How to estimate the Green's function of a heterogeneous medium between two passive sensors? Application to acoustic waves

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The exact Green's function of a heterogeneous medium can be retrieved from the crosscorrelation of the fields received by two passive sensors. We propose a physical interpretation based on time-reversal symmetry. We address the issue of causality and show the role of multiple scattering for the reconstruction of the Green's function. Ultrasonic experimental results are presented to illustrate the argument. Applications to geophysics and ocean acoustics are discussed. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1617373]

In most applications of wave physics (imaging, detection, communication), it is essential to know the Green's function (GF) of the medium under investigation. When possible, the GF (or impulse response) h_{AB} between two points A and B is determined by a direct pulse/echo measurement. Recent results[1–3] exploited an other idea: when A and B are both passive sensors, h_{AB} can be recovered from the cross-correlation of the fields received in A and B, the wave field being generated either by deterministic sources or by random noise. In a closed reverberant medium[1] mathematical arguments were given, based on a discrete random modal expansion. An ensemble-averaged GF is fundamentally different from the *actual* GF of one realization of disorder. We propose a simple physical interpretation of the emergence of the exact GF in the correlations based on reciprocity, with no reference to a random modal expansion. We particularly address two issues: (1) Physically, the GF h_{AB} is causal, but the correlation between the wave fields received in A and B may be noncausal, therefore, should one keep the causal part, the anticausal part of the correlation, or both to estimate h_{AB}? (2) In an inhomogeneous medium, what is the role of scattering in the reconstruction of the GF from field–field correlations? We also present experimental results to support the argument.

To begin with, let us consider two receiving points A and B and a source C. We will note $h_{IJ}(t)$ the scalar wave field sensed in I when a Dirac $\delta(t)$ is sent by J. If $e(t)$ is the excitation function in C, then the wave fields ϕ_A and ϕ_B received in A and B will be $e(t)\otimes h_{AC}(t)$ and $e(t)\otimes h_{BC}(t)$, $\otimes$ representing convolution. The cross-correlation C_{AB} of the fields received in A and B is then

$$C_{AB}(t)=\int \phi_A(t+\theta)\phi_B(\theta)d\theta$$
$$=h_{AC}(-t)\otimes h_{BC}(t)\otimes f(t)$$

with $f(t)=e(t)\otimes e(-t)$. A physical argument based on time-reversal (TR) symmetry indicates that the direct GF h_{AB} may be entirely recovered from C_{AB}.

As long as the medium does not move, the propagation is reciprocal, i.e., $h_{IJ}(t)=h_{JI}(t)$. So when we crosscorrelate the impulse responses received in A and B, the result $C_{AB}(t)$ is also equal to $h_{CA}(-t)\otimes h_{BC}(t)$. Now, imagine that we do a fictitious TR experiment: A sends a pulse, C records the impulse response $h_{CA}(t)$, time reverses it and sends it back; the resulting wave field observed in B would then be $h_{CA}(-t)\otimes h_{BC}(t)$ which, because of reciprocity, is exactly the cross-correlation $C_{AB}(t)$ of the impulse responses received in A and B when C sends a pulse. We would like the GF h_{AB} to appear in this crosscorrelation. But in the most general case, C_{AB} has no reason to be equal to h_{AB}. Yet we can go beyond: imagine now that we use several points C, and that we place them in such a way that they form a *perfect* TR device: such would be the case if the sources C were continuously distributed on a surface surrounding A, B, and the heterogeneities of the (lossless) medium. Then a TR operation would be perfect. During the "forward" step, at time $t=0$ A sends a pulse that propagates everywhere in the medium [including in B where the field received is $h_{AB}(t)$], may be scattered many times and is eventually recorded on every point C, with no loss of energy. After the TR, the wave should exactly go backwards: it should hit B first and refocus on A at time $t=0$,[4] which implies that the field received in B (at times $t<0$) is exactly $h_{AB}(-t)$, the time-reversed version of the GF. Once the pulse has refocused on A, it does not stop[5] but diverges again from A and gives rise, at times $t>0$, to $h_{AB}(t)$ in B. If there is frequency-dependent attenuation, TR invariance is broken but reciprocity still holds: a TR operation would only yield a filtered version of $h_{AB}(t)$. Thus the impulse response $h_{AB}(t)$ can be retrieved from either the causal $(t>0)$ or the anticausal part $(t<0)$ of the sum of field–field correlations $C_{AB}(t)$, provided that the sources C are placed so that they would form a perfect TR device.

In real life, whatever the type of waves involved, this

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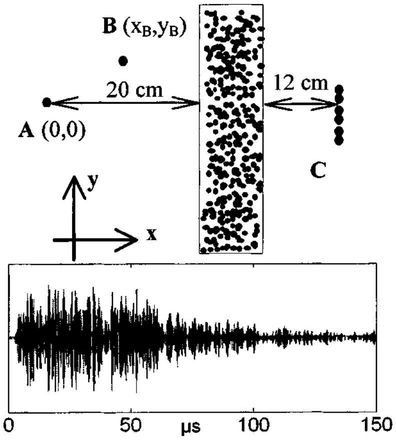

FIG. 1. Experimental setup (a). The receiver A is fixed at the origin, the experiment is done for various position of B ranging from $x_B=-50$ mm, $y_B=-15$ mm to $x_B=50$ mm, $y_B=15$ mm. Twenty-one sources C are used (size: 0.39 mm, pitch: 0.42 mm, frequency 3.1 MHz). The distance between A and C is 35 cm. (b) Wave form received by A when a 1 μs pulse is sent by one of the sources.

condition is hard to meet. In seismology for instance the displacement field at the earth surface is recorded by seismic stations (A,B) but the sources (C) of the earthquakes are far from being arranged as a perfect TR device, they are mostly aligned along faults. Yet the elastic GF can be partially retrieved using correlations of the late seismic codas produced by distant earthquakes.[3] Why is this possible when the TR criterion is not fulfilled? A laboratory experiment can help us find an answer and shed light on the role played by multiple scattering (Fig. 1). Piezoelectric transducers (A,B) record the wavefields generated by 21 ultrasonic sources C successively firing a broadband pulse (1 μs, central frequency 3.1 MHz). The experiment takes place in water tank ($c=1.5$ mm/μs), and a scattering slab is placed between the sources and the receivers. It is made of randomly distributed steel rods (29.5 rods/cm^2); the transport mean-free path[6] ℓ^* was measured to be 3 mm, while the thickness of the slab is $L=30$ mm, the medium is therefore highly scattering as can be seen from the waveform plotted on Fig. 1(b). Frequency-dependent dissipation is negligible. The experiment is repeated for various positions of the second receiver B, each time we crosscorrelate the 21 pairs of fields received in A and in B. $C_{AB}(t)$ is calculated by summing the 21 crosscorrelations.

The coherent field is totally extinct in the received wave forms h_{AB} and h_{AC}. Since $L \gg \ell^*$, the correlation length of the field emerging from the slab is $\sim\lambda/2$, and from the Van Cittert–Zernike theorem the fields sensed in A and B are spatially incoherent since the transverse size of the slab is 25 cm while the distance between B and the slab varies between 15 and 25 cm.

Here, the GF between A and B is a well-defined pulse arriving at time $|AB|/c$, followed later by lower reflections on the rods. The experimental results show that the emergence of the GF from $C_{AB}(t)$ highly depends on the position of B, and on the number of sources employed. With only one source [Fig. 2(a)] $C_{AB}(t)$ is too noisy to see the emergence of the GF. But at the same point B, with 21 sources [Fig. 2(b)] instead of one, $C_{AB}(t)$ shows a strong peak at time $t=33.5$ μs, which is exactly the travel time $|AB|/c$. Yet for a different position of B [Fig. 2(c)], $C_{AB}(t)$ shows a peak at time $t=-33.35$ μs, the *opposite* of the expected travel time. And for a third position of B [Fig. 2(d)], even with 21 sources, the GF does not emerge in $C_{AB}(t)$. So it appears that the GF (at least its first arrival) can indeed be recognized in the correlation $C_{AB}(t)$, but only at certain times (causal or anticausal), and for certain positions of B relatively to A. Why is that?

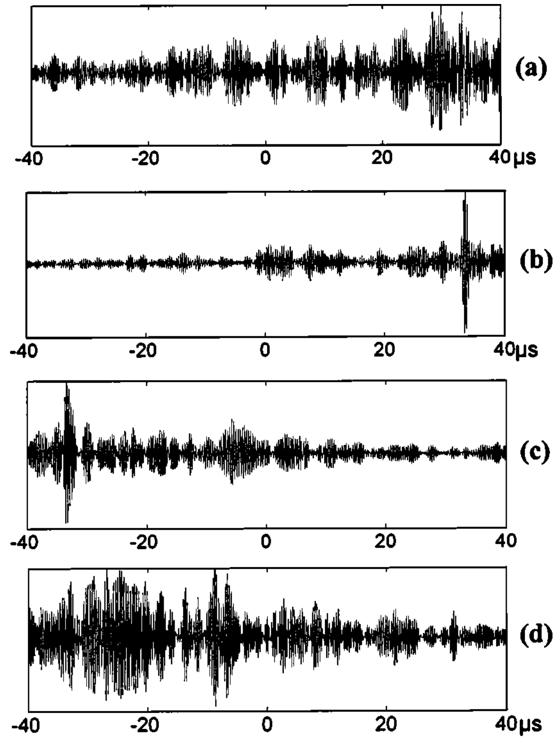

FIG. 2. Cross-correlation $C_{AB}(t)$ for: (a) 1 source, $x_B=-50$ mm, $y_B=-5$ mm; (b) 21 sources, $x_B=-50$ mm, $y_B=-5$ mm; (c) 21 sources, $x_B=50$ mm, $y_B=-5$ mm; (d) 21 sources, $x_B=25$ mm, $y_B=15$ mm.

The TR analogy gives the answer: for this particular setup, in a fictitious TR experiment where A would be the source and the 21 C points a *finite-size* TR device, the time-reversed pulse would hit B at times $t<0$ only if B is between A and C (i.e., $x_B>0$), and at times $t>0$ only if it is behind A ($x_B<0$). Consequently, when one crosscorrelates two wave fields in order to reconstruct the GF of an unknown medium, one has to know the location of the receivers relatively to the sources and to the scatterers in order to keep only the relevant part of $C_{AB}(t)$.

It also appears that h_{AB} cannot be properly reconstructed for any position of B, as was shown on Fig. 2(d). Figure 3 compares $C_{AB}(t)$ to the theoretical travel times for 61 positions of B ($x_B=-50$ mm, $y_B=-15$ to 15 mm), with and without the rods: the curvature of the GF emerges only in the former case. This emphasizes the role of multiple scattering: the region for which the arrival time is well retrieved is much smaller in a homogeneous medium (water) than through the forest of rods. This too can be interpreted via the TR analogy: a TR experiment works better (meaning that it reconstructs better the ''initial scene'') through a multiple scattering medium[7] than in a homogeneous medium (here the initial scene would be the propagation of a spherical pulse emitted by A).

In our experiment, there is no statistical average over disorder. The only average is an average over the sources,

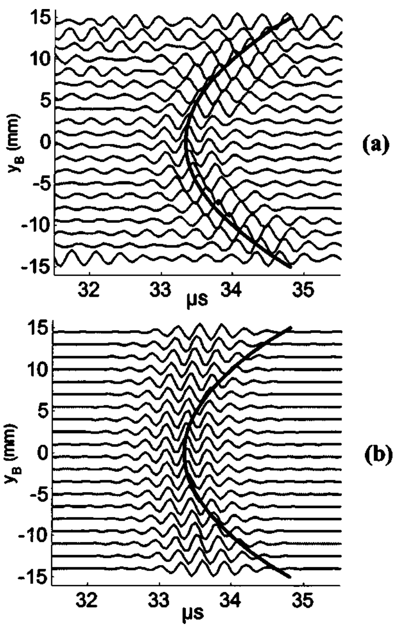

FIG. 3. Cross-correlation $C_{AB}(t)$ for 21 sources, the position of B is $x_B = -50$ mm, y_B ranging from -15 to 15 mm, with (a) and without (b) the multiple scattering slab. The thick line shows the theoretical arrival time of the GF. It is properly reconstructed (within 0.05 μs, which is the sampling time) in an angular sector of 28° with the multiple scattering slab, vs only 6° in water.

when we stack 21 field–field correlations. Also, the equipartition of energy between discrete modes cannot be attained here, since the medium is open; besides even if the sources C were truly ominidirectional, only half of their angular spectrum would actually excite the scattering slab. This is very different from other works.[1] Here we do not fulfill the equipartition condition, neither do we not fulfill the ''perfect TR device'' condition, but we try to come closer to it using several sources instead of one. The TR approach implies that if the sources C were completely surrounding the medium, the sum of the crosscorrelations would give the *exact* Green's function of the medium,[8] not an ensemble-averaged GF. Our experimental results show that with a limited number of sources it is possible to estimate at least the first arrival of the GF, not everywhere at every time but in a limited area whose size is larger in the presence of multiple scattering.

These results can be extrapolated to various applications of wave physics, e.g., ocean acoustics[2] or seismology. When an earthquake occurs, seismograms can show a long coda due to multiple scattering in the Earth's crust. It was recently shown[3] that the crosscorrelation of coda waves received on two seismic stations (A,B), averaged over a hundred earthquakes (i.e., sources C), exhibited a pulse arriving at the same time and with the same polarization as a direct Rayleigh wave that would travel from A to B. In future developments, since the epicenter of earthquakes can be known, it would be possible to determine whether it is the causal part, the anticausal part or both that have to be taken into account in order to have a better estimation of the GF. A more detailed publication on this subject is on hand. The prospects are wide: the idea developed here are applicable to every domain of wave physics where one can measure directly the field (amplitude and phase, not only the intensity) and perform a crosscorrelation. Acoustic, elastic, or even radio waves could be employed. Whatever the type of waves, the TR analogy provides an elegant way to interpret the emergence of the GF from the correlation of the fields.

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