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Phase and Correlation in 'Random' Seismic Fields and the Reconstruction of the Green Function

MICHEL CAMPILLO[1]

Abstract—We first present a summary of recent results on coda interpretation. We emphasize the observation of the stabilization of P to S energy ratio indicating the modal equipartition of the wavefield. This property clearly shows that the coda waves are in the regime of multiple scattering. Numerical solutions of the elastic radiative transfer equation are used to illustrate the evolution of the wave-field towards P-to-S energy stabilization, and asymptotically to complete isotropy. The energy properties of the coda have been widely studied but the phase properties have often been neglected. The recently observed coherent backscattering enhancement, an expression of the so-called 'weak localization', demonstrates that interference effects still persist for multiple diffracted waves. Another manifestation of the persistence of the phase is the possibility to reconstruct the Green function between two stations by averaging the cross correlation of coda waves produced by distant earthquakes and recorded at those two stations. This reconstruction is directly related to the properties of reciprocity and time reversal of any wavefield. Using broadband seismic coda waves, we show that the dominant phases of the Green function in the band 2 s–10 s, namely fundamental mode Rayleigh and Love waves, are reconstructed. We analyze the time symmetry of the cross correlation and show how the level of symmetry evolves with the isotropization of the diffuse field with lapse time. Similarly we investigate the correlation in continuous ambient noise records. Whereas the randomness of the coda results from multiple scattering by randomly distributed scatterers, we assume that the seismic noise is random mostly because of the distribution of sources at the surface of the Earth. Surface waves can be extracted from long time series. The dispersion curves of Rayleigh waves are deduced from the correlations. On paths where measurements from earthquake data are also available, we show that they are in good agreement with those deduced from noise correlation. The measurement of velocities from correlation of noise along paths crossing different crustal structures opens the way for a 'passive imaging' of the Earth's structure.

Key words: Random fields, coda, seismic noise, imaging.

1. Introduction

Keiiti Aki played a fundamental role in teaching generations of seismologists the need to consider that the actual complexity of the Earth is translated into the complexity of the seismograms. The studies presented here have long-ranging roots

[1]Laboratoire de Géophysique Interne et Tectonophysique, Observatoire de Grenoble, UniversitéJoseph Fourier and CNRS, BP 53, 38041 Grenoble, France. E-mail: michel.campillo@obs.ujf-grenoble.fr

in Aki's works and remarks. His pioneering 1957 study of noise has been the foundation of numerous applications in earthquake engineering and has paved the way for the present developments. On the other hand, Aki also continuously encouraged the investigation of the properties of coda waves. This paper summarizes certain results on multiple scattering in seismology and related fields. The emphasis is given to the works done in Grenoble where Aki made several visits and had a profound influence.

2. Coda

AKI (1969, 1980) interpreted coda waves as scattered waves from the inhomogeneities within the Earth. A major advance was made by defining coda Q (AKI and CHOUET, 1975). The envelope decay of coda waves was found to be in the form

$$E(t) = t^{-n} \exp\left(\frac{-\omega t}{Q_c}\right), \tag{1}$$

where ω is the circular frequency, t is time, and Q_c describes the rate of decay of the seismogram envelopes. n is another phenomenological parameter that is found to be between 1 and 2. The exponential decay is therefore controlled by Q_c which is a regional parameter, mostly independent from source mechanism, depth and epicentral distance when a sufficient lapse time is considered. Q_c has been measured in many regions of the world (see e.g., HERRAIZ and ESPINOSA, 1987) and shown to be dependent of the tectonic regime (e.g., SINGH and HERRMANN, 1983). The physical meaning of Q_c in terms of medium properties is still debated.

It is important to notice that the model expressed in (1) does not intend to describe the elastic displacement field but a scalar integrated variable such as the kinetic energy, or the intensity. Two end-member models were proposed by AKI and CHOUET: the single scattering model, which is pertinent only for the early coda as we will see later, and the diffusion model, defined as a solution of a diffusion equation in a full space. An extensive description of the numerous observations and theoretical developments that followed Aki's pioneering work is given in the book by SATO and FEHLER (1998) (see also WU and AKI, 1988, 1990).

We show in the following that coda waves are relevant to multiple scattering, a regime which can be approximated by diffusion for long lapse times. At shorter lapse time the energy can be described by the radiative transfer equation (RTE), an integro-differential expression of the local balance of energy taking into account all orders of scattering (e.g., CHANDRASEKHAR, 1960). It assumes an incoherent summation of waves, i.e., it neglects the phase of the waves and the related effects of interference. WU (1985) introduced radiative transfer in seismology. MARGERIN (2005) gives a comprehensive review of the subject. The radiative transfer equation can be derived from an ensemble average of the wave equation (e.g., RYZHIK *et al.*, 1996). It can only be solved numerically except in very specific cases (WU, 1985; ZENG, 1991; SATO, 1993). Numerical simulations have been done with the Monte-Carlo method (GUSEV and ABUBAKIROV, 1990; HOSHIBA, 1991, 1995, 1997; SATO, 1995; MARGERIN *et al*., 1998; YOSHIMOTO, 2000). It relies on the analogy between radiative transfer and Boltzmann's equation in the kinetic theory of gases. The propagation of energy is represented as the random walk of a very large number of particles. The elastic treatment requires allowing for the polarized nature of P and S waves and numerical solutions can be computed (MARGERIN *et al*., 2001).

The main parameter that describes the propagation in the multiple scattering regime is the mean free path l, or its counterpart the mean free time (that is simply l divided by V the wave velocity). The scattering mean free path l is the central quantity to describe the propagation in a scattering medium. It is the typical length scale of attenuation by scattering of a wave propagating in a given direction. It is related to the more common scattering quality factor Q_{sc} as follows:

$$l = \frac{Q_{sc} V}{\omega}. \tag{2}$$

The way the initial direction of a wave is affected by scattering is related to the 'transport mean free path' l^*, which is equal to l only for isotropic scattering (see e.g., SATO and FEHLER, 1998).

The different regimes of propagation are illustrated in Figure 1 where the full numerical solution of the RTE is compared with single scattering and diffusion approximations. It shows that the diffusion approximation is accurate to give the decay at large lapse times. We will discuss this point later from a more detailed example. The dashed line gives the solution for the single scattering approximation. The validity of the single scattering approximation is limited to the very beginning of the coda, i.e., within one or two mean free times. In practice it is difficult to know exactly the value of the mean free time. The measurement of the mean free path is actually ambiguous in most cases since both scattering and dissipation may contribute to the decay of a propagating wave. To overcome this difficulty, HOSHIBA (1991, 1993) proposed to use the RTE to interpret energy measurements performed for different lapse times and different source-station distances.

Since the Earth's structure is characterized by strong velocity variations with depth, wave propagation is strongly affected by the stratification. Assuming that the Moho is separating the heterogeneous crust and the more homogeneous mantle, a leakage of the diffuse energy of the crust into the mantle can be postulated. The strength of this leakage is controlled by the velocity contrast at the base of the crust. A wave resulting from a diffraction in the crust and incident upon the Moho will be trapped if its incidence angle is supercritical, while it mostly leaks into the transparent mantle if it is subcritically incident. The process was theoretically described under the diffusion approximation by MARGERIN *et al*. (1998, 1999) who advanced an interpretation of coda Q in the form $Q_c = Q_0 f$ with

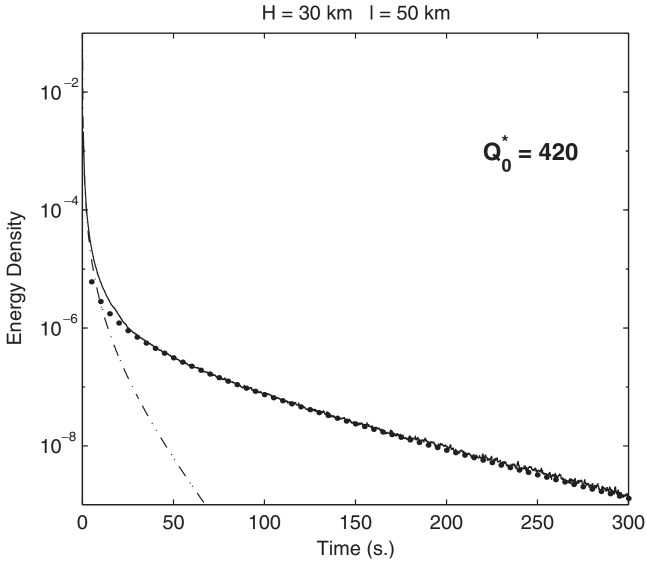

Figure 1
Coda decay obtained in a model consisting of a heterogeneous crust of thickness $H = 30$ km over a transparent mantle. The mean free path of the crust l is equal to 50 km. The solid line shows the numerical solution of the Radiative Transfer Equation obtained by Monte-Carlo modeling. The thin dashed line is the result of the single scattering approximation. The black circles indicate the behavior expected from a diffusion approximation (from CAMPILLO *et al.* 1999).

$$Q_0 = \frac{6\pi H^2}{Vl\xi^2}, \tag{3}$$

where H is the thickness of the crust, V the S-wave velocity, and ξ a parameter representing the effect of reflection at the Moho, dependent on the mean free path and the crustal thickness. Note that the frequency $f = 1$ Hz is assumed in the definition of Q_0. This formula is valid when the mean free path is smaller than the thickness of the crust. Otherwise one must rely on numerical or approximate RTE solution to infer Q_0. In this model Q_c can be regarded as the time of residence of diffuse waves in the crust. The same type of conceptual stratified model was used by LACOMBE *et al.* (2003) who modelled simultaneously the spatial decay of direct Lg waves and the envelopes of regional coda waves with only the mean free path and absorption length in the crust.

3. *Diffusion and Equipartition*

The validity of the Q_c model above is strongly limited by the assumption of the diffuse character of the wavefield that we have not yet demonstrated. It is difficult to assess the regime of propagation of the waves in a medium with absorption. With elastic waves we can rely on the concept of equipartition that leads eventually to a measurable marker of the diffusive regime, namely the ratio of P to S wave energies. The predominance of S waves in the coda was discussed in AKI (1992). He demonstrated that the scattering from P to S is much stronger than from S to P. Using an elegant reciprocity argument, he showed that the ratio of scattering coefficients is

$$\frac{g_{PS}}{g_{SP}} = \frac{\alpha^4}{\beta^4}, \tag{4}$$

where α and β are the velocity of P and S waves, respectively. We therefore expect a predominance of S waves in the coda. This was also observed by DAINTY and TOKSÖZ (1990) who have shown using array analysis of NORSAR data, that the coda is dominated by waves with apparent velocities less than 4 km/sec, that is, by S waves. More generally, in the diffusive regime that emerges after several scatterings, the wavefield is expected to consist of contributions of all possible modes of propagation. For example in a full space, modes of propagation are P and S plane waves.

In this case equipartion means that the wavefield consists of waves propagating in all directions and polarizations, with equal weight in average. A fixed weight implies that the relative contribution of P and S waves to the local total energy stabilizes to a constant ratio, whereas the total energy is continuously decaying due to the spatial expansion. A simple mode counting argument allows to compute the ratio of P to S energy at equipartition in a full space (WEAVER, 1982, 1990; RYZHIK *et al.*, 1996):

$$\frac{E_S}{E_P} = 2\frac{\alpha^3}{\beta^3}. \tag{5}$$

Assuming a ratio of velocities of $\sqrt{3}$, the energy ratio is 10.4. When there is preferential absorption of one of the modes (P or S), a stabilization of the energy ratio occurs in the multiple scattering regime (MARGERIN *et al.*, 2001). The effect of absorption is to shift the ratio in favor of the mode that is less absorbed. With realistic values of dissipation in rocks, this effect is not expected to strongly affect the observations. It is useful to further clarify the concept of equipartition and the definition of the underlying set of modes. The set of modes we consider is the one of a reference model on which the disorder is added. The disorder is assumed to be large enough to provoke the coupling between the modes but not to change drastically the structure of the spectrum and eigenfunctions. In other words, the modes we consider are not formally the ones of the actual Earth with all its complexity, but the ones of a fictitious model close enough to essentially share the same spectrum, i.e., essentially the same propagation properties. For a first-order computation of the energy ratio, it could be a simple stratified model. Furthermore, we deal here with ‘local’ propagating modes of the lithosphere, that is fundamental and higher modes of surface waves, and body waves leaking into the mantle. It is important to notice that equipartition is expected in the phase space so that it does not mean that the local distribution of energy is the same everywhere in the real space. Locally the ratio is governed by the eigenfunctions of the modes. For example, in the equipartition regime, the contribution of surface waves at the free surface will be larger than at depth due to the decay of fundamental mode eigenfunctions (see HENNINO *et al.*, 2001).

What is the significance of the stabilization of S-to-P energy ratio with respect to the validity of the diffusion approximation and the equipartition itself? Is there an unambiguous relation between stabilization of energy ratio and equipartition? To answer these questions, Figure 2 sets forth numerical solutions of the elastic radiative transfer equation computed with the Monte Carlo approach of MARGERIN *et al.* (2001) compared with solutions of the diffusion equation (PAUL *et al.*, 2005). They indicate that the stabilization occurs when the total energy is well approximated by a solution of the diffusion equation (Fig. 2, upper panels). On the contrary, when considering the angular dependence of the energy flow (Fig. 2, lower panel), it is clear that the stabilization occurs very early in the evolution of the wavefield toward isotropy. At a time when the stabilization is reached (60 s in Fig. 2), the anisotropy of the field is very strong with a ratio of more than four between energies propagating in the forward and backward directions. The diffusion solution itself includes a flow of energy from the source, and therefore an anisotropy of the field. Figure 2 also indicates that, when using the diffusion approximation, the anisotropy is underestimated with respect to the radiative transfer equation. We must conclude that the stabilization of S-to-P energy ratio is a good indication that the field is entering a regime in which the total energy is described by the diffusion equation and therefore will evolve towards equipartition and isotropy. Note that although the computations we just presented are performed in a full space, the argument of the remnant flow still will be valid in a stratified medium like the Earth.

It is possible to evaluate the S-to-P energy ratio from field records from a small aperture array. The energies are measured from the divergence and curl of the field. Divergence and curl can be evaluated from an array at the surface by using the extra information given by the free surface conditions. Such an experiment was performed in Mexico (CAMPILLO *et al.*, 1999; SHAPIRO *et al.*, 2000). The energy ratio was computed in moving windows along the seismograms. The P to S ratio stabilizes in the coda at a value of about 7.3, independently of source depth and epicentral distance (Fig. 3). This value is indeed far from the full space expectation (10.4). This theoretical ratio is obtained by neglecting reflections and the generation of Rayleigh waves while the measurement is made at the free surface itself. Considering a model that includes Rayleigh waves, HENNINO *et al.* (2001) showed that the energy ratio at the free surface of an elastic half space is 7.2, a value very close to the average observation. Note that at depth the ratio tends to the value obtained in a full space

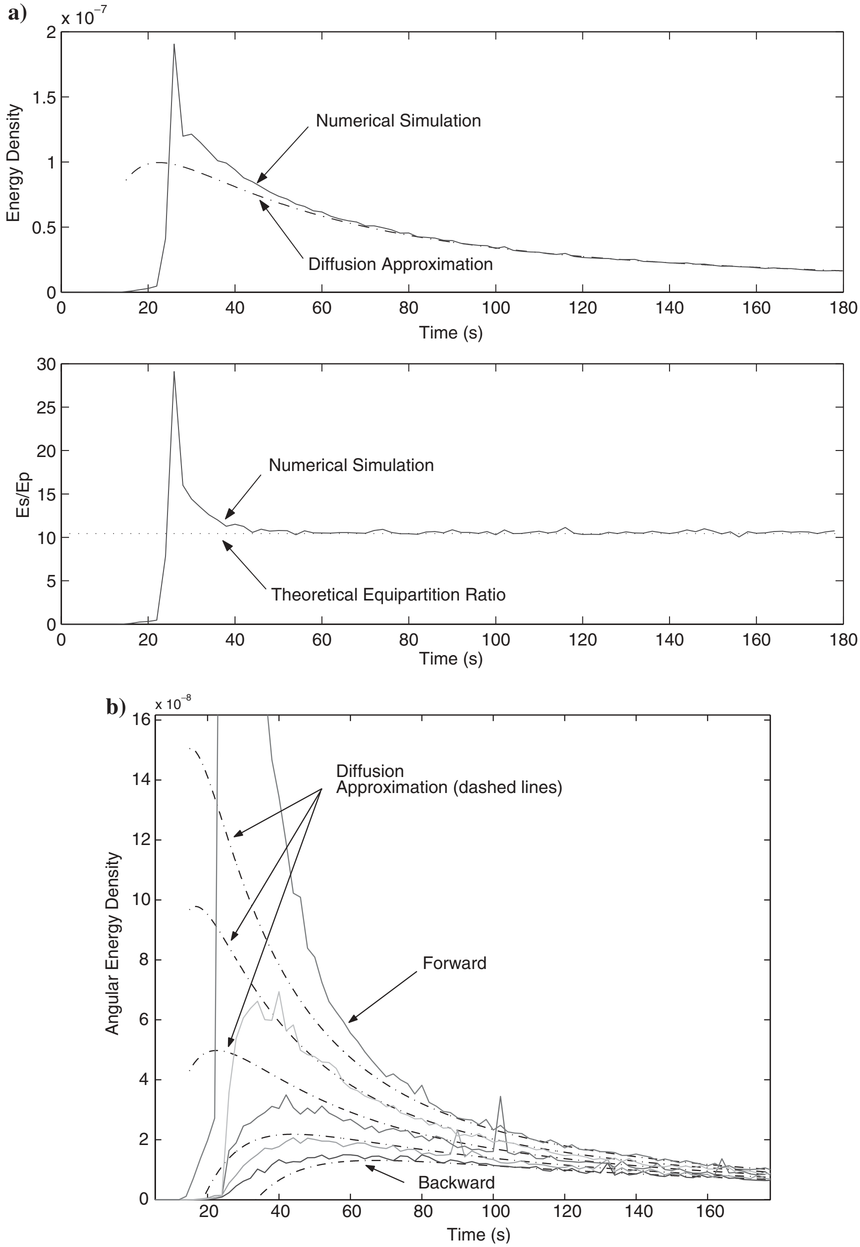

Figure 2

Comparison between numerical (Monte-Carlo) solutions of the radiative transfer equation, and analytical solutions of the diffusion equation. Energy density (top), *P*-to-*S* energy ratio (middle) and angular distribution of elastic energy flux (bottom). The dashed and solid lines show the results of the diffusion approximation and radiative transfer equation, respectively. The energy flux decreases monotonically from $\theta = 0$ (forward direction) to $\theta = \pi$ (backward direction), where θ denotes the angle between the propagation direction and the source-observer vector. The results for $\theta = \pi/4$, $\pi/2$ and $3\pi/4$ are also plotted.

(10.4). In field data the stabilization occurs in the early coda (Fig. 3), indicating that the late coda is clearly diffusive. This result is another indication that coda waves have the robust properties of diffusive waves and are particularly well adapted for measuring earthquake magnitudes (e.g., MAYEDA and WALTER, 1996) and site effects (e.g., PHILLIPS and AKI, 1986).

4. Interference Effects with Multiply-scattered Waves

One of the underlying assumptions with the use of RTE or diffusion equation is that the energy of the signal is made up of a summation of incoherent contributions. These theories account for the macroscopic behavior of averaged quantities such as the energy density. At the same time, the displacement is always a local solution of wave equations, and therefore the existence of phase effects, such as interferences, occurring at the scale of the wavelength must not be ruled out. In this sense, multiply-scattered seismic waves are relevant to mesoscopic physics. The effect of interferences within a diffuse field is illustrated by the so called coherent backscattering enhancement, also named weak localization. This effect appears only in the multiple scattering regime. It is a consequence of reciprocity that results in the constructive interference between long reciprocal paths in wave scattering. The probability to return to the source is increased by a factor of exactly 2 with respect to other paths. Eventually it results in a local energy density enhancement by the same factor 2 (e.g., AKKERMANS *et al.*, 1986).

In seismic experiments performed at the surface, weak localization appears as an enhancement of seismic energy in the vicinity of a source for long lapse times. MARGERIN *et al.* (2001) studied the scalar case in the configuration of a seismic experiment. Note that special care must be given to rules of reciprocity with polarized waves. The elastic case was treated by VAN TIGGELEN *et al.* (2001). A field experiment was performed in a volcanic environment using a sledge hammer as a source recorded along a profile (LAROSE *et al.*, 2004). The energy of the signal is computed in different time windows of 0.4 s duration. In each window, the energy is normalized by the maximum over the array, and then averaged over 12 different configurations. Figure 4 shows the average energy enhancement along the profile computed in time windows at different lapse times after the passing of the direct waves. As expected from the theory, the enhancement shows up progressively for the longer lapse times. This simple experiment indicates a nonintuitive mesoscopic effect that demonstrates that the phase effect cannot be neglected for seismic waves in the multiply-reflected regime. The characteristic time for the onset of the enhancement spot (approximately 0.7 s in the case of Fig. 4) is the scattering mean-free time, which is a measurement of the heterogeneity of the medium. This type of experiment is a way to measure the mean free time independently of dissipation.

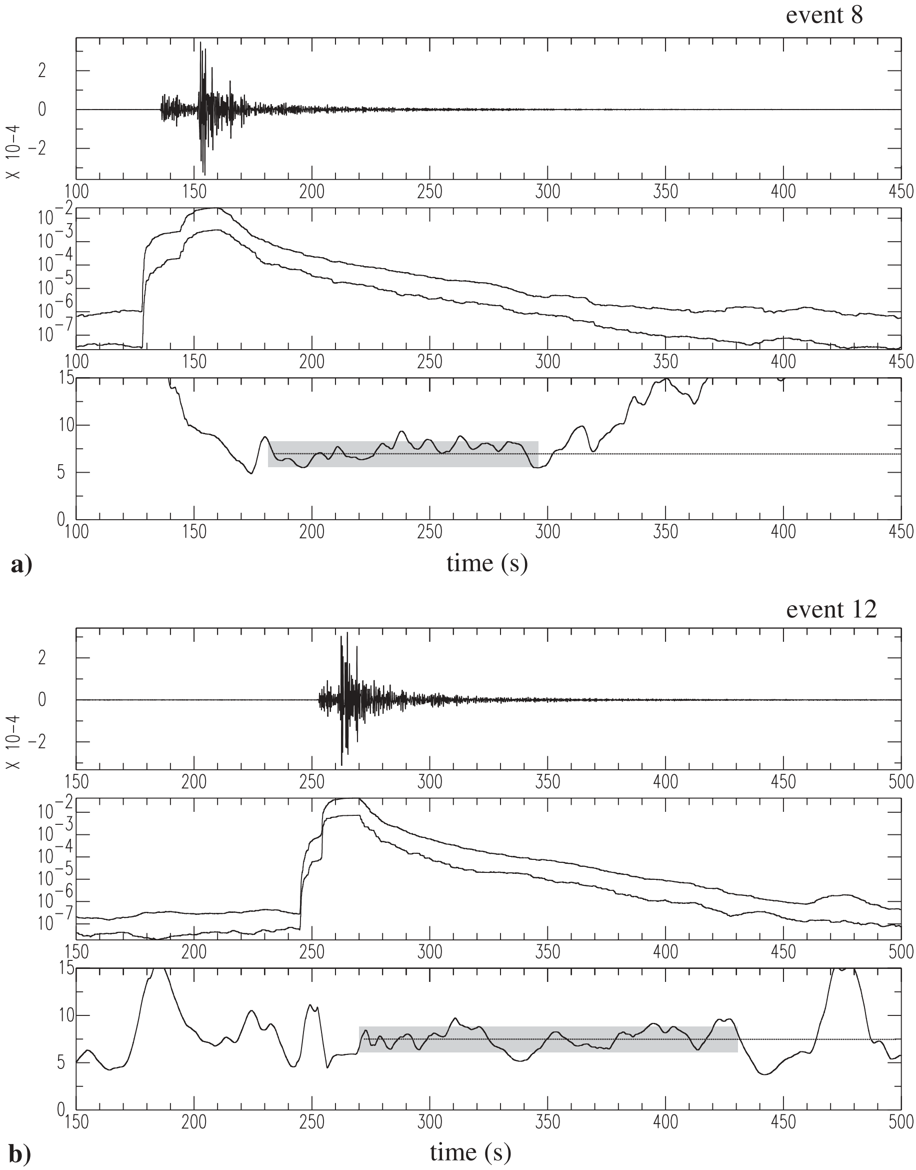

Figure 3

Examples of stabilization of the energy ratio for two earthquakes at different epicentral distances. In each case the panel shows the seismograms on the vertical component (top), the P and S energies computed in the frequency range 1 to 3 Hz (middle) and S-to-P energy ratio (bottom). Note that the energies are plotted on a logarithmic scale while the ratio is plotted on a linear scale (from CAMPILLO *et al.*, 1999).

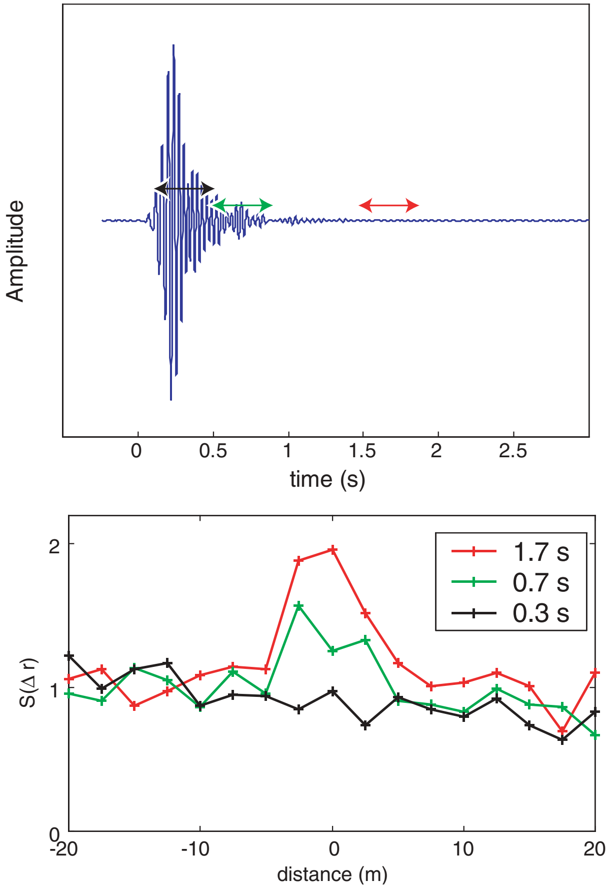

Figure 4
Emergence of weak localization. The upper panel shows an example of a signal produced by a hammer strike. The normalized average energy in the different time windows is plotted as a function of the distance from the source (lower panel). Note the absence of an enhancement spot for the early coda and the progressive onset of the weak localization.

5. Correlation and Diffuse Field

A consequence of interference effects such as the weak localization is that we can expect to extract deterministic wave propagation between two points where diffuse fields are recorded. Such an approach follows a long trend of attempts to reconstruct the Green's function from complex fields such as diffuse fields or noise, that were otherwise considered as random (e.g., AKI, 1957; TOKSÖZ, 1964; CLEARBOUT, 1968). The reconstruction of deterministic arrivals from the correlation of wavefields is not specific to diffuse waves. We will discuss this in the next section, however let us first consider the implications of multiple scattering. Formally, an implication of the presence of all modes is that the information regarding any possible path is represented in the coda records. Let us follow a conceptual approach to understand further the role of scattering and equipartition. Mathematically, a wavefield inside a finite elastic body can be expressed in modal form

$$u(\mathbf{x}, t) = \sum_n a_n \phi_n(\mathbf{x}) e^{i\omega_n t}, \tag{6}$$

where $\mathbf{x}$ is position, t is time, ϕ_n are eigenfunctions, ω_n are eigenfrequencies, and a_n are modal excitation functions that depend only on the source. As in the section on equipartition, the modes correspond to a reasonable reference model, close to the actual medium, and to which a disorder is added in the form of a perturbation of the elastic parameters for example. When disorder is added and after a sufficiently long time, the field becomes diffuse and the coefficients a_n become random functions of time. Equipartition means that the modal amplitudes are uncorrelated random variables

$$\langle a_n a_m^* \rangle = \delta_{nm} F(\omega_n), \tag{7}$$

where $F(\omega)$ is the spectral energy density of the source in the frequency band $[\omega - \delta\omega, \omega + \delta\omega]$. An implication of equation (7) is that all modes in a narrow frequency range are excited at the same energy level. Note that here we make the drastic hypothesis that, except for the total energy emitted, the signature of the source properties (mechanism, precise location, ..) has completely disappeared in the diffusion process. The brackets in equation (7) mean either an ensemble average over the disorder or a time/frequency average of a single realization. Another type of average can be performed by considering the fields produced by a set of sources inside the body. Since we can compute the correlation over limited time windows only in practice, source averaging is required. We will consider this kind of averaging in the following sections.

The average correlation between the fields at locations x and y simply becomes

$$\langle C(\mathbf{x}, \mathbf{y}, \tau) \rangle = \sum_n F(\omega_n) \phi_n(\mathbf{x}) \phi_n(\mathbf{y}) e^{-i\omega_n \tau}, \tag{8}$$

since the cross terms disappear on average due to (2). This expression, actually its time derivative, is close to the Green function between x and y defined for positive times as

$$G(\mathbf{x}, \mathbf{y}, t) = \Re\left(\sum_n \frac{\phi_n(\mathbf{x}) \phi_n(\mathbf{y})}{-i\omega_n} e^{-i\omega_n t} \right). \tag{9}$$

This reasoning must be regarded as a plausibility argument. LOBKIS and WEAVER (2001) gave a complete argument that does not rely on the equipartition hypothesis. In this case, the correlation depends also on the source whose influence was discarded when equipartition was assumed. The correlation is actually proportional to $G(\mathbf{x}, \mathbf{y}, t)$ convolved by the local response at the source $G_{ss} = G(\mathbf{s}, \mathbf{s}, t)$. The same formal result was already in DRAEGER and FINK (1999). If the medium is locally homogeneous, G_{ss} can be approximated by $\delta(t)$ and this factor is neglected. In case of a strong heterogeneity in the vicinity of the source, as the free surface in seismology, G_{ss} exhibits secondary arrivals. They result in spurious contributions in the correlation. G_{ss} can nevertheless be neglected if we assume a homogeneous distribution of sources throughout the medium. In practice, when dealing with seismograms from earthquakes at different depths and in different environments, we expect that the spurious contributions cancel out with source averaging nonetheless their role in the eventual 'noise' in the reconstruction cannot be ruled out. The correlation of two signals given by the general form (8) does not have the dimension of a displacement. Depending on the quantity considered in practice, displacement or velocity for example, we expect to recognize the waveshape of the response from the time derivative, or the correlation itself.

Encouraged by the positive results obtained in acoustics, CAMPILLO and PAUL (2003) tested the feasability of the reconstruction with coda waves recorded in Mexico. The first striking observation is that long-range correlations of (multiply-scattered) coda waves can be measured between two stations. As intuitively expected, the correlation obtained for a single coda window does not exhibit any clear pulse. The surface wave part of the Green function is accurately reconstructed only after source averaging. The correlation of records from a single earthquake exhibits no visible arrivals associated with the Green function but, due to the limited time averaging performed with actual seismograms, only remnant fluctuations of the diffuse field. In the source averaging, the deterministic Green function emerges since it increases with N the number of time windows while the incoherent fluctuation varies only with the square root of N. Rayleigh and Love waves can be identified without ambiguity in the correlations, both from their speeds and from their polarizations. Furthermore, the average cross correlations between the different components of motions present the symmetries of the Green tensor. Depending on the couple of stations considered, it was observed that the correlations are symmetric or asymmetric in time, a feature that was interpreted by CAMPILLO and PAUL (2003) as the mark of the flow of diffuse energy from the source. This point will be discussed in section 7.

In the following we discuss how the Green function can emerge from the correlations for configurations close to the ones encountered in seismology. Two different approaches are presented. First, the reconstruction can be performed from the correlations of signals produced by a set of sources. The Green function will emerge through source averaging. This approach is independent of the regime of propagation given a sufficient distribution of sources. The presence of scattering is not required. Second, the reconstruction can be performed from a diffuse field, in theory even from a single source. Finally, it will be shown that the reconstruction can be performed with a limited number of sources if sufficiently strong scattering occurs.

6. *Analogy with Time Reversal Experiments*

Before entering into a detailed discussion of the results, let us propose a simple analogy that facilitates understanding the physical process leading to the reconstruction. DERODE *et al*. (2003) showed the relation between the reconstruction from correlations and time reversal experiments. In the original time reversal experiment (FINK, 1992), a series of active devices are used to collect the acoustic signals, time reverse them and re-emit them in the medium, forming a so-called time-reversed mirror. Because of time symmetry of the wave equation, the waves naturally back propagate to focus on the original source. This operation is similar to the concept of wave-equation migration developed in seismic exploration while in this case numerical propagation is achieved (e.g., TARANTOLA, 1999). The Time Reversal

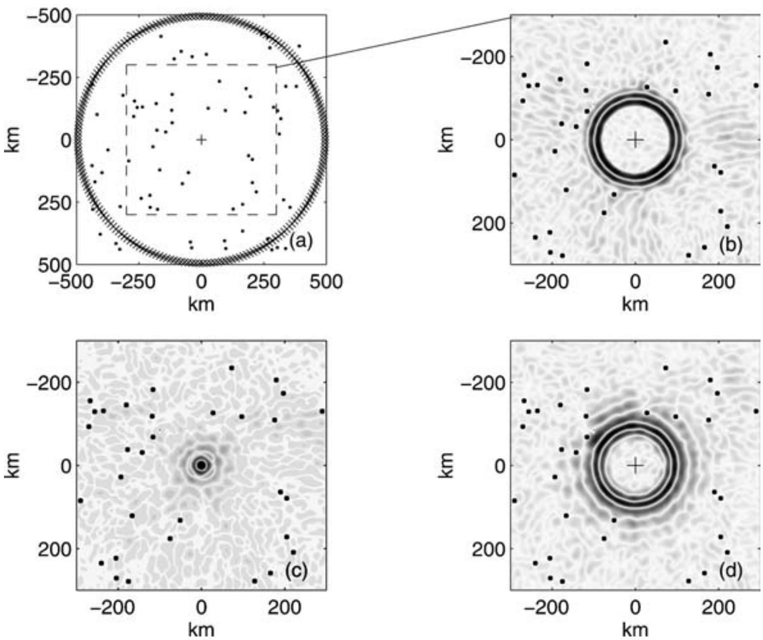

Figure 5
Numerical simulation of the reconstruction of the causal and anti-causal parts of the Green function from cross correlations. (a) Configuration of the numerical experiment. 1000 sources S ($\times$) surrounding the reference point A (+) are considered successively . The black dots indicate the point scatterers. (b) Snapshot of the cross correlation between the field in A with the field at location $(\mathbf{x}, \mathbf{y})$ after averaging over the sources S for correlation time −30 s. The weakly diffusive medium is characterized by the transport mean free path $l^* = 640$ km which is larger than the distance between the points where the correlations are computed. A converging wavefront is well-defined and constitutes the anti-causal part of the Green function. (c) Snapshot for correlation time $t = 0$ s : the wavefront is focused on A. (d) Snapshot for $t = 30$ s: the diverging wavefront corresponds to the causal part of the Green function (from PAUL *et al*., 2005).

Mirror (TRM) technique was a breakthrough for practical applications in ultrasonics (e.g. FINK *et al.*, 2000) and marine acoustics (e.g., PARVULESCU, 1995; ROUX *et al.* 2004).

TRM can be understood easily with the Huyghens' principle. Imagine that a signal emitted by a source at $\mathbf{x}_a$ is recorded at every point $\mathbf{s}$ on a close surface Σ surrounding the source. For the sake of simplicity, we assume in the following that source functions are ideally impulsive as well as receiver responses. After time reversal and re-emission, the waves will naturally converge to the original source. The validity of this idea is independent of the medium in the vicinity of the point $\mathbf{x}_a$, which can be heterogeneous.

The relation of TRM and emergence of Green function from correlations is straightforward and based on reciprocity only. If the primary source is now at the point $\mathbf{s}$ on Σ, we can compute the cross correlation between the signals produced in $\mathbf{x}_a$ and another location $\mathbf{x}_b$. Because of reciprocity and by definition of correlation and convolution, the correlation $C(\mathbf{x}_a, \mathbf{x}_b)$ is strictly equal to the signal that would be produced if the source was in $\mathbf{x}_a$, recorded in $\mathbf{s}$ where it was time-reversed and re-emmitted to be finally recorded in $\mathbf{x}_b$. Reproducing this operation with a distribution of sources on Σ leads to the exact equivalent of a perfect time-reversal experiment. This analogy corresponds to visualizing the correlation between the field at a reference point and at surrounding points as a reconstructed physical field. Figure 5 gives a simple example that illustrates that reconstruction by correlation is not solely an artificial signal processing but results from physical wave propagation. The correlation map depicts the convergent (anti-causal solution: G(−t)) and divergent (causal G(t)) wave fronts. These two contributions are expected (LOBKIS and WEAVER, 2001) and observed in seismology (CAMPILLO and PAUL, 2003; PAUL *et al.*, 2005) and ultrasonics (e.g., MALCOLM *et al.*, 2004; LAROSE *et al.*, 2005).

Note that there is an implicit assumption in the time reversal and the use of the concept of the Green function. To consider the simplest case of acoustics, a signal, typically a particle velocity, cannot be directly time-reversed and re-emmitted as a pressure without a hypothesis about the impedance. Even though the proportionality of velocity and pressure, i.e., a constant impedance, is valid only for a homogeneous medium, it is a good approximation if the medium is locally homogeneous. If not, one must consider the local response at the source itself, that is the Green function *Gss*. We mentioned this point in the previous section and indicated that it can be reasonably ruled out by averaging over a source set since in every case *Gss* is dominated by the pulse at time 0.

The analogy allows to directly apply the results of the analysis of time reversal experiments to the reconstruction of the Green function by correlations. Most of the analysis was done for acoustic waves. They demonstrate that the identification of the Green function to source averaged field-field correlations is correct when the uncorrelated sources are distributed on a closed surface as for the perfect TRM, or if they are uniformly distributed in the volume (e.g., ROUX *et al.*, 2004; WEAVER and LOBKIS, 2004). SNIEDER (2004) used a stationary phase argument to demonstrate that the surface waves can be reconstructed by correlation when assuming an isotropic distribution of incoming scattered waves. WAPENAAR (2004) recently gave a formal demonstration of a generalization in the elastic case for a distribution of sources on a closed surface. These formal demonstrations cannot be directly applied to actual configurations in seismology since sources are not uniformly distributed, neither in the volume nor on a closed surface. In exploration seismics with a sufficiently large coverage of sources, the correlation method can be used to provide Green functions between receivers at depth (BAKULIN and CALVERT, 2004) in the same way as in laboratory.

7. Correlation and Diffusion Approximation

It was known for years from experiments with ultrasounds that time reversal focusing, or conversely correlation reconstruction can be achieved with a very limited mirror surface, owing the medium to be heterogeneous enough to produce a diffuse field (e.g., DERODE *et al.*, 1998; ROUX *et al.*, 2000). Following the analogy between source average correlation and TRM, the same argument holds for Green function reconstruction in seismology. First let us review a couple of theoretical results exclusively concerning diffuse waves. The point here is to isolate a single source. If multiple scattering occurs, the resulting wavefield evolves towards an equipartitioned state as discussed above. Under this condition, it is easy to show that the average correlation is proportional to the Green function (LOBKIS and WEAVER, 2001). The argument is based on a time average over a long lapse time and with the limitations of the equipartition principle on the role played by disorder in mixing the modes without changing the structure of the spectrum as already mentioned. VAN TIGGELEN (2003) used the diffusion approximation to derive an asymptotic relation between the field-field correlation and the ensemble averaged Green function of the medium, i.e., the response in a medium with the effective properties of the actual heterogeneous medium. The correlation of the fields at two points separated by **r** and located at **R** from the source can be written as

$$C_{\mathbf{R}}(\mathbf{r},\tau) = \rho(\mathbf{R},\tau)\frac{\partial}{\partial\tau}[\langle G_B(\mathbf{r},\tau)\rangle - \langle G_B(\mathbf{r},-\tau)\rangle] - 3\mathbf{J}(\mathbf{R},\tau)\cdot\nabla[\langle G_B(\mathbf{r},\tau)\rangle - \langle G_B(\mathbf{r},-\tau)\rangle] \quad (10)$$

where the brackets $\langle\cdot\rangle$ denote an ensemble average, and G_B is the retarded causal Green function filtered in frequency band B. ρ is the diffuse energy density and **J** the energy current. With D the diffusion constant, **J** is given by the Fourier's law:

$$\mathbf{J}(\mathbf{R},t) = -D\nabla\rho(\mathbf{R},t). \quad (11)$$

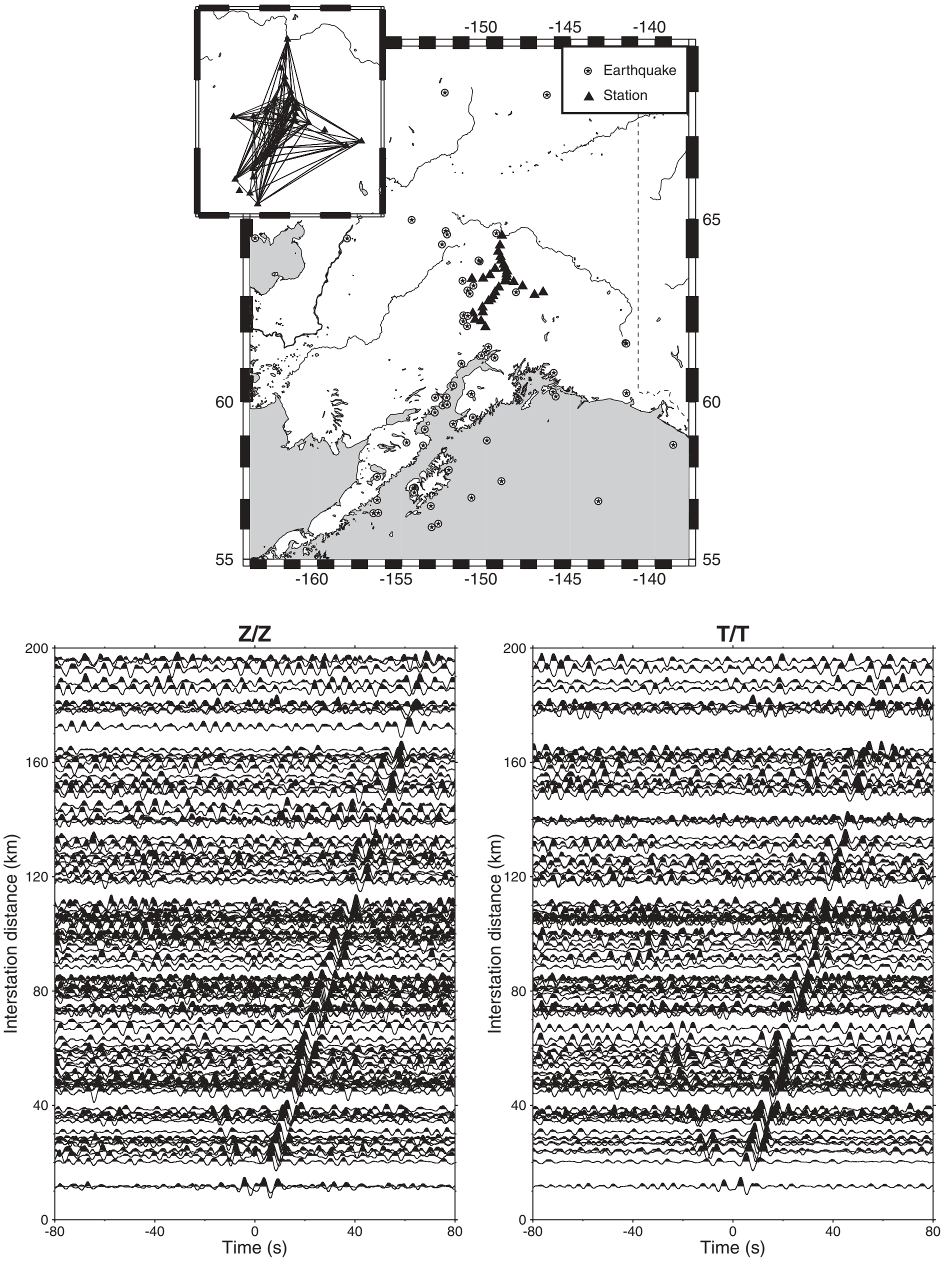

Figure 6

Top: Map of stations and earthquakes used in the analysis of Alaskan data. The paths between couples of stations for which average cross correlations have been computed are shown in the inset. Bottom: Average cross correlations as a function of interstation distance. The correlations have been computed for vertical (Z) components of motion (left), and transverse (T) (right).

The relation is valid for long times of correlation and ensemble averaged quantities. Indeed it is not directly relevant for seismological applications nonetheless it provides a useful relation between correlation of the diffuse field produced by a single source and the (time-symmetric) Green function.

These results on the correlation of diffuse waves provide another perspective on the reconstruction of Green function which is independent of the source distribution and which suggests that if the time of observation is long enough, a single source could be sufficient. To conclude this short summary on the theoretical approaches, we have two different arguments which can be invoked: Firstly, source averaging over an (almost) uniform volumic distribution of sources, or upon a uniform source distribution on an enclosing surface, and secondly infinite time averaging over a perfectly diffuse wavefield. In practical seismological applications neither of the two proper theoretical arguments can be used directly how ever both can be exploited simultaneously. Since the duration of the records is limited by the presence of noise and by absorption, averaging over a set of sources is required to expect the emergence of the Green function. It can be stated both ways: Multiple scattering (and the consequent equipartition) compensates the lack of sources or conversely source averaging compensates for the limited duration of the actual signals. DERODE *et al.* (2003b) and LAROSE *et al.* (2004) showed the role of multiple scattering in enhancing the efficiency of the reconstruction of the Green function with a limited number of sources and finite durations of recording, in conditions approximating those of seismology. Examples of numerical simulations of 2-D snapshots of correlations are presented by PAUL *et al.* (2005). They illustrate that even a discrete distribution of sources is enough to reconstruct the Green function as soon as multiple scattering is involved.

8. Applications to Coda Waves

CAMPILLO and PAUL (2003) reconstructed the surface wave part of the Green's functions between three pairs of stations in Mexico. To confirm these results in a different environment and to show that they are not dependent upon a particular azimuth of the path between the stations, PAUL *et al.* (2005) used the data from a temporary broadband experiment in Alaska (FERRIS *et al.*, 2003). Figure 6 (upper panel) shows the locations of stations and earthquakes. Correlations have been computed for every available pair of stations. The resulting paths span a large range of azimuths (Fig. 6). The records from about 100 regional earthquakes have been used. To obtain the source averaged correlations we first filtered the original broadband seismograms in the band 0.08–0.3 Hz. The horizontal components of the seismograms were then rotated assuming the inter station great circle path to be the radial direction. We used coda records starting 20 s after the arrival of the *S* wave and ending when the signal-to-noise ratio was smaller than 4. The cross correlation between coda records cannot be performed directly. In fact the exponential coda amplitude decay would result in a strong overweighting of the earliest part of the coda. In order to overcome this difficulty, we disregard the amplitudes completely and consider only one-bit signals. Since the phases are preserved when transforming the original data into 1-bit signals, this operation does not alter significantly the correlation of band-passed time series. CAMPILLO and PAUL (2003) checked that using 1-bit signals leads to the same results as a compensation for coda Q in successive time windows. This procedure has been successfully applied in time reversal experiments (DERODE *et al.*, 1999). The cross correlation of one-bit coda records at each pair of stations was computed for each earthquake, normalized to a maximum amplitude of one, and averaged over the entire set of events.

The resulting time-distance sections of the cross correlation of vertical with vertical and transverse with transverse components are also presented in Fig. 6. As for the Mexican data, the cross correlations present the symmetries of the Green tensor (PAUL *et al.*, 2005). The dominant phases are Rayleigh and Love waves as expected for the response at the surface of a source at the surface. The quality of the reconstruction is not enough to observe the body waves. Besides, it is possible that the coda at the period range we consider is dominated by surface waves and that the body waves are more difficult to reconstruct. Traces with a maximum amplitude at a negative time have been time-reversed to improve the identification at positive times and to attain a better view of the presence, or the absence, of symmetry in time. This experiment shows the anti-causal waves, which are expected from the symmetry of the wave equation to be valid solutions, but are not present in the signals produced by earthquakes or other actual sources in the real causal world.

We interpret the fact that causal and anti-causal Green functions have different amplitudes, i.e., time asymmetry, as the effect of the persistence of a net flux of energy from the source (Fig. 3), even in the late coda as it was discussed above. Since the sources are not evenly distributed, a preferential direction of transport remains after averaging. The data indicate that the time asymmetry is less pronounced when the correlation is performed in the later windows of the coda. This confirms that coda waves are behaving as multiply-scattered waves evolving towards equipartition and isotropy.

The relatively low frequency content (0.08–0.3 Hz) of the reconstructed signals has several possible causes. First of all the high frequency waves are attenuated more rapidly. It is therefore natural that they are more difficult to observe in noisy signals such as correlations. Furthermore the reconstruction at a given distance is more difficult for high frequency waves. It can be understood easily with a ray argument. Following a stationary phase argument (e.g., AKI and RICHARDS, 1980; ROUX *et al.*, 2004, and SNIEDER, 2004 for application to the reconstruction of Green function) the coherent constructive contributions of the direct wave part of the Green functions can be seen as the diffracted waves propagating in a limited 'endfire lobe' around the direction of the two stations. Indeed the width of this contributing region is proportional to the square root of the wavelength (e.g., SPETZLER and SNIEDER, 2004). Conversely, it is also decreasing with the distance, showing that it is naturally difficult to reconstruct high frequencies at large distances, i.e., a large number of wavelengths. It is also important to note that the signal-to-noise-ratio obtained with the correlation depends on the total duration of the signals which were correlated. Because of absorption, the available duration of coda is strongly decreasing with frequency varying between 0.1 to 5 Hz. This is an important limitation of the use of coda waves. The bandwidth of the signals is also important since an average upon frequency is also implicitly performed. Actually we know that the diffuse field remains correlated over a frequency interval called the Thouless frequency. The Thouless frequency can be written in our case as: $\omega_T \approx Vl/3R^2$ where V is the shear-wave velocity, l the mean free path and R the characteristic distance between the source and the pair of receivers. There is no advantage in frequency averaging when the bandwidth is small with respect to the Thouless frequency. In our example, ω_T is much smaller than the bandwidth and therefore our processing takes advantage of the independent information available over the entire frequency band.

9. Field Correlation in the Seismic Noise

In the previous section, we discussed the correlations between fields that are produced by deterministic sources such as earthquakes. We saw that, in our applications, the reconstruction of the Green function relies both on a source average over a large number of sources and on the (partial) isotropy of the field due to multiple scattering. It is therefore appealing to use the ambient noise although its actual origin is not fully known. The same principles must apply for diffuse coda waves from transient sources or for noise. The idea of using noise for exploration purposes is not new. Since 1957, Keiiti Aki proposed the use of noise to retrieve the propagation properties of the subsurface. CLAERBOUT (1968) also suggested the use of ambient noise to reconstruct the reflectivity profile. Later on, a similar concept of extraction of the response by correlation was successful in helioseismology (e.g., DUVALL *et al.*, 1993; RICKETT and CLAERBOUT, 1999, 2000).

Ambient noise has also been a long standing subject of interest in marine acoustics (KUPERMANN and INGENITO, 1980) and correlations of noise produced by the wavebreaking and ships have been used to reconstruct the acoustic response of the ocean (SABRA *et al.*, 2003). This calls to mind the fluctuation-dissipation theorem (e.g., KUBO, 1966) which states a relation between the Green function between two points and the correlation of the random fluctuations of the field at these two points. Originally, this relation was developed for thermal noise. An application in acoustics was shown by WEAVER and LOBKIS (2001) who successfully retrieved reflected paths in an aluminium block.

The thermal noise in the seismic frequency band is totally negligible with respect to the effects of ocean-continent coupling, wind and human activity and therefore the concept of fluctuation-dissipation does not seem to apply at the first order to what we call seismic noise. On the other hand, the property can be more robust and holds for a variety of wavefields. Indeed, there is no guarantee that the ambient seismic noise has the same behavior as thermal noise but we expect that the distribution of the sources of the seismic noise randomizes when averaged over sufficiently long times. Furthermore, the waves that compose the noise are randomized by scattering from heterogeneities and a form of equipartition is therefore expected.

Aki (1957) proposed the use of noise records from a seismic array to evaluate the phase velocity of the predominant surface wave, a technique often referred to as the SPAC method. The method is based on short distance correlation and azimuthal averaging. At short distance, the correlation in the frequency domain is expressed as a Bessel function of order zero, J_0, and a fit of the observed correlation makes it possible to measure locally the wave number assuming a dominant mode. The use of noise records to extract local propagation properties below the array has been successfully applied to investigate shallow layers, especially soft layers that could be responsible for seismic amplification during earthquakes (e.g., Kudo *et al.*, 2002). Note that there is a close relation between the SPAC method, where the identification of the Green function in the correlation is not explicit, and our reconstruction. Considering a single surface mode, the problem reduces to a 2-D scalar homogeneous problem. In this case (2D, SH) the Green function in the Fourier domain is:

$$G(x,y) = \frac{1}{4i\mu}[J_0(kr) - iY_0(kr)], \tag{12}$$

where Y denotes the Neuman function and μ the rigidity. Note that this expression gives the causal Green function: The Fourier transforms of imaginary and real part contribute equally in the positive times and cancel out exactly in negative times. The correlation ($J_0(kr)$) is proportional to the imaginary part of the Green function. It therefore gives a signal that is proportional to the causal Green function with an even contribution in the negative times, similar to what we described previously (e.g., Fig. 5). Indeed the assumptions under this reasoning, lateral homogeneity and single mode, cannot be put forward for long-range correlation. The canonical problem of a random distribution of plane waves within a homogeneous elastic medium was treated in 2-D and 3-D by Sánchez-Sesma and Campillo (2006) who showed that equipartition between P and S is required for retrieving the exact Green function.

The use of long-range correlations for practical applications in seismology is recent. Shapiro and Campillo (2004), Sabra *et al.* (2005), Shapiro *et al.* (2005) and Pedersen *et al.* (2005) demonstrated the validity of the approach. Shapiro and Campillo (2004) performed group velocity measurements on reconstructed signals in the period range 10–125 s and for interstation distances from a few hundreds to several thousands of kilometers. SABRA *et al.* (2005) further described the emergence of the Green function. Nevertheless, the limits of application of the method in seismology are not fully understood since the origin of the noise itself is not precisely known in the different frequency bands.

In the low frequency domain ($f < 0.3$ Hz), the ambient noise seems widely dominated by the interaction of the ocean with the solid Earth (LONGUET-HIGGINS, 1950; FRIEDERICH *et al*, 1998; WEBB, 1998; RHIE and ROMANOWICZ, 2004; TANIMOTO, 2005; KEDAR and WEBB, 2005). The noise amplitude is strongly correlated with storm activity (e.g., BROMIRSKI, 2001; 2005) and even with climate change (GREVEMEYER *et al.*, 2000). At higher frequencies, the noise is produced locally by human activity and wind and, because of attenuation, cannot propagate over extended distances. In any case the noise is produced by surface sources which generate predominantly surface waves. It is therefore expected that the signals that can be extracted from noise records are also predominantly made of surface waves. Furthermore one must remember that the noise spectrum has marked peaks, and consequently the correlation (which spectrum is the product of the spectra of the records) is yet more strongly peaked. While the capability of reconstructing the body waves cannot be ruled out, it has not been convincingly demonstrated to date. In spite of the locations of the noise sources at the free surface, the scattering associated with the strong heterogeneity of the surface layers and with topography results in a coupling between surface waves and body waves. The efficiency of this process in the Earth is illustrated by the observation of the equipartition between the different modes of body and surface waves. It is therefore expected that body waves could be retrieved from correlation of records at closely located stations.

Even though only surface waves have been reconstructed from noise thus far, it is a valuable result since dispersive surface waves are widely used for imaging crustal and lithospheric shear velocity structures. SHAPIRO *et al.* (2005) presented the first example of the use of the ocean-generated noise to map Rayleigh wave group velocity. Here we present an example of this mapping. We use continuous noise records from 62 broadband stations in California. We selected 30 days from August to September 2004 during which no $M > 5.8$ earthquake occurred. The data were bandpass-filtered between 10 and 20 s and all pairs of crosscorrelations were computed. In most of the cases, we can clearly identify the fundamental Rayleigh wave. We computed the amplitude ratio between the emerging Rayleigh and the surrounding fluctuations. When the ratio is less than 4, the waveforms are rejected. This processing results in 785 group speed measurements at 18 s for paths larger than two wavelengths (Fig. 7). Velocity dispersion is measured by frequency-time analysis (LEVSHIN *et al.*, 1989; RITZWOLLER and LEVSHIN, 1998; SHAPIRO and SINGH, 1999). We finally applied a tomographic inversion (BARMIN *et al.*, 2001) to obtain a group speed map on a 28 km $\times$ 28 km grid across California.

The group velocity map (Fig. 8) shows the different geological features. The Sierra Nevada and the Peninsular Ranges, composed principally of Cretaceous

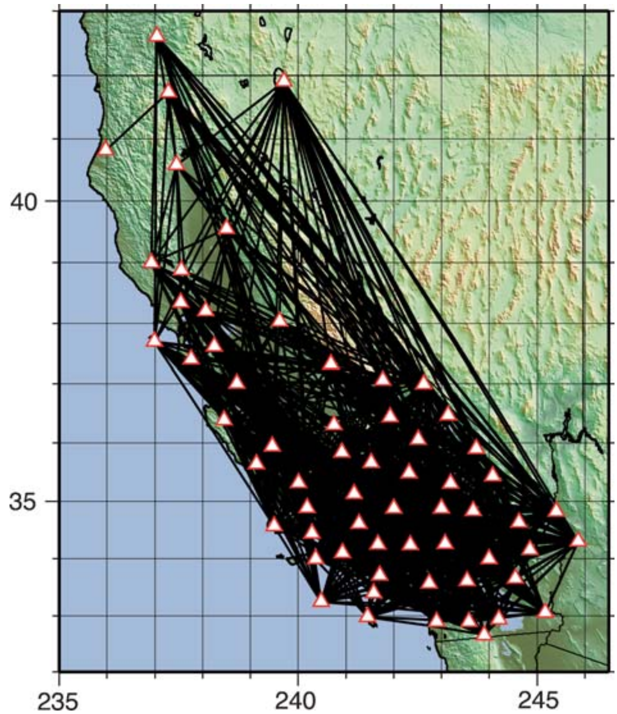

Figure 7
Paths where 18 s Rayleigh wave group speed measurements were obtained from cross correlations of ambient seismic noise. White triangles show locations of USArray stations used in this study.

granitic batholiths, are characterized by high group velocities. The group speeds are lower in the Great Basin and in the Mojave Desert, indicating that the middle crust in these areas is probably hotter and weaker than in the Sierra Nevada. Group speeds are low in the sedimentary mountain ranges, e.g., the Transverse Ranges, the southern part of the Coast Ranges, and the Diablo Range, while fast wave speeds are observed for the Salinian block. The signature at depth of the San Andreas fault is clearly visible in this area. The western wall is characterized by the high velocities of the plutonic rocks of the Salinian block while the low-speed eastern wall is associated with the sedimentary rocks of the Franciscan formation.

This example illustrates that the noise on Earth in the period band considered has the required properties for allowing the emergence of the Green function from averaging of correlation over long time series. Surprisingly, LAROSE *et al.* (2005b) succeeded in retrieving the Green function at stations distant of several wavelengths in a very different context. They used the high frequency noise recorded on the moon by a group of sensors installed during the Appolo 17 mission. Although the noise is of completely different origin on the Moon, namely heat-generated cracking, the

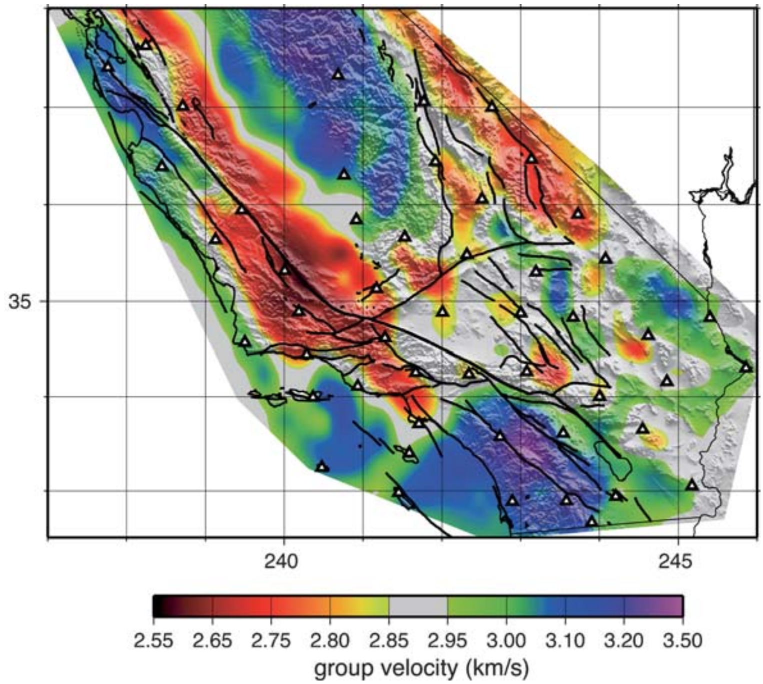

Figure 8
Rayleigh wave group speed map at 18 s period constructed by cross correlating 30 days of ambient noise between stations. Black solid lines show known active faults. White triangles show locations of the stations used in this study.

technique proved to be efficient to probe the shallow layers. It could be a valuable tool for planetary exploration.

10. Conclusion

We show that the average properties of the time decay of coda energy can be explained by the theory of radiative transfer assuming an uncoherent sum of contributions. This theory is valid only for intensity or total energy. Coda waves exhibit the transition from low-order scattering in the early coda to diffusion in the late coda. The stabilization of the ratio of S-to-P energies is a marker of the diffusive regime. It indicates that the waves are evolving toward equipartition, although equipartition is an asymptotic regime. The diffuse field keeps a significant preferential direction of flow even at extended lapse times.

Radiative transfer and diffusion neglect the phase effects in the combination of the contribution. We demonstrate the presence of mesoscopic effects that require consideration of the very nature of the waves, i.e., their property of producing interference effects. Weak localization can be observed with simple experiments in the field. It results in the so-called coherent backscattering enhancement at the source, a phenomenon characteristic of multiple scattering. This observation shows the importance of phase effects for fields that we used to consider to be ‘random’. Since phase information is preserved, we used spatial correlation properties of the fields to infer deterministic propagation characteristics. Different approaches to the problem are related to the type of averaging performed to extract the deterministic Green function from the field fluctuations. Seismological applications rely on source averaging. The presence of scattering has an important positive effect by making the wavefield more isotropic. The results presented here concern coda waves produced by a set of earthquakes or long-time series of noise records for which we expect the source of the noise to randomize partially. In both cases, it is demonstrated that the field-field correlations contain, at least partially, the deterministic response of the Earth between the stations, offering the possibility of an empirical construction of seismograms based on the actual wave propagation in the complex Earth. They can be used for imaging the velocity structure of the Earth interior without waiting for earthquakes. As the measurements from the correlations are performed for paths between stations and for short periods, such methods are likely to contribute to improved resolution in tomographic imaging. Correlation of diffuse wavefields overall opens up a wide range of potential applications.

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