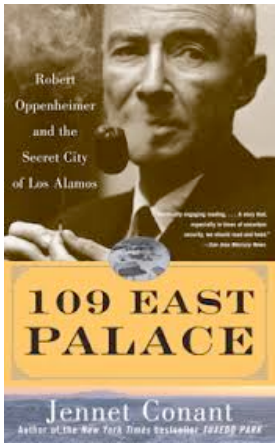
Nonlinear elasticity: new observations and implications

Paul Johnson

Los Alamos

Ambient Noise Imaging and Monitoring Workshop, Cargese 2013

oppenheimer



teller

fermi

feynman

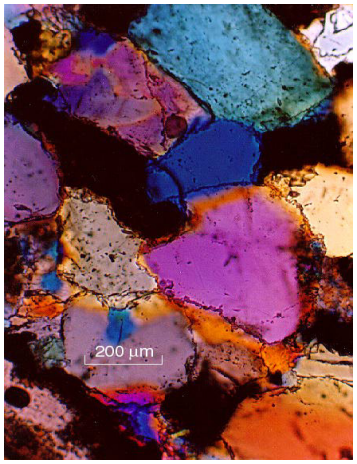
- All materials are elastically/acoustically nonlinear.

- Earth materials and damaged solids in general exhibit characteristic and strong elastic nonlinear behavior.

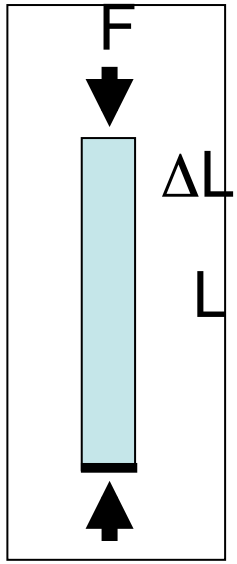
- One can ignore the nonlinearity, or you can use it to characterize the medium.

Outline

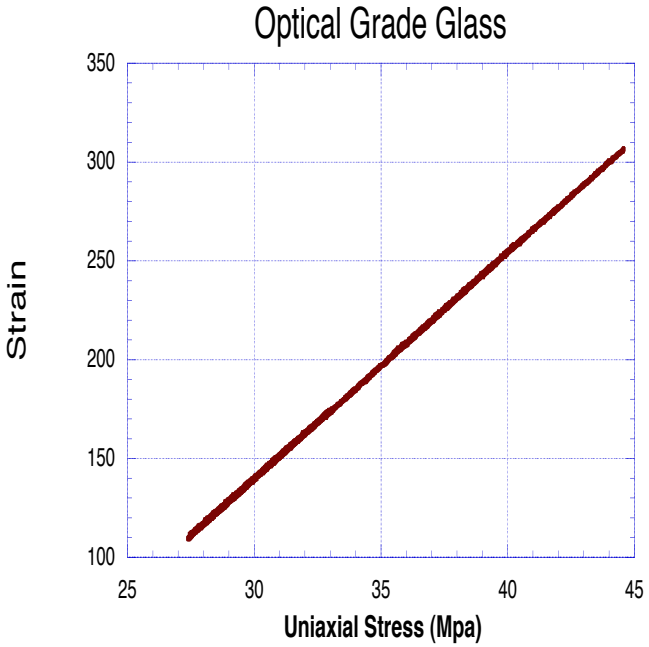
- quastistatic behavior
- dynamical behavior
- a phenomenological model
- physical origin
- a new model in development
- some applications related to passive noise imaging
- laboratory experiments of sheared granular media
    - acoustically triggered slip
    - precursors to slip
    - DEM simulation
- summary



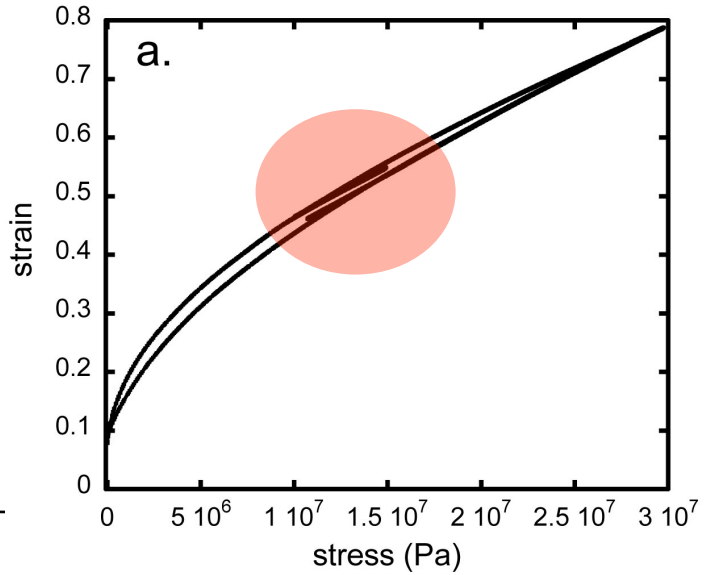
Stress-strain measurements in the lab

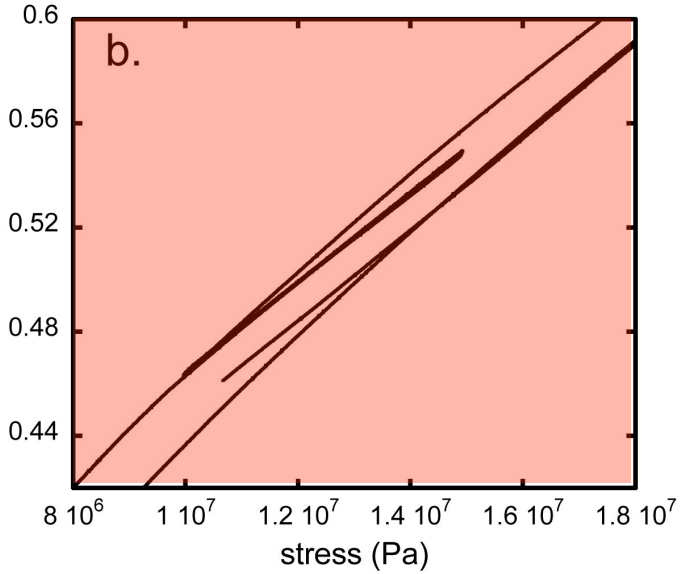


$\varepsilon = \Delta L / L$

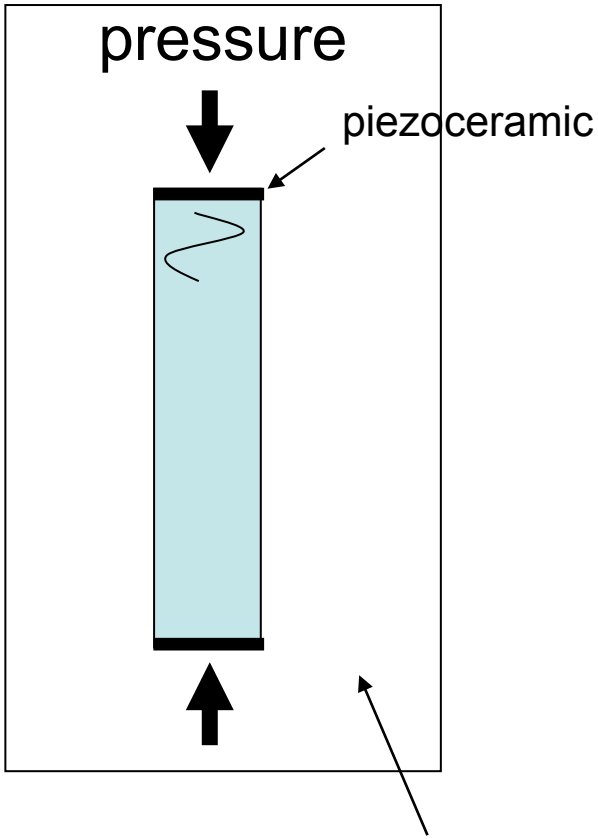


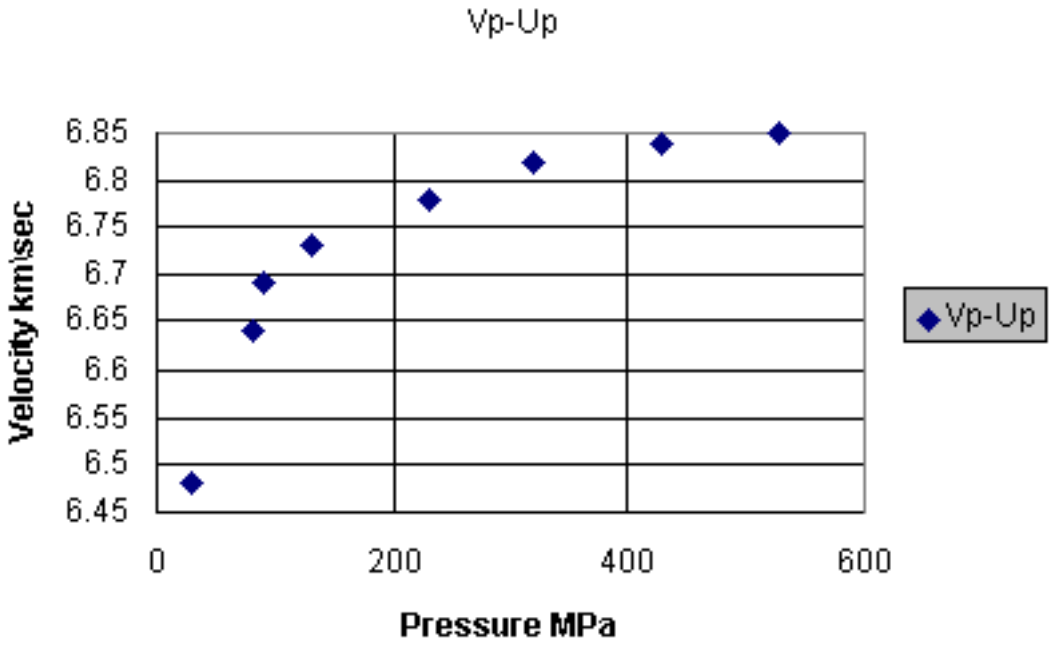
Berea sandstone



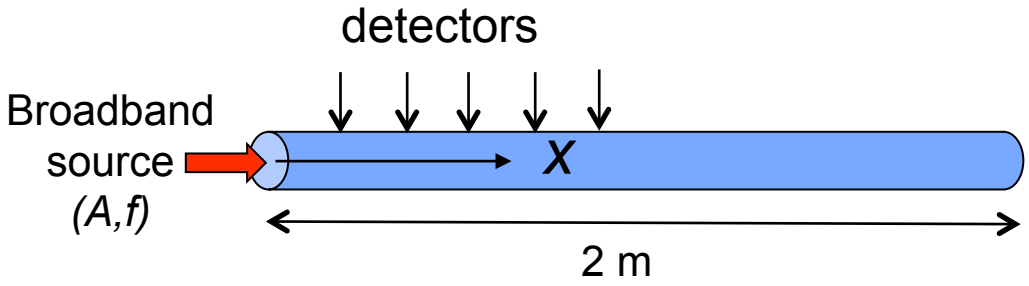


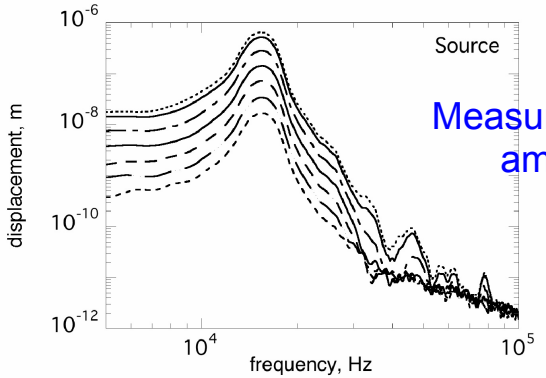
Acousto-elasticity in the lab



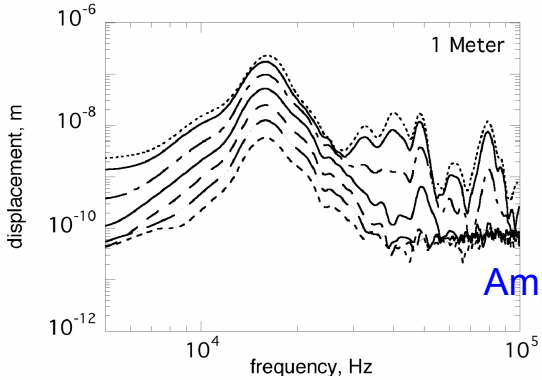


Nonlinear dynamical experiments: generation of new frequencies

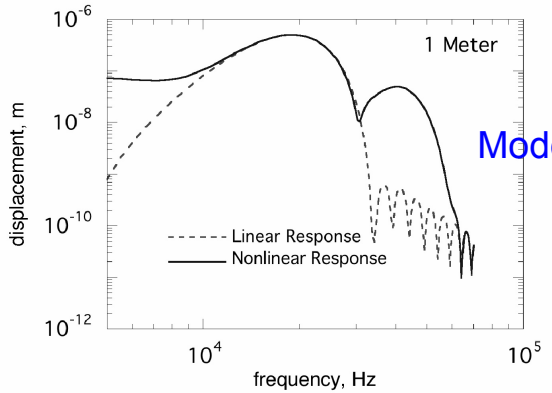




Measured source amplitude



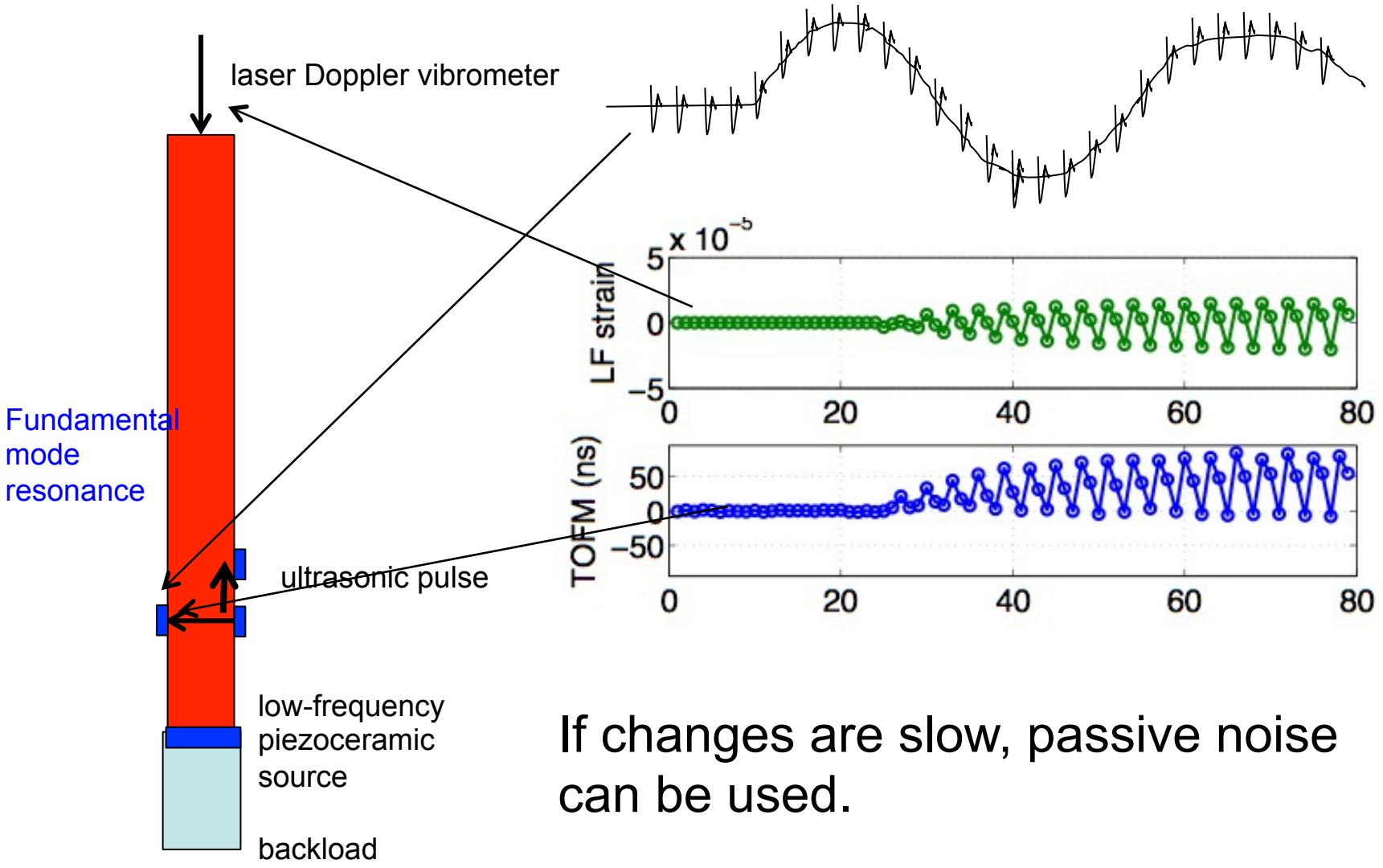
Amplitude at 1 m



Model amplitude at 1 m

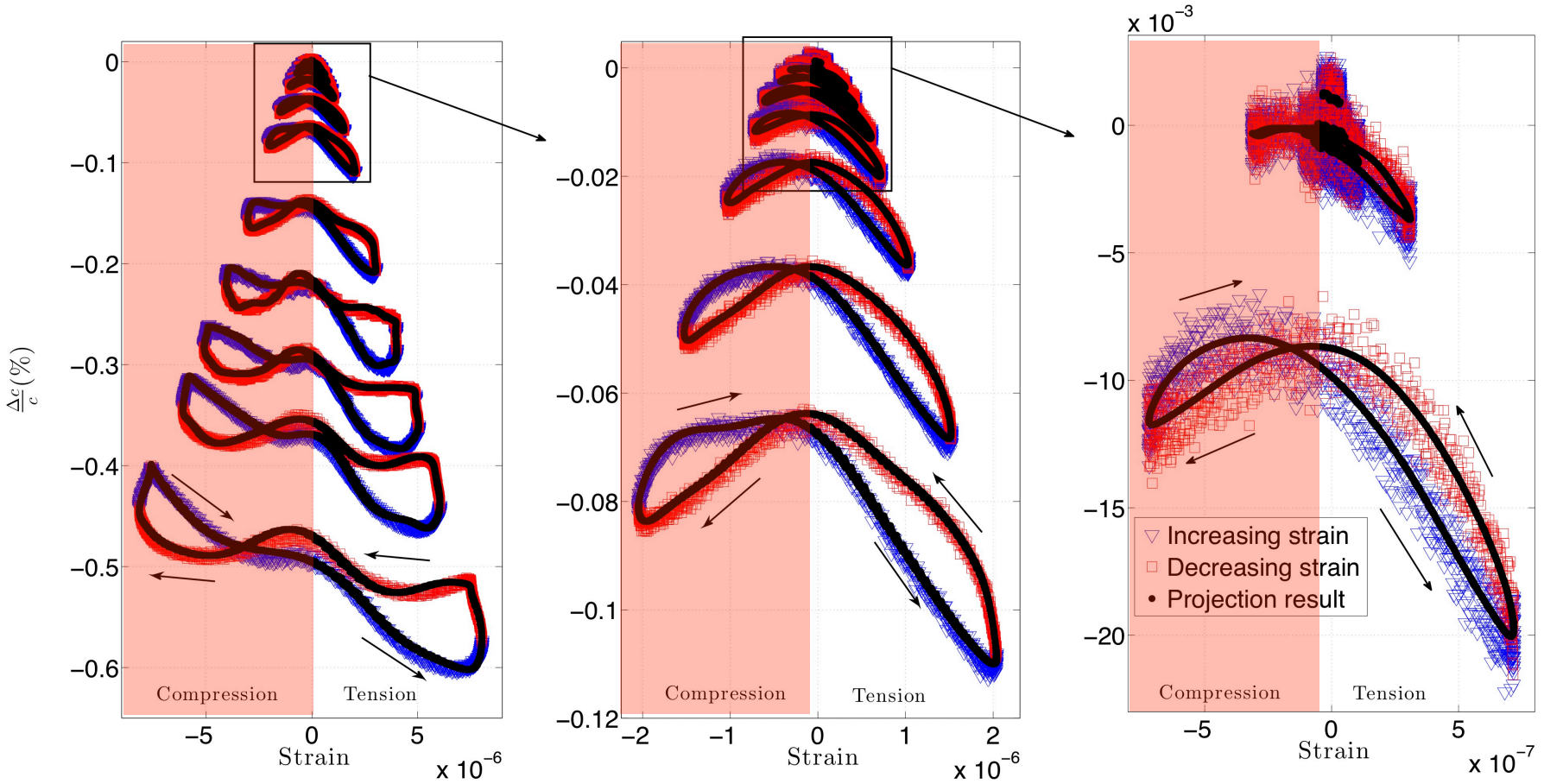
Johnson and McCall, GRL 1994

Dynamic acousto-elasticity (DAE)



If changes are slow, passive noise can be used.

Dynamic acousto-elasticity reveals exotic behavior



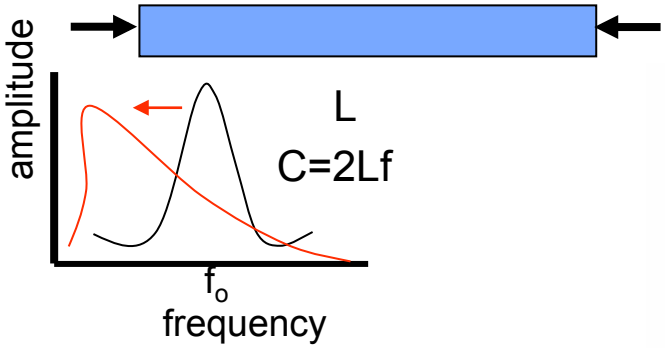
Note progressive offset with amplitude —'conditioning'

Highly complex behavior that can be described with additional harmonics

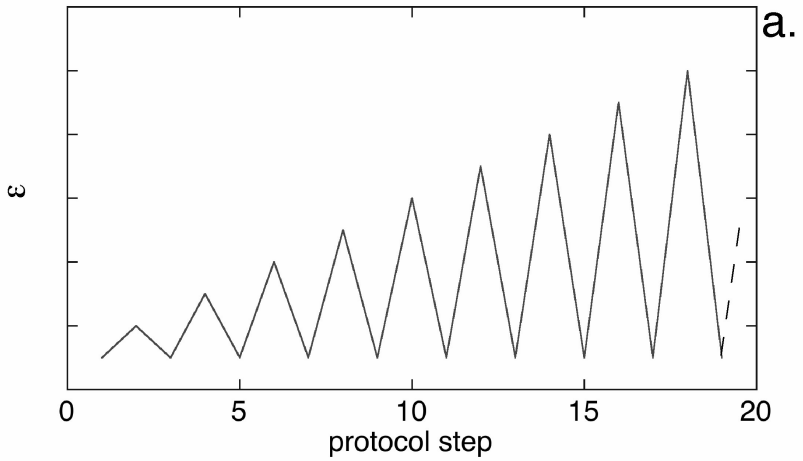
Riviere et al., in review JAP, 2013

regimes of nonlinear elastic behavior

Regime i & ii, iii: linear and classical nonlinear in sandstone

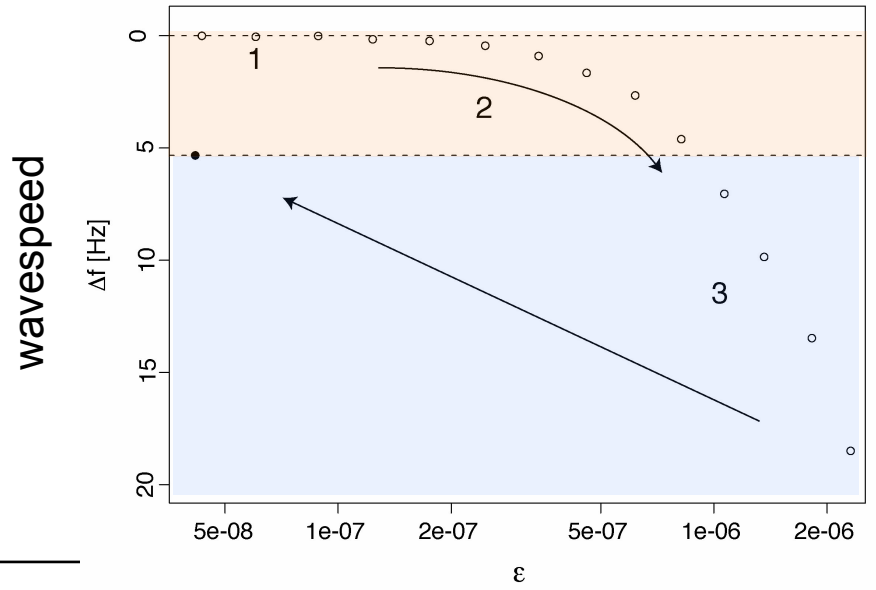


Young's mode experiments in a bar



Experimental Protocol

results



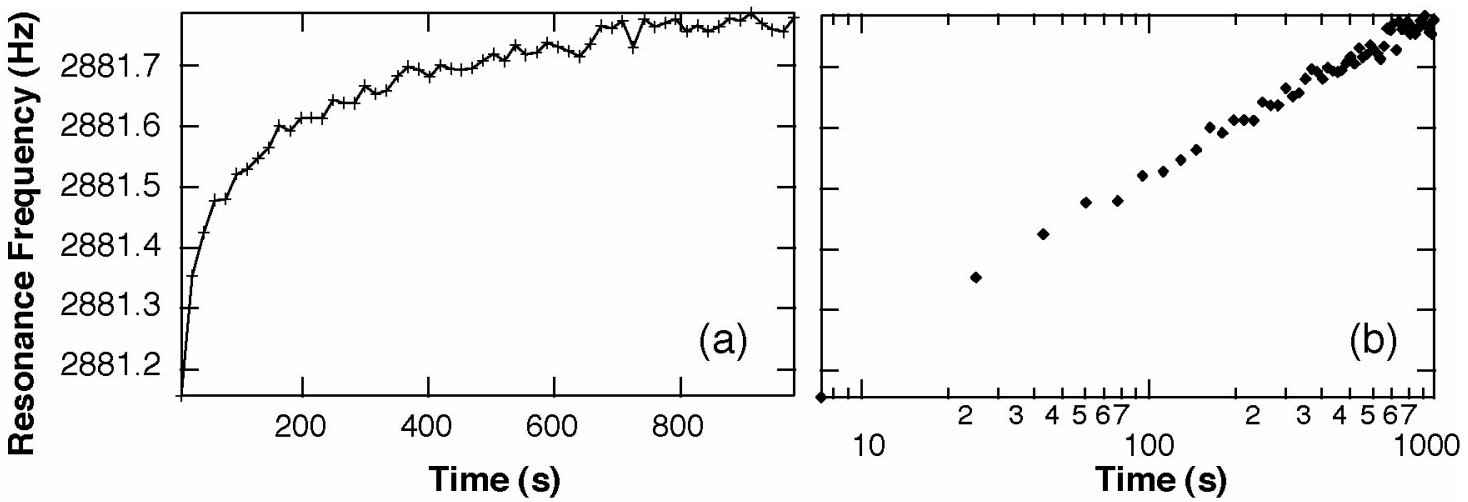
classical' behavior

Nonclassical, nonequilibrium

Pasqualini et al., Jour. Geophys. Res. 2007

Regime iii: nonequilibrium slow dynamics

Recovery in Berea sandstone



Linear with the log of time

Modified from TenCate and Smith, PRL 1997

1-D phenomenological stress-strain relation

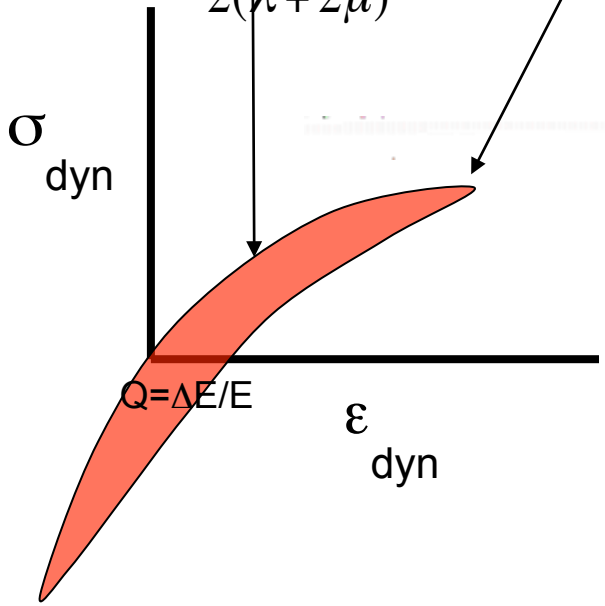
Stress in elastically nonlinear material is,

$$\sigma = K_0\left(\varepsilon + \beta\varepsilon^2 - \alpha\left[(\Delta\varepsilon)\varepsilon + sign(\dot{\varepsilon})\frac{\varepsilon^2 - (\Delta\varepsilon)^2}{2}\right]\right)$$

$$\varepsilon = \frac{du}{dx}, \beta = \frac{3(\lambda+2\mu)+2(l+2m)}{2(\lambda+2\mu)}$$

M=modulus, ε= strain, λ.μ = Lamé coeffs.
l, m = 3rd order [Murnaghan] coefficients

Preisach-Mayergoyz model of elasticity



The elastic nonlinear is much *larger and different in origin* to that in ordinary materials (e.g.,

β_{H2O}~5 α=0

β_{rock}~10^2-10^3, α=10^2-10^5.

1-D phenomenological wave equation

Normal form:

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma}{\partial x} + F$$

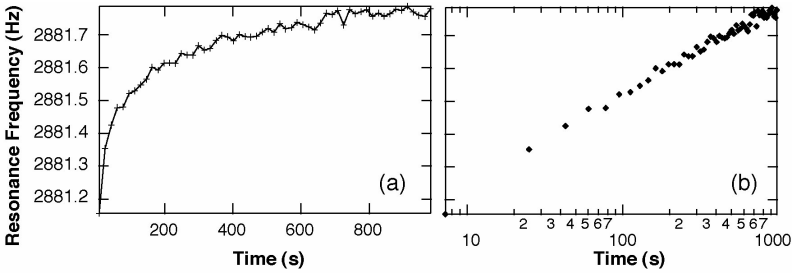
substituting the nonlinear stress relation

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial K_0 \left(\varepsilon + \beta \varepsilon^2 - \alpha \left[(\Delta\varepsilon)\varepsilon + sign(\dot{\varepsilon}) \frac{\varepsilon^2 - (\Delta\varepsilon)^2}{2} \right] \right)}{\partial x} + F$$

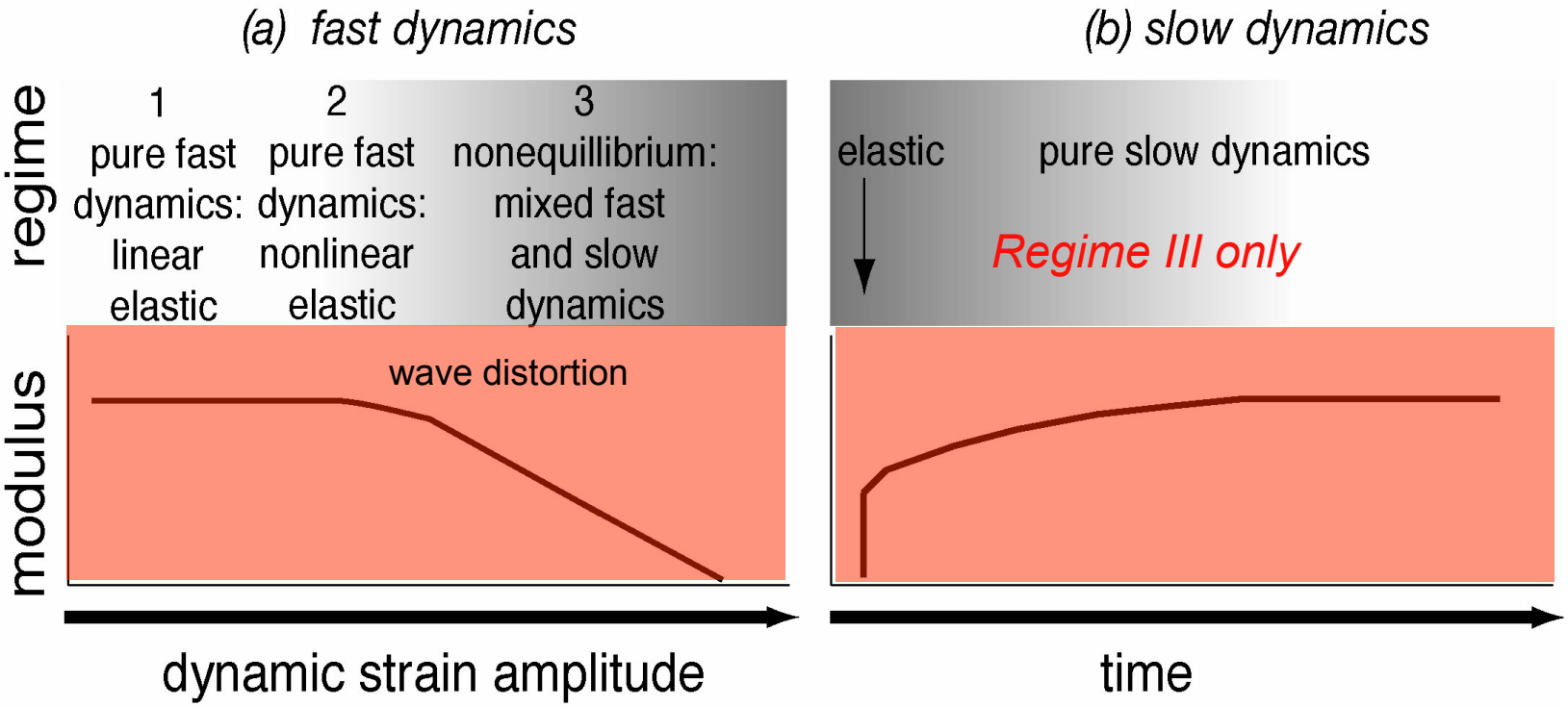
Problems:

i. Phenomenologic

ii. Contains no rate dependence

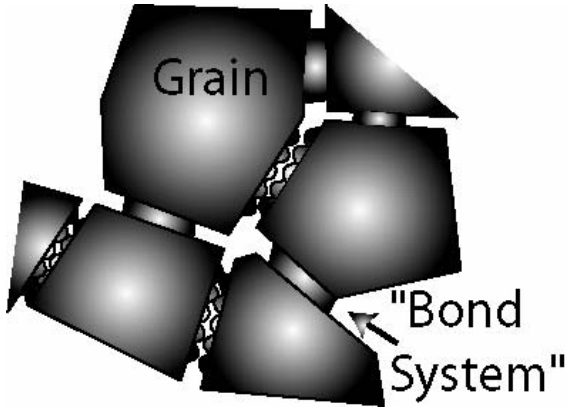


Review: nonlinear fast and slow dynamics

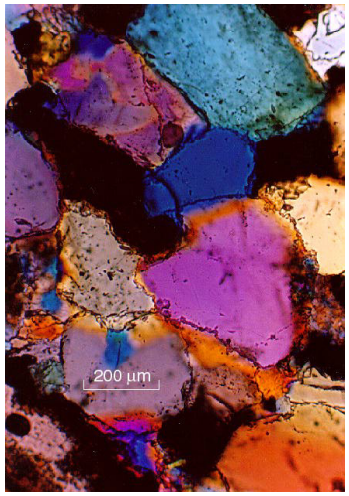


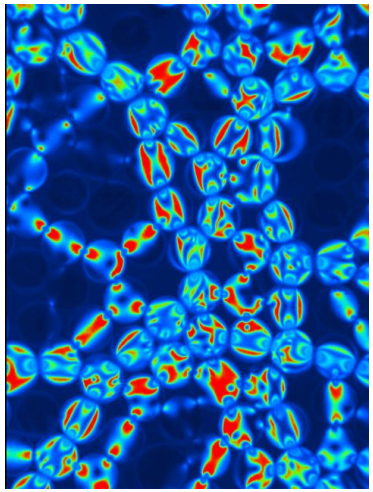
The same energy landscape is traversed for the fast and slow dynamics
based on measurement of the Reid parameter and the material elastic response is the same before and after measurement.

Physical origin of behavior

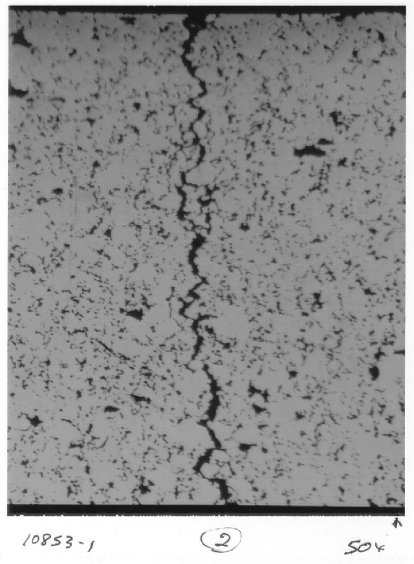


"Distributed"





"Localized"

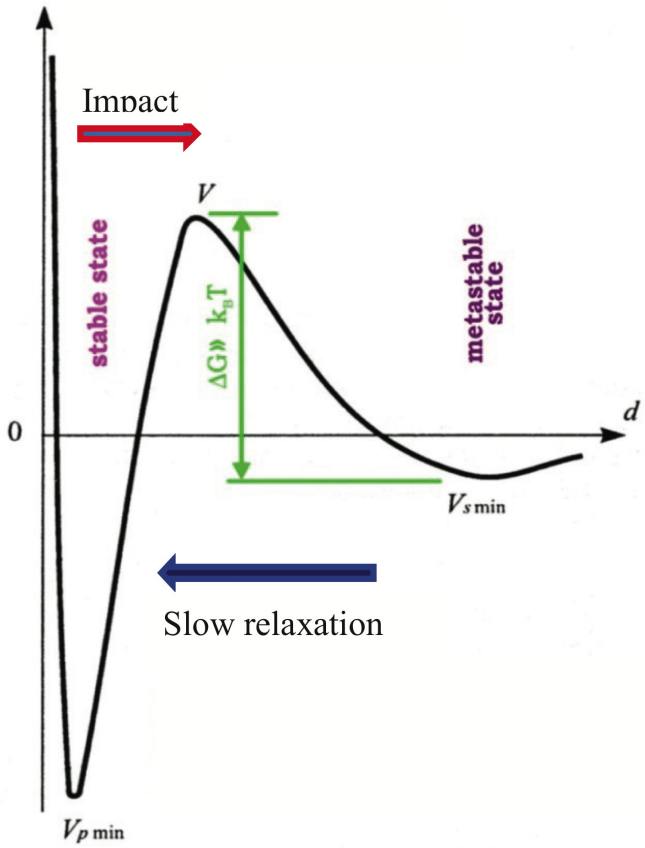


The grains or material around damaged region remain rigid.
The bond system is soft.

New model approach

Arrhenius model I

- An acoustical wave forces contacts over potential energy barriers (the fast nonlinear dynamics)
- contacts relax back to equilibrium states via thermal motions (the slow dynamics)

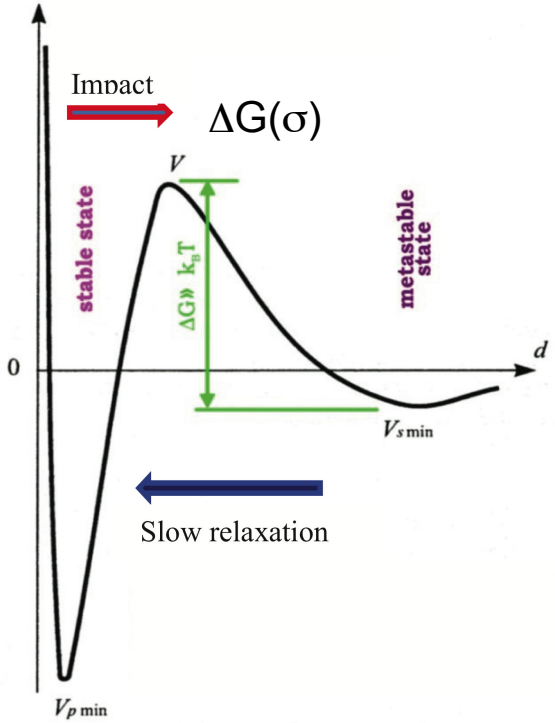


Arrhenius model II

The Arrehenius law of strain change is,

$$\dot{\varepsilon} = \dot{\varepsilon}_0 \exp\left[-\frac{\Delta G(\sigma)}{kT}\right] \qquad 1.$$

$\Delta G(\sigma)$ the Gibbs change of the Gibbs thermodynamic potential at the potential barrier
σ = dynamic stress magnitude
k = Boltzmann constant
T = temperature.



the characteristic activation volume V is,

$$V = \frac{kT}{\Lambda},$$

$$d = V^{1/3}$$

Where d is the characteristic dimension of the contact and
Λ – comes from experimental measurement

Ostrovsky and Lebedev, in development, 2013

Arrhenius model III

Λ comes in here:
The change in stress due to an impulse is,

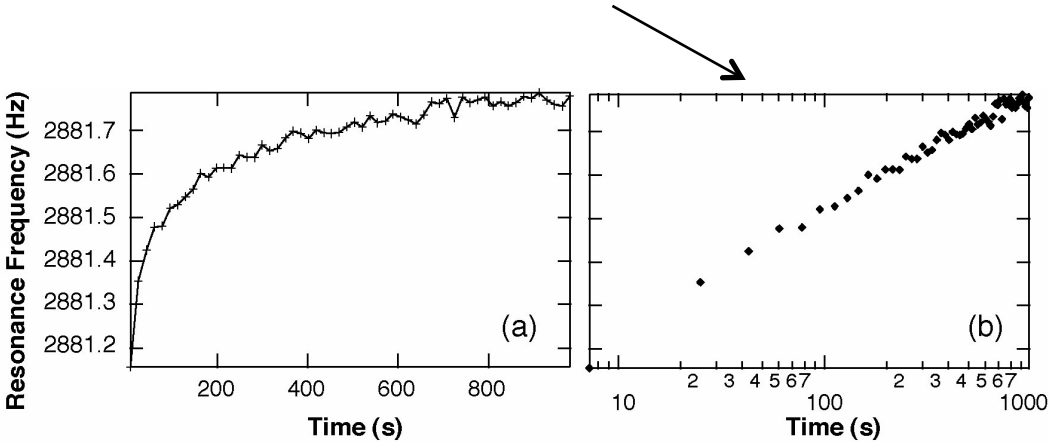
$$d\sigma = Md\varepsilon + \Lambda d(\ln \varepsilon_1) \quad 2.$$

M is the linear elastic modulus (longitudinal or shear) & $\Lambda = \left[\frac{kT}{V}\right]$

after a dynamic excitation there is no stress acting (dσ=0), the solution to eqn(2) is logarithmic:

$$\varepsilon = \varepsilon_0(t) = \left[\frac{\Lambda}{M}\right]\ln(t) \quad 3.$$

Λ can be estimated from the slope of the material relaxation (the slow dynamics) obtained from experiment.



Ostrovsky and Lebedev, in development, 2013

Arrhenius model preliminary results

Material	c_p, m/s	ρ, g/cm^3	M, GPa	Λ, MPa	V^*, m^3	d, Å
Pearlite (grey iron)	3225	1.1	11.4	3.8	$1.1\cdot10^{-27}$	**10**
Alumina ceramics	4940	3.9	95	4.3	$0.9\cdot10^{-27}$	**9.8**
Quartzite	3826	2.35	34	6.7	$0.59\cdot10^{-27}$	**8.4**
Pyrex with cracks	4700	2.23	49.3	4.4	$0.9\cdot10^{-27}$	**9.7**
Marble	4030	2.56	41.6	1.9	$2.1\cdot10^{-27}$	**12.8**
Perovskite ceramic	1670	4.1	11.5	7.8	$0.5\cdot10^{-27}$	**8**
Soil	$c_S \cong 150$	$\cong 1.3$	0.029	0.26	$15.4\cdot10^{-27}$	**24.8**
Glass beads	P-mode		E=0.23	0.0037	10^{-24}	**100**
Glass beads	S-mode		G=0.082	0.0005	$8\cdot10^{-24}$	**200**

Even macrocracks fall into this range (so far). Meaning the fundamental asperitiy size is atomic dimension. This is very useful information about the origin of NL elasticity.

Ostrovsky and Lebedev, proceedings Int. Symp. NL Acoust, Tokyo, 2012

Other forcing fields

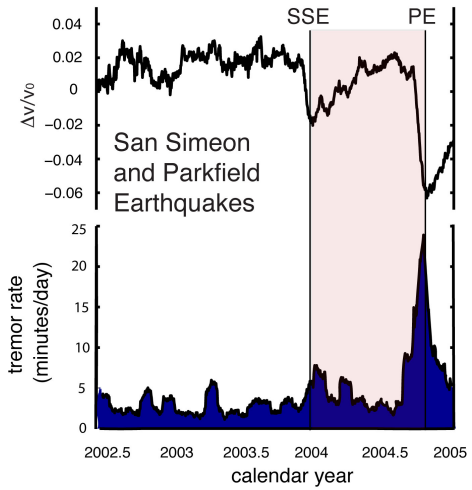
We've seen the effects of acoustic waves and static stress fields. Other fields have profound effects:

Water and thermal forcing….. (no time here to discuss)

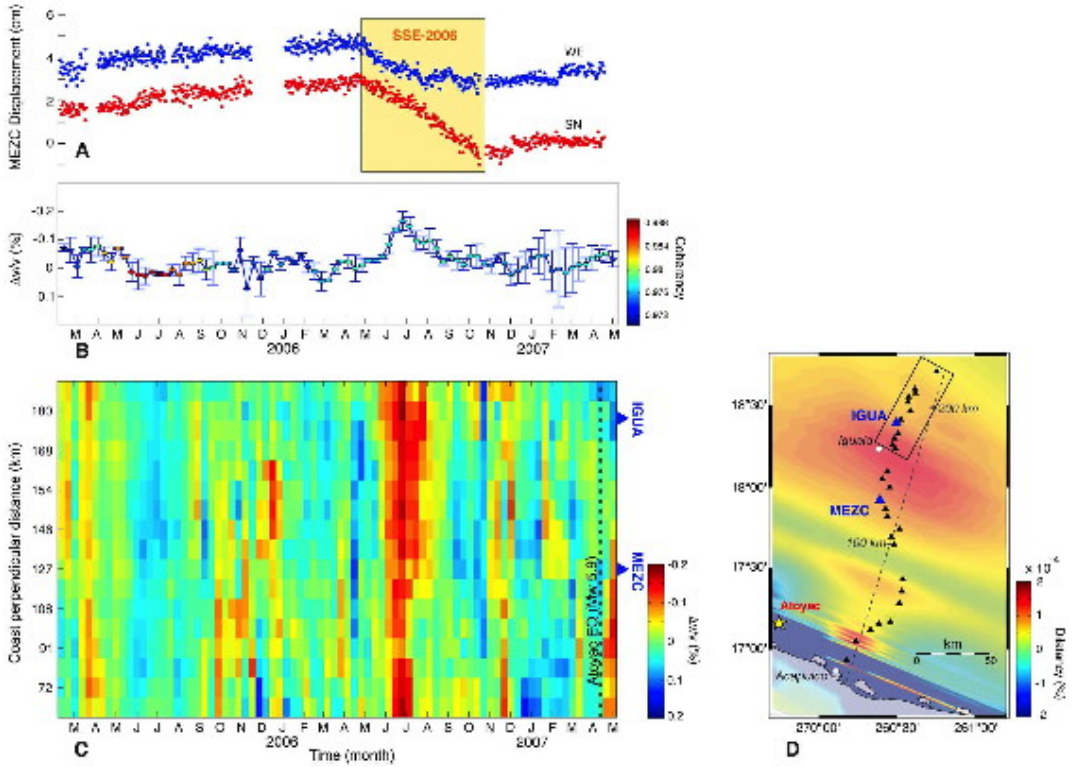
Many topics in earth where elastic nonlinearity is important (earthquake source (*the* Ben Zion), strong ground motion…

Applications to structures (Gueguen)

Applications using passive noise:
Post fault relaxation (Brenguier), slow slip, earth tides (Hillers, River, Daub)….



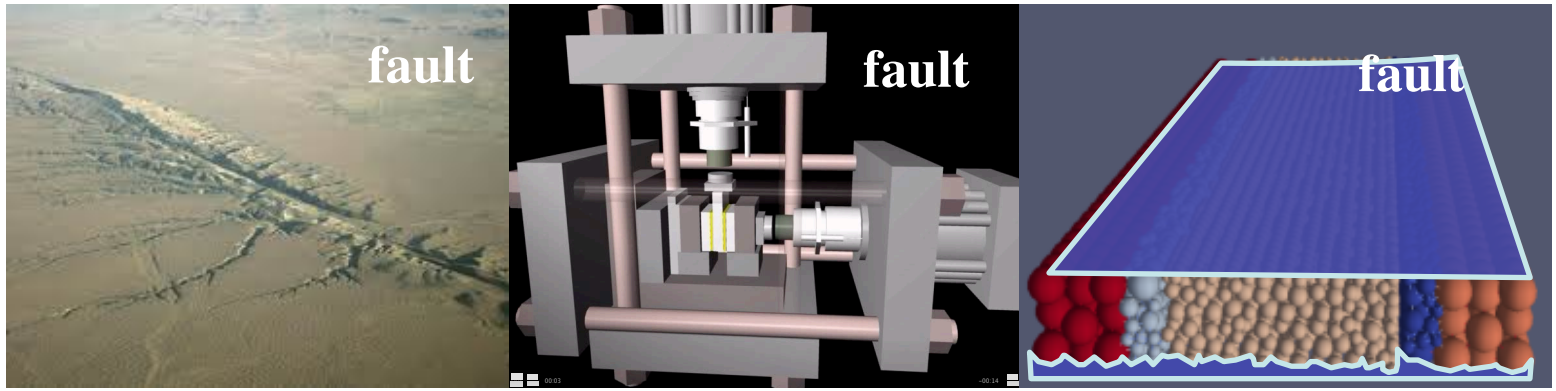
e.g., relaxation post parkfield applying passive noise–Brengieur et al.



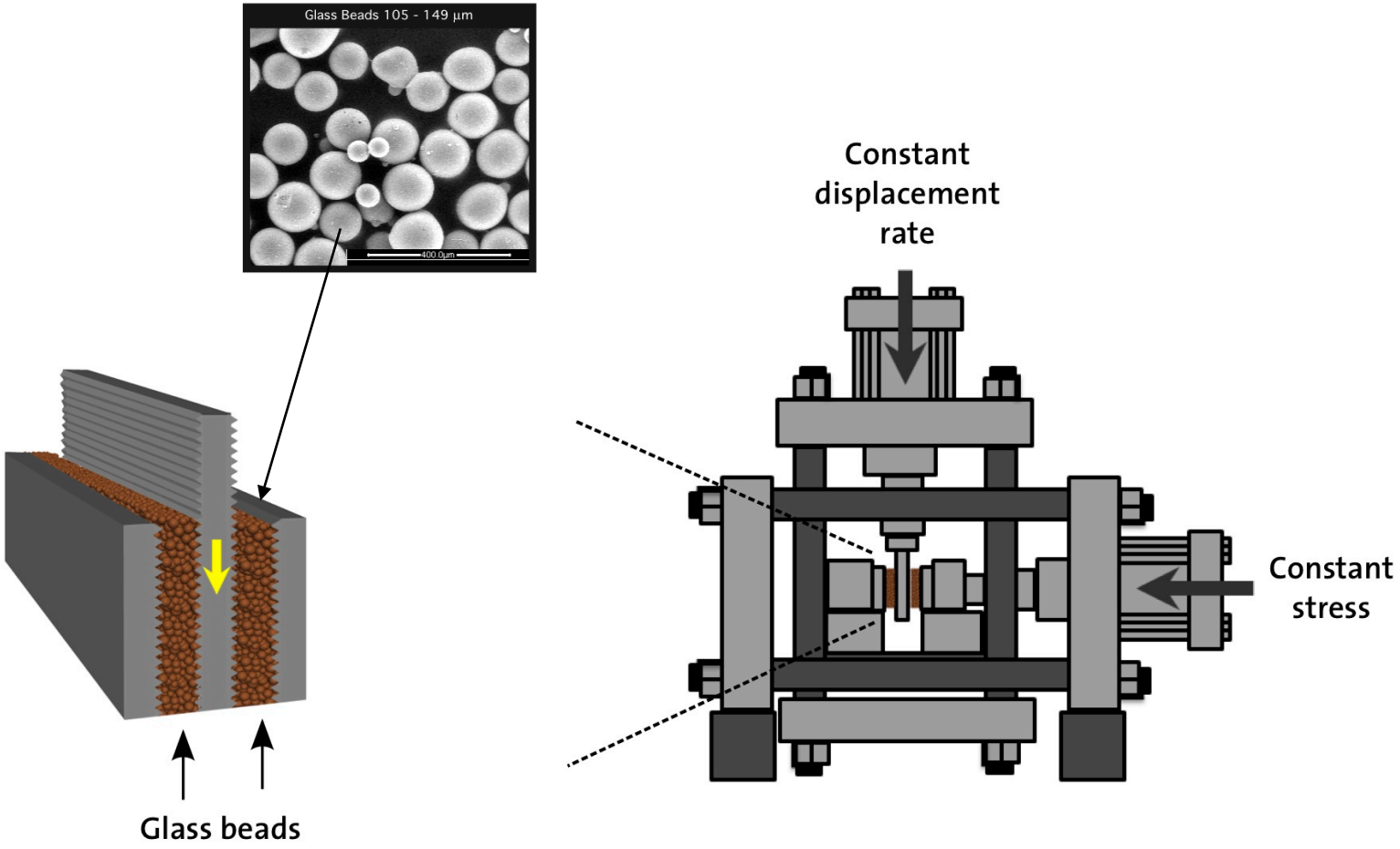
Changes in velocity due to slow slip applying passive noise in Mexico (Rivet et al.)

Elastic (nonlinear) effects in fault studies

in the field | **in the laboratory** | **numerical**

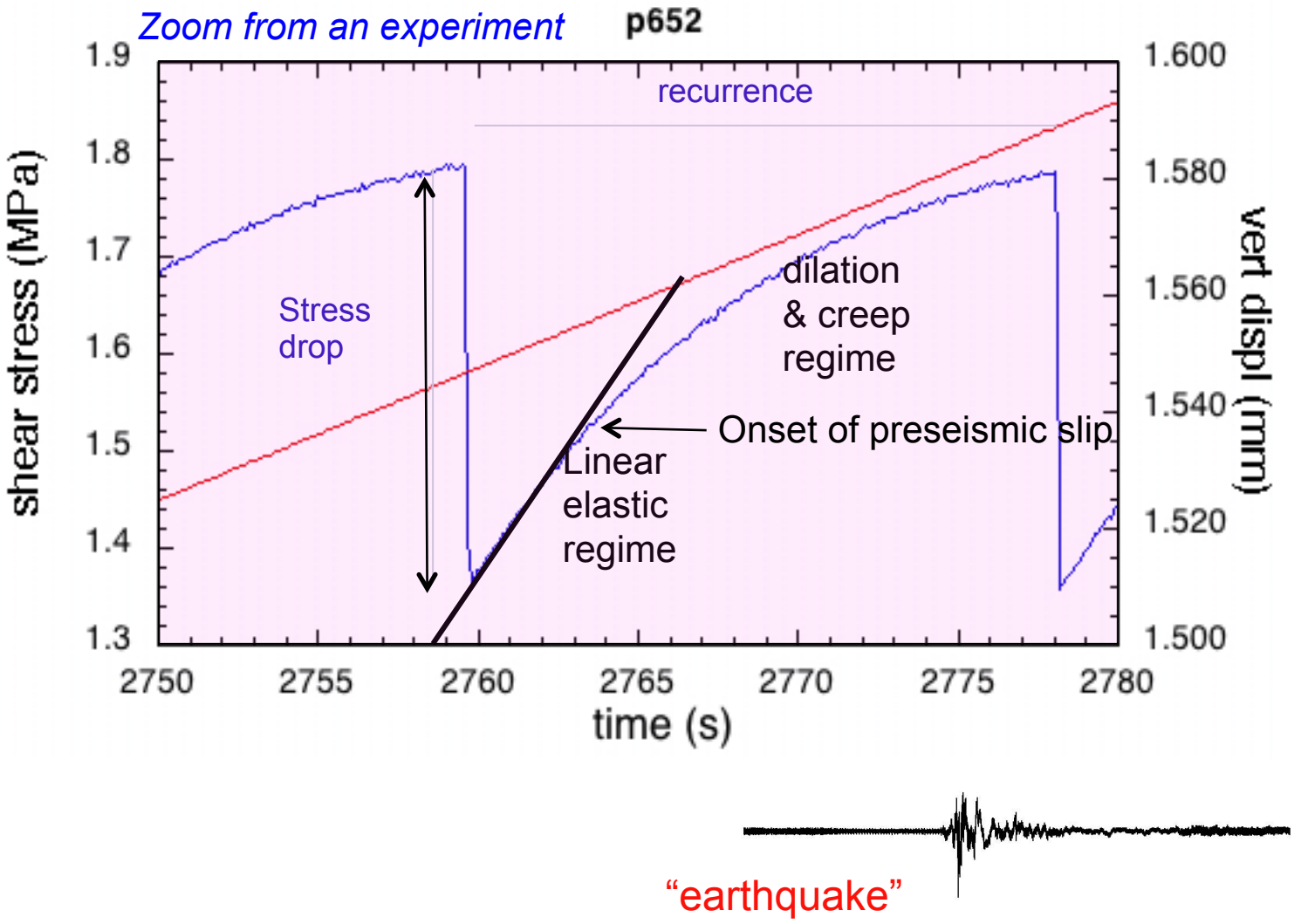


The Marone earthquake apparatus at Penn State

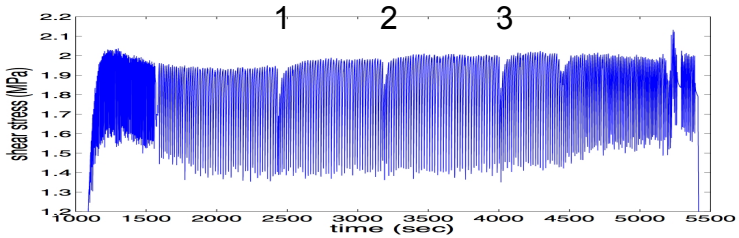


Stick-slip instabilities

4 Mpa load



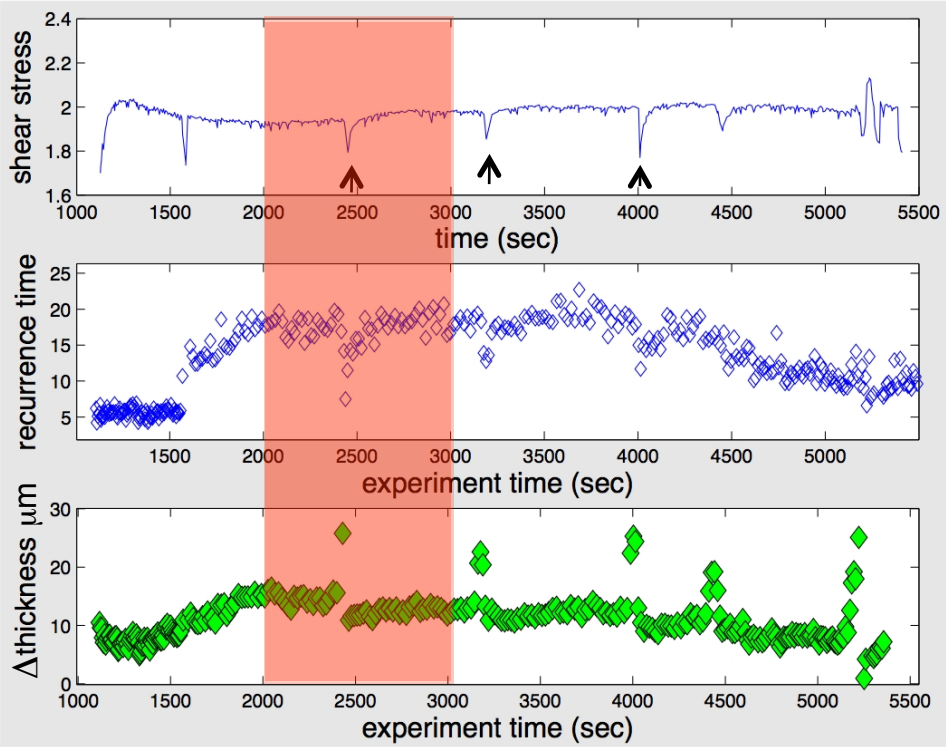
triggered slip and long recovery

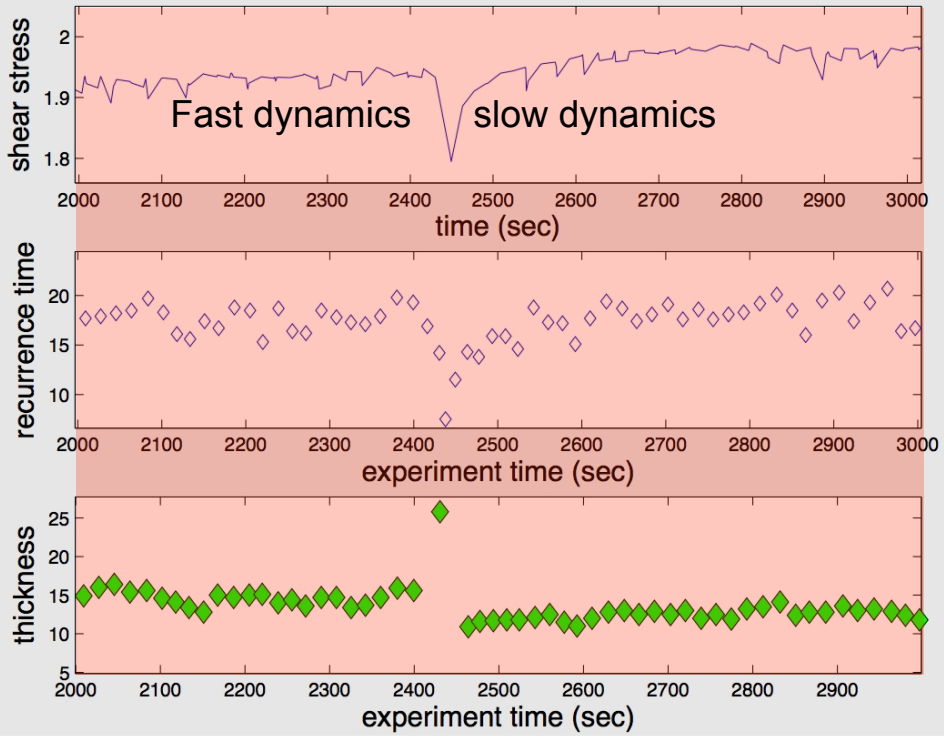


p1894, 5MPa

Without dynamic waves, recurrence and thickness are ~constant

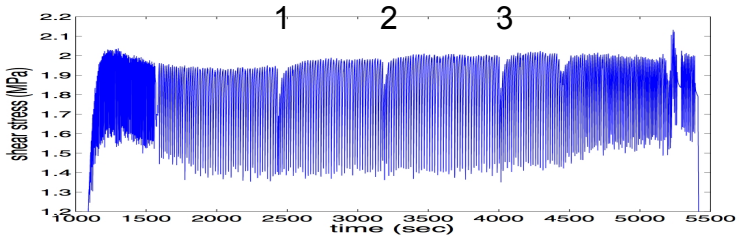
Dynamic strain ~10^{-6}

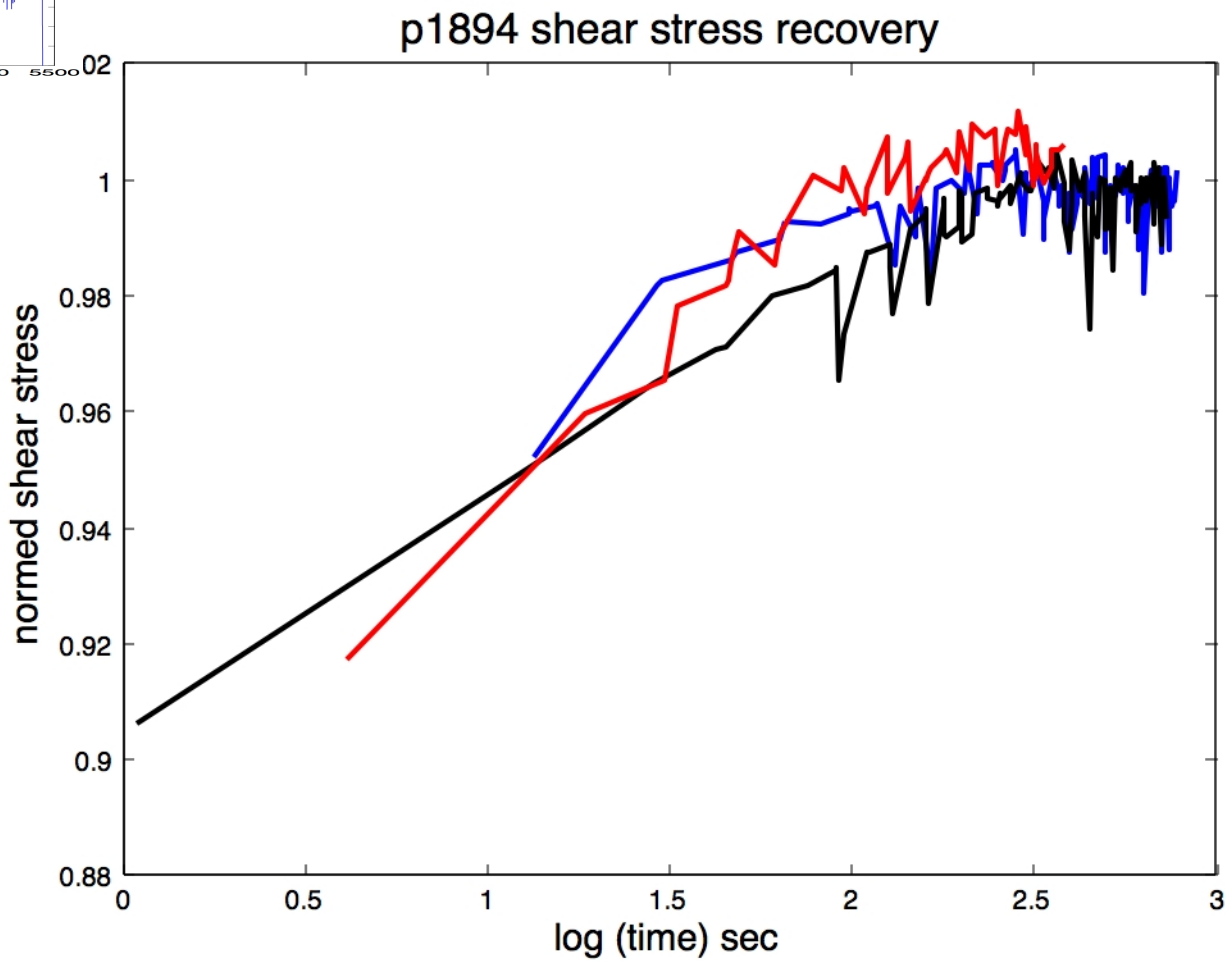




Johnson, Proc. Int. Symp NL Acoust., Tokyo, 2012

Recovery phenomena: shear stress

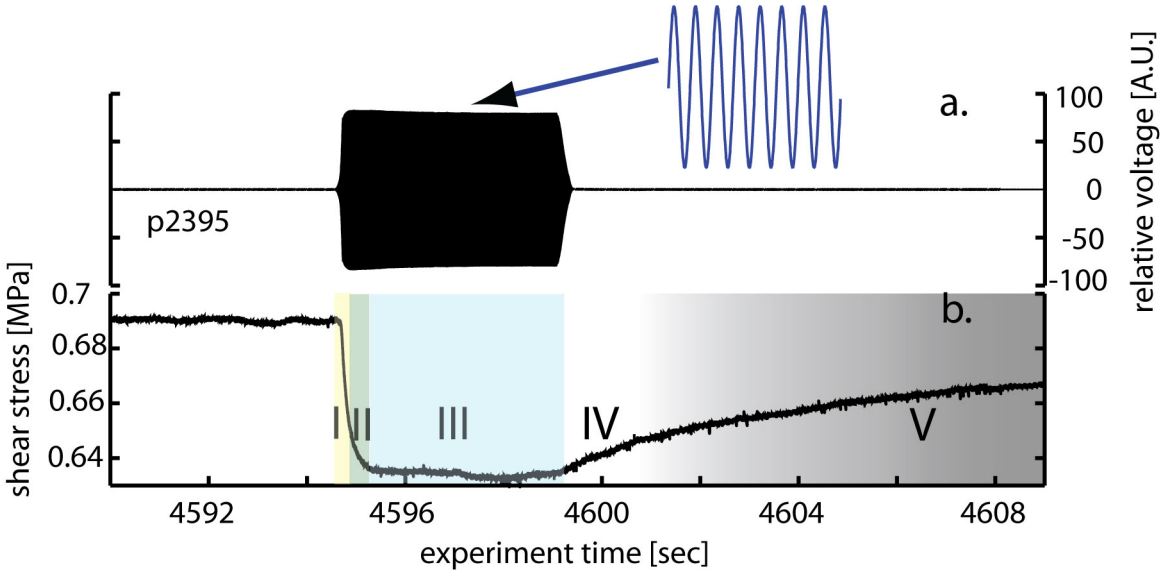




Johnson, Proc. Int. Symp NL Acoust., Tokyo, 2012

Triggered slow, ~silent slip

2 Mpa load

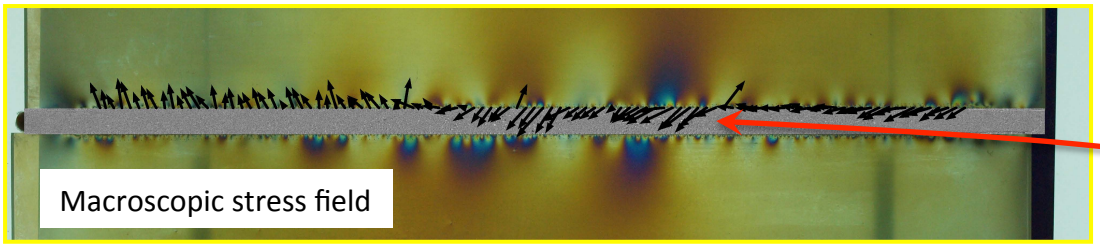


Triggering slow slip only occurs at loads <=3 Mpa; we are approaching plasticity at these pressures.

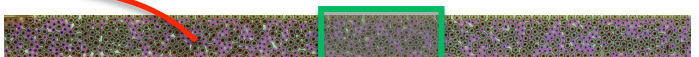
Johnson et al., JGR 2012

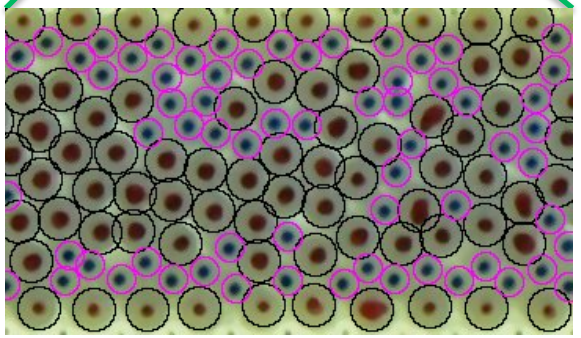
Experimental System applying photoelasticity

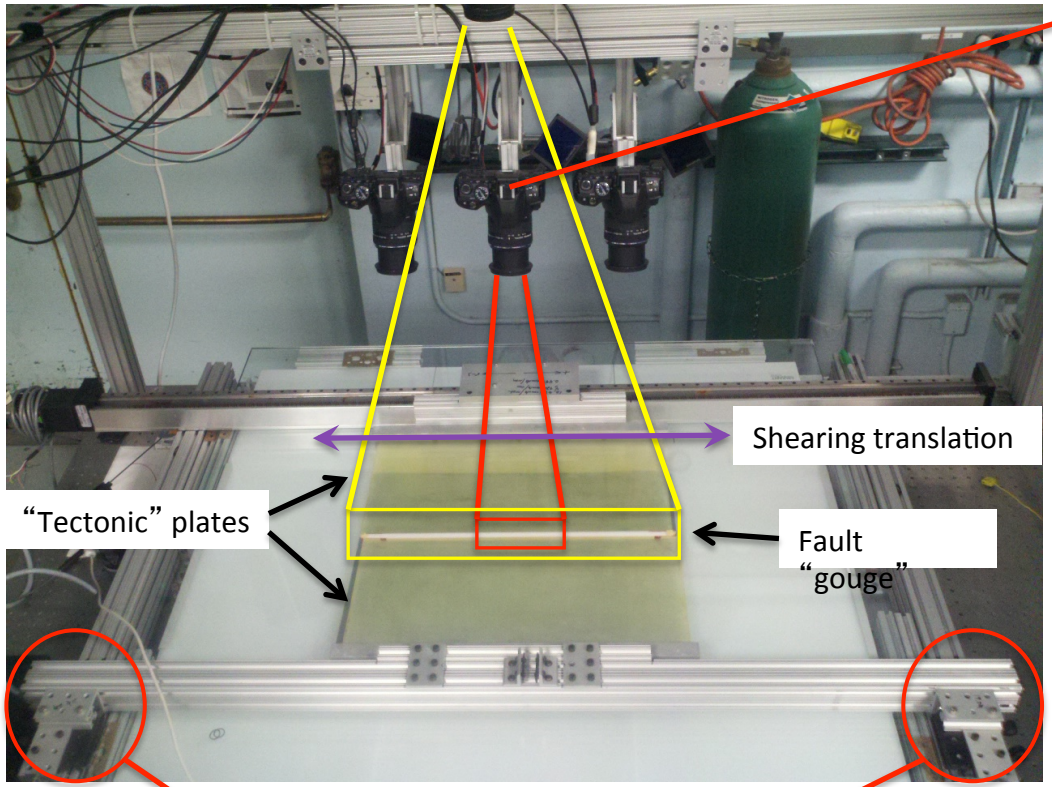
Backhaus, Geller, Ecke—
Los Alamos



Interface displacement and strain

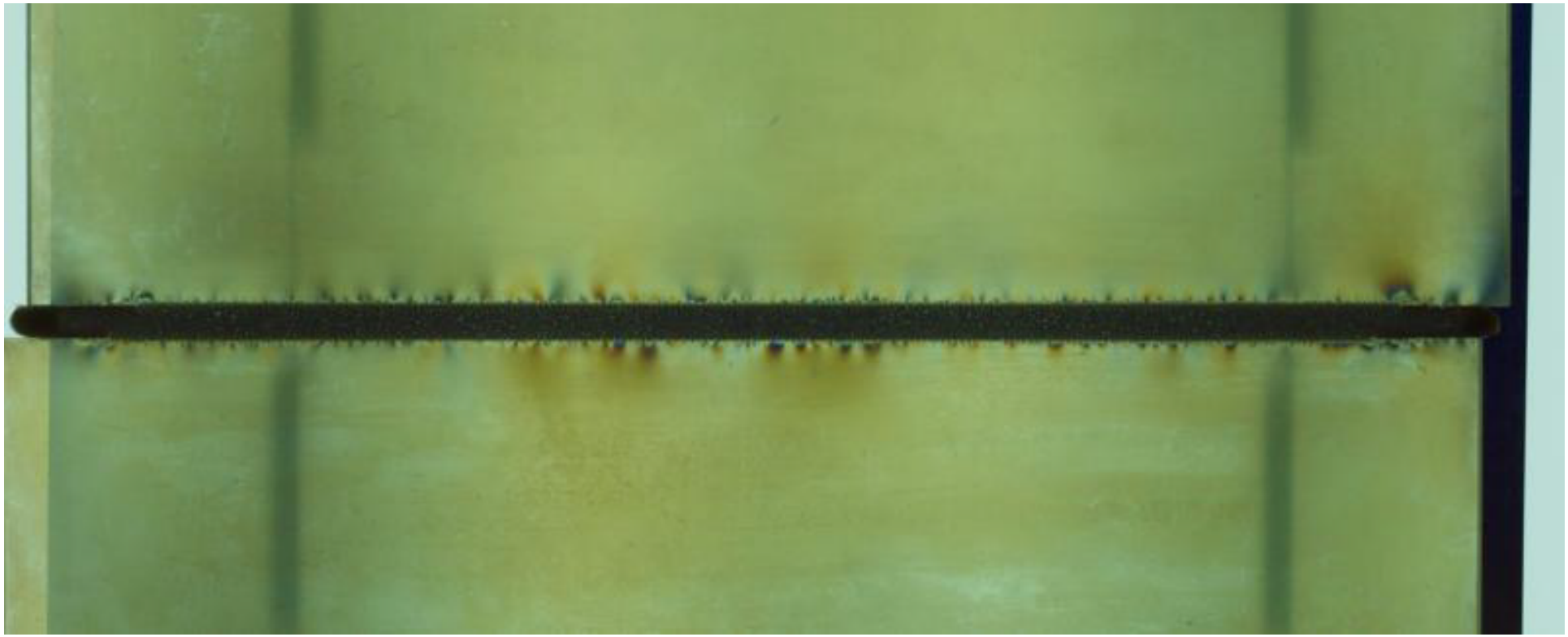




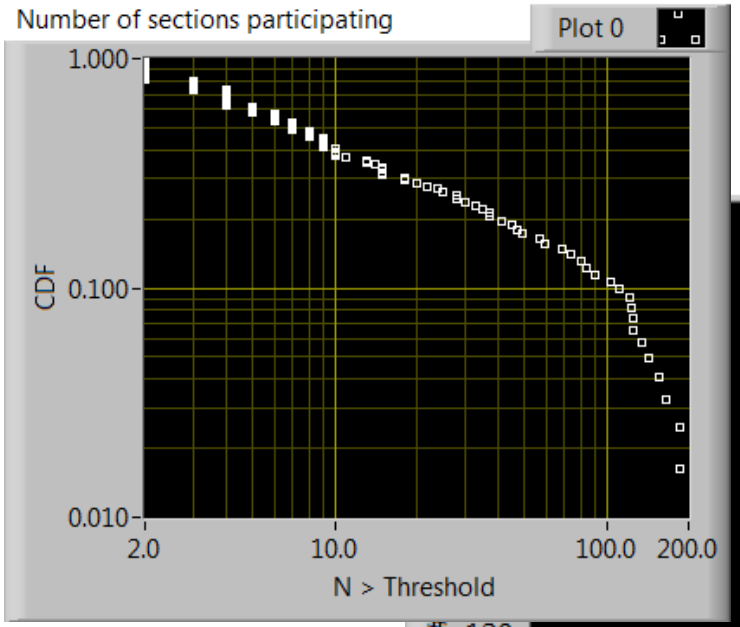


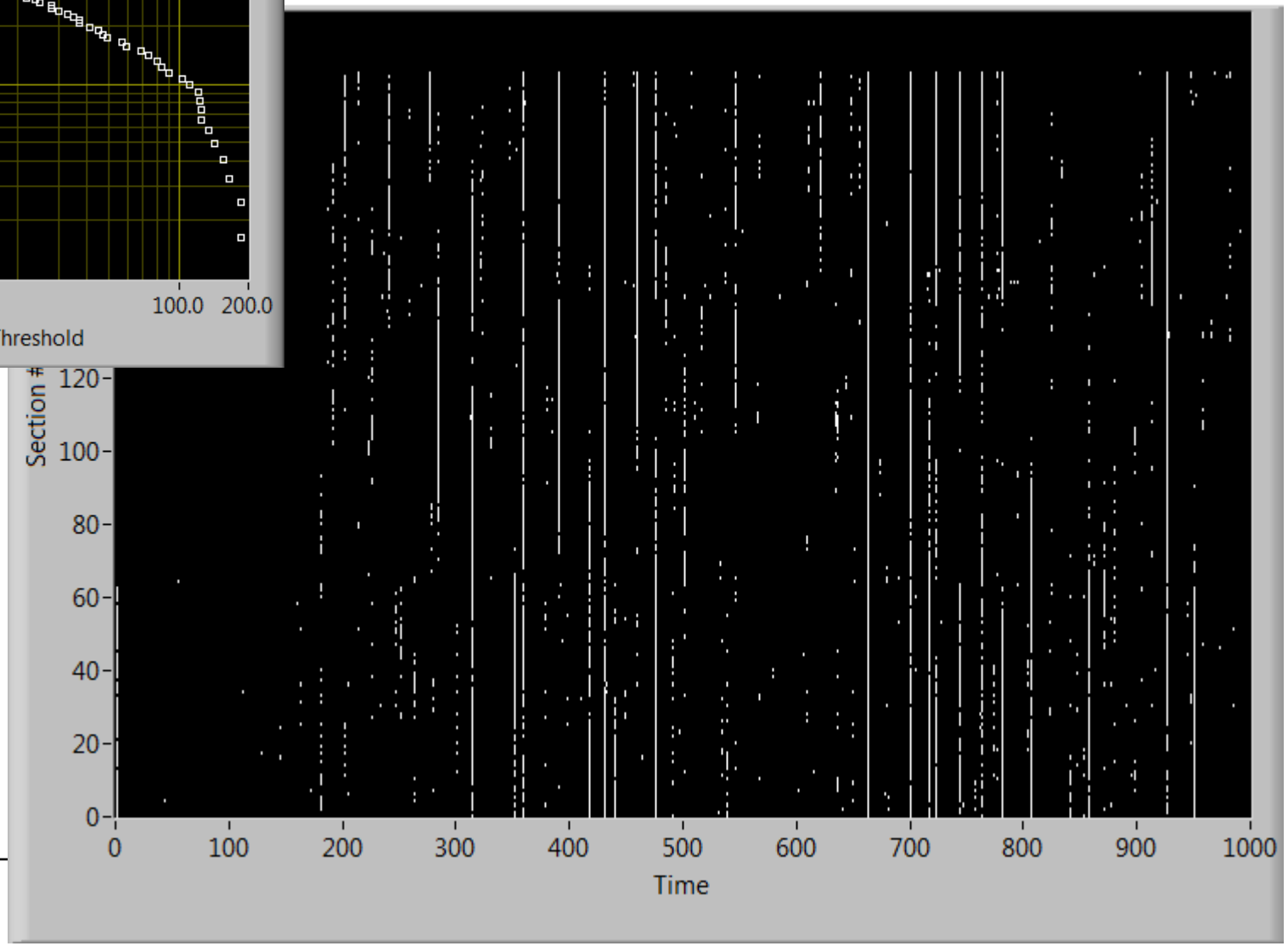
Normal force sensors

Backhaus, Geller, Ecke—
Los Alamos



Compute Fault Displacements and Threshold Noise at 0.35 pixels

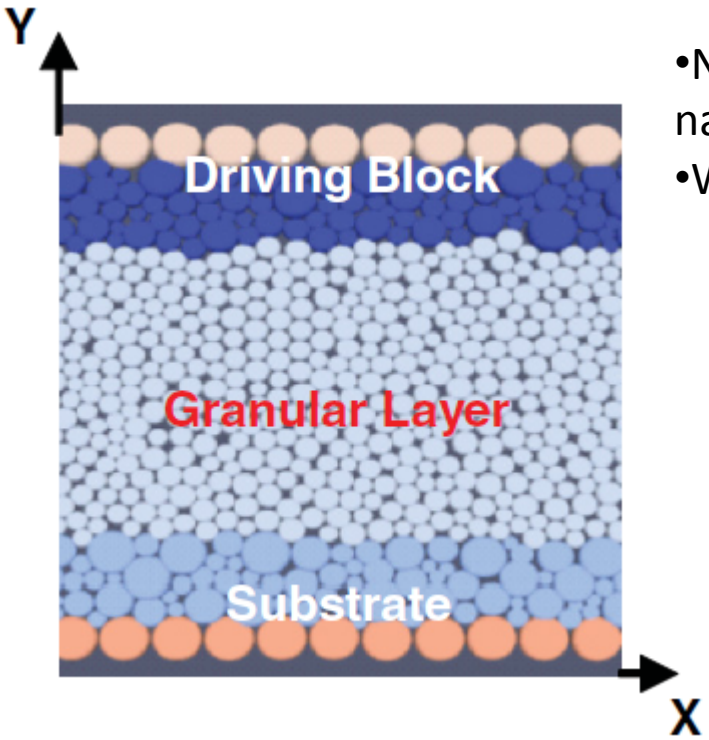


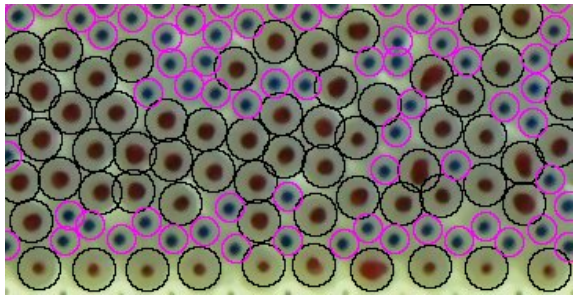


122 Events

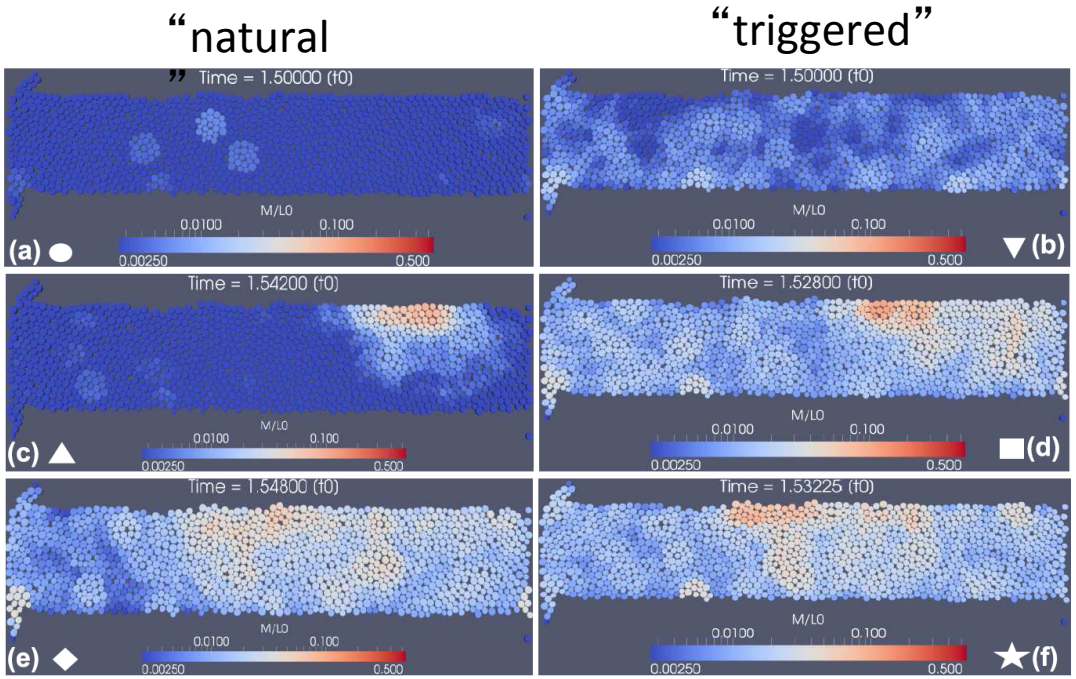
~ 12 Events that span the full length

Working Towards Detailed Comparison With DEM



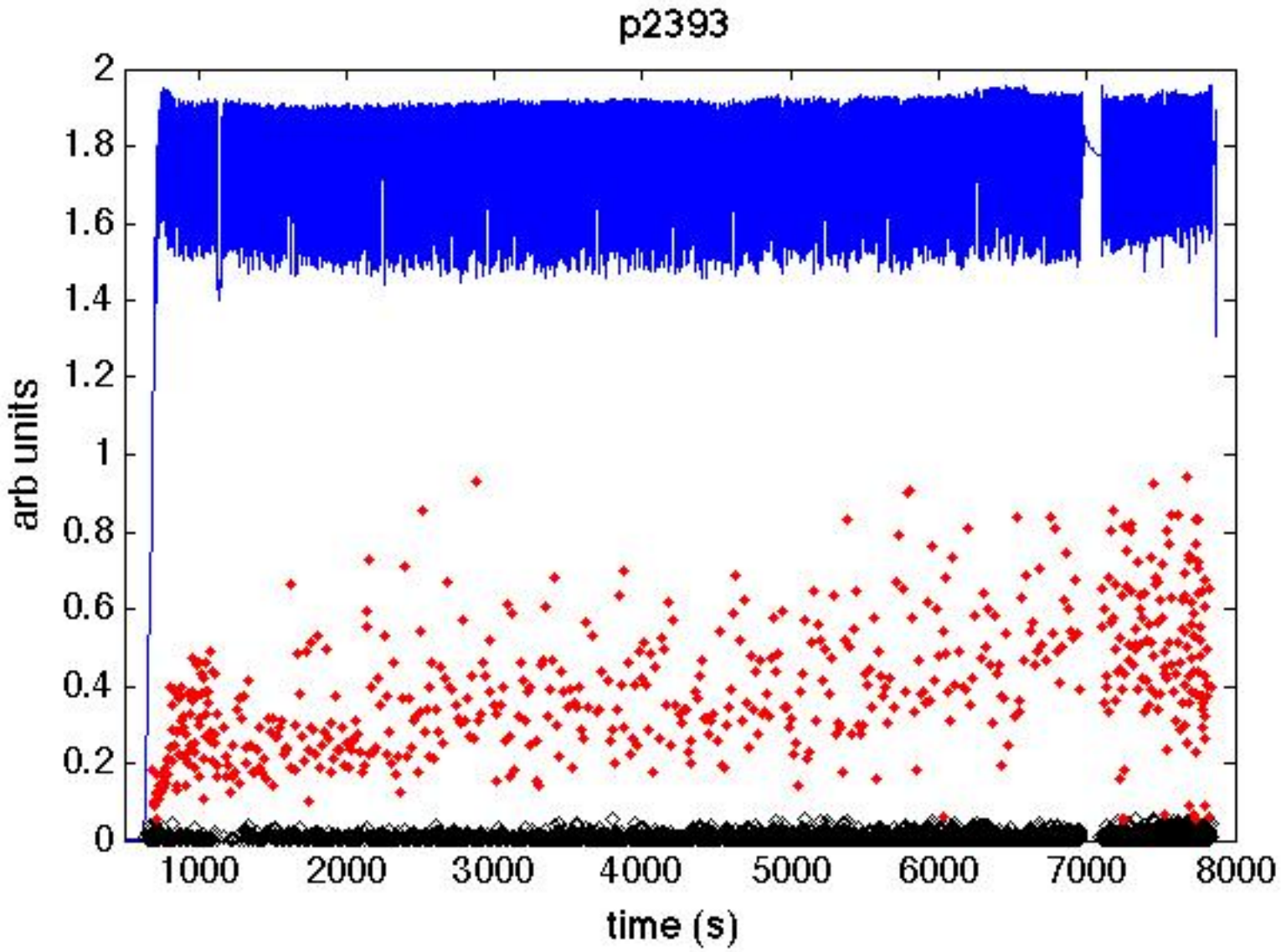


- Non-affine particle displacement seems to distinguish natural and triggered events
- Working to extract particle trajectories

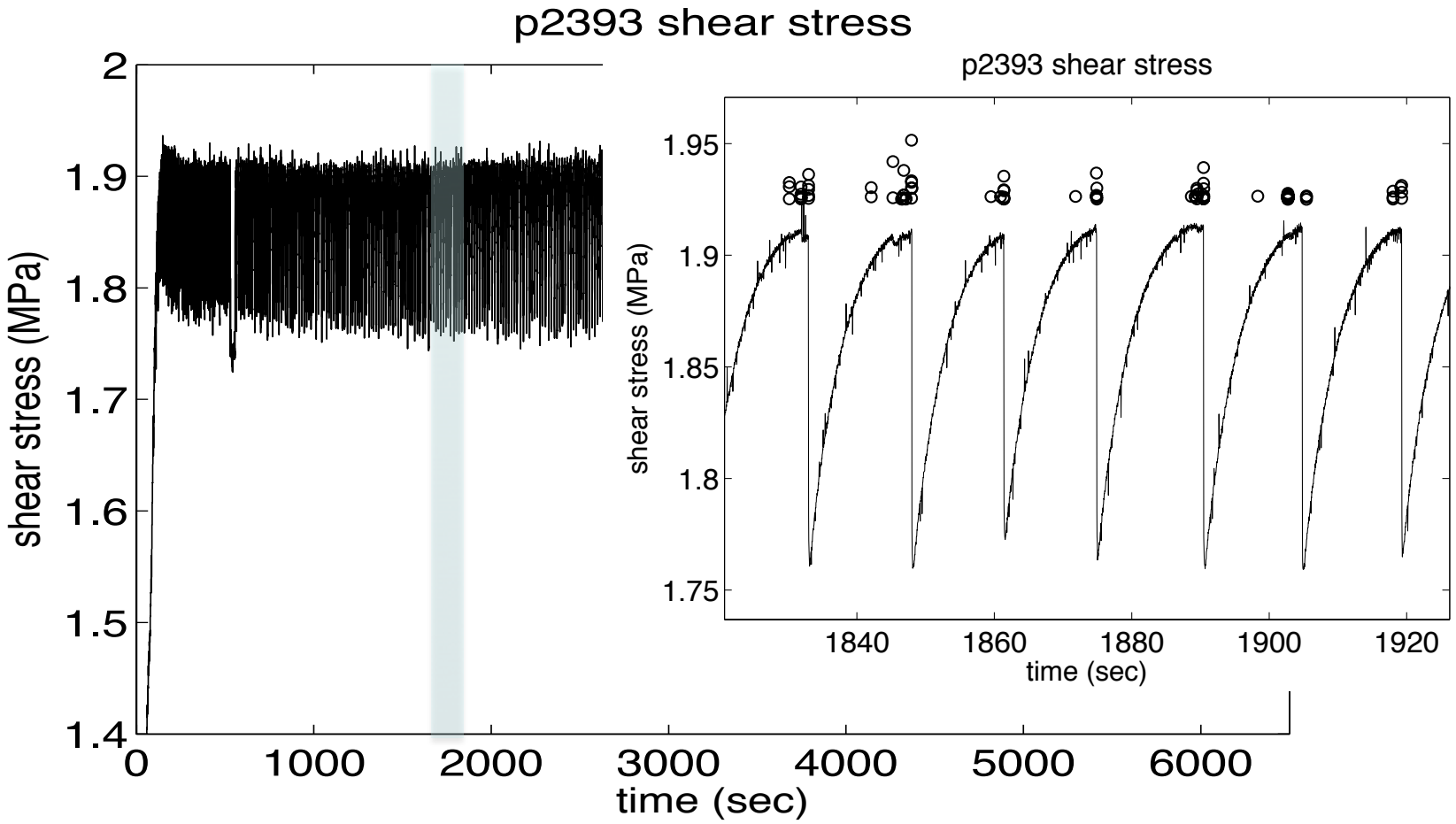


Precursors to slip—earthquake machine at Penn State

Slip events and acoustic emission in a typical experiment

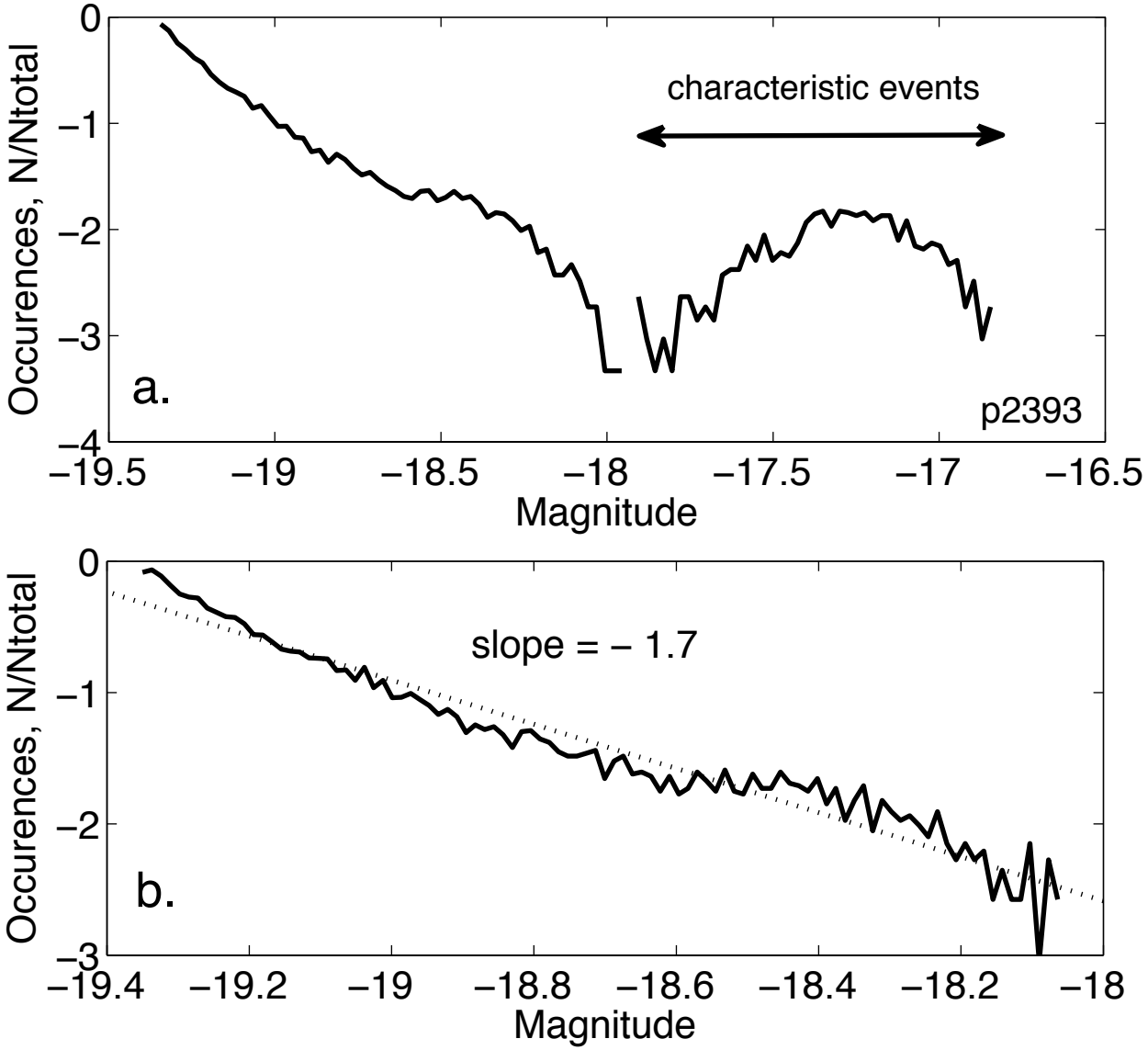


Slip events and acoustic emission in a typical experiment



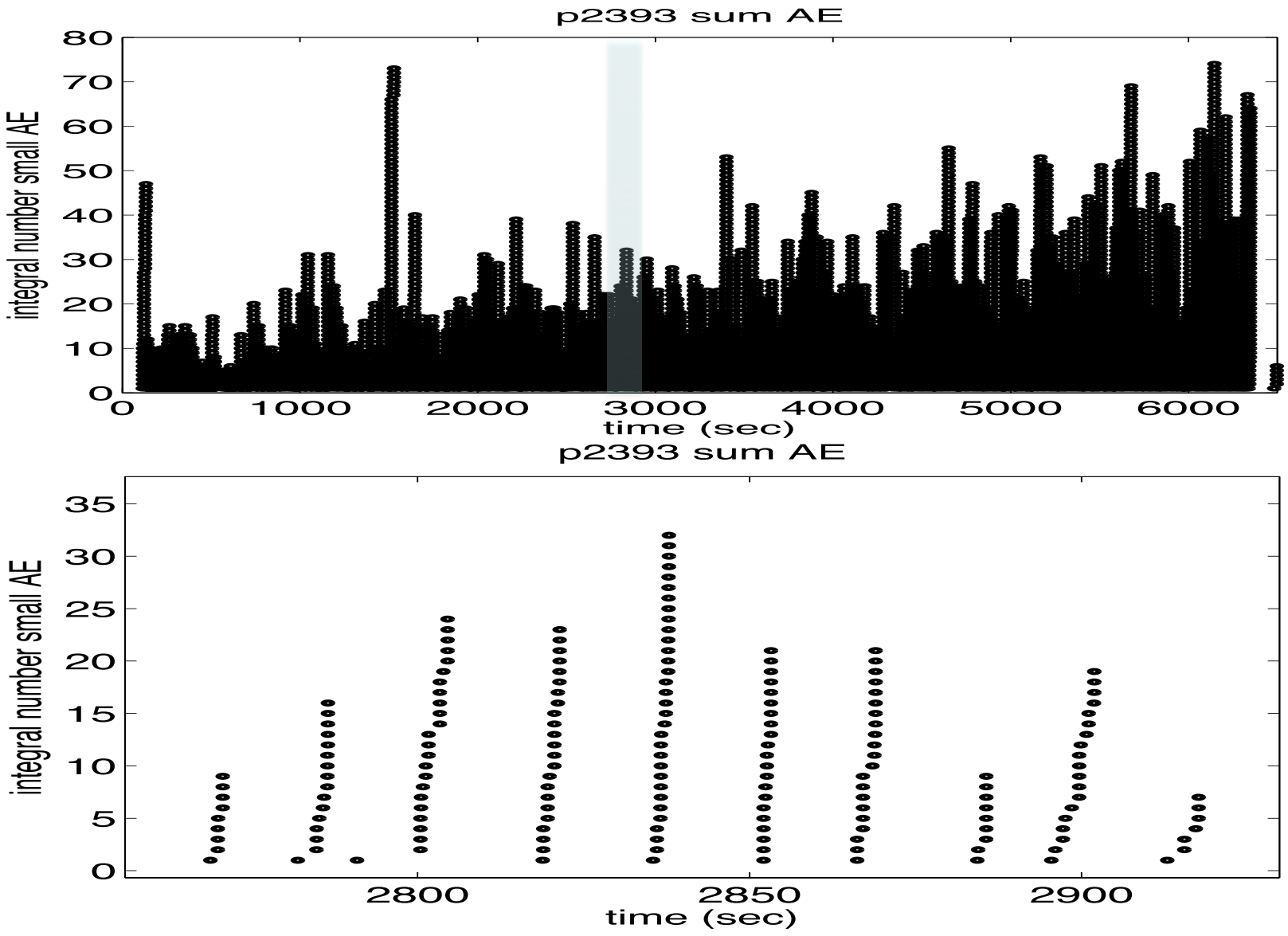
Johnson et al., in preparation, 2013

AE occurrence



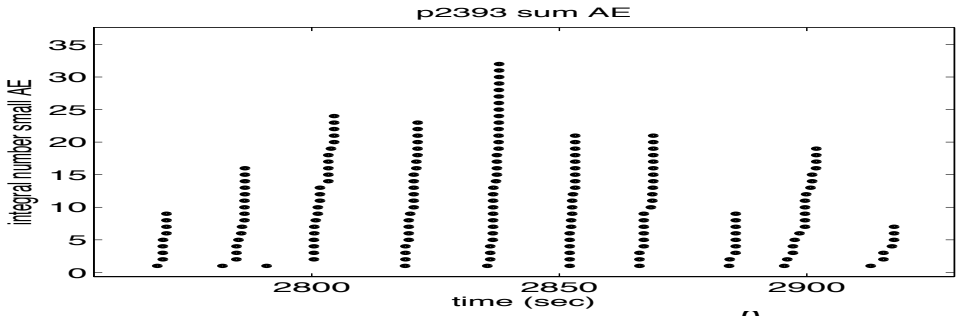
Johnson et al., in preparation, 2013

AE each slip event



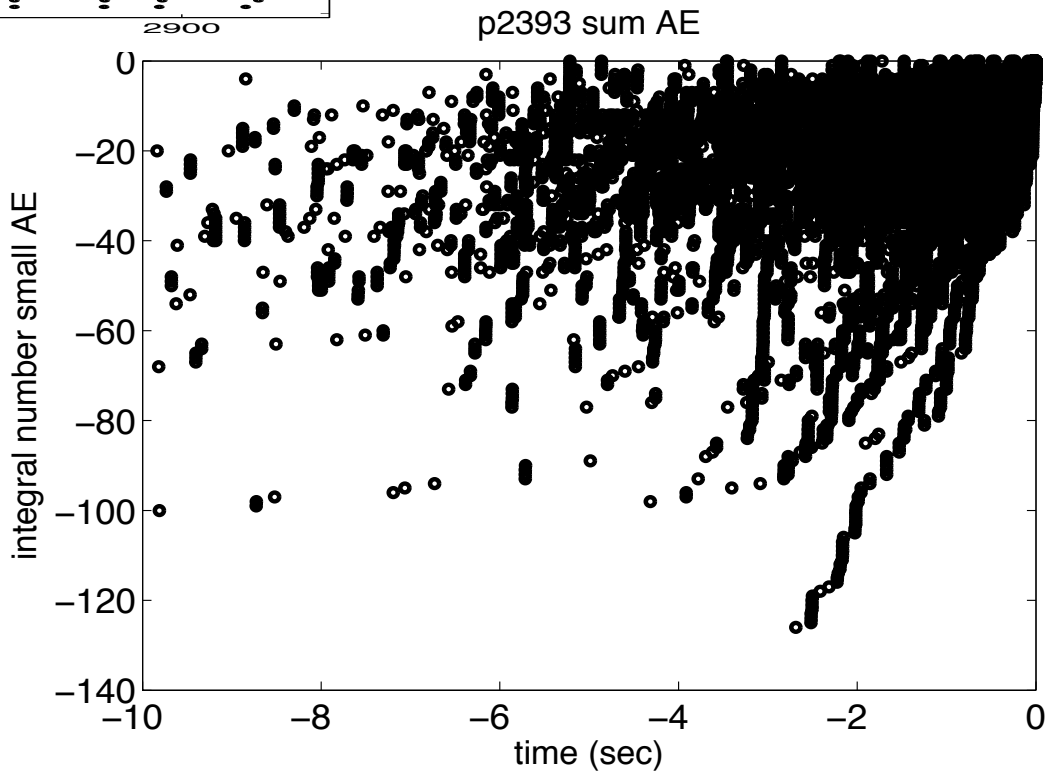
Johnson et al., in preparation, 2013

Sum of AE, each slip event



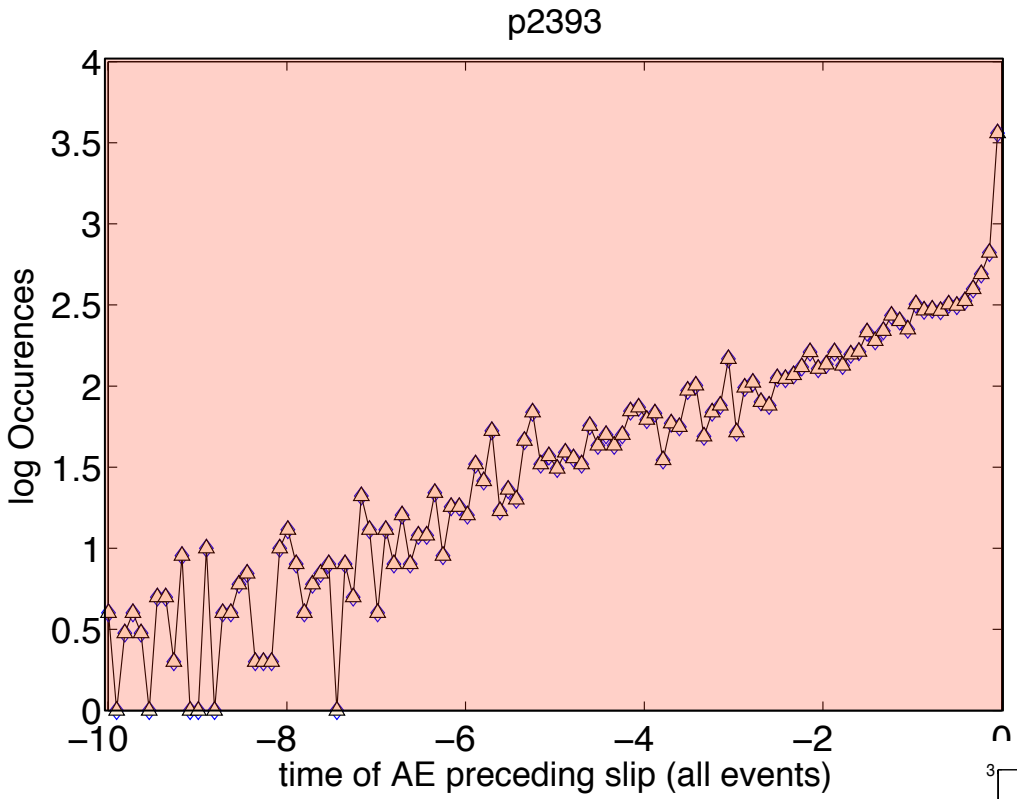
0 0 0 0

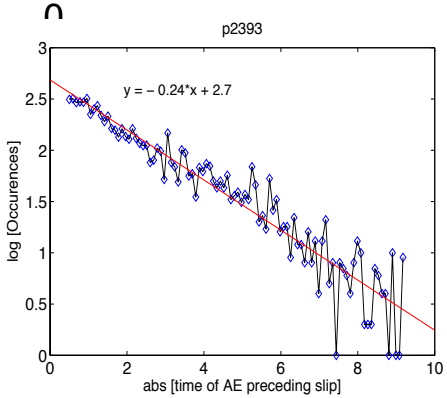
0.....



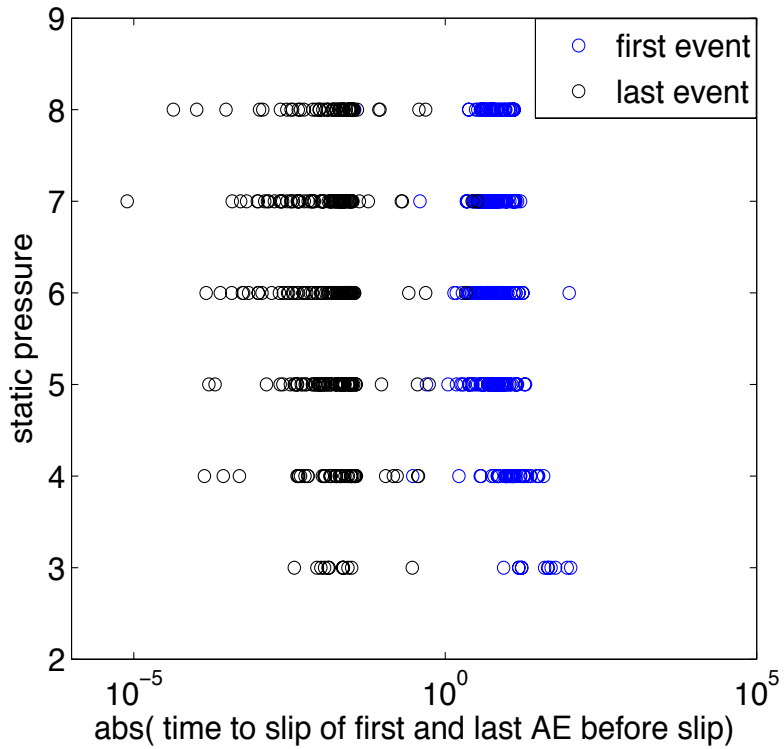
Johnson et al., in preparation, 2013

Histogram of small magnitude AE preceding slip





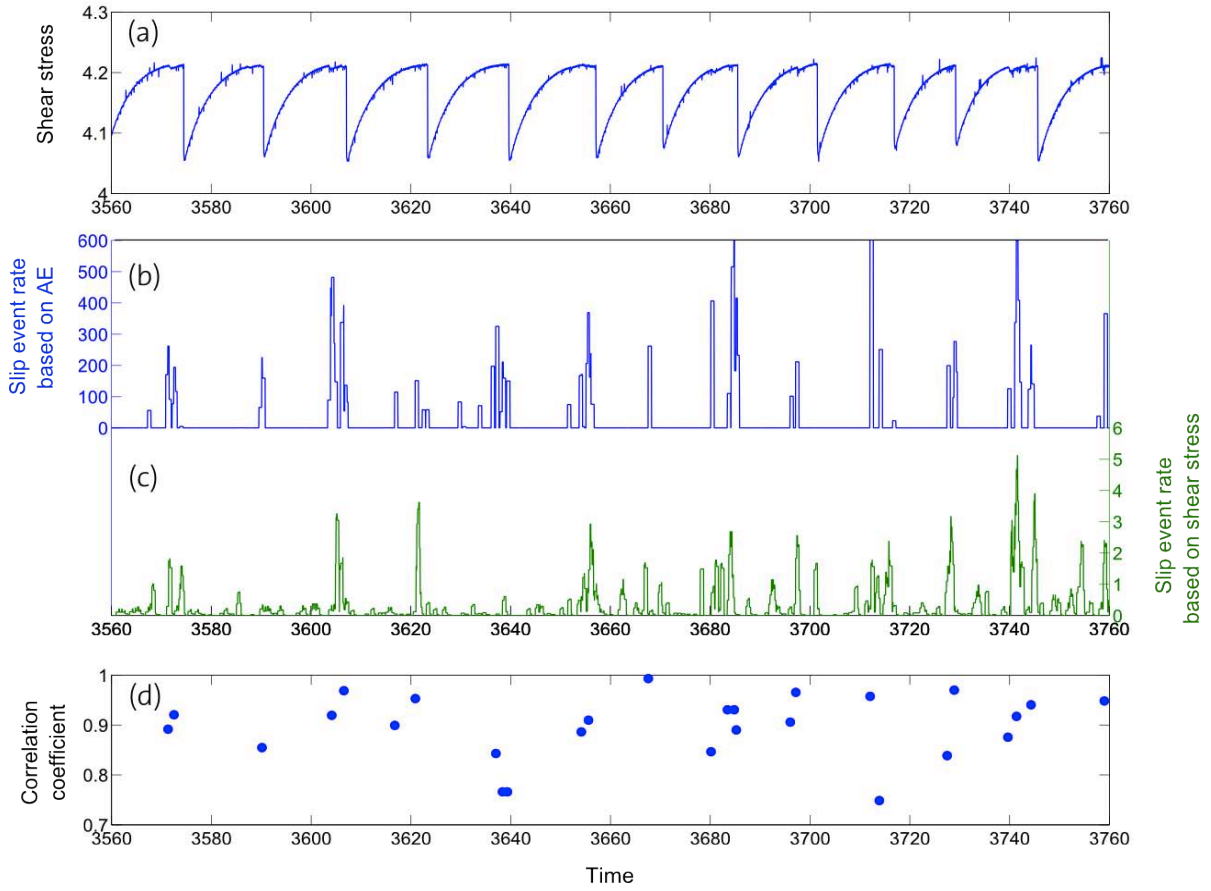
Johnson et al., in preparation, 2013



Insert dat from p2393….

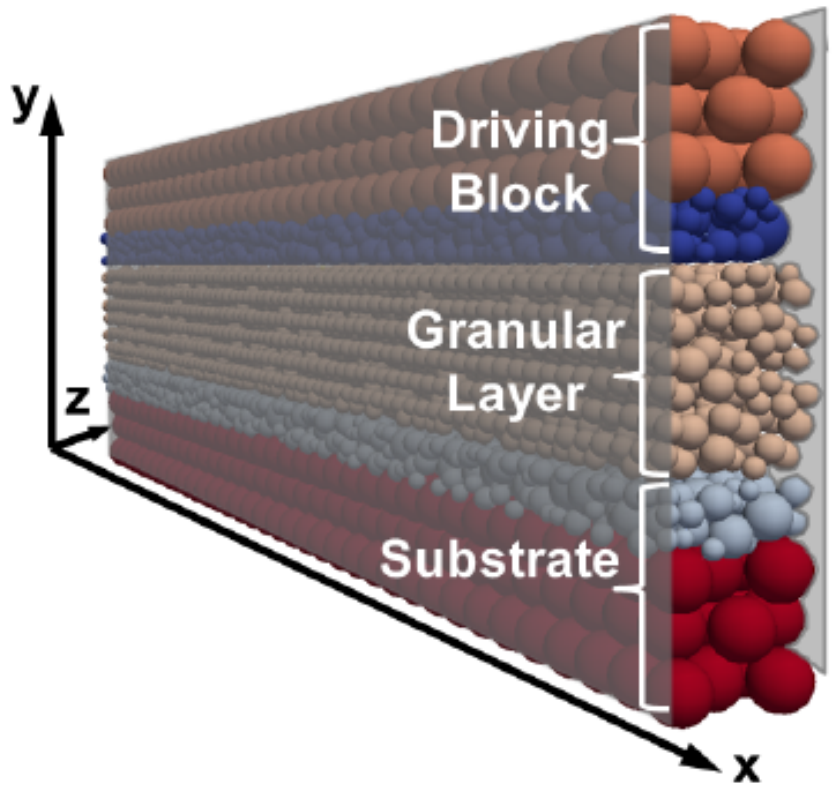
Johnson et al., in preparation, 2013

AE and microslips correlate



Johnson et al., in preparation, 2013

DEM model of Gouge layer



Hexagonal
Close Packing
driving layer

roughness layer

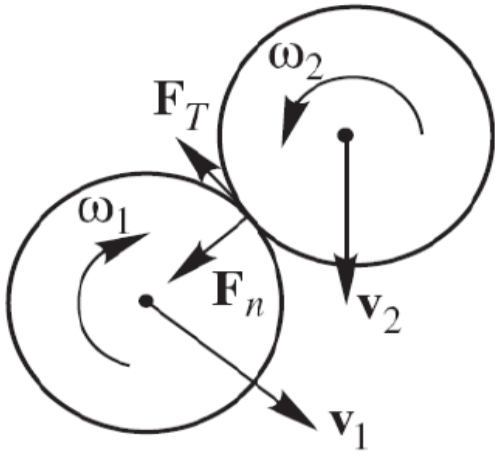
gouge layer
unbonded particles
with size distribution

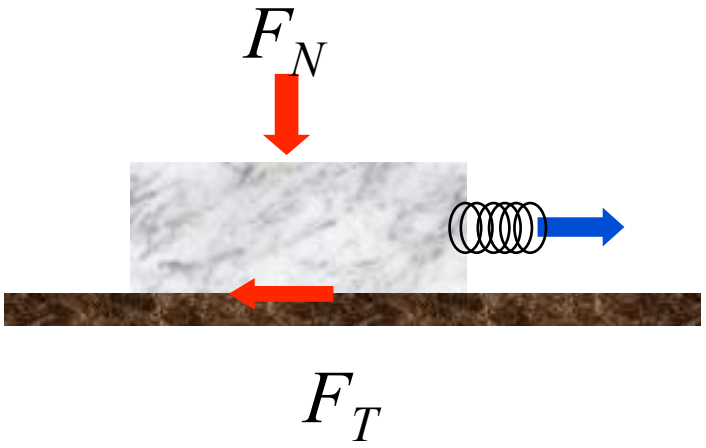
100 – 150 μm

Ferdowsi et al., in review 2013

DEM modeling

Motion of particles solving Newton´s equations





- Normal contact force
- Tangential contact force

Coulomb friction law $\left|F_T\right| \leq \mu \cdot \left|F_N\right|$

μ_S: static friction

μ_D: dynamic friction

Ferdowsi et al., in review 2013

Particle kinetic energy

$$K_{lin}{}^{i} \equiv \frac{1}{2} \cdot m_i \cdot \left\| \dot{\overrightarrow{u_i}} \right\|^2$$ translational kinetic energy

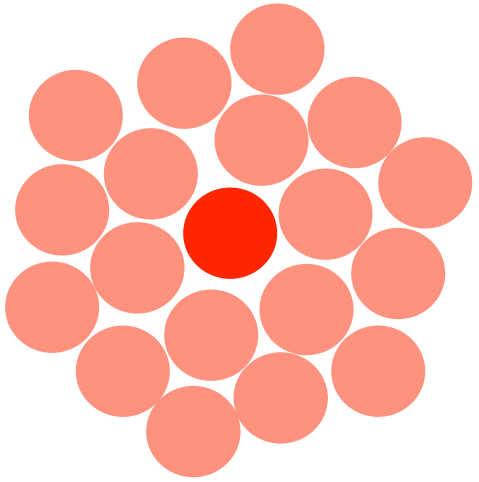
$$K_{rot}{}^{i} \equiv \frac{1}{2} \cdot I_i \cdot {\omega_i}^2$$ rotational kinetic energy

$m_i \equiv$ mass

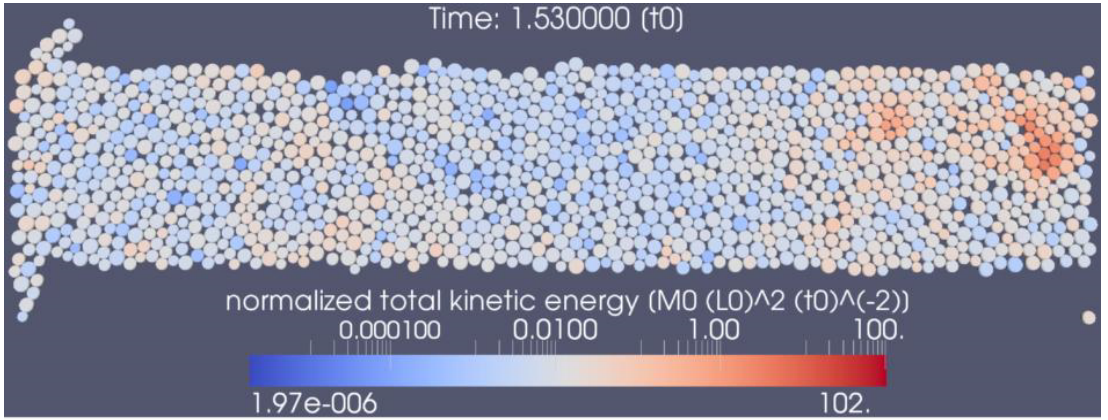
$\overrightarrow{u_i} \equiv$ displacement vector

$I_i \equiv$ moment of inertia

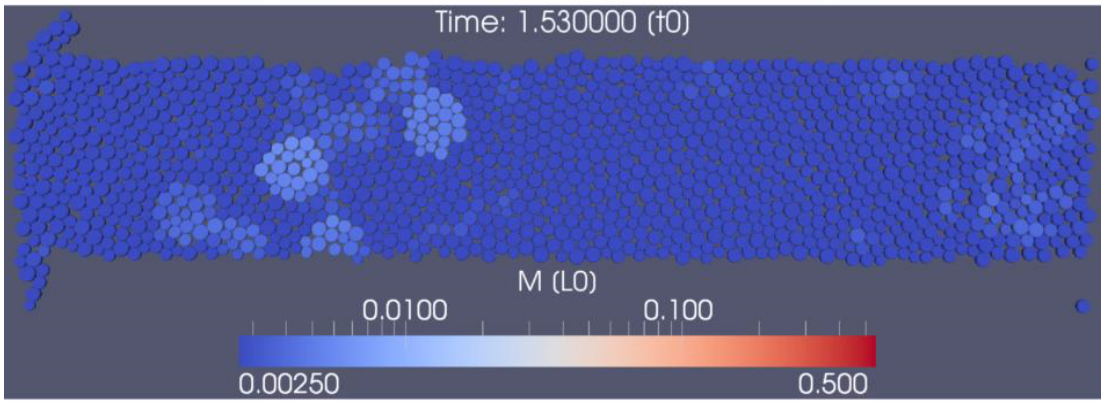
$\omega_i \equiv$ angular velocity

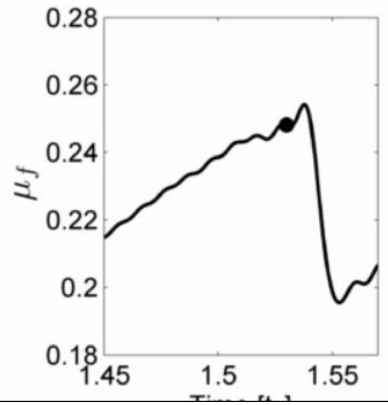


Total kinetic energy (normalized)

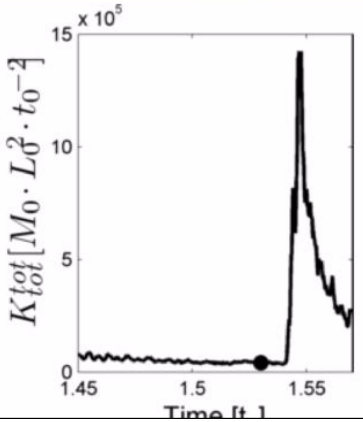


Non-affine deformation

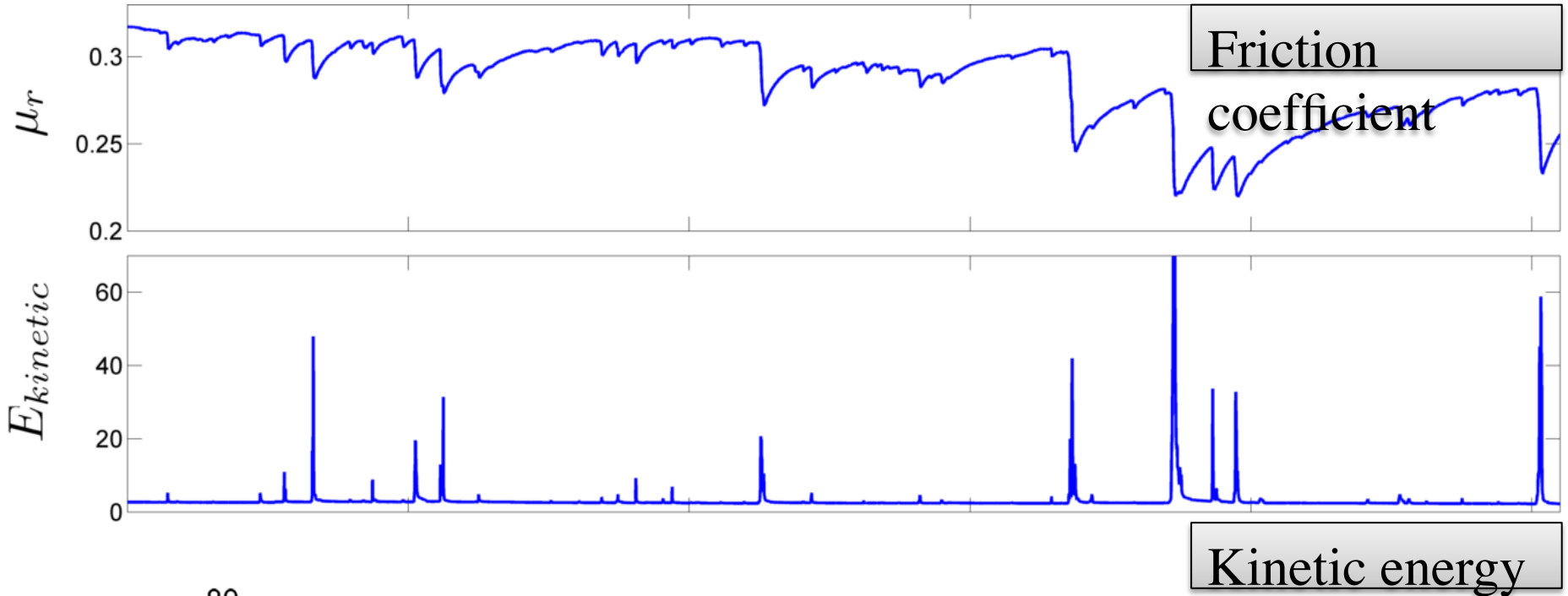


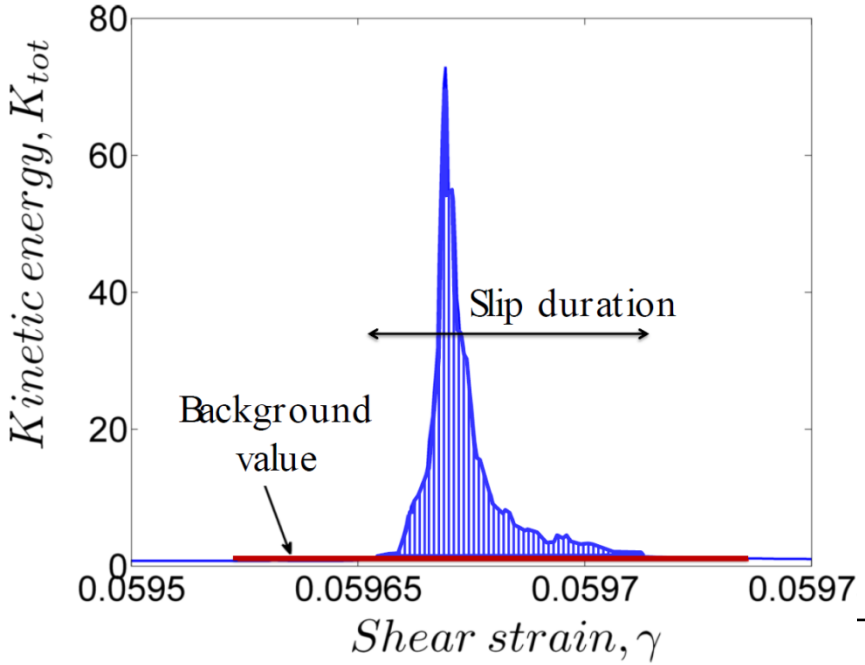


friction coefficient

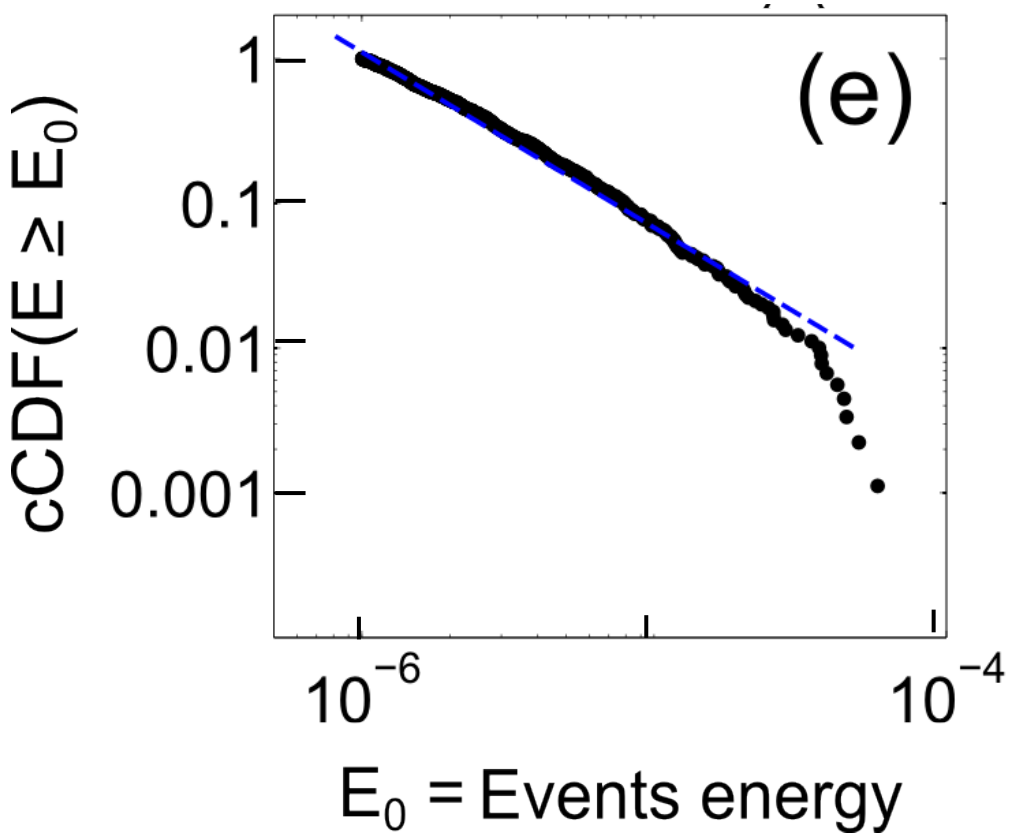


total kinetic energy


μ_r
0.3
0.25
0.2
$E_{kinetic}$
60
40
20
0
Friction coefficient
Kinetic energy


$Kinetic\ energy, K_{tot}$
80
60
40
20
0
Slip duration
Background value
0.0595
0.05965
0.0597
0.0597
$Shear\ strain, \gamma$

DEM Distribution of events



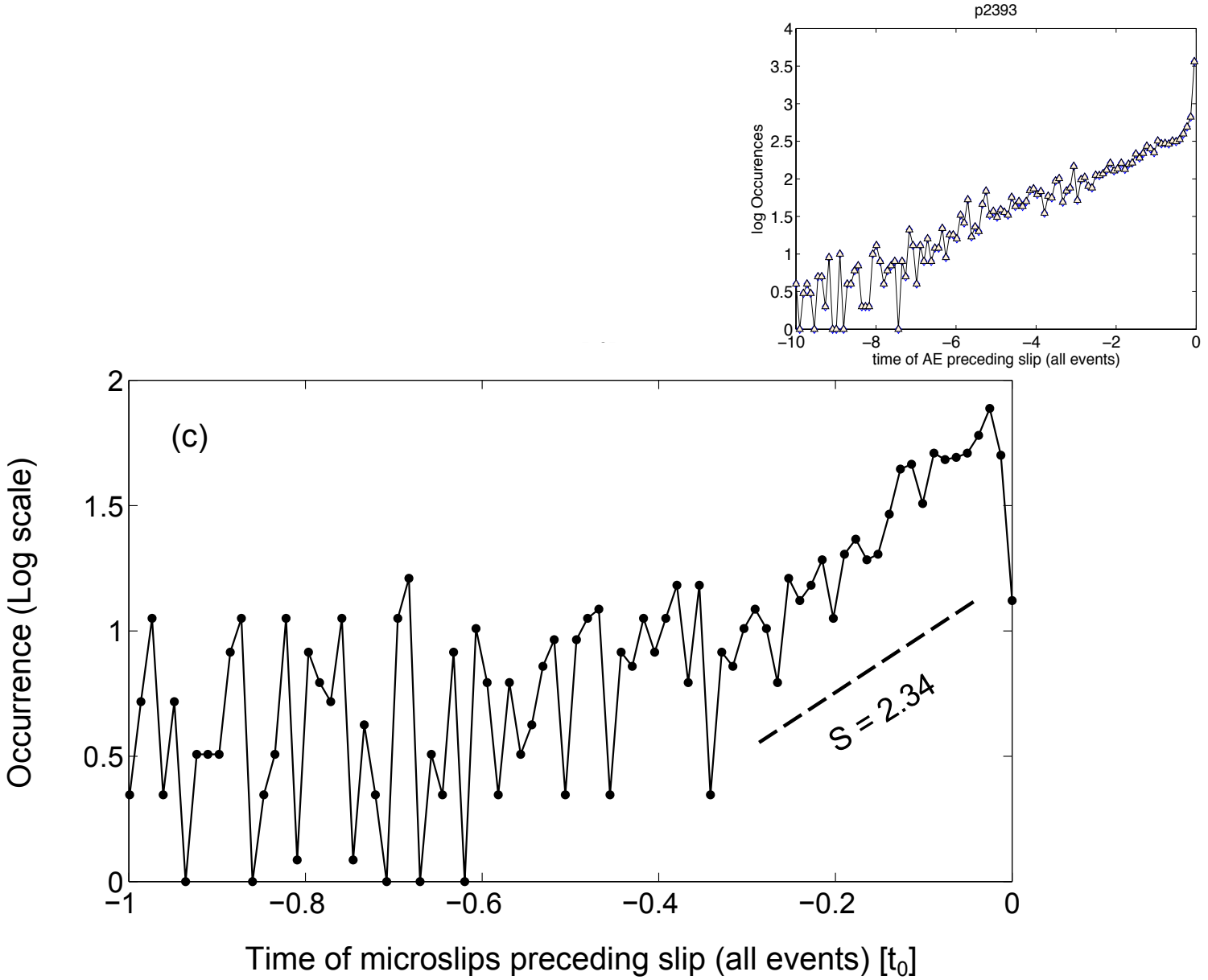
$$cCDF(E) \propto E^{-\beta}$$

$$\beta \simeq 1.23$$

GR - type

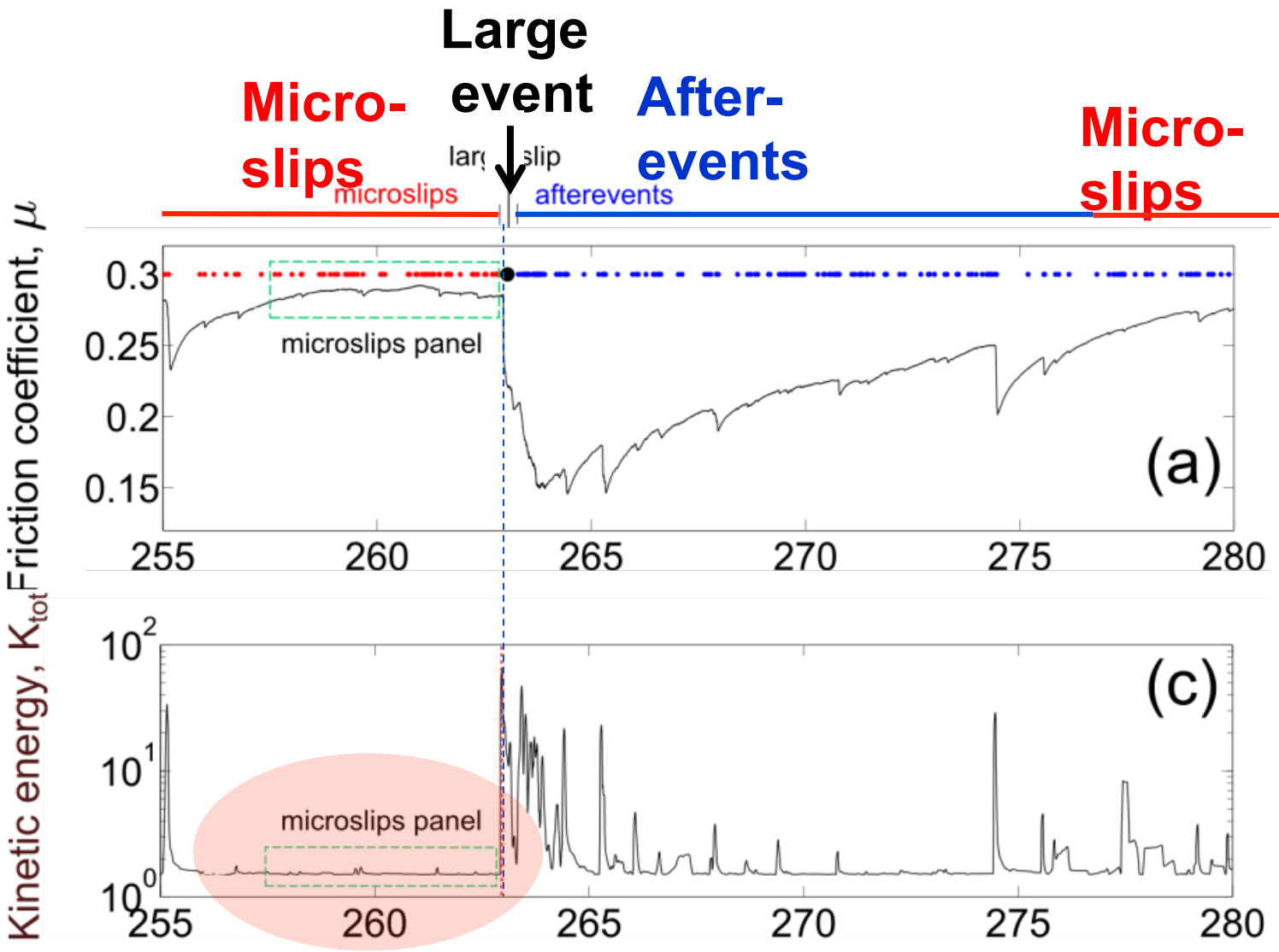
Ferdowsi et al., in review 2013

DEM event occurrence preceding stick slip events



Johnson et al., in preparation, 2013

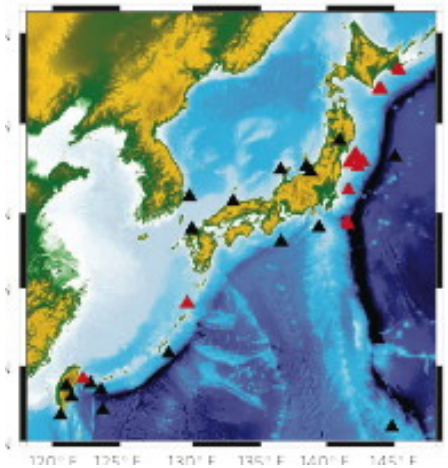
Slips associated with kinetic energy release

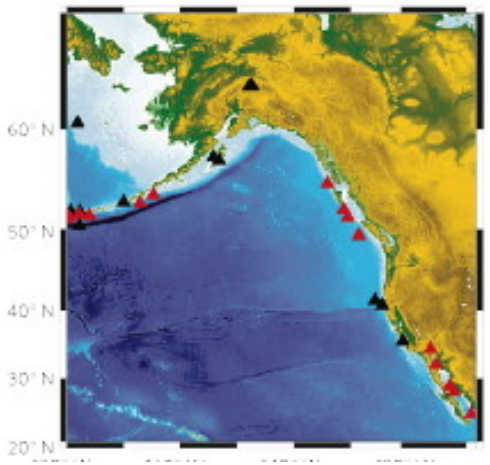


We infer that grain rearrangements are responsible for micrslips/ AE in the lab experiments

Ferdowsi et al., in review 2013

Pecursors in earth





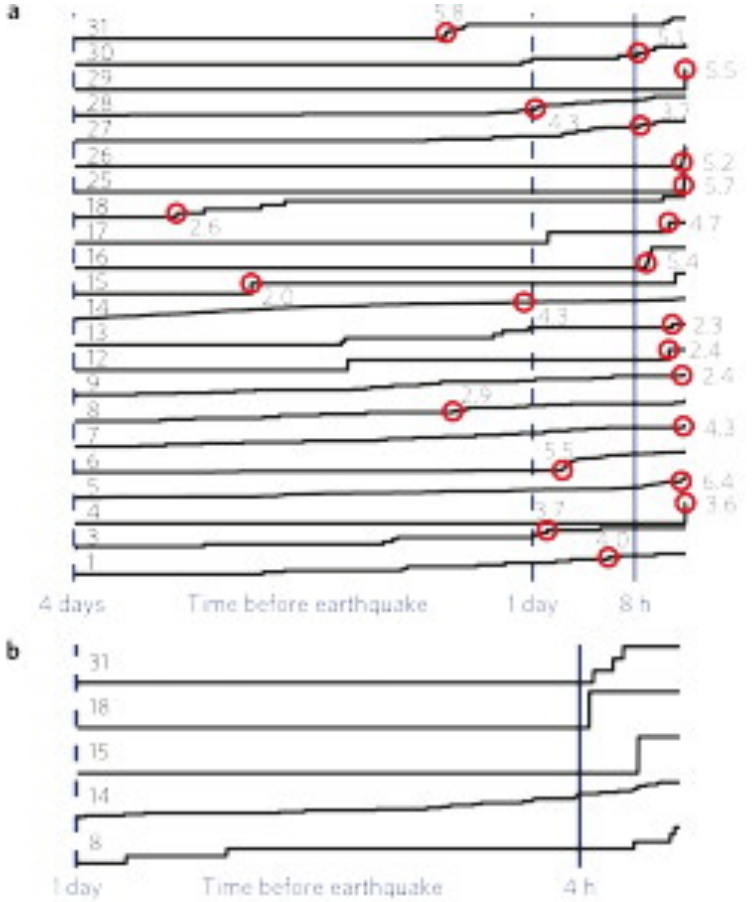


Figure 2 | Evolution of the cumulative numbers of events in the few days before 22 interplate earthquakes (~70% of the data set). All of the events

- Precursors are relative common for interplate earthquakes.
- Mechanism is uncertain, but may represent slow slip preceding stick-slip.
- We observe no pre-slip in experiments (sensitivity issue?).

Bouchon et al., Nature Geo., 2013

Summary/conclusions

Nonlinear effects are strong in earth materials, and significant in many earth processes

NL response is comprised of fast and slow dynamics

Nonlinear response is involved in triggering in lab studies

Precursors are ubiquitous in lab studies and supported by DEM simulation, due to grain re-arrangement

Passive noise can be used to follow elastic changes in Earth