SEISMIC WAVE DIFFUSION IN THE EARTH LITHOSPHERE

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Abstract. This paper is devoted to the study of the time decay of the coda of seismograms. We consider a conceptual model of the Earth upper layers: a diffractive crust overlying an almost homogeneous mantle. We simulate the multiple scattering of the seismic waves using the classical radiative transfer equation in a scalar approximation. We solve the equation using the Monte Carlo method and give a particular attention to the asymptotics of the solution. Under the condition that the ratio mean free path/layer thickness is less than one , we can give an analytical form of the time decay of the coda energy as the solution of a diffusive equation. Otherwise, our transcription of the boundary condition is not valid. The asymptotic form is similar to the one that was proposed by Aki and Chouet (1975) to fit their observations. We checked numerically that, even when the mean free path is larger than the layer thickness, the asymptote of the radiative transfer solution has the same functional form than the one obtained when we are able to compute an analytical solution of the diffusion equation. We show a direct comparison of observations made in Mexico with the results of our model in which we included the effect of a weak absorption. The measurements are well predicted by the model, both for the absolute level of apparent attenuation and for its frequency dependence. We stress the

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importance of the leakage of diffuse energy in the mantle and propose to define a time of residence of diffuse waves in the crust to characterize the temporal decay of the coda energy. An important aspect of our interpretation is the fact that the coda decay corresponds to the decay of scattered waves in the diffusive regime. To demonstrate the diffuse character of the coda, we look for the energy partitionning that is expected for elastic waves in the diffusive regime. Considering a series of records from earthquakes at different epicentral distances, we show that the equilibration between the two modes (that is the energy partitioning) appears very early in the coda. The ratio between compressional and shear energies is very stable while the energy level changes by several orders of magnitude. The energy ratio is independent of the earthquake considered. This experiment tends to confirm that the seismic coda corresponds to waves in the diffusive regime and therefore supports our interpretation that the decay observed in Mexico at low frequency is strongly governed by the rate of leakage of diffuse energy in the mantle.

1. Introduction

The seismograms consist of records of the 3 component motion of the free surface of the Earth during earthquakes. The seismograms begin with a series of arrivals of compressional, shear and surface waves that can be interpreted in terms of the ray theory. The very deep structure of our planet has been explored by using this type of waves. On short period seismograms (in the range 1-10 Hz), after these arrivals the envelope decays slowly to reach the level of seismic noise after a delay that can be currently several tens of times the travel time of direct waves. This long tail is called the seismic coda. The coda consists of waves that are arriving to the receiver from almost all directions (see for example Campillo at al, 1998 for recent references). Applying a frequency-wave number analysis to the data of the NORSAR array, Dainty and Toksoz (1990) have shown that the coda is dominated by waves with apparent velocities less then 4 km/s, that correspond in the Earth crust to the shear elastic waves called S waves in seismology.

In their pioneering work, Aki and Chouet (1975) showed that the time decay of energy in the coda is a regional characteristic, independent of the source depth or magnitude. They measured this constant decay rate after a lapse time that is larger than two times the travel time of direct shear waves. They found that the coda envelope can be fitted by the formula:

$$\rho(f,t) = \frac{S_0(f) g_\pi(f)}{2\pi\beta^2 t^{2\gamma}} \exp\left(-\frac{2\pi f t}{Q_c}\right) , \qquad (1)$$

where $S_0(f)$ is the shear energy emitted by the source, $g_\pi(f)$ is the backscattering coefficient and β the shear wave velocity. γ is a factor of spreading chosen to be 1 since it is assumed that the coda is made up of body waves. For large lapse time the decay is governed essentially by the exponential term. Q_c, the "coda quality factor" characterizes the decay rate (through the exponential term) and was introduced by analogy with the classical quality factor associated with absorption.

The parameter Qc (coda Q) has been measured in a large number of regions and the different authors reported essentially the Aki and Chouet conclusions about the stability of the decay rate. Qc appeared to be the easiest amplitude parameter to measure while the peak amplitude or attenuation of direct waves are in most cases very difficult to evaluate in the Earth due to focusing and defocusing effects.

It was found that Qc is increasing with frequency with a rate that is larger when its value is small at low frequency. At high frequency, the values observed are very high almost everywhere while around 1 Hz Q_c exhibits regional variations correlated with the tectonic style.

Aki and Chouet interpreted their result using either the single scattering approximation or the diffusion approximation for scalar waves in a full space. Later, Wu (1985) introduced the stationary radiative transfer equation for scalar waves. Abubikarov and Gusev (1987) and Hoshiba (1991) used the Monte Carlo method to solve the radiative transfer equation for scalar waves in the time domain. In all these studies, a constant background velocity and a homogeneous distribution of scatterers were assumed. The use of this type of model in the interpretation of the observed coda decay rate suggests a strong absorption that would be the dominant process in the apparent attenuation of seismic waves in the crust. On the other hand, the study of the decay of direct waves with distance led to the inverse conclusion (see Campillo and Plantet, 1991): when Qs the quality factor of S waves is small (significantly less than 1000), it has a strong frequency dependence characteristic of the scattering effect on primary waves. The measurements of Qs in tectonically active regions suggest that the scattering effect is dominant over the absorption. This apparent contradiction between these two interpretations can be solved by considering a more realistic model of the Earth structure to interpret the coda decay.

2. Radiative transfer in the crust

The major characteristic of the structure of the first tens of kilometers of the solid Earth is the existence of the crust overlying the mantle. The crust and the mantle have very different physical properties and chemical composition. The mantle has a very homogeneous composition with mostly olivine (Mg,Fe)SiO4 while the crust is the result of the differentiation of light elements during the Earth existence. In geologically active provinces, the compositionnal heterogeneity of the crust makes it possible the progressive build up of mechanical heterogeneity during the long history of the deformations associated with the tectonic processes. Besides the mantle has a ductile mechanical behavior while the upper crust is brittle. This results in the development of crack networks in the crust. Experiments of deep reflection seismic soundings in the continental domain provided images of a reflective crust above a transparent mantle (see for example Allmendinger et al., 1987 or Meissner, 1989).

It is therefore reasonable to set up a model in which an upper layer containing numerous scatterers (impedance fluctuations, cracks) overlies a half space that is weakly diffusive. There is another special feature of this structure that cannot be neglected. The wave velocities are significantly higher in the mantle than in the crust (typically $4700m/s$ and $3500m/s$ for S waves). As a consequence, the wavefield produced by a source located in the crust comprises strong guided waves due to post critical reflections on the boundary. This particularity of the Green function has to be incorporated in our treatment of the multiple diffraction. A simple conceptual model that accounts for the specificities of the Earth was studied by Margerin et al. (1998a). The configuration is shown in Fig 1. Let us begin with a purely elastic model. Considering the observation that coda waves are mostly S waves, we neglect the compressional P waves and assume that, after a few diffractions the S waves are depolarized. We therefore limit our analysis to a scalar problem. This model may appear too simple but our goal at this stage is to investigate the consequences of the existence of the layering. After a series of numerical studies using Monte Carlo simulation, it was shown that the energy decay in such a model varies with time according to the regime of diffraction.

We present in Fig 2 an example of the results for a configuration in which the crustal thickness is assumed to be 30 km while the mean free path in the crust is 50 km. For very short lapse time, the solution is asymptotic to the single scattering solution. Nevertheless, the divergence of the two curves is almost instantaneous, demonstrating the importance of multiple scattering and the extreme limitation of the single scattering approximation in this problem. Rapidly, the solution of the radiative transfer equation

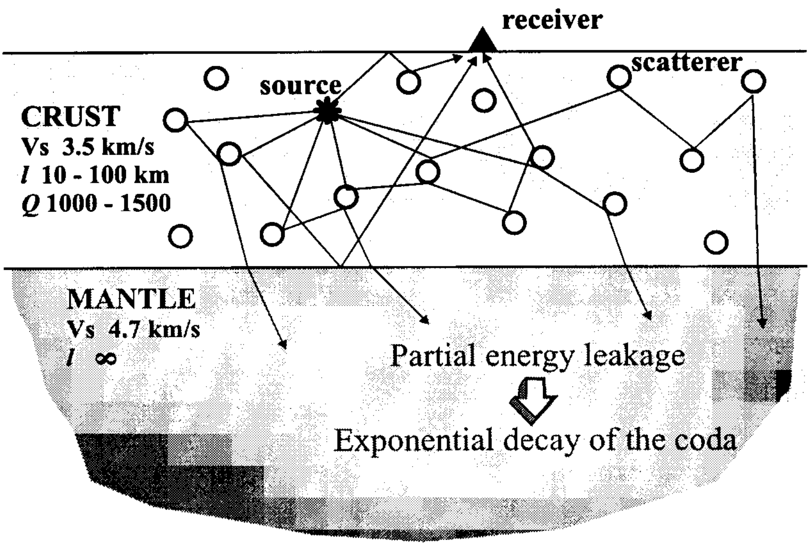

Figure 1. Sketch of our simplified model with the physical parameters used in the computations.

becomes asymptotic to a curve which has the expression proposed by Aki and Chouet from their data analysis. We fit the curve using equation (1) with Qc being the free parameter. This result indicates that the simple purely elastic model we propose is able to reproduce this very strong property of the observation, that is the existence of an exponential decay term. Margerin et al. (1998a,b) showed that the fonctional form of the decay is identical for a broad range of mean free path and crustal thickness.

We interpret this exponential behavior as the consequence of the leakage of the diffuse energy from the crust to the mantle. This leakage is nevertheless governed by the reflection coefficient at the base of the crust. In the following we shall use the diffusion approximation to understand this behavior and to give a formal interpretation of Q_c.

3. Diffusion equation and asymptotics

The asymptote of the energy envelope of the multiply scattered waves is given, for large lapse times, by the solution of the diffusion equation. Although this is a general statement, the manner of writing the diffusion equation with its initial conditions is far to be obvious. Margerin et al. (1998a) studied explicitely a particular case where the mean free path is smaller than the crustal thickness ($l < H$). The source is assumed to be at the surface. In this case, one can assume that the wavefield is almost diffusive when it reaches the base of the crust. It is therefore possible to write

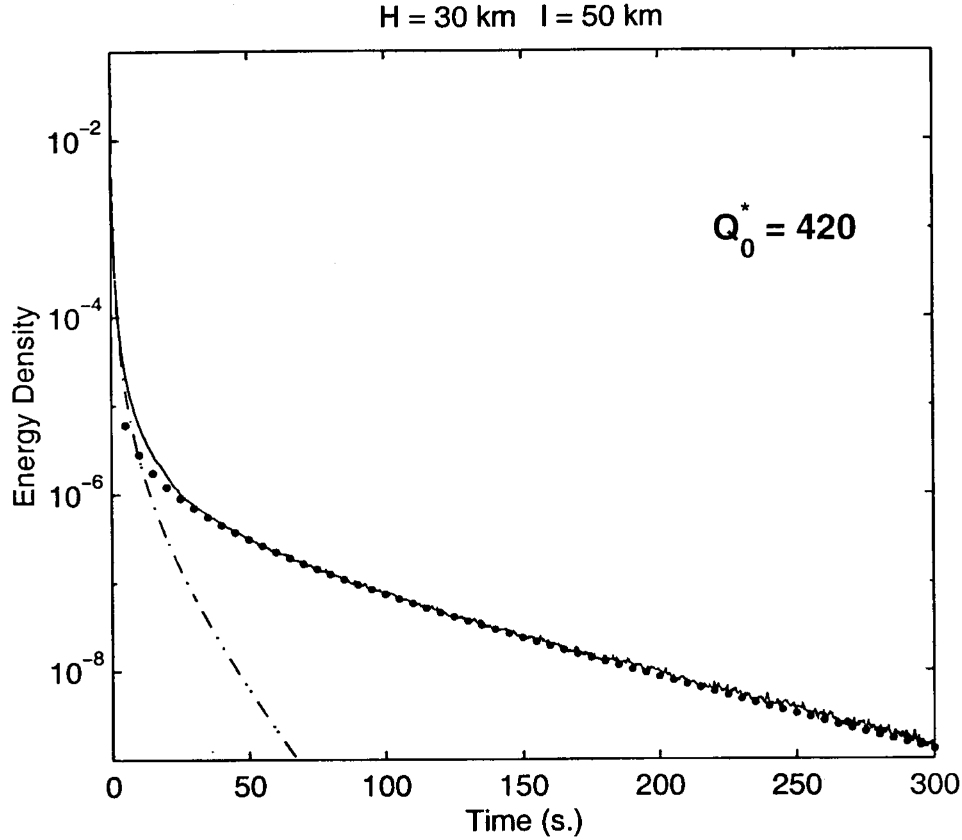

Figure 2. Coda decay obtained in our model with $H = 30$ km and the mean free path of the crust l equal to 50 km. Solid lines show the numerical solution of the Radiative Transfer Equation obtained by Monte Carlo modeling. The thin dotted lines shows the result of the single scattering approximation. The black circles indicate an approximation of the radiative transfer solution obtained with the formula $\frac{1}{t}\exp(-\frac{2\pi t}{Q_0^*})$. The Q_0^* value corresponding to the best approximation is 420. The maximum standard deviation of the Q_0^* is $\Delta Q_0^* \approx 30$.

directly a boundary condition for diffuse waves and to solve the diffusion equation analytically. It was checked that, under the condition $l < H$, the agreement between the solution of the diffusion and radiative transfer equations is perfect for large lapse times. The complete solution corresponding to the conceptual model presented above is:

$$\rho(r,t)=\frac{\exp(-\frac{r^2}{4D_1t})}{2\pi HD_1t}\sum_n\frac{\sin\xi_n+\frac{t_n\gamma}{H}\cos\xi_n}{(1+\frac{\gamma}{H})\sin\xi_n+\frac{t_n\gamma}{H}\cos\xi_n}\exp(-D_1\frac{\xi_n^2}{H^2}t) \qquad (2)$$

where ρ is the energy density in the coda, t, the time elapsed since the energy release at the source, f, the frequency of waves. The ξ_n are the roots of the equation :

$$\xi_n\tan\xi_n=\frac{H}{\gamma}, \qquad \xi_n\in]n\pi,n\pi+\frac{\pi}{2}[\qquad n\in N. \qquad (3)$$

In this expression γ is a variable that depends on the differential cross section of the scatterer in the crust, their density and the reflection coefficient at the boundary (Margerin et al, 1998a).

For a configuration where the source and the receiver are close, the leading term of the solution has the form:

$$\rho(t)=\frac{1}{t}\exp(-\frac{t}{\tau_d}) \qquad (4)$$

and

$$\tau_d=\frac{Q_c^\star}{2\pi f}\simeq\frac{3H^2}{vl\xi^2}, \qquad (5)$$

where ξ is the root of the equation :

$$\xi\tan\xi=\frac{H}{\gamma}, \qquad \xi\in\left]0,\frac{\pi}{2}\right[. \qquad (6)$$

$Q_c^\star$ is a parameter that describes the decay rate of the coda in our model. We denote by $Q_0^\star$ the value of $Q_c^\star$ at the frequency $f=1$ Hz. We maintain in the notation the analogy with the classical quality factor because of the wide practice of this parametrization in experimental seismology. It allows to compare directly the output of our model with the measurements made on actual records. However, the use of τ_d, the time of residence of diffuse energy in the crust would be more appropriate to characterize the leakage effect. Whatever the parametrization chosen, the important result is the fact that the solution of the diffusion equation in a model without absorption gives a functional decay identical to the one that has been observed by Aki and Chouet and widely confirmed later on. The solution of the diffusion equation makes it possible to give an explicit expression of $Q_c^\star$ or τ_d in terms of the physical parameters of the model, again under the condition $l<H$. We are not able to write down a similar problem of diffusion when $l>H$. In this case the field that reaches initially the boundary is not diffusive and therefore we cannot incorporate simply the boundary conditions in our model. Nevertheless, the numerical simulation of the radiative transfer equation for different ratios l/H indicates that the same functional form of the asymptotic decay is expected when $l < H$ or $l > H$ (Margerin et al., 1998b).

4. Comparison with observations

In the previous sections we studied a simple conceptual model in which we assume no absorption. Indeed two effects can govern the actual decay of the coda of the seismograms. One is the leakage that we just discussed and another is the anelastic absorption which can be represented by the quality factor Q_i. In order to test the relative importance of these effects, we present a comparison between the decays observed at a series of seismic stations and the results of our simulations using realistic values for the parameters of the model. The stations are located along the Pacific Coast of Mexico, a region which is tectonically active.

We use the records at the seismological stations PNIG, HUIG, CAIG and ZIIG of the Mexican Seismological National Network (see Singh et al., 1997). We consider the records of N earthquakes which occured at epicentral distances between 30 and 150 km. At each station, the decay at a given frequency f is parametrized through the measure of $Q_c(f)$ in the way proposed by Aki and Chouet (1975) (equation 1). The results are shown in Fig 3.

Between 1 and 5 Hz, they exhibit a strong frequency dependence as expected from our model. In the region where these data are collected, the crustal thickness is between 20 and 30 km (Kostoglodov et al., 1996). The mean free path is not objectively determined. Therefore, we consider values of l between 20 and 70 km, a range that covers the values proposed for the Earth crust in the frequency range considered here.

The results of the numerical simulations in absence of absorption are plotted in continuous lines in Fig 3 for the extreme values of l 20 and 70 km. The results obtained at low frequency (1Hz) are close to the observed values, while at high frequency Q_c is widely overestimated by our purely elastic model. We interpret this discrepancy by the effect of anelasticity. According to laboratory studies, the quality factor inherent to dry rocks is independent of frequency (Johnston, 1981). For crustal rocks at depth and the frequency range considered here, it is difficult to evaluate directly a possible frequency dependence of Q_i. In absence of further evidence, we consider the model with the smallest number of free parameters: a constant Q_i. We add to our model a small absorption term described by $Q_i = 1200$. This value corresponds to the high frequency limit of the measured Q_c. This

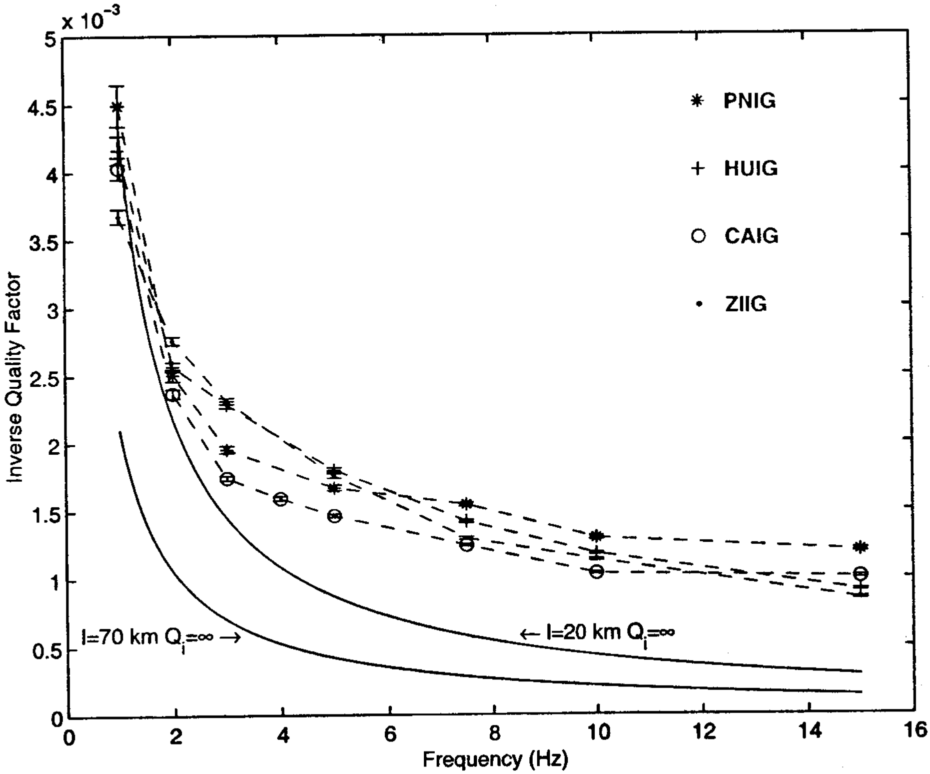

Figure 3. The symbols correspond to Q_c as a function of frequency measured at the different stations. The continuous lines correspon d to the solution of the radiative transfer equation in a purely elastic medium for the extreme values of mean free path 20 km and 70 km.

is a reasonable guess since this value is similar to the one measured in geologically stable regions (Singh and Herrmann, 1983, Hasegawa, 1985) where the crust was not affected by tectonic motions since the early stages of the Earth history and is therefore probably very homogeneous. The apparent attenuation of direct waves measured in these regions is a good evaluation of the intrinsic attenuation of rocks that has to be taken into account in the modelling. After adding to our model a weak attenuation term described by $Q_i = 1200$, we compute again the coda decay parameter $Q_c^\star$ for different frequencies and for the extreme values of l of 20 km and 70 km. The results are compared with the measurements in Fig 4.

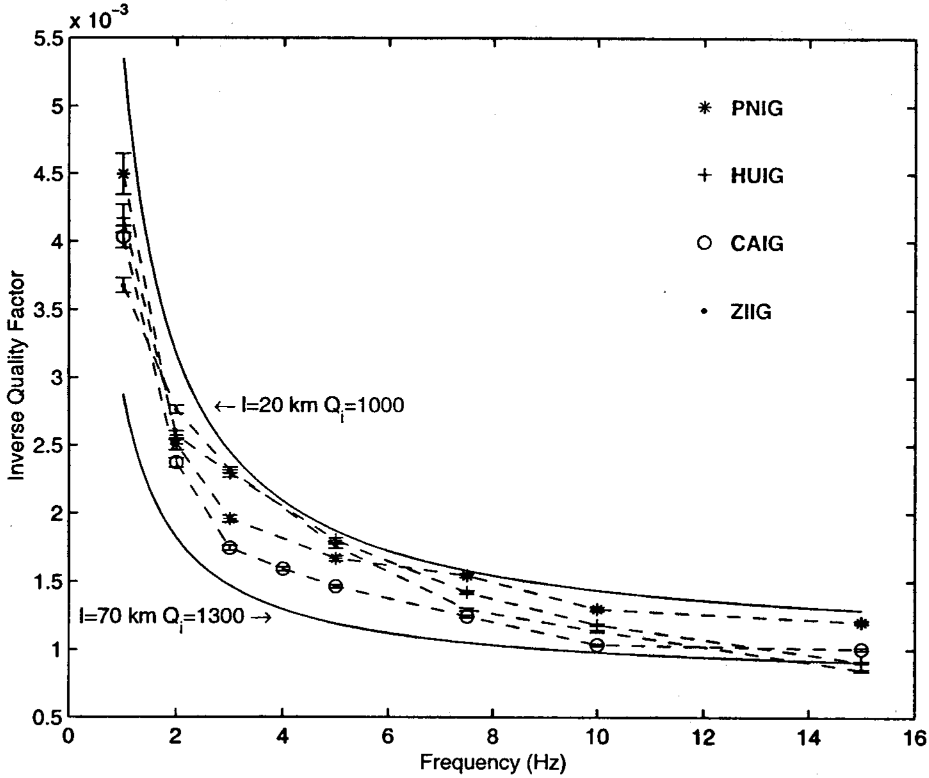

Figure 4. A comparison of the measured values of Q_c with the ones deduced from our model including a weak intrinsic absorption.

The numerical results are now in agreement with the observations in the complete range of frequency.

This agreement obtained with a very simple model shows that we catch the essential of the physical processes that govern the coda decay in this example: the leakage of diffuse energy in the mantle, that dominates at low frequency and leads to a linear frequency dependence of $Q_c^\star$ and the intrinsic absorption of rocks that dominates the attenuation in the high frequency limit. Indeed such a simple conceptual model is not expected to represent the various aspects of the Earth structure but it illustrates the importance of the leakage, a process that was neglected in previous interpretations of Q_c. This effect has to be considered in tectonically active regions where the scattering in the crust is strong (l of the order of H) while in stable regions, we expect a weak scattering in the crust (l large) and therefore a predominance of the intrinsic attenuation over the leakage effect in the exponential decay of the coda energy.

In the example of the records in Mexico, the conclusion of our analysis is that the decay rate we measure at low frequency is governed by the leakage into the mantle of diffuse waves of the crust. A strong support to this interpretation should be to demonstrate the diffusive character of coda waves. It is difficult to assess explicitely in what regime is the wavefield from conventional measurements. In the case of seismic waves, we can use the properties of the elastic waves. In the following we use an energy balance property of the elastic waves as a marker of the onset of the diffusive regime.

5. Partition of P and S energy

The understanding of the regime of the scattering of seismic waves in the Earth is fundamental for their interpretation in terms of the properties of the Earth materials. Most of previous theoretical studies of the seismic coda (including our study of the effect of the layering) used the acoustic approximation and did not consider the different polarizations of the elastic waves and the mode conversions that occur at each scattering event. As a consequence, the experimental studies were essentially concerned with the measurement of the decay rate of the coda envelopes. However, such kind of measurements is insufficient to distinguish unambiguously between the different scattering regimes.

Recently, the elastic radiative transfer equation has been derived (Weaver, 1982, 1990, Ryzhik et al., 1996; Papanicolaou et al., 1996; Turner, 1998). This equation takes into account the wave polarization and the mode conversion between P and S waves in an infinite elastic body. A fundamental property expected from these studies is that, in the diffusion regime, R, the ratio of energy densities of S and P waves, becomes constant:

$$R = \frac{W_s}{W_p} = \frac{2\alpha^3}{\beta^3} \tag{7}$$

where α is the P-wave velocity and β is the S-wave velocity. This property can be regarded simply as a complete randomization of the field in the phase space. For a medium with roughly $\alpha/\beta = \sqrt{3}$, which is the case in the lithosphere, this ratio is 10.39. Therefore, the elastic diffusion approximation predicts that the seismic coda is dominated by S waves as it was observed with actual records. A constant ratio between P and S energy densities is not expected for deterministic arrivals for which this ratio depends on the nature of the source and of the path. The measurement of the repartition of the S and P wave energies and its possible equilibration can give a very strong indication on the regime of seismic wave scattering. The problem is that S and P waves cannot be separated using a record of ground displacement at one point. Particularly in the coda, the wavefield consists of numerous simultaneous arrivals from unknown directions, making the separation between P and S waves impossible using standard signal processing techniques.

Shapiro et al. (1998) present a new approach to separate P and S wave energies. This technique is based on the processing of the data from a small-aperture array. It implies the measurements of the spatial derivatives and the calculation of the curl and divergence of the displacement $\vec{u}$. We have:

$$\vec{curl}\vec{u} = 0 \tag{8}$$

for P waves and

$$div\vec{u} = 0 \tag{9}$$

for S waves. We use these fundamental properties to estimate P and S waves energies separately.

Following Aki and Richards (1980) we can write the density of energy associated with the deformation in elastic medium:

$$W = \frac{1}{2}\sigma_{ij}\chi_{ij} \tag{10}$$

where χ is the deformation tensor and σ is the stress tensor. In an isotropic medium, the equation (10) can be rewritten as:

$$W = (\frac{\lambda}{2} + \mu)(div\vec{u})^2 + \frac{\mu}{2}(\vec{curl}\vec{u})^2 + \mu I \tag{11}$$

where

$$I = 2(\frac{\partial u_x}{\partial y}\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial z}\frac{\partial u_z}{\partial x} + \frac{\partial u_y}{\partial z}\frac{\partial u_z}{\partial y}) - 2(\frac{\partial u_x}{\partial x}\frac{\partial u_y}{\partial y} + \frac{\partial u_x}{\partial x}\frac{\partial u_z}{\partial z} + \frac{\partial u_y}{\partial y}\frac{\partial u_z}{\partial z}) \tag{12}$$

and λ and μ are Lamé constants. In this study we are looking for a property of a diffuse wavefield, i.e. a random and almost isotropic field. We calculate the average energy $\overline{W}$ in a sufficiently large time window. In this case, average of cross-products of a-priory non correlated functions become zero and we obtain:

$$\overline{I} = 0 \tag{13}$$

and

$$\overline{W} = \overline{W_P} + \overline{W_S} \tag{14}$$

where:

$$\overline{W_P} = \overline{(\frac{\lambda}{2} + \mu)(div\vec{u})^2} \tag{15}$$

$$\overline{W_S} = \overline{\frac{\mu}{2}(\vec{curl u})^2} \tag{16}$$

According to the theory (7) of the infinite body, if the wavefield is diffusive, the ratio between these two energies is a constant given by the Lamé parameters, whatever the total energy of the field is and for any type of source. For non-random field, this ratio evolves rapidly according to the arrivals of deterministic waves. For deterministic arrivals, the ratio indeed looses its signification since we neglect the interference term I in the evaluation of energy. We will evaluate this ratio with actual data using the equations (15) and (16). The direct way to do it would be to install seismic receivers in closely located boreholes at different depths relatively far from the earth surface. This would allow to calculate the partial derivatives with respect to the three spatial coordinates, to estimate the curl and the divergence and hence the S and P wave energies. However, installing numerous seismic stations in bore-holes is extremely expensive and the actual bore-holes are not deep enough to allow to neglect the effect of the free surface. So far, there is no theoretical result to describe the equilibration at the surface of a half-space. The problem is indeed more difficult than for the infinite body because the free surface implies deterministic reflection and conversion.

At a depth below the surface larger than the transport mean free path, and because of the scatterings, the reflected wavefield is diffuse and therefore presents the same equilibration between the modes as for the infinite medium. The effect of the surface will be confined to a limit layer. At the surface, the effect of reflection and conversion affects the ratio of energies. This effect is independent of the amplitude of the incident diffuse field. Therefore, as long as an equilibration at a constant ratio occurs in the bulk, a form of equilibration is expected at the free surface. The value of the ratio of the energies at the surface can be different from the one in the bulk. Our goal here is to investigate the existence of the equilibration rather than to discuss the value of the ratio.

We set up an experiment to estimate the S-to-P energy ratio and its temporal evolution using only receivers located at the surface.

We installed a temporary small aperture array close to the city of Chilpancingo (the capital of the Guerrero state, Mexico). The high seismicity rate in Mexico made it possible to record rapidly a series of local earthquakes with magnitudes larger than 4. The location of the array is shown in Fig 5.

The geometry of the array is a 50 m side square (Fig 6). The sensors are CMG-40T seismometers connected to Reftek digitizers. The absolute time was provided by the radio signals of the GPS satellites.

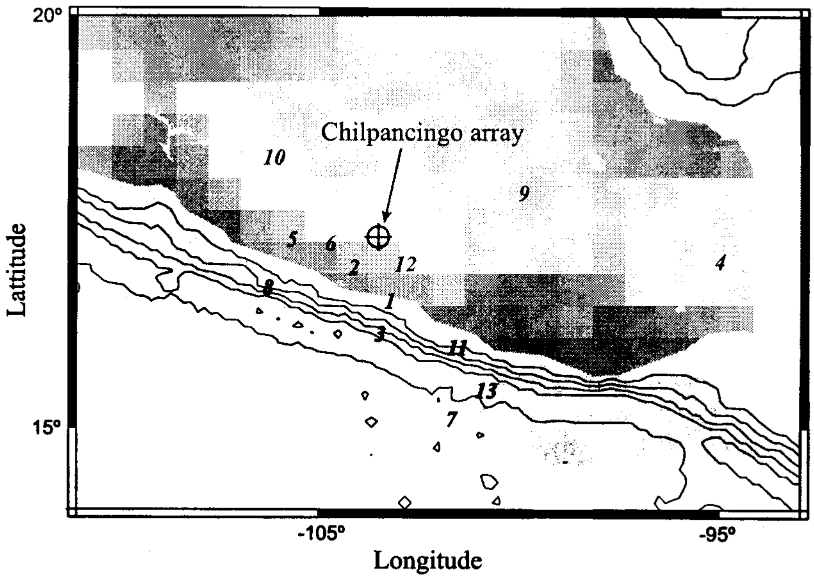

Figure 5. Map of Southern Mexico. The crossed circle shows the location of the Chilpancingo array. The italic numbers indicate the epicenters of the events used in Shapiro et al., 1998.

The spatial derivatives of the displacement with respect to horizontal coordinates were estimated by the relations:

$$\frac{\partial u_i}{\partial x} = \frac{u_i^3 - u_i^1}{d} \qquad \frac{\partial u_i}{\partial y} = \frac{u_i^2 - u_i^4}{d} \qquad i = x, y, z \tag{17}$$

where u_i^n is the displacement recorded at station n of the array (see Fig 4) and d is the distance between the two stations.

We estimated spatial derivatives of the displacement with respect to the vertical coordinate using the boundary condition at the surface. The free stress condition is:

$$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0 \tag{18}$$

In an isotropic medium, it leads to the relations between the derivatives:

$$\left\{ \begin{array}{lcl} \frac{\partial u_y}{\partial z} & = & -\frac{\partial u_z}{\partial y} \\ \frac{\partial u_x}{\partial z} & = & -\frac{\partial u_z}{\partial x} \\ \frac{\partial u_z}{\partial z} & = & -\frac{\lambda}{\lambda + 2\mu}(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}) \end{array} \right. \tag{19}$$

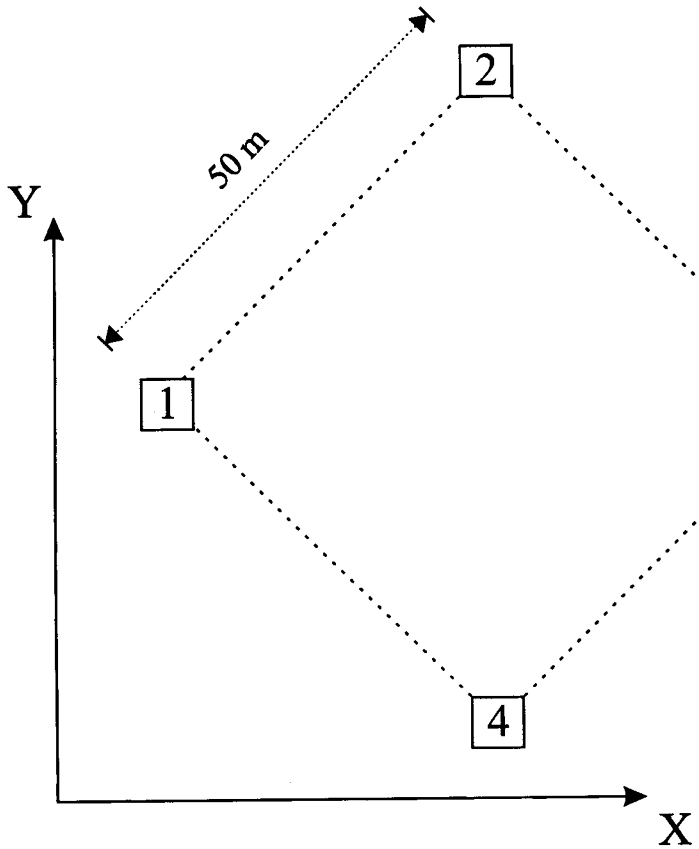

Figure 6. Configuration of the Chilpancingo array.

This last equation can be written:

$$\frac{\partial u_z}{\partial z} = (2(\beta/\alpha)^2 - 1)(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}) \qquad (20)$$

Using equations (17), (19), and (20) we can now estimate the whole set of spatial derivatives of the displacement. We can therefore rewrite equations (15) and (16) and estimate the ratio of S and P wave energy densities:

$$R = \frac{1}{4}(\frac{\alpha}{\beta})^2 \frac{4\frac{\partial u_z}{\partial x}^2 + 4\frac{\partial u_z}{\partial y}^2 + (\frac{\partial u_x}{\partial y} - \frac{\partial u_y}{\partial x})^2}{(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y})^2} \qquad (21)$$

TABLE 1. Parameters of earthquake used in the study. Locations and magnitudes are given by Mexican Seismological Survey.

N	yy:mm:dd	lat	lon	H(km)	M
1	97:06:21	16.47	-99.18	5	4.5
2	97:06:28	16.88	-99.63	7	4.1
3	97:06:29	16.07	-99.30	23	4.4
4	97:06:29	16.96	-95.07	64	4.5
5	97:07:19	17.22	-100.4	56	4.9
6	97:07:21	17.17	-99.92	24	4.5
7	97:07:22	15.02	-98.42	5	5.1
8	97:07:24	16.63	-100.7	16	4.5
9	97:07:28	17.78	-97.51	126	4.0
10	97:07:29	18.21	-100.7	74	4.4
11	97:07:30	15.9	-98.4	?	?
12	97:08:01	16.92	-99.06	35	4.3
13	97:08:03	15.37	-98.05	27	4.7

6. Experimental results

During the three months of the experiment we recorded 13 earthquakes located in southern Mexico and with magnitudes large enough to produce a coda with a good signal-to-noise ratio. Event locations are listed in Table 1 and are shown in Fig 5.

We selected events not too far away in order to have late arrivals associated with backscattered waves rather than deterministic low velocity guided modes or critical deep reflections. The energy ratio was computed for each record according to equation (21). We calculated the average S-to-P energy ratio (R) in a moving window of width 16 s. The results of the processing for two events are shown in Fig 7. The two events have different locations. The time window corresponding to the coda with a signal to noise ratio greater than 4 is indicated by grey shading.

The time evolution of the energy ratio shows that the ratio tends to stabilize in the coda. The ratio is widely variable in the noise and at the onset of direct waves. In the coda, in spite of some fluctuations, it is impor-

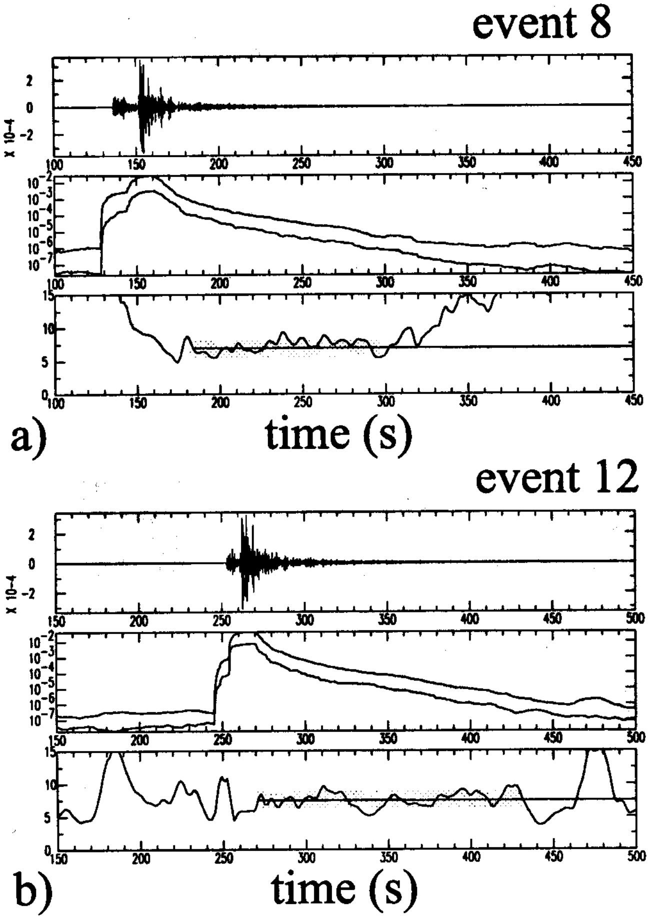

Figure 7. Examples of measurements . We selected events with different epicentral distances and backazimuths: a) event 8; b) event 12. For each event, we present the vertical component seismogram band-passed between 1 and 3 Hz (upper part), the P and S energies smoothed in a 16 s moving window (central part) and the S-to-P energy ratio (lower part).

tant to note that the ratio varies of about 25% of the mean value for the largest fluctuations while, in the same time, the energy varies by a factor of 10,000. An average value of R for the whole coda window is about 7.5 with a standard deviation of 1. We present in Fig 8 the results of essentially the same analysis but limited to the time window corresponding to the arrivals of the body waves.

The grey shading indicates the arrival times of the direct P and S waves. The mean value of the energy ratio R is calculated on 1s intervals. Its evolution is now plotted on a logarithmic scale. The energy ratio shows very

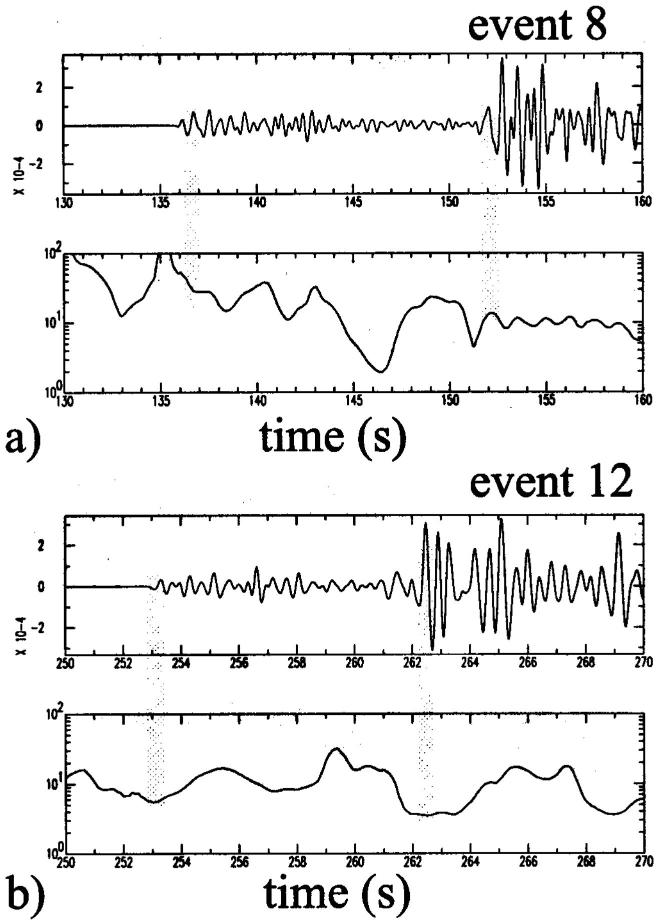

Figure 8. Examples of the measurement of the S-to-P energy ratio for direct waves: a) event 8; b) event 12. For each event, we present the vertical component seismogram band-passed between 1 and 3 Hz (upper part), the P and S energies smoothed in a 1 s moving window (central part) and the S-to-P energy ratio (lower part). Note the difference of scale in the plot of the energy ratio with respect to Fig 7.

rapid and huge variations during the deterministic arrivals. It is expected from the properties of reflection and conversion of body waves at a free surface that the energy ratio has great variations depending upon the angle of incidence and the polarization of the incident wave. The presence of a low velocity layer at the surface provokes strong P to S conversions that can explain the surprisingly high values of R observed at the onset of P waves. We performed the same measurements for the 13 earthquakes. Concerning the coda, we found in every case that the two modes equilibrate very rapidly after the arrival of the direct waves. We extract the values of R for the coda and for the direct P and S waves in order to compare how these measures

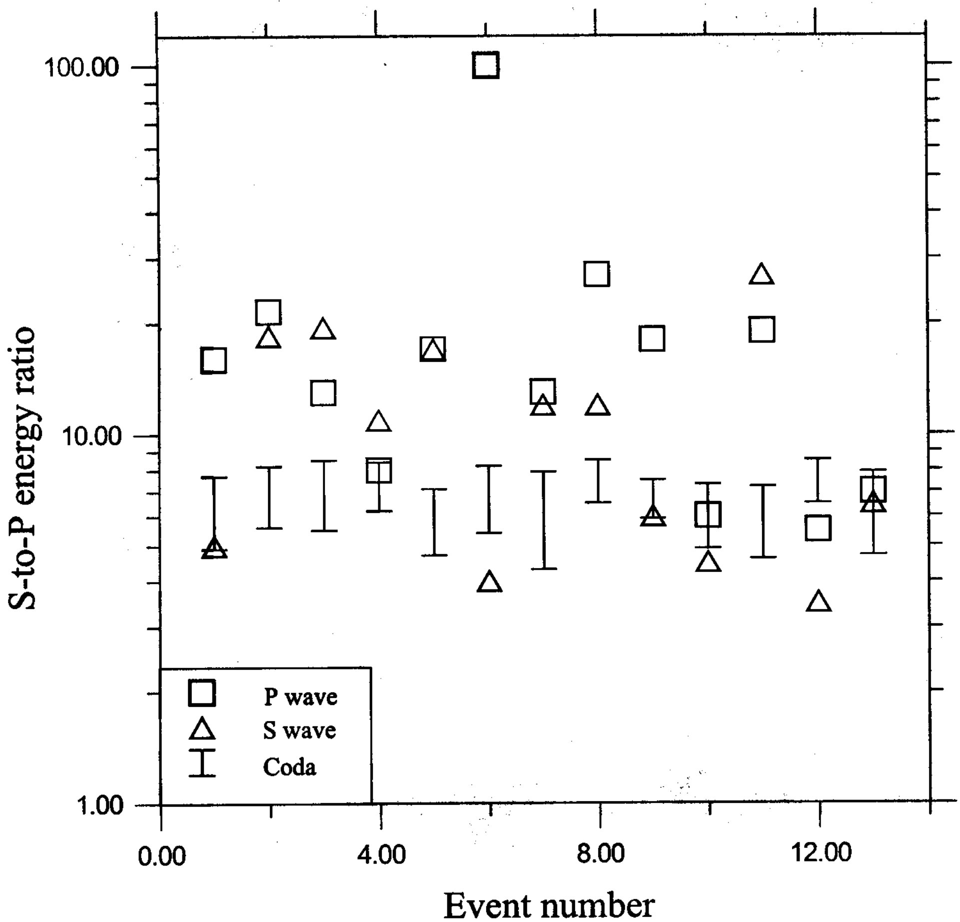

Figure 9. S-to-P energy ratios for the 13 events. The squares and the triangles indicate the ratios measured for P and S waves, respectively. Vertical bars show average values plus minus standard deviation of the S-to-P energy ratio measured in the coda window.

vary from one event to another. The results of the individual measurements are shown in Fig 9. For all events we obtain almost the same R value in the coda: about 7 in average. It means that the S-to-P energy ratio in the coda stabilizes at a values that does not depend on the seismic source nor on the earthquake location. The picture is completely different for direct waves. The ratio is much more variable from an event to the other with mean values which are significantly higher than for the coda.

The stabilization of R in the coda at the same level for all earthquakes is a good indication that the diffusion regime is reached. The theoretical value expected for diffuse waves in an infinite body is given by the equation (7). The value which we have observed at the free surface is lower than the one theoretically predicted for the poissonian solid (7 and 10.39, respectively). This difference can be due to the fact that we did not take into account the presence of the reflective surface. Obviously the reflection is a deterministic effect that breaks the isotropy of the diffuse field. This is a fundamental problem that is not yet fully solved as already noticed and we do not know exactly what is the equilibration ratio at the free surface. One can also claim that in a real situation a part of the energy of the P and S waves which are diffuse in the bulk are diffracted into surface waves at the free surface, a process that is not considered in the theory so far. These different arguments show that the important point in our analysis is not the value of the ratio but the equilibration itself which is a marker of the diffusive regime. In this sense, our experiment confirms the results of the analysis of the coda decay for which we found that it follows a law characteristic of the diffusive regime. We need to develop a simulation technique including the proper polarization and coupling effects to give a quantitative interpretation of the value of the ratio and of the short lapse time needed for the waves to equibrate. It is the object of further works. At this present stage, we retain from the results of this experiment that their are in agreement with our conclusion that the decay of the coda energy observed in Mexico is related to the leakage into the mantle of waves which are in the diffusive regime in the crust.

7. Conclusion

We studied the time decay of the coda of seismograms. We set up a simplified model of the Earth upper layers: a diffractive crust overlying an almost homogeneous mantle. We used the scalar approximation of the radiative transfer equation to model the multiple scattering of the seismic waves. We solved the equation using the Monte Carlo method. When the ratio mean free path/layer thickness is less than one, we can give an analytical form of the time decay of the coda energy as the solution of a diffusive equation. Otherwise, our transcription of the boundary condition is not valid. The asymptotic form is similar to the one that was proposed by Aki and Chouet (1975) to fit the observations. When the mean free path is larger than the layer thickness, we found numerically that the asymptote of the radiative transfer solution has the same functional form as the one obtained when our diffusion approximation is valid. The decays computed in our model in which we included the effect of a weak absorption are in a very good agreement with the observations. The leakage of diffuse energy into the mantle plays a prominent part at low frequency and controls the frequency dependence of Q_c when, as in our case, the mean free path is of the order of the crustal thickness. We propose to characterize the temporal decay of the coda energy by the time of residence of diffuse waves in the crust. We showed the diffuse character of the coda by looking for the energy partitionning that is expected for elastic waves in the diffusive regime. Considering a series of records from earthquakes at different epicentral distances, we show that the equilibration between the two modes (that is the energy partitioning) appears very early in the coda. The ratio between compressional and shear energies is very stable while the energy level changes by several orders of magnitude. The energy ratio is independent of the earthquake considered. This experiment tends to confirm that the seismic coda corresponds to waves in the diffusive regime and therefore that the decay is strongly governed by the rate of leakage of diffuse energy in the mantle.

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