Institut Langevin
ONDES ET IMAGES

Ultrasound propagation in complex media under a random matrix approach

Alexandre Aubry

Institut Langevin – CNRS UMR 7587, ESPCI ParisTech – Paris, France

Collaborators:
Arnaud Derode (*Institut Langevin, Paris, France*)
Sharfine Shahajahan, Fabienne Rupin (*EDF R&D, France*)
John Page, Laura Cobus (*University of Manitoba, Winnipeg, Canada*)
Sergey Skipetrov, Bart van Tiggelen (*LPMMC, Grenoble, France*)

Introduction

Experimental flexibility of ultrasound

Multi-element array

Time-resolved measurement of the amplitude and phase of the wave

Numerous applications

Ultrasound Imaging: Medical diagnosis – Non destructive evaluation

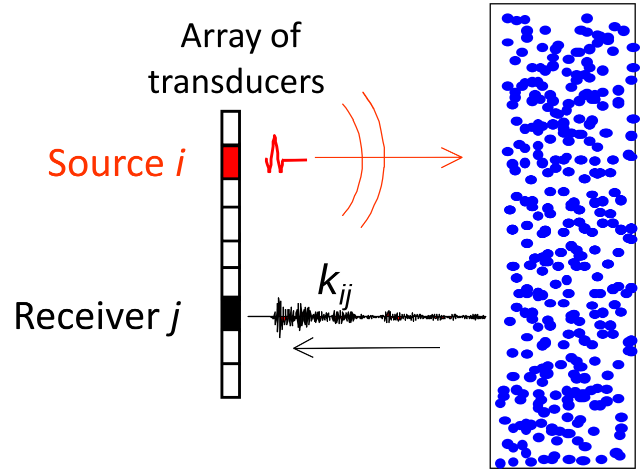
Focusing: Medical therapy – Telecommunications

Time reversal: Imaging and focusing through complex media

Interest of a matrix approach

Acquisition of the inter-element matrix **K**.

This matrix contains all the information available on the medium under investigation



Matrix **K** in « simple » media

C. Prada and M. Fink, *Wave Motion*, **20**: 151-163, 1994

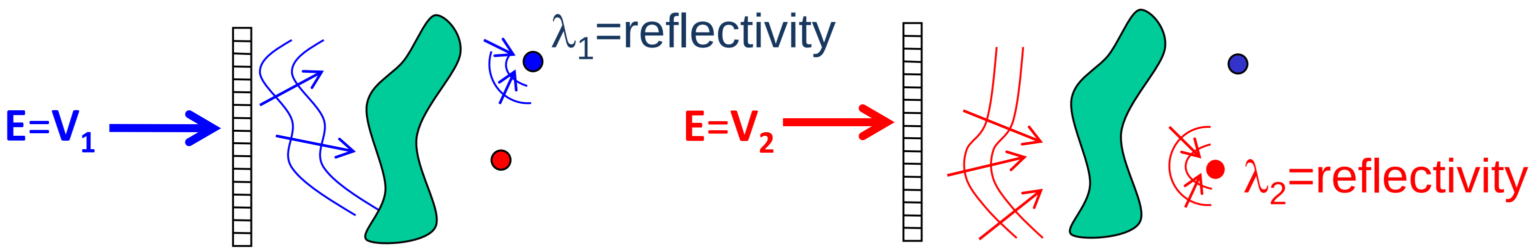
D.O.R.T method: decomposition of the time-reversal operator $\mathbf{KK}^{\dagger}$

Singular value decomposition of **K**

$$\mathbf{K} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{V}^{\dagger}$$

- $\boldsymbol{\Lambda}$: Diagonal matrix containing the *N* singular values ($\lambda_1 > \lambda_2 > \ldots > \lambda_N$)
- **U** and **V** : Unitary matrices whose columns are the singular vectors

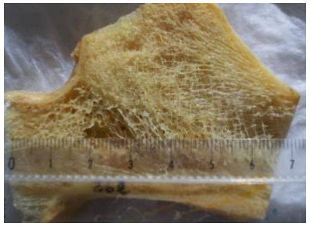
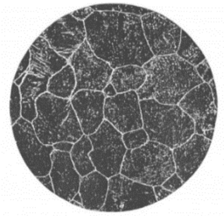
Simple media: one eigenstate ⟷ one scatterer



Each eigenvector V_i back-propagates respectively towards each scatterer of the medium

Introduction

Matrix **K** in complex media?

Practical interest:

Interest for detection and imaging (separation single / multiple scattering)

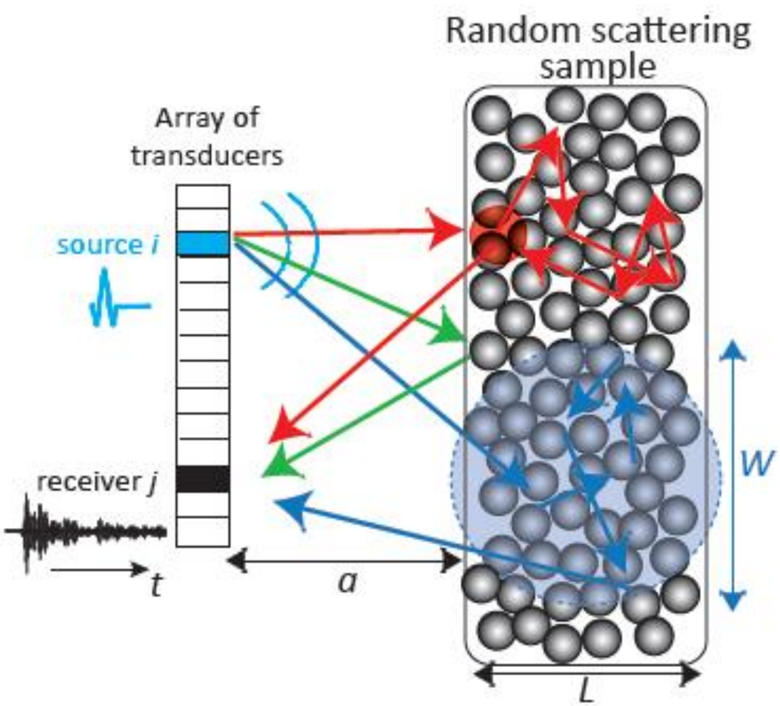
Characterization of scattering media

Fundamental interest:

Link with random matrix theory

Study of Anderson localization (recurrent scattering)

One relevant parameter: the scattering mean free path l_e



Weak disorder ($kl_e \gg 1$)

Single scattering ($t \ll l_e/c$)

→ Classical imaging techniques (ultrasound imaging, D.O.R.T method, Kirchhoff migration)

Multiple scattering ($t \gg l_e/c$)

→ Nightmare for imaging

→ Statistical approach, diffusion equation

→ Measurements of transport parameters (l_e, D…)

Strong disorder ($kl_e \sim 1$)

Anderson localization ($D(L) \to 0$)

→ Strong interference effects halt the wave within the scattering medium

→ Scattering loops

Introduction

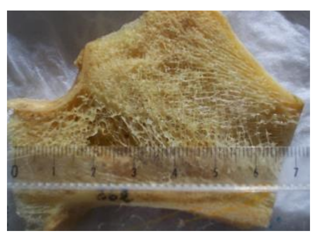
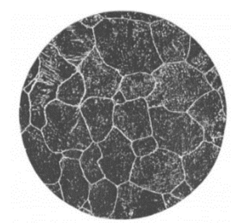
Which frequency?

Ultrasound 1 → 5 MHz

Which medium, which scattering regime?

Ballistic regime	*Diffusive regime*		*Anderson localization*
$kl_e \sim 1000$	$kl_e \sim 100$		$kl_e \sim 1$

Disorder →

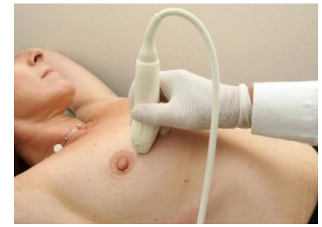
Soft tissues	**Bone**	**Coarse grain steels**	***Mesoglasses***
			
$l_e \sim 100$ mm	$l_e \sim 10$ mm	$l_e \sim 25$ mm	$l_e \sim 1$ mm
$\lambda \sim 0.5$ mm	$\lambda \sim 1$ mm	$\lambda \sim 2$ mm	$\lambda \sim 2$ mm

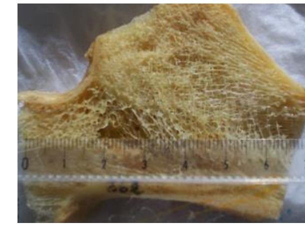
Outline

Propagation operator in random media

Statistical properties

Deterministic coherence of single scattering

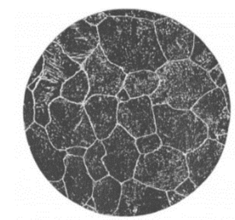




Target detection in multiple scattering media

Separation of single and multiple scattering

Detection of flaws in coarse grain steels



Coll. A. Derode, S. Shahjahan, F. Rupin

Recurrent scattering in strongly scattering media

Recurrent scattering, Memory effect

Manifestation of Anderson localization

Coll. J.H. Page, L.C. Cobus, S. Skipetrov, A. Derode, B.A. van Tiggelen

Propagation operator in random media

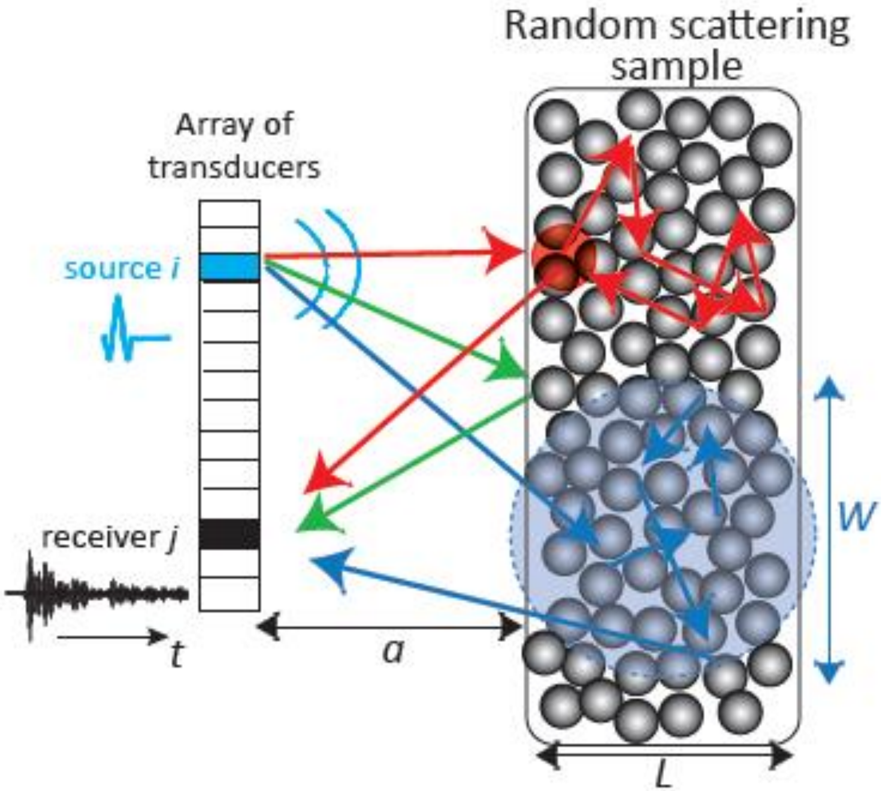
Alexandre Aubry, Arnaud Derode

A. Aubry and A. Derode, *Phys. Rev. Lett.* **102**, 084301, 2009

A. Aubry and A. Derode, *Waves Random Complex Media* **20**, 333-363, 2010

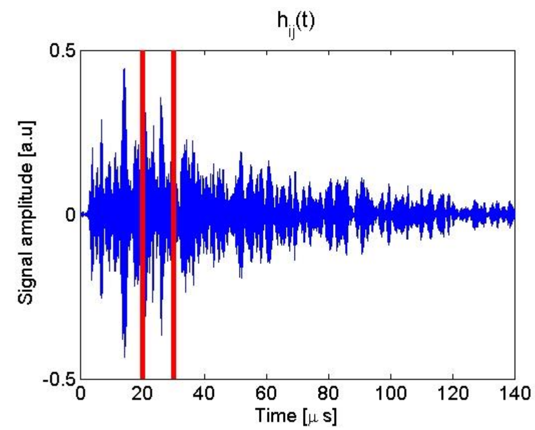
A. Aubry and A. Derode, *J. Acoust. Soc. Am.* **129**, 225-234, 2011

Experimental procedure



Keep the temporal resolution provided by ultrasonic measurements while working in the Fourier domain

1/ Acquisition of the inter-element matrix



2/ Time-frequency analysis

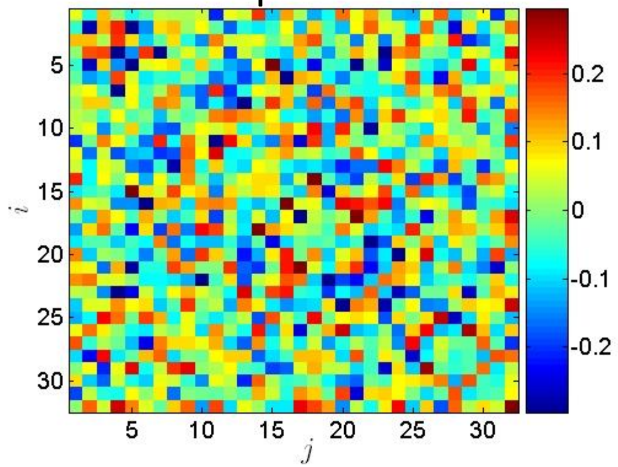
$$\mathbf{K}(t,f)$$

time frequency

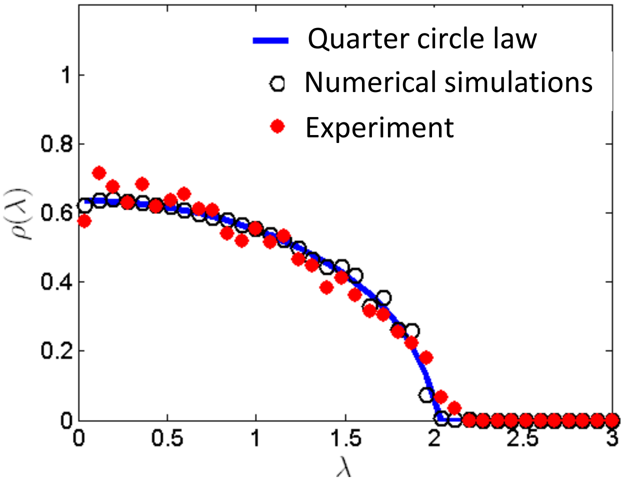
Statistical properties of **K** / multiple scattering

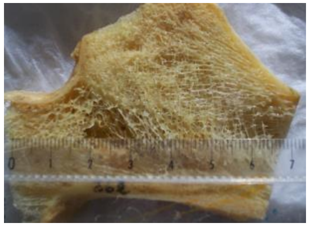
Experiment in a multiple scattering medium

Real part of **K**



Distribution of singular values





Random distribution of scatterers

↓

K(t,f) = random matrix

↓ SVD

No equivalence between eigenstates and scatterers of the medium

↓

Statistical approach

↓

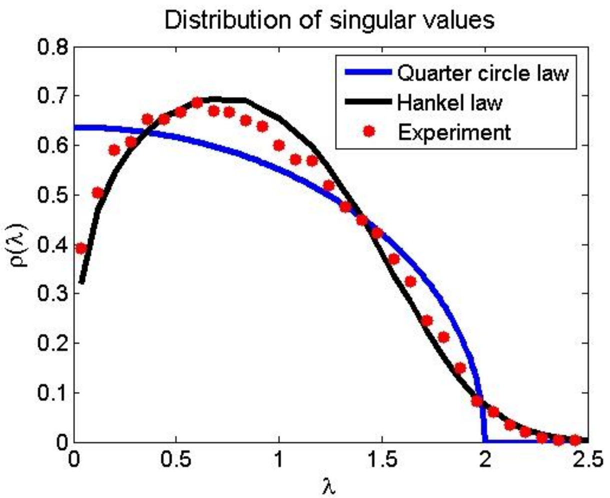
Distribution of singular values = Quarter circle law

V. Marcenko and L. Pastur, Math. USSR-Sbornik **1**, 457, 1967

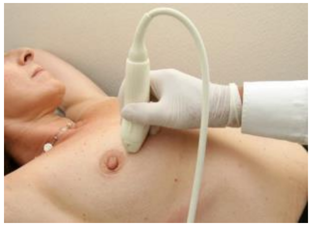
Statistical properties of **K** / single scattering

Experiment in a single scattering medium

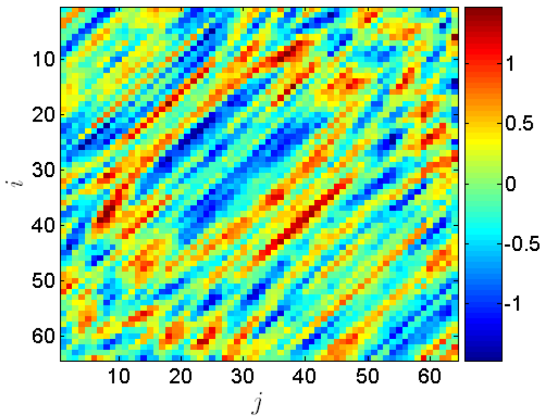
In the single scattering regime, we are far from the expected quarter circle law…



Soft tissues

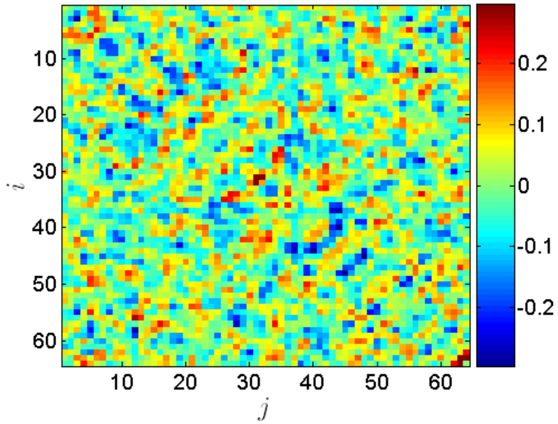


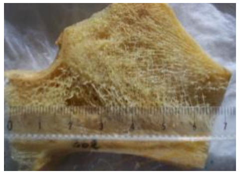
Single scattering



Occurrence of a deterministic coherence along the antidiagonals of **K** in the single scattering regime

Multiple scattering

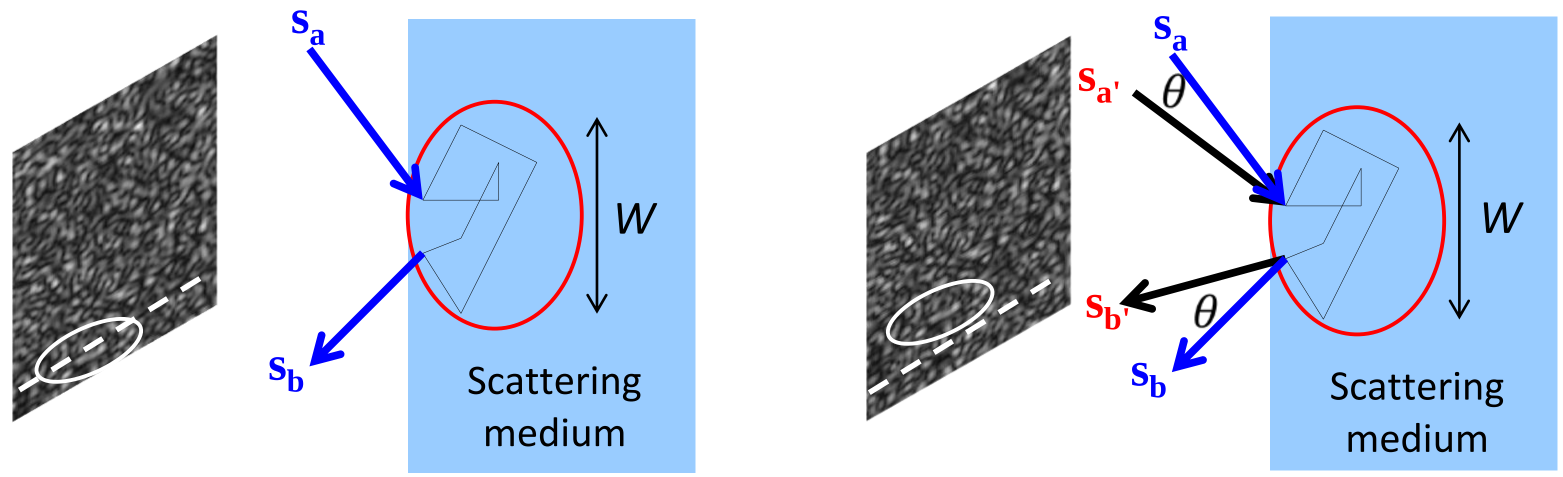




Random feature in the multiple scattering regime

Optical memory effect in backscattering

Speckle is not random at it might seems



Small rotation of the incident field → **Small rotation of the speckle**

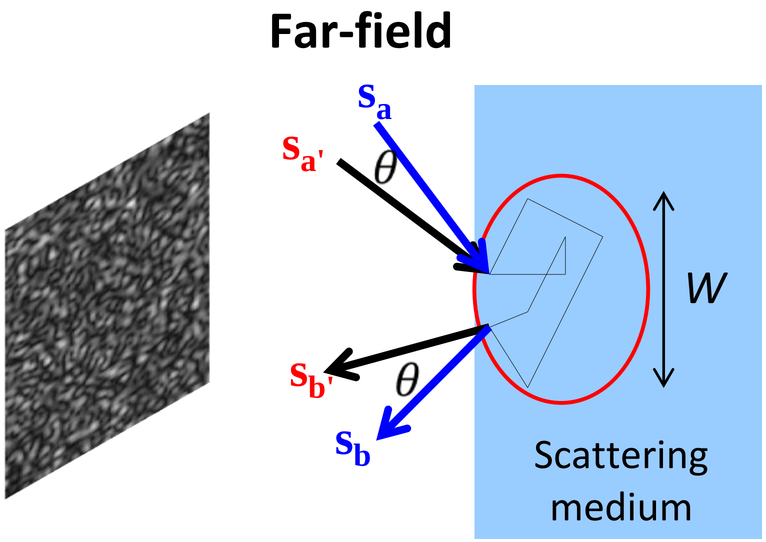
Despite disorder, it remains an information on the nature of the incident beam.

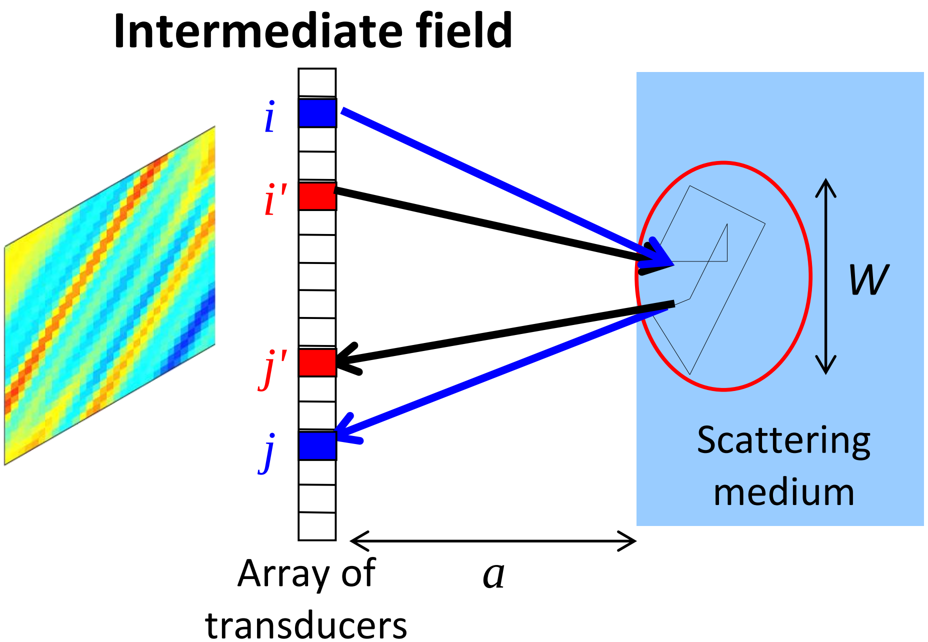
Single scattering: the memory effect persists **over the whole angular domain**

Multiple scattering: The memory effect is restricted to a **small angular domain** $\Delta\theta \sim \lambda / W$

Freund et al., Phys. Rev. Lett., 1988 – Feng *et al.*, Phys. Rev. Lett., 1988

Memory effect in backscattering





Memory effect condition: $\Delta s_a = \Delta s_b$ → $i+j=i'+j'$ (Antidiagonals of **K**)

Single scattering: Deterministic coherence whatever the distance between *i* and *j*

Multiple scattering: Short-range correlations governed by the size of the diffusive halo

Coherence length $l_c \sim \lambda a / W$

Memory effect → Spatial coherence along the antidiagonals of **K**

Target detection in multiple scattering media

Alexandre Aubry, Arnaud Derode (IL)

Sharfine Shahajahan, Fabienne Rupin (EDF R & D)

A. Aubry and A. Derode, *Phys. Rev. Lett.* **102**, 084301, 2009

A. Aubry and A. Derode, *J. Appl. Phys.* **106**, 044903, 2009

S. Shahjahan, A. Aubry, F. Rupin, B. Chassignole, and A. Derode, Rev. Prog. Quant. Nondestruc. Eval. **32**, 2013

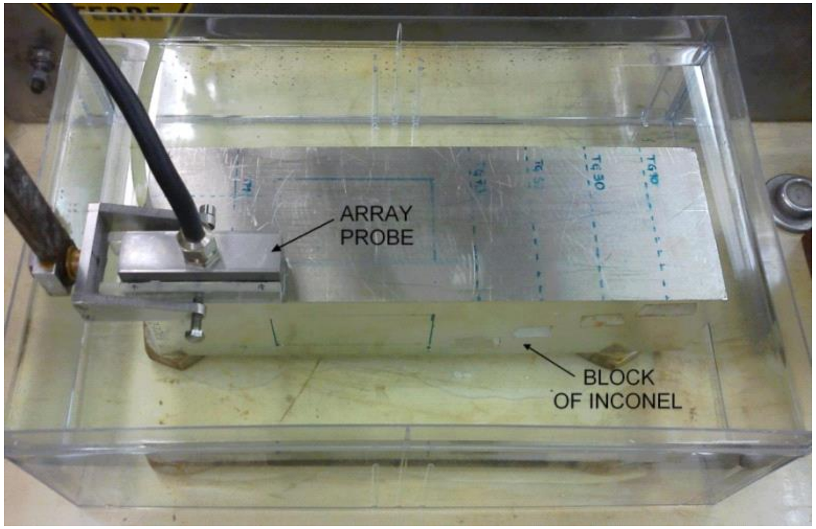
A *real* multiple scattering medium

Material : Inconel600®

Nickel-based coarse-grain alloy

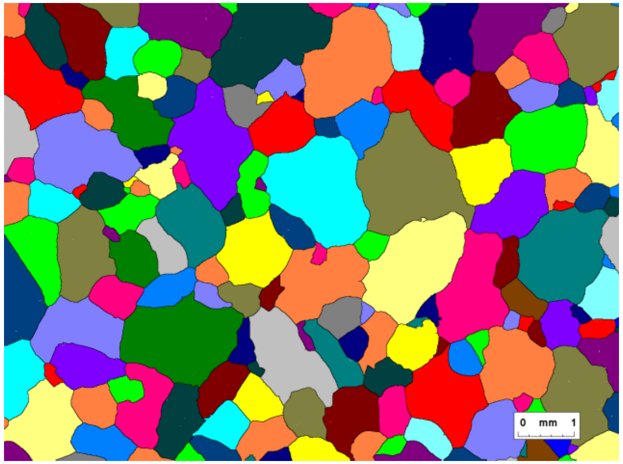
High resistance to corrosion and heat

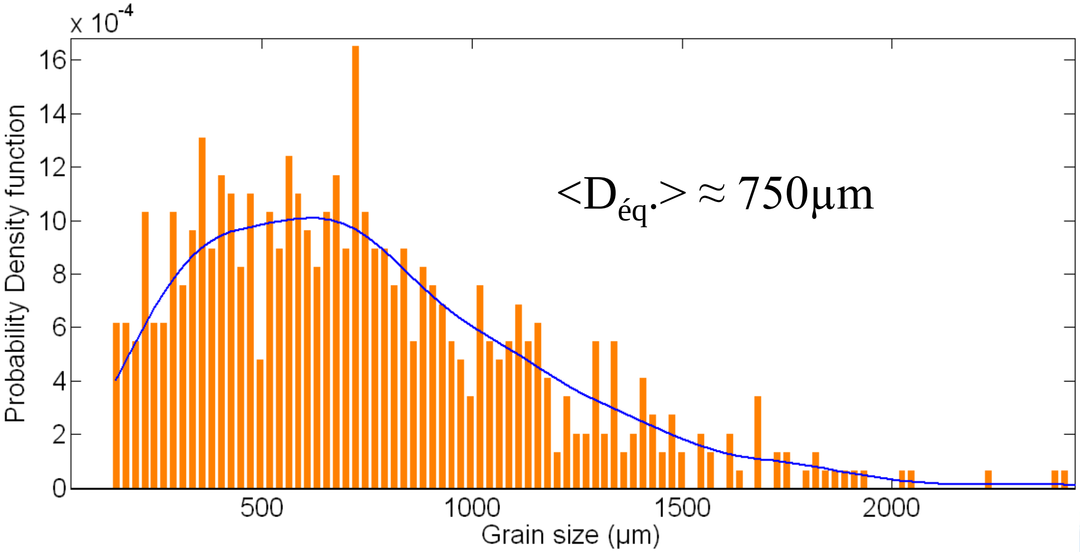
Nuclear and aeronautics industries



Array of 64 transducers, central frequency 3 MHz, bandwidth 50%

Polycristalline medium with statistically isotropic grains

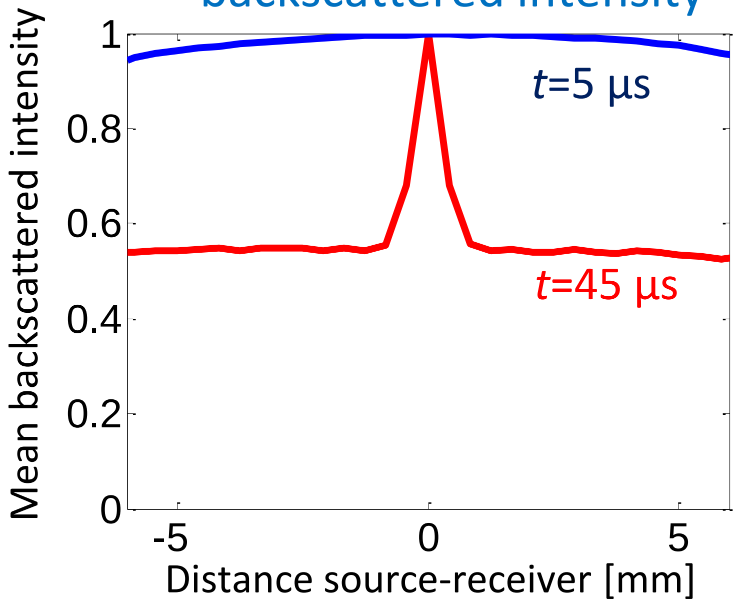




Multiple scattering / Coherent backscattering

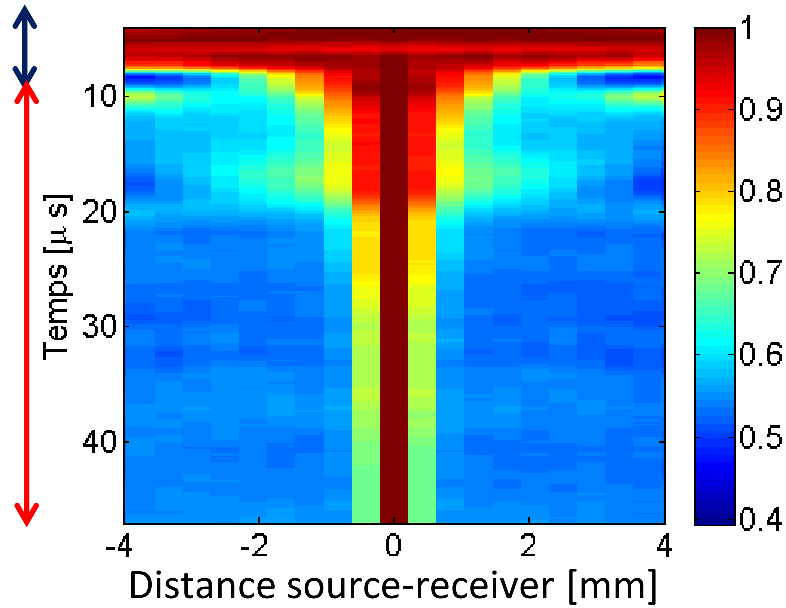
Space-time evolution of the mean backscattered intensity

Wolf and Maret, *Phys. Rev. Lett.*, 1985
van Albada and Lagendijk, Phys. Rev. Lett., 1985



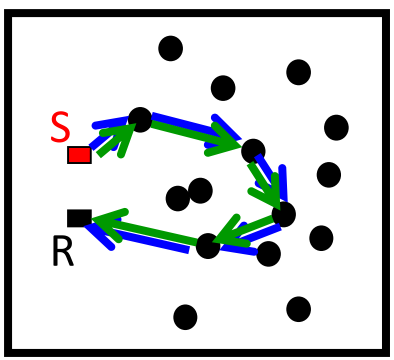
Single scattering (Flat plateau)

Multiple scattering: coherent backscattering peak on top of a flat plateau

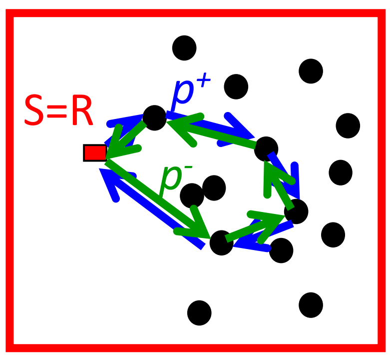


Multiple scattering becomes predominant around *t*=10 μs

Scattering mean free path $l_e \sim 25$ mm



Incoherent contribution



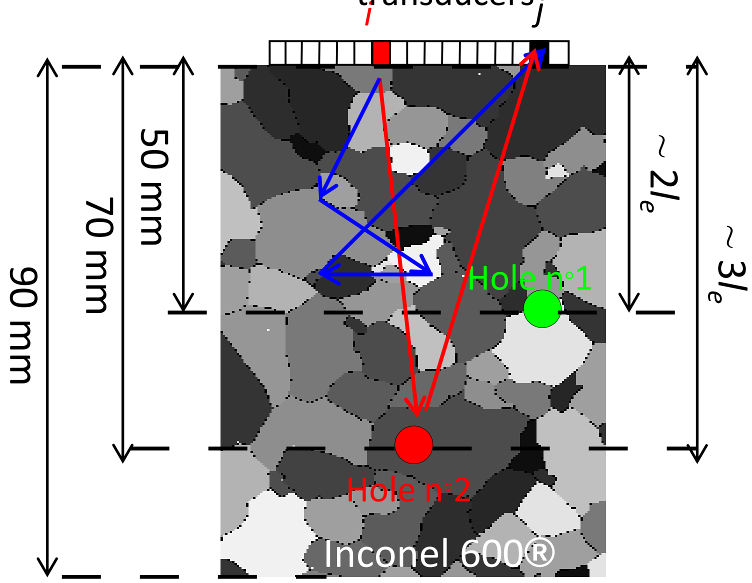
Coherent backscattering peak

Classical ultrasound imaging

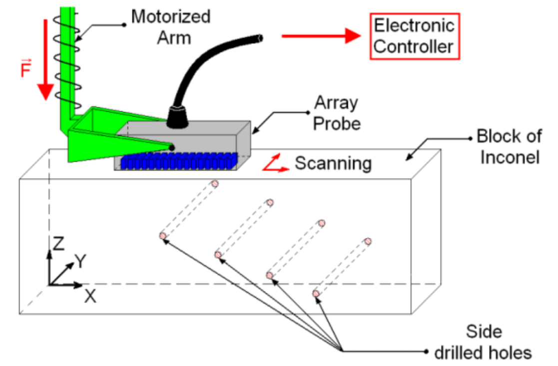
Experimental configuration

Central frequency 3 MHz, bandwidth 50%

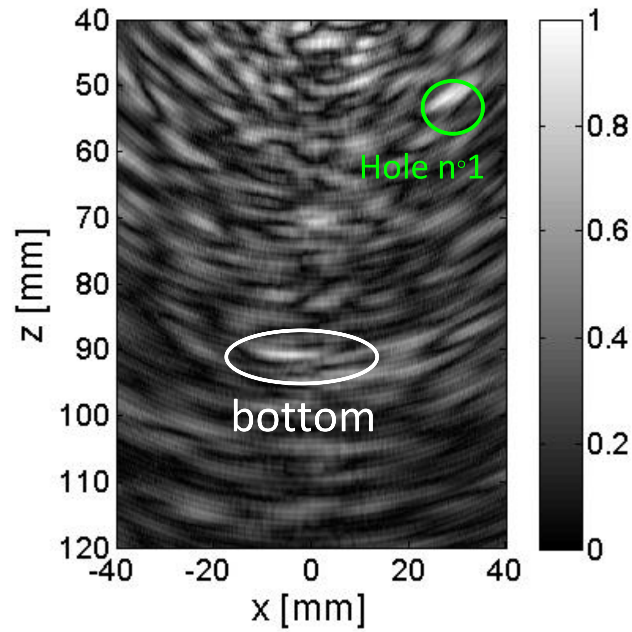
64-element array of transducers



φ_{hole}=2 mm



Classical ultrasound imaging



Detection of hole n° 1 placed at Z=50 mm

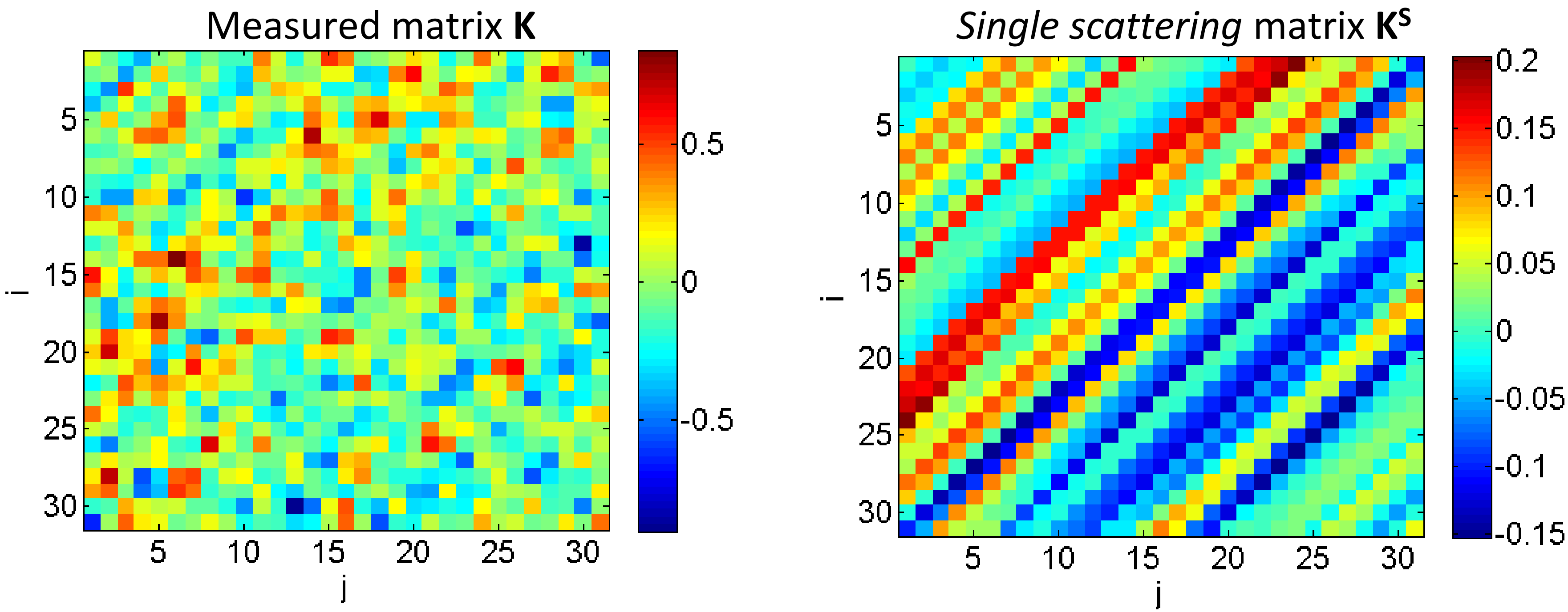
No detection of the hole n° 2 (The echo has to go through $6l_e$)

Image of speckle due to multiple scattering → False alarms

Building of a smart radar/sonar which separates single scattered echoes from the multiple scattering background

Smart radar filtering multiple scattering

t=23.4 µs (expected time-of-flight for the hole) - f = 2.7 MHz

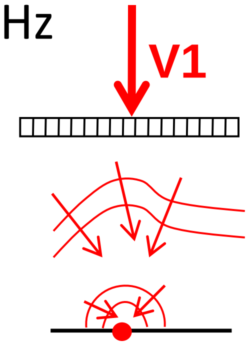


Need to combine the smart radar with an imaging technique

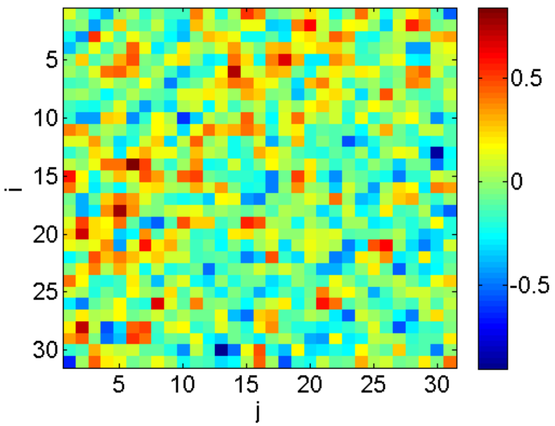
→ **D.O.R.T method** C. Prada and M. Fink, *Wave Motion* **20**, 151-163, 1994

Smart radar filtering multiple scattering

t=23.4 µs (expected time-of-flight for the hole) - f = 2.7 MHz



Measured matrix **K**



DORT method applied to **K**
1^{st} eigenspace

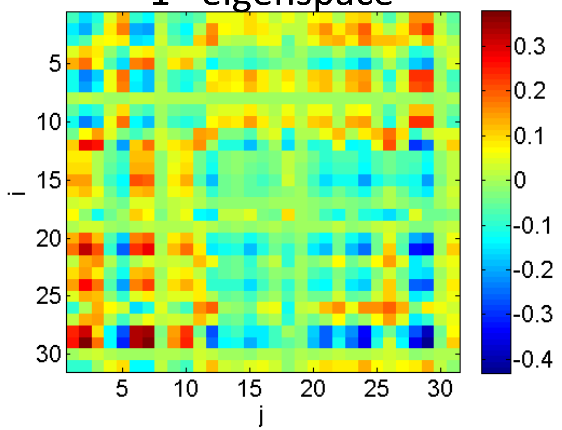
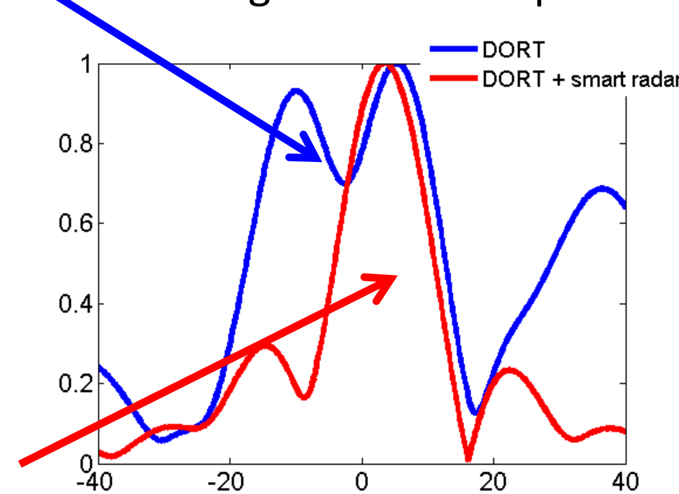
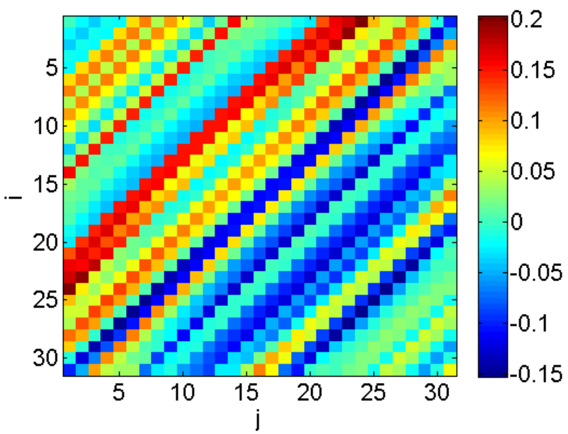


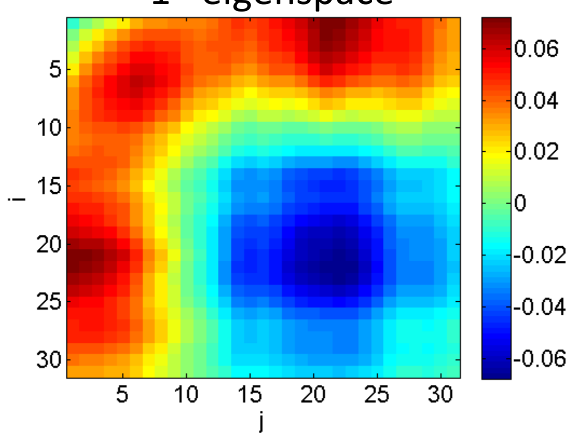
Image in the focal plane



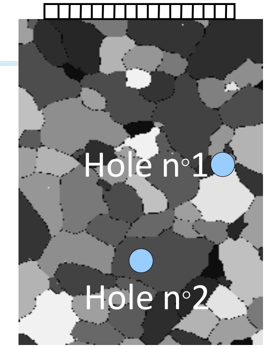
Single scattering matrix $\mathbf{K^S}$



DORT method applied to $\mathbf{K^S}$
1^{st} eigenspace



Smart radar filtering multiple scattering



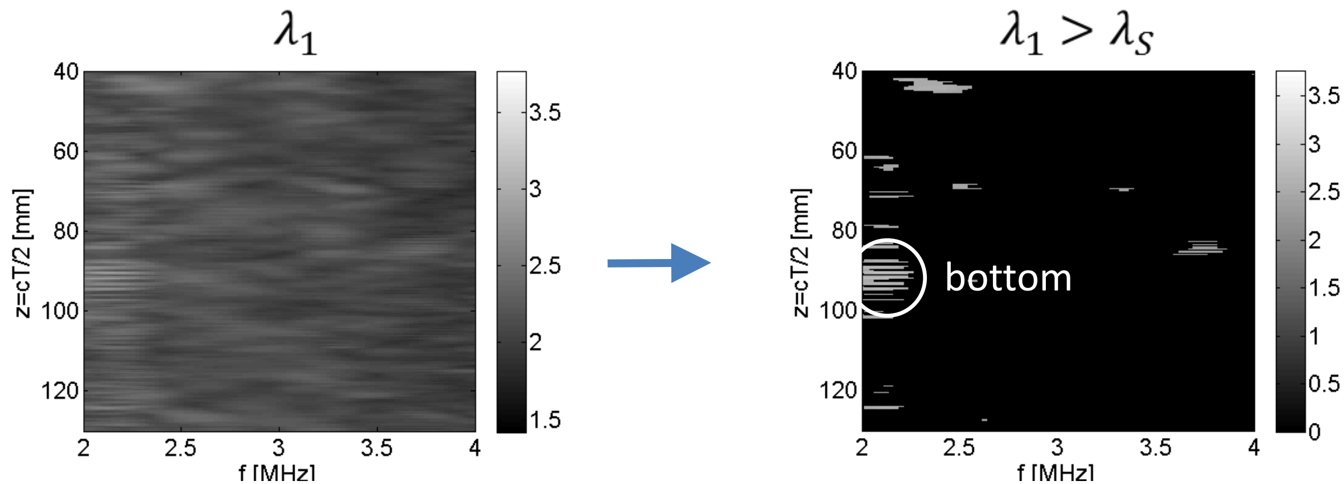
Systematic detection based on random matrix theory (RMT)

if $\lambda_1 > \lambda_S$, a target is detected

if $\lambda_1 < \lambda_S$, we cannot conclude about the presence of a target

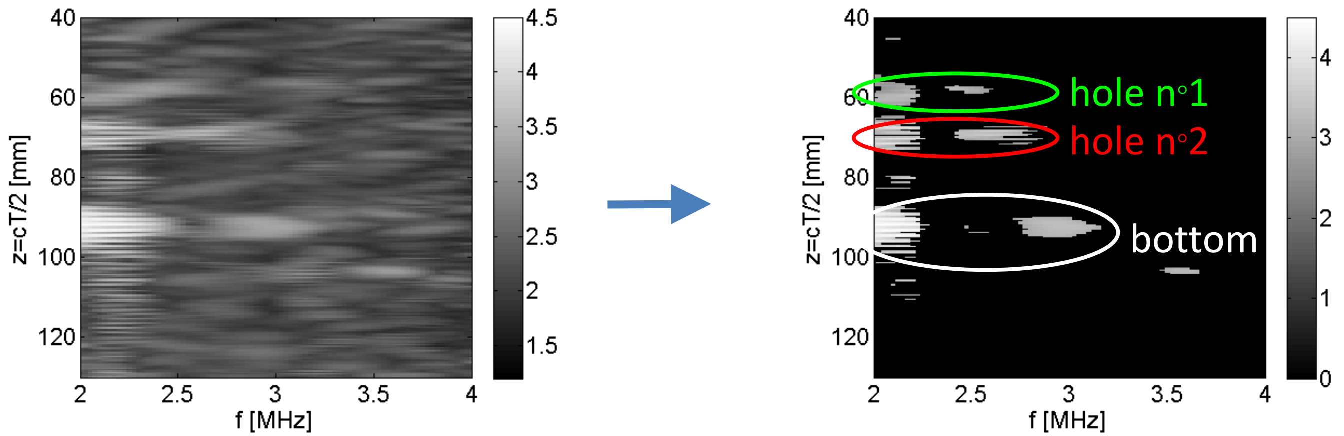
Measured matrix **K**

PFA=1%

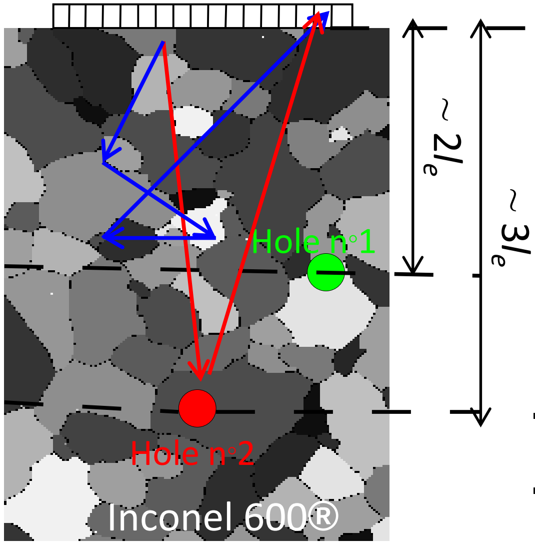


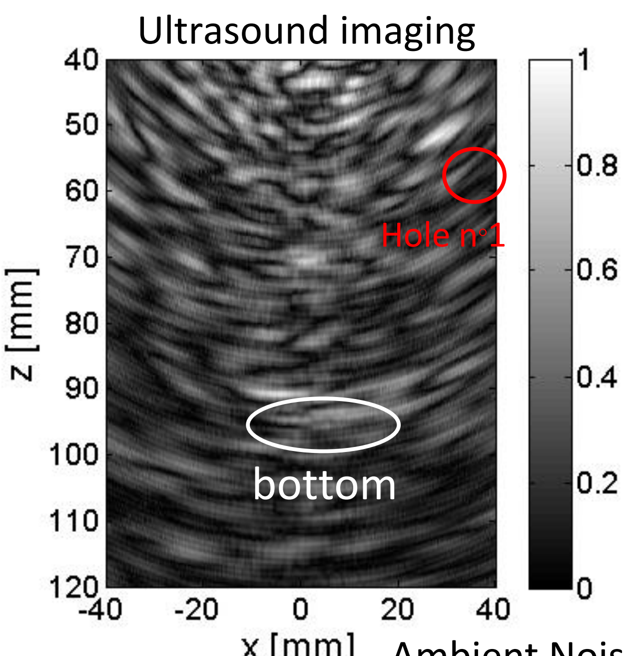
Single scattering matrix **K^S**

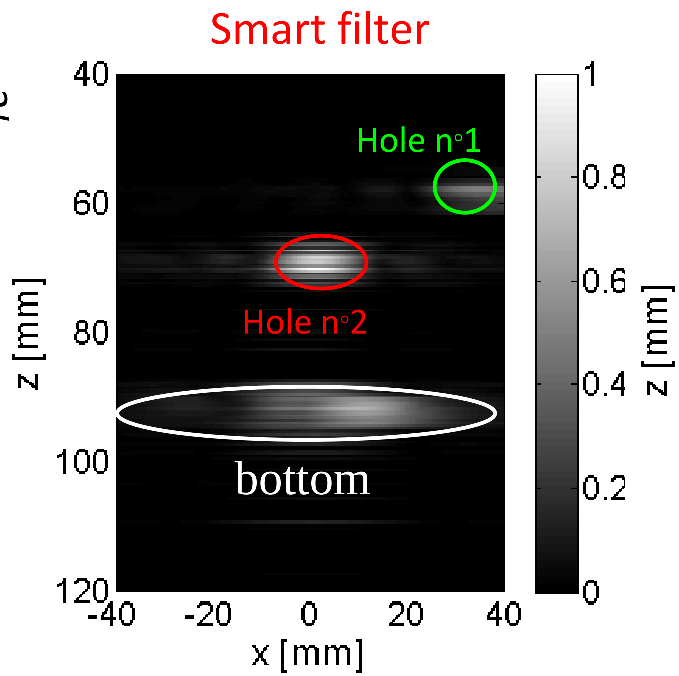
PFA=1%

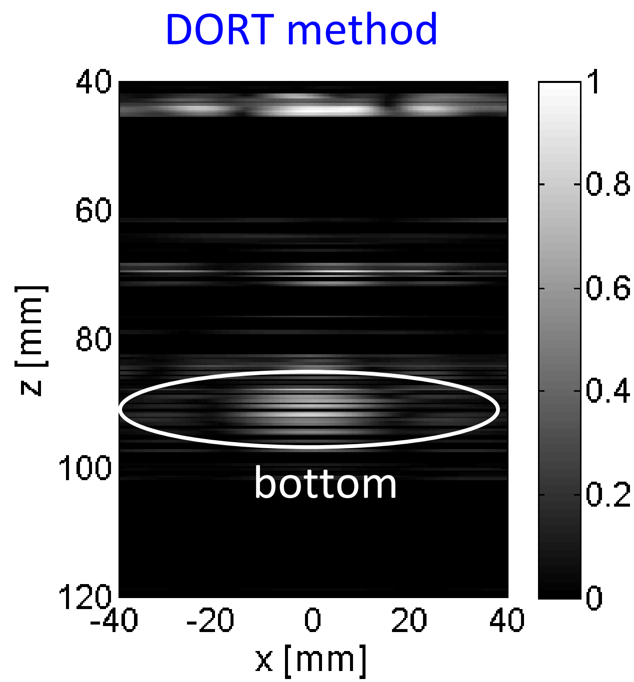


Smart radar filtering multiple scattering







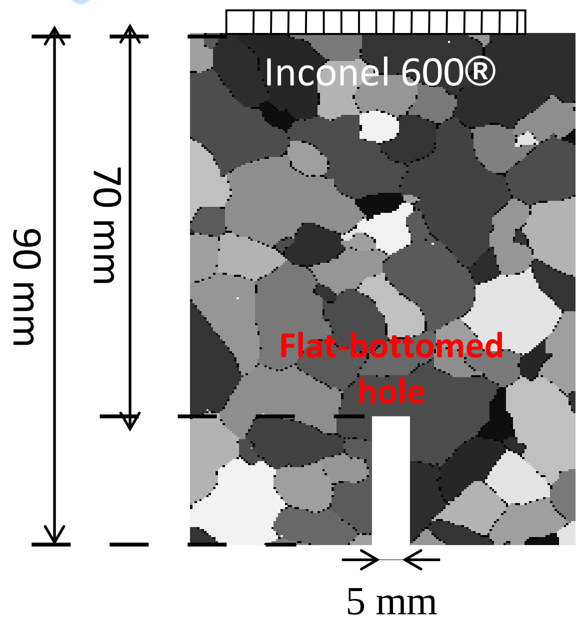


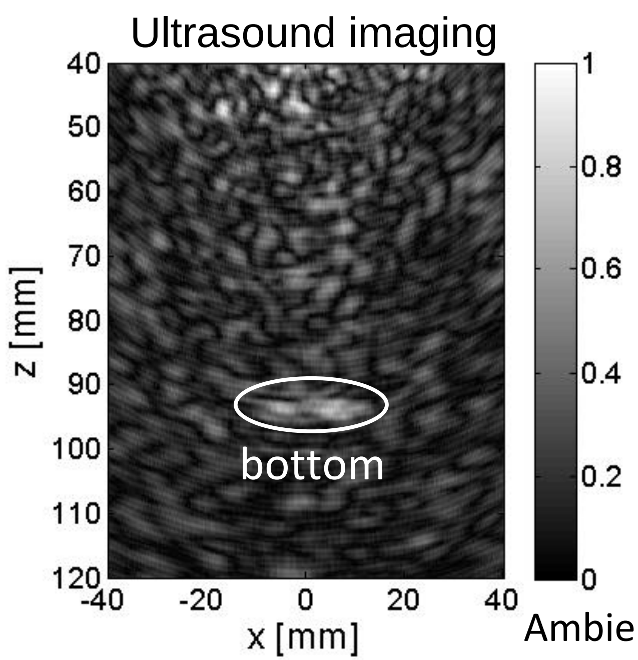
Our approach allows to:

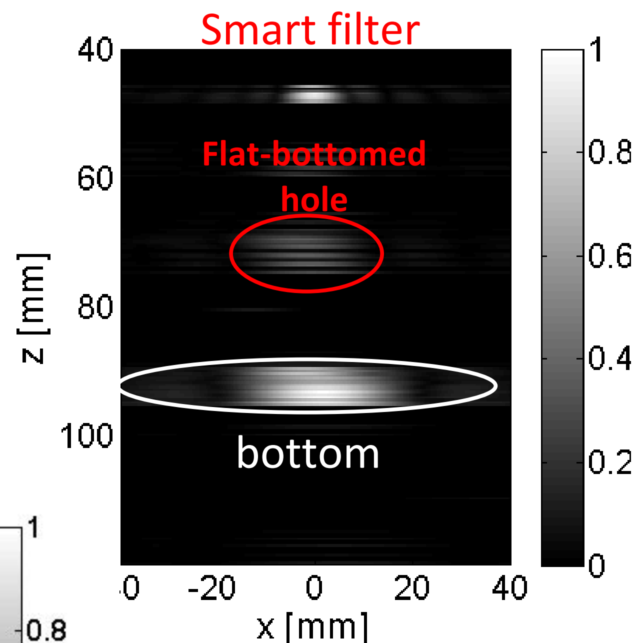
Strongly diminish the multiple scattering noise

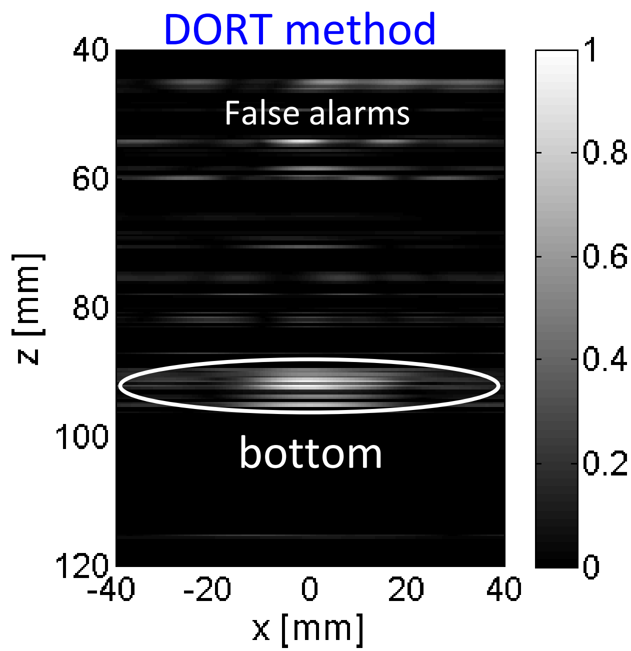
Smooth aberration effects

Smart radar filtering multiple scattering









A perfect image of the hole is obtained as if the heterogeneities of the alloy had disappeared

Recurrent scattering in strongly scattering media

Alexandre Aubry, Arnaud Derode (IL)
John Page, Laura Cobus (University of Manitoba)
Sergey Skipetrov, Bart van Tiggelen (LPMMC, Grenoble)

Anderson localization of elastic waves

H. Hu, A. Strybulevych, J.H. Page, S.E. Skipetrov, and B. Van Tiggelen, Nature Physics 4, 945, 2008

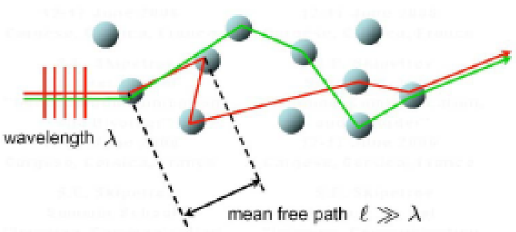
Mesoglasses fabricated by brazing aluminum beads together to form a solid porous 3D elastic network.

Pulsed transmission measurements:

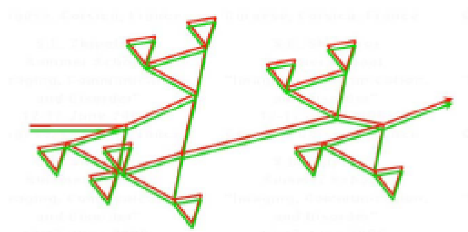
→ Localization between 1.2 and 1.25 MHz

ϕ=4 mm, l_e=1.1 mm,
$v_p = 2.5\ \mathrm{mm/\mu s}$, $kl_e \sim 3$

Weak disorder ($k\ell$ >> 1):
Diffuse propagation
$D_B = ⅓\, v_E \ell_B^*$ (neglect interference)



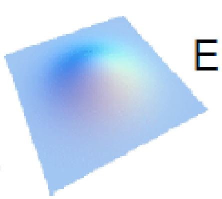
Strong disorder ($k\ell$ ~ 1):
Anderson localization
(interference is important!)



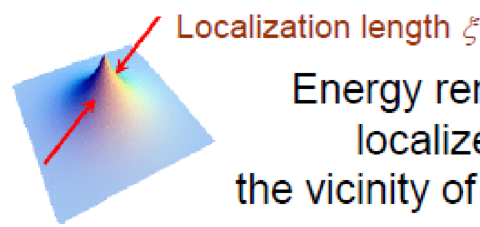
P.W. Anderson, *Phys. Rev.*, 1959

scattering loops → constructive interference

e.g., After a short pulse of ultrasound is incident on the medium…



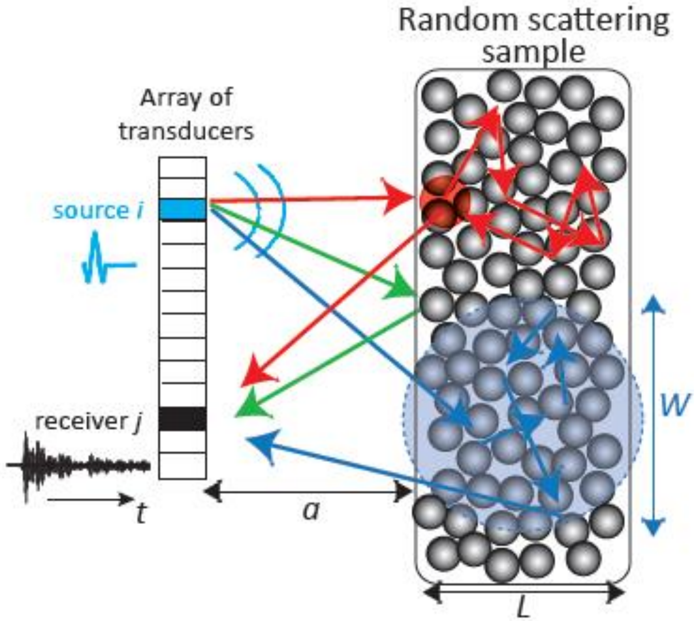
Energy density spreads diffusively from the source



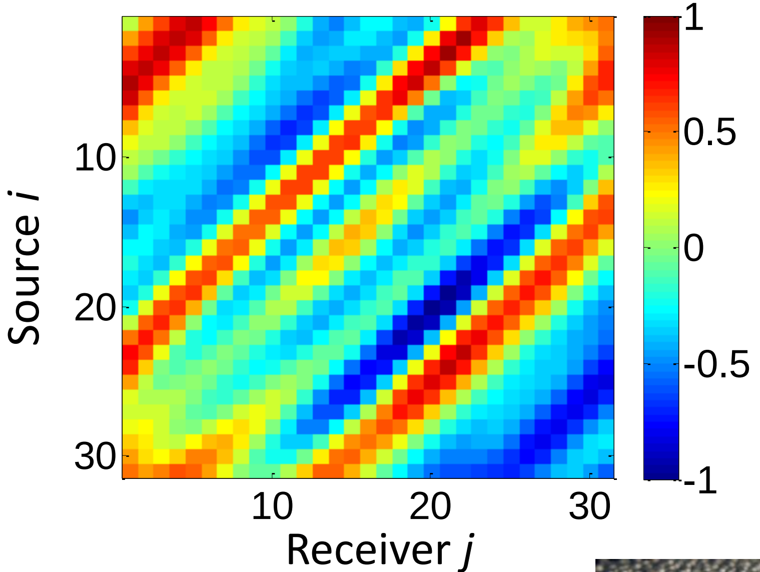
Energy remains localized the vicinity of the source

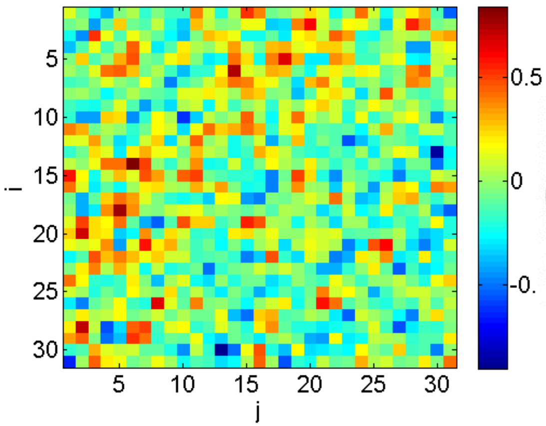
Figure courtesy of John Page

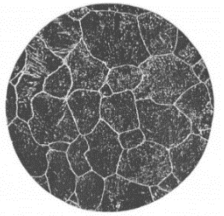
Statistical properties of **K** / strong disorder



Real part of **K** – *t*=185 µs, *f*=1.25 MHz





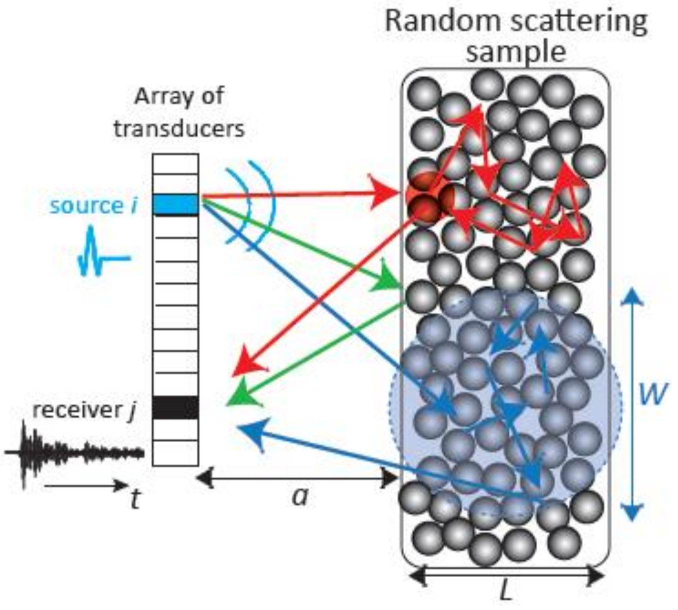


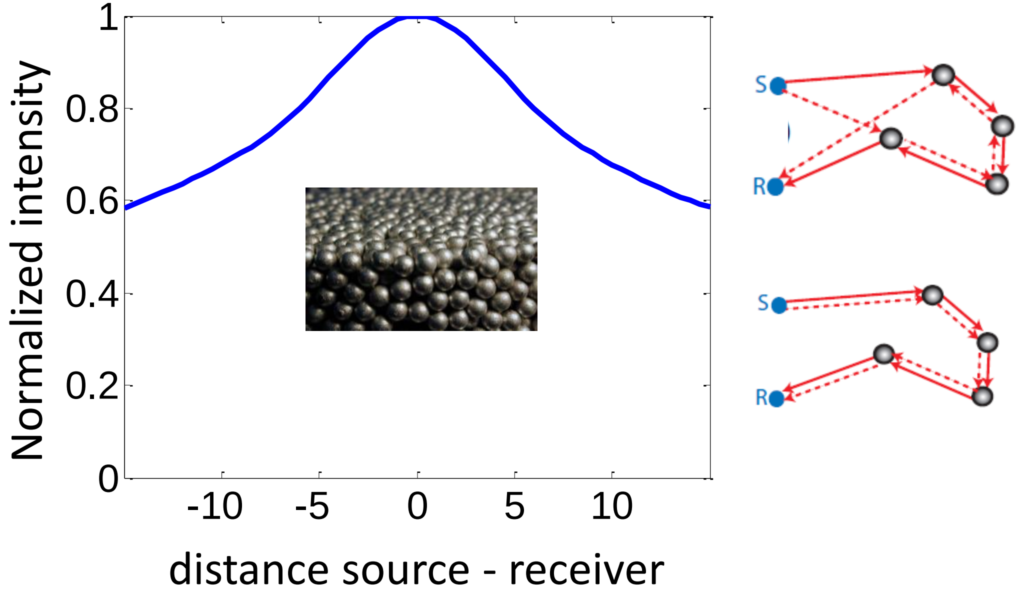
Long-range correlations even at **long times** of flight in the strongly scattering regime

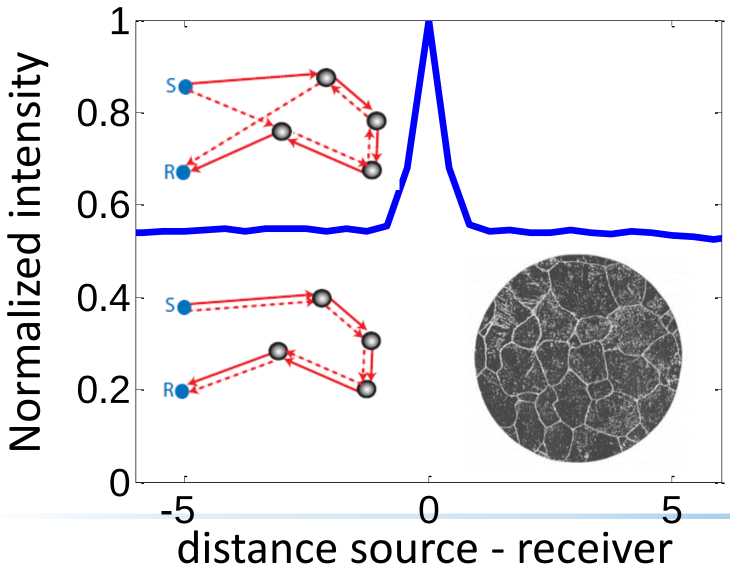
↓

Recurrent scattering

Spatial intensity profile



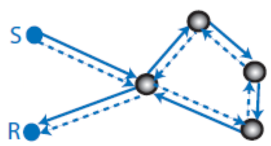




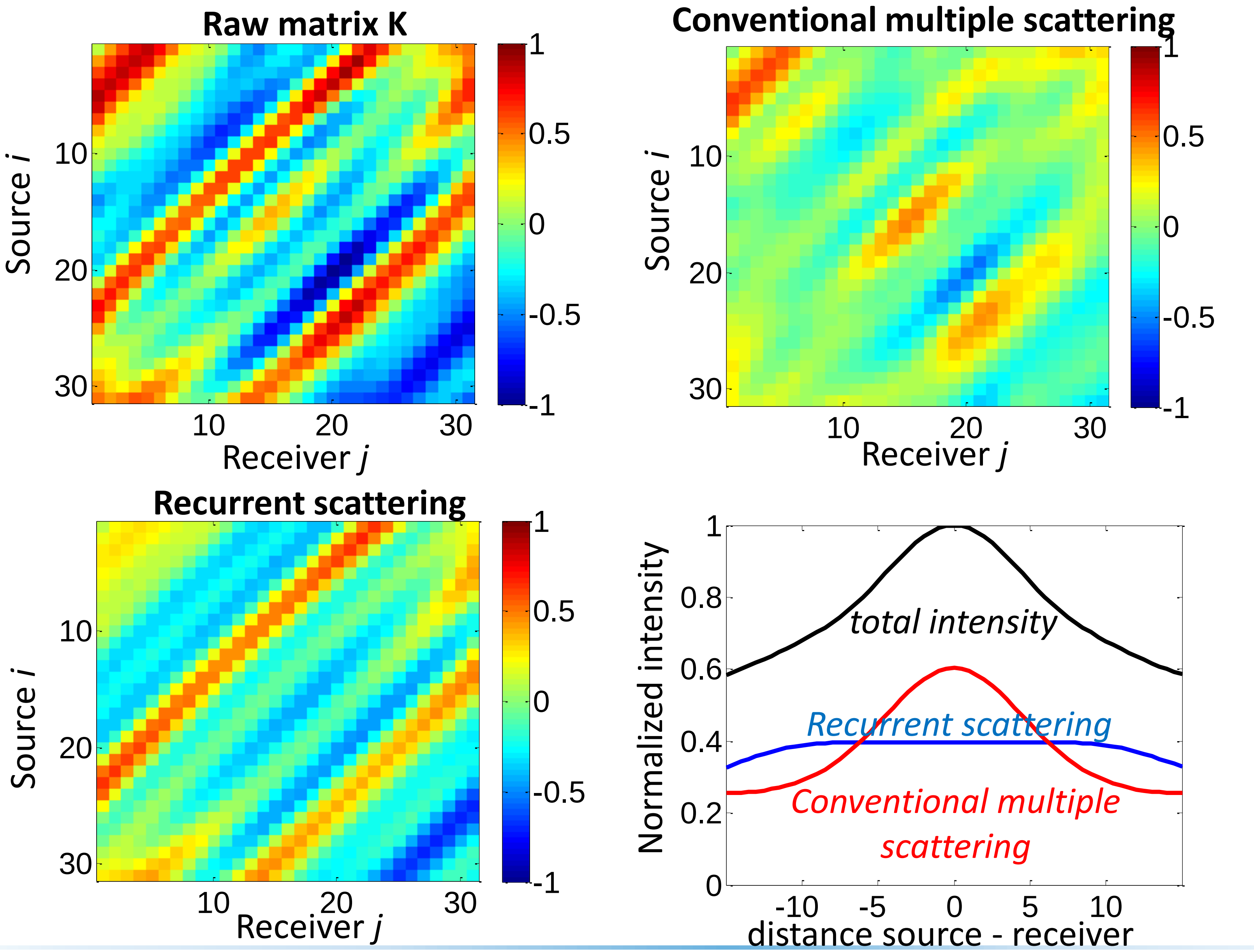
Coherent backscattering enhancement is below 2 even at long time of flight

↓

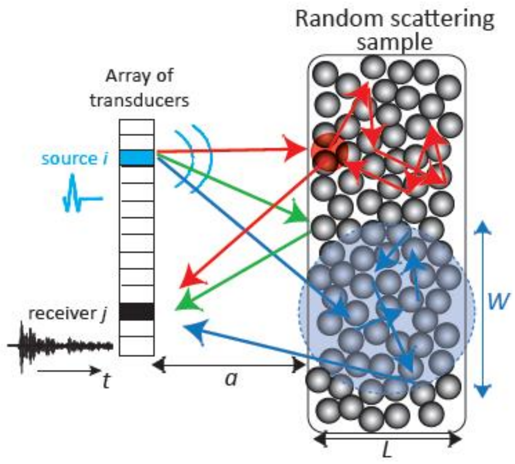
Recurrent scattering



Separation recurrent scattering / conventional multiple scattering



Conventional multiple scattering / Coherent backscattering



Diffusive regime

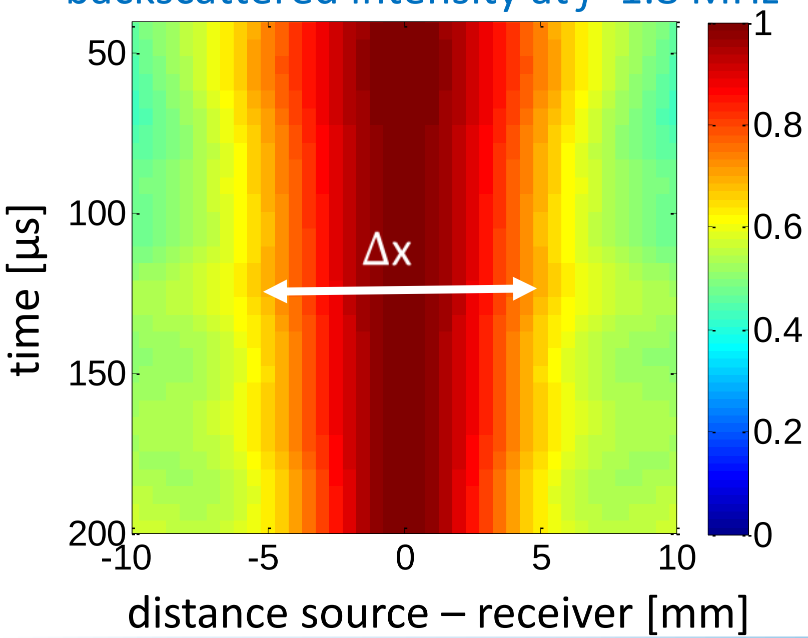
The growth of the diffusive halo scales as $W \sim \sqrt{Dt}$

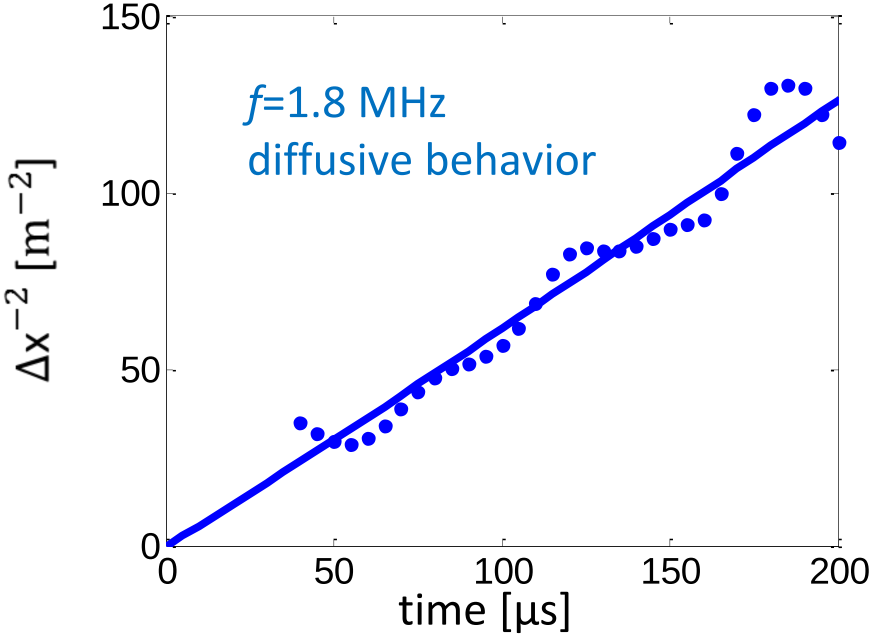
$$\Delta x \sim \lambda a / W$$

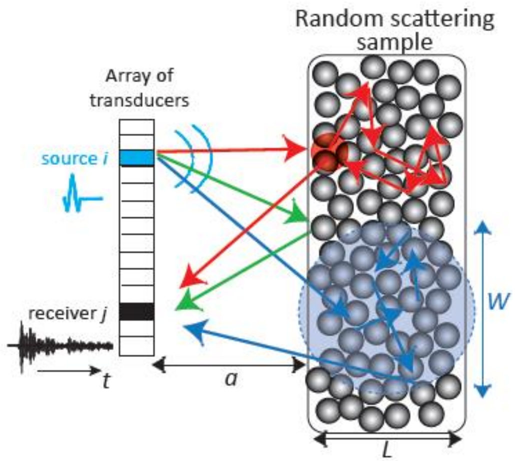
The CB peak width scales as

$$\Delta x^{-2} \propto W \sim Dt$$

Space-time evolution of the mean backscattered intensity at *f*=1.8 MHz







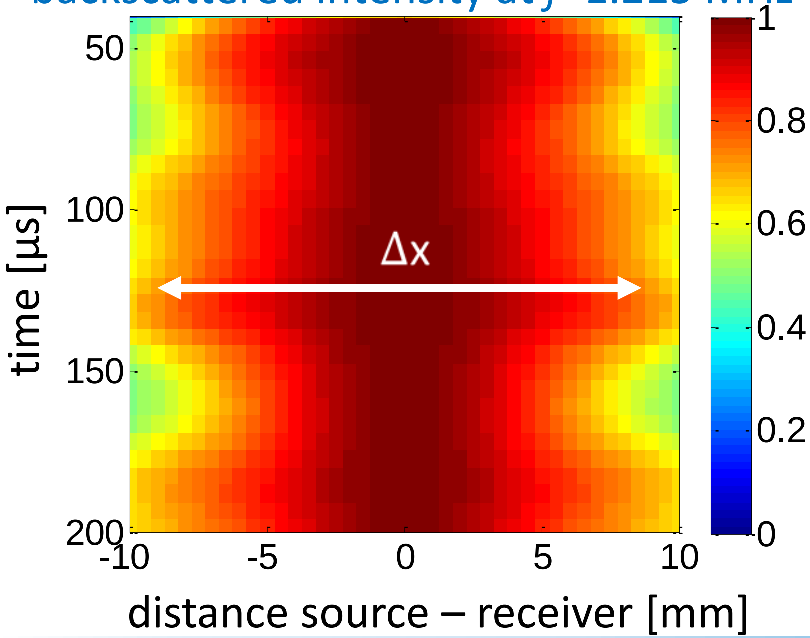
Manifestation of Localization

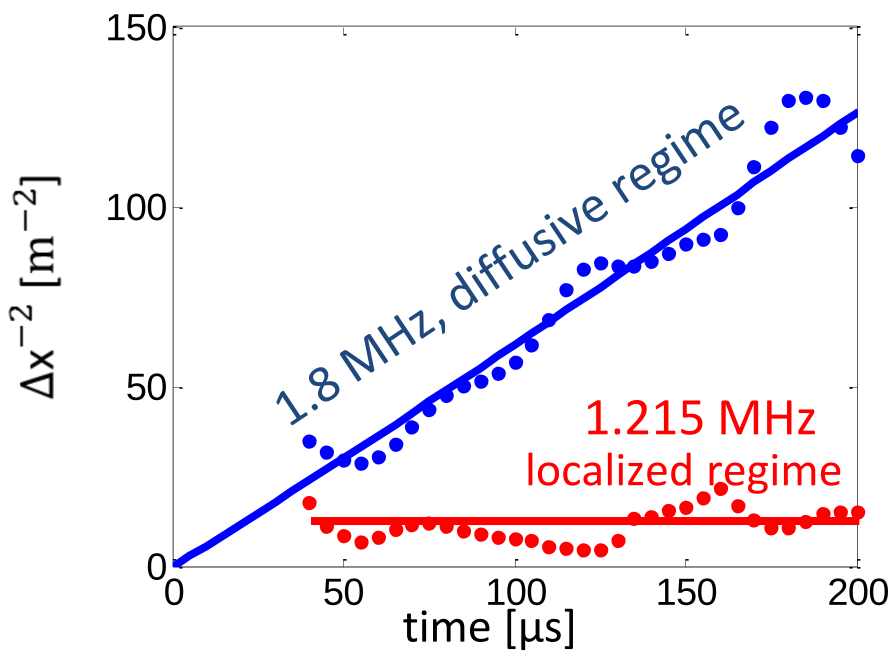
Saturation of the growth of the diffusive halo

$\Delta x \sim \lambda a / W$

The CB peak narrowing saturates

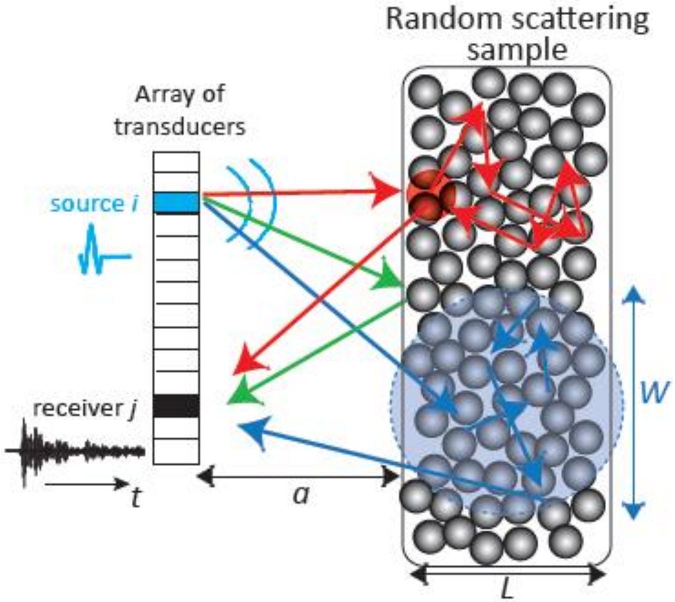
Space-time evolution of the mean backscattered intensity at *f*=1.215 MHz



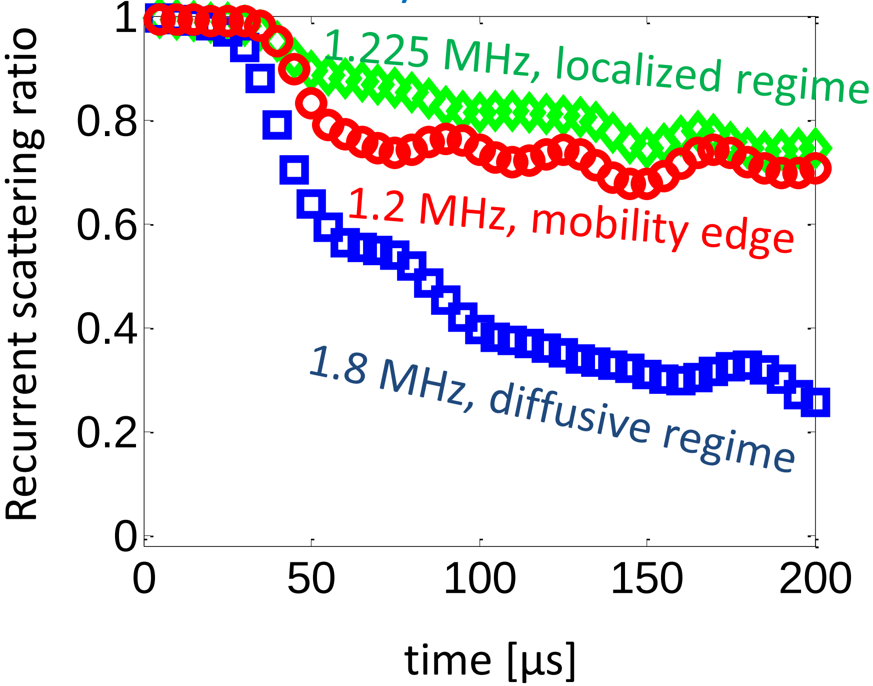


Recurrent scattering ratio

Recurrent scattering intensity ratio = $\frac{\text{recurrent scattering intensity}}{\text{total backscattered intensity}}$



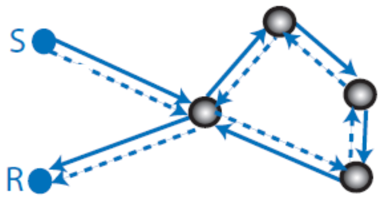
Recurrent scattering intensity ratio VS time



The **recurrent scattering** contribution is **substantial** even at **long times of flight** in the **localization band** (>70% at t=200 µs)

Time decay of the return probability

Recurrent scattering intensity



Return probability = Probability for a wave to come back at its starting spot

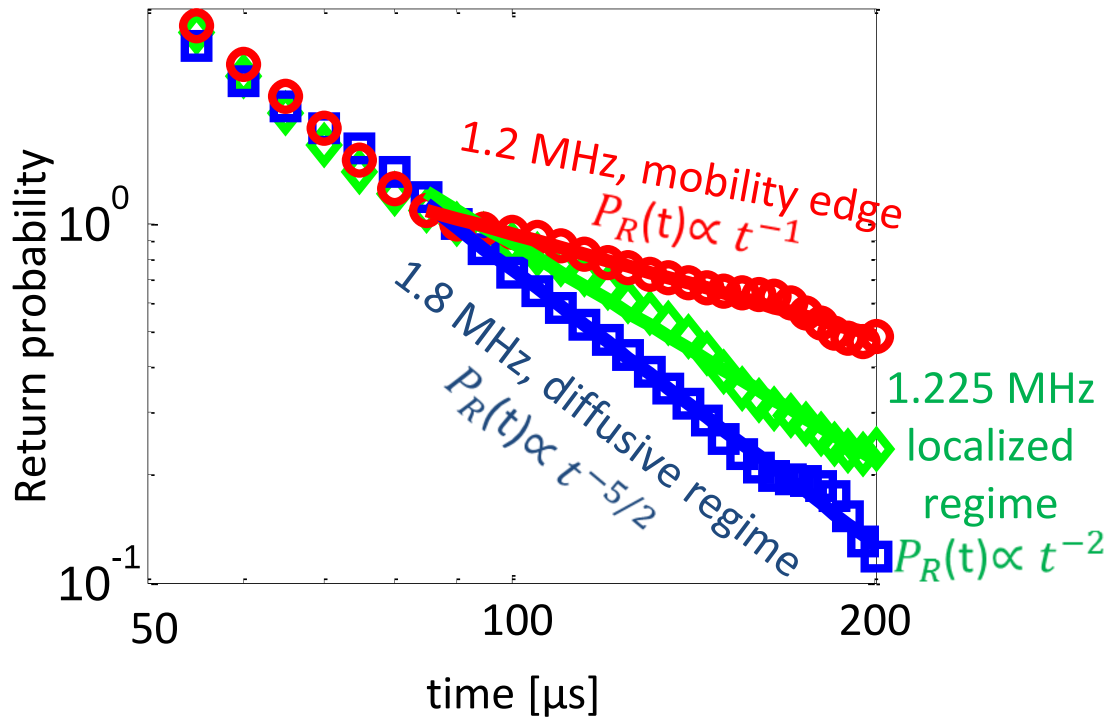
↓

Key quantity in self-consistent theory of Anderson localization (renormalization of the diffusion constant)

↓

Diffusive regime: $P_R(t) \propto t^{-5/2}$

Localized regime: $P_R(t) \propto t^{-2}$

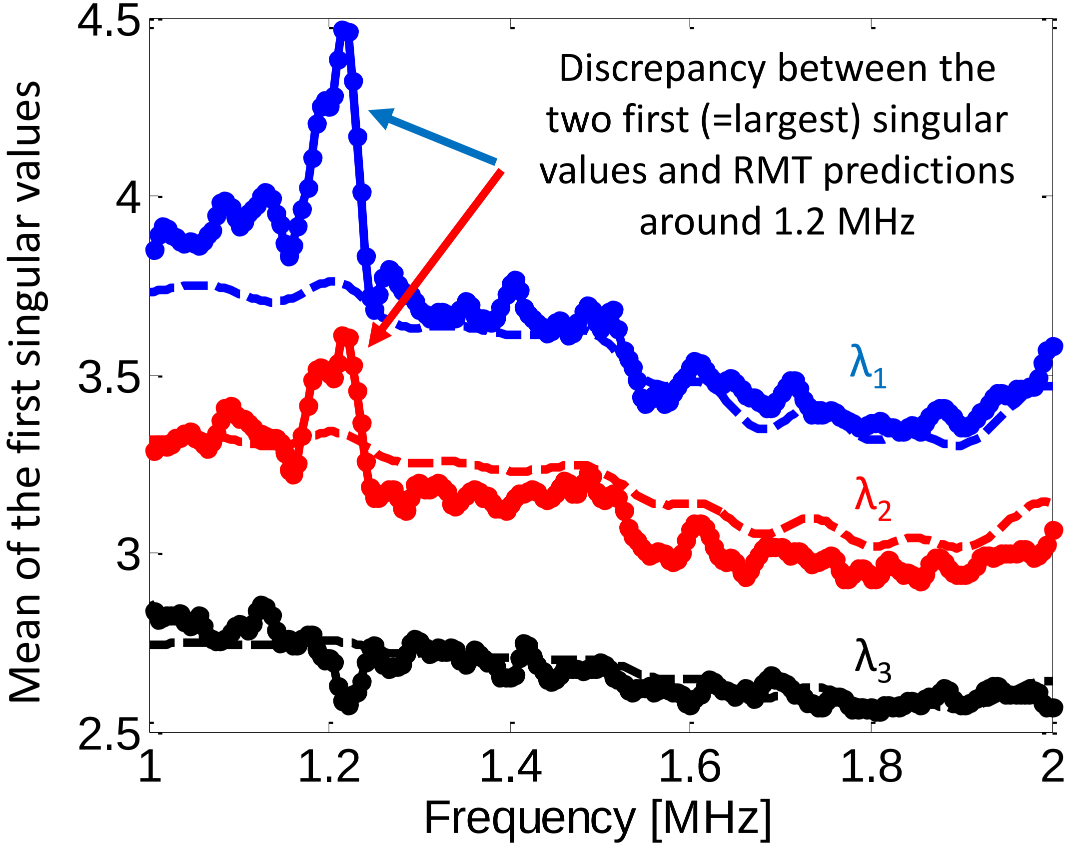


The measured **return probability** displays **a very slow decay** around the **mobility edge** (unpredicted by theory)

S.E. Skipetrov and B.A. van Tiggelen, *Phys. Rev. Lett.* , 2006

Intense recurrent scattering paths in the localized band

DORT analysis of the array response matrix **K** (t=150 μs)



Singular value decomposition of **K**

$$\mathbf{K} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{V}^{\dagger}$$

dots: experiment

dashed line:
random matrix theory (Hankel)

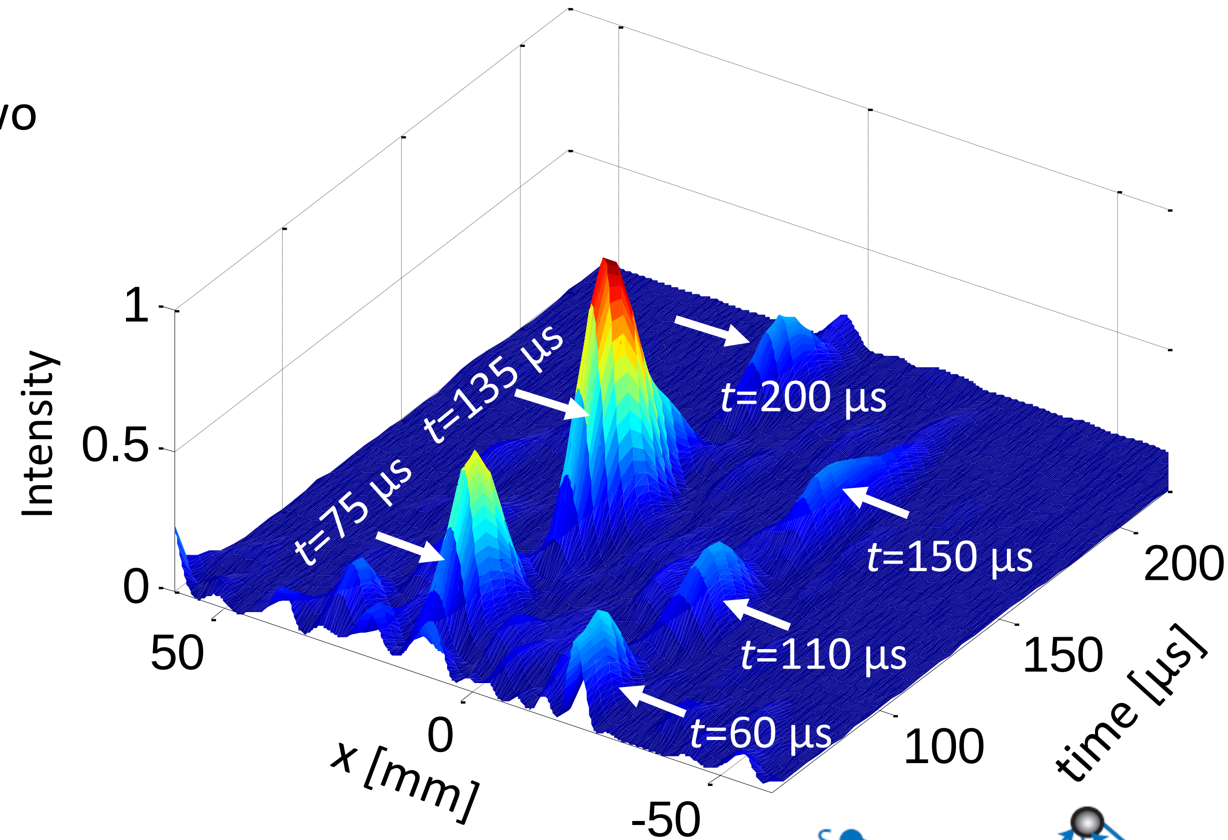
The **largest singular values** may be associated to **intense recurrent scattering paths**

Intense recurrent scattering paths in the localized band

Weakly scattering media: one eigenstate ⟷ one scatterer

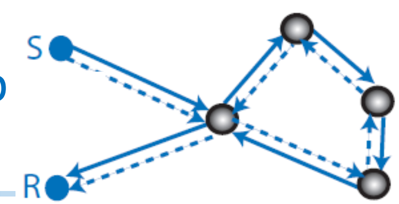
Strongly scattering media: one eigenstate ⟷ one recurrent scattering path

Back-propagation of the two first eigenvectors at the surface of the scattering sample (*f*=1.2 MHz)



Hot spot = entry-exit point of a recurrent scattering path

Hot spots switch on at regular intervals of time → The same scattering loop travelled several times



Conclusion & Perspectives

Propagation operator in random media

Statistical behaviour of the array response matrix (Random Matrix Theory)

Deterministic coherence of the single scattering contribution → Memory effect

Perspectives: Transmission (open/closed channels)

Separation single scattering / multiple scattering

Detection and imaging of flaws in coarse grain steels

Performance much better than classical imaging techniques

Perspectives: Extension to optics → S.M. Popoff *et al.*, Phys. Rev. Lett., 2011

Extension to seismology ?

Recurrent scattering in strongly scattering media

Recurrent scattering and memory effect

Manifestation of Anderson localization: Coherent backscattering, Return probability

Correspondence between eigenstates of **K** and scattering loops

Perspectives: Theoretical understanding, Random lasers

Thanks for your attention!

Alexandre Aubry

Institut Langevin – CNRS UMR 7587, ESPCI ParisTech – Paris, France

Collaborators:

Arnaud Derode (*Institut Langevin, Paris, France*)

Sharfine Shahajahan, Fabienne Rupin (*EDF R&D, France*)

John Page, Laura Cobus (*University of Manitoba, Winnipeg, Canada*)

Sergey Skipetrov, Bart van Tiggelen (*LPMMC, Grenoble, France*)