Correction Pb 2

For a given solution at ω, the null Lagrangian writes:

$$\omega^2 I_1 = k^2 I_2 + I_3 \qquad (1)$$

and its stationarity:

$$\omega^2 \delta I_1 = k^2 \delta I_2 + \delta I_3 \qquad (2)$$

For a perturbation $\delta\omega$, the new solution has a null Lagrangian:

$$(\omega + \delta\omega)^2 (I_1 + \delta I_1) = (k + \delta k)^2 (I_2 + \delta I_2) + I_3 + \delta I_3$$

Keeping the terms of first order, we substract (1), then 2 and obtain:

$$2\omega \; \delta\omega I_1 = 2k \; \delta k I_2$$

and finally

$$U = \frac{\delta\omega}{\delta k} = \frac{k}{\omega} \frac{I_2}{I_1} = \frac{1}{C} \frac{I_2}{I_1}$$

This is a useful relation between C and U.

In the case of the homogeneous plate, the application is immediate knowing the definition of the I integrals (no need to perform any algebra).