# On the correlation of non-isotropically distributed ballistic scalar diffuse waves

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Theorems indicating that a fully equipartitioned random wave field will have correlations equivalent to the Green's function that would be obtained in an active measurement are now legion. Studies with seismic waves, ocean acoustics, and laboratory ultrasound have confirmed them. So motivated, seismologists have evaluated apparent seismic travel times in correlations of ambient seismic noise and tomographically constructed impressive maps of seismic wave velocity. Inasmuch as the random seismic waves used in these evaluations are usually not fully equipartitioned, it seems right to ask why it works so well, or even if the results are trustworthy. The error, in apparent travel time, due to non-isotropic specific intensity is evaluated here in a limit of large receiver-receiver separation and for the case in which the source of the noise is in the far field of both receivers. It is shown that the effect is small, even for cases in which one might have considered the anisotropy to be significant, and even for station pairs separated by as little as one or two wavelengths. A formula is derived that permits estimations of error and corrections to apparent travel time. It is successfully compared to errors seen in synthetic waveforms.
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## I. INTRODUCTION

Much recent research has focused on the correlations of seismic, ocean acoustic, and laboratory ultrasonic noise. Theorems indicating that a fully equipartitioned noise field will have correlations $C(\tau)$ essentially equivalent to the Green's function $G(\tau)$ that would be obtained in an active measurement are now legion.[1–4] These have been supported by laboratory experiments[5–10] and analysis of ocean acoustic and seismic field data.[11–13] The identity promises to facilitate the acquisition of acoustic information without the use of a controlled source. Applications in seismology and exploration geophysics where sources are earthquakes or thumper trucks or explosives are especially interesting. The conditions on acoustic noise such that its correlations will indeed converge to the Green's function, and the rate and quality of that convergence, remain active areas for inquiry. Many questions relate to the robustness of the identity for the case of imperfectly diffuse noise fields, the effect of only partial equipartition, and the effect of dissipation. Questions as to how one might compensate in the case of imperfectly diffuse noise are also arising.[14] Reviews may be found in the special June 2006 issue of *Geophysics*.[15,16] A tangential literature has entertained sundry generalizations of the basic identity, for example, to media lacking time-reversal invariance,[17,18] to dissipative media[19] and to the diffusion equation,[20] and to structural acoustics and discrete media.[21,22]

It has long been recognized that field-field correlations of narrow-band diffuse waves have a universal *local* short range structure equal to a Bessel function of order zero. Rollwage *et al.*[23] showed that diffuse fields with wavenumber $k$ in a shallow water tank have correlations $\langle\psi(\mathbf{x})\psi(\mathbf{y})\rangle \sim J_o(\mathbf{k}|\mathbf{x}-\mathbf{y}|)$. It is well known that the ensemble average (over different realizations of a multiple scattering medium) of field-field correlations in three dimensions is a spherical Bessel function of order zero as attenuated by scattering $\langle\psi(\mathbf{x})\psi(\mathbf{y})\rangle \sim J_o(k|\boldsymbol{x}-\boldsymbol{y}|)\exp\{-|\boldsymbol{x}-\boldsymbol{y}|/\text{meanfreepath}\}$. The microtremor survey method[24,25] suggested by Aki in 1957 and in wide use in seismology today is based on this notion of local correlations being essentially Bessel functions. In all this work it seems to have been little recognized that the Bessel-character of the short range local correlations has an extension to long ranges and to the time domain. It transpires that such fields have correlations equal to the *imaginary part of Green's function*[21] in turn equal to a Bessel function only at short range in an unbounded continuous homogeneous scalar medium. Field-field correlations are thus in general richer than simple Bessel functions; they depend on the type of wave, and include effects from the structure and geometry of the medium.

Proof of the identity between correlations and Green's function varies with definition of a diffuse field. In finite bodies it is convenient to take a modal perspective, in which a diffuse field has uncorrelated normal mode amplitudes but with equal expected energies. This definition is commonly adopted in room acoustics and structural vibration[26,27] and in thermal physics.

In the so-called ballistic case, with few scatterers, the proofs are especially simple and intuitive.[2,4] It is this case that pertains to travel time tomography and attracts the most attention in seismology.[16,28,29] Seismologists have constructed high resolution maps of Rayleigh and Love wave velocities from tomographic analyses of travel times seen in correlations of ambient seismic noise.[28,29]

An imperfectly equipartitioned diffuse field precludes confident application of field correlation methods, or so one imagines. Ambient seismic noise as used in Refs. 28 and 29 at frequencies below 1 Hz, usually has its origin in ocean storms and thus has a preferred direction. Seismic coda, as used in Ref. 30 has a degree of isotropy that develops slowly as the coda ages. Laboratory experiments[5–10] with ultrasound can be designed to better conform to the demands of the theorems but, except for the case of thermal noise,[5,31,32] even their correlations fail to perfectly match conventional waveforms obtained actively. Nevertheless noise correlations continue to provide useful information. They have been used to detect changes in material properties,[33,34] and most strikingly have been used in spite of the imperfect equipartition to generate high resolution maps of seismic velocity.[28,29] These maps appear to be robust, but doubts remain. How reliable are they? Why do they appear so robust in spite of the anisotropy of the diffuse field upon which they are based? The familiar condition that the noise field be fully equipartitioned in order to recover Green's function is perhaps over restrictive for the purpose of estimating travel times.[2]

Snieder[2] showed that, in an asymptotic limit in which the two receivers are separated by a distance long compared to a wavelength, and when the distribution of diffuse ballistic intensity is smooth, albeit not necessarily isotropic, the correlation is the Green's function; one need not have a fully equipartitioned isotropic noise field. There were in that discussion, however, no indications as to the errors that might follow from a nonsmooth intensity distribution, or a less than fully asymptotic receiver separation. Recently Malargia and Castellaro[35] argued that the apparent robustness is due to the probability density function for the direction of an incident plane wave corresponding to a probability density function for apparent seismic velocity that is strongly peaked at the actual seismic velocity. In Sec. II, we readdress the derivation of the relation between $G$ and $C$, but for a non-isotropic distribution of incident incoherent intensity. Attention is confined to the two dimensional case, as the case of three dimensions is both analytically simpler and of less practical importance. Sections III–VIII analyze the effect of that nonisotropy on practical estimates of travel time at finite receiver separations. It is shown that a travel time estimate is corrupted only slightly at realistic values for these parameters. An expression for the travel time correction is derived.

## II. FIELD CORRELATIONS IN A NONISOTROPIC DISTRIBUTION OF DIFFUSE PLANE WAVE INTENSITY

Consider two receivers as in Fig. 1: one at origin, and the other a distance $x$ from the origin. We distribute incoherent impulsive sources $s(\theta)$ over an annular region of (large) radius $R$ around the receivers. With $\langle s\rangle=0$, and $\langle s(\theta)s^*(\theta')\rangle=B(\theta)\delta(\theta-\theta')$.

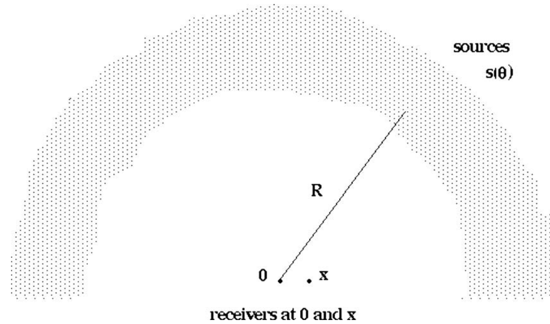


FIG. 1. Incoherent sources distributed at large distance from a pair of receivers.

These give rise to a field $\psi(\boldsymbol{r},t)$ whose Fourier transform

$$\tilde{\psi}(\boldsymbol{r},\omega)=\int \psi(\boldsymbol{r},t)\exp(-i\omega t)dt$$

is a superposition of cylindrical waves from the many sources $s$.

$$\tilde{\psi}(x,\omega)=\int s(\theta)d\theta\exp(i\omega R-i\omega x\cos\theta)/\sqrt{Ri(\omega-i\varepsilon)}. \quad (1)$$

The angle $\theta$ is defined relative to the strike line connecting the receivers. Here and below integrals over $\theta$ are over a full $2\pi$ and integrals over $t$ and $\omega$ are from $-\infty$ to $\infty$, unless otherwise noted. The usual $-i\varepsilon$ has been inserted to analytically continue $\omega$ to its complex plane; it resolves ambiguities at real $\omega$, and the choice of sign assures causality.[36] The cylindrical waves have been written in a form valid for asymptotically large $\omega R$. Thus the present derivation is restricted to noise whose sources are in the far field. The receivers at 0 and $x$ are not required to be in each other's far fields; indeed, attention is chiefly directed to the case in which they are separated by distances of order one wavelength. Units are used such that wave speed is unity.

The field has correlation defined by

$$C(\boldsymbol{r},\boldsymbol{r}',\tau)=\int \psi(\boldsymbol{r}',t)\psi(\boldsymbol{r},t+\tau)dt. \quad (2)$$

By the Wieiner–Khinchin cross-correlation theorem, $C$ is the inverse Fourier transform of $\langle\tilde{\psi}^*\tilde{\psi}\rangle$

$$C(\boldsymbol{r},\boldsymbol{r}',\tau)=\frac{1}{2\pi}\int d\omega\langle\tilde{\psi}(\boldsymbol{r}',\omega)^*\tilde{\psi}(\boldsymbol{r},\omega)\rangle\exp(i\omega\tau). \quad (3)$$

The time derivative of the correlation between receivers at $\mathbf{r}=0$ and $\mathbf{r}'=\mathbf{i}x$ is

$$C'_{0,x}(\tau) \equiv \partial_\tau \int \psi(0,t)\psi(x,t+\tau)dt,$$

$$\tilde{C}'_{0,x}(\omega) \equiv i\omega \int C_{0,x}(\tau)\exp(-i\omega\tau)d\tau$$
$$= i\omega\langle \tilde{\psi}(0,\omega)^* \tilde{\psi}(x,\omega)\rangle \quad (4)$$

The relevant expectation is

$$\langle \tilde{\psi}(0)^* \tilde{\psi}(x)\rangle = \left\langle \int s(\theta')d\theta' \times \int s(\theta)d\theta \exp(-i\omega x \cos\theta)/R\sqrt{\omega^2+\varepsilon^2} \right\rangle$$
$$= \int B(\theta)d\theta \exp(-i\omega x\cos\theta)/R\sqrt{\omega^2+\varepsilon^2}. \quad (5)$$

We now return to the time domain, first multiplying by $i\omega$ (to impose the $\tau$ derivative needed for equivalence to $G$) and also inserting a rescaling factor $-R/4\pi$ for simplification of the final expressions.

$$C'_{0,x}(\tau) = \frac{-1}{4\pi}\int B(\theta)d\theta i\,\mathrm{sgn}(\omega) \times \exp(-i\omega x\cos\theta)\exp(i\omega\tau)d\omega. \quad (6)$$

The factor sgn($\omega$) arises from $\lim_{\varepsilon\to 0+} \omega/\sqrt{\varepsilon^2+\omega^2}$. $B(\theta)$ is now written in a Fourier series (cosines only, by symmetry, as the receiver correlation does not distinguish between positive and negative $\theta$)

$$B(\theta) = B_0 + B_1\cos\theta + B_2\cos 2\theta + \cdots. \quad (7)$$

The odd harmonics could be removed by choosing to consider only a lapse time-symmetrized version of $C$.

The integration over $\theta$ is found in Abramowitz and Stegun 9.1.21 of Ref. 37. Thus

$$C'_{0,x}(\tau) = -\frac{1}{2}\sum_q (-i)^q B_q \int_{-\infty}^{\infty} i\,\mathrm{sgn}(\omega)\exp(i\omega\tau)J_q(\omega x)d\omega. \quad (8)$$

The evenness or oddness of $J$ corresponds to that of $q$ and so one may write

$$\int i\,\mathrm{sgn}(\omega)\exp(i\omega\tau)J_q(\omega x)d\omega = \begin{cases} 2i\int_0^\infty J_q(\omega x)\cos(\omega\tau)d\omega & \text{for } q \text{ odd} \\ -2\int_0^\infty J_q(\omega x)\sin(\omega\tau)d\omega & \text{for } q \text{ even.} \end{cases} \quad (9)$$

Thus,

$$C'_{0,x}(\tau) = \sum_{\text{even } q} (-1)^{q/2}B_q\int_0^\infty \sin(\omega\tau)J_q(\omega x)d\omega + \sum_{\text{odd } q}(-1)^{(q+1)/2}B_q\int_0^\infty \cos(\omega\tau)J_q(\omega x)d\omega. \quad (10)$$

These integrals are found in Abramowitz and Stegun 11.4.37 and 38 of Ref. 37. We take $x \geq 0$ without loss of generality, and $\tau \geq 0$ by recognizing that the expressions below can be evaluated for negative $\tau$ by replacing it with $|\tau|$ and changing the signs of the terms $B_q$ for which $q$ is even.

The expressions differ depending on whether or not $x > \tau$.

$$C'_{0,x}(\tau) = [-B_1/x - 2B_2\tau/x^2 + B_3(1-4\tau^2/x^2)/x + \cdots], \quad x > \tau$$
$$= \frac{1}{\sqrt{\tau^2-x^2}}[B_0 + xB_1/(\tau+\tau\sqrt{\tau^2-x^2}) + x^2B_2/(\tau+\sqrt{\tau^2-x^2})^2 + x^3B_3/(\tau+\sqrt{\tau^2-x^2})^3 + \cdots], \quad x < \tau. \quad (11)$$

In the special case of isotropy, where $B_q = 0$ except for $q = 0$, one recovers the well known result that $C'$ is equal to the time-symmetrized Green's function, where $G$ is

$$G = \begin{cases} 0 & \text{for } |x| > |\tau| \\ \mathrm{sgn}(\tau)/\sqrt{\tau^2-x^2} & \text{for } |x| < |\tau|. \end{cases} \quad (12)$$

These expressions have been evaluated numerically for a few choices $\{B_q\}$. Two of these are plotted in Fig. 2. A few points are evident: Nonisotropy leads to $C'$ having support at times before the arrival of the Green's function; the waveform includes not just the trivial anticausal part at negative $\tau$, but also a noncausal part at $|\tau| < |x|$. One also observes that every plot has a singularity at the arrival time $\tau = \pm x$. In most cases the singularity is of the form $1/\sqrt{(\tau^2-x^2)}$, but in some cases, notably Fig. 1(d) where $B(\pi)$ and $B(0)$ are close to zero, the singularities at the arrival times are less severe. Nevertheless, identification of arrival time is not difficult in any of these waveforms; lack of isotropy does not degrade estimation of wavespeed in these broad-band signals.

## III. BAND-LIMITED CORRELATIONS

In practice, correlation waveforms have finite bandwidth. To address practical questions, we must therefore convolve the above waveforms with the time domain version of the square of the spectrum, a necessarily even function of time. In this case, Eq. (6) is modified:

$$C'_{0,x}(\tau) = \frac{-1}{4\pi}\int B(\theta)d\theta i\,\mathrm{sgn}(\omega) \times \exp(-i\omega x\cos\theta)\exp(i\omega\tau)|\tilde{a}(\omega)|^2 d\omega \quad (13)$$

If $B$ is expanded in a Fourier series, Eq. (7), one has

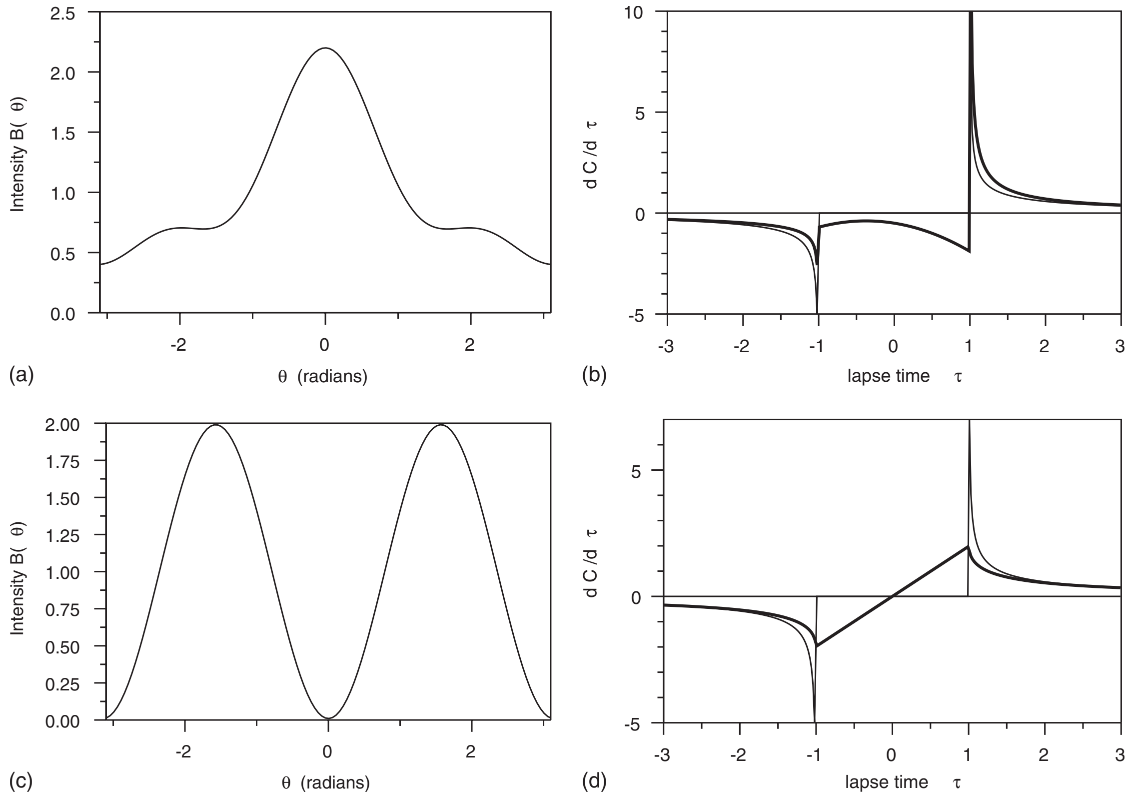


FIG. 2. Two examples of non-isotropic $B(\theta)$ [(a) and (c)] and the corresponding broadband field correlation [(b) and (d)] for distance $x=1$. Light line is Green's function, and the bold line is the correlation. The two cases correspond, respectively, to $\{B_q\}=\{1,0.7,0.3,0.2,0,0,0,\ldots\}$ in (a) and (b), and $\{1,0,-0.99,0,0,\ldots\}$ in (c) and (d).

$$C'_{0,x}(\tau) = -\frac{1}{2}\sum_q (-i)^q B_q \int_{-\infty}^{\infty} i\,\mathrm{sgn}(\omega)\exp(i\omega\tau)J_q(\omega x) \times |\tilde{a}(\omega)|^2 d\omega. \quad (14)$$

The new factor $|a|^2$ is the spectral power density of the noise, assumed here to have been independent of direction $\theta$. It is necessarily even and real. It is useful to take it in the form of a (nonunique) sum of real contributions centered on positive and negative frequencies $\pm\omega_o$:

$$|\tilde{a}(\omega)|^2 = \tilde{A}(\omega-\omega_0) + \tilde{A}(\omega+\omega_0). \quad (15)$$

In particular, it can be useful to take these contributions to be Gaussian

$$\tilde{A}(\varpi) = \exp(-\varpi^2 T^2) \quad (16)$$

and to be real with chief support near $\varpi=0$. Its inverse Fourier transform is

$$\frac{1}{2\pi}\int |\tilde{a}(\omega)|^2 \exp(i\omega t)d\omega = \exp(i\omega_0 t)A(t) + \exp(-i\omega_0 t)A(-t), \quad (17)$$

where $A$ is the inverse Fourier transform of $\tilde{A}$ and $A(-t)=A^*(t)$. If $\tilde{A}(\omega)$ is the Gaussian described above, then $A(t)$ is

$$A(t) = \frac{1}{2T\sqrt{\pi}}\exp(-t^2/4T^2). \quad (18)$$

If as before we take $B$ in a Fourier series, we may confine our integration to positive $\omega$ if we assume $\tilde{A}(\omega-\omega_0)$ has no support at negative $\omega$. Then

$$\int i\,\mathrm{sgn}(\omega)\exp(i\omega\tau)J_q(\omega x)|\tilde{a}(\omega)|^2 d\omega = \begin{cases} 2i\int_0^{\infty} J_q(\omega x)\cos(\omega\tau)\tilde{A}(\omega-\omega_0)d\omega & \text{for } q \text{ odd} \\ -2\int_0^{\infty} J_q(\omega x)\sin(\omega\tau)\tilde{A}(\omega-\omega_0)d\omega & \text{for } q \text{ even.} \end{cases} \quad (19)$$

So that,

$$C'_{0,x}(\tau) = \sum_{q\text{ even}} (-1)^{q/2}B_q\int_0^{\infty}\sin(\omega\tau)J_q(\omega x)\tilde{A}(\omega-\omega_0)d\omega + \sum_{q\text{ odd}}(-1)^{(q+1)/2}B_q\int_0^{\infty}\cos(\omega\tau)J_q(\omega x)\tilde{A}(\omega-\omega_0)d\omega. \quad (20)$$

## IV. BAND-LIMITED HIGH FREQUENCY CORRELATIONS WAVEFORM NEAR NOMINAL ARRIVAL TIME

At asymptotically large $\omega x$, Eq. (20) may be simplified by recalling

$$J_q(\omega x) \sim \sqrt{2/\pi \omega x} \cos\{\omega x - \pi q/2 - \pi/4\}.$$

With the definition $\phi_q = \pi q/2 + \pi/4$, we find

$$C'_{0,x}(\tau) = \frac{1}{\sqrt{2\pi\omega_0 x}} \sum_{q \text{ even}} (-1)^{q/2} B_q \int_0^\infty [\sin(\omega\tau + \omega x - \phi_q) + \sin(\omega\tau - \omega x + \phi_q)] \tilde{A}(\omega - \omega_0) d\omega + \frac{1}{\sqrt{2\pi\omega_0 x}} \sum_{q \text{ odd}} (-1)^{(q+1)/2} B_q \int_0^\infty [\cos(\omega\tau + \omega x - \phi_q) + \cos(\omega\tau - \omega x + \phi_q)] \tilde{A}(\omega - \omega_0) d\omega. \quad (21)$$

We again confine attention, without loss of generality, to $\tau$ and $x$ both positive, and recognize that the first terms oscillate rapidly and contribute negligibly to the integration. Then

$$C'_{0,x}(\tau) = \frac{1}{\sqrt{2\pi\omega_0 x}} \sum_{q \text{ even}} (-1)^{q/2} B_q \operatorname{Im}\{\exp(i\omega_0(\tau - x) + i\phi_q) \times A(\tau - x)\} + \frac{1}{\sqrt{2\pi\omega_0 x}} \sum_{q \text{ odd}} (-1)^{(q+1)/2} B_q \operatorname{Re} \times \{\exp(i\omega_0(\tau - x) + i\phi_q) A(\tau - x)\}. \quad (22)$$

By choosing $\omega_0$ such that $\tilde{A}(\varpi)$ is maximum at $\varpi = 0$, $A(t)$ can be approximated as real; its leading order imaginary part would be only of order time cubed (units of time being inverse bandwidth, $T$).

Thus in the vicinity of the arrival at $\tau = x$, and for $z = \tau - x$, we write

$$C'_{0,x}(\tau) = \frac{A(z)}{\sqrt{2\pi\omega_0 x}} \sum_{q \text{ even}} (-1)^{q/2} B_q \sin(\omega_0 z + \phi_q) + \frac{A(z)}{\sqrt{2\pi\omega_0 x}} \sum_{q \text{ odd}} (-1)^{(q+1)/2} B_q \cos(\omega_0 z + \phi_q). \quad (23)$$

This expression for the band-limited correlation waveform is asymptotically valid for large $\omega_0 x$. The ostensible arrival time might be determined by examining the location of the first peak, near $z = 0$. If only the $B_0$ term is present, then the peak is at $z = (\pi/2 - \phi_0)/\omega_0 = (\pi/4\omega_0)$. The peak is slightly delayed, by one-eighth of a cycle, beyond the true arrival time at $z = 0$. This observation could used to estimate arrival time from a band-limited Green's function. Alternatively, true arrival time could be estimated by picking the point that is one-eighth of a cycle *after* the zero that precedes the peak. The identification of a zero crossing is often used for high precision estimates of ultrasonic propagation time.[38]

If we have two or more terms $B$, then after inserting for the $\phi_q = \pi q/2 + \pi/4$, Eq. (23) becomes

$$C'_{0,x}(\tau) = \frac{A(z)}{\sqrt{2\pi\omega_0 x}} (B_0 + B_1 + B_2 + B_3 + \cdots) \times \sin(\omega_0 z + \phi_0). \quad (24)$$

The correlation is proportional to the full intensity on strike, $B(0) = B_0 + B_1 + B_2 + \cdots$. Asymptotically, it has the same temporal form that it has in the isotropic case. One concludes that the presence of an anisotropic diffuse field does not, at least in the asymptotic limit $\omega x \gg 1$, impair estimation of arrival time beyond the $1/8$ cycle delay noted above that is present even with an isotropic field. This was Snieder's conclusion also.[2]

## V. CORRECTIONS TO ZERO CROSSING TIME

It is of interest to inquire how, short of the full asymptotic limit $\omega x \to \infty$, apparent arrival time might be affected by non-isotropic $B(\theta)$. Without appealing to a Fourier decomposition of $B$, or to the Bessel function identities, we may write [Eq. (13)]

$$C'_{0,x}(\tau) = \frac{-1}{4\pi} \int B(\theta) d\theta i \operatorname{sgn}(\omega) \exp(-i\omega x \cos\theta) \times \exp(i\omega\tau) |\tilde{a}(\omega)|^2 d\omega.$$

Again we set $\tau = z + x$ and confine attention to positive $\omega$

$$C'_{0,x}(\tau) = \frac{-i}{4\pi} \int B(\theta) d\theta \exp(i\omega x(1 - \cos\theta)) \times \exp(i\omega z) \tilde{A}(\omega - \omega_0) d\omega + \text{c.c.} \quad (25)$$

The asymptotically high $\omega x$ behavior near the arrival time is dominated by $\theta$ near 0:

$$C'_{0,x}(\tau) \sim \frac{-1}{4\pi} \int \left\{ B(0) + \frac{1}{2} B''(0) \theta^2 + \cdots \right\} \times \left\{ 1 - \frac{1}{24} i\omega x \theta^4 + \cdots \right\} \exp(i\omega x \theta^2/2) d\theta \times \int_0^{+\infty} d\omega i \exp(i\omega z) \tilde{A}(\omega - \omega_0) d\omega + \text{c.c.}, \quad (26)$$

$$= \frac{-1}{4\pi} \int_0^{+\infty} \left\{ B(0) \sqrt{\frac{2\pi}{-i\omega x}} + \frac{1}{4} B''(0) \sqrt{\frac{8\pi}{(-i\omega x)^3}} - \frac{i\omega x}{24} \left(\frac{3}{4}\right) B(0) \sqrt{\frac{32\pi}{(-i\omega x)^5}} \right\} d\omega i \exp(i\omega z) \times \tilde{A}(\omega - \omega_0) + \text{c.c.}, \quad (27)$$

or

$$= \frac{-1}{4\pi} \int_0^{+\infty} \left\{ B(0) \sqrt{\frac{2\pi}{\omega x}} e^{i\pi/4} + \frac{1}{4} B''(0) \sqrt{\frac{8\pi}{(\omega x)^3}} e^{3i\pi/4} - \frac{i\omega x}{24} \left(\frac{3}{4}\right) B(0) \sqrt{\frac{32\pi}{(\omega x)^5}} e^{5i\pi/4} \right\} \times d\omega i \exp(i\omega z) \tilde{A}(\omega - \omega_0) + \text{c.c.} \quad (28)$$

There are two distinct methods by which one might in practice attempt to identify arrival time. It is not uncommon in ultrasonics to identify a zero crossing. It is more common in seismology, where signals tend to be more contaminated by noise, to cross correlate a waveform against a reference wavelet and select the time shift which maximizes the cross correlation. Here we shall investigate both methods.

At leading order, i.e., neglecting all but the first term, the correlation waveform is

$$B(0)A(z)\sqrt{\frac{2\pi}{\omega_0 x}}\sin(\omega_0 z+\pi/4), \tag{29}$$

which has a zero at $z=-\pi/4\omega_0$. This is the same waveform as in Eq. (24). We thus, as above, estimate "arrival time" ($z=0$) as one-eighth cycle later than this zero. To study corrections to the waveform near the zero, and therefore the shift of the zero, we analyze the expression (28) at $z=-\pi/4\omega_0$ and change integration variable: $\omega=\omega_0+\delta$, $d\omega=d\delta$, so $\exp(i\omega z)=\exp(-i\delta\pi/4\omega_0)\exp(-i\pi/4)=\exp(i\delta z)\times\exp(-i\pi/4)$.

Then, for $z=-\pi/4\omega_0$,

$$C'_{0,x\sim}=\frac{-i}{4\pi}\int_{-\infty}^{+\infty}d\delta\exp(i\delta z)\tilde{A}(\delta)\left\{B(0)\sqrt{\frac{2\pi}{\omega_0 x}}\left(1-\frac{\delta}{2\omega_0}\right)+\frac{1}{4}B''(0)\sqrt{\frac{8\pi}{(\omega_0 x)^3}}e^{i\pi/2}\left(1-\frac{3\delta}{2\omega_0}\right)-\frac{i\omega_0 x}{24}\left(\frac{3}{4}\right)B(0)\sqrt{\frac{32\pi}{(\omega_0 x)^5}}e^{i\pi}\left(1-\frac{3\delta}{2\omega_0}\right)+\cdots\right\}+\text{c.c.} \tag{30}$$

The terms independent of $\delta$ integrate to $2\pi A(z)$. The terms linear in $\delta$ integrate to $2\pi i\partial_z A(z)$. Then, for $z=-\pi/4\omega_0$,

$$\begin{aligned}C'_{0,x}&\sim\frac{1}{2}\left\{B(0)\sqrt{\frac{2\pi}{\omega_0 x}}\left(-iA(z)+\frac{A'(z)}{2\omega_0}\right)+\frac{1}{4}B''(0)\sqrt{\frac{8\pi}{(\omega_0 x)^3}}\left(A(z)+\frac{3iA'(z)}{2\omega_0}\right)+\frac{\omega_0 x}{24}\left(\frac{3}{4}\right)B(0)\sqrt{\frac{32\pi}{(\omega_0 x)^5}}\left(A(z)+\frac{3iA'(z)}{2\omega_0}\right)\right\}+\text{c.c.}\\&=\left\{B(0)\sqrt{\frac{2\pi}{\omega_0 x}}\left(\frac{A'(z)}{2\omega_0}\right)+\frac{1}{4}B''(0)\sqrt{\frac{8\pi}{(\omega_0 x)^3}}A(z)+\frac{\omega_0 x}{24}\left(\frac{3}{4}\right)B(0)\sqrt{\frac{32\pi}{(\omega_0 x)^5}}A(z)\cdots\right\}\\&=\sqrt{\frac{2\pi}{\omega_0 x}}\left\{B(0)\frac{A'(z)}{2\omega_0}+\frac{1}{2\omega_0 x}B''(0)A(z)+\frac{1}{8\omega_0 x}B(0)A(z)\cdots\right\}.\end{aligned} \tag{31}$$

Equation (31) represents the value of the correlation waveform $C'$ at the nominal zero at $z=-\pi/4\omega_0$. The $z$-derivative of $C'$ at the zero is [see Eq. (29)]

$$B(0)A(z)\sqrt{\frac{2\pi}{x\omega_0}}\omega_0.$$

Thus the waveform *near* its nominal zero at $z_0=-\pi/4\omega_0$ is

$$B(0)A(z)\sqrt{\frac{2\pi}{x\omega_0}}\omega_0(z-z_0)+\sqrt{\frac{2\pi}{\omega_0 x}}\left\{\frac{1}{2\omega_0 x}B''(0)A(z)+\frac{1}{8\omega_0 x}B(0)A(z)+B(0)\frac{A'(z)}{2\omega_0}\right\}, \tag{32}$$

which has its zero at

$$z=z_0-\left(4\frac{B''(0)}{B(0)}+1+4xA'(z)/A(z)\right)\bigg/8\omega_0^2 x. \tag{33}$$

For the assumed $A(z)$, $4xA'/A=-2zx/T^2$ so arrival time, as evaluated by examining the time of this zero, is earlier than the true arrival time, by an amount

$$\left\{4\frac{B''(0)}{B(0)}+1+\pi(x/2\omega_0 T^2)\right\}\bigg/8\omega_0^2 x. \tag{34}$$

Equation (33) serves as a higher order asymptotic estimate for the shift of the zero relative to its location as determined by Snieder[2] or by Eqs. (24) and (29).

The second and third terms in Eq. (34) are present even if $B$ is constant, i.e., even if the correlation waveform is $G$ itself. This is an indication that travel time assessment by identifying the time of the zero crossing and adding one-eighth of a cycle is only correct asymptotically; there are corrections at finite $\omega x$ and finite $x/T$.

The first term in Eq. (34) is the more interesting. It gives the leading order effect of nonisotropic diffuse intensity. By way of illustration, take $T$ large, $B''(0)=-2B(0)$ [as would be the case if $B(\theta)=1+\cos(2\theta)$], and $\omega_0 x=6$ (one wavelength separation); then the zero occurs $7/48\omega_0$ later than one would have supposed, or about 1/40 of a period. Velocity estimate would be erroneously low by about 2.5%.

## VI. ARRIVAL TIME AS ESTIMATED BY CORRELATION WITH A REFERENCE WAVELET

Seismologists often evaluate arrival time by correlating the signal with a reference wavelet. Thus it is of interest also to cross correlate the waveform (13) with the same waveform obtained for the case $B$=const (i.e., with the band-limited Green's function itself). We again write Eq. (13) as

$$C'_{0,x}(\tau)=\frac{-1}{4\pi}\int B(\theta)d\theta i\,\text{sgn}(\omega)\exp(-i\omega x\cos\theta)\times\exp(i\omega\tau)|\tilde{a}(\omega)|^2d\omega,$$

whose Fourier transform is (an unimportant factor of −2 has been dropped)

$$\begin{aligned}\tilde{C}'_{0,x}(\omega)&\sim\int B(\theta)d\theta i\,\text{sgn}(\omega)\exp(-i\omega x\cos\theta)|\tilde{a}(\omega)|^2\\&=\int\left\{B(0)+\frac{1}{2}B''(0)\theta^2\right\}d\theta i\,\text{sgn}(\omega)\times\exp\left(i\omega x\frac{1}{2}\theta^2\right)(1-i\omega x\theta^4/24)\exp(-i\omega x)|\tilde{a}(\omega)|^2\\&=\int\left\{B(0)+\frac{1}{2}B''(0)\theta^2-B(0)i\omega x\theta^4/24\right\}d\theta i\end{aligned}$$

$$\times \mathrm{sgn}(\omega)\exp\left(i\omega x\frac{1}{2}\theta^2\right)\exp(-i\omega x)|\tilde{a}(\omega)|^2$$

$$= \left\{ B(0)\sqrt{\frac{2\pi}{i\omega x}} + \frac{1}{2}B''(0)\sqrt{\frac{8\pi}{(i\omega x)^3}} - B(0)\frac{i\omega x}{24}\frac{3}{4}\sqrt{\frac{32\pi}{(i\omega x)^5}} \right\} i\,\mathrm{sgn}(\omega)\exp(-i\omega x)|\tilde{a}(\omega)|^2. \quad (35)$$

By expanding $\cos\theta$ near $\theta=0$ we have implicitly focused on the arrival at positive lapse time. We wish to form the cross correlation between this and its version with $B=1$. At an offset of $\Delta$, this is

$$X(\Delta) = \int \tilde{C}_{B\neq 1}(\omega)\tilde{C}^*_{B=1}(\omega)\exp(-i\Delta\omega)d\omega$$

$$= \int \left\{ \sqrt{\frac{2\pi}{i\omega x}} + \frac{B''(0)}{2B(0)}\sqrt{\frac{8\pi}{(i\omega x)^3}} - \frac{i\omega x}{24}\frac{3}{4}\sqrt{\frac{32\pi}{(i\omega x)^5}} \right\} \times \left\{ \sqrt{\frac{2\pi}{-i\omega x}} + \frac{i\omega x}{24}\frac{3}{4}\sqrt{\frac{32\pi}{(-i\omega x)^5}} \right\}\exp(-i\omega\Delta) \times |\tilde{a}(\omega)|^4 d\omega$$

$$= \int \frac{2\pi}{|\omega| x}\left\{ 1 + \frac{B''(0)}{2B(0)}\frac{1}{(i\omega x)} - \frac{1}{(8i\omega x)} \right\} \times \left\{ 1 + \frac{1}{(8i\omega x)} \right\}\exp(-i\omega\Delta)|\tilde{a}(\omega)|^4 d\omega$$

$$= \int \frac{2\pi}{|\omega| x}\left\{ 1 + \frac{B''(0)}{2B(0)}\frac{1}{(i\omega x)} + \cdots \right\} \times \exp(-i\omega\Delta)|\tilde{a}(\omega)|^4 d\omega. \quad (36)$$

We expand this for small $\Delta$ and find that it achieves its maximum, $\partial X/\partial\Delta=0$, at

$$\Delta = \frac{B''(0)}{2xB(0)}\int \frac{1}{|\omega|}|\tilde{a}(\omega)|^4 d\omega \Big/ \int |\omega||\tilde{a}(\omega)|^4 d\omega \sim \frac{B''(0)}{2x\omega_0^2 B(0)}, \quad (37)$$

which is identical to the expression derived above (34), after its terms unrelated to anisotropy are removed.

Equations (37) and (34) provide an asymptotically valid estimate, $B''(0)/2x^2\omega_0^2 B(0)$, for the apparent fractional increase in wavespeed occasioned by having constructed a correlation waveform from smooth but not isotropic diffuse intensity. In Sec. VII, this estimate is compared to results from numerical simulations.

## VII. COMPARISON WITH SYNTHETIC EXPERIMENTS

We compare the above predictions with numerical experiments, some purely synthetic and based on an assumed homogeneous medium, as pictured in Fig. 1, and others based on field data from a many component array of seismograms in Oman.

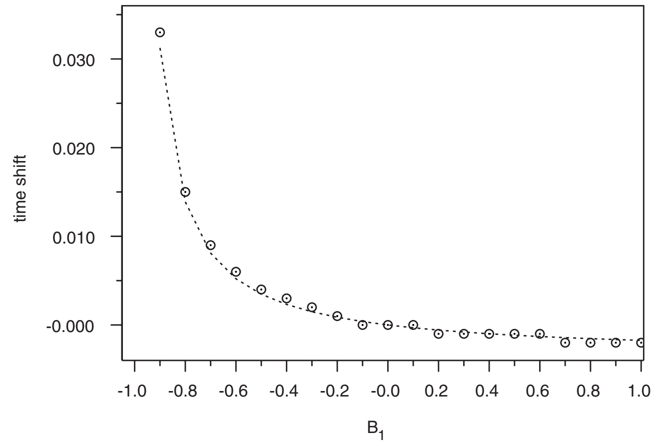


FIG. 3. Comparison of predicted time shift (dashed line) and time shift as obtained by correlation against the actual Green's function (symbols), for a set of intensity distributions given by $B(\theta)=1+B_1\cos(\theta)$ and a range of values for $B_1$.

For purposes of the tests with synthetic data, Eq. (11) for a distance $x=1$ was convolved with a tone burst of the form given in Eqs. (15)–(18) with $T=1/4$ and $\omega_0=12$. Thus we consider the case in which the receiver pair is separated by almost two wavelengths. This was done for a variety of choices $B(\theta)$. Each such waveform was isolated into its positive lapse time part ($\tau>0$) and cross correlated with the actual band-limited Green's function obtained by taking $B_0=1$, $B_{q\neq 1}=0$. The relative shift between these two was taken as the error in apparent arrival time as predicted in Sec. VI. As seen in Figs. 3–8, the theoretical expression (37) does a good job of predicting the time shift. This suggests that Eq. (37) may (i) be used in practice to estimate the error in apparent velocity, (ii) be used to correct for a non-isotropic distribution, or (iii) reassure a practitioner that such anisotropy does not significantly impact estimations of seismic velocity.

Figure 3 compares time shifts, for the arrival at positive lapse time, for an intensity distribution given by $B(\theta)=1+B_1\cos\theta$, for a range of $B_1$ values between $-1$ and $+1$. Except for the singular case $B_1=-1$ where the intensity on

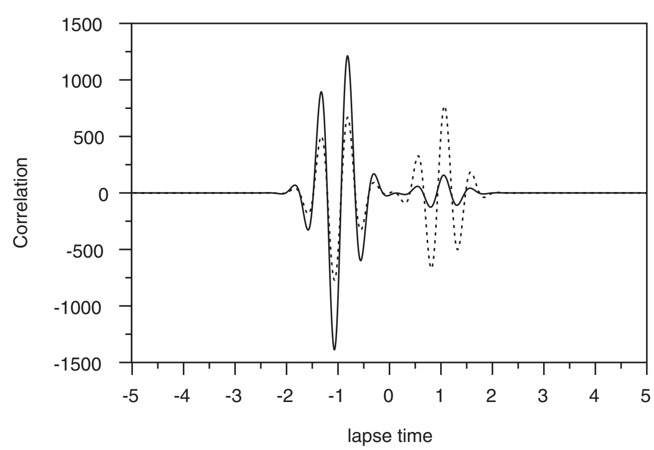


FIG. 4. Band-limited waveform (solid line) obtained from Eq. (13) using an intensity distribution $B(\theta)=1-0.8\cos\theta$. Dashed line is the time-symmetrized Green's function for the same spectrum, as obtained from Eq. (13) using $B(\theta)=1$.

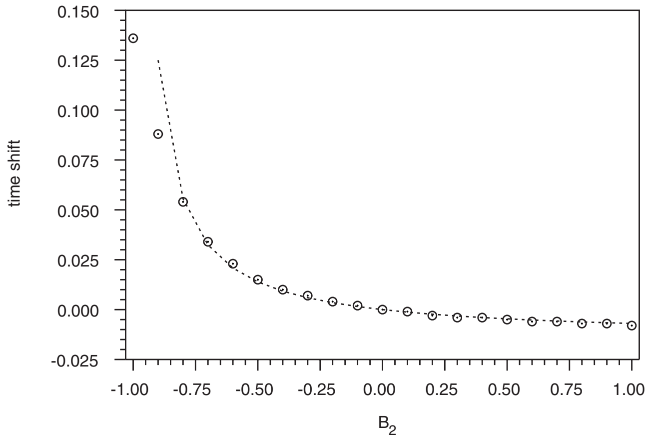


FIG. 5. Time shifts for $B(\theta)=1+B_2\cos 2\theta$, as a function of $B_2$.

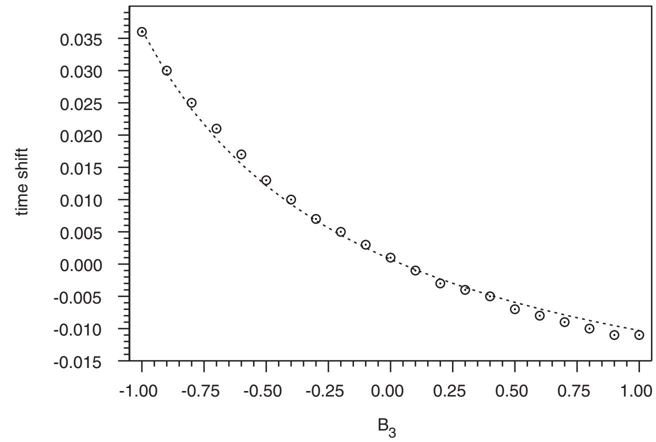


FIG. 7. Time shifts for $B(\theta)=1.7+0.4\cos\theta-0.2\cos 2\theta+B_3\cos 3\theta$.

strike $B(0)$ vanishes and Eq. (37) predicts an infinite correction, theory does an excellent job. Figure 4 shows an example of the waveforms used for this comparison, for the case $B_1=-0.8$. Figures 5–8 are similar to Figs. 3 and 4, but for more strongly varying intensity distributions.

## VIII. COMPARISON USING DATA FROM FIELD MEASUREMENTS

A data set of 2 560 000 seismic responses courtesy of Petroleum Development Oman has been discussed elsewhere.[39,40] These responses were obtained from 1600 geophones in a square array of 25 m spacings, as due to 1600 active sources in a similar square array offset by 12.5 m. Here we use this data set to construct correlation waveforms from arbitrary distributions of sources. Two receivers ($\alpha=1,2$) in the center of the array were selected. They were separated by a distance of 155 m, corresponding to a surface wave transit time of about 0.13 s at 15 Hz. Signals were studied from sources $j$ in an annulus centered on the receivers, an annulus of inner radius 300 m, and thickness 69 m. Waveforms $\psi_{\alpha j}$ from the data set were windowed into the range 0–1.5 s so as to minimize contributions from scattered waves and emphasize ballistic waves, and correlated. The result was then summed with an angular weighting $B(\theta)$. The resulting $C_{12}(\tau)=\Sigma_j B(\theta_j)\int \psi_{1j}(t)\psi_{2j}(t+\tau)dt$ was then averaged over a set of 18 receiver pairs with different absolute orientations. This gave a correlation waveform for each of several choices $B(\theta)$.

Figure 9 compares the theoretical prediction to the time shift (divided by the nominal arrival time) as obtained by cross correlating the arrivals as constructed with $B=\text{const}$, and $B$ as indicated in the captions. Theoretical predictions were based on power spectra having their support between 10 and 20 Hz. Arrival time, at 15 Hz, was about 0.13 s. Thus we took $\omega_0=94$ rad/s and $\omega_0 x=12$. The spectrum was only approximately Gaussian, but it shared a second moment with a Gaussian of $T=0.04$ s. The plots examine the same three cases of weighting $B$, examined in Figs. 3, 5, and 7. The asymptotic theory continues to describe the apparent time shift. The theory is quantitatively less accurate. The difference may be ascribed to the presence of some nonballistic waves (there is some scattering), to not being in the far field of the original sources [their emissions are not plane waves in the vicinity of the receivers $R=\infty$, as assumed for Eq. (1)], and to geometric dispersion (the soil is layered; surface waves have a frequency dependent speed.)

## IX. SUMMARY

Non-isotropic distributions of ballistic specific intensity violate the assumptions behind the identification of ambient

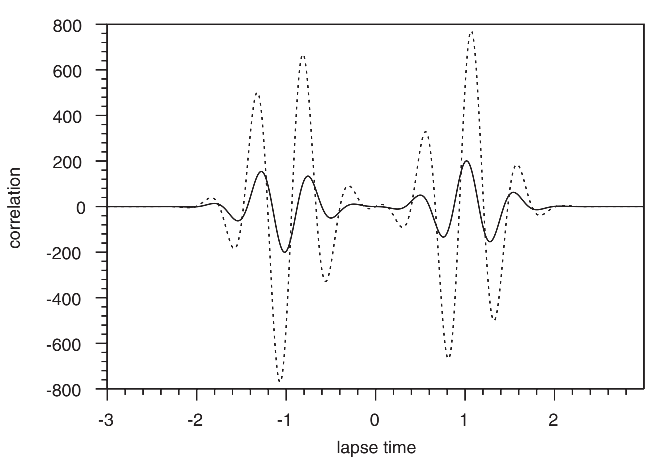


FIG. 6. Example of waveforms used in Fig. 5.

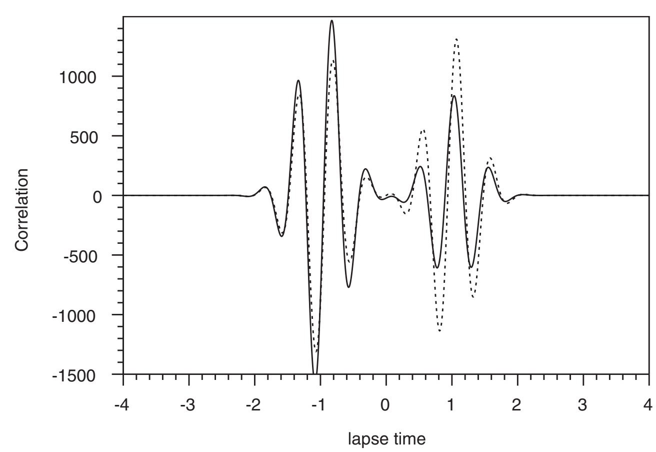


FIG. 8. Example of waveforms used in Fig. 7, the case $B_3=-1$.

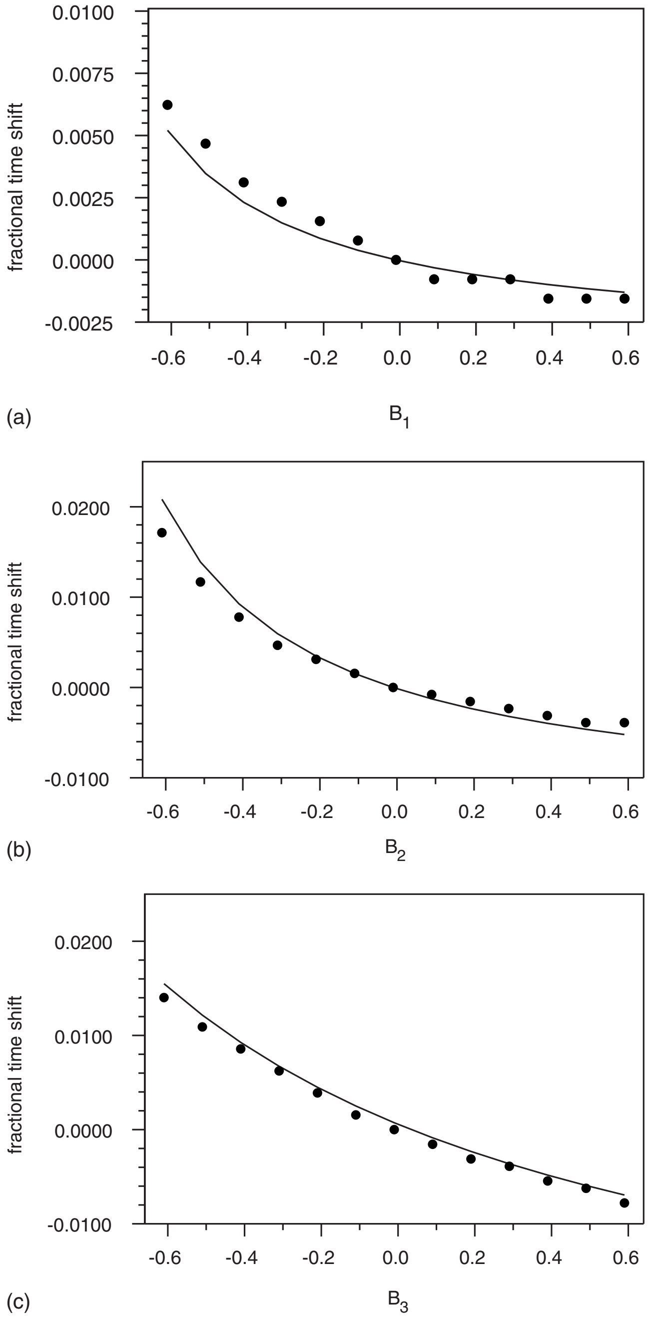


FIG. 9. (a) Comparison of predicted (solid line) and measured arrival time difference (as a fraction of the nominal arrival time 0.13 s) between cross correlations obtained by using $B=\text{const}$ and $B=1+B_1\cos\theta$ (symbols). Raw data taken from the Oman data set. (b) Comparison of predicted (solid line) and measured arrival time difference between cross correlations obtained by using $B=\text{const}$ and $B(\theta)=1+B_2\cos 2\theta$ (symbols). Raw data taken from the Oman data set. (c) Comparison of predicted (solid line) and measured arrival time difference between cross correlations obtained by using $B=\text{const}$ and $B(\theta)=1.7+0.4\cos\theta-0.2\cos 2\theta+B_3\cos 3\theta$ (symbols). Raw data taken from the Oman data set.

noise correlations and Green's functions. Here an asymptotically valid formula is derived that permits estimations of the consequent error in estimates of travel time based on such correlations. The formula is successfully compared with apparent travel times seen in synthetic waveforms. Although based on simple assumptions, the formula derived here was shown to be a good approximation when dealing with actual records of surface waves from an exploration experiment. The presence of different types of waves and of heterogeneities in that medium did not significantly degrade the accuracy of the theoretical prediction.

It is found that for sufficiently smooth distributions, and for sufficiently large receiver-receiver separations, the error in apparent arrival time is small, thus removing corresponding concern over possible inaccuracies in modern maps of seismic velocity based on arrival times seen in ambient seismic noise correlations.

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[1]O. I. Lobkis and R. L. Weaver, "On the emergence of the Greens function in the correlations of a diffuse field," J. Acoust. Soc. Am. **110**, 3011–3017 (2001).
[2]R. Snieder, "Extracting the Greens' function from the correlation of coda waves: A derivation based on stationary phase," Phys. Rev. E **69**, 046610 (2004).
[3]R. L. Weaver and O. I. Lobkis, "Diffuse waves in open systems and the emergence of the Greens' function," J. Acoust. Soc. Am. **116**, 2731–2734 (2004).
[4]P. Roux, K. G. Sabra, W. A. Kuperman, and A. Roux, "Ambient noise cross correlation in free space: Theoretical approach," J. Acoust. Soc. Am. **117**, 79–84 (2004).
[5]R. L. Weaver and O. I. Lobkis, "Ultrasonics without a source; Thermal fluctuation correlations at MHz frequencies," Phys. Rev. Lett. **87**, 134301 (2001).
[6]A. Derode, E. Larose, M. Campillo, and M. Fink, "How to estimate the Green's function of a heterogeneous medium between two passive sensors? Application to acoustic waves," Appl. Phys. Lett. **83**, 3054–3056 (2003).
[7]R. L. Weaver and O. I. Lobkis, "The mean and variance of diffuse field correlations in finite bodies," J. Acoust. Soc. Am. **118**, 3447–3456 (2005).
[8]E. Larose, O. I. Lobkis, and R. L. Weaver, "Coherent backscattering of ultrasound without a source," Europhys. Lett. **76**, 422–428 (2006).
[9]E. Larose, O. I. Lobkis, and R. L. Weaver, "Passive correlation imaging of a buried scatterer," J. Acoust. Soc. Am. **119**, 3549–3552 (2006).
[10]A. E. Malcolm, J. A. Scales, and B. A. van Tiggelen, "Retrieving the Green function from Diffuse, equipartitioned waves," Phys. Rev. E **70**, 015601 (2004).
[11]N. M. Shapiro and M. Campillo, "Emergence of broadband Rayleigh waves from correlations of the ambient seismic noise," Geophys. Res. Lett. **31**, L07614 (2004).
[12]S. E. Fried, W. A. Kuperman, K. G. Sabra, and P. Roux, "Extracting the local Green's function on a horizontal array from ambient ocean noise," J. Acoust. Soc. Am. **124**, EL183–EL188 (2008).
[13]P. Roux and W. A. Kuperman, "Extracting coherent wave fronts from acoustic ambient noise in the ocean," J. Acoust. Soc. Am. **116**, 1995–2003 (2004).
[14]L. Stehly, M. Campillo, B. Froment, and R. L. Weaver, "Reconstructing Green's function by correlation of the coda of the correlation ($C^3$) of ambient seismic noise," J. Geophys. Res. **113**, B11306 (2008).
[15]K. Wapenaar, D. Draganov, and J. Robertsson, "Seismic interferometry supplement," Geophysics **71**, SI1–SI102 (2006).
[16]P. Gouedard, L. Stehly, F. Brenguier, M. Campillo, Y. Colin de Verdiere, E. Larose, L. Margerin, P. Roux, F. J. Sanchez-Sesma, N. M. Shapiro, and R. L. Weaver, "Cross-correlation of random fields: mathematical approach and applications," Geophys. Prospect. **56**, 375–393 (2008).
[17]K. Wapenaar, "Nonreciprocal Green's function retrieval by cross correlation," J. Acoust. Soc. Am. **120**, EL7–EL13 (2006).
[18]O. A. Godin "Recovering the acoustic Green's function from ambient

noise cross correlation in an inhomogeneous moving medium," Phys. Rev. Lett. **97**, 054301 (2006).
[19]R. Snieder, "Extracting the Greens function of attenuating acoustic media from uncorrelated waves," J. Acoust. Soc. Am. **121**, 2637–2643 (2007).
[20]R. Snieder, "Retrieving the Greens function of the diffusion equation from the response to a random forcing," Phys. Rev. E **74**, 046620 (2006).
[21]R. L. Weaver "Ward identities and the retrieval of Greens functions in the correlations of a diffuse field," Wave Motion **45**, 596–604 (2008).
[22]K. G. Sabra, E. S. Winkel, D. A. Bourgoyne, B. R. Elbing, S. L. Ceccio, M. Perlin, and D. R. Dowling, "Using cross correlations of turbulent flow-induced ambient vibrations to estimate the structural impulse response. Application to structural health monitoring," J. Acoust. Soc. Am. **121**, 1987–1995 (2007).
[23]M. Rollwage, K. Ebeling, and D. Guicking, "Experimental investigation of two dimensional diffuse sound fields in a shallow water basin," Acustica **58**, 149–161 (1985).
[24]K. Aki, "Space and time spectra of stationary waves, with special reference to microtremours," Bull. Earthquake Res. Inst., Univ. Tokyo **35**, 415–456 (1957).
[25]H. Okada, "The microtremor survey method," Geophysical Monograph Series No. 12, Society of Exploration Geophysicists (2003).
[26]R. H. Lyon, "Statistical analysis of power injection and response in structures and rooms," J. Acoust. Soc. Am. **45**, 545–565 (1969).
[27]R. H. Lyon and R. G. DeJong, *Theory and Application of Statistical Energy Analysis*, 2nd ed. (Butterworth-Heinemann, Boston, 1995).
[28]N. M. Shapiro, M. Campillo, L. Stehly, and M. H. Ritzwoller, "High resolution surface wave tomography from ambient seismic noise," Science **307**, 1615–1618 (2005).
[29]K. G. Sabra, P. Gerstoft, P. Roux, W. A. Kuperman, and M. C. Fehler, "Surface wave tomography from microseisms in southern California," Geophys. Res. Lett. **32**, L14311 (2005).
[30]M. Campillo and A. Paul, "Long-range correlations in the diffuse seismic coda," Science **299**, 547–549 (2003).
[31]R. L. Weaver and O. I. Lobkis, "Elastic wave thermal fluctuations, ultrasonic waveforms by correlation of thermal phonons," J. Acoust. Soc. Am. **113**, 2611–2621 (2003).
[32]O. Godin, "Retrieval of Green's functions of elastic waves from thermal fluctuations of fluid-solid systems," J. Acoust. Soc. Am. **125**, 1960–1970 (2009).
[33]F. Brenguier, M. Campillo, C. Hadziioannou, N. M. Shapiro, R. M. Nadeau, and E. Larose, "Postseismic relaxation along the San Andreas fault at Parkfield from continuous seismological observations," Science **321**, 1478–1481 (2008).
[34]F. Brenguier, N. M. Shapiro, M. Campillo, V. Ferrazzini, Z. Duputel, O. Coutant, and A. Nercessian, "Towards forecasting volcanic eruptions using seismic noise," Nature Geoscience **1**, 126–130 (2008).
[35]F. Mulargia and S. Castellaro, "Passive imaging in nondiffuse acoustic wavefields," Phys. Rev. Lett. **100**, 218501 (2008).
[36]P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill, New York, 1953), Sec. 4.8.
[37]*Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables*, edited by M. Abramowitz and I. A. Stegun (Dover, New York, 1972).
[38]H. F. Nicholson, G. Lowet, C. M. Langton, J. Dequeker, and G. Van der Perre, "A comparison of time-domain and frequency-domain approaches to ultrasonic velocity measurement in trabecular bone," Phys. Med. Biol. **41**, 2421–2435 (1996).
[39]P. Gouédard, P. Roux, M. Campillo, and A. Verdel, "Convergence of the two-point correlation function toward the Green's function in the context of a seismic prospecting dataset," Geophysics **73**, V47–V53 (2008).
[40]G. C. Herman and C. Perkins, "Predictive removal of scattered noise," Geophysics **71**, V41–V49 (2006).