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Statistical Measures of Crustal Heterogeneity from Reflection Seismic Data: The Role of Seismic Bandwidth.

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Abstract.
In recent years there has been a growing realisation that geological media vary over a broad scale range. As such heterogeneity cannot be described in a deterministic way, a parallel growth in stochastic analyses of geological/geophysical data has emerged. The stochastic description of these media is usually through some form of correlation function, of which the von Karman is the most widely employed. Using this form, media can be described in terms of a characteristic scale size (or correlation length), L and a coloured scaling regime, with scaling described by H, the Hurst exponent. Beyond the correlation length, the material follows a white noise spectrum where material average properties dominate, below the correlation length local heterogeneity dominates. Hence, the correlation length is a fundamental parameter in a range of geodynamical problems. *In situ* information about stochastic properties of deep crustal rocks can be obtained from the statistical analysis of reflection seismic data. Typical correlation distances within the crust are found to be several hundred metres. Here we show that correlation distances derived from reflection seismic data are strongly influenced by the spectral content of the source. In particular we conclude that there is no reliable evidence for hundred metre scale correlation lengths for crustal heterogeneity.

1. Introduction

It is now widely recognised that the crust is heterogeneous over a broad range of scales. Seismic velocity fluctuations exhibit variability from the borehole scale [e.g. *Holliger,* 1996] to crustal scale, as seen on refraction/reflection experiments. In particular the geology of the crystalline crust usually consists of a geometrically complex distribution of lithology, which varies over a wide scale range. A distinction can be drawn between the upper and lower crust. The upper crust is known, in general, to be heavily faulted and fractured, hence acoustic impedance heterogeneity has contributions from both fractures and lithology. In the lower crust it is more probable that variability is primarily lithologically controlled. In the absence of "large" discrete tectonic horizons or large faults, such heterogeneous media can produce complicated reflectivity patterns which, when illuminated by a seismic wave, may not be easy to interpret using conventional methods, such as line drawing.

In order to characterise a geometrically complex distribution of lithology (with or without superimposed fractures), it is often necessary to take a statistical approach. This approach has been successfully applied to a variety of geological maps [e.g. *Holliger and Levander,* 1992] and superdeep borehole wireline data [*Leary,* 1991; *Holliger,* 1996; *Dolan et al.,* 1998]. Inverting both geological maps and deep borehole data for spatial statistics often involves minimising the difference between correlation functions characterising field data and the analytical expression for some type of known correlation function. The most widely employed correlation function is the von Karman [e.g. *Holliger,* 1996; *Dolan and Bean,* 1997], which has three tuneable parameters, the Hurst exponent, H, the correlation length, L and the variance. For scale sizes smaller than the correlation length (or characteristic scale size), the medium exhibits power-law (fractal) scaling, controlled by the value of H. For scale sizes larger than L, fluctuations in the medium behave as a white noise. $H \simeq 0.3$ and L of the order of hundreds of metres are typically obtained from geological maps. Borehole data usually yields $H \simeq 0.2$ and L of the order of 10 to 100's of metres [cf. *Holliger,* 1996]. However, Dolan and Bean 1997 have suggested that such small values of L are strongly controlled by the nature of borehole data pre-processing (detrending), making it difficult to interpret the physical (geological) significance of the published correlation lengths for borehole data. In contrast to the short correlation length model, Leary 1991 has also suggested that L far exceeds borehole data lengths. Although borehole data give us detailed *in situ* information about the uppermost crust (albeit in 1D), deep seismic data are needed in the search for equivalent *in situ* information about deeper crustal rocks. In a similar way to geological maps and borehole data, seismic sections can be analysed statistically in order to determine values of H and L, describing the underlying velocity field. In this paper we look at the role of seismic bandwidth when estimating such statistical measures of crustal heterogeneity from reflection seismic sections.

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2. Seismic Data in Complex Geology

Over the past fifteen years or so, considerable effort has been invested in trying to understand how scaling heterogeneity affects wave propagation. In this approach numerical wave simulations are generally performed in random media and the results compared with real seismic data [e.g. *Holliger and Levander,* 1992; *Roth and Korn,* 1993]. This approach has added considerably to our understanding of how wave scattering affects seismic imagery and has, for example, important consequences for traditional interpretations of controlled source refraction data [*Levander and Holliger,* 1992]. Recently, the more difficult problem of inverting seismic data for statistical measure of crustal heterogeneity (e.g. values of H and L) has been addressed [e.g. *Line et al.,* 1998; *Pullammanappallil et al.,* 1997]. Again, this work has opened up new avenues for investigating spatial variation in crustal scaling in otherwise inaccessible geology.

Several methods have been employed for determining the spatial statistics directly from seismic sections. These include the autocorrelation, power spectra and variograms [e.g. *Hurich,* 1996; *Pullammanappallil et al.,* 1997; *Rea and Knight,* 1998]. In essence these methods access the same information on the seismic section although there may be implementation advantages for a given method over another. In this paper we employ the $k-t$ spectrum [for details, see *Pullammanappallil et al.,* 1997], which yields a measure of the scaling (both L and H) on seismic sections. We carry out our analysis on Primary Reflection Sections (PRS) [*Claerbout,* 1985]. PRS's are noise free, perfectly migrated synthetic seismic section where a source wavelet is convolved with the derivative of a velocity field.

Unlike previous workers here we assume that the underlying velocity field has an effectively infinite correlation length. In effect this means that the correlation length exceeds the data length. We then illuminate this medium with a range of source frequencies and invert for H and L. Our choice of an 'infinite' correlation length allows us to assess the effects of limited source bandwidth in the inversion procedure for velocity field statistics. In particular, any finite value of L will only be apparent, as L goes to infinity in our original velocity models.

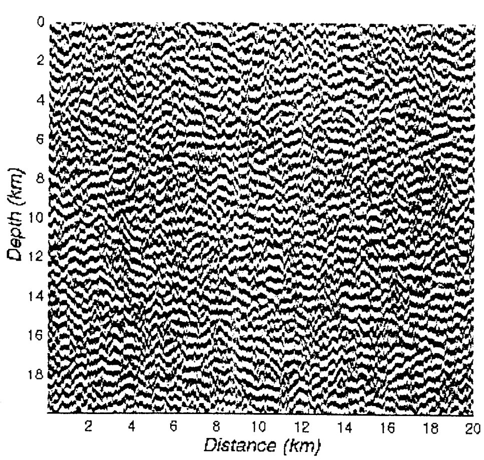

Figure 1. Primary Reflection Section (PRS) derived for a 20x20km velocity model with a Hurst exponent of 0.2, a mean velocity of 6km/s and an 'infinite' correlation length ($Lx = Lz = 2000$km), illuminated by a 10Hz source wavelet. See text for details.

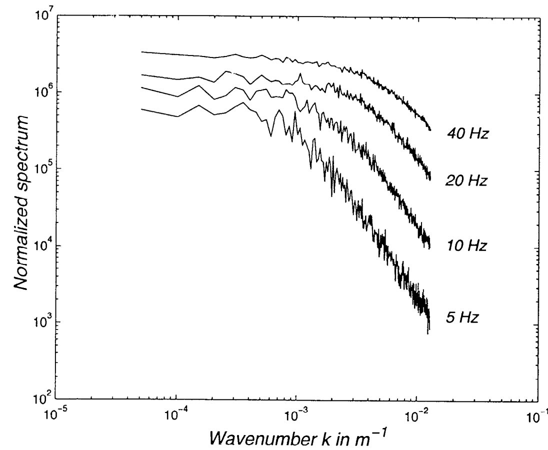

Figure 2. k-t spectra for four example PRS's based on a velocity model with a Hurst exponent of 0.2, a mean velocity of 6km/s and an 'infinite' (2000km) correlation length. Source central frequencies are given on the figure. Note the roll-over in the spectra, giving apparent correlation lengths which vary with source frequency.

3. Results

We start with a 20x20km statistically isotropic velocity model. We choose a Hurst exponent, $H = 0.2$, which is typical of the values commonly determined from borehole data. The correlation length in our model tends towards infinity (see section 2). The standard deviation of velocity fluctuations about a mean velocity of 6km/s is 3%. Our pixel resolution is 39x39m. The Primary Reflection Section (PRS) for this velocity field, illuminated with a source frequency of 10Hz is shown in figure 1. We analyse the data in figure 1 for scaling by looking at the spectral characteristics of horizontal slices through the data, averaged over all depths [cf. *Pullammanappallil et al.,* 1997]. The results are shown in figure 2, for four different central frequencies (5, 10, 20 and 40Hz). Two things are worth noting about figure 2. First of all, different corner frequencies clearly exist in all the spectra, leading to different apparent correlation lengths. Second, for each spectrum the data scale as a power-law for frequencies above its corner, the slope of which is consistent with the statistics of the underlying velocity field, given by $H = 0.2$. To help identify the approximate wavelengths at which this corner occurs we multiply the spectrum by k, and pick the maximum (figure 3). Note our aim here is to check for the existence of a scaling relationship, not to invert for

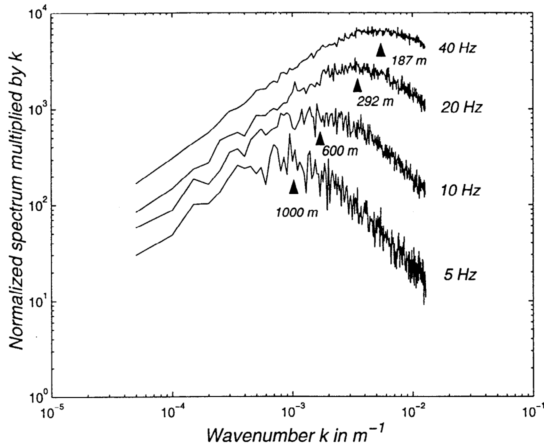

Figure 3. Spectra shown in figure 2, multiplied by k to aid identification of the corners. Approximate apparent correlation lengths are indicated.

true corner wavenumbers, which would require fitting a von Karman function to the spectra in figure 2. Corner picks of 1000, 600, 292 and 187m are associated with source central wavelengths of 1200, 600, 300, and 150m respectively, yielding a scaling relationship. In figure 4 we show this relationship between apparent correlation length (inverse of corner k) and source central frequency for a range of source frequencies. It is clear from figure 4 that the source frequency plays a central role in controlling the position of the rollover, leading to an apparent characteristic scale size or correlation length for the underlying velocity field.

In the stochastic modelling of crustal velocity fluctuations it is usually assumed that horizontal correlation lengths are larger than vertical correlation distances [e.g. *Hurich*, 1996]. In such a case, the apparent correlation length is also controlled by the anisotropy aspect ratio in addition to the source central frequency. In figure 4 we show the effects of aspect ratio on the relationship between apparent correlation length, L and source central frequency. For larger anisotropy aspect ratios the apparent correlation length is pushed to larger values, for a given source central frequency. However, it should be noted that the derived values of L are still only apparent as the true L values significantly exceed the model size (see caption, figure 4). Hence for real field data although we expect the derived correlation length to be primarily controlled by the seismic wavelength, we do not predict a one-to-one relationship as the existence of any angular variations in scaling bandwidth would introduce a multiplicative constant. In Appendix A we support the above result analytically, for the case of isotropic scaling.

4. Discussion and Conclusions

For any material, the correlation length plays an important role in determining its physical behaviour. At scale sizes smaller than the correlation length local heterogeneity dominates material behaviour, whereas material average properties dominate over local heterogeneity for scale sizes larger than the correlation length. In this paper we show that band limited controlled source seismic data cannot give us reliable measures of crustal correlation lengths which are of the order of or larger than the seismic wavelength. We can, however, recover velocity scaling laws for sub seismic wavelength heterogeneity, using noise free synthetic data. Band limited reflection seismic wavelets filter broad band impedance heterogeneity. In Appendix A, we give the filter response. This result will also hold for other methods often employed in the statistical analysis of seismic sections, it is not confined to the k-t spectrum. As correlation lengths play a pivotal role in crustal strength and permeability estimates (in geodynamical problems and crustal heterogeneity modelling, for example), we suggest that this important parameter can only be investigated using broad band records.

5. Appendix A: k - t spectrum for fractal media, with varying source frequency

We look at the k - t spectrum for a medium characterised by a fractal velocity, with a Hurst exponent H. This medium is explored by a source wave with a cen-

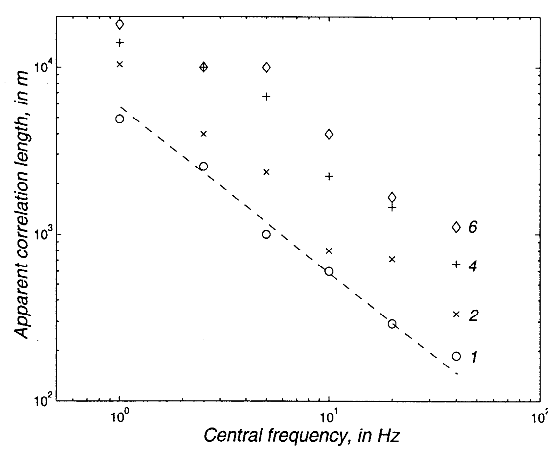

Figure 4. Relationship between calculated apparent correlation lengths, L and source frequency based on PRS's, 20x20km in dimension. We show results for four different models. Each model is probed by a suite of source frequencies. The four model types are indicated by the numbers on the right hand side in the figure. 1 - the heterogeneity is isotropic with a correlation length $Lx = Lz = 2000$km. The dashed line, showing how the apparent correlation length varies as a function of source central frequency, is based on the analytical result in Appendix A. 2 - horizontal/vertical aspect ratio = 2, $Lx = 2000$km, $Lz = 1000$km. 4 - $Lx = 2000$km, $Lz = 500$km. 6 - $Lx = 2000$km, $Lz = 333$km. Note that the limited model size has an effect on the scaling for low frequencies and large scaling anisotropy aspect ratios.

tral frequency f and bandwidth Δf. The velocity field $v(x,t)$ has a spectral density (i.e. a 2D power spectrum) E_v of the form

$$E_v(k,\omega) \sim |k,\omega|^{-2-2H} \quad (1)$$

where the function $|x,y|$ is any positively defined function such that $|\lambda x, \lambda y| = \lambda |x,y|$, $\forall \lambda \geq 0$. The reflectivity field $r(x,t)$ is taken as the derivative along the t-axis of v, and has a spectral density E_r such that $E_r(k,\omega) \sim \omega^2 E_v(k,\omega) \sim \omega^2 |k,\omega|^{-2-2H}$. This reflectivity field is probed by a wave of central frequency f, propagating along the t-axis at all x. The corresponding PRS, $s_f(x,t)$, has a spectral density E_{s_f} such that $E_{s_f}(k,\omega) \sim \mathcal{B}_f(\omega) E_r(k,\omega)$ where $\mathcal{B}_f(\omega)$ is the power spectrum of the source, centered at f. The k - t spectrum $E_f(k)$ is the projection of E_{s_f} onto the k-axis $E_f(k) = \int d\omega \, E_{s_f}(k,\omega)$, hence

$$E_f(k) \sim \int d\omega \, \omega^2 \, |k,\omega|^{-2-2H} \, \mathcal{B}_f(\omega) \quad (2)$$

We will consider a source characterised by $\mathcal{B}_{f,\Delta f}(\omega \in [f-\Delta f; f+\Delta f]) = 1$ and null otherwise, for Δf small enough compared to f ($\Delta f \ll f$). Before doing so, we first derive the PRS spectrum in the simple case of a low-pass source.

Low-pass source: We take a source with a power spectrum

$$\mathcal{B}_f(\omega) = \begin{cases} 1 & \text{if } \omega < f \\ 0 & \text{if } \omega > f \end{cases} \quad (3)$$

Then the k - t spectrum is $E_f(k) \sim \int_0^f d\omega \, \omega^2 \, |k,\omega|^{-2-2H}$. We divide the interval of integration into two: **(1)** $E_f(k \gg f) \sim k^{-2-2H} \int_0^f d\omega \, \omega^2 \sim k^{-2-2H} f^3$ and **(2)** $E_f(k \ll f) \sim \int_0^k d\omega \, \omega^2 k^{-2-2H} + \int_k^f d\omega \, \omega^{-2H}$, giving $E_f(k \ll f) \sim k^{1-2H} + \frac{1}{1-2H}(f^{1-2H} - k^{1-2H})$. We limit the derivation to the $H < 0.5$ case (a more involved derivation would show that the same final result is obtained for $H > 0.5$). For $H < 0.5 \Rightarrow 1 - 2H > 0$, and with $k < f$, we get that $E_f(k \ll f) \simeq k^{1-2H} + f^{1-2H} \simeq f^{1-2H} k^0$.

Band-pass source: The source with spectrum

$$\mathcal{B}_{f,\Delta f}(\omega) = \begin{cases} 1 & \text{if } f \in [f-\Delta f; f+\Delta f] \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

can be obtained by substracting the two sources with low-pass spectra cutting off at $f+\Delta f$ and at $f-\Delta f$. We therefore distinguish the two regimes: **(1)** $k \gg f + \Delta f$: here, we find that $E_{f,\Delta f}(k) \sim (f+\Delta f)^3 k^{-2-2H} - (f-\Delta f)^3 k^{-2-2H}$ and, for $\Delta f \ll f$:

$$E_{f,\Delta f}(k \gg f + \Delta f) \sim f^2 \, \Delta f \, k^{-2-2H} \quad (5)$$

(2) $k \ll f - \Delta f$: for $H < 0.5$, we obtain that $E_{f,\Delta f}(k) \sim (f+\Delta f)^{1-2H} - (f-\Delta t)^{1-2H}$, and thus

$$E_{f,\Delta f}(k \ll f - \Delta f) \sim f^{-2H} \, \Delta f \, k^0 \quad (6)$$

Finally, we have that the k - t spectrum is of the form:

$$\begin{cases} E_{f,\Delta f}(k \ll f - \Delta f) \sim \Delta f \, f^{-2H} \, k^0 \\ E_{f,\Delta f}(k \gg f + \Delta f) \sim \Delta f \, f^2 \, k^{-2-2H} \end{cases} \quad (7)$$

showing that the corner frequency separating the 'flat' regime k^0 and the colored regime k^{-2-2H} is given by the central frequency f of the source.

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