INITIATION OF FRICTION INSTABILITY ON A PLANE FAULT SYSTEM

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Abstract We study an unstable elastodynamic process during the initiation phase (i.e. the period between a perturbation of a unstable state and the onset of rupture propagation associated with the seismic wave radiation). We consider the elastic antiplane problem for a system of finite faults under a slip-weakening friction law. A spectral analysis is used to determine the existence, or not, of a catastrophic evolution of the slip. We find that long initiation durations are expected. We also investigate the possibility of defining an effective friction law for a finite fault with a small scale heterogeneity. The "spectral equivalence" between an heterogeneous fault system and an homogeneous fault is pointed out. Surprisingly good agreements are found between the heterogeneous fault model and the homogeneous fault with an effective friction law. Finally we analyze the initiation pattern as a possible signature of instability and we show how the weakening rate is transmitted in the elastic medium through a "domain of confidence".

J.A.C. Martins and M.D.P. Monteiro Marques (eds.), Contact Mechanics, 99-108.

1. Introduction

Laboratory experiments on friction (Dieterich (1979), Ohnaka et al. (1987), Ohnaka and Kuwahara (1990)) have pointed out the existence of a phase of slow but accelerating motion, called the initiation or nucleation phase. This initiation stage precedes the propagation of the dynamic instability along the fault surface. The preseismic slip associated with the nucleation process should be recognized as a manifestation of some phenomena precursor to the rupture. Following these experiments, we shall consider here a slip-weakening friction law.

The elastic quasi-static problem with slip dependent friction was studied in Ionescu and Paumier (1996) where results concerning non homogeneous bifurcation of the static equilibrium positions were obtained. Having in mind the multiplicity of the equilibrium positions they concluded that it is difficult to predict the new equilibrium position with a quasi-static analysis and that a dynamic analysis is required.

We shall concentrate here on the elastodynamic analysis of the friction in the anti-plane case. More precisely, we focus on the initiation of the shear process during the weakening stage to point out simple mathematical properties of its unstable evolution as a result of a slip dependent friction law. Our aim is to describe the growth of the instability in a form very simple to evaluate and to interpret. The present paper does not discuss the long term evolution of the system as done in Cochard and Madariaga (1994), Geubelle and Rice (1995).

2. The evolution problem

We consider the 2D anti-plane shearing on a bounded fault region Γ_f included in the plane $y = 0$ in an homogeneous linear elastic space. The fault region can be composed of a set of simple faults on which the contact is described by a slip dependent friction law. We assume that the displacement field is 0 in directions Ox and Oy and that u_z does not depend on z. The displacement is therefore denoted simply by $w(t, x, y)$. The elastic medium has the shear rigidity G, the density ρ and the shear velocity $c = \sqrt{G/\rho}$. The non-vanishing shear stress components are $\sigma_{zx} = \tau_x^\infty + G\partial_x w(t, x, y)$ and $\sigma_{zy} = \tau_y^\infty + G\partial_y w(t, x, y)$, and the normal stress on the fault plane is $\sigma_{yy} = -S$ ($S > 0$). Let us assume in the following that the slip and the slip rate are nonnegative. Having in mind that we deal with a fault plane and with the evolution of one initial pulse, we may put (for symmetry reasons) $w(t, x, y) = -w(t, x, -y)$, hence we consider only one half-space $y > 0$. The equation of motion is

$$\partial_{tt}^2 w(t, x, y) = c^2 \nabla^2 w(t, x, y) \tag{1}$$

for $t > 0$ and $y > 0$. The boundary conditions on $y = 0$ are:

$$
\begin{aligned}
w(t,x,0) &= 0 \ \text{ if } \ x \notin \Gamma_f, & (2)\\
G\partial_y w(t,x,0) + \tau_y^\infty &= \mu(x, w(t,x,0)))S, \ \text{ if } \ \partial_t w(t,x,0) > 0, & (3)\\
G\partial_y w(t,x0) + \tau_y^\infty &\le \mu(x, w(t,x,0))S \ \text{ if } \ \partial_t w(t,x,0) = 0, & (4)
\end{aligned}
$$

where $\mu(x,s)$ is the coefficient of friction which is a function of the slip s and may be non-homogeneous. The initial conditions are denoted by w_0 and w_1, that is,

$$w(0,x,y) = w_0(x,y), \quad \partial_t w(0,x,y) = w_1(x,y). \tag{5}$$

Since our intention is to study the evolution of the elastic system near an unstable equilibrium position, we shall suppose that $\tau_y^\infty = S\mu(x,0)$. We remark that taking w as a constant satisfies (1)-(4); hence $w \equiv 0$ is an (unstable) equilibrium position, and w_0, w_1 may be considered as small perturbations of the equilibrium.

In order to give a non dimensional formulation we introduce a, the characteristic length, and we put $x_1 = x/a, \quad x_2 = y/a$.

Suppose that the initial perturbation is small and the nonlinear function μ may be approached in a neighborhood of $s = 0$ by its linear approximation i.e.

$$\mu(x_1, s) \approx \mu(x_1, 0) + \partial_s \mu(x_1, 0)s \tag{6}$$

and we introduce the non-dimensional function:

$$\beta(x_1) = a\alpha(x_1), \quad \text{where} \quad \alpha(x_1) = -\frac{S}{G}\partial_s \mu(x_1, 0). \tag{7}$$

We can state now the following linearized evolution problem:

$$
\begin{aligned}
\partial_{tt}^2 w(t,x_1,x_2) &= (c/a)^2 \nabla^2 w(t,x_1,x_2) & (8)\\
w(t,x_1,0) &= 0, \quad \text{for} \quad x_1 \notin \Gamma_f, & (9)\\
\partial_{x_2} w(t,x_1,0) &= -\beta(x_1) w(t,x_1,0), \quad \text{for} \quad x_1 \in \Gamma_f, & (10)\\
w(0,x_1,x_2) = w_0(x_1,x_2), & \qquad \partial_t w(0,x_1,x_2) = w_1(x_1,x_2). & (11)
\end{aligned}
$$

3. The spectral problem

Let us consider the following eigenvalue problem connected to (8)-(11): find $\Phi : R \times R_+ \to R$ and λ^2 such that:

$$
\begin{aligned}
\nabla^2 \Phi(x_1,x_2) &= \lambda^2 \Phi(x_1,x_2) \quad \text{for} \quad x_2 > 0, & (12)\\
\Phi(x_1,0) &= 0, \quad \text{for} \quad x_1 \notin \Gamma_f, & (13)\\
\partial_{x_2} \Phi(x_1,0) &= -\beta(x_1)\Phi(x_1,0) \quad \text{for} \quad x_1 \in \Gamma_f. & (14)
\end{aligned}
$$

Since we deal with a symmetric operator we have real-valued eigenvalues λ^2, i.e. λ is real or purely imaginary.

Two techniques are used to solve the above eigenvalue problem. The first one is based on the equivalence with the following hyper-singular integral equation for $\Phi(x_1, 0)$:

$$\beta(x_1)\Phi(x_1,0) = -\frac{\lambda}{\pi}FP\int_{\Gamma_f}\Phi(s,0)\frac{K_1(\lambda \mid s - x_1 \mid)}{\mid s - x_1 \mid}ds, \qquad (15)$$

where $\lambda \geq 0$, K_1 is the modified Bessel function of the second kind and the integral is taken in the finite-part sense. In the case of homogeneous single fault system (i.e. $\Gamma_f = [-a, a]$, and $\beta(x_1) = const$) this integral equation has been solved by Dascalu et al. (2000) for small values of λ using a semi-analytical method. Recently, this method have been improved by Dascalu and Ionescu (2001) to work in the case of a system of multiple and homogeneous faults (i.e. $\beta(x_1) = \beta^k$ on the fault k) and for arbitrary λ.

The second technique, developed in Voisin et al (2001) uses a finite element approach. To do this the finite fault zone is embedded in a bounded elastic domain $\Omega =]-L, L[\times]0, L[$. The infinite elastic half space is limited by a fictitious boundary all over which the displacement is negligible, i.e. a null displacement all along Γ_d, the part of the boundary of Ω which is not on the fault Γ_f, is imposed. The variational formulation of (12)-(14) is

$$\int_\Omega \nabla\Phi \cdot \nabla v dx_1 dx_2 - \int_{\Gamma_f} \beta\Phi v dx_1 = -\lambda^2 \int_\Omega \Phi v dx_1 dx_2,$$

for all functions $v \in V_h$ (V_h is a finite element space of dimension N, composed of continuous and affine functions over each triangle) such that $v = 0$ on Γ_d.

4. Spectral expansion and the dominant part

Let us denote by (λ_n^2, Φ_n) the associated eigenvalues and the eigenfunctions of (12)-(14) and let N be such that $\lambda_0^2 > \lambda_1^2 > ... > \lambda_{N-1}^2 > 0 > \lambda_N^2 > ....$ The solution of (8)-(11) can be generically written (in its spectral expansion) as:

$$w = w^d + w^w,$$

where w^d is the "dominant part" and w^w is the "wave part", given by:

$$w^d = \sum_{n=0}^{N-1}[\cosh(c|\lambda_n|t/a)W_n^0 + a\frac{\sinh(c|\lambda_n|t/a)}{c|\lambda_n|}W_n^1]\Phi_n(x_1,x_2), \qquad (16)$$

$$w^w = \sum_{n=N}^{\infty}[\cos(c|\lambda_n|t/a)W_n^0 + a\frac{\sin(c|\lambda_n|t/a)}{c|\lambda_n|}W_n^1]\Phi_n(x_1,x_2). \qquad (17)$$

where W_n^0, W_n^1 are the projections of the initial data on the eigenfunctions. We remark that the part of the solution associated with positive eigenvalues

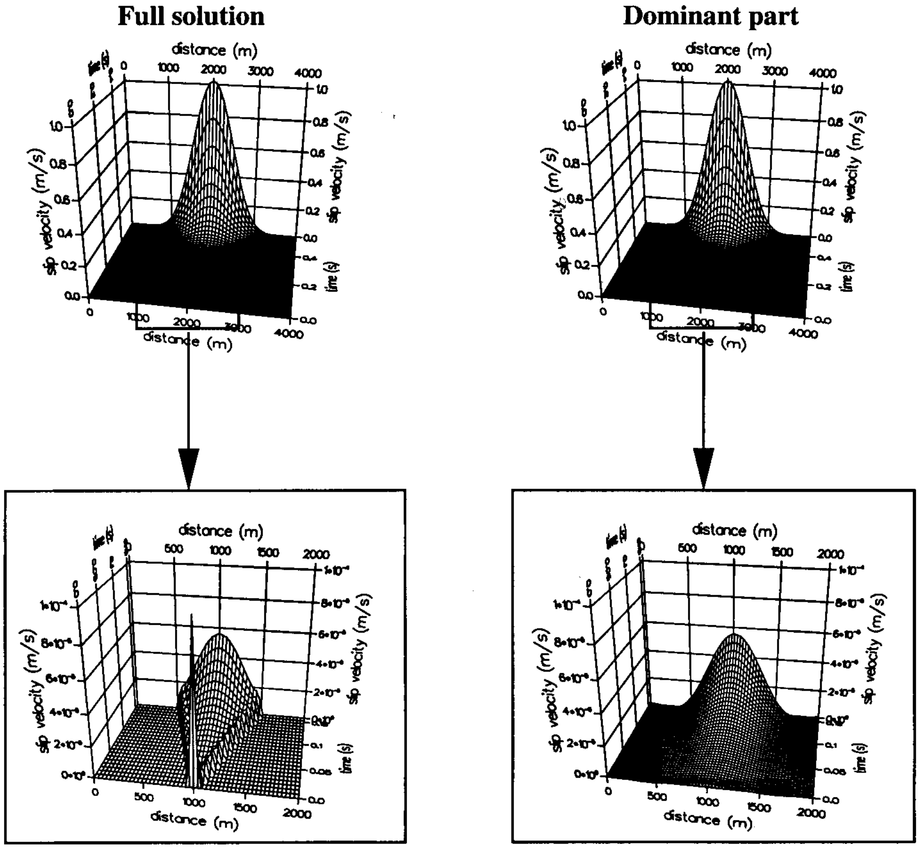

Figure 1. Comparison between the "Full Solution" $\partial_t w(t, x, 0)$ (computed with a FD method) and the "Dominant Part" $\partial_t w^d(t, x, 0)$ (analytical expression) during initiation $t \in [0, T_c]$. Note that the two solutions are indistinguishable (upper plots). The lower plots are enlarged views of the upper ones in a time window just after the application of the initial conditions (t=0). Note that in the lower plots the slip rate is of order of $10^{-4} msec^{-1}$ and in the upper ones is of order of $1 msec^{-1}$. The violation of causality is clear, but the difference between complete and dominant solution has an amplitude less than the initial perturbation.

λ^2 will have an exponential growth with time. Hence, after a while this part will completely dominate the other part which has a wave-type evolution. This behavior is the expression of the instability caused by the slip weakening friction law. The use of the expression of the dominant part leads to a solution in which the perturbation has been severely smoothed by the finite wavenumber integration. The propagative terms are rapidly negligible and the shape of the slip distribution is perfectly described by the dominant part.

The spectral approach has the advantage to be extended relatively easily to other cases. Indeed, for an infinite fault (which implies a continuous spectrum) the analytical expression of the dominant part was first obtained by Campillo and Ionescu (1997) in the anti-plane case, and thereafter by Favreau et al. (2000), (2001) in the in-plane and in the 3D cases.

The accuracy of the approximation of the dominant part is illustrated by the numerical comparison. The dominant part was compared in Campillo and Ionescu (1997), Favreau et al. (2000),(2001), Dascalu et al. (2000) and Voisin et al. (2001) with the full solution computed by a finite differences method described in Ionescu and Campillo (1999). In all these cases the difference was found to be of the order of the initial perturbation, which is negligible with respect to the final amplitude of the solution (see Figure 1 top).

The dominant part is not a complete solution but a part of the solution associated with the real positive eigenvalues. Since all propagating terms are omitted, the dominant part is not causal, and is not expected to be so. In the case of the initiation, the amplitude of the mismatch between complete and dominant solution at the causality limit scales with the initial perturbation (see Figure 1 bottom).

Recently, Knopoff et al. (2000) presented an analytical study of the initiation of shear instability under slip weakening friction for an infinite homogeneous fault in the anti-plane case. They have considered the perturbation on the fault (i.e. of the friction constitutive equation) and obtained an elegant and complete solution of w. More recently, Ampuero et al. (2001), which used a different technique, confirmed the qualitative behavior of the solution given by Campillo and Ionescu (1997) via the dominant part w^d.

5. Stability analysis

One can easily remark that $w \equiv 0$ is a stable position if $\lambda_0^2 < 0$ (i.e. $N = 0$). In this case the dominant part w^d vanishes and the system has a stable behavior. Hence it is important to obtain a simple condition on the distribution $\beta(x_1)$ which determines the positiveness of the eigenvalue λ_0^2.

Let us suppose in the following that we deal with a homogeneous fault system (i.e. $\beta(x_1) = const$). In this case the spectrum $(\lambda_n^2(\beta))_{n\geq 0}$ is a function of the non-dimensional parameter β. Let $0 < \beta_0 \leq \beta_1 \leq \cdots$ be the intersection points of the curves $\beta \longrightarrow \lambda_k^2(\beta)$ with the axis $\lambda^2 = 0$, i.e. $\lambda_k^2(\beta_k) = 0$. The constant β_0 depends only on the geometry of the anti-plane problem (i.e. the distribution of the faults) and it is independent on all physical entities involved in our problem. This non dimensional parameter gives quantitatively the limit between the stable ($\beta < \beta_0$) and unstable ($\beta > \beta_0$) behaviors of the fault.

In the case of a homogeneous single fault system (i.e. $\Gamma_f = [-a, a]$) the value of β_0 was computed by Dascalu et al. (2000) to be $\beta_0 = 1.15777388.....$

For each representative physical quantity which is included in the non dimensional parameter β (the friction weakening slope $S\mu'(0) = (\mu_s - \mu_d)S/L_c$, the interface stiffness G/a, the fault half length a, the elastic bulk modulus G, etc...) we can define a "critical" value.

6. Renormalisation of a heterogeneous fault

Friction is a phenomenon that concerns both microscopic and macroscopic scales. The phenomenon is observed in seismology at the scale of the seismic waves, that is kilometric. The smallest scales of heterogeneity cannot be obtained directly. Even the laboratory measurements Dieterich (1979), Ohnaka and Kuwahara (1990) do not represent the local boundary condition at the microscopic scale but the macroscopic frictional behavior of the elastic bodies in contact at the scale of the samples. We aim to check the assumption that there exists an equivalent macroscopic friction law for the problem of a fault with small scale strength heterogeneity. By equivalent, we mean that this "macroscopic" effective law is sufficient to describe the global behavior of the fault. Our analysis concerns primarily the initiation phase which is an unstable and highly dynamic stage of rupture.

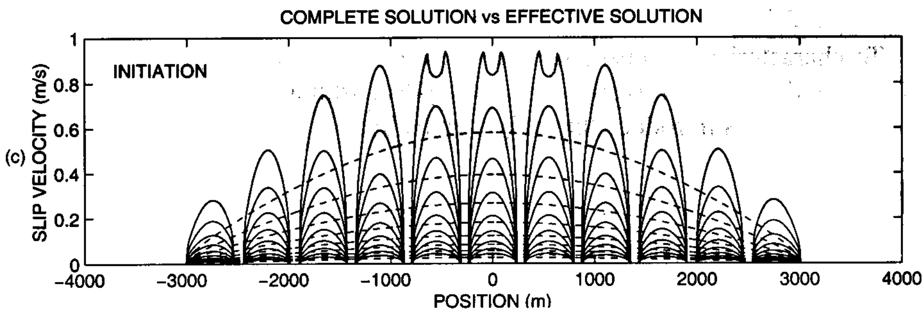

Figure 2. Comparison between two slip rate distributions $\partial_t w(t, x, 0)$ on the fault computed with a FD method at the same times during the initiation phase. The slip rate corresponding to the heterogeneous model of the local friction (continuous line) vs the slip rate corresponding to the homogeneous model with the effective friction law (dashed lines).

Let us suppose in the following that we deal with linear slip-weakening friction, i.e. the constitutive friction is well defined by the parameter β during the initiation phase. We consider a fault Γ_f^{loc} composed of N_f segments $[b_k, d_k]$, $k = 1, 2, .., N_f$, all identical and of weakening rate β_{loc}, separated by rigid barriers. We aim to show that the dynamic behavior of this complex fault system is equivalent to the dynamic behavior of a simple homogeneous fault $\Gamma_f^{equiv} = [b_1, d_n]$ with a weakening rate β_{equiv}.

The initiation develops according to a finite set of eigenfunctions associated with positive eigenvalues that govern the exponential evolution of the instability. The process evolution is dominated by the greatest positive eigenvalue λ_0^2. Indeed, after a period of time the term which involves $exp(ct\lambda_0)$ completely dominates all other terms in the series, hence we can write:

$$w(t,x,y) \approx [ch(ct\lambda_0)W_0^0 + sh(ct\lambda_0)]W_1^0]\Phi_0(x,y). \tag{18}$$

We define the effective or equivalent friction as the slip dependent function which generates the same first positive eigenvalue as the one associated with the heterogeneous problem. This means that we look for β_{equiv} such that

$$\lambda_0^{equiv}(\beta_{equiv}) = \lambda_0^{loc}(\beta_{loc}). \tag{19}$$

In Figure 2 we have plotted the spatial distribution of the slip rate at different times during the initiation phase for a heterogeneous fault (solid lines) with a weakening parameter β_{loc}. With dashed lines we have plotted the spatial distribution of the slip rate at the same times on a homogeneous fault with a weakening parameter β_{equiv} computed from (19). Note that the homogeous fault gives a good description of the heterogenous fault at a macroscopic scale.

7. The initiation pattern

To characterize the unstable behavior of a fault will be a step in the earthquake prediction. Let us suppose that we deal with a slow initiation (i.e. (18) holds) and let us introduce the ratio γ given by:

$$\gamma(x_1,x_2) = -\frac{\partial_{x_2}\Phi_0(x_1,x_2)}{\Phi_0(x_1,x_2)}, \tag{20}$$

This ratio represents the information about the weakening rate of the fault, when it is defined. Let us analyze now the parameter γ in three different cases. If we use the expression of the dominant part (see Campillo and Ionescu (1997)) in the case of the homogeneous and infinite fault we deduce that the parameter γ is constant in all the elastic space, i.e. we have

$$\gamma(x,y) = -a\frac{\partial_y w^d(t,x,y)}{w^d(t,x,y)} = \beta, \quad \text{everywhere in} \quad R \times R_+, \tag{21}$$

In the case of a single finite fault the function γ has its support in a narrow band of the size of the fault length. The general shape of γ defines an initiation pattern that qualitatively characterizes the unstable behavior of the fault. The most interesting point is the existence of a domain, including Γ_f, over which $\gamma(x,y) = \beta$ (see Figure 3). We now define the "domain of confidence" as

$$D_c(\beta) = \{(x_1,x_2); \gamma(x_1,x_2) = \beta\}.$$

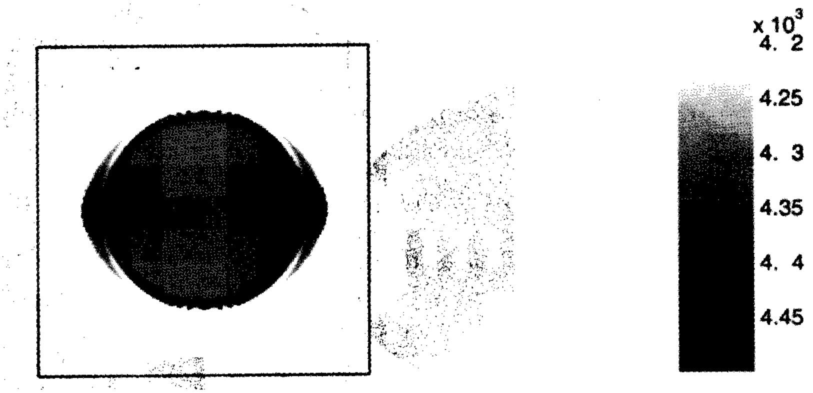

Figure 3. Spatial distribution of the derivative of strain with respect to displacement, parameter $\gamma(x, y)$ computed with a FD method, during the initiation on a single finite homogeneous fault. Note the constant value of γ in a broad region around the fault $D_c(\beta)$ called the "domain of confidence".

As it follows from Campillo et al. (2001), Voisin et al. (2001) in the case of a heterogeneous fault, described in previous section, close to each individual fault segment, an initiation pattern develops in the elastic medium, associated with a local domain of confidence over which $\gamma(x_1, x_2) = \beta_{loc}$. But now, the striking feature is the existence of a wide domain over which $\gamma(x_1, x_2)$ is nearly constant, independent of the individual fault segments but closely related to the whole fault system (see Figure 4). All over this domain, we have $\gamma(x_1, x_2) = \beta_{equiv}$. That is, over this wide domain, it is possible to measure the collective behavior of all the fault segments, similar to the behavior of a homogeneous fault.

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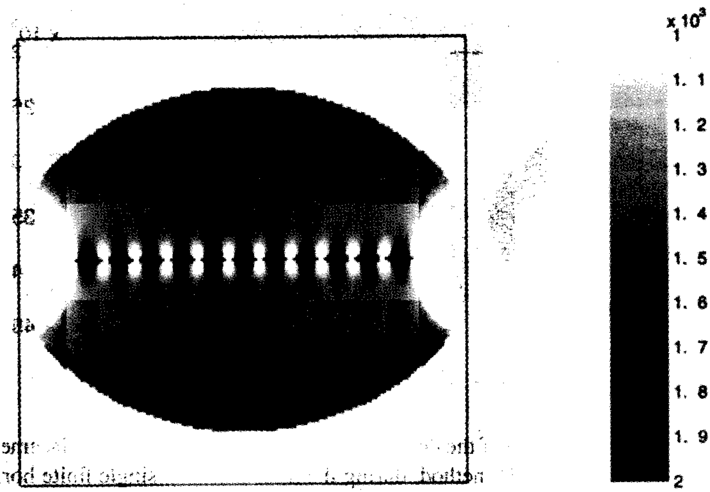

Figure 4. Spatial distribution of the derivative of strain with respect to displacement, parameter $\gamma(x, y)$ computed with a FD method, during the initiation on a heterogeneous fault. Note the constant value of γ in a "collective domain of confidence" $D_c(\beta_{equiv})$.

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