Active and passive elastography: a medical imaging tool of elastic waves

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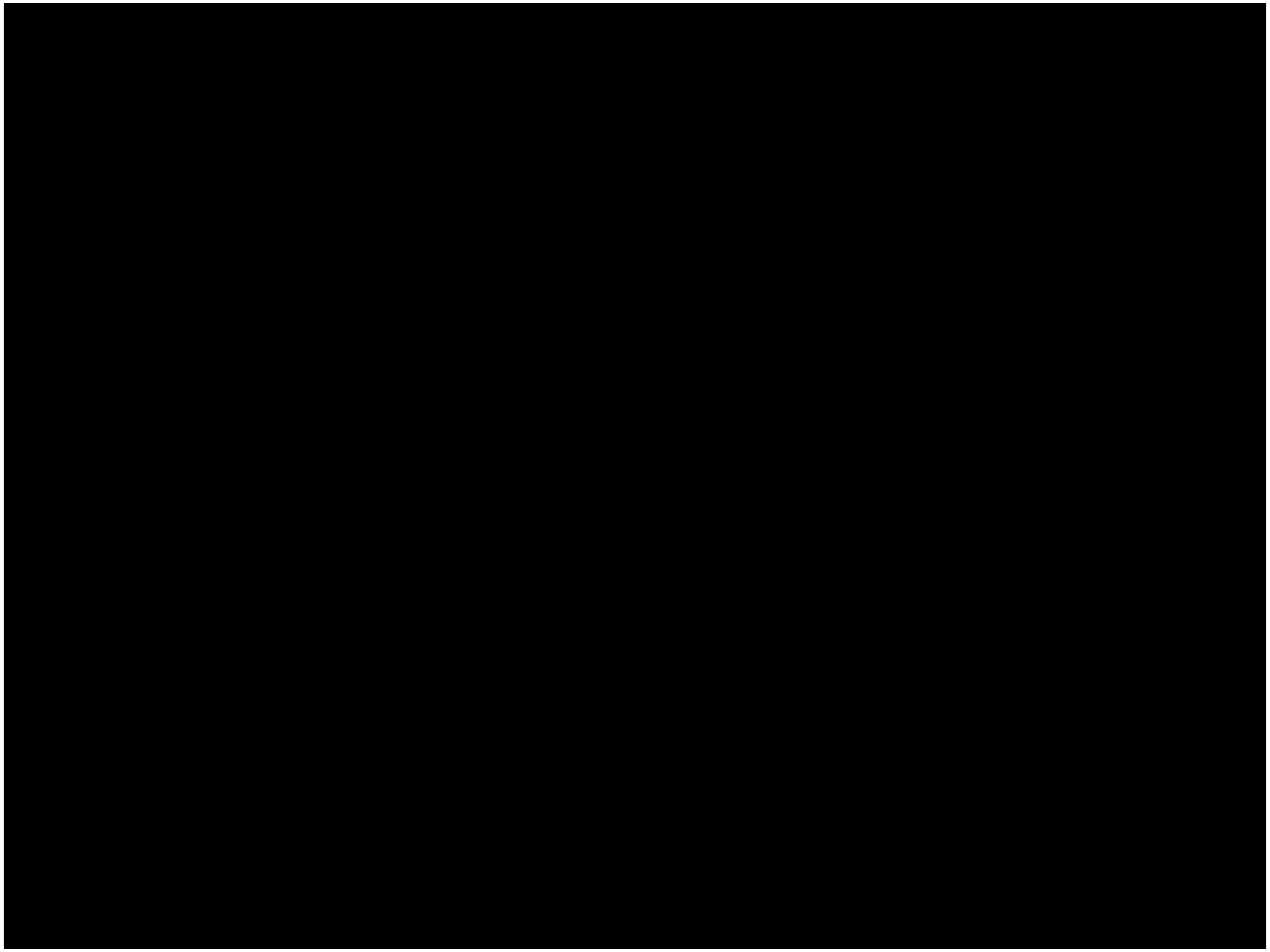
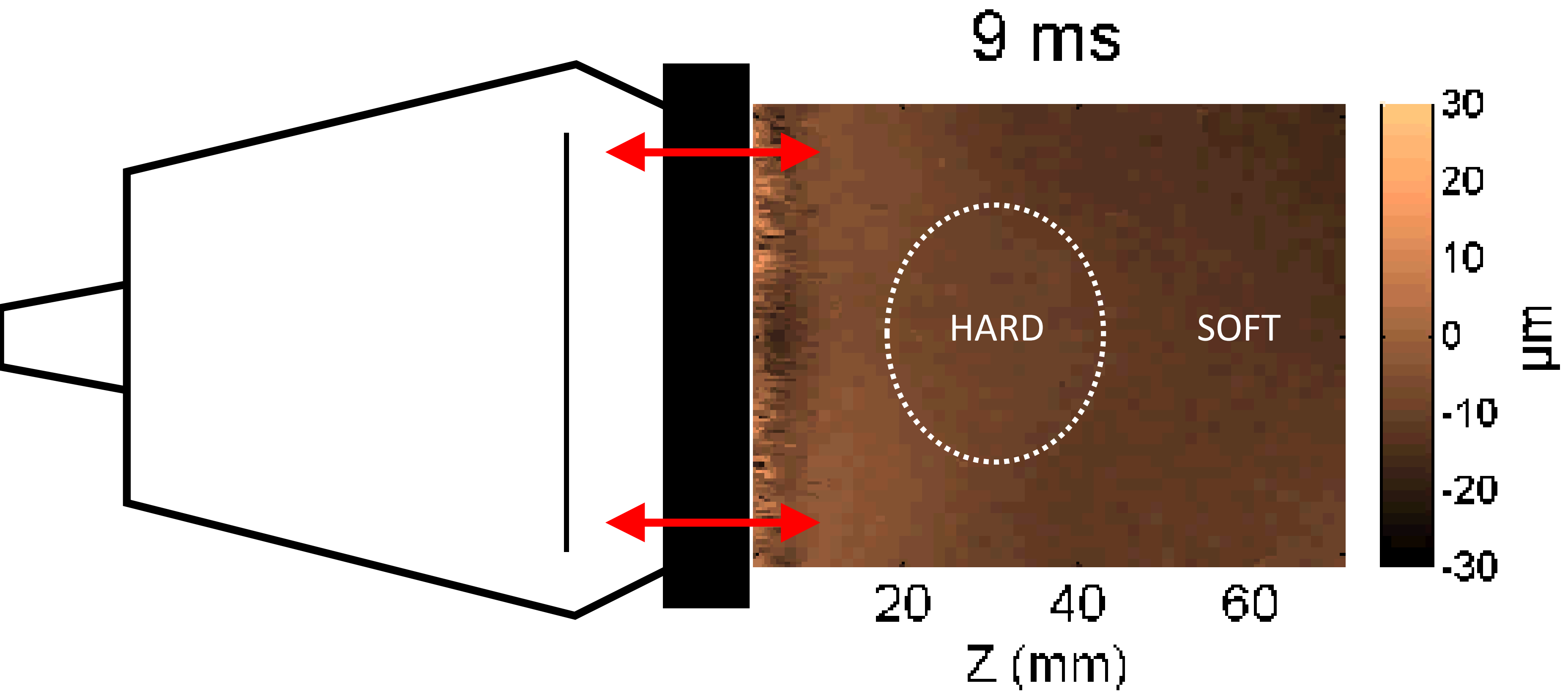
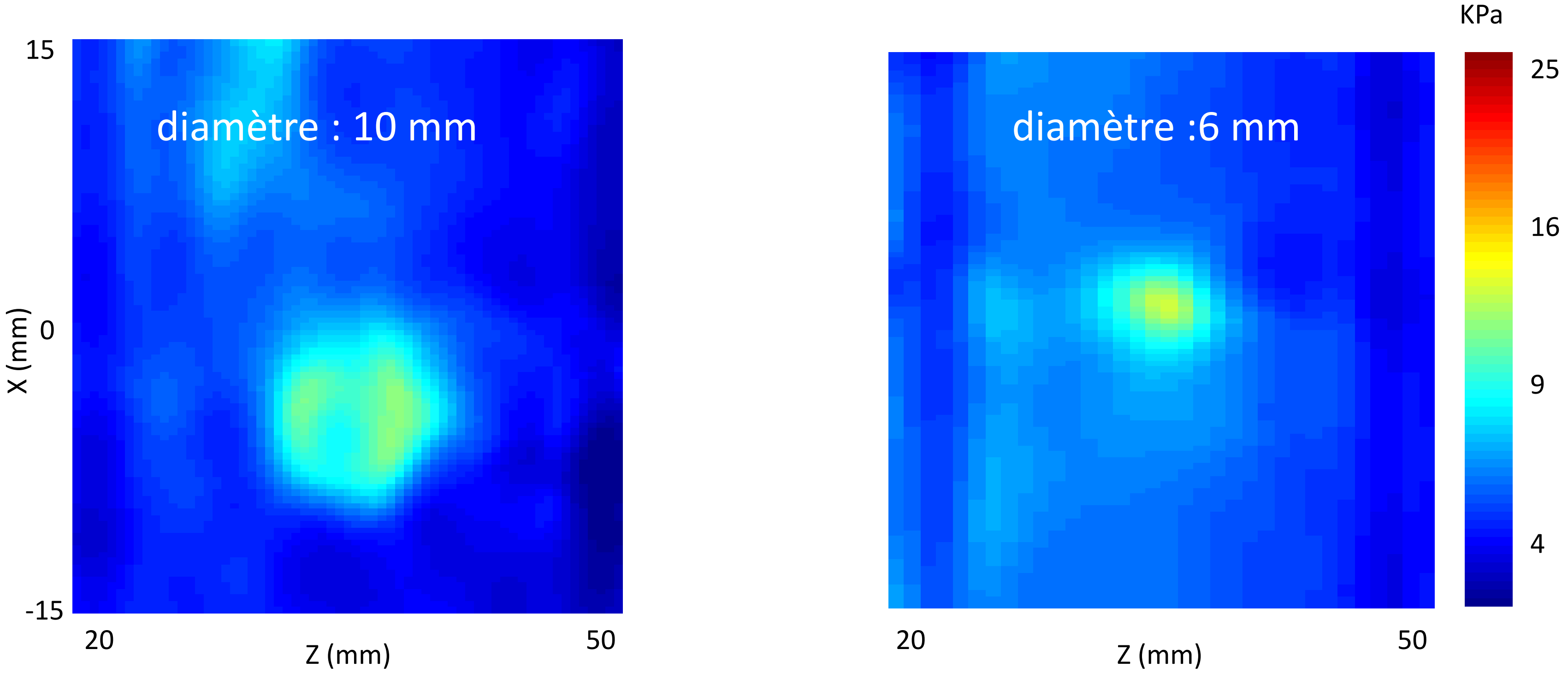
Part I: Overview of elastography



Experimental movie of the z component of the displacements



Example of inclusions in gels



Brevet n° FR99 03157 déposé le 16 Mars 1999 : "Imagerie sismique des ondes de cisaillement",
Laurent Sandrin, Mickael Tanter, Stefan Catheline, Mathias Fink

Echosens (2003): le Fibroscan

Supersonic Imagine (2008): l'Aixplorer

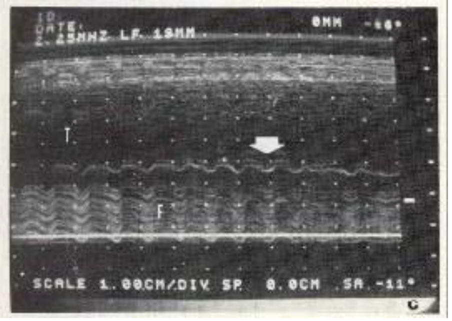
Now: Philips, Siemens…

Years

Qualitatif

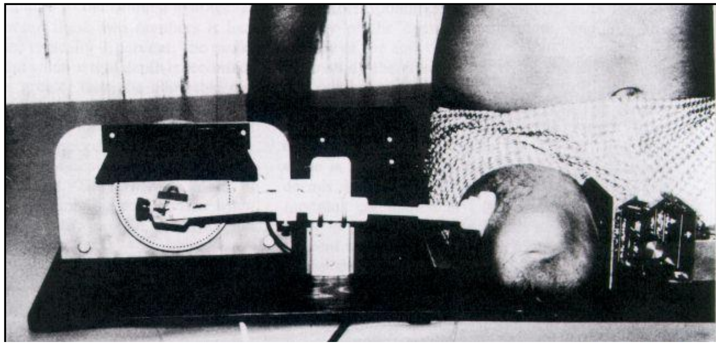
1981 Natural motion (Dickinson)

1983 Vibrator (Eisencher Echosismography)



Quantitatif

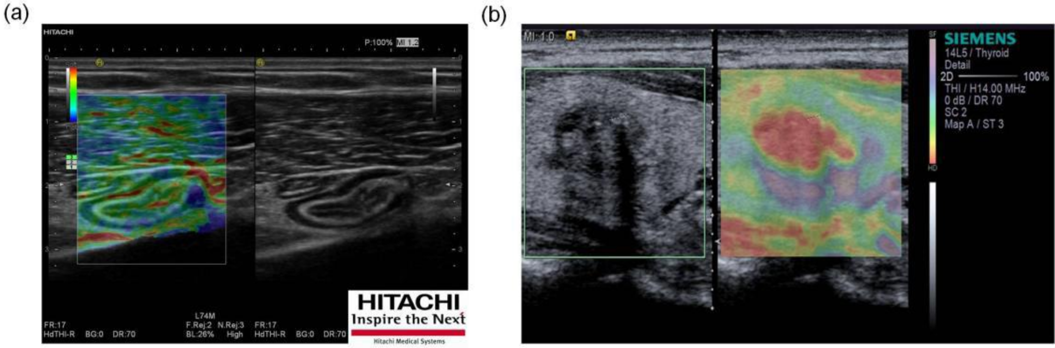
1987: **Monochromatic** + Doppler (Krouskop)



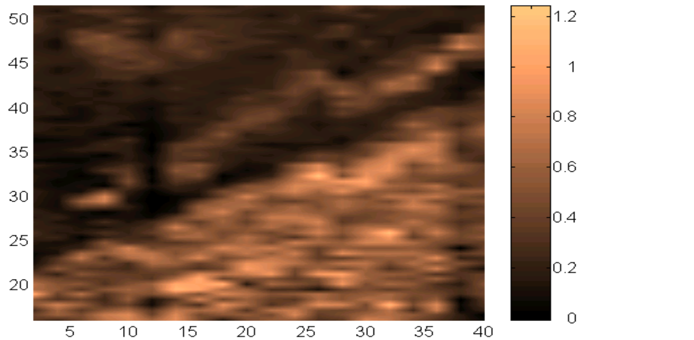
1991: **Static** (Ophir)

Hooke's law:

$$T_{ij} = C_{ijkl} S_{kl}$$



1998: **Pulse** (Fink, Catheline)



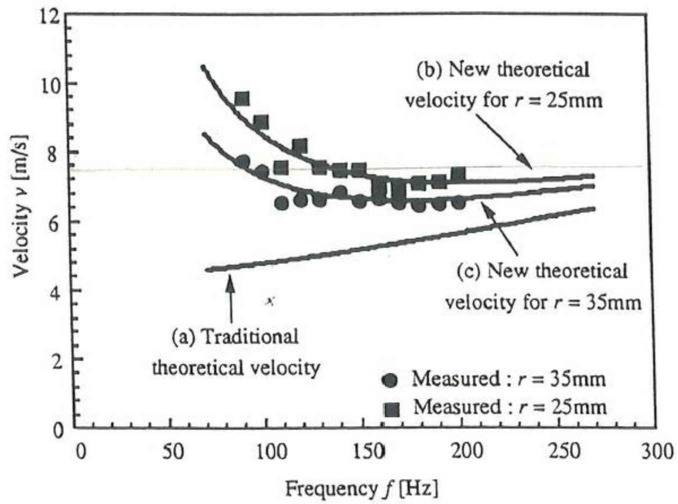
Sonoelasticity in soft tissues

IMAGING SYSTEM OF PRECISE HARDNESS DISTRIBUTION IN SOFT TISSUE IN VIVO USING FORCED VIBRATION AND ULTRASONIC DETECTION

Katsunori Fujii*, Takuso Sato*, Keisuke Kameyama*, Toshikazu Inoue*, Katsunori Yokoyama* and Koichi Kobayashi**

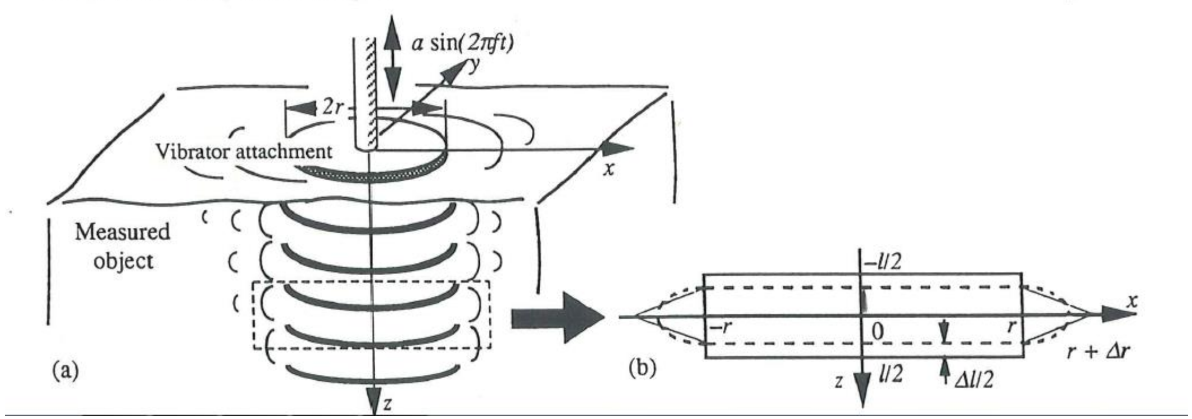
* Interdisciplinary Graduate School of Science and Engineering, Tokyo Institute of Technology , 4259 Nagatsuta, Midori-ku, Yokohama 227, Japan

** University of Tokyo, Japan



Acoustical Imaging **21**, 253 (1994)

Fig. 1. Propagating velocity of the low frequency vibration. (a) Traditional formulation calculated from Eq. (2), New formulation calculated for two kinds of attachments ((b) r=25 and (c) 35mm) and the experimental results obtained in the setup shown in Fig. 3.



What is learnt at school about shear and compression waves

Elastic wave equation $(\lambda+2\mu)\vec{\nabla}(\vec{\nabla}\vec{u})-\mu\vec{\nabla}\wedge\vec{\nabla}\wedge\vec{u}-\rho\frac{\partial^2\vec{u}}{\partial t^2}=\vec{0}$ ⟷ (Math/Physics) $(\lambda+2\mu)\overrightarrow{grad}.div\,\vec{u}-\mu\,\overrightarrow{rot}\,\overrightarrow{rot}.\vec{u}-\rho\frac{\partial^2\vec{u}}{\partial t^2}=\vec{0}$

Separation in 2 independent wave equations

- $\boxed{\overrightarrow{rot}.\vec{u}=\vec{0}}$ $\overrightarrow{grad}.div\,\vec{u}-\frac{1}{C_P^2}\frac{\partial^2\vec{u}}{\partial t^2}=\vec{0}$ with $C_P=\sqrt{\frac{\lambda+2\mu}{\rho}}$

Rotational-free strain
(without rotation)

$$\begin{cases}\frac{\partial^2 u_x}{\partial x^2}-\frac{1}{C_P^2}\frac{\partial^2 u_x}{\partial t^2}=0\\ \frac{\partial^2 u_y}{\partial y^2}-\frac{1}{C_P^2}\frac{\partial^2 u_y}{\partial t^2}=0\\ \frac{\partial^2 u_z}{\partial z^2}-\frac{1}{C_P^2}\frac{\partial^2 u_z}{\partial t^2}=0\end{cases}$$

Harmonic plane wave decomposition ⟷

$$\begin{cases}-k_x^2u_x+(\frac{\omega}{C_P})^2u_x=0\\ -k_y^2u_y+(\frac{\omega}{C_P})^2u_y=0\\ -k_z^2u_z+(\frac{\omega}{C_P})^2u_z=0\end{cases}$$

$\vec{k}$ is parallel to $\vec{u}$

Indeed if k_y=k_z=0 then u_y=u_z=0

→ The compression wave is longitudinal

- $\boxed{div.\vec{u}=0}$ $-\overrightarrow{rot}.\overrightarrow{rot}\,\vec{u}-\frac{1}{C_S^2}\frac{\partial^2\vec{u}}{\partial t^2}=\vec{0}$ with $C_S=\sqrt{\frac{\mu}{\rho}}$

Divergence-free strain
(without change of volume)

$$\begin{cases}\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2}-\frac{1}{C_S^2}\frac{\partial^2 u_x}{\partial t^2}=0\\ \frac{\partial^2 u_y}{\partial x^2}+\frac{\partial^2 u_y}{\partial z^2}-\frac{1}{C_S^2}\frac{\partial^2 u_y}{\partial t^2}=0\\ \frac{\partial^2 u_z}{\partial x^2}+\frac{\partial^2 u_z}{\partial y^2}-\frac{1}{C_S^2}\frac{\partial^2 u_z}{\partial t^2}=0\end{cases}$$

Harmonic plane wave decomposition ⟷

$$\begin{cases}-k_y^2u_x-k_z^2u_x+(\frac{\omega}{C_S})^2u_x=0\\ -k_x^2u_y-k_z^2u_y+(\frac{\omega}{C_S})^2u_y=0\\ -k_x^2u_z-k_y^2u_z+(\frac{\omega}{C_S})^2u_z=0\end{cases}$$

$\vec{k}$ is perpendicular to $\vec{u}$

Indeed if k_y=k_z=0 then u_x=0

→ The shear wave is transverse

The importance of the source term

Full elastic wave equation $$(\lambda+2\mu)\overrightarrow{grad}.div\,\vec{u}-\mu\,\overrightarrow{rot}\,\overrightarrow{rot}.\vec{u}-\rho\frac{\partial^2\vec{u}}{\partial t^2}=\delta(\vec{r})\delta(t)\vec{n}$$

~~Separation in 2 independent wave equations~~ Much trickier

The complete solution was first given by Stockes in 1849. ("Quantitative seismology" Aki & Richards, "Waves in elastic solids"Achenbach…)
The Green's function is:

$$G_{mn}(\mathbf{0},\mathbf{r})=\frac{1}{4\pi\rho\alpha^2}\frac{\gamma_m\gamma_n}{r}e^{iqr}+\frac{1}{4\pi\rho\beta^2}\frac{\delta_{mn}-\gamma_m\gamma_n}{r}e^{ikr}+\frac{1}{4\pi\rho}\frac{3\gamma_m\gamma_n-\delta_{mn}}{r^3}\left[\frac{e^{ikr}}{i\omega}\left(\frac{r}{\beta}-\frac{1}{i\omega}\right)-\frac{e^{iqr}}{i\omega}\left(\frac{r}{\alpha}-\frac{1}{i\omega}\right)\right]$$

L-wave T-wave Near field term

$$G_{mn}(\mathbf{0},\mathbf{r}) = G^{L}_{mn}(\vec{0},\vec{r}) + G^{T}_{mn}(\vec{0},\vec{r}) + G^{NF}_{mn}(\vec{0},\vec{r})$$

The far-field culture dominates wave physics, specially in ultrasounds.
But, in seismology and in elastography, the near field cannot be avoided.

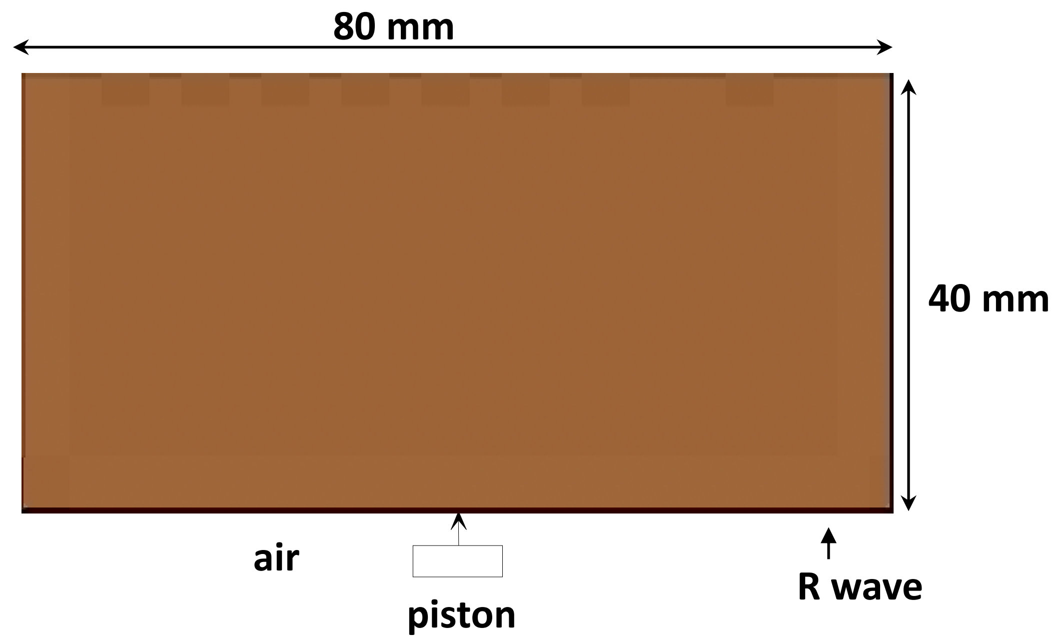
The near field term is responsible for the longitudinal shear wave.
Commercial applications by Echosens for the liver fibrosis started in 2003.

The pulsed approach and the Green's function solids

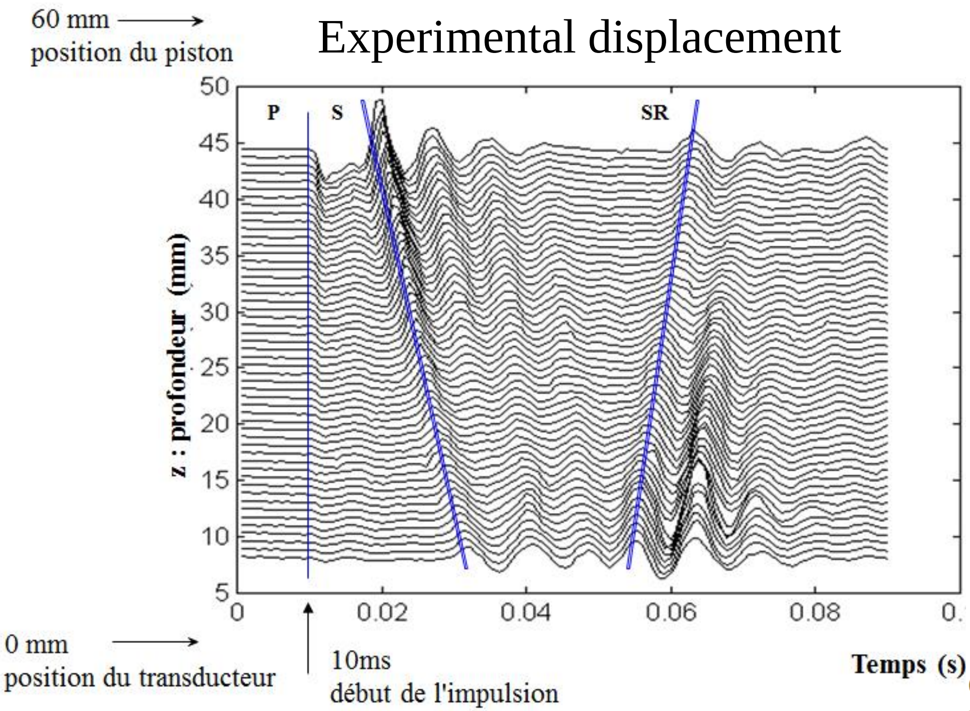
Numerical simulation of the Green 's function *(Gakenheimer &Miklowitz)*

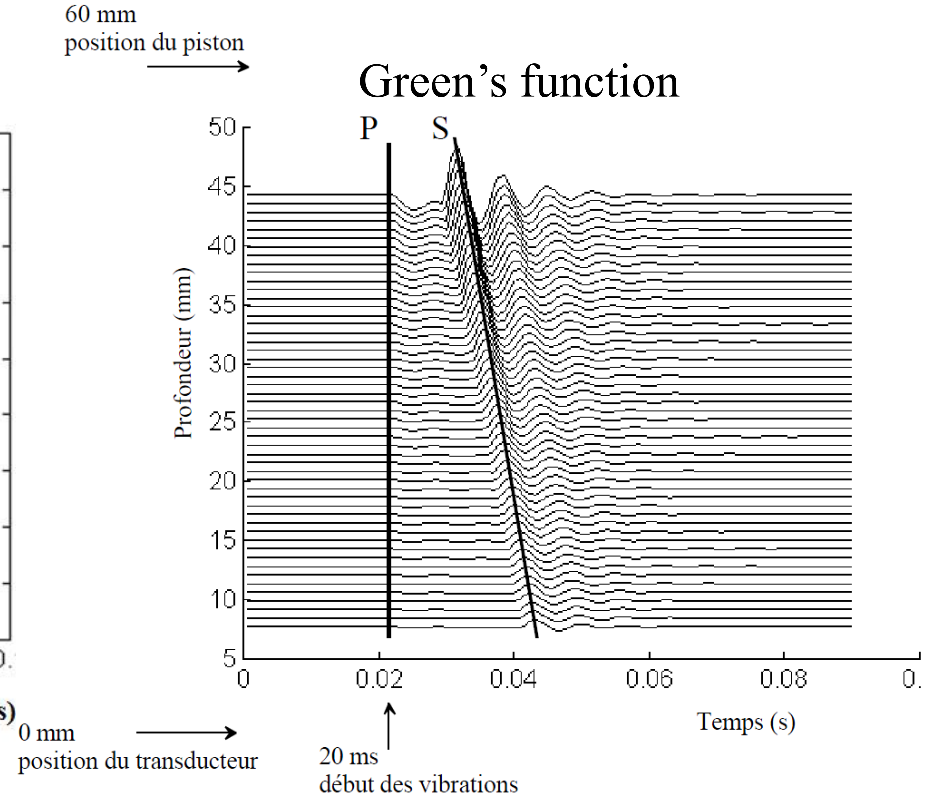
Longitudinal component : u_z

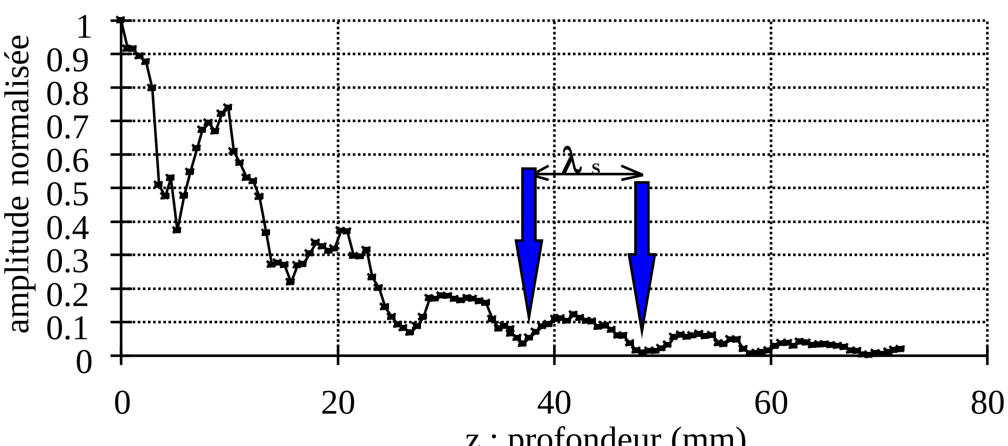
Central frequency: 200 Hz

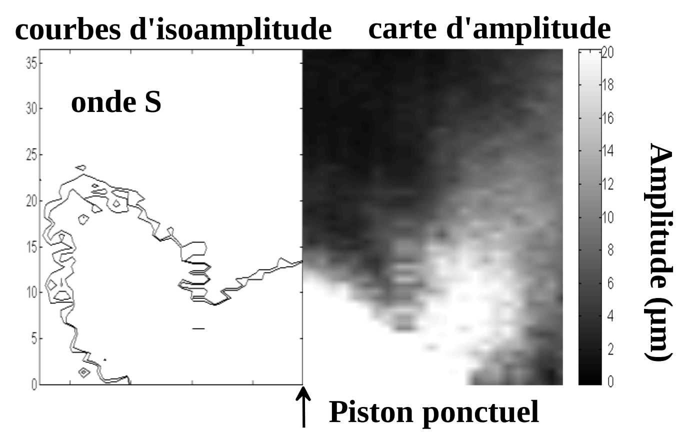


Theory versus experiment









Questions

$$G_{mn}(\mathbf{0},\mathbf{r}) = \frac{1}{4\pi\rho\alpha^2}\frac{\gamma_m\gamma_n}{r}e^{iqr} + \frac{1}{4\pi\rho\beta^2}\frac{\delta_{mn}-\gamma_m\gamma_n}{r}e^{ikr} + \frac{1}{4\pi\rho}\frac{3\gamma_m\gamma_n-\delta_{mn}}{r^3}\left[\frac{e^{ikr}}{i\omega}\left(\frac{r}{\beta}-\frac{1}{i\omega}\right)-\frac{e^{iqr}}{i\omega}\left(\frac{r}{\alpha}-\frac{1}{i\omega}\right)\right]$$

L-wave T-wave Near field term

$$G_{mn}(\mathbf{0},\mathbf{r}) = G^L_{mn}(\vec{0},\vec{r}) + G^T_{mn}(\vec{0},\vec{r}) + G^{NF}_{mn}(\vec{0},\vec{r})$$

Questions: $\begin{cases} div\ G^{NF} \neq 0 \\ \overrightarrow{rot} G^{NF} \neq \vec{0} \end{cases}$

The near field term is not rotational-free neither divergence free. This is why it is sometimes called ~~coupling term~~. ?#1

Is the longitudinal shear wave really a shear wave? ?#2

$$\overrightarrow{rot}\, G^L = \frac{e^{iqr}}{4\pi\rho\alpha^2 r^2}\left[\gamma_1\hat{\mathbf{1}}+\gamma_2\hat{\mathbf{2}}\right] \neq 0$$

➡ The L-wave slightly rotates: it is not a P-wave

?#3

$$div\ G^T = \frac{-e^{ikr}}{4\pi\rho\beta^2 r^2}\gamma_3 \neq 0$$

➡ The T-wave slightly dilates: it is not a S-wave

The importance of the near field term

Separation in 2 independent near field terms

$$\begin{cases} G_{mn}^{NFP}(\mathbf{0},\mathbf{r}) = \dfrac{1}{4\pi\rho}\dfrac{3\gamma_m\gamma_n - \delta_{mn}}{r^3}\dfrac{e^{iqr}}{i\omega}\left(\dfrac{r}{\alpha} - \dfrac{1}{i\omega}\right) \\ G_{mn}^{NFS}(\mathbf{0},\mathbf{r}) = \dfrac{1}{4\pi\rho}\dfrac{3\gamma_m\gamma_n - \delta_{mn}}{r^3}\dfrac{e^{ikr}}{i\omega}\left(\dfrac{r}{\beta} - \dfrac{1}{i\omega}\right) \end{cases}$$

The residuals can be perfectly compensated for by the near field term

$$\begin{cases} div(G^T + G^{NFS}) = 0 \\ \overrightarrow{rot}(G^L + G^{NFP}) = \vec{0} \end{cases}$$

The two terms Green's function is:

$$G_{mn}(\mathbf{0},\mathbf{r}) = \underbrace{G_{mn}^L + G_{mn}^{NFP}}_{P} + \underbrace{G_{mn}^T + G_{mn}^{NFS}}_{S} = G_{mn}^P + G_{mn}^S$$

- The P-wave is the rotational-free solution of the wave equation but is composed of a longitudinal term and a near field term.

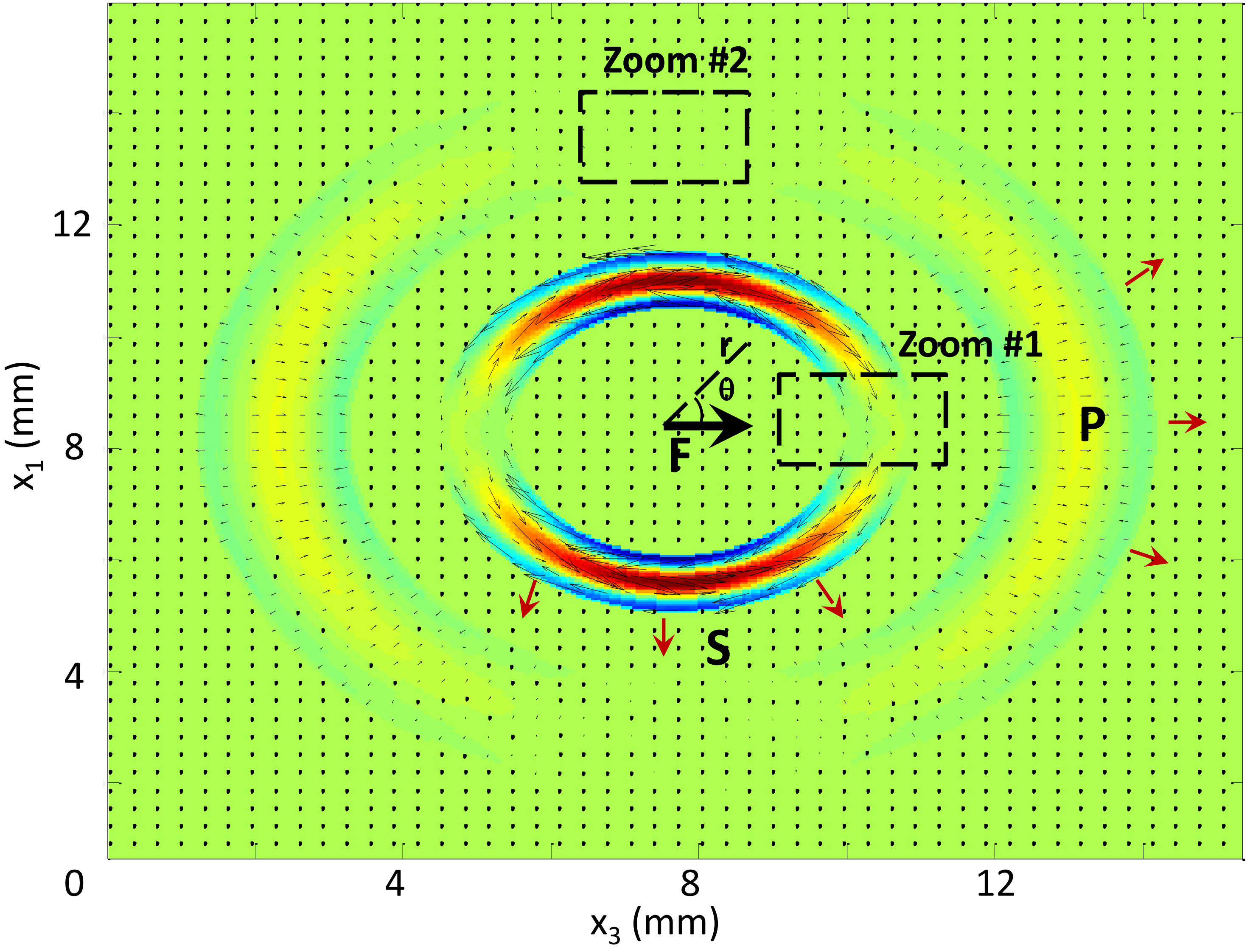
- The S-wave is the divergence-free solution of the wave equation but is composed of transverse term and a near field term.

- Because the near field term can be split into a shear term and a compression term, there is no reason to call it "coupling".

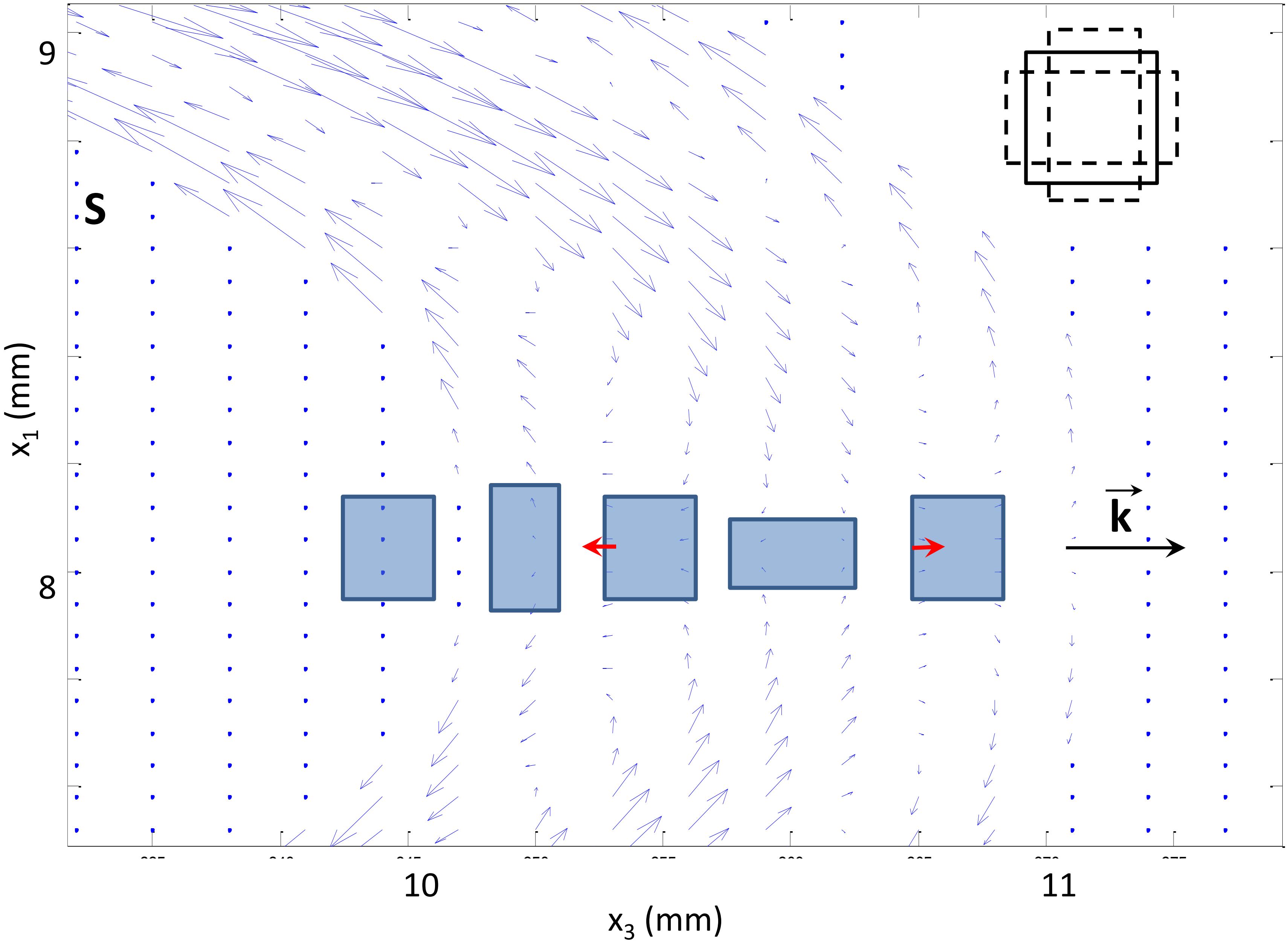
Let's take a look at the near field terms.

The elastic Green's function

Numerical computation of a solid with C_P=2000m.s^{-1} and C_S=1000m.s^{-1}, the central frequency pulse is 1MHz.



Zoom #1: The longitudinal shear wave



Zoom #1: The longitudinal shear wave

- Direction parallel to the source $\theta=0$

Longitudinal polarization

?#2

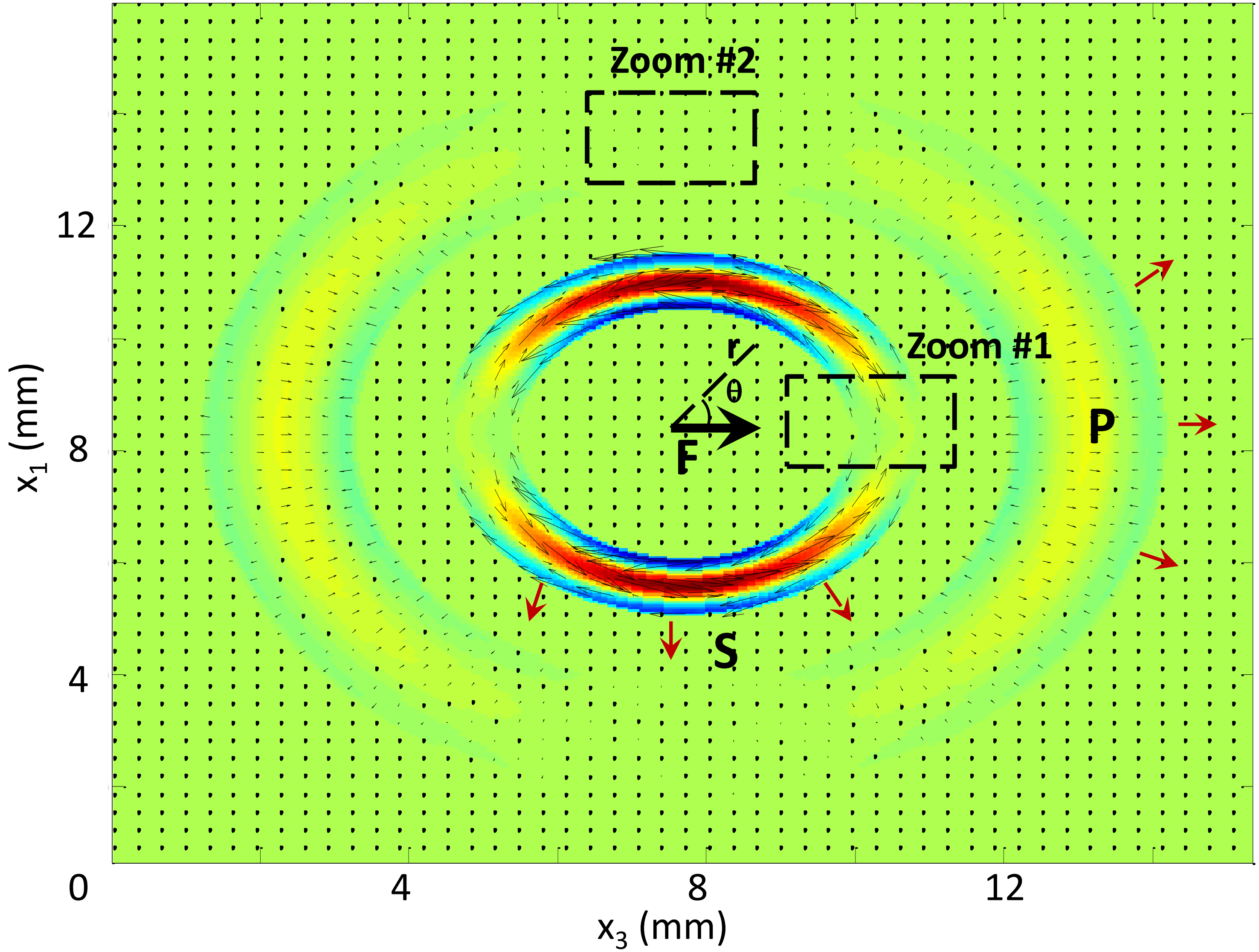
→ This longitudinal shear wave deserves its name, it is indeed a shear wave. More precisely it is the near field part of the shear wave.

→ Yamakoshi and Sato had the right intuition.

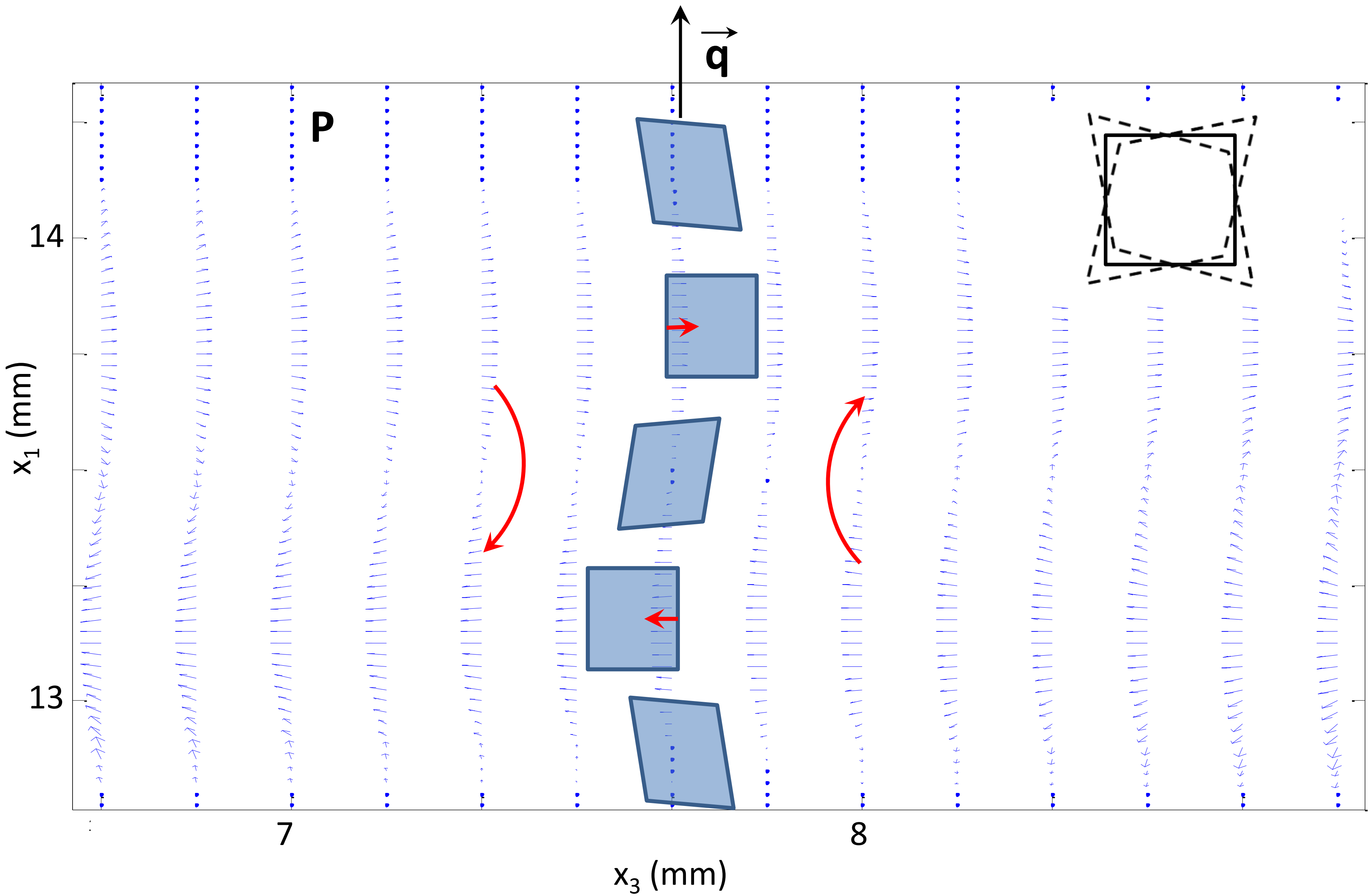
→ This longitudinal shear wave is divergence-free and curl-free. However, the strain tensor is still non-zero.

The elastic Green's function

Numerical computation of a solid with C_P=2000m.s^{-1} and C_S=1000m.s^{-1}, the central frequency pulse is 1MHz.



Zoom #2: The transverse dilatation wave



Zoom #2: The transverse dilatation wave

- Direction perpendicular to the source $\theta=\pi/2$ transversally polarized

➡ This transverse dilatational wave is described for the first time

➡ It is curl-free, divergence-free

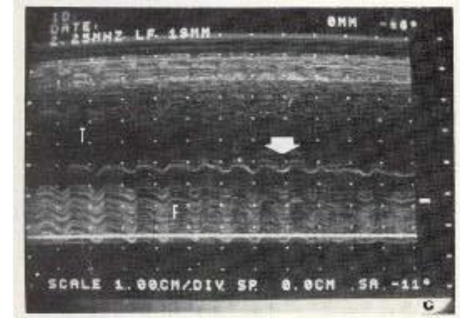
S. Catheline and N.Benech
Longitudinal shear wave and transverse dilatational wave in solids
J. Acoust. Soc. Am. ***137*** *, EL200 (2015).*

Years

Qualitatif

1981 Natural motion (Dickinson)

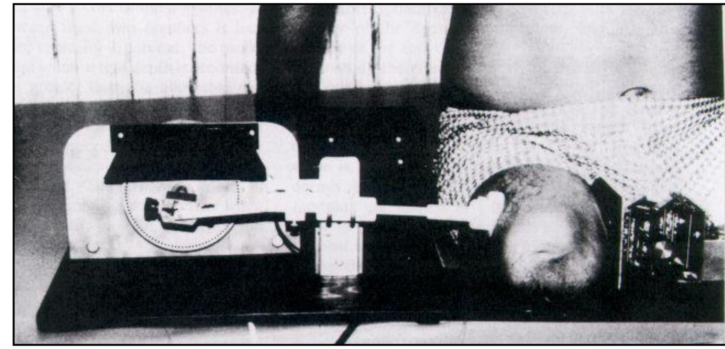
1983 Vibrator (Eisencher Echosismography)



Quantitatif

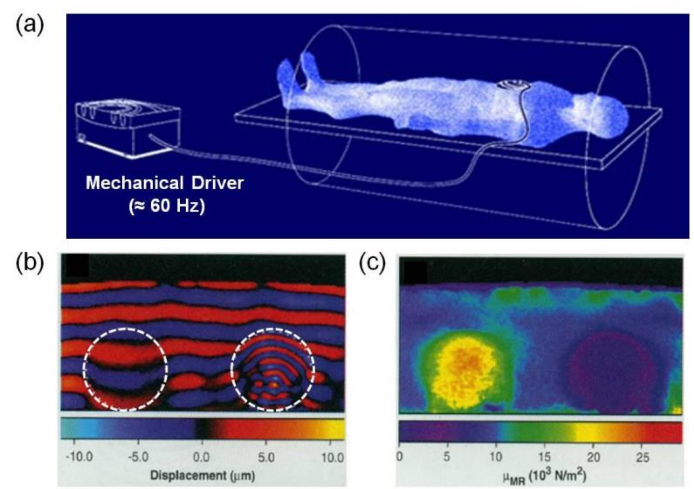
1987: **Monochromatic** + Doppler (Krouskop)

1990: **Monochromatic** + Doppler (Levinson, Parker, Sato)



1991: **Static** +Ultrasound (Ophir)

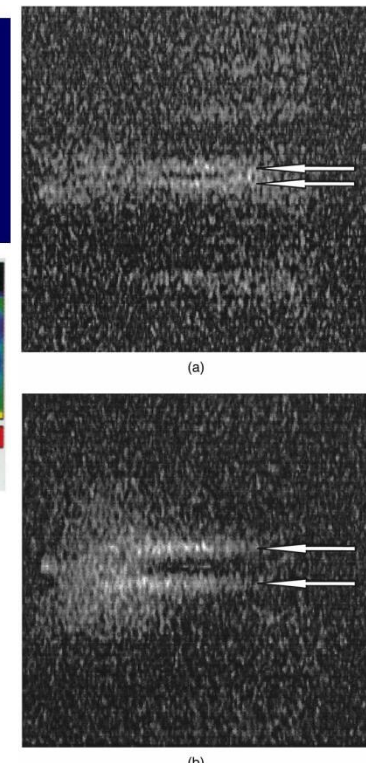
1995: **Monochromatic**+MRI (Ehman&Greenleaf)



1998: **Pulse**+ultrasound (Fink, Catheline)

1998: **Pulse**+MRI+Radiation pressure (Sarvazian)

2004: **Pulse**+ultrasound (Fink, Tanter, Bercoff, Nightingale)



Shear waves induced by Lorentz force in soft tissues

S. Catheline, P. Grasland-Mongrain, R. Souchon,
F. Cartellier, A. Zorgani, C. Lafon and J.-Y. Chapelon

LabTAU INSERM U1032, University of Lyon, France

Université Claude Bernard Lyon 1

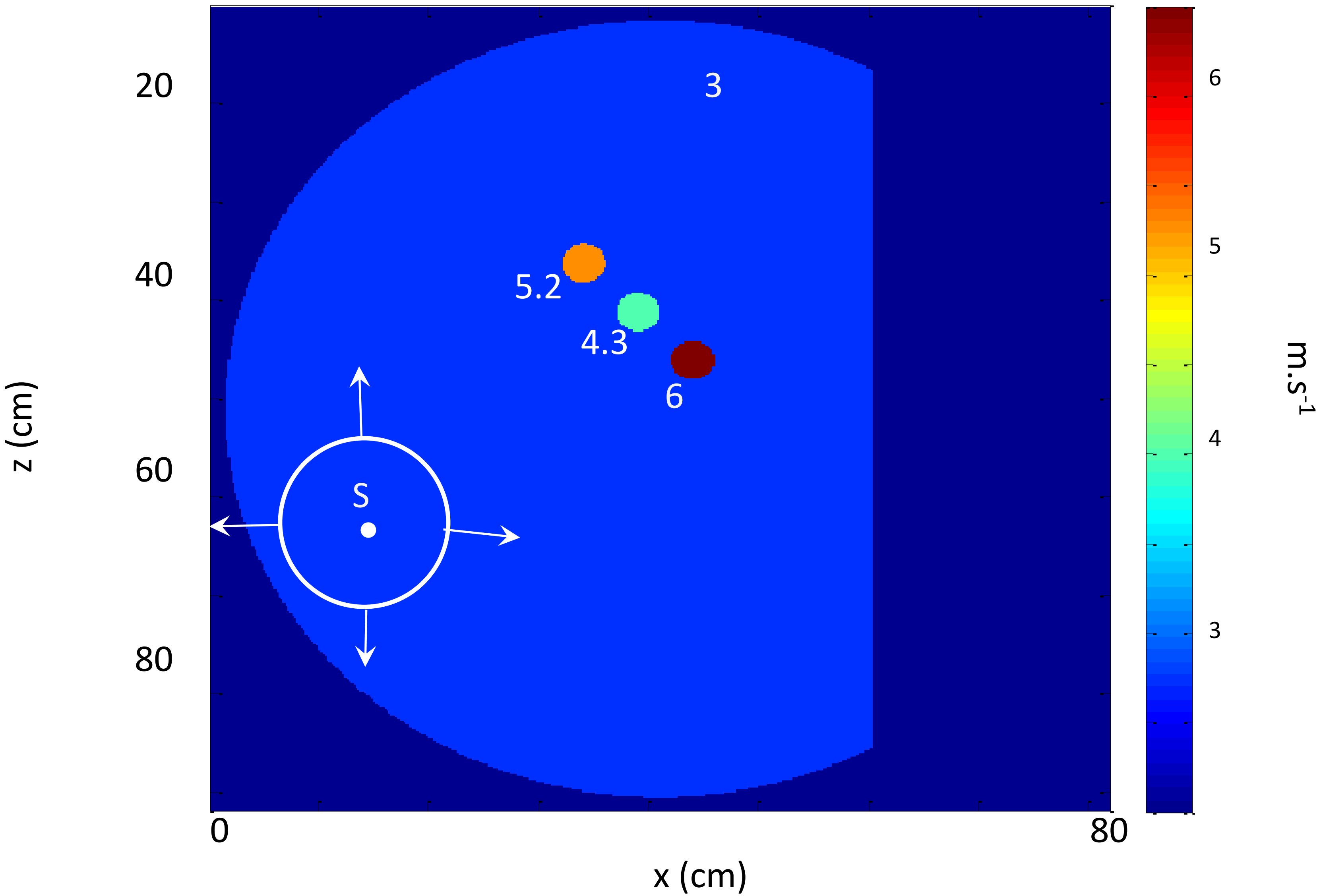
Inserm

Passive elastography approach

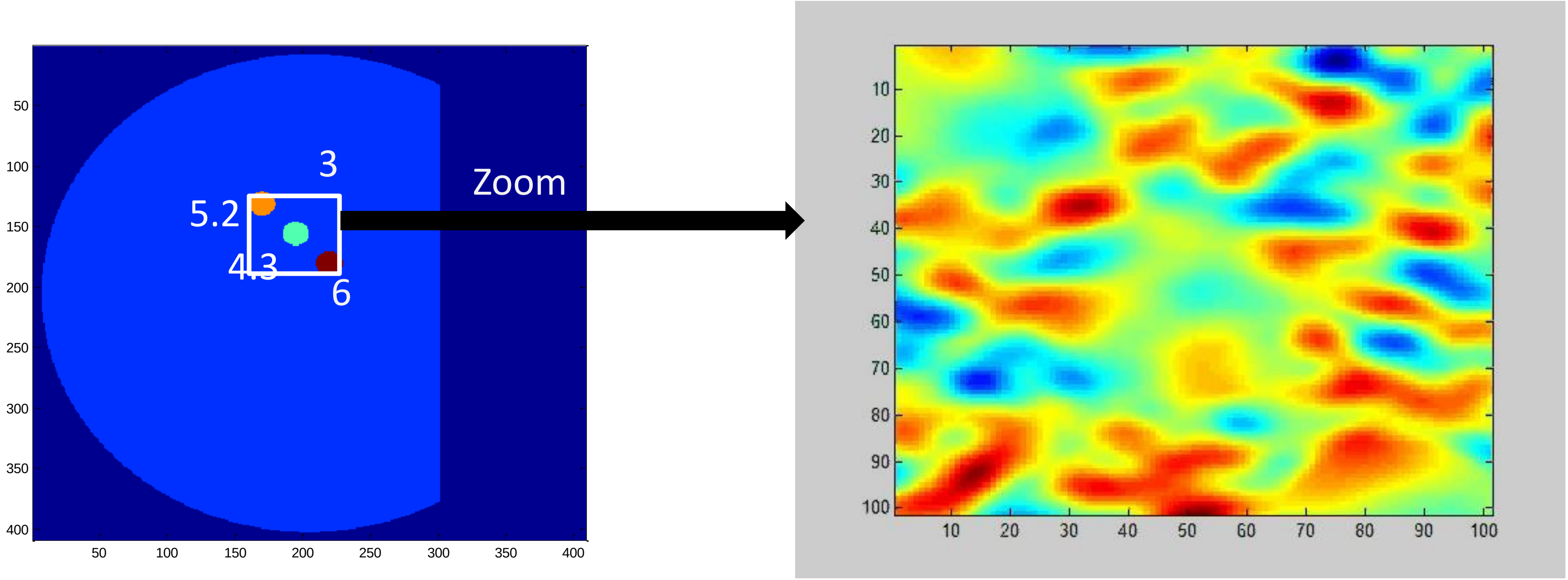
Ali Zorgani, R.Souchon, A. Hoang-Dinh, J-Y Chapelon, S.Catheline

INSERM U1032, LabTAU, University of Lyon

The diffuse field approach: finite difference simulation



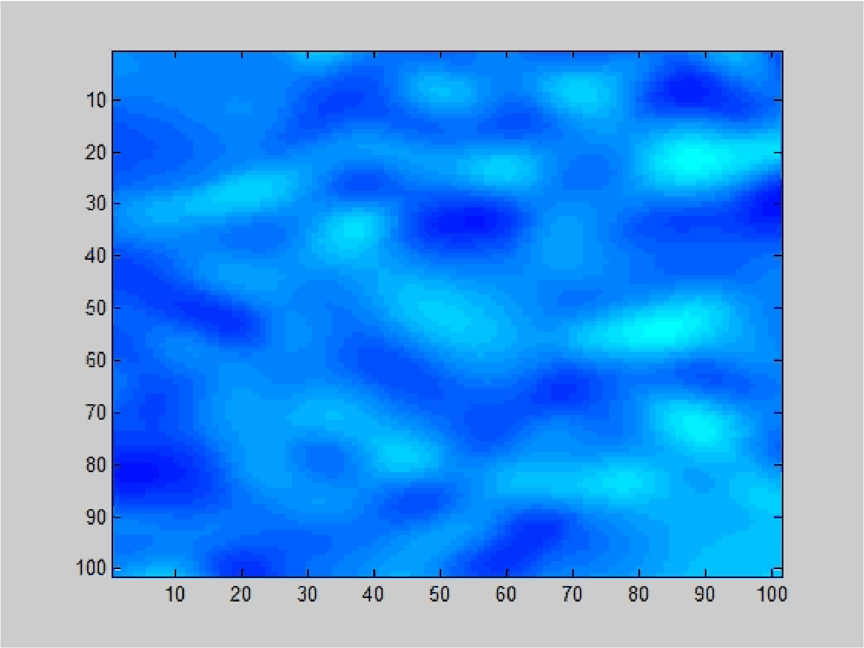
The diffuse field approach



Time reversal

Key for speed extraction=TR

TR=spatio-temporal correlation
(coda wave interferometry)



S.Catheline, N. Benech, X. Brum, and C. Negreira, *Phys.Rev.Letter.* **100**, 064301 (2008).

T.Gallot, S. Catheline, P. Roux, J. Brum, N. Benech, C. Negreira, *IEEE UFFC*, vol.58,6,p.1122 (2011)

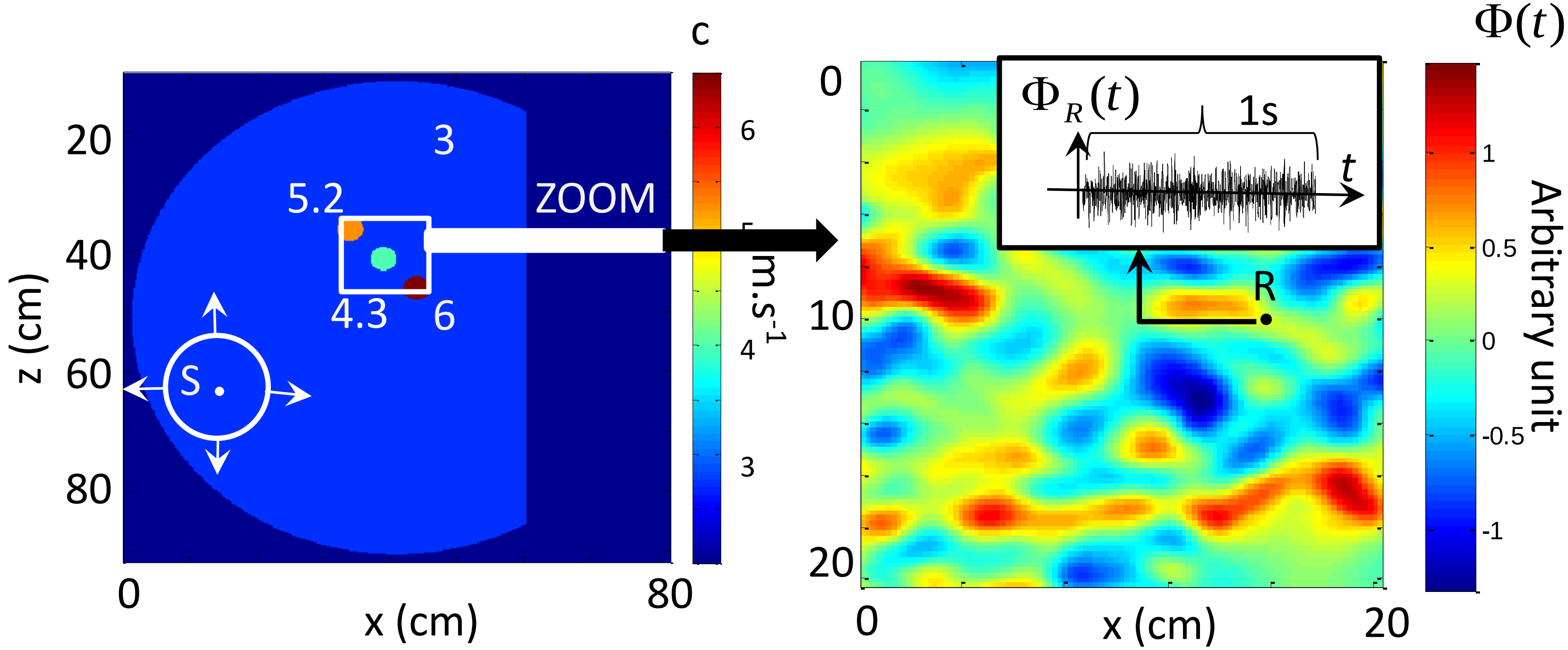
$$\Delta\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = 0 \rightarrow \psi^{RT} = \Phi(\vec{r},t)\otimes\Phi(\vec{r}_0,-t) = \int \Phi\,\Phi^*\,dt$$

Plane wave

$$\varepsilon_z = \frac{\partial\Phi}{\partial z} \rightarrow \Delta\varepsilon_z - \frac{1}{c^2}\frac{\partial^2\varepsilon_z}{\partial t^2} = 0 \rightarrow \xi^{RT} = \int \varepsilon_z\varepsilon_z{}^*\,dt \rightarrow \xi^{RT} \approx -k^2\psi^{RT}$$

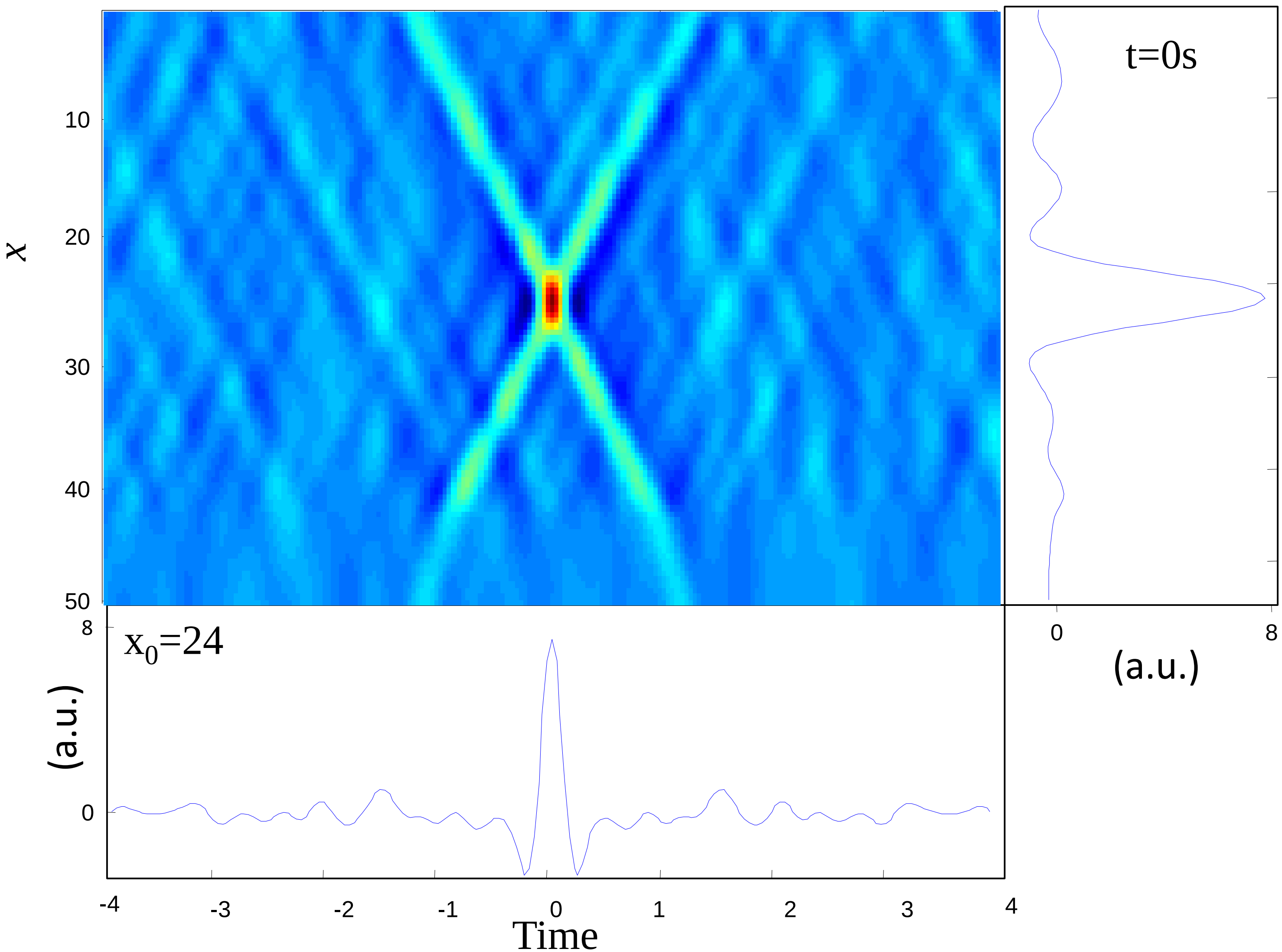
$$v = \frac{\partial\Phi}{\partial t} \rightarrow \Delta v - \frac{1}{c^2}\frac{\partial^2 v}{\partial t^2} = 0 \rightarrow V^{RT} = \int v v^*\,dt \rightarrow V^{RT} \approx -\omega^2\psi^{RT}$$

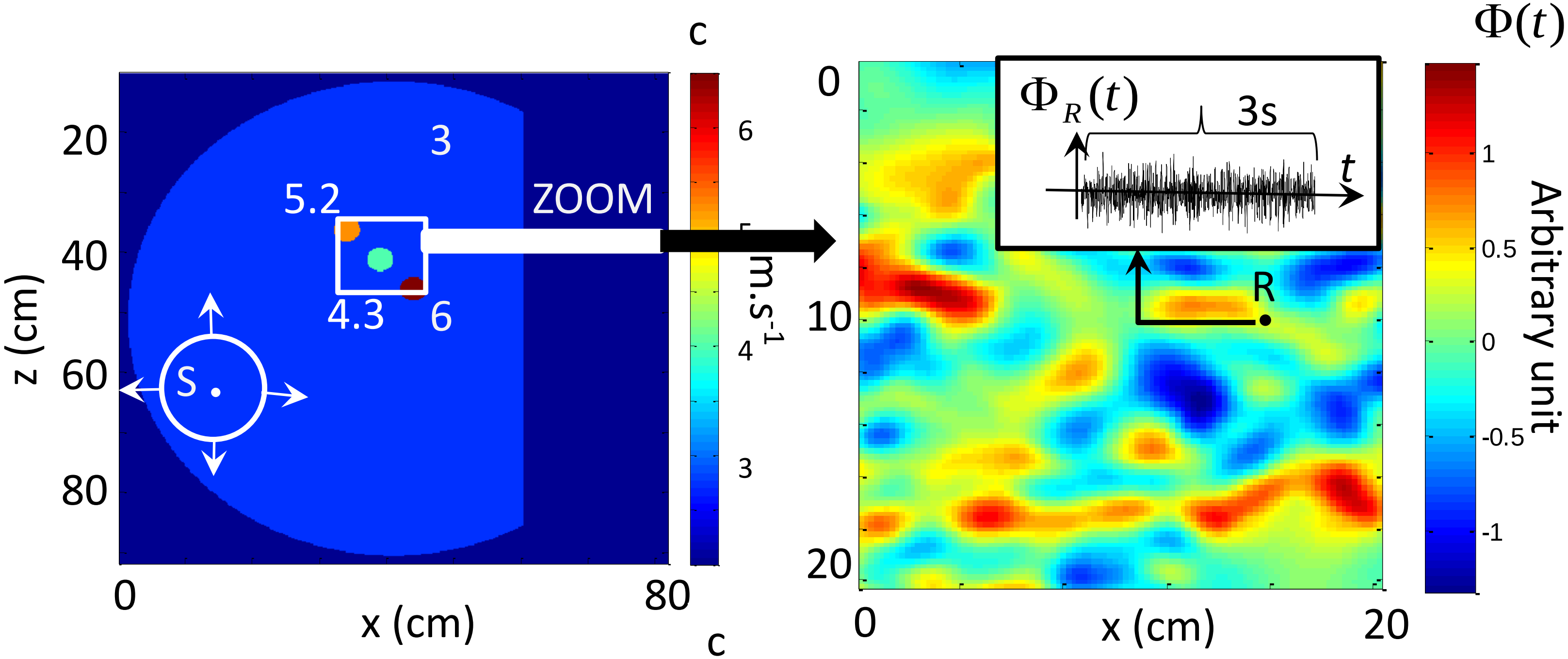
$$\boxed{c = \frac{\omega}{k} = \sqrt{\frac{V^{RT}}{\xi^{RT}}}}$$

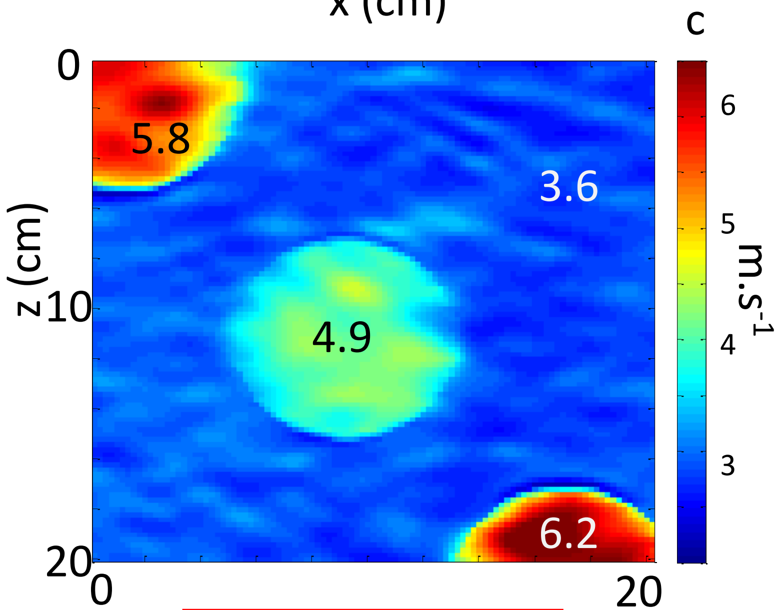


$F_{sampling}$=1000Hz

Over sampling

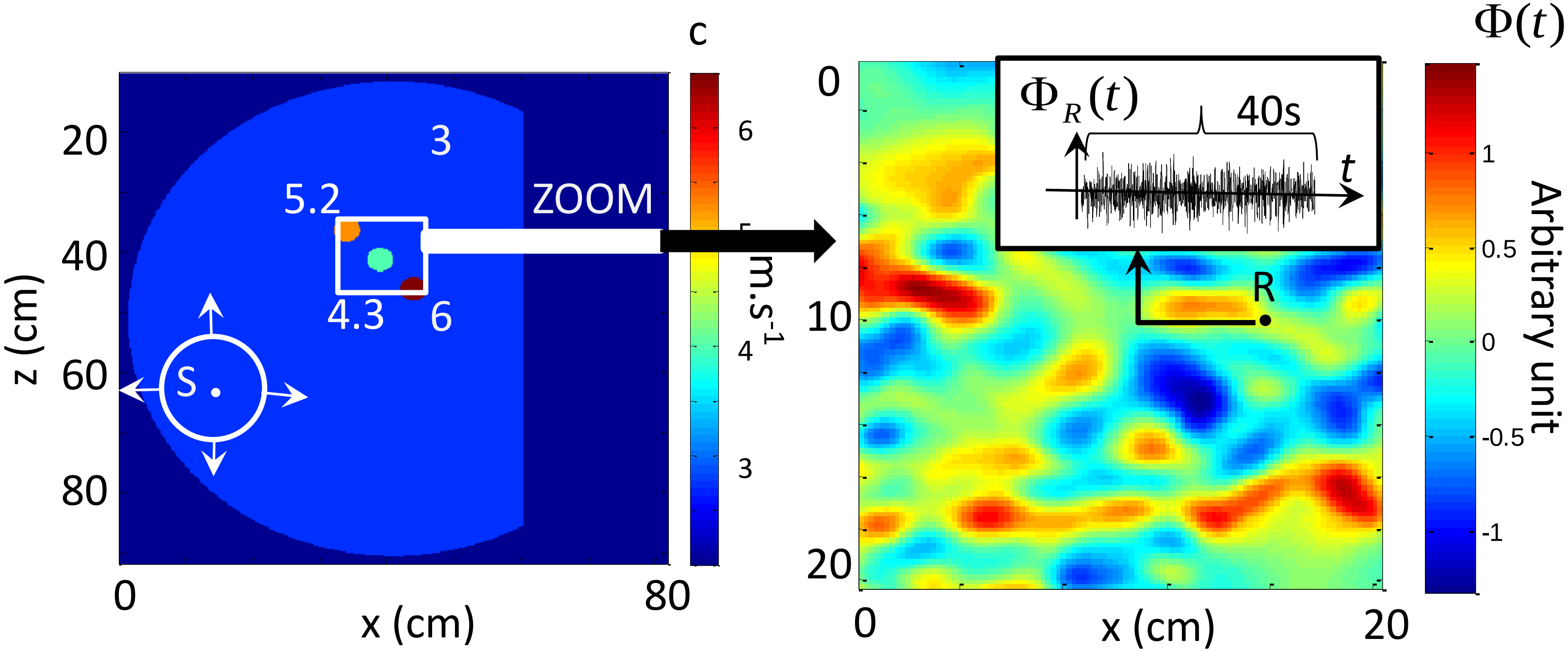






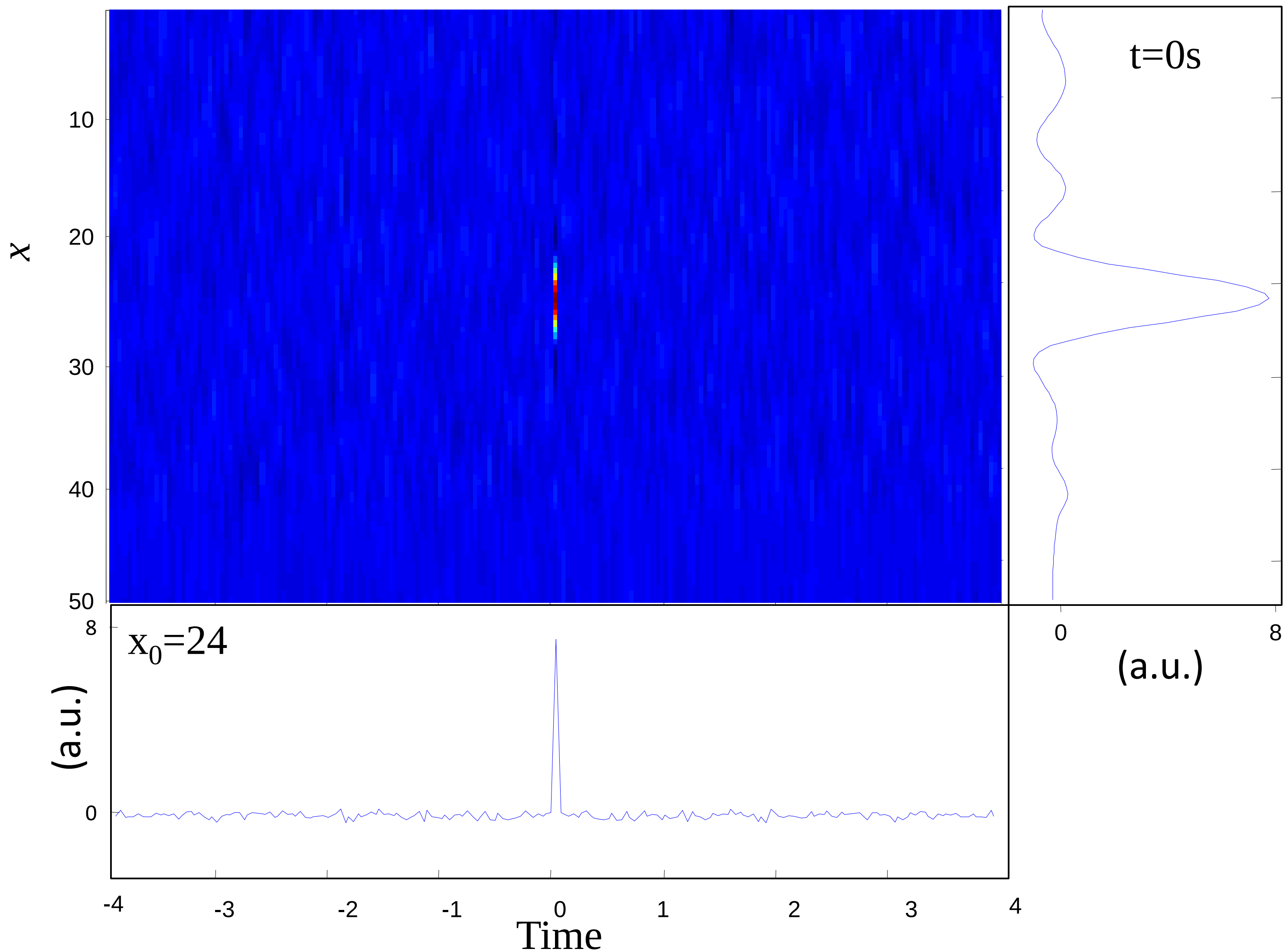
$$c = \frac{\omega}{\mathrm{Re}(k)} = \sqrt{\frac{V^{RT}}{\xi^{RT}}}$$

T.Gallot, S. Catheline, P. Roux, J. Brum, N. Benech, C. Negreira
Passive elastography: Shear wave tomography from physiological noise correlation in soft tissues
IEEE Transactions on UFFC, vol. 58, no. 6, June 2011.



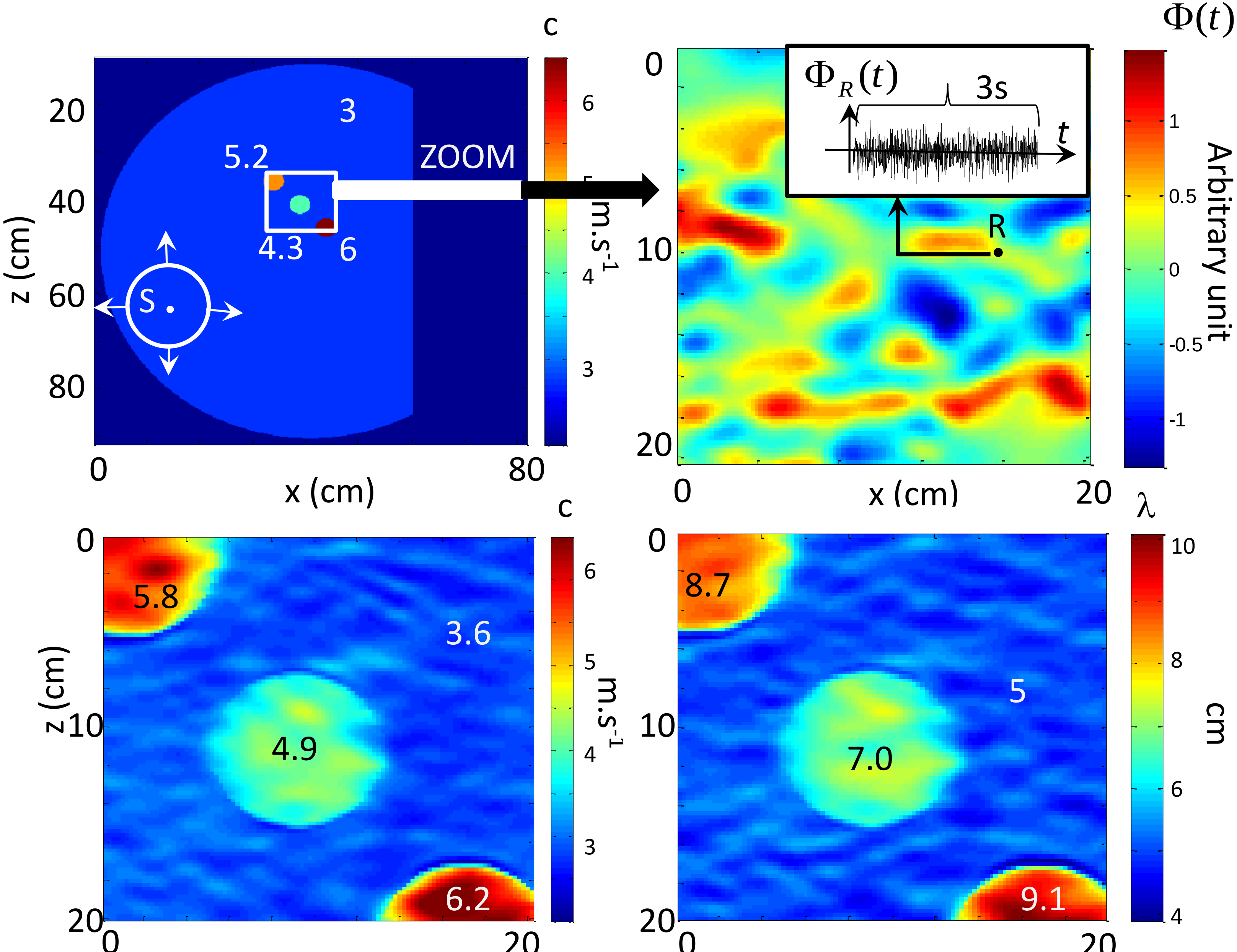
$F_{sampling}$=25Hz

Under sampling



$$C(\vec{r}_0, \vec{r}; 0) = \int_0^T \psi_z(\vec{r}_0, \tau) \cdot \psi_z(\vec{r}, \tau) \mathrm{d}\tau.$$

$$C_\phi(\vec{r}_0, \vec{r}; 0) = \int_0^T \psi_z(\vec{r}_0, \phi(\tau)) \cdot \psi_z(\vec{r}, \phi(\tau)) \mathrm{d}\tau.$$

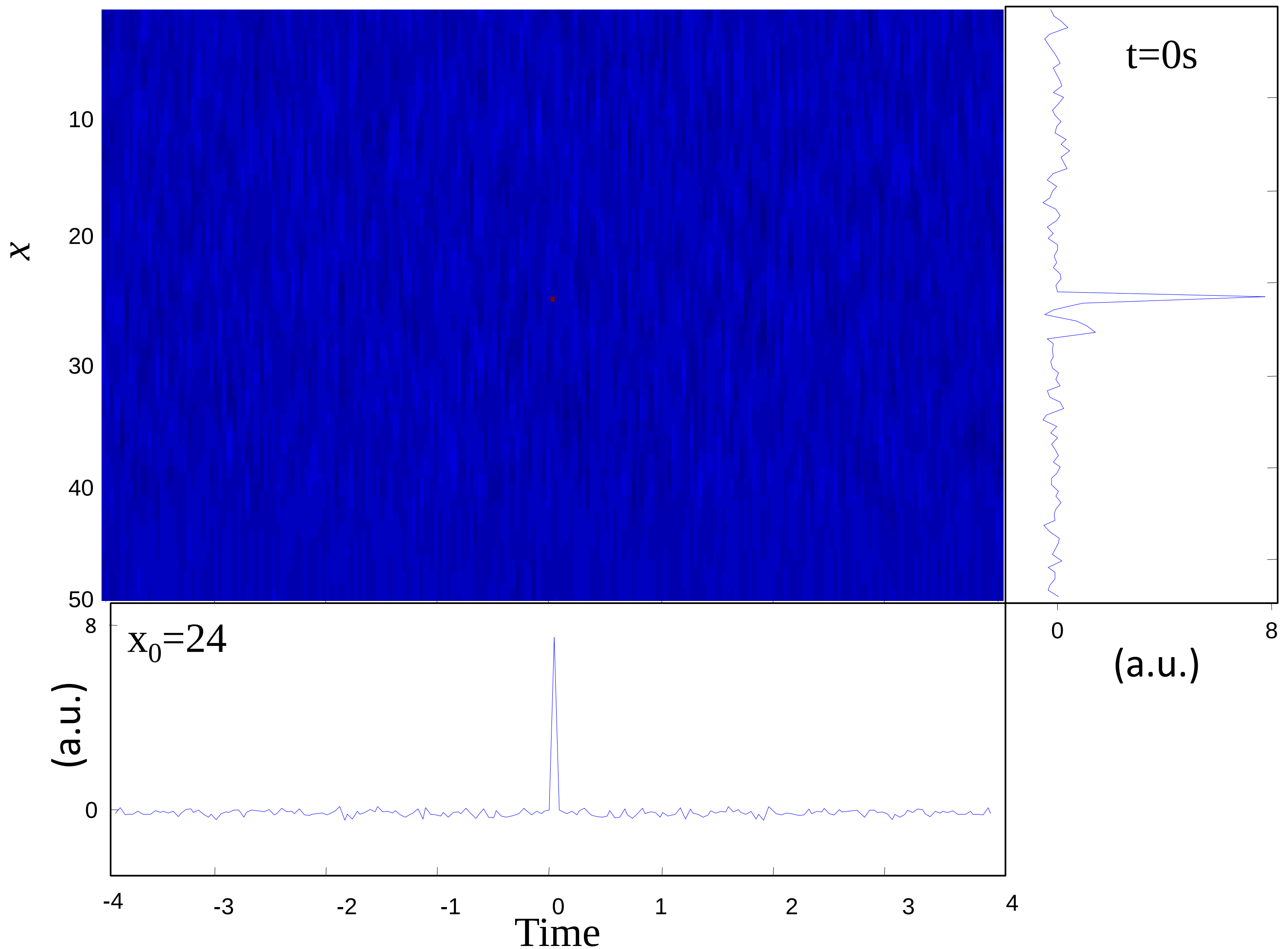


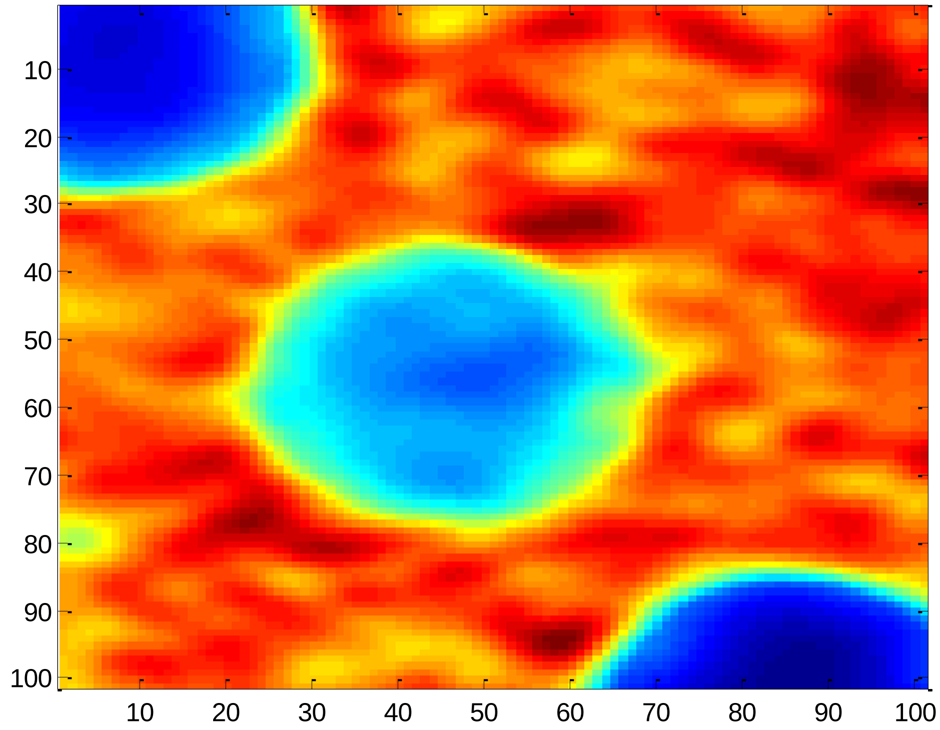
$$c = \frac{\omega}{\mathrm{Re}(k)} = \sqrt{\frac{V^{RT}}{\xi^{RT}}}$$

$$k = \sqrt{\frac{\xi^{RT}}{\psi^{RT}}}$$

S. Catheline, R. Souchon, M. Ruppin, J. Brum, A. H. Dinh, J-Y Chapelon
Appl. Phys. Lett. 103, 014101 (2013)

Elasticity imaging: under sampling experiments





$$\mathrm{Im}\left[G_{mn}(\mathbf{0},\mathbf{r})\right]=\frac{k}{12\pi\mu}\left\{\left[\left(\frac{\beta}{\alpha}\right)^{3}\left(j_{0}(qr)+j_{2}(qr)\right)+2j_{0}(kr)-j_{2}(kr)\right]\delta_{mn}+\left[3j_{2}(kr)-3\left(\frac{\beta}{\alpha}\right)^{3}j_{2}(qr)\right]\gamma_{m}\gamma_{n}\right\}$$

$$\mathrm{Im}\left[G_{mn}(0,0)\right]=\frac{k}{12\pi\mu}\left[\left(\frac{\beta}{\alpha}\right)^{3}+2\right]\cong\frac{k}{6\pi\mu}=\frac{f}{3\rho c_{S}^{3}}$$

The softer, the higher the amplitude

Is it always true? Not sure. Bar, plate, string

$$G^{plate}(0,x)=\frac{ic^{2}}{8\omega^{2}}[j_{0}(kr)+N_{0}(kr)-j_{0}(i\gamma r)-iN_{0}(i\gamma r)]$$

$$G^{bar}(0,x)=\frac{ic^{3}}{4\omega^{3}}e^{ikx}$$

$$G^{string}(0,x)=i\frac{c}{2\omega}e^{ikx}$$

MRI Results to be coming…