Recovering the Green's function from field-field correlations in an open scattering medium (L)

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The possibility of recovering the Green's function from the field-field correlations of coda waves in an open multiple scattering medium is investigated. The argument is based on fundamental symmetries of reciprocity, time-reversal invariance and the Helmholtz-Kirchhoff theorem. A criterion is defined, indicating how sources should be placed inside an open medium in order to recover the Green's function between two passive receivers. The case of noise sources is also discussed. Numerical experiments of ultrasonic wave propagation in a multiple scattering medium are presented to support the argument.
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Wave propagation in a multiple scattering or reverberating environment has been a subject of interest in a wide variety of domains ranging from solid state physics to optics or acoustics. Ultrasound is particularly interesting because it allows a direct measurement of the field fluctuations, both in amplitude and in phase. In connection with this, a remarkable work by Weaver and Lobkis[1-3] recently showed that the Green's function between two points could be recovered from the field-field correlation of a diffuse ultrasonic field. This amounts to do "ultrasonics without a source" since they showed that thermal noise could be used instead of a direct pulse/echo measurement between the two points. The experiment was carried out in an aluminium block, and the theoretical analysis was based on discrete modal expansion of the field, with random modal amplitudes. Applications are promising : it would be possible to recover the Green's function of a complex medium just by correlating diffuse fields received on passive sensors (application to shallow water ocean acoustics, where the field is not diffuse but propagates in a wave guide, was also evoked[4]).

However, the basic assumption in the theoretical analysis is that the medium is closed and free of absorption. In a real medium, absorption will tend to cut out the longest scattering (or reverberating) paths, and discrete modes will not be resolved any more. Similar problems are expected if the medium is open rather than closed (actually, in an open medium, the fluctuation-dissipation theorem[3] establishes the result, as long as the field is diffuse in the thermal sense). The aim of this letter is to examine whether the Green's function can still be recovered from the correlations of an ultrasonic wave field in an open scattering medium, when a discrete expansion on orthogonal modes is no longer relevant and the field is not thermally diffuse.

To that end, we present 2-D numerical experiments of acoustic scattering on rigid inclusions randomly located either in a closed cavity or in a open medium. The wave equation is solved by a finite differences simulation (centered scheme); the boundary conditions is implemented following Collino's work[5]. Naturally, a finite-difference scheme shows numerical dispersion. However, the essential point is that the fundamental symmetries of reciprocity and time-reversal still hold in the numerical experiments.

To begin with, let us consider two receiving points A and B and a source C placed amongst a random collection of scatterers, as represented in Fig 1. The scatterers are in water; only lossless acoustic waves are considered here. At the edges of the grid, the boundary conditions may be either perfectly reflecting (Dirichlet) as in a closed cavity or absorbing (open medium). The signal transmitted by C is a pulse with a center frequency 1 MHz and a gaussian envelope ($\sigma = 0.7$ µs).

We will note $h_{IJ}(t)$ as the impulse response between I and J, i.e., the wave field sensed in I when a Dirac $\delta(t)$ is sent by J. If $e(t)$ is the excitation function in C, then the wave field ϕ_A and ϕ_B received in A and B will be respectively $e(t)\otimes h_{AC}(t)$ and $e(t)\otimes h_{BC}(t)$, $\otimes$ representing convolution. The cross-correlation of the fields received in A and B is then

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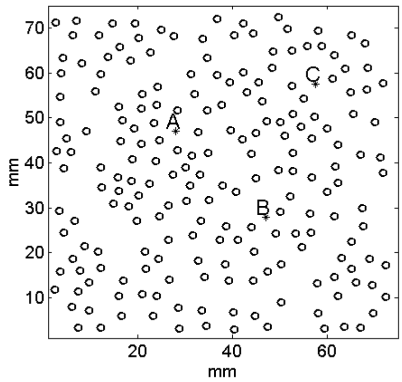

FIG. 1. Two hundred perfectly rigid scatterers (diameter 2.1 mm) are randomly distributed over a 7.5×7.5 cm^2 area. A point source is placed in *C*; *A* and *B* are receiving points. The boundary conditions may be perfectly reflecting (Dirichlet) as in a closed cavity or absorbing (open medium).

$$C_{AB}(t) = \int \phi_A(\theta)\phi_B(t+\theta)d\theta = h_{AC}(t) \otimes h_{BC}(-t) \otimes f(t)$$

with $f(t) = e(t) \otimes e(-t)$. $f(t)$ depends only on the excitation imposed at the source, whereas the information regarding the impulse response between A and B is hidden in $h_{AC}(t) \otimes h_{BC}(-t)$. Indeed, the impulse responses of a closed cavity satisfy a remarkable property, as shown by Carsten Draeger in 1999, [6] which he termed the "cavity equation" :

$$h_{AC}(t) \otimes h_{BC}(-t) = h_{AB}(t) \otimes h_{CC}(-t)\,. \qquad (1)$$

For this relation to hold, the cavity must be lossless and its eigenmodes not degenerate. Note that, in practice, the correlations cannot be performed over an infinite time interval (the ring time of a cavity is infinite if it is lossless); therefore the cavity equation can be compared to experimental results if the integration time ΔT is sufficiently large compared to 1/Δω, with Δω the characteristic distance between modes, so that the modes are resolved. 1/Δω is sometimes referred to as the Heisenberg time, or break time. Figure 2 illustrates the validity of the cavity equation; here the impulse responses have been recorded during an integration time of 80 ms (2 $\times 10^6$ time steps), and the Heisenberg time is ~ 5 ms. From Draeger's cavity equation, the correlation between the fields received in *A* and *B* is:

$$C_{AB}(t) = h_{AB}(t) \otimes h_{CC}(-t) \otimes f(t)\,.$$

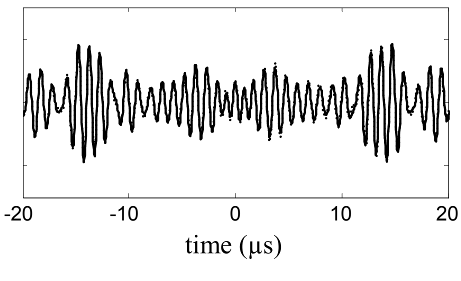

FIG. 2. Comparison between $C_{AB}(t)$ (thick continuous line) and $h_{AB}(t) \otimes h_{CC}(-t) \otimes f(t)$ (dotted line). The overall correlation coefficient between the two waveforms is 98.7%.

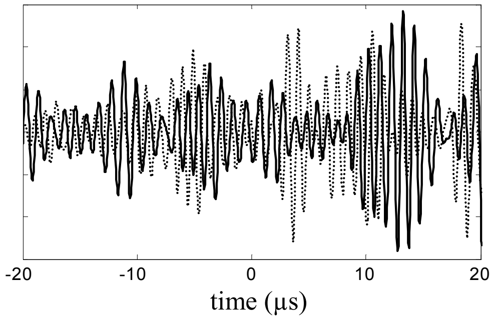

FIG. 3. Comparison between $C_{AB}(t)$ (thick continuous line) and $h_{AB}(t) \otimes h_{CC}(-t) \otimes f(t)$ (dotted line) in the open scattering medium. The impulse responses were recorded during 800 μs until they became negligible. The overall correlation coefficient between the two waveforms is 0.48%.

Therefore, similarly to Weaver's results,[1-3] the direct Green's function h_{AB} *is* present in the correlations of the field within a closed cavity and can be recovered from C_{AB} provided that the h_{CC} term can be properly deconvolved, at least in the frequency domain limited by the spectrum of $f(t)$.

Is this valid in an open medium? We have conducted the same numerical experiment, with the same distribution of scatterers, but with absorbing instead of reflecting boundary conditions. As a result, the cavity equation is no longer valid and the correlation of the scattered field C_{AB} shows no resemblance whatsoever with the Green's function (the correlation coefficient function between wave forms represented on Fig. 3 is less than 0.5%).

However, a physical argument indicates that the Green's function can *still* be recovered from the correlations C_{AB}, even in an open medium, if several judiciously distributed sources are used instead of a single point *C*. To that end, we propose to analyze the experiment in terms of time-reversal symmetry. Indeed, there is a strong link between correlations of a diffuse field and time reversal.[7]

Because the scatterers do not move and there is no flow within the medium, the propagation is reciprocal, i.e. $h_{IJ}(t)=h_{JI}(t)$. When we cross-correlate the impulse responses received in *A* and *B*, the result $h_{AC}(t) \otimes h_{BC}(-t)$ is equal to $h_{CB}(-t) \otimes h_{AC}(t)$. Now, imagine that we do the following time-reversal experiment: *B* sends a pulse, *C* records the impulse response $h_{CB}(t)$, time-reverses it and sends it back; the resulting wave field observed in *A* would then be $h_{CB}(-t) \otimes h_{AC}(t)$, which, because of reciprocity, is exactly the cross-correlation $h_{AC}(t) \otimes h_{BC}(-t)$ of the impulse responses received in *A* and *B* when *C* sends a pulse. We would like the direct Green's function h_{AB} to appear in the cross-correlation. But in the most general case, $h_{CB}(-t) \otimes h_{AC}(t)$ has no reason to be equal to h_{AB}, as was shown in Fig. 3. Yet we can go beyond: imagine now that we use several points C to perform the time-reversal operation, and that we place them in such a way that they form a *perfect* time-reversal-device, with no loss of information. Following the Helmholtz-Kirchhoff theorem, such would be the case if the sources *C* were continuously distributed on a surface surrounding the scattering medium. Then the time-reversal operation should be perfect. During the "forward" propagation, *B* sends a pulse that propagates everywhere in the medium

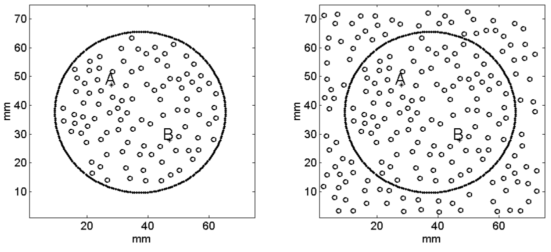

FIG. 4. A and B are receiving points. Two hundred fifty source points are placed regularly around a circle with radius 18.7 mm, 100 scatterers being inside the circle. They completely surround the medium (a), or only partially (b). The boundary conditions on the edges of the grid are absorbing (open medium), in both cases.

[including in A where the field received is $h_{BA}(t)$], it may be scattered many times and is eventually recorded on every point of the time-reversal device, with no loss. When the field is time-reversed, since nothing of it has been lost, it should exactly go backwards in time (and refocus on B at time t=0) everywhere in the medium, which implies that the field received in A after the time-reversal is exactly $h_{BA}(-t)$, the time-reversed version of the direct Green's function. Then, once the wave has refocused on B (at time t=0), it does not stop since there is no "acoustic sink" in B:[8] the wave diverges again from B and gives rise, at times t>0 to $h_{BA}(t)$ in A.

Thus, if we use a collection of sources C arranged in such a way that they form a perfect time-reversal device, we should have

$$\sum_{C} h_{AC}(-t) \otimes h_{CB}(t) = h_{BA}(t) + h_{BA}(-t) \qquad (2)$$

A more detailed analysis, taking into account the monopolar or dipolar nature of the source/receivers is given by Didier Cassereau.[9] Equation (2) implies that the impulse response $h_{BA}(t)$ can still be recovered from the correlation of a diffuse field, even in an open medium, provided that the sources C are distributed judiciously, and all the correlation functions are summed over the source positions. Unlike the case of a closed medium, no additional deconvolution by h_{CC} is needed. From this time-reversal analogy, we deduce a condition for the Green's function to emerge from cross correlations in open media: the sources C must be placed so that they form a perfect time-reversal device.

We have checked this in the numerical experiments depicted in Fig 4. The results are in excellent agreement with Eq. (2), as is shown in Fig. 5: the degree of correlation between waveforms is 97.4 %. Of course, when the sources are not placed as a perfect mirror, as presented in Fig. 4(b), the results are less good (the degree of correlation between waveforms is 81.9%) because one part of the waves are not recorded by the time-reversal device due to the presence of scatterers outside the sources. Yet the main features of the Green's function can still be recognized, even at late times. If the number of sources is decreased, the reconstruction of the Green's function is less satisfactory, as shown in Fig. 6. With

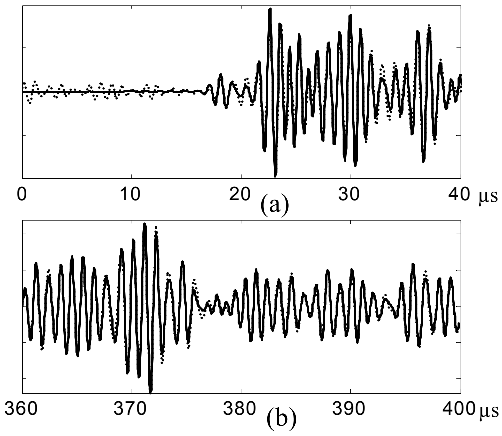

FIG. 5. Comparison between $\Sigma_C\, h_{AC}(t)\otimes h_{BC}(-t)\otimes f(t)$ (dotted line) and h_{AB}-$t)\otimes f(t)$ in the open scattering medium surrounded by 250 sources C as depicted in Fig. 4(a), at early times (a) and in the late coda, 360 µs later (b). The overall correlation coefficient between waveforms is 97.4%.

only 50 sources (instead of 250 previously) regularly spaced every ~5λ/3 all along a circle as in Fig. 4(a), the correlation coefficients between waveforms is 70 %. However if the 50 sources are gathered together in a 72° angular sector (pitch λ/3), it drops to 53%. Indeed, since the coherence length of a diffuse wave field is ~λ, it is useless to place the sources closer.

So far, we have considered that the origin of the field measured in A and B was an active and coherent source transmitting a short pulse (or a collection of such sources). What if there are no such sources in the medium, but a diffuse continuous noise? The physical origin of this noise may be thermal vibrations.[3] In seismology, noise in the seismograms comes from a variety of different sources (traffic, sea waves, weather, human activity…) continuously and (allegedly) randomly pumping energy into the earth and essentially exciting surface waves. In ocean acoustics, noise may originate from boats, surf, wind, animals etc. By definition, the noise sources cannot be controlled. In the light of the discussion above, in order to recover the Green's function from the cross-correlation of the noise received in A and B, the most favorable situation would be that in which noise can be con-

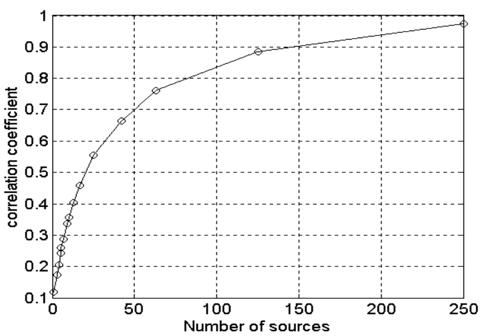

FIG. 6. Correlation coefficient between $\Sigma_C\, h_{AC}(t)\otimes h_{BC}(-t)\otimes f(t)$ (dotted line) and $h_{AB}(-t)\otimes f(t)$ versus the number of sources employed.

sidered as coming from virtual point sources C randomly distributed everywhere in the medium and continuously generating uncorrelated white noises $n_C(t)$. In that case, the cross-correlation between waveforms sensed in A and B would be

$$\sum_{C}\sum_{C'} h_{AC}(-t) \otimes n_C(-t) \otimes h_{C'B}(t) \otimes n_{C'}(t)$$

If the observation time ΔT is long enough compared to the correlation time of the noise, then $n_C(t) \otimes n_{C'}(-t)$ will converge to $\delta(t)\ \delta_{C,C'}$. Moreover, if the virtual noise sources C are distributed everywhere in the medium (in other words, if each degree of freedom is excited randomly and independently) then the C-points would necessarily constitute a perfect time-reversal device, so Eq. (2) should be verified again.

We have carried out a numerical experiment based on this idea. Two hundred forty sources where distributed at random inside the open scattering medium shown in Fig. 1, and 240 uncorrelated white noises, convolved by $e(t)$, were transmitted by these sources during 40 ms. The resulting wave forms are received in A and B. Their cross-correlations $C_{AB}(t)$ is compared to the direct Green's function $h_{AB}(t) \otimes f(-t)$: the agreement is still very good (61% correlation coefficient) even at late times.

The emergence of the Green's function in the field-field correlations in a closed cavity with discrete modes is now well established.[1-3] In this letter, we have argued that recovering the Green's function was also possible in an open multiple scattering medium and we have proposed a criterion based on reciprocity, time-reversal symmetry and the Helmholtz-Kirchhoff theorem: if sources are placed as if they were to form a perfect time-reversal device, then the Green's function can be recovered by summing the cross correlations. This has been validated by numerical experiments. The reduction of the number of sources was also discussed and the possibility of using noise sources was illustrated.

There is still much food for thought, particularly regarding the role of scatterers in the reconstruction of the Green's function. The argument we developed here is valid for any medium (homogeneous, high-order multiple scattering, reverberant…) where reciprocity and invariance under time-reversal hold. The field does not need to be thermally diffuse for the Green's function to emerge from the correlations, as long as there are enough well-positioned sources. Another approach is to consider the scatterers as secondary sources which are necessary to truly randomize the wave field emanating from a *single* original source. Correlating the late part of the coda would then permit us to reconstruct at least the early arrivals of the Green's function. Recent seismologic results support this idea.[10] The influence of absorption is also to be investigated.

ACKNOWLEDGMENTS

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