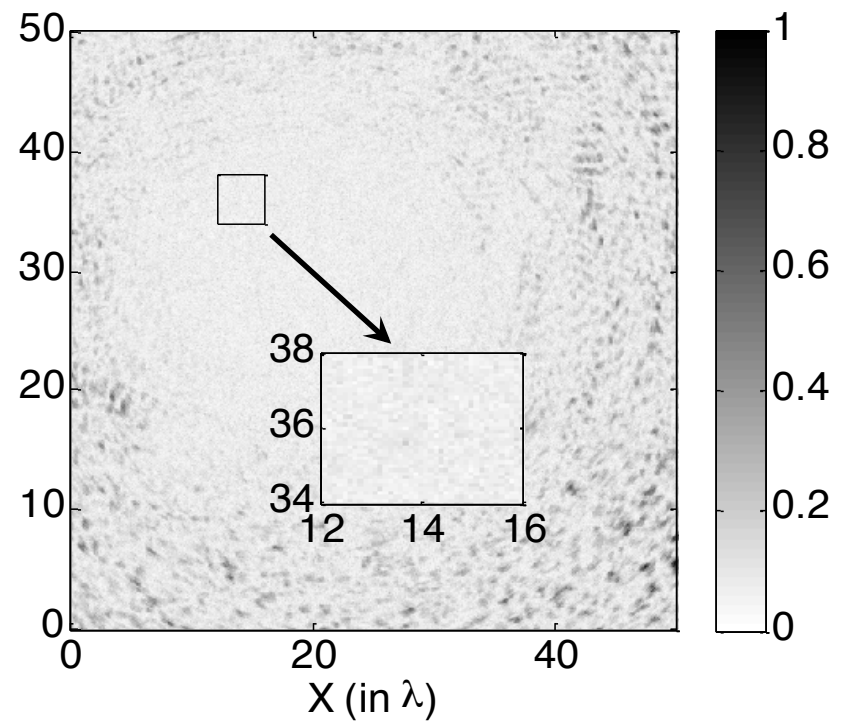
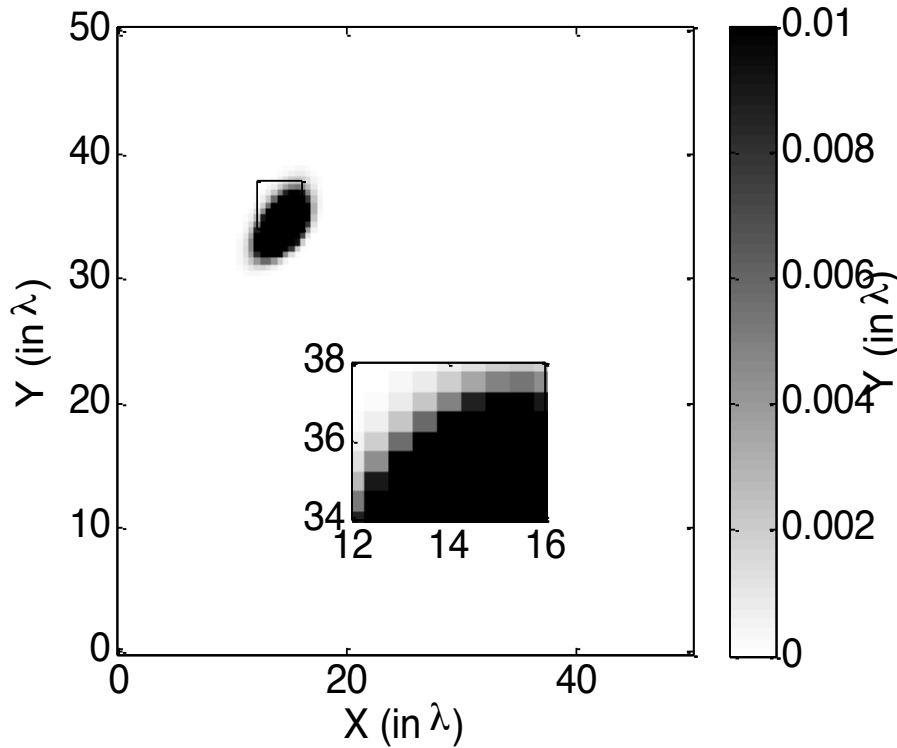
# classical imaging



**Simple scattering**  
1 change



**Multiple scattering**  
800 scatterers+ 1 change

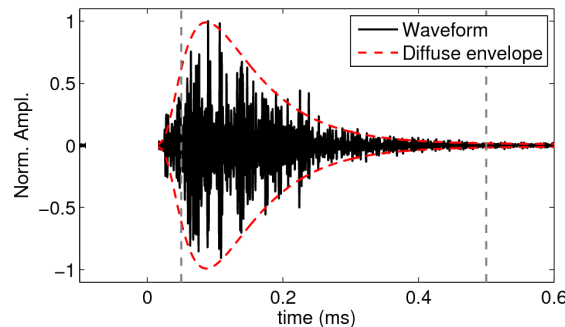
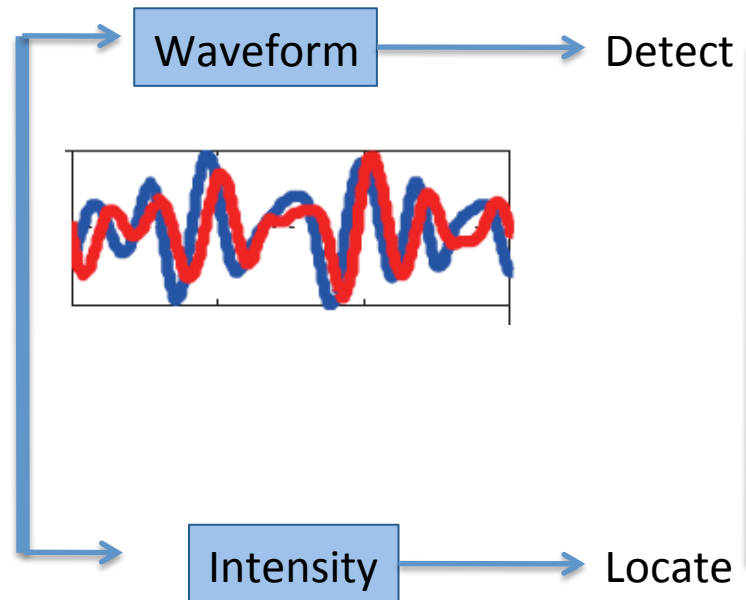


*Locadiff* (2010) // classical imaging

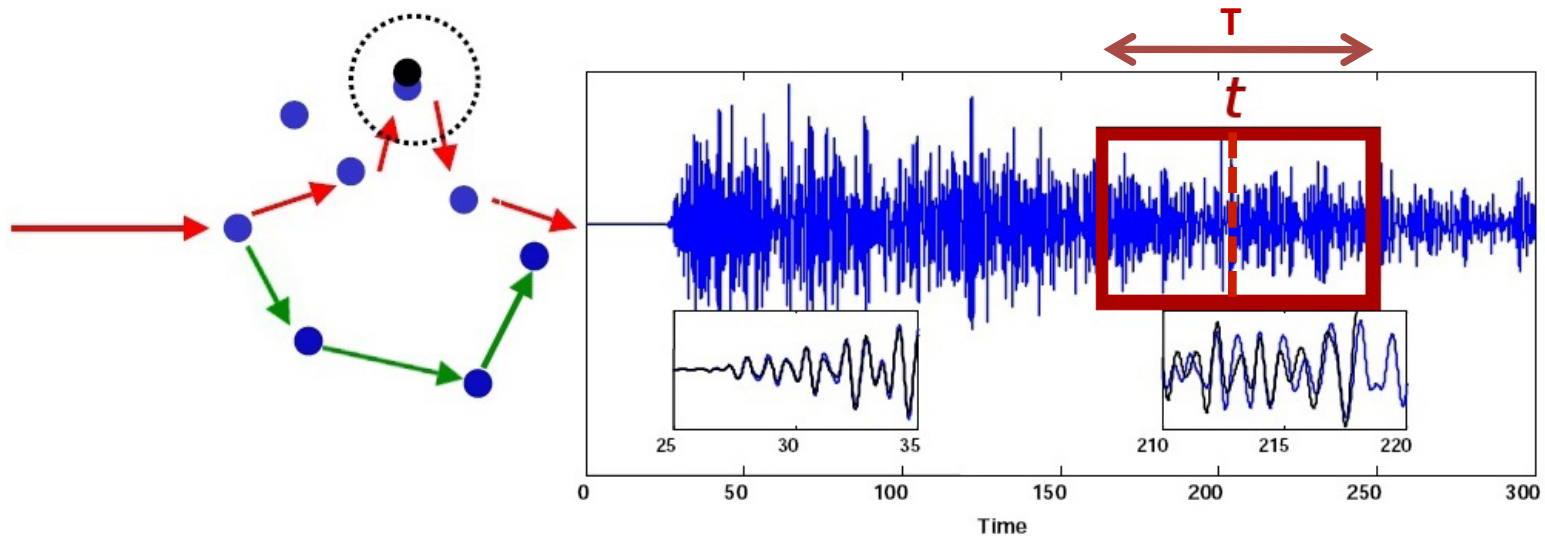
# Locadiff :

Larose et al, Appl. Phys. Lett. (2010)  
Rossetto et al, J. Appl. Phys. (2011)  
Planes et al, 2013, 2014, 2015... Obermann et al, 2013, 2014, 2015

Mesososcopic waves



# Signature of a change in the coda

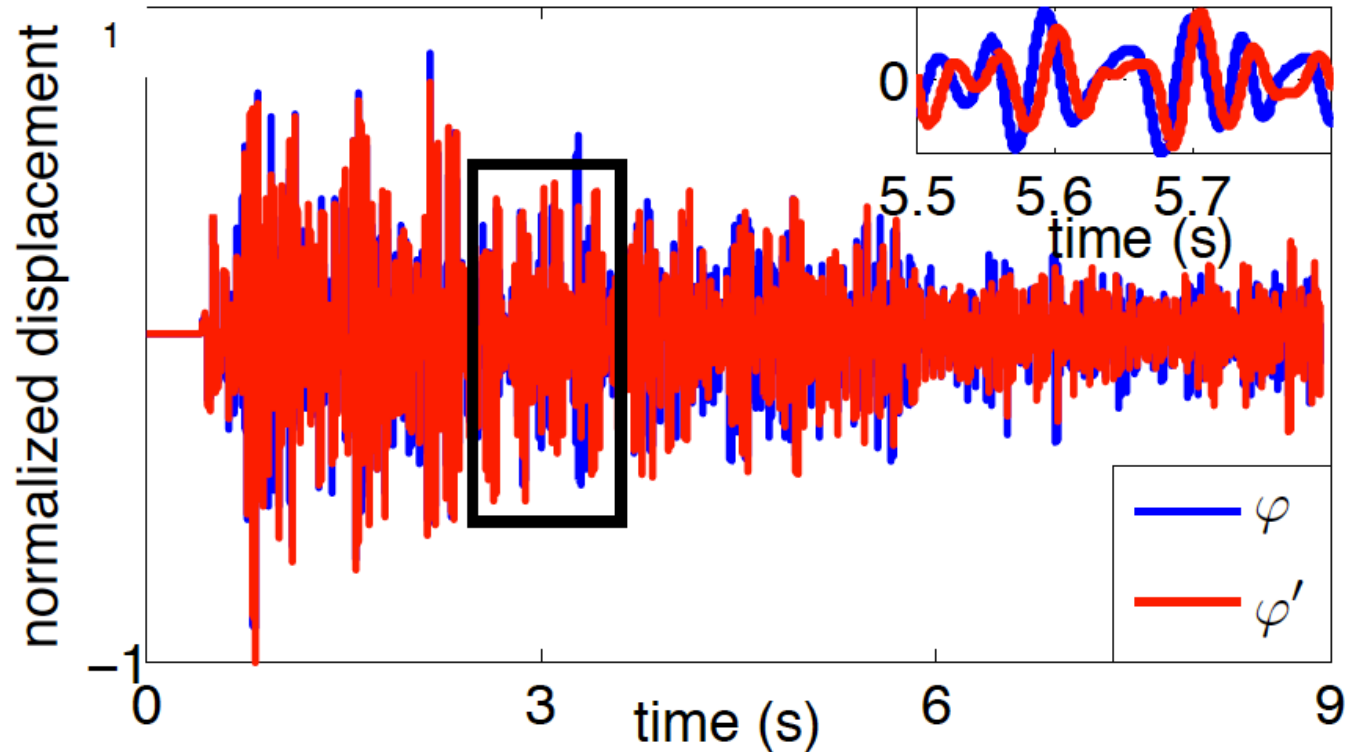


**Very sensitive to weak changes**

$$\text{Decorrelation: } DC(t) = 1 - \frac{\langle \phi_0(t) \cdot \phi_1(t) \rangle_T}{\sqrt{\langle \phi_0(t)^2 \rangle_T \langle \phi_1(t)^2 \rangle_T}}$$

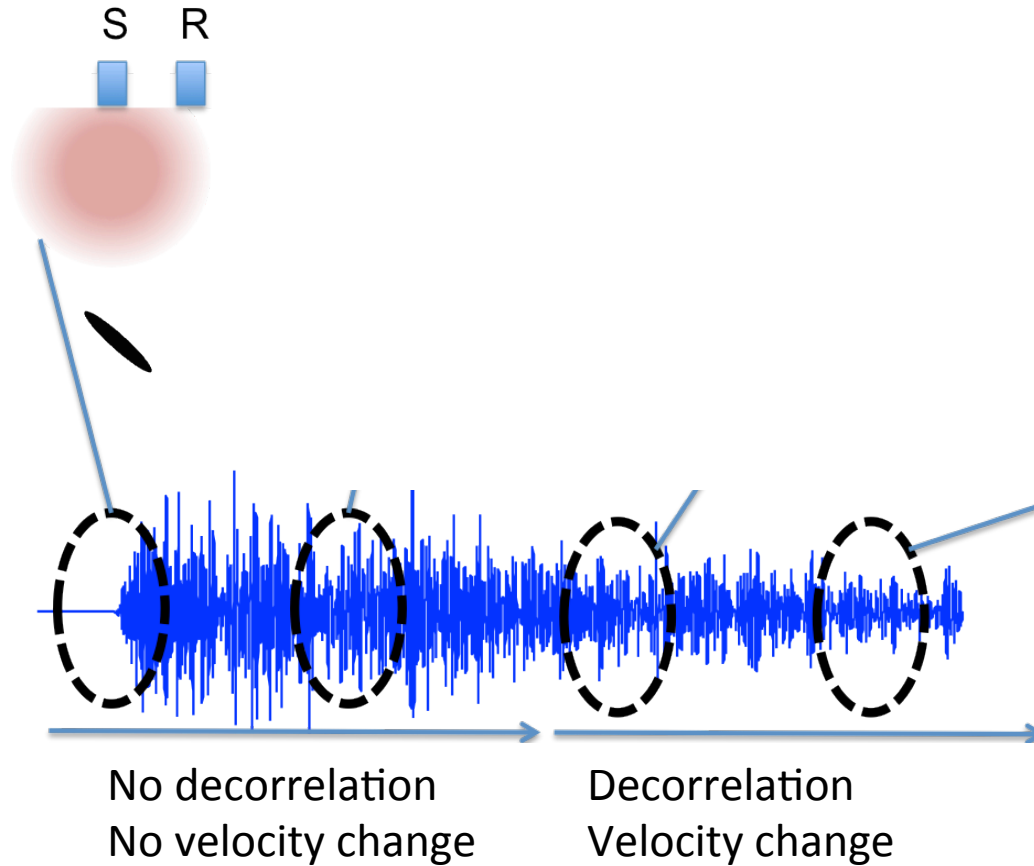
Stretching factor:  $\epsilon(t) = \epsilon$  that maximises  $\langle \phi_0(t) \cdot \phi_1(t(1 - \epsilon)) \rangle_T$

# Signature of a change in the coda

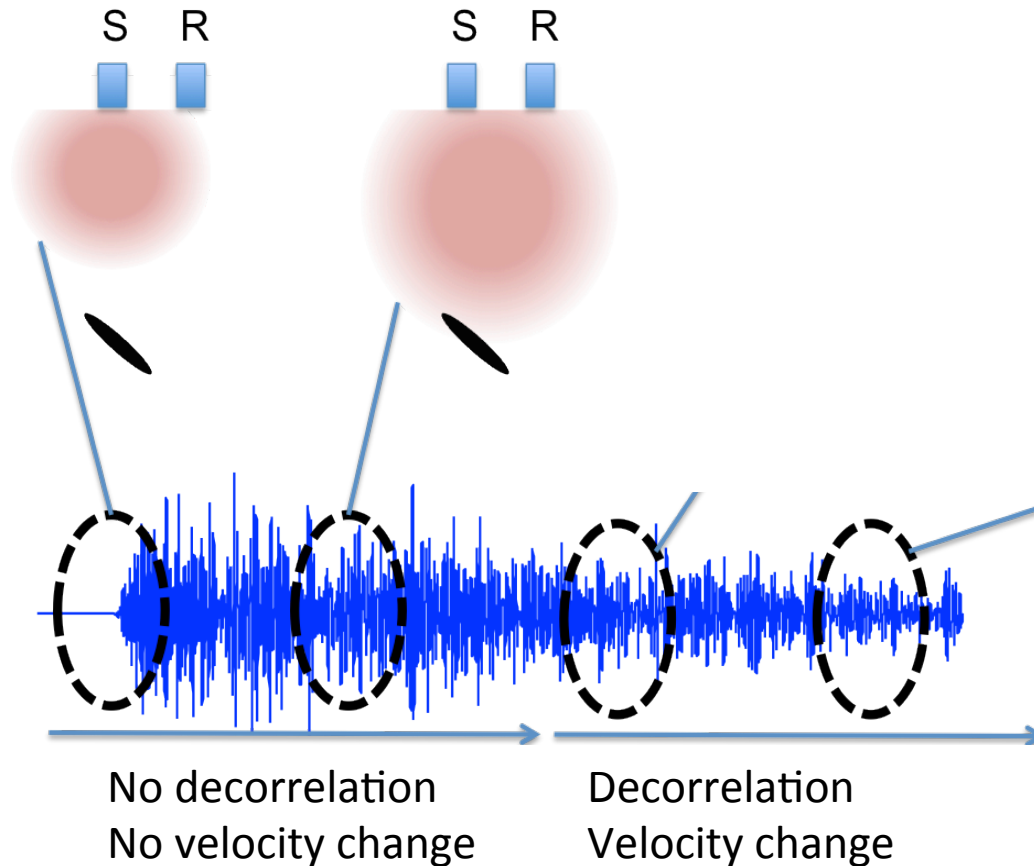


$$CC(\varepsilon) = \frac{\int_{t_1}^{t_2} \varphi' [t(1 - \varepsilon)] \varphi [t] dt}{\sqrt{\int_{t_1}^{t_2} \varphi'^2 [t(1 - \varepsilon)] dt \int_{t_1}^{t_2} \varphi^2 [t] dt}},$$

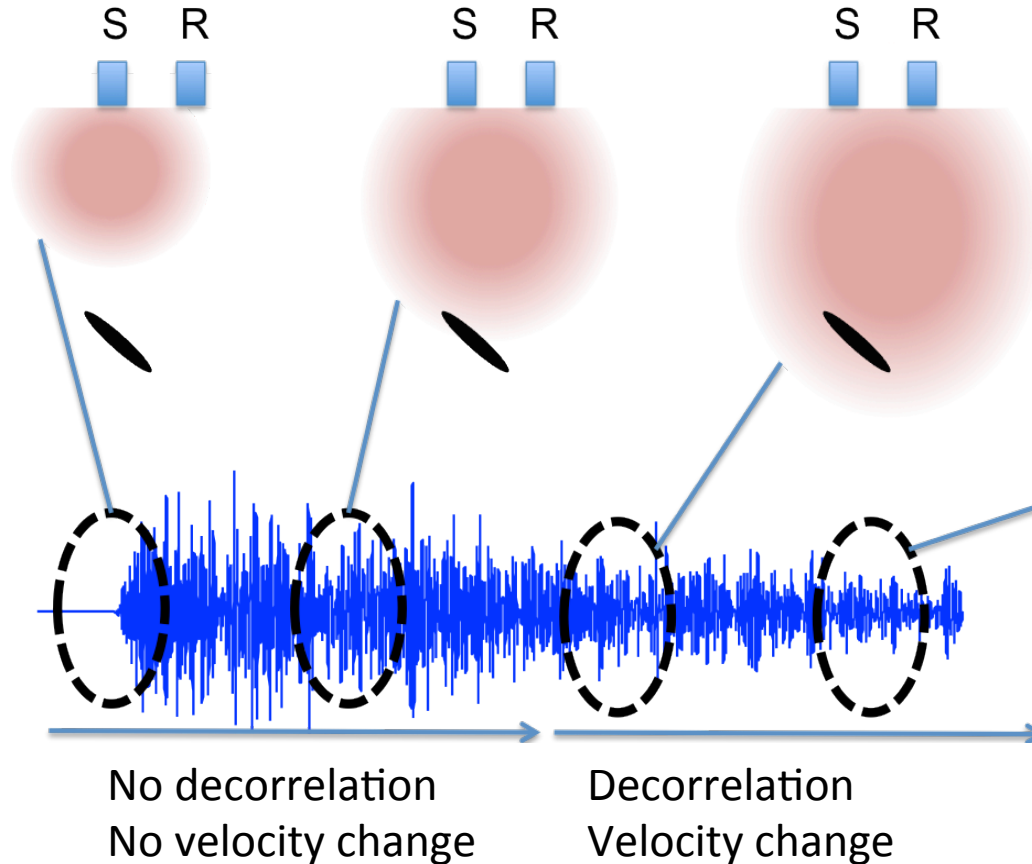
# Signature of a change in the coda



# Signature of a change in the coda

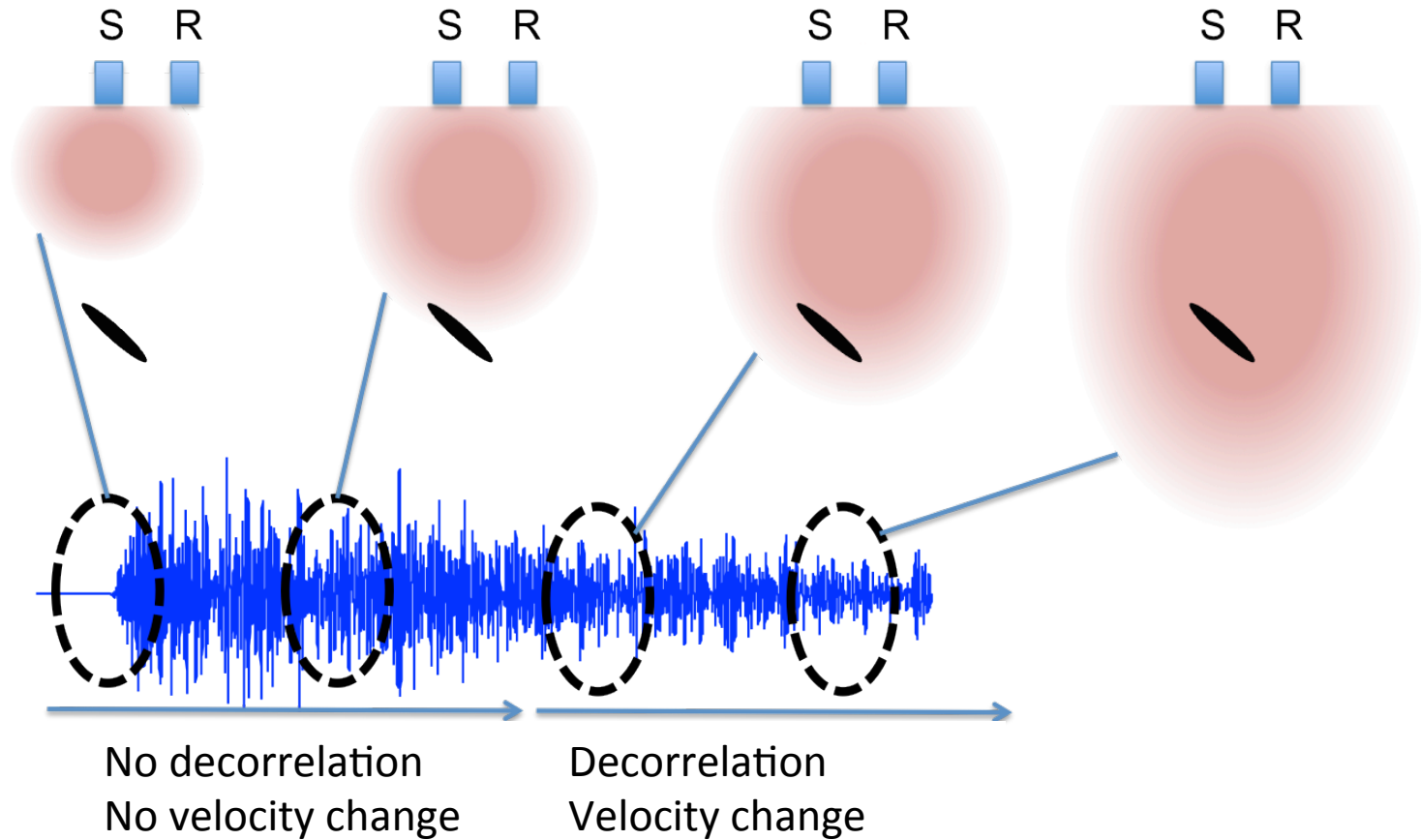


# Signature of a change in the coda



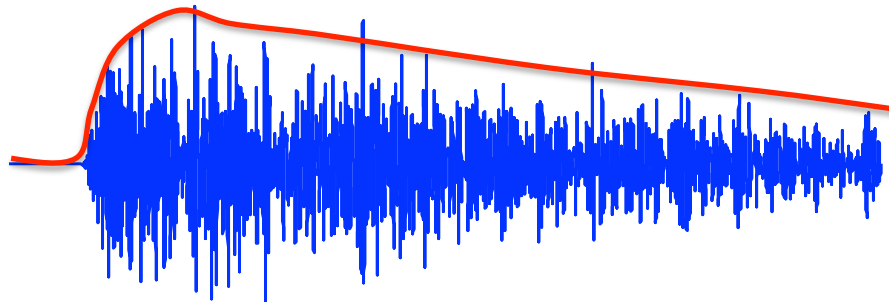


# Signature of a change in the coda



THE INTENSITY  
 =  
 PROBABILITY OF TRANSPORT

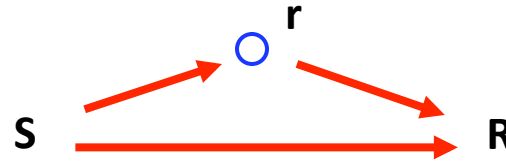
$$I(S, R, t)$$



Diffusion (heat)

# Decorrelation induced by an extra scatterer :

## Theoretical model



### Theoretical decorrelation

$$DC^{th}(\mathbf{S}, \mathbf{R}, \mathbf{r}, t) = \frac{c\sigma}{2} \frac{\int_0^t I(\mathbf{S}, \mathbf{r}, u) I(\mathbf{r}, \mathbf{R}, t - u) du}{I(\mathbf{S}, \mathbf{R}, t)}$$

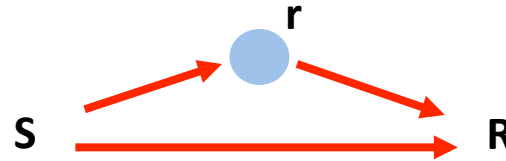
Rossetto et al. [J. Appl. Phys. 2011]

$I$  : Intensity propagator (Diffusion solution, Radiative Transfer)

$\sigma$  : Scattering cross section of the new defect

# Local relative velocity change $dV/V$ :

## Theoretical model



Pacheco & Snieder [2005]

### Theoretical relative velocity change

$$\varepsilon^{\text{app}}(S, R, r, t) = \frac{dv}{v} \frac{\Delta V}{t} \frac{\int_0^t I(S, r, u) I(r, R, t - u) du}{I(S, R, t)}$$

$I$  : Intensity propagator (Diffusion solution, Radiative Transfer)

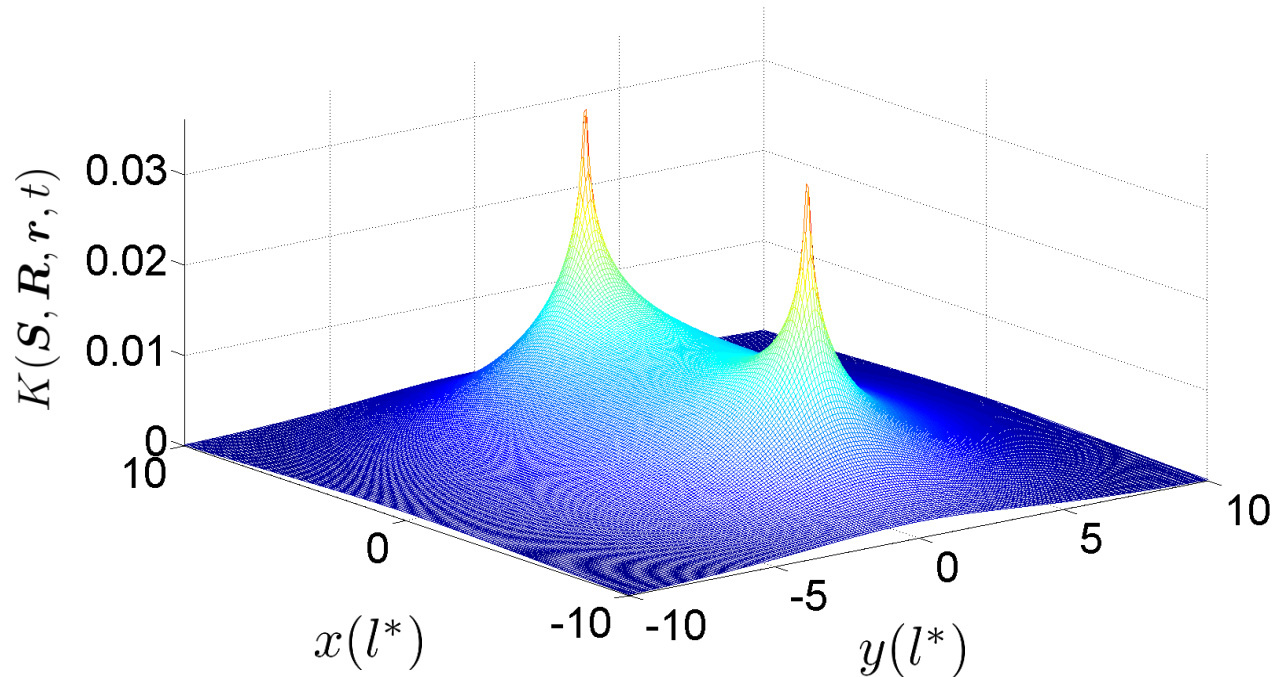
$dv/v$  : Scattering cross section of the new defect

# Sensitivity kernel

## decorrelation

$$DC^{th}(S, \mathbf{R}, \mathbf{r}, t) = \frac{c\sigma}{2} K(S, \mathbf{R}, \mathbf{r}, t)$$

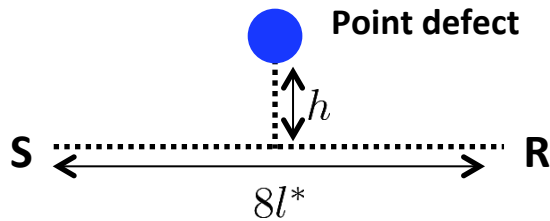
$$K(S, \mathbf{R}, \mathbf{r}, t) = \frac{\int_0^t I(S, \mathbf{r}, u) I(\mathbf{r}, \mathbf{R}, t - u) du}{I(S, \mathbf{R}, t)}$$



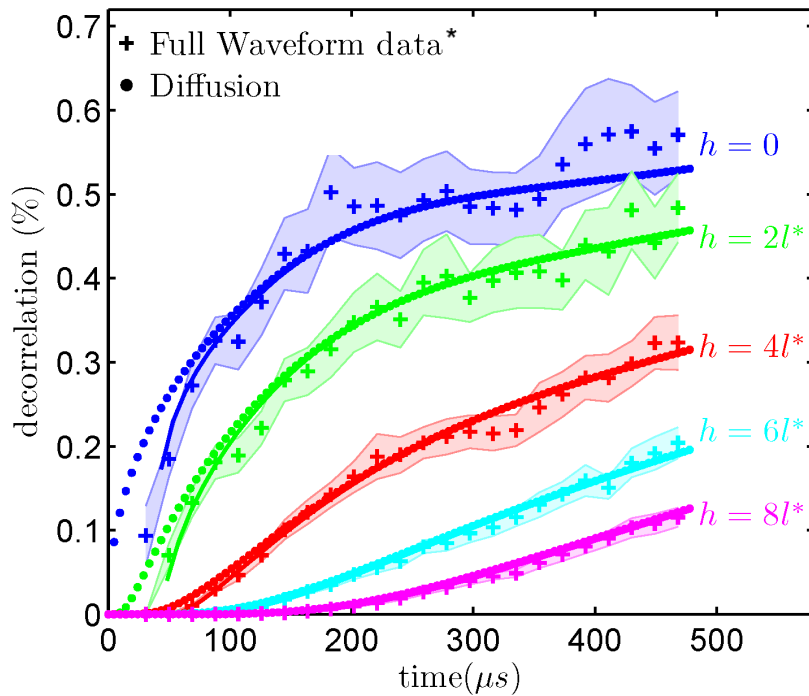
$I(S, \mathbf{R}, t) =$  Diffusion solution

# Forward problem validation

Far field



$$DC^{th}(\mathbf{S}, \mathbf{R}, \mathbf{r}, t) = \frac{c\sigma}{2} \frac{\int_0^t I(\mathbf{S}, \mathbf{r}, u) I(\mathbf{r}, \mathbf{R}, t - u) du}{I(\mathbf{S}, \mathbf{R}, t)}$$



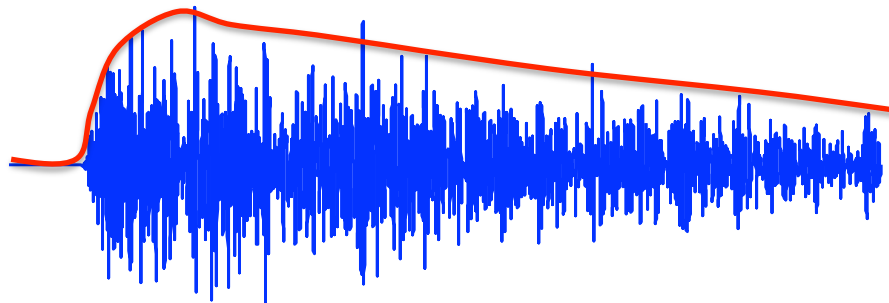
$$I(\mathbf{s}, \mathbf{r}, t) = \frac{I_0}{(4\pi Dt)} e^{-\frac{\|\mathbf{s}-\mathbf{r}\|^2}{4Dt}}$$

T. Planes et al, 2014 & 2015

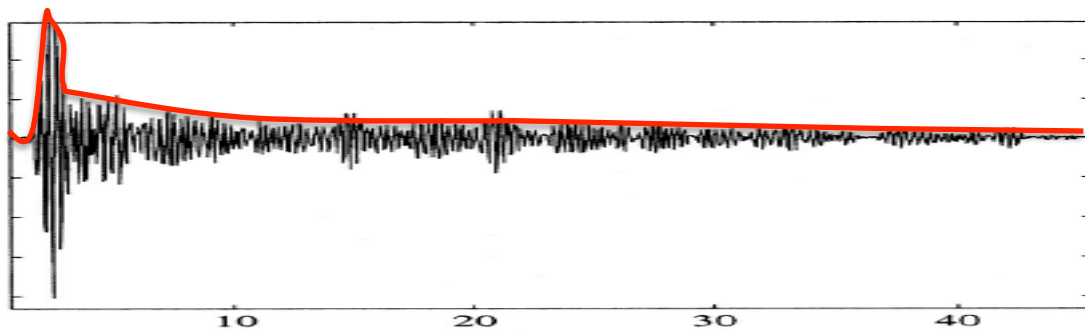
Sensitivity Kernels NEED:

Predict the TRANSPORT OF THE INTENSITY

$$I(S,R,t)$$

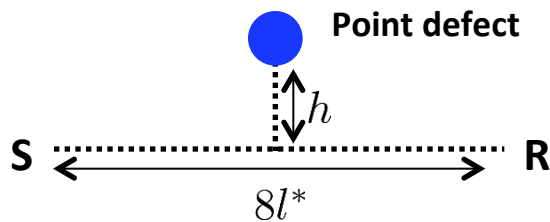


Diffusion (heat)

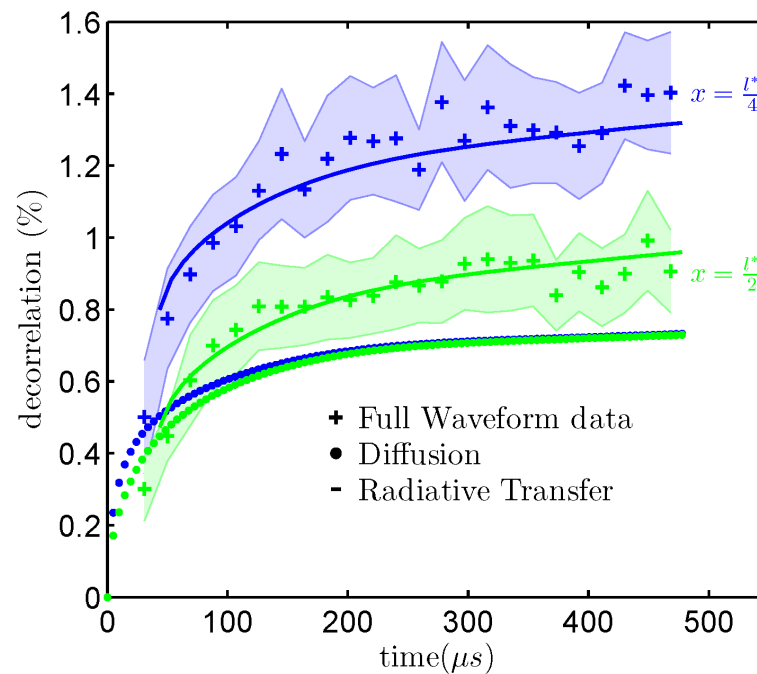
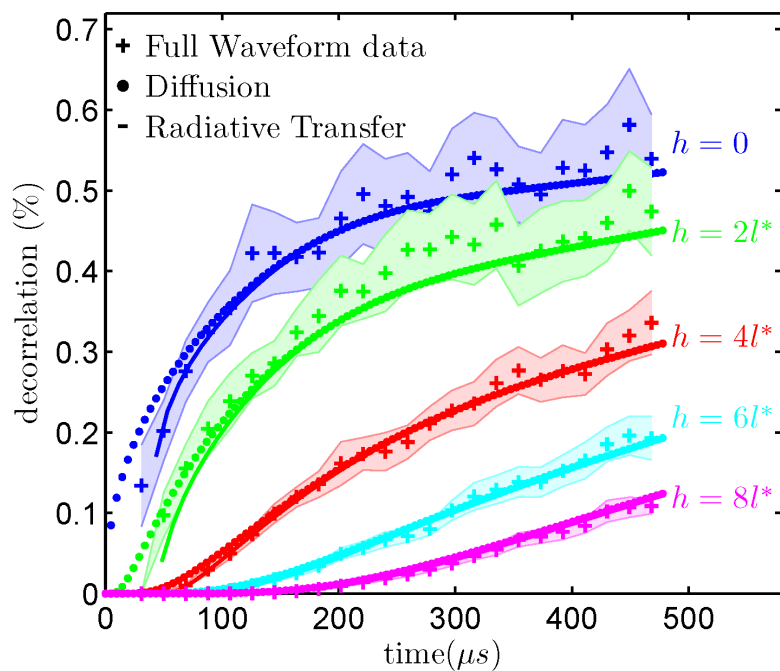
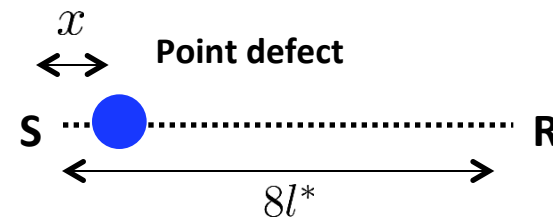


Radiative Transfert  
Sato 1993,  
Passchens 1997...

Far field



Near field





# Forward problem validation

2D Radiative Transfer Solution  
[Paasschens PRE 1997] :

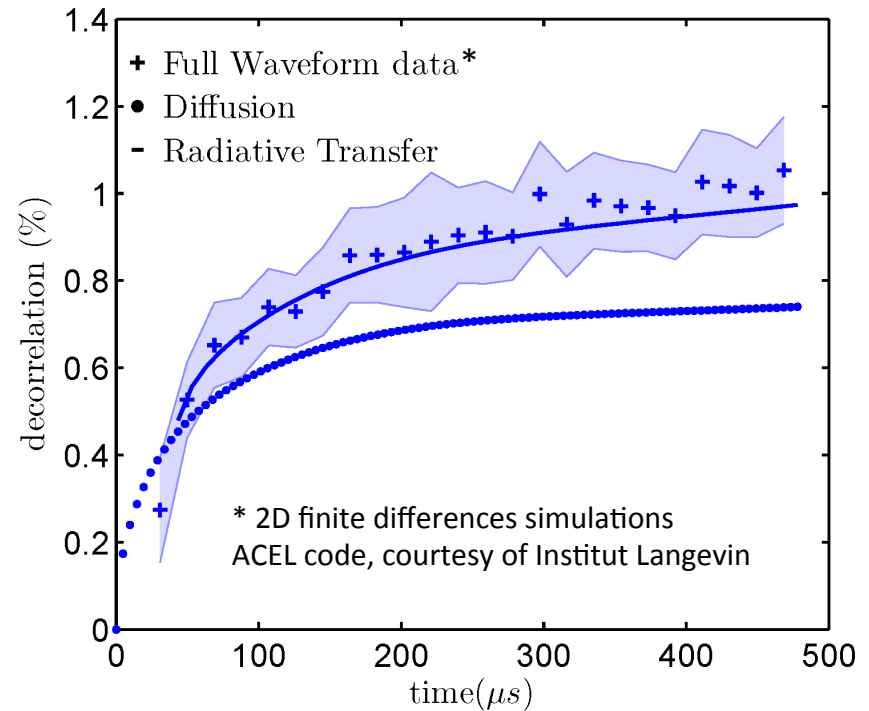
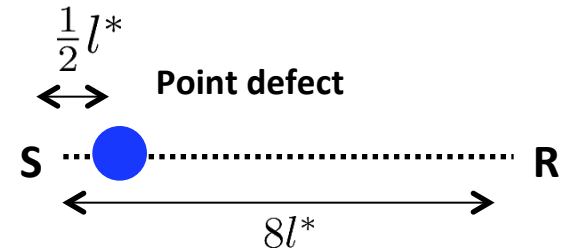
$$I(\mathbf{r}, t) = \underbrace{\frac{e^{-ct/l}}{2\pi r} \delta(ct - r)}_{\text{cohérent}}$$

cohérent

$$+ \underbrace{\frac{1}{2\pi l ct} \left(1 - \frac{r^2}{c^2 t^2}\right)^{-\frac{1}{2}} e^{[l^{-1}(\sqrt{c^2 t^2 - r^2} - ct)]} \Theta(ct - r)}_{\text{incohérent}}$$

incohérent

Near field

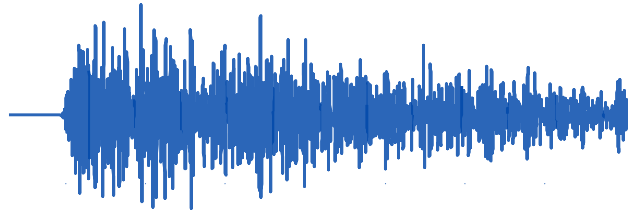


T. Planes et al, 2014 & 2015

# Application to ACTIVE data



# Workflow



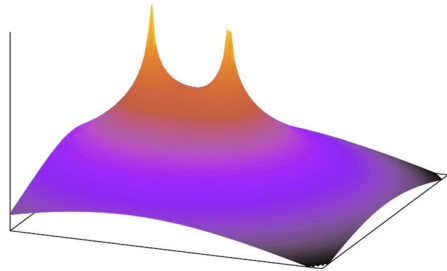
Active records



Measure  $dV/V$  and DC

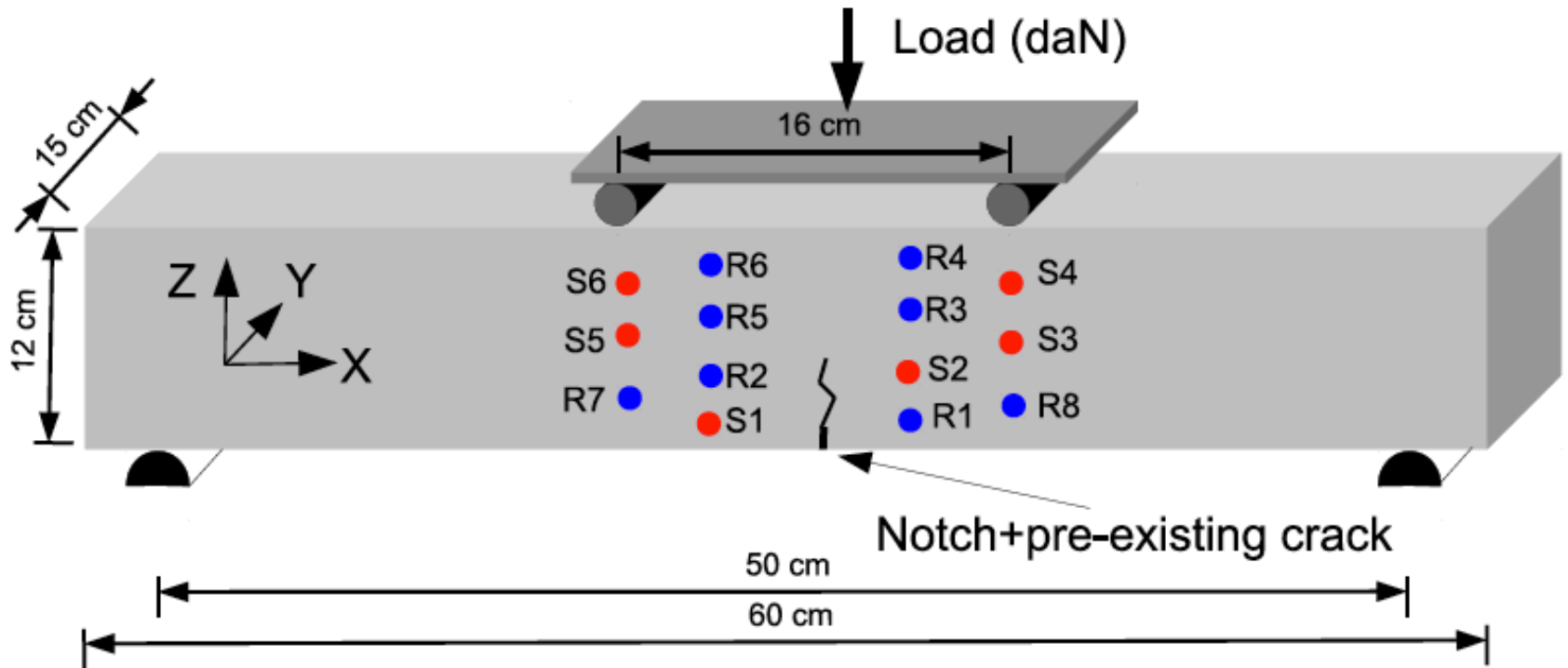


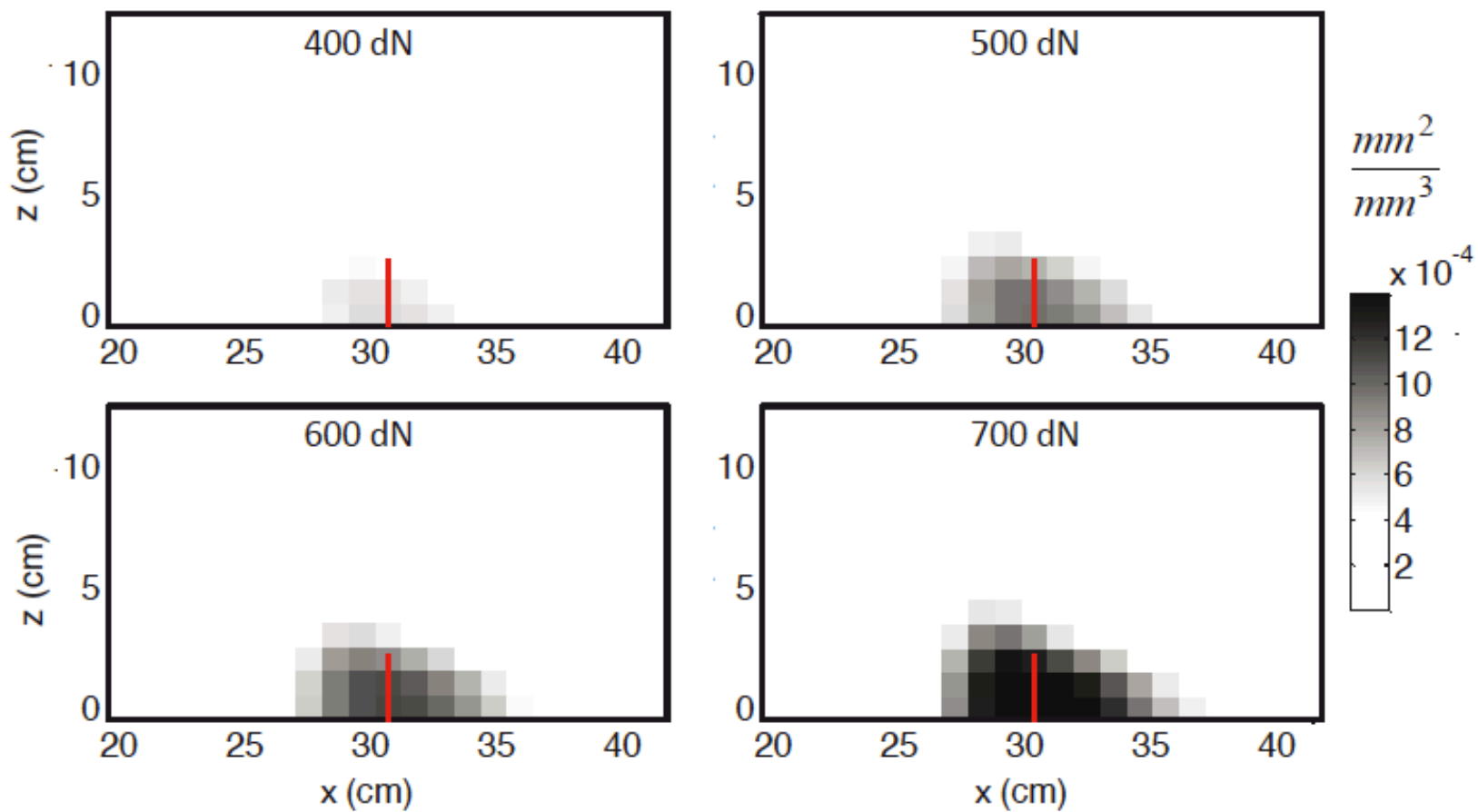
**LOCATE**  
the changes

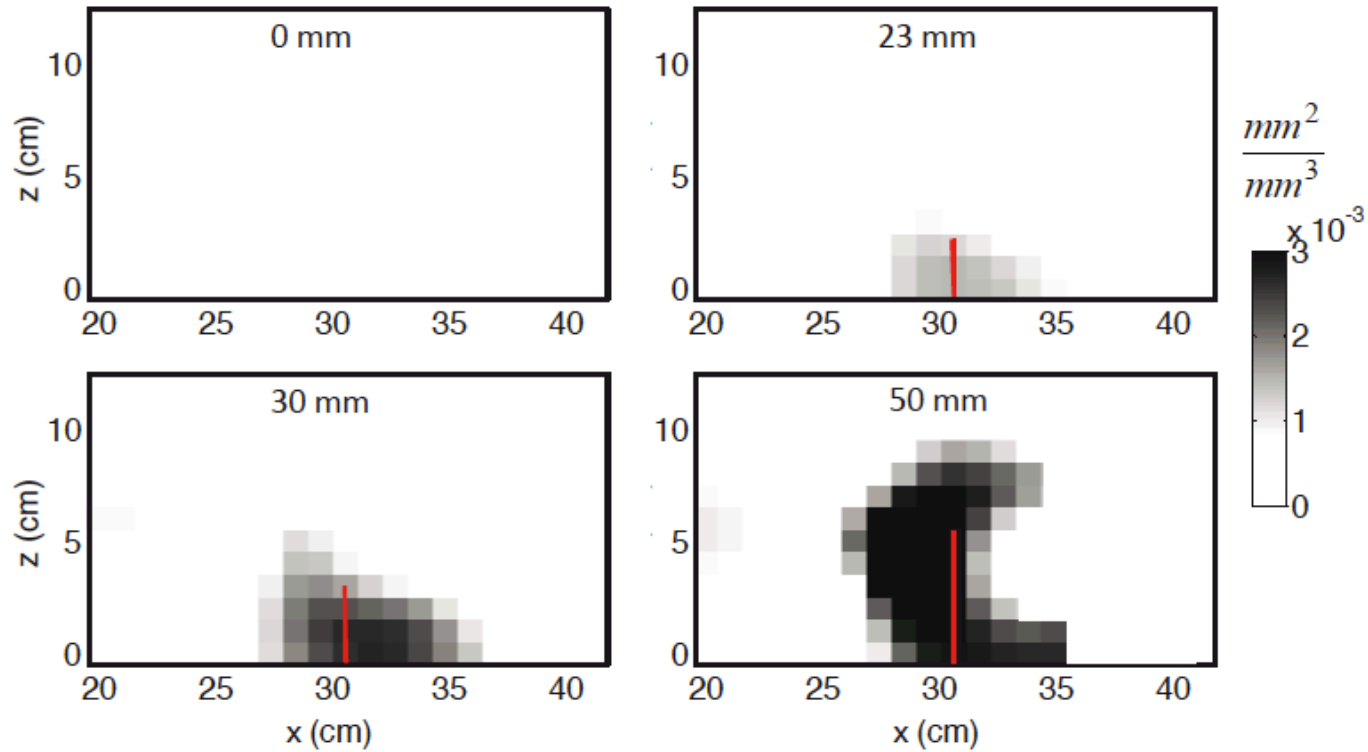


Evaluate sensitivity  
kernels

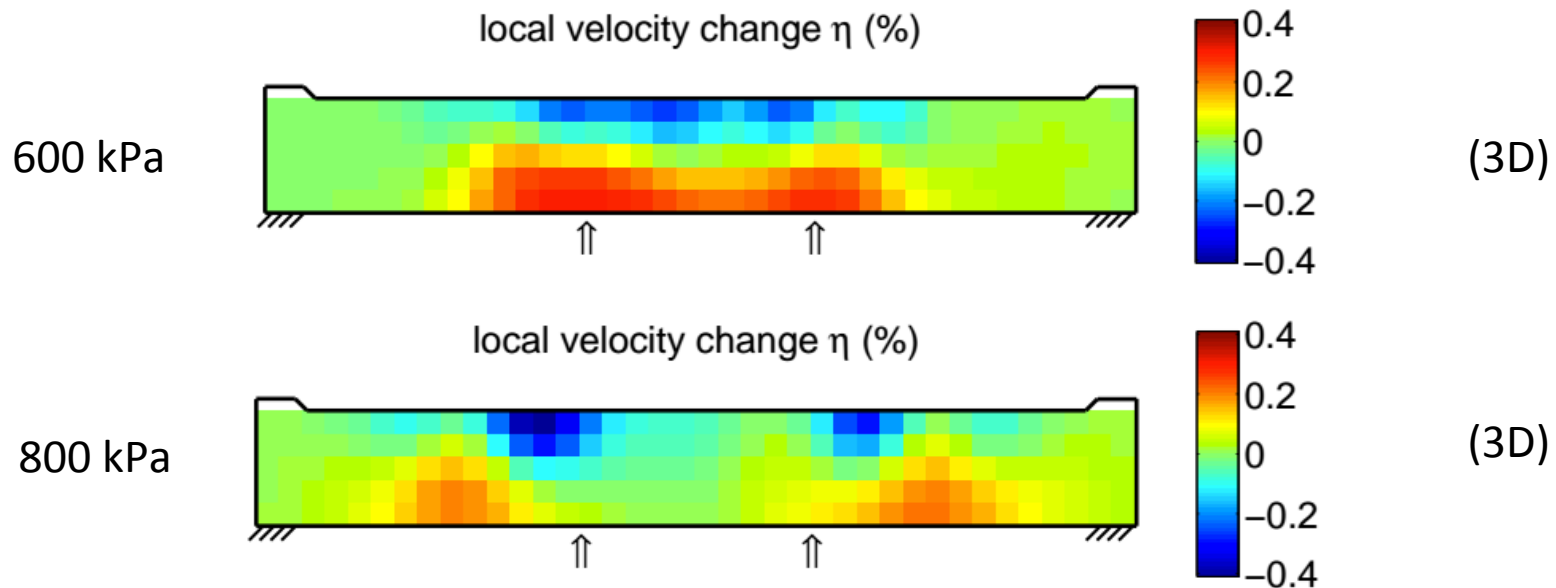






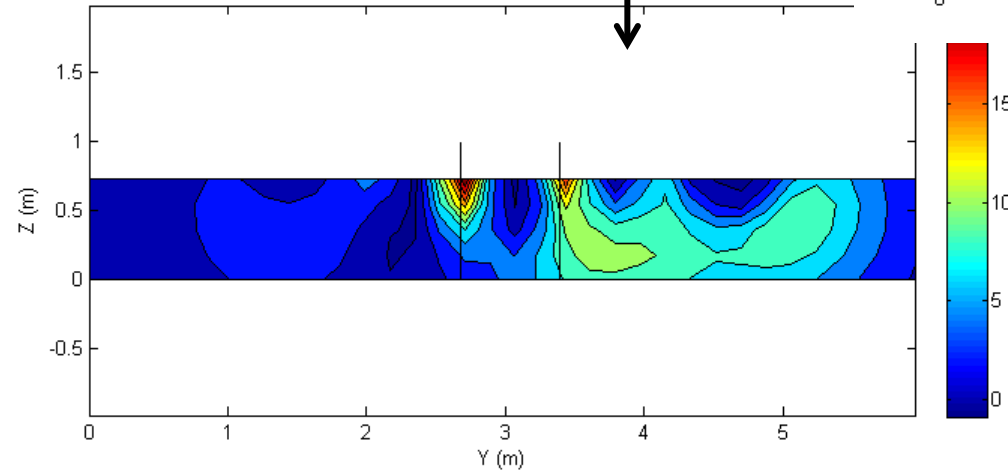
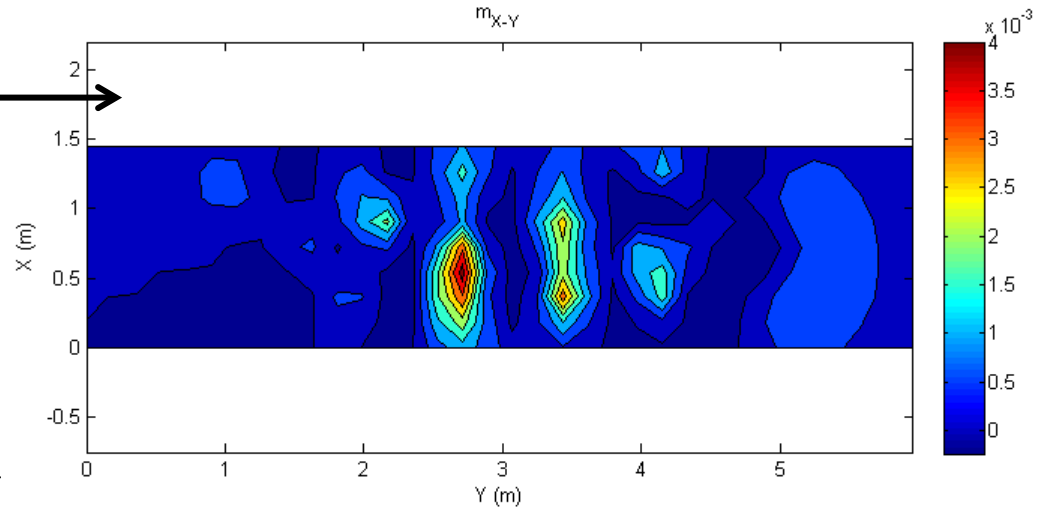
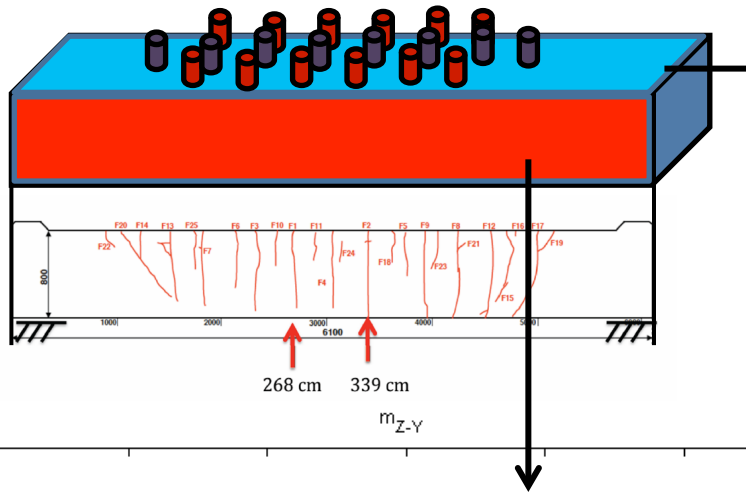


# Stress map (3D) *In Situ & non-destructive*



Zhang et al (2015)

# Damage/crack localisation



Zhang et al (2015)



# Application to PASSIVE data



# Workflow

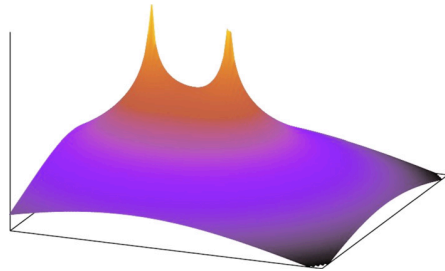
Ambient noise



Daily cross-correlation



Measure  $dV/V$  and  $K_d$

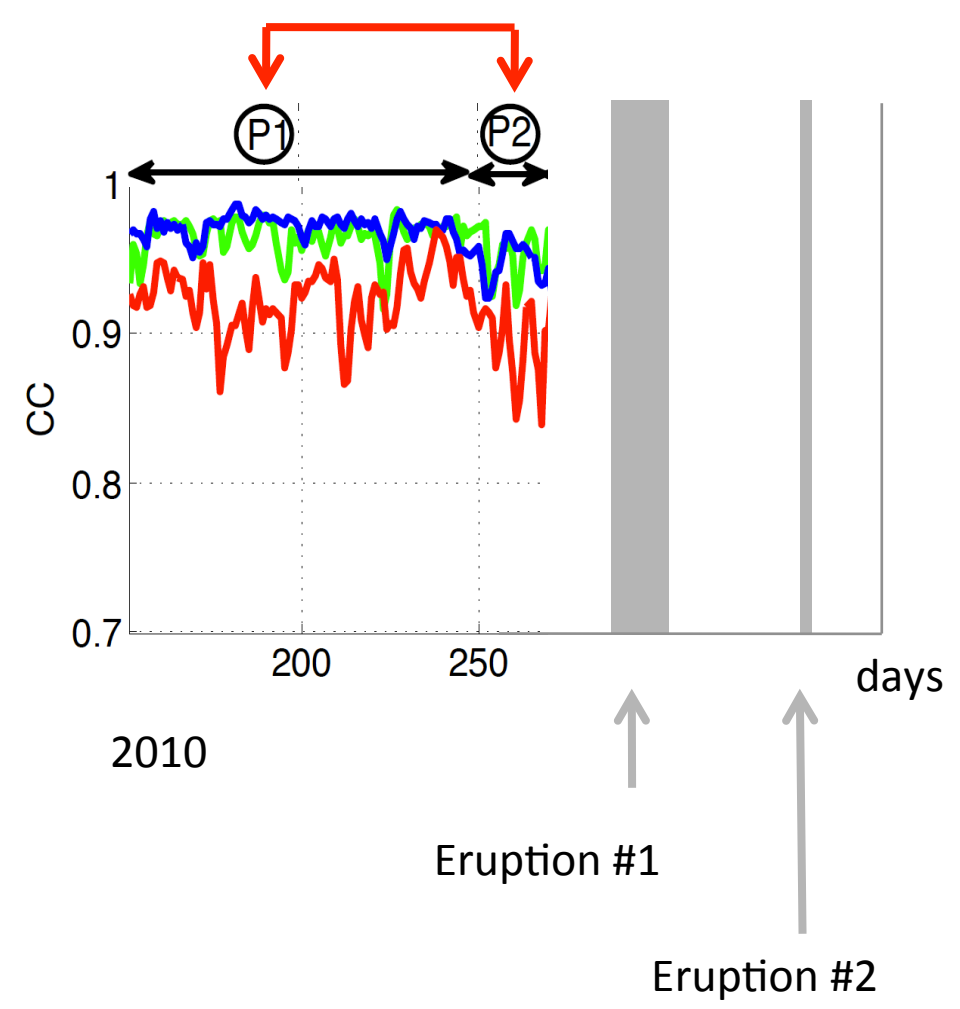
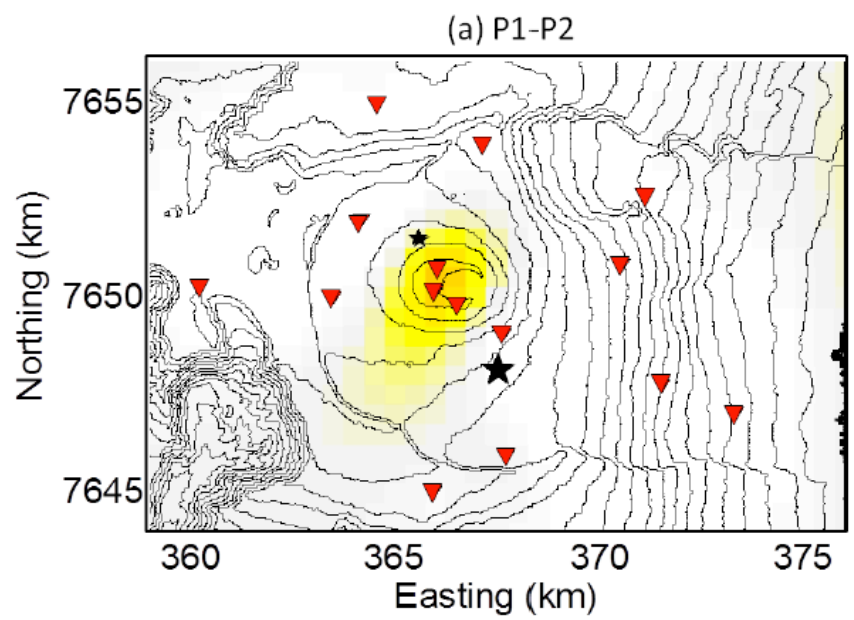


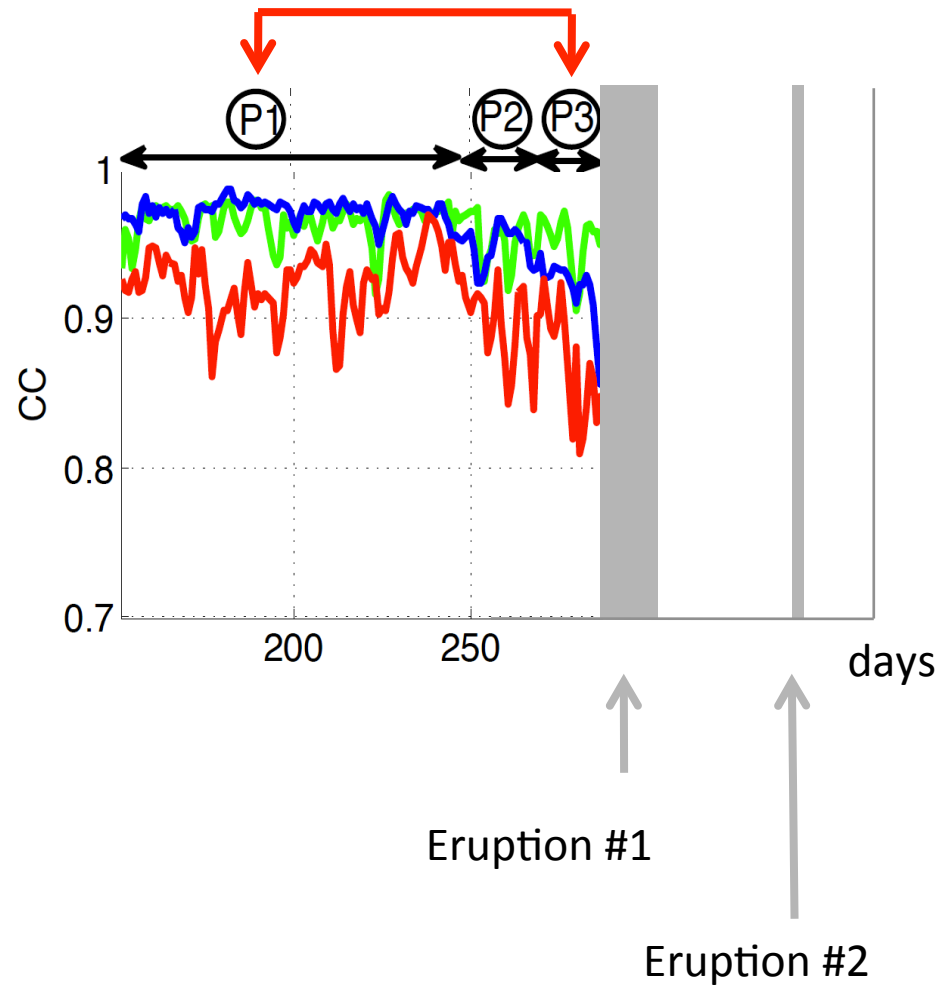
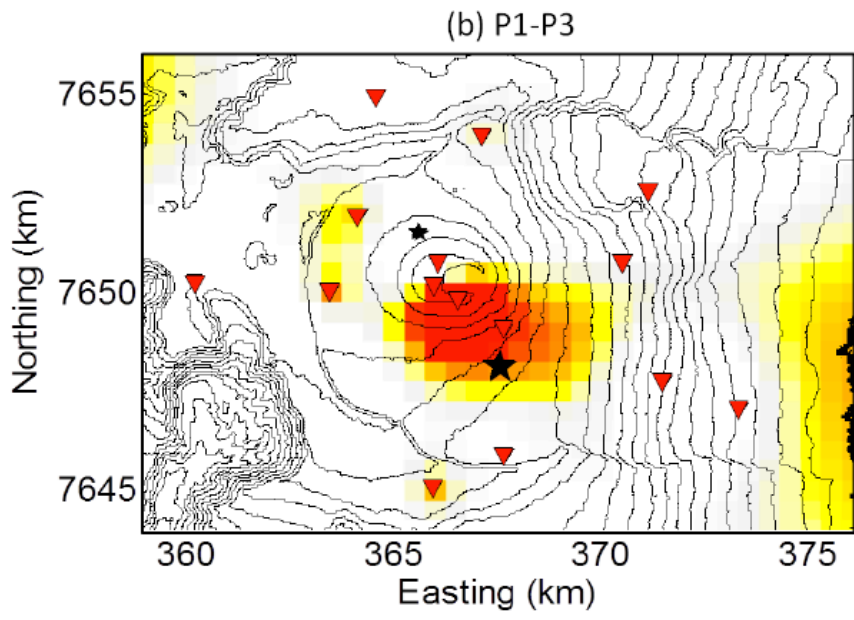
Evaluate sensitivity kernels

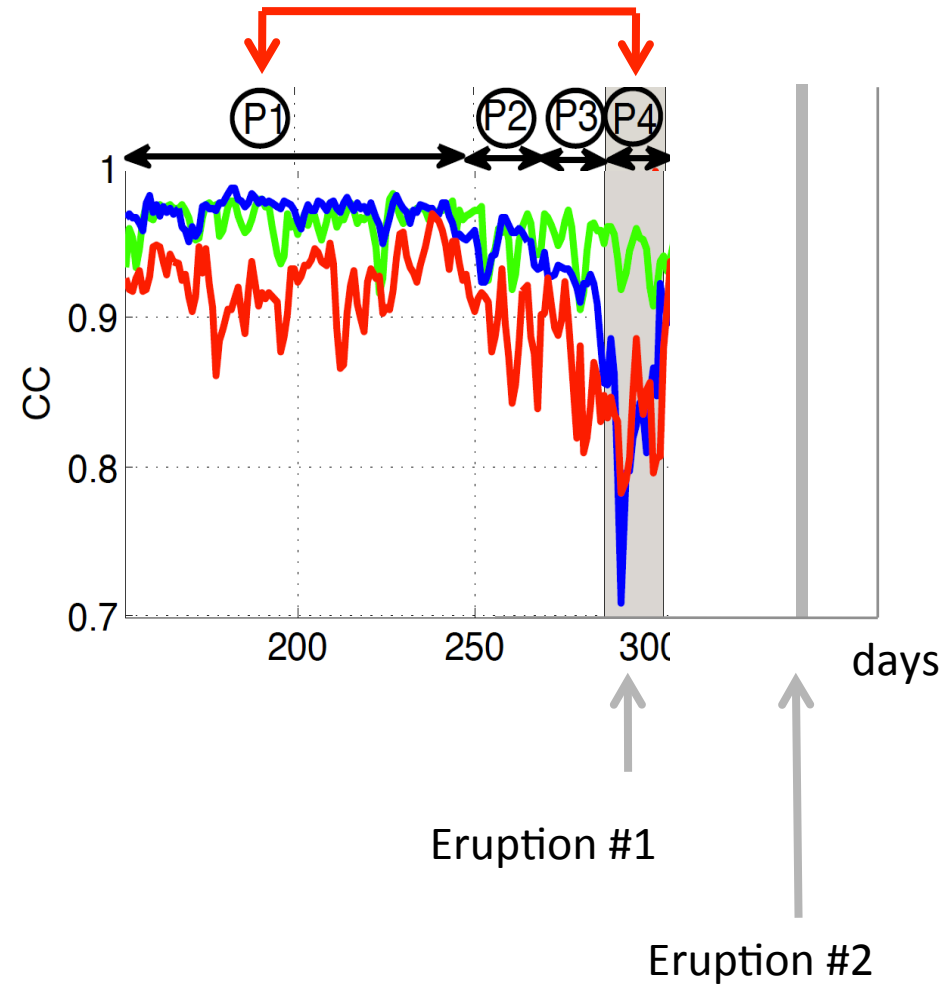
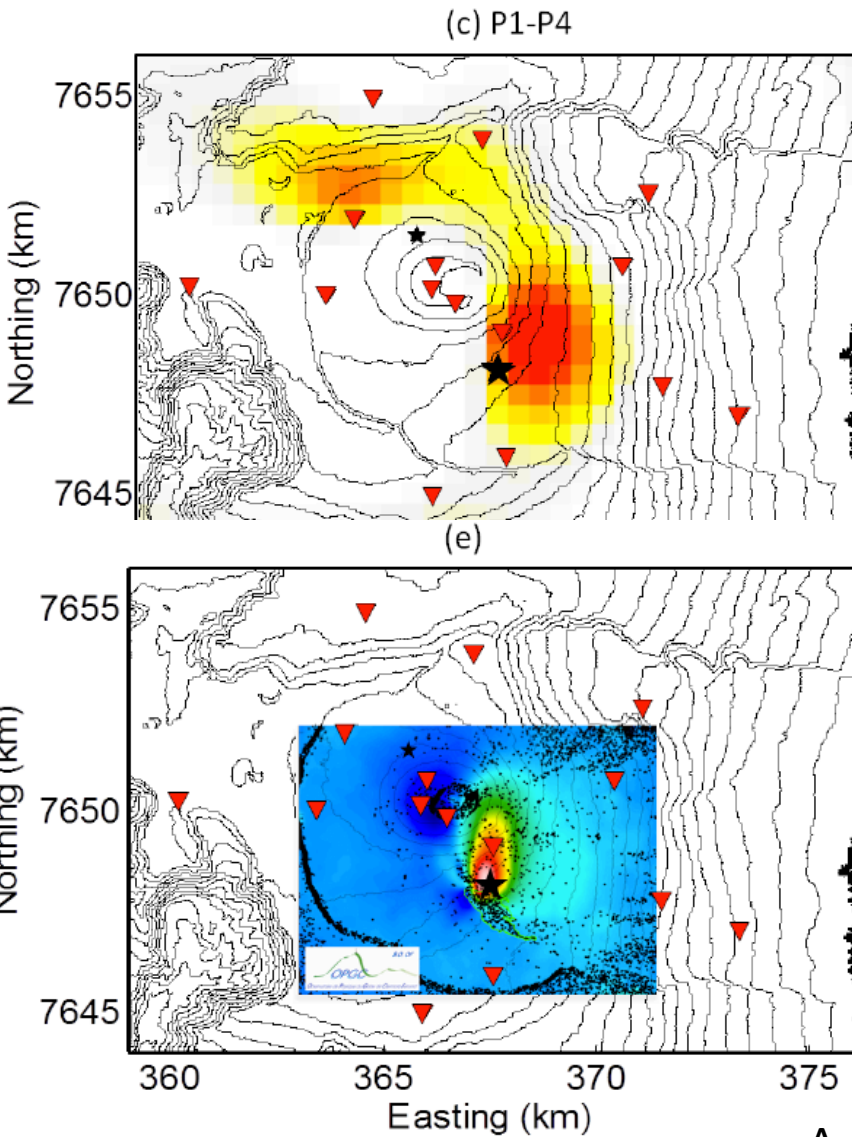


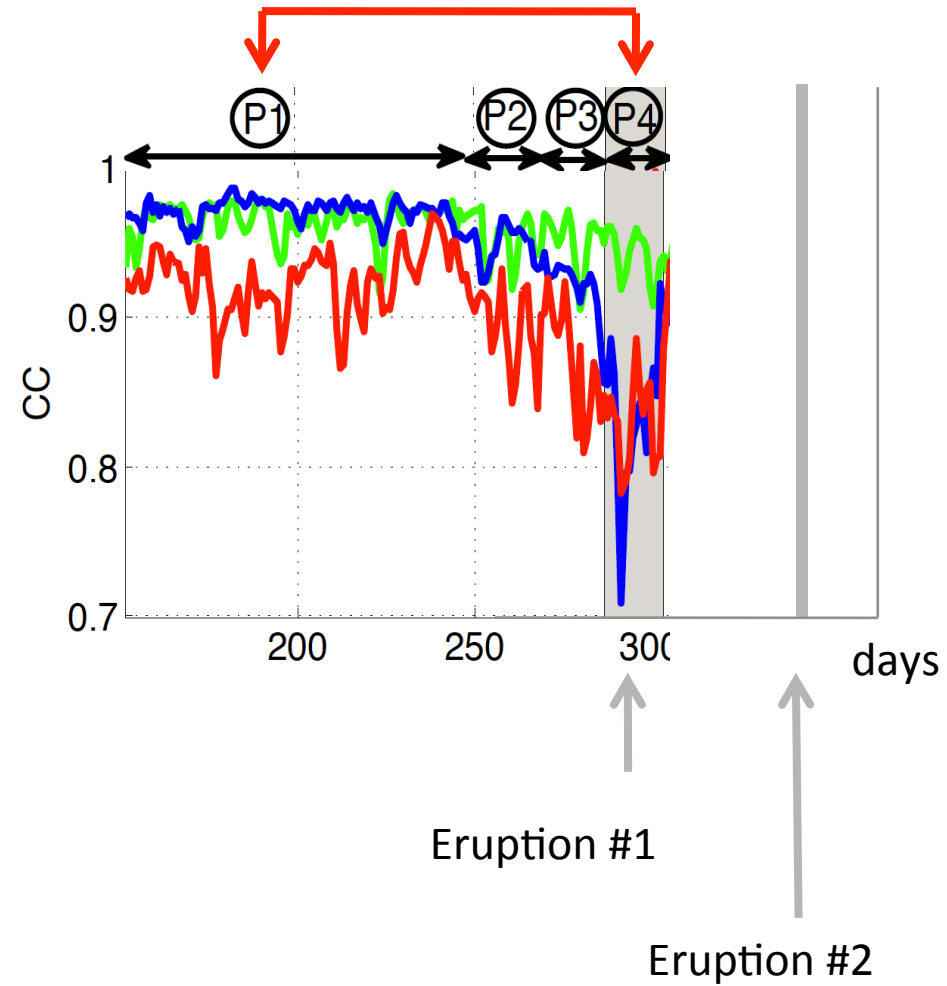
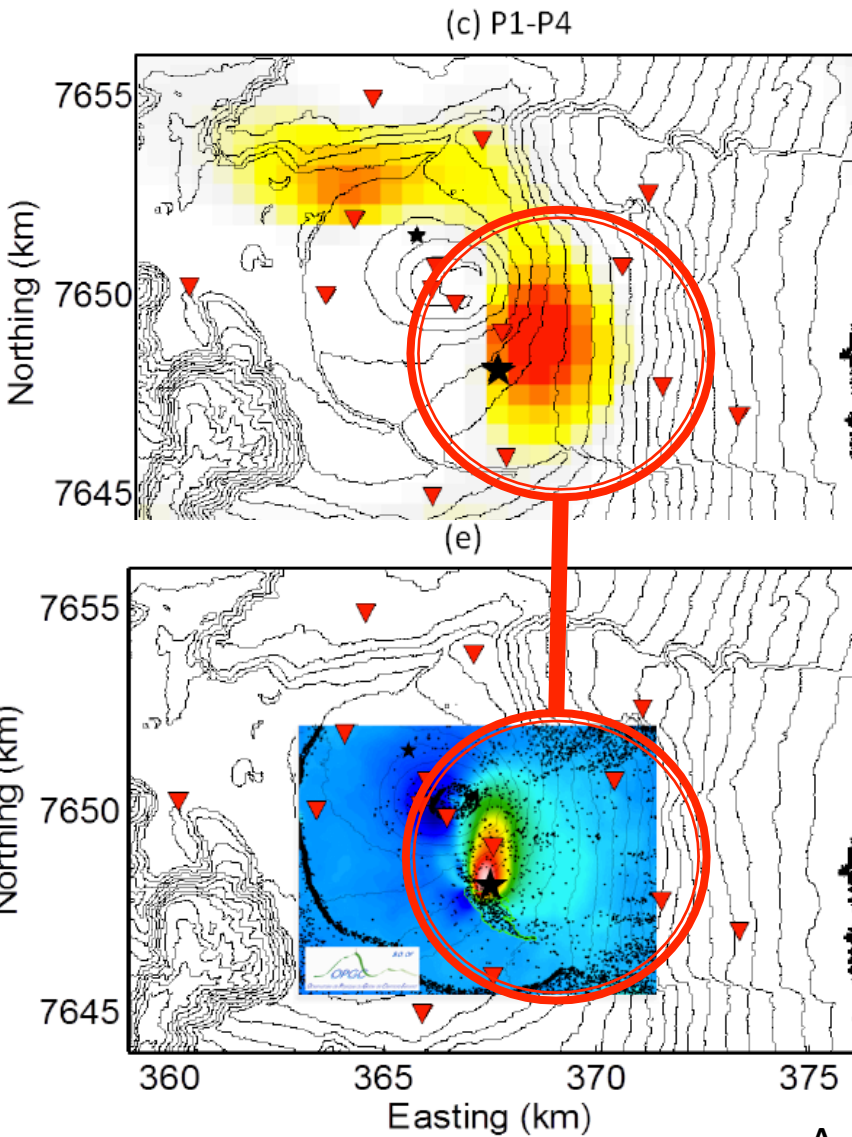
**LOCATE**  
the changes



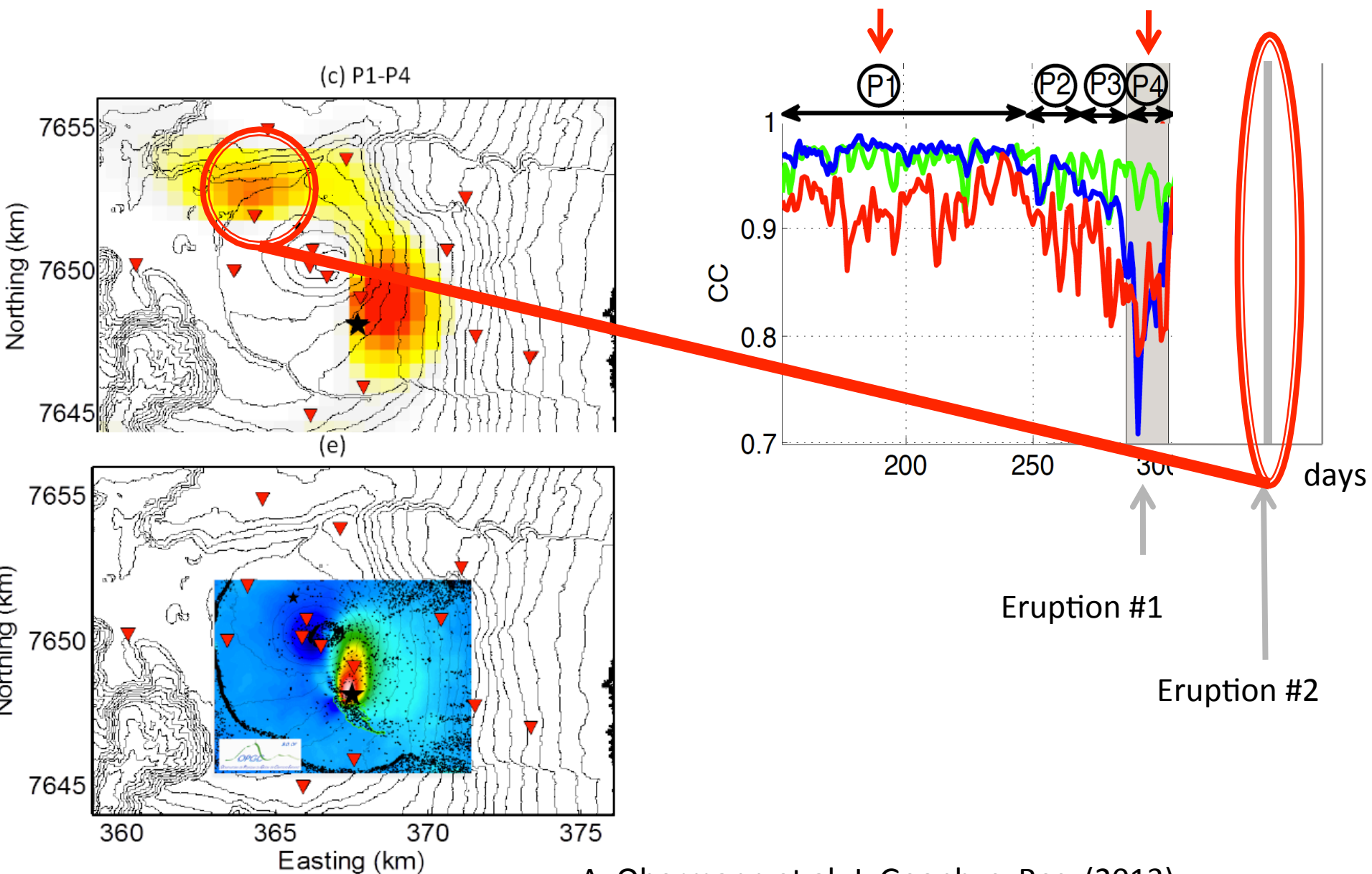






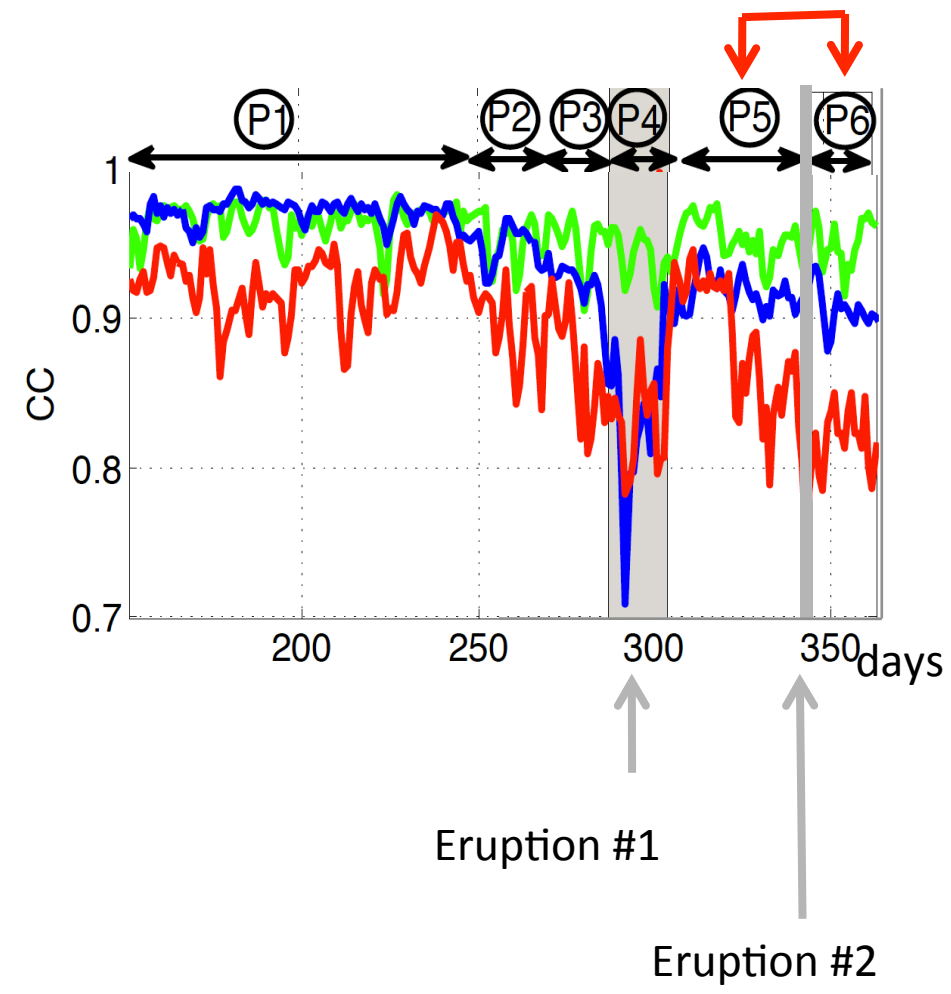
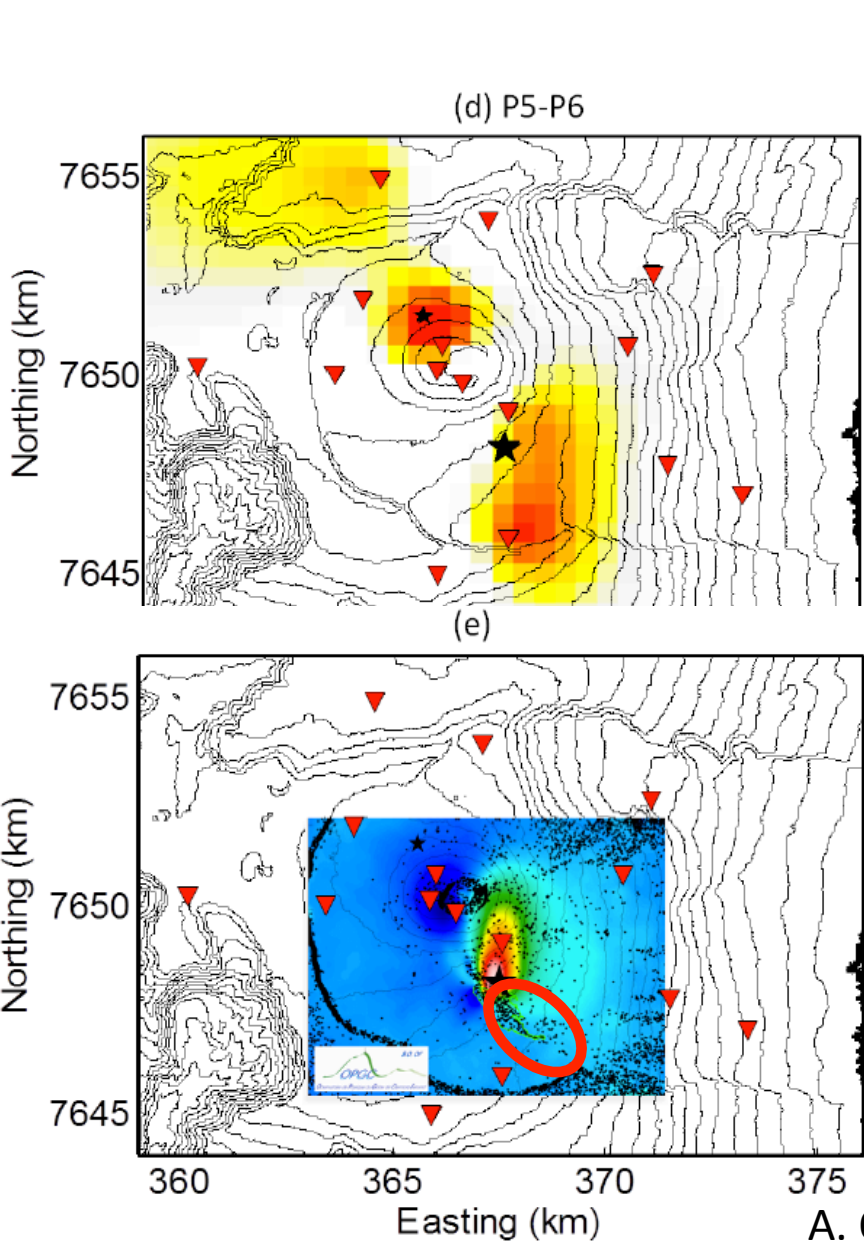


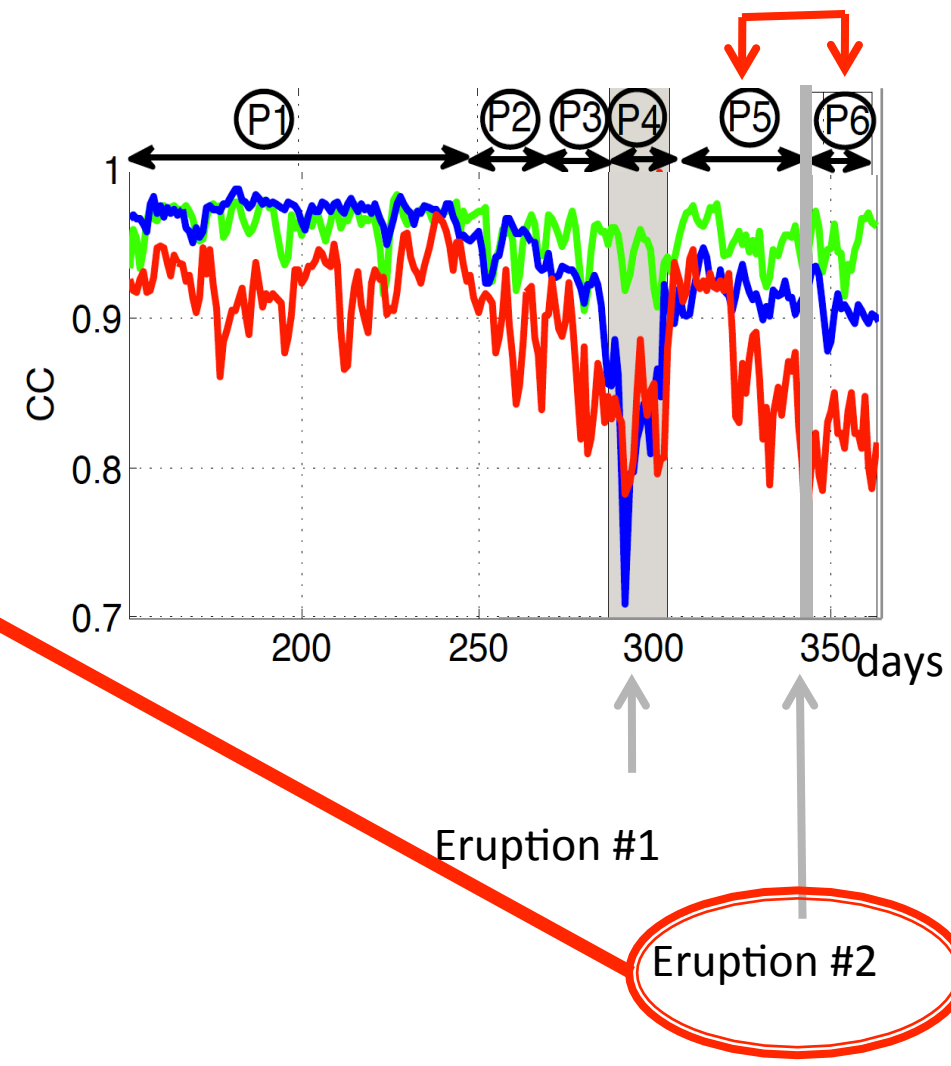
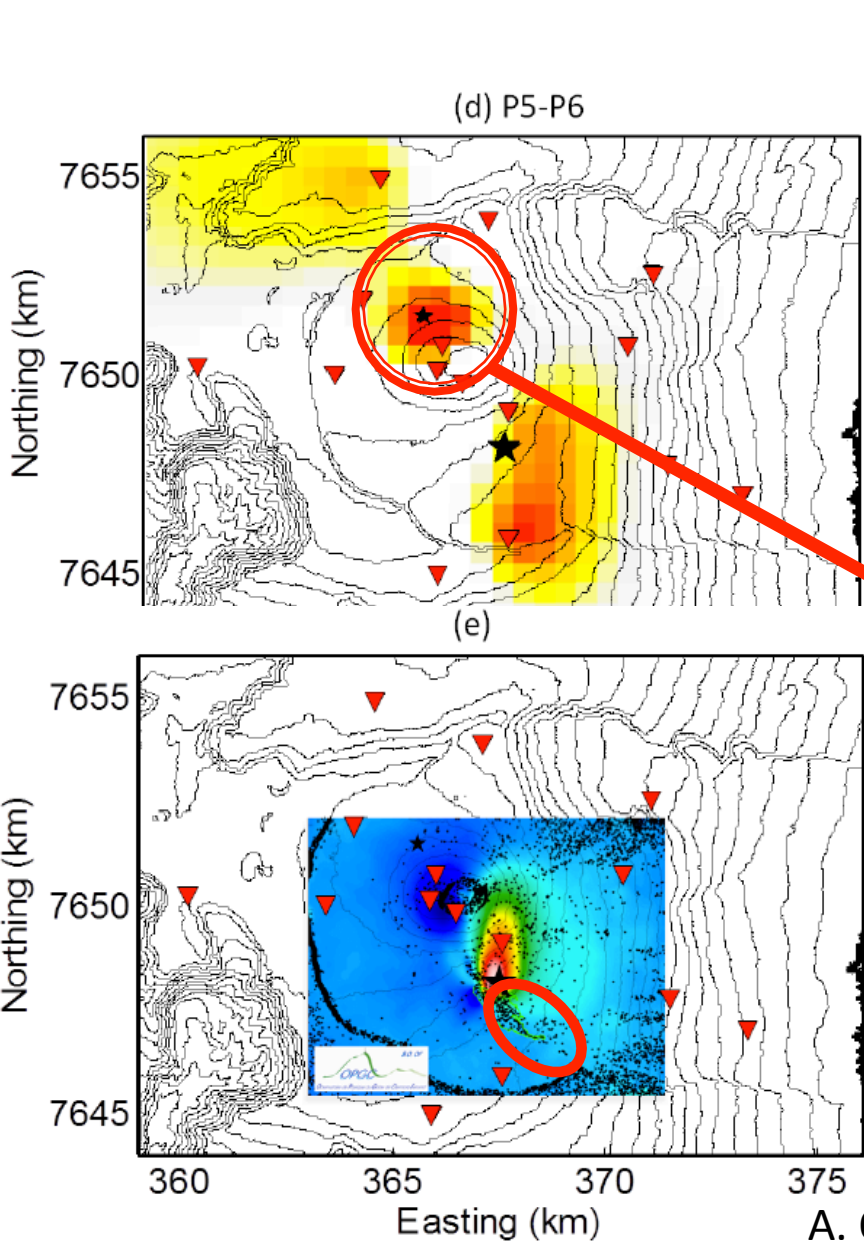


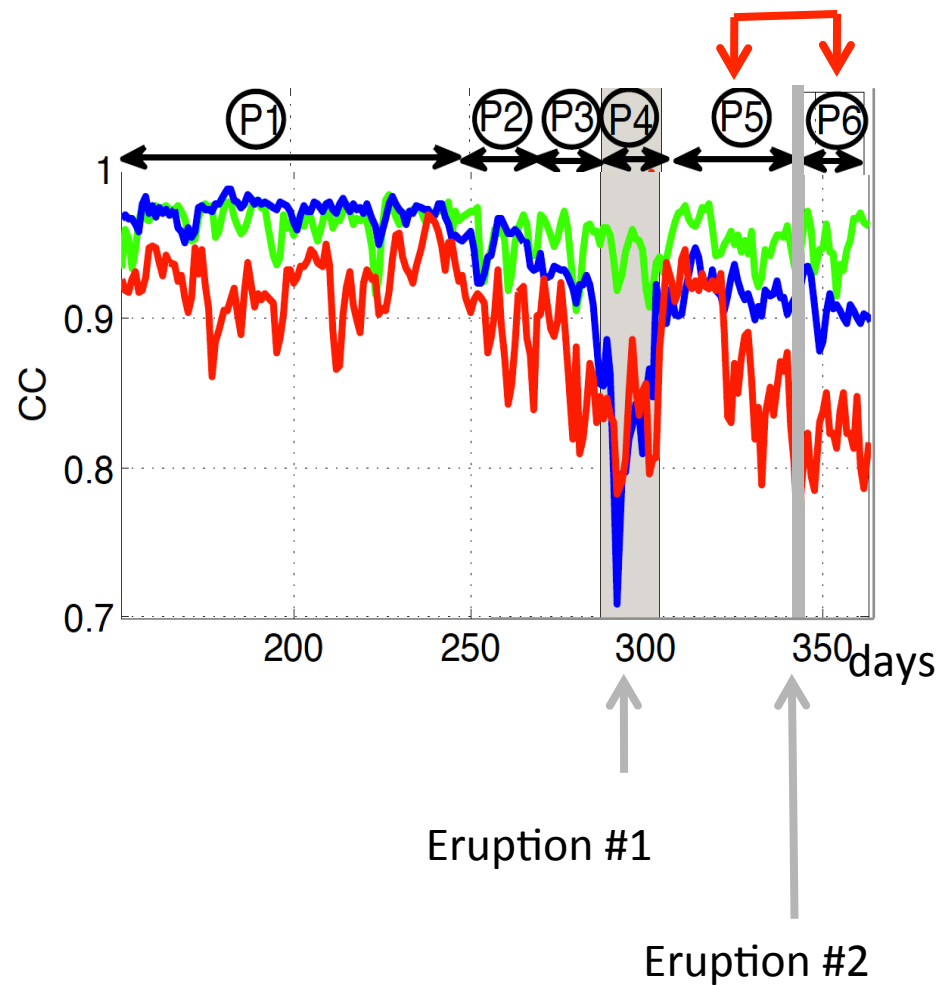
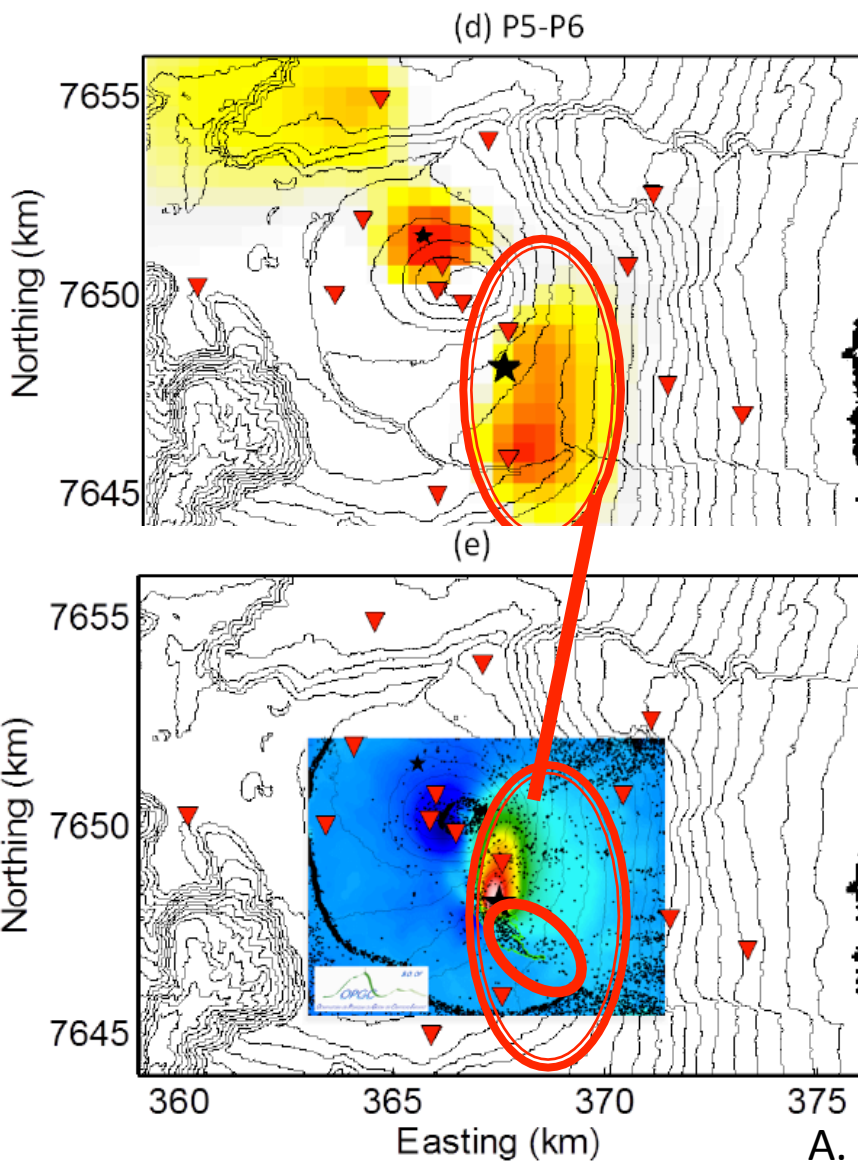


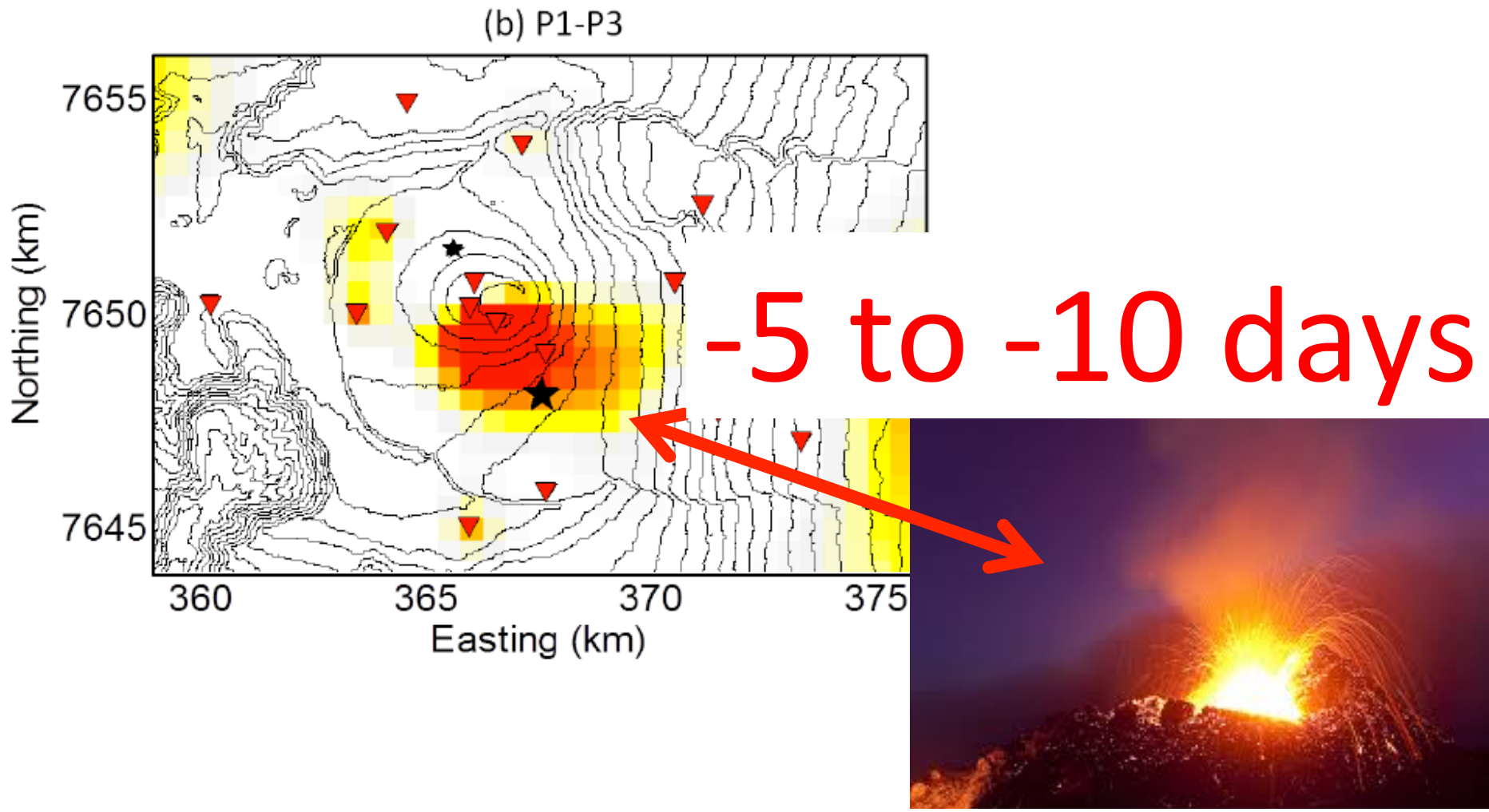
A. Obermann et al. J. Geophys. Res. (2013)



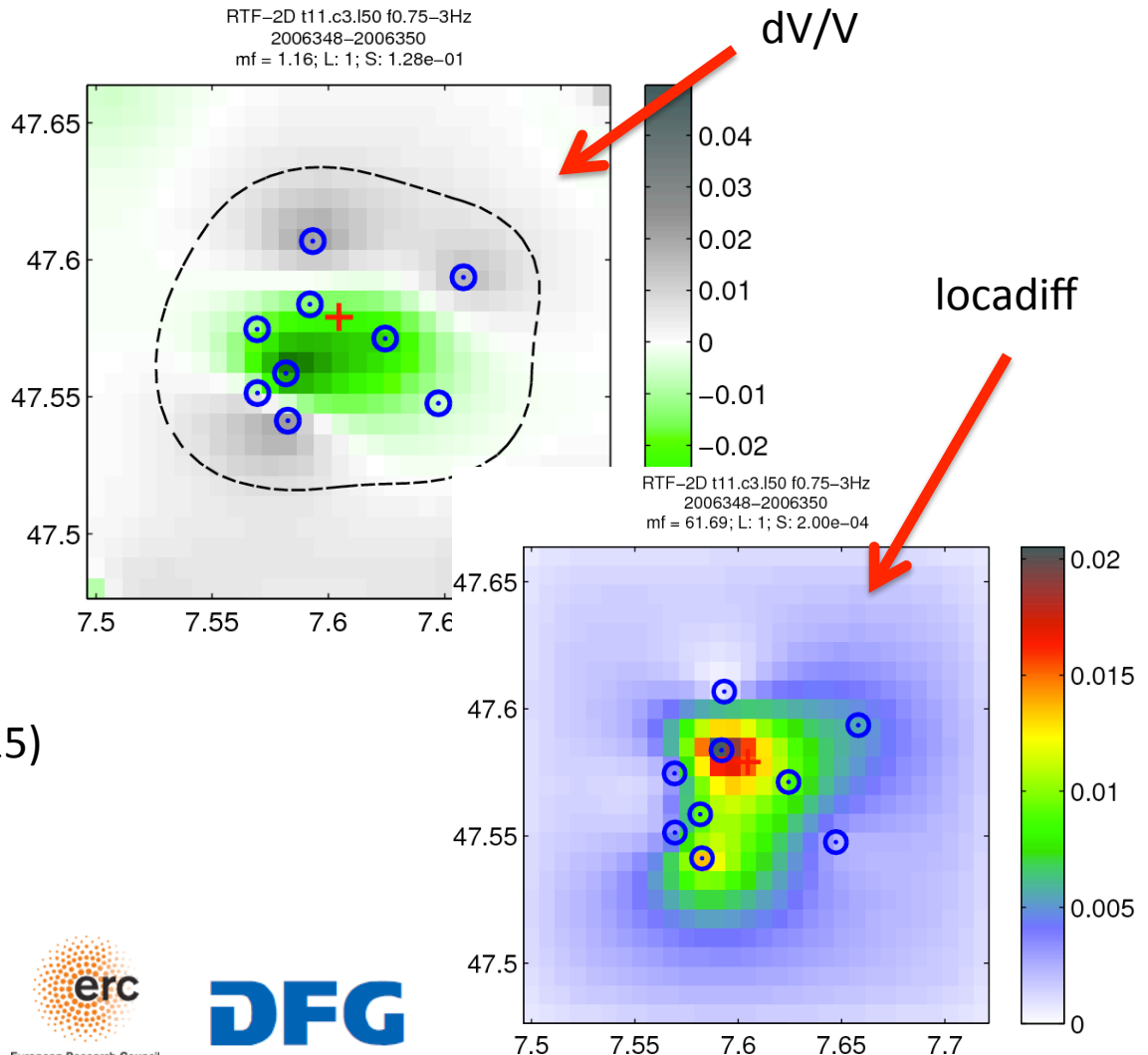
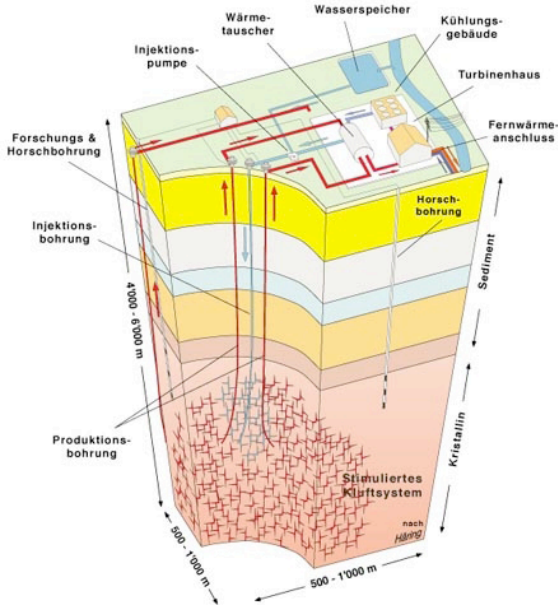








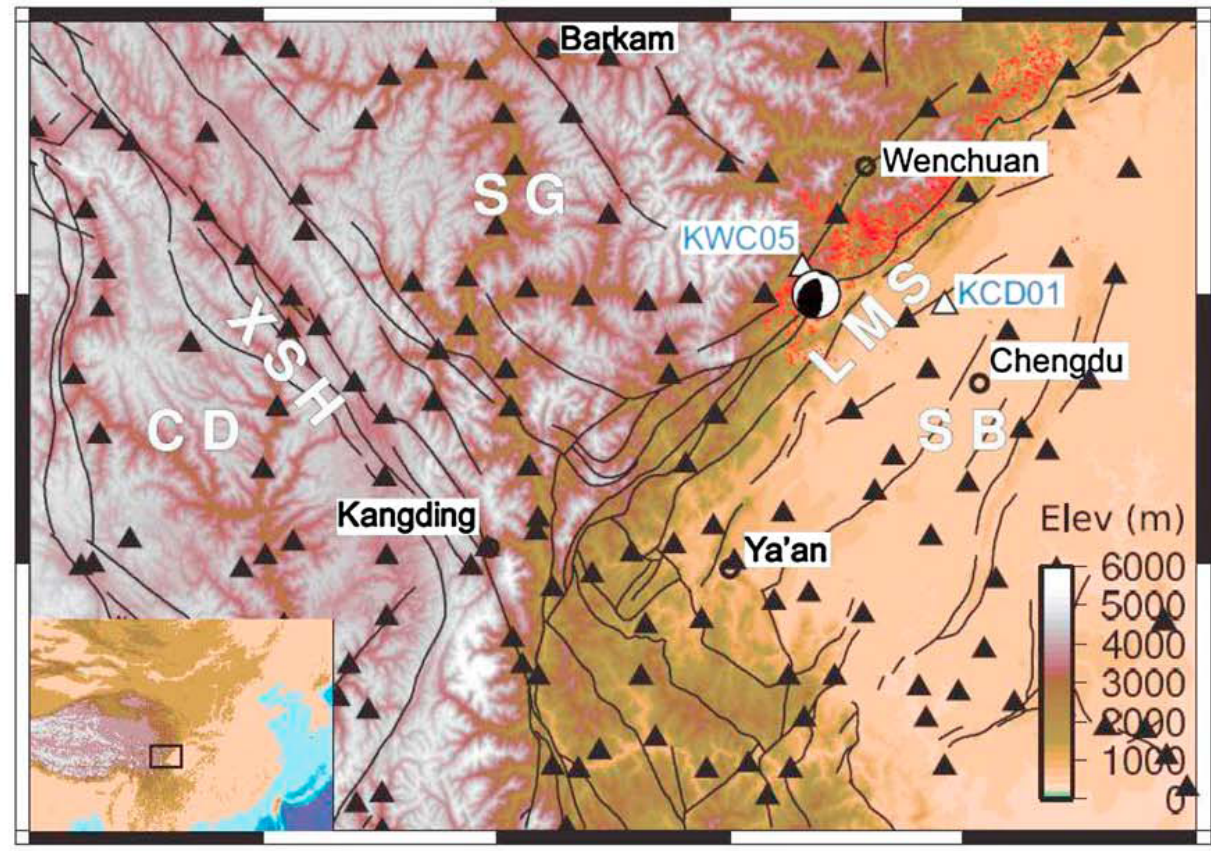
# 2006 Basel geothermal injection experiment



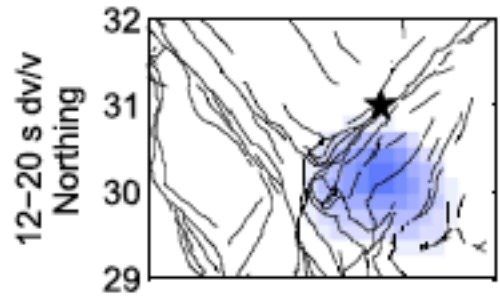
G. Hillers et al... Geophysics (2015)



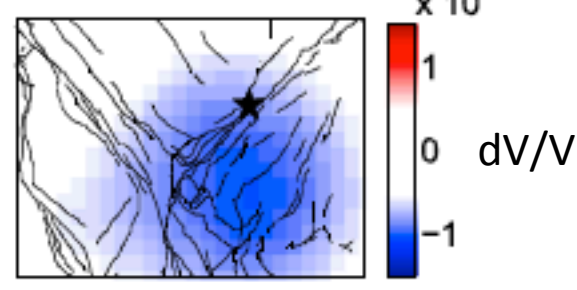
At large scale :



Monsoon 2007



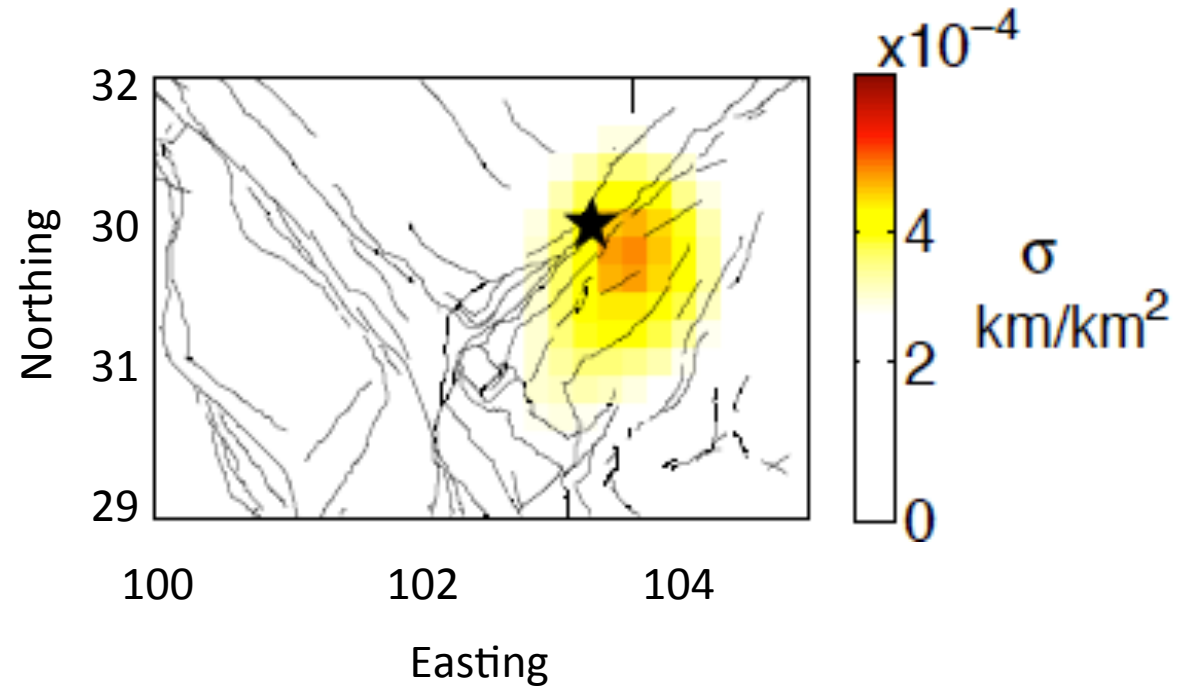
Monsoon 2008



# Wenchuan Earthquake

@ 1-3 s

50 days Before // 50 days after



Obermann et al, 2014