Ia sheet.

from notes

$$u_2 = C' \exp\left(ik\left(-s'z + x - ct\right)\right) + F' \exp\left(ik\left(s'z + x - ct\right)\right)$$

Boundary conditions $\quad \tau = 0 \quad z = h$

$\qquad\qquad\qquad\qquad\quad \tau = 0 \quad z = 0$

$$-C' \exp(-iks'h) + F' \exp(iks'h) = 0$$

$$-C' + F' = 0$$

determinant $= 0 \Rightarrow$ dispersion relation

$$-\exp(-iks'h) + \exp(iks'h) = 0$$

$$\sin ks'h = 0 \iff ks'h = n\pi$$

$$2\pi \frac{f}{c} \left(\frac{c^2}{\beta^2} - 1\right)^{1/2} h = n\pi$$

$$\frac{1}{\beta^2} - \frac{1}{c^2} = \frac{n^2}{4f^2h^2}$$

$$\frac{1}{c^2} = -\frac{n^2}{4f^2h^2} + \frac{1}{\beta^2} \Rightarrow c^2 = \frac{1}{\frac{1}{\beta^2} - \frac{n^2}{4f^2h^2}} = \frac{4h^2\beta^2}{4h^2 - \frac{n^2\beta^2}{f^2}}$$

$$\boxed{C(f) = \frac{2h\beta}{\sqrt{4h^2 - \frac{n^2\beta^2}{f^2}}}}$$

cut off frequencies of the modes: $\quad 4h^2 - \frac{n^2\beta^2}{f^2} > 0$

$$4h^2 > \frac{n^2\beta^2}{f^2} \Rightarrow \boxed{f > \frac{n\beta}{2h}}$$

$n = 0 \quad C(f) = \beta$

$n = 1 \ldots$