GEOPHYSICAL RESEARCH LETTERS, VOL. 16, NO. 4, PAGES 279-282, APRIL 1989

THE INFLUENCE OF THE SOURCE ON THE HIGH-FREQUENCY BEHAVIOR OF THE NEAR-FIELD ACCELERATION SPECTRUM : A NUMERICAL STUDY

Jean–Christophe Gariel[1] and Michel Campillo

Observatoire de Grenoble, IRIGM, Grenoble, France

Abstract. The hypothesis that particular physical properties of the earth at the source can affect the radiation of high-frequency waves was numerically investigated. Two factors have been considered : a non-elastic behavior of the material at the crack tip (i.e presence of a cohesive zone) and the kinematics of the rupture front. Using the discrete wave number method, we calculated complete near-source synthetic spectra radiated by a circular shear crack model. The two processes can explain an exponential decrease of the acceleration spectrum at high frequencies. Moreover, they can be differentiated. In the case of a final smooth deceleration of the rupture front, the rate of decay of the acceleration spectra is equivalent in all the azimuths. A clear azimuthal dependence exists in the presence of a cohesive zone, with the maximum rate of decay being observed in the direction of the fault strike. The parameter f_{max} varies as the ratio of the rupture velocity over the length of the cohesive zone for the non-elastic model and as the inverse of the duration of the deceleration of the rupture front when the rupture front stops smoothly. These two models result in a decay of spectral amplitude at high frequency that mimics the effect of the anelastic attenuation.

Introduction

All accelerograms from large and moderate earthquakes recorded throughout the world are characterized by a strong decrease of their spectrum at high frequencies beyond a frequency generally called f_{max} (Hanks, 1982). The origin of this decrease is still a controversial subject because it can be attributed to path, source, or local site effects (see Papageorgiou, 1988 for a review). It is, however, important to isolate the factors which may influence the behavior of the acceleration spectrum at high frequency. In the following, we shall consider only the radiation in the near-source region. If the idealization of the ω^{-2} source model was valid at high frequencies (i.e. the source acceleration spectrum remains flat), the study of the rate of decay of the spectrum could give information about the attenuation in the crust or in the shallow layers. On the contrary, if the high-frequency behavior of the spectrum is strongly affected by source characteristics, it will be very difficult to interpret the actual decrease as it will depend on both source process and attenuation within the shallow layers.

In this study, we will investigate theoretically the implications of particular physical properties of the earth material at the source, for the radiation of high frequencies. We will consider successively two factors :

1/ The existence of a non-elastic behavior of the material at the crack tip which can be represented by a distribution of "cohesive forces" along the "cohesive zone" (Ida, 1972). This hypothesis was suggested by Papageorgiou and Aki (1983) who assumed that an estimate of the classical parameter f_{max} is given by the ratio v/d where v is the rupture velocity and d is the size of the cohesive zone.

2/ The kinematics of the rupture front which can be related to a change of fracture energy. The existence of a smooth deceleration in place of a sudden stop, for example, can generate a strong fall-off of the high frequency content of the radiation.

In this report, we present numerical calculation of synthetic spectra in order to evaluate the effect of these two factors on the high-frequency behavior of the acceleration spectrum, using the simple circular shear crack model.

Effect of a Cohesive Zone

An important feature of the elastic shear crack model is the elastic behavior of the material at the crack tip. The classical solution for this problem (Kostrov, 1964) is a displacement discontinuity of the form $\sqrt{x}$ and a singularity of $1/\sqrt{x}$ for the shear stress just behind the crack tip (x is the distance to the crack tip). As the slip velocity is the Hilbert transform of the shear stress times a constant (Ida and Aki , 1972), this singularity appears also for the slip velocity. Thus, this model is not realistic as it supposes that the material can sustain an infinite stress.

In order to eliminate these singularities, Barrenblat (1959) considered non-elastic behavior of the material at the crack tip. This behavior is represented by a distribution of "cohesive forces" inside the crack working across the fault plane. Ida (1972) has evaluated the stress field and the displacement discontinuity at the crack tip for several models of cohesives forces in the case of a two-dimensional longitudinal shear crack. For the simplest model, i.e. a constant cohesive force along the cohesive zone, the problem can be solved analytically, whereas other models require numerical solutions.

The model : We start with the simple circular shear crack model where rupture nucleates at a point and propagates radially at constant velocity until it suddenly stops. In the elastic hypothesis, the high frequency content of the acceleration spectrum generated by this crack is related to the stopping phases radiated when the crack stops (Madariaga, 1976). To evaluate the effect of a cohesive zone on the high frequency radiation of the circular crack model, we introduce the slip time history given in analytical form by Ida (1972). Denoting by $\vec{r}$ the position vector from the center of the crack, we define the slip $\Delta u(\vec{r},t)$ by the following expressions :

$$\frac{\Delta u(\vec{r},t)}{u_0} = 0 \qquad t < t_0(r)$$

$$\frac{\Delta u(\vec{r},t)}{u_0} = (1/8D_0)\left[\,(vt-4D_0^2)\ln\left|\frac{\sqrt{vt}+2D_0}{\sqrt{vt}-2D_0}\right| + 4D_0\sqrt{vt}\right] \qquad t_0(r) < t < t_1$$

$$\frac{\Delta u(\vec{r},t)}{u_0} = (1/8D_0)\left[\,(vt_1-4D_0^2)\ln\left|\frac{\sqrt{vt_1}+2D_0}{\sqrt{vt_1}-2D_0}\right| + 4D_0\sqrt{vt_1}\right] \qquad t > t_1$$

---

[1] Now at Lamont-Doherty Geological Observatory

Paper number 89GL00444.
0094-8276/89/89GL-00444$03.00

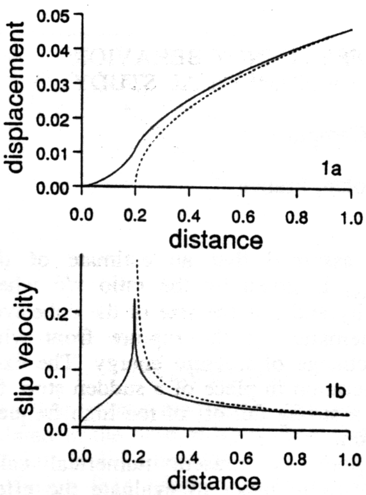

Fig. 1. (1a) Displacement discontinuity at the crack tip for the elastic case (dashed line) and the non-elastic case (solid line). (1b) Slip velocity at the crack tip for the elastic case (dashed line) and the non-elastic case (solid line).

with $t_0(r) = \frac{r}{v} \quad t_1 = \frac{R}{v}$

R = Final radius of the crack.
v_r = rupture velocity.
D_0 = Slip at the end (away of the crack tip) of the cohesive zone.
u_0 = Final slip at the center of the crack.

Figure 1a and 1b show respectively the slip function and the slip velocity for the two cases : elastic and non-elastic. It should be noticed that the latter model eliminates the second order discontinuity in the final spatial distribution of slip and the discontinuity in the temporal evolution of slip velocity.

Model parameters : We calculate the acceleration spectra produced by a vertical circular crack with a radius of 1 km located at a depth of 3 km in a half space. The medium velocities are : β=3km/s, α=5.2km/s and ρ=2.8. The receivers are located at an epicentral distance of 4 km and at azimuths of 20, 50 and 80^0 from the fault strike. The rupture velocity is $0.75.\beta$=2.66 km/s. To calculate synthetic spectra, we use the method of Bouchon (1981) which consists of discretizing the wavefield with respect to horizontal wavenumber. Following Campillo and Bouchon (1983), the fault is represented by an array of point sources and the superposition of the elastic field radiated by all elementary point sources is calculated in the frequency horizontal wavenumber domain. The interval between elementary sources is chosen to be smaller than one-fifth of the shortest wavelength considered (i.e. 25m). The spectra are calculated at equally spaced frequencies ranging from 0 to 20 Hz.

Results : We only present spectra for the transverse component of the acceleration. We verified that the results were similar for the other components. Figure 2a compares the S-wave acceleration spectra in the case of a cohesive zone with a length of 0.2 km with the elastic case (reference model) for the receiver located at 20^0. More details about the calculation in the elastic case can be found in Campillo and Bouchon (1983). In presence of a cohesive zone, the impulses related to the stopping phases are emitted by the end of the cohesive zone (Figure 1b) (in our case, at 0.8 km from the center of the crack). So, in order to have equivalent arrival times for the two models, the reference model is a circular crack with a radius of 0.8 km with a rupture velocity given by :

$$V_r=(R-coh).v/R=2.12\text{km/s}$$

where coh is the size of the cohesive zone.

At low frequencies (below the corner frequency), the two spectra coincide as the seismic moment of the two models are equivalent. At intermediate frequencies (between the corner frequency and a frequency ranging between 8 and 10 Hz), the spectra are slightly different because the two rupture velocities differ. Above 8 to 10 Hz, for the elastic model the spectrum remains flat, while the spectrum of the non-elastic model shows a clear decay. Similar observations can be made for the receiver located at an azimut of 50^0 (Figure 2b). On the other hand, the spectrum calculated for a receiver at 80^0 (Figure 2c) still presents a decrease from about 10 Hz, but the rate of the decay is very weak compared to the other receivers. Figure 3 compare the time series for receivers located at 10^0 and 80^0. For the receiver located at 10^0, the cohesive zone has an important effect on the stopping phases. It smooths the impulse emitted by the stopping phases (S_1 and S_2) and therefore, it acts as a low-pass filter whose cut-off frequency may be interpreted as f_{max}. This result can be related to that of Achenbach and Harris (1978) who, studying cracks with curvilinear wavefronts, have shown that the acceleration spectra of the stopping phases is of the form $o(\omega^{-(\kappa-1)/2})$ where κ is a parameter which controls the shape of the displacement discontinuity at the crack tip (κ=1 is the case of a brittle fracture). Moreover, the effect of the cohesive zone is not equivalent in all the azimuths. It exhibits a clear azimuthal dependence of the rate of decay, the maximum effect is observed in the direction of the fault strike and the minimum deviation from the elastic reference model corresponds to the perpendicular direction. So, the spectral ratio between non-elastic and reference spectra varies according to receiver location. Although the slip function of the non-elastic model and that of the elastic one are related by a filtering operation, we cannot consider the non-elastic acceleration spectrum as the result of a simple convolution of the reference spectrum by an unique low-pass filter.

As pointed out by Papageorgiou and Aki (1983), the two important parameters which must control the emission of high frequencies in presence of a cohesive zone should be the rupture velocity and the size of the cohesive zone.

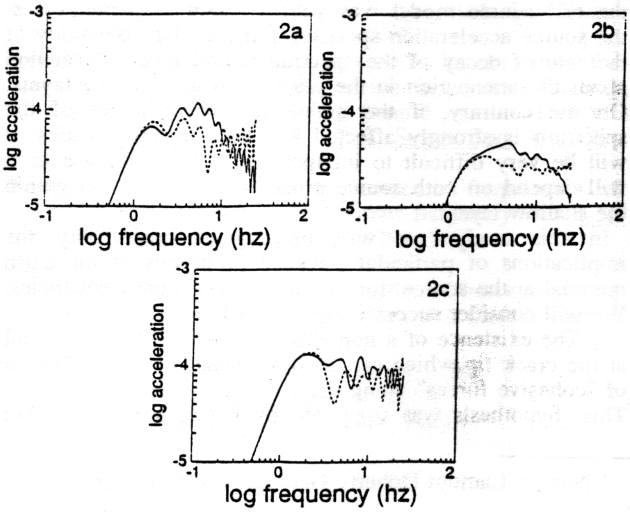

Fig. 2. Comparaison between synthetic acceleration spectra calculated in the elastic case (dashed line) and in the non-elastic case (solid line). The receivers are located at an azimuth of 20^0 (2a), 50^0 (2b) and 80^0 (2c).

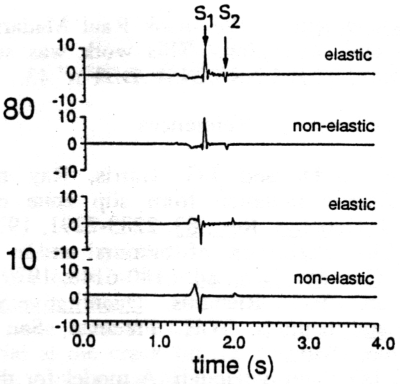

Fig. 3. Comparaison between synthetic acceleration time-series calculated in the elastic case (top) and in the non-elastic case (bottom). The receivers are located at an azimuth of 10° and 80°. S_1 and S_2 indicate the two stopping phases.

Figure 4 shows the acceleration spectra at a receiver located at 20^0, for 4 different rupture velocities (2., 2.5, 3. and 3.5 km/s). Although f_{max} is a parameter rather difficult to measure precisely, it appears clearly that in the presence of a cohesive zone f_{max} is proportional to rupture velocity. If we now consider a constant rupture velocity (2.66 km/s) and 3 different sizes (0.1, 0.2 and 0.3 km) of the cohesive zone as in Figure 5, f_{max} varies as the inverse of the size of the cohesive zone. These numerical results show, as assumed by Papageorgiou and Aki (1983), that the cut-off frequency f_{max} varies proportionaly to the ratio v/d. Due to the azimuthal dependence of the rate of decay of the spectrum, this variation will be very difficult to observe for stations located near the perpendicular direction to the fault strike.

Effect of the Kinematics of the Rupture Front

In the context of elastic shear crack theory, it can be demonstrated (Aki, 1979, Aki and Richards, 1980) that a change in the fracture energy can be related to a change in the rupture velocity. Campillo (1983) has shown numerically that a smooth deceleration of the rupture front strongly affects the high frequency behavior of the acceleration spectrum and has suggested a relation between the duration of the deceleration and the cut-off frequency f_{max}. In order to compare the effects due to the kinematics of the rupture front to those due to a cohesive zone, we introduce now an elastic crack model with a smooth deceleration of the rupture front (cosine shape).

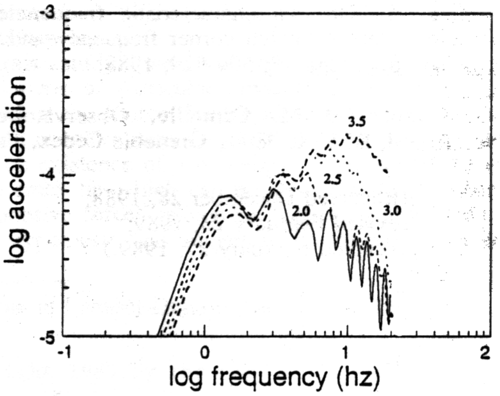

Fig. 4 Synthetic acceleration spectra calculated for 4 different rupture velocities (2., 2.5, 3. and 3.5 km/s). The size of the cohesive zone is 0.2 km.

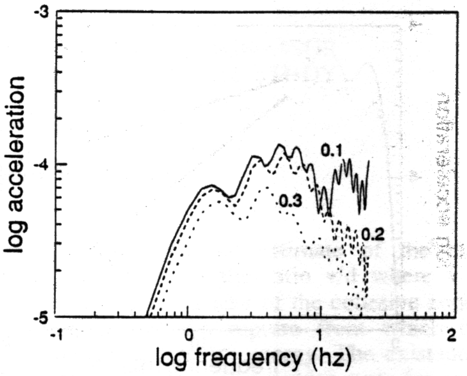

Fig. 5. Synthetic acceleration spectra calculated for 3 different sizes of cohesive zone (0.1, 0.2 and 0.3 km). The rupture velocity is 2.66 km/s.

The geometrical characteristics of the crack are similar to those considered in the previous section. The only change concerns the end of the rupture. The rupture front does not stop abruptly but decelerates in 0.1 s. Figure 6 presents the acceleration spectra for 3 receivers located at 20, 50 and 80^0. They are characterized by a clear decrease from a frequency ranging between 5 and 7 Hz. In contrast with the case of a cohesive zone, we do not observe any azimutal dependence of the rate of decay of the acceleration spectrum. So, if the source effect is finally proved to be at the origin of f_{max}, this difference could be used to discriminate between the two hypotheses investigated in this study.

Discussion and Conclusion

Studying about one hundred accelerograms recorded in California, Anderson & Hough (1984) have shown that, at high frequencies, the acceleration spectrum is characterized by a trend of exponential decay of the form $e^{-\pi\kappa f}$. Under the assumption that the Fourier spectrum of acceleration at the source is flat above the corner frequency (ω^{-2} model), they attributed this decay to an attenuation model in which Q increases rapidly with depth. They also observed

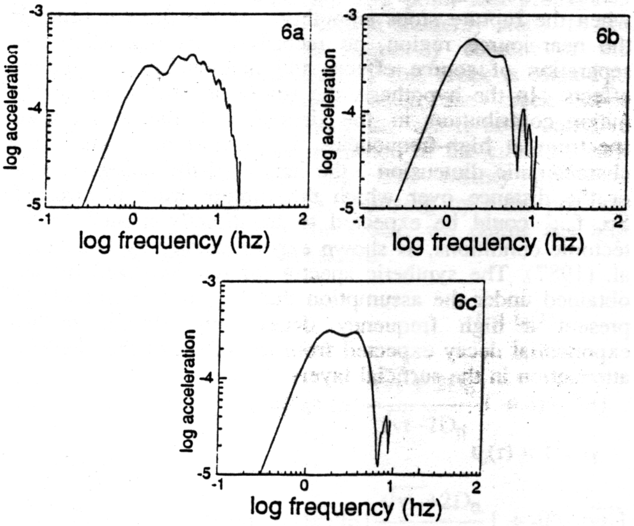

Fig. 6. Synthetic acceleration spectra calculated for an elastic model with a 0.1s final deceleration. The receivers are located at 20^0 (6a), 50^0 (6b) and 80^0 (6c).

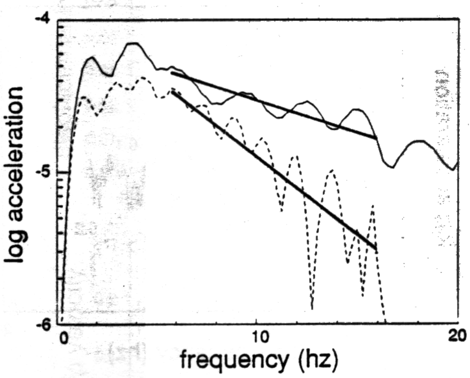

Fig. 7. Linear-log representation of the acceleration spectra calculated in the case of a cohesive zone with a length of 0.2 km (solid line) and in the case of final deceleration during 0.1s (dashed line). Receivers are located at an azimuth of 20^0.

that the spectral decay parameter κ tends toward a finite value as the epicentral distance approaches zero. They have related this observation to a characteristic of the subsurface geological structure.

Figure 7 presents similar spectra to those depicted on figures 2a and 6, but now the spectra are plotted as a linear function of frequency. The dominant trend for the two spectra is a linear decrease of the log of spectral amplitude with frequency. Thus, the presence of a cohesive zone or a smooth deceleration of the rupture front can fully explain an exponential decay of the acceleration spectrum. If we had consider that the decay of the two synthetic acceleration spectra was the result of an attenuation process of the form $e^{\omega t/2Q}$, we would have obtained Q_β values of about 30 and 70 when our calculations were performed assuming an infinite value of Q_β.

This numerical study has shown that two factors of the source, can explain the decrease of the Fourier acceleration spectrum at high frequency : the presence of a cohesive zone and a smooth deceleration of the rupture front. These two processes can be distinguished. While the effect of a deceleration is equivalent in all the azimuths, the effect of the cohesive zone presents a clear azimuthal dependence; the maximum effect being in the fault strike direction. The frequency f_{max} varies as the ratio v/d in presence of a cohesive zone and as 1/t (t=duration of the deceleration) when the rupture stops smoothly. At the present time, in the near-source region, no records exists that allow the separation of source effects and near-surface attenuation effects. In the hypothesis the source is proved to have a major contribution in the decrease of the acceleration spectrum at high-frequencies, f_{max} could be related to a characteristic dimension : the length of the cohesive zone or the distance over which the rupture front is arrested. So, f_{max} could be expected to be directly related to the tectonic conditions, as shown experimentally by Ohnaka et al. (1987). The synthetic spectra for our idealized models obtained under the assumption that f_{max} is a source effect, present a high frequency decay very similar to the exponential decay expected from the effect of the anelastic attenuation in the surficial layers.

Acknowledgments. We thank Raul Madariaga for his comments and suggestions. This work was supported by CNRS, Contribution CNRS/INSU/DBT n^0 43.

References

Achenbach, J. D. and J.G. Harris, Ray method for elastodynamic radiation from slip zone of arbitrary shape, J. Geophys. Res., 83, 2283-2291, 1978.

Aki K., Characterization of barriers on an earthquake fault, J. Geophys. Res., 84, 6140-6148, 1979.

Aki, K. and P.G. Richards, Quantitative Seismology: Theory and methods, W.H. Freeman, San Francisco, California, 984pp.

Anderson, J.G. and S. Hough, A model for the shape of the Fourier amplitude spectrum of acceleration at high frequencies, Bull. Seism. Soc. Am., 74, 1969-1993, 1984.

Barenblat, G., The formation of equilibrium cracks during britlle fracture. General ideas and hypotheses. Axially symmetric cracks, Appl. Math. Mech., 23, 622-636, 1959.

Bouchon, M., A simple method to calculate Green's functions for an elastic layered media, Bull. Seism. Soc. Am., 71, 959-971, 1981.

Campillo, M., Numerical evaluation of near-field, high-frequency radiation from quasi-dynamic circular fault, Bull. Seism. Soc. Am., 73, 723-734, 1983.

Campillo, M. and M. Bouchon, A theoretical study of the radiation from small strike-slip earthquakes at close distances, Bull. Seism. Soc. Am., 73, 83-96, 1983.

Hanks, T.C., f_{max}, Bull. Seism. Soc. Am., 72, 1867-1880, 1982.

Ida, Y., Cohesive forces across the tip of a longitudinal shear crack and Griffith's specific surface energy, J. Geophys. Res, 77, 3796-3805, 1972.

Ida, Y. and K. Aki, Seismic source time function of propagating longitudinal shear-cracks, J. Geophys. Res., 77, 2034-2044, 1972.

Kostrov, B.V., Self-similar problems of propagation of shear cracks, J. Appl. Math. Mech., 28, 1077-1078, 1964.

Madariaga, R., Dynamics of an expanding circular fault, Bull. Seism. Soc. Am., 66, 639-666, 1976.

Ohnaka, M., Y. Kuwahara and K. Yamamoto, Constitutive relations between dynamic physical parameters near a tip of the propagating slip zone during stick slip failure. Tectonophysics, 144, 109-125, 1987.

Papageorgiou, A. and K. Aki, A specific barrier model for the quantitative description of inhomogeneous faulting and the prediction of strong ground motion. Part I : Description of the model, Bull. Seism. Soc. Am., 73, 693-722, 1983.

Papageorgiou, A., On two characteristic frequencies of acceleration spectra : patch corner frequency and f_{max}, Bull. Seism. Soc. Am, 78, 509-529, 1988.

J-C. Gariel and M. Campillo, Observatoire de Grenoble, IRIGM, BP53X, 38041 Grenoble Cedex, France.

(Received December 28, 1988;
revised February 23, 1989;
accepted February 27, 1989.)