$$u(\lambda) = c(\lambda) - \lambda \frac{dc}{d\lambda}$$

$$\lambda = cT = \frac{c}{f} \qquad \overset{\text{dispersion}}{\Longrightarrow} \quad c(\lambda) = \frac{2h\beta}{\sqrt{4h^2 - n^2\beta^2 \frac{\lambda^2}{c^2}}}$$

$$c^2(\lambda) = \frac{4h^2\beta^2 + n^2\beta^2\lambda^2}{4h^2} \qquad c(\lambda) = \beta\sqrt{1 + \frac{n^2\lambda^2}{4h^2}}$$

$$\frac{dc}{d\lambda} = \beta \frac{1}{2} \frac{n^2}{4h^2} 2\lambda \frac{1}{\sqrt{1 + \frac{n^2\lambda^2}{4h^2}}} = \frac{n^2\lambda\beta^2}{4h^2} \frac{1}{c}$$

$$u = c - \lambda \frac{dc}{d\lambda} = c - \frac{n^2\beta^2}{4h^2}\lambda^2 \frac{1}{c}$$

note that $\frac{n^2\lambda^2}{4h^2} = \frac{c^2}{\beta^2} - 1$ $\qquad \Rightarrow u = c - \left(\frac{c^2}{\beta^2} - 1\right)\frac{\beta^2}{c}$

$$\boxed{u = \frac{\beta^2}{c}}$$

Result of problem 2:

$$I_1 = \frac{1}{2}\int_0^\infty \rho(z)\, h'^2(z)\, dz .$$

$$I_2 = \frac{1}{2}\int_0^\infty \mu(z)\, h'^2(z)\, dz$$

$\rho(z) = \rho_0 \quad ; \quad \mu(z) = \mu_0$

$$I_1 = \frac{1}{2}\rho_0 \int_0^h h'^2(z)\, dz .$$

$$I_2 = \frac{1}{2}\mu_0 \int_0^h h'^2(z)\, dz$$

$$\frac{I_2}{I_1} = \frac{\mu_0}{\rho_0} = \beta^2$$

$$u = \frac{1}{c}\frac{I_2}{I_1} = \underline{\underline{\frac{\beta^2}{c}}}$$