Energy partitions among elastic waves for dynamic surface loads in a semi-infinite solid

By

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Abstract

We examine the energy partitions among elastic waves due to dynamic normal and tangential surface loads in a semi-infinite elastic solid. While the results for a dynamic normal load on the surface of a half-space with Poisson ratio of 1/4 is a well known result by Miller and Pursey (1955), the corresponding results for a dynamic tangential load are almost unknown. The partitions for the normal and tangential loads were computed independently by Weaver (1985) against Poisson ratio ($0 \le \nu \le 1/2$) using diffuse field concepts within the context of ultrasonic measurements. The connection with the surface load point was not explicit, which partially explains why these results did not reach the seismological and engineering literature. The characteristics of the elastic radiation of these two cases are quite different. For a normal load about 2/3 of energy leave the loaded point as Rayleigh surface waves. On the other hand, the tangential load induces a similar amount in the form of body shear waves. It is established that the energies injected into the elastic half-space by concentrated normal and tangential harmonic surface loads are proportional to the imaginary part of the corresponding components of the Green's tensor when both source and receiver coincide. The relationship between the Green's function and average correlations of motions within a diffuse field is clearly established.

Key Words: Energy partitions, Surface loads, Elastic waves, Diffuse fields

Introduction

In their pioneering work, Miller and Pursey (1955) computed the power radiated in the form of dilatational, shear, and surface waves after a harmonic normal load is applied on the free surface of a semi-infinite isotropic solid. Their results where verified with the admittance method and are often cited in the literature ranging over a variety of applications (see *e.g.* Richart *et al.*, 1970).

The partitions for the normal and tangential loads were computed independently by Weaver (1985) against Poisson ratio ($0 \le \nu \le 1/2$) using diffuse field concepts. He studied the participation of the free surface in the general disturbance of a diffusely vibrating elastic body and showed that in the vicinity of a free surface a diffuse acoustic field may be viewed as the superposition of incoherent, homogeneous plane waves, incident upon the surface and including their outgoing reflected consequences. He used the connection between diffuse fields and Green's function, but not all the implications were established then. In fact, the connection with the surface point load was not explicit and this partially explains why these results did not reach the seismological and engineering literature. Here we emphasize the deterministic nature of his results and point out the connection to diffuse field concepts. Of course, the citations to Weaver (1985) work in seismology are relatively recent and are related with diffuse fields. One last point, Weaver (1985) also verified his calculations using an admittance method similar to the one by Miller and Pursey (1955).

The partition of energy within a diffuse field was introduced in dynamic elasticity by Weaver (1982) using statistical ideas from room acoustics. Based on this approach, Weaver (1985) computed the partitions for the half-space at the free surface. This is worth emphasizing - because this in turn implies: (a) that we can measure them in the field (i.e. *H/V* spectral ratios related to energy partitions) if the seismic noise is diffuse, and (b) that there is another way to calculate it - like Weaver did in 1985.

In this communication we examine these results under a new perspective. We point out how average measurements of ambient vibrations may reveal intrinsic properties of systems. Calculations based on the diffuse fields concepts permit to obtain deterministically the energy partitions of the energy injected on the elastic half-space by surface loads. The canonical results discussed herein can be of interest in several fields.

The Concentrated Surface Loads

Miller and Pursey (1955) calculated the partition from a normal point load in two ways: (1) they evaluated the energy radiated into all types of waves at infinity, and (2) evaluating the real part of admittance (*i.e*. the imaginary part of Green functions at the source, in modern terms). That these should be equal is just a simple matter of conservation of energy. Specifically, they found that the total power radiated by a vertical harmonic load $Pe^{i\omega t}$ is given by $\Pi=1.209\omega^2 P^2/\pi\rho\alpha^3$, where ω = circular frequency, ρ = mass density, and α = propagation velocity of P waves. Such an amount is partitioned among P, SV and Rayleigh waves in 6.89, 25.76 and 67.35 per cent, respectively. They assumed a Poisson ratio ν =1/4 and verified their results using the admittance method. With today's perspective, it is clear they were looking at the imaginary part of Green function at the loaded point. In what follows the load P will be assumed unitary.

Miller and Pursey (1954), Cherry (1962) and Gupta (1965) studied the tangential load but the emphasis in their work was restricted to radiation patterns. They pointed out that such a source produce an azimuthally variation of both body (P, SV, and SH) and surface waves. Cherry (1962) found that the generated Rayleigh waves had amplitudes smaller than those for normal load. However, no attempt to compute energies was made.

The Relationships of the Green Function and Field Correlations

It has been demonstrated that the Green's function can be retrieved from averaging cross correlations of the recorded motions in a diffuse field. If a 3D diffuse, equipartitioned, harmonic displacement vector field $u_i(\mathbf{x},\omega)$ is established within an elastic medium, the average cross-correlations of motions at points $\mathbf{x_A}$ and $\mathbf{x_B}$ can be written as (*e.g.* Sánchez-Sesma *et al.*, 2008):

$$\left\langle u_i(\mathbf{x_A},\omega)u_j^*(\mathbf{x_B},\omega)\right\rangle = -2\pi E_S^{FS} k^{-3}\,\mathrm{Im}\,G_{ij}(\mathbf{x_A},\mathbf{x_B},\omega) \tag{1}$$

In this equation, the Green´s function $G_{ij}(\mathbf{x_A},\mathbf{x_B},\omega) =$ displacement at $\mathbf{x_A}$ in direction i produced by a unit harmonic point force, $\delta_{ij}\delta(|\mathbf{x}-\mathbf{x_B}|)\exp(\mathrm{i}\omega t)$, acting at $\mathbf{x_B}$ in direction j, $\mathrm{i}=\sqrt{-1}=$ imaginary unit, t = time, $k=\omega/\beta=$ shear wave number, $\beta=$ shear wave propagation velocity, and E_S^{FS} = average energy density of shear waves in the full-space (*FS*) which is a measure of the available diffuse illumination. The asterisk * implies the complex conjugate and the angular brackets mean average.

If we assume $\mathbf{x_A} = \mathbf{x_B}$ we can rewrite Eq. 1 in terms of the total energy density at point $\mathbf{x_A}$ by means of

$$E(\mathbf{x_A}) = \rho\omega^2\left\langle u_m(\mathbf{x_A})u_m^*(\mathbf{x_A})\right\rangle = -2\pi\mu E_S^{FS} k^{-1}\times \mathrm{Im}\,G_{mm}(\mathbf{x_A},\mathbf{x_A})\,, \tag{2}$$

where $\mu=$ shear modulus. The total energy density is the sum of the energy due to shear, dilatational, and surface waves (if any), $E(\mathbf{x}) = E_S(\mathbf{x}) + E_P(\mathbf{x}) + E_{SW}(\mathbf{x})$. Eq. 2 shows the total energy density at a point as proportional to the imaginary part of the trace of the Green tensor for coincident receiver and source. The singularities of Green´s functions trace components are restricted to the real part. The imaginary parts are finite and regular; each one represents the power injected by the unit harmonic load at that point. These quantities also reveal energies that are both radiated and coming back to the source as they modify the work done by the load. This property may be useful to characterize the system. If the summation convention is ignored, the energy density associated to a given direction is simply $E_m(\mathbf{x_A})$, see Perton *et al*. (2009). For the homogeneous elastic space we have $E_m(\mathbf{x_A}) \equiv E(\mathbf{x_A})/3$. The relationships among energy densities and its partitions have been recently studied by Perton *et al*. (2009), by Margerin (2009) and by Margerin *et al.* (2009). The connection of the imaginary part of the Green´s function at the source with the optical theorem has been explored by Snieder *et al.* (2009).

Power Radiated by a Harmonic Point Force and Partitions

From Eq. 2 we can write $\mathrm{Im}[G_{jj}^{FS}(\mathbf{x},\mathbf{x};\omega)] = -kE(\mathbf{x})/6\pi\mu E_S^{FS}$ which is the imaginary part of the elastodynamic Green function for full-space when source and receiver coincide. In this case $E(\mathbf{x}) = E^{FS} = E_P^{FS} + E_S^{FS}$, which means that energies are independent of position. No summation is implied in this result. Therefore, the rate at which work is done by a unit harmonic load at any point $\mathbf{x}$, within a full-space, in direction j is the time average of the force times the velocity (Weaver, 1985):

$$\Pi_j^{FS}(\omega) = \frac{-\omega}{2} \times \mathrm{Im}[G_{jj}^{FS}(\mathbf{x},\mathbf{x};\omega)] = \frac{\omega^2}{24\pi\rho\alpha^3}\left(1 + \frac{2\alpha^3}{\beta^3}\right), \text{ no sum.} \tag{3}$$

The one in the parenthesis represents the energy associated to P waves, the term $2\alpha^3/\beta^3$ corresponds to the energy associated to S waves. In fact, $E_S^{FS} / E_P^{FS} = 2\alpha^3 / \beta^3 = R$ which is the well known equipartition ratio in 3D (Weaver, 1982).

From the results by Sánchez-Sesma and Campillo (2006) for the full-space one can establish the partitions in terms of directions and wave type in terms of the power of a single force in full-space:

$$\Pi_{j,W}^{FS}(0,\omega) = \Pi_j^{FS}(\omega) \times p_{j,W}^{FS}, \text{ no sum.} \tag{4}$$

where $p_{j,W}^{FS}$ = partition coefficient in the full-space for direction j and wave type W (either P, SV, or SH). Table 1 displays these partitions. The sum of partitions for each direction is one, as expected. To illustrate the meaning of these values, consider a unit horizontal load (j = 1 or 2). The power associated to SH waves is $\Pi_{1,SH}^{FS}(\omega) = (\omega^2/24\pi\rho\alpha^3)(1+R)\,p_{1,SH}^{FS}$ = $(\omega^2/24\pi\rho\alpha^3) \times (3R/4) = \omega^2/16\pi\rho\beta^3$. For a Poisson solid ($\alpha/\beta = \sqrt{3}$), the partition factor $p_{1,SH}^{FS}$ = $7.695 \mathrm{x}(1+2\mathrm{x}5.196)^{-1}$ implies that more than 68% of the injected energy by a horizontal load in a full space corresponds to SH waves. On the other hand, the energy associated to SV waves is more than 22%.

Table 1. Values of normalized energy partitions for the full-space: [$(1+R)\,p_{j,W}^{FS}$]

Wave type *W*	*j* = 1	*j* = 2	*j* = 3
P	1	1	1
SV	*R*/4	*R*/4	*R*
SH	3*R*/4	3*R*/4	0

Dealing with diffuse waves at the free surface, Weaver (1985) examined the participation of the surface in the general disturbance of a diffusely vibrating elastic body and demonstrated that in the vicinity of a free surface a diffuse acoustic field may be regarded as a sum of incoherent isotropic homogeneous independent plane waves incident upon the surface together with their respective outgoing reflected consequences. Using this idea, Weaver (1985) computed the energy densities for motions at the free surface and obtained partition factors in terms of directions and wave types for various Poisson ratios ($0 \le \nu \le 1/2$). For horizontal and vertical surface loads we can express these results as:

$$\Pi_j^{HSS}(0,\omega) = \Pi_j^{FS}(\omega) \times p_{j,W}^{HSS} \text{, no sum.} \quad (5)$$

where $p_{j,W}^{HSS}$ = partition coefficient at the half-space surface (*HSS*) for direction *j* and wave type *W* (either P, SV, SH, or Rayleigh). From the curves of Fig. 1b, for $\nu = 1/4$, a Poisson solid, the power associated to the vertical load is then

$$\Pi_z^{HSS}(0,\omega) = \Pi_1^{FS}(\omega) \times (p_{z,R}^{HSS} + p_{z,SV}^{HSS} + p_{z,P}^{HSS}) = \frac{\omega^2(1+R)}{24\pi\rho\alpha^3} \times (1.71 + 0.66 + 0.20) = 1.209\frac{\omega^2}{\pi\rho\alpha^3}, \quad (6)$$

which is the value computed by Miller and Pursey (1955).

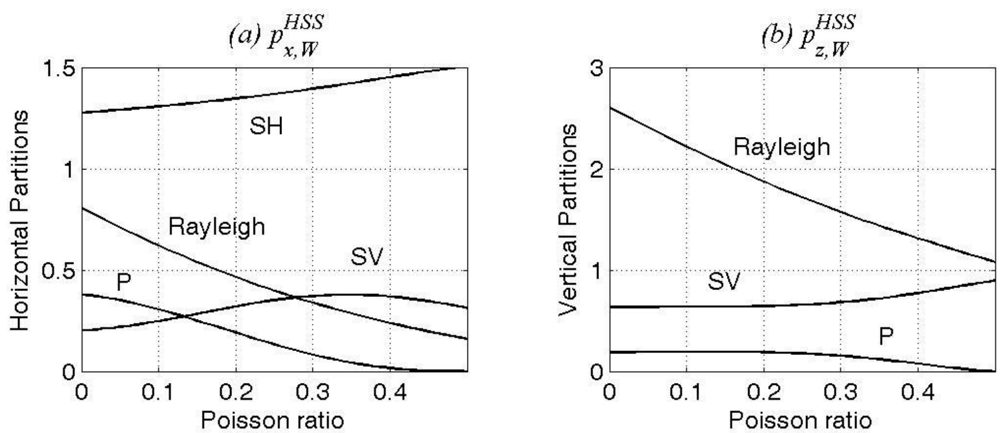

Figure 1. Partition coefficients for direction *j* (*x*, or *z*) and wave type (P, SV, SH, or Rayleigh) *vs* Poisson ratio. Horizontal and vertical partitions are given in a) and b), respectively.

Partitions may be grouped to present energy in their various forms. For instance, summing over wave types for a given direction we have the energy density associated to such direction at the free surface. This is measured in terms of the available energy density away from the free surface. Alternatively, such partitions represent the radiated energy among the various types of waves for a dynamic load in the said direction. This is a deterministic fact and the focus of this communication.

Energy Partitions and the *H/V* Spectral Ratio

In Fig. 2a the total partitions for both vertical and horizontal loads are given as functions of Poisson ratio. This allows obtaining the theoretical value for the horizontal to vertical ratio on the surface of a half-space. Considering that horizontal components are equal we can obtain the *H*/*V* spectral ratio using Eq. 2 and invoking the partitions by means of

$$\frac{H}{V} = \sqrt{\frac{2\langle u(0,0;\omega)^2 \rangle}{\langle w(0,0;\omega)^2 \rangle}} = \sqrt{\frac{2\,\mathrm{Im}\,G_{11}^{HS}(0,0;\omega)}{\mathrm{Im}\,G_{33}^{HS}(0,0;\omega)}} = \sqrt{\frac{2\,p_x^{HSS}}{p_z^{HSS}}}\,, \tag{7}$$

which is frequency independent and is given in Fig. 2b *vs* Poisson ratio. A useful approximation for these results is $H/V \approx 1.245 + 0.348\nu$. The *H/V* ratio is then a characteristic property of the medium and it depends on Poisson ratio. If a diffuse field is established, such a simple measurement may allow rapid determination of this elastic property. For a layered system the theoretical half-space *H/V* ratio provides asymptotic values for high and low frequencies that will depend on the Poisson ratio of the uppermost layer and of the basement, respectively (see, Margerin *et al.*, 2009).

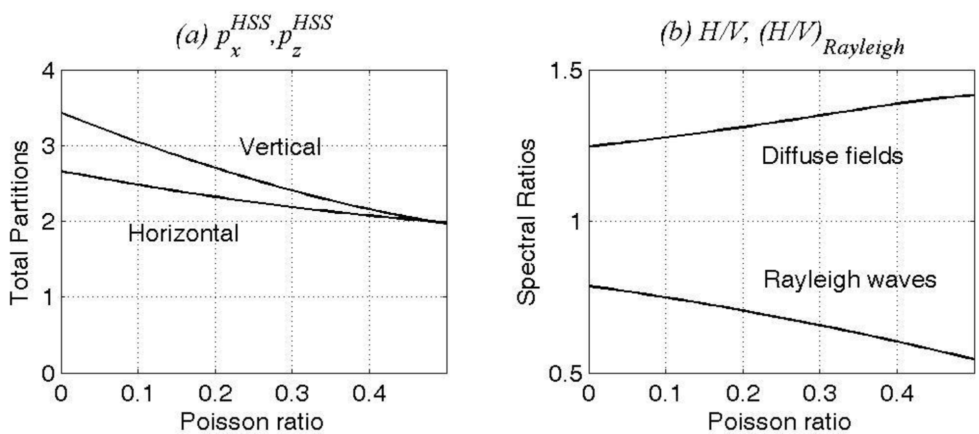

Figure 2. (a) Total partition coefficients for horizontal and vertical directions (*x*, or *z*) against Poisson ratio. (b) Theoretical *H/V* ratios for both diffuse fields and Rayleigh waves.

When dealing with microtremors it is frequent to compare *H/V* with the ellipticity of Rayleigh waves, $(H/V)_{Rayleigh}$, which has clear meaning if the direction of propagation is known. Within the diffuse field theory the *H/V* ratio includes, in addition to Rayleigh waves, P, SV and SH waves as well. In order to gauge the differences with the ellipticity of Rayleigh waves we depict in Figure 2b the *H/V* ratio for a diffuse field and the ellipticity for Rayleigh waves propagating along a given direction, say x, which can be obtained (*e.g.* Aki and Richards, 1980) from the expression:

$$(H/V)_{Rayleigh} = \frac{2\delta\sqrt{\delta^2 - 1}}{2\delta^2 - 1}, \tag{8}$$

where $\delta = \beta/c_R$ and c_R = velocity of Rayleigh waves. The differences are significant and show that for the half-space case the diffuse field *H/V* is far away of the ellipticity of Rayleigh waves. In layered systems the resemblance of measured *H/V* with $(H/V)_{Rayleigh}$ has been used for inversion.

The extension of these concepts to a multilayered layered medium implies numerical treatment. With this in mind, it is convenient to consider the formal solutions by Lamb (1904) and Chao (1960) for normal and tangential loads, respectively, in the frequency-wavenumber domain and check the total partitions. The corresponding analytical expressions are given in the Appendix.

From the total partitions (Figure 2a) it is possible to obtain the normalized percentages of total energy partitions for both horizontal and vertical load cases. Figure 3 depicts these percentages.

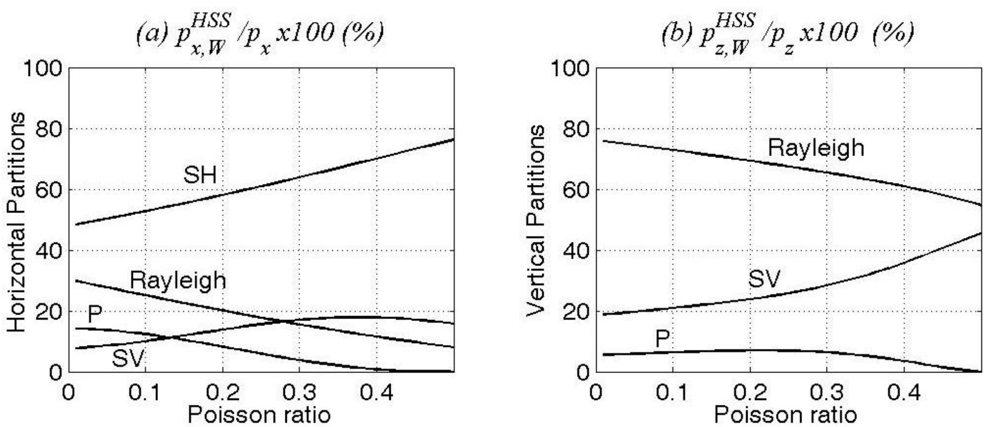

Figure 3. Percentages of partitions for (a) horizontal and (b) vertical load cases (*x*, or *z*) against Poisson ratio.

Discussion and Conclusions

The power emitted by a unit harmonic vertical force acting at the surface of a half-space can be computed using standard integral transform techniques. In fact, Weaver (1985) verified his results calculating the vertical response at the point of load. This latter method does not rely on the concept of diffusely and isotropically incident wave fields. It is a deterministic fact.

For the half-space cases there is some amplification due to the free boundary and the horizontal and vertical sum are larger than one. For a Poisson solid (ν =1/4) the total vertical and horizontal partitions give 1.71+0.66+0.20=2.57 and 1.37+0.41+0.36+0.14=2.28, respectively. Thus, the energy density at the free surface is (2x2.28+2.57)/(1+1+1)=2.37 times the full-space value and the ratio of horizontal to vertical energy densities is $E_H/E_V=H^2/V^2$=2x2.28/2.57≈1.774 and H/V=1.332. For a Poisson ratio ν=1/3 we have H/V=1.349. The relative energy densities at the free surface and other ratios are frequency independent. Their variations with normalized depth, z/Λ_R, where Λ_R = wavelength of Rayleigh waves, have been reported by Perton *et al.* (2009) for a Poissonian half-space.

From Figure 1b we can obtain the approximate theoretical values of partitions for an elastic half-space with Poisson ratio of 1/4 and a normal load on the surface: 67% for Rayleigh waves, 26% for SV waves and only 7% for P waves. We see that almost 2/3 of energy leaves the loaded region as Rayleigh waves and of the rest, only a relatively small amount of body waves of P type is collimated with the load. The energy pumped toward depth in SV waves has radiation lobes and lateral spread. For a normal load the most significant radiation is lateral propagations of Rayleigh waves.

The tangential load case is given in Figure 1a. If Poisson ratio is 1/4, the approximate figures for the energy share are 18% to Rayleigh waves, 60% and 16% to SH and SV waves, respectively, and only 6% for P waves. Here there is a more complicated spatial radiation pattern but the distinctive feature is that 2/3 of the energy leaves the loaded point as shear body waves. Particularly important is the radiation toward depth of horizontally polarized, SH, shear waves.

These two cases clearly exhibit the distinct character of the radiated energy implicit in $\mathrm{Im}G_{11}$ and $\mathrm{Im}G_{33}$ at the surface. This behavior allows us to draw a coherent picture useful to deal with layered media partitions. Both the normal and the tangential loads produce almost 2/3 of energy in the form of surface and body waves, respectively. While the former case gives essentially surface waves that in high frequency ”do not see” deep layers, the horizontal load produces body waves that may interact with the layering. These facts have some bearing on the interpretation of the diffuse field *H/V* spectral ratio. But this issue will be discussed elsewhere.

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Appendix. Analytical Formulae in Frequency-Wavenumber Domain

It is possible to write formally, from Lamb (1904) and Chao (1960) solutions, the expressions for vertical and horizontal surface displacement at the point of application of a harmonic vertical or horizontal unit load, respectively, by means of (see *e.g.* Kausel, 2006):

$$u_z(r,z;\omega) = \frac{-\mathrm{i}}{2\pi\mu}\int_0^\infty \frac{(\nu^2-k^2)\gamma\exp(-\mathrm{i}\gamma z)+2k^2\exp(-\mathrm{i}\nu z)}{F(k)}kJ_0(kr)dk \text{, and} \tag{A1}$$

$$u_r(r,z,\theta;\omega) = \frac{-\mathrm{i}}{2\pi\mu}\cos\theta\int_0^\infty\left(\frac{2\nu k^2\exp(-\mathrm{i}\gamma z)+(\nu^2-k^2)\nu\exp(-\mathrm{i}\nu z)}{F(k)}\left(J_0-\frac{J_1}{kr}\right)+\frac{\exp(-\mathrm{i}\nu z)}{\nu}\times\frac{J_1}{kr}\right)kdk\,. \tag{A2}$$

In these equations $\mu =$ shear modulus, k = radial wavenumber, and $\nu = \sqrt{\omega^2/\beta^2-k^2}$ = vertical wavenumbers for P and S waves, respectively, α,β = P, S wave speeds, $\mathrm{i}=\sqrt{-1}$ = imaginary unit, $F(k)=(k^2-\nu^2)^2+4\gamma\nu k^2$ = Rayleigh function, and $J_n(kr)\equiv J_n$ = Bessel function of the first kind and order n with argument kr, and finally $r, \theta,$ and z are the usual cylindrical coordinates.

These displacements correspond to elements of the Green tensor and are singular at the origin. The singularity is restricted to the real part. Thus we can write the expressions for the $\mathrm{Im}G_{33}(0,0;\omega)$ and $\mathrm{Im}G_{11}(0,0;\omega)$ at the source in terms of the given integrals in the radial wavenumber domain by taking the imaginary part in equations A1 and A2:

$$\mathrm{Im}\,G_{33}(0,0;\omega) = \frac{-1}{2\pi\mu}\mathrm{Re}\int_0^\infty \frac{\omega^2/\beta^2\gamma}{F(k)} k\,dk, \text{ and} \tag{A3}$$

$$\mathrm{Im}\,G_{11}(0,0;\omega) = \frac{-1}{4\pi\mu}\mathrm{Re}\int_0^\infty \left(\frac{\omega^2/\beta^2\nu}{F(k)} + \frac{1}{\nu}\right) k\,dk\,. \tag{A4}$$

Numerical integration allows verifying the linear variation with frequency of the Im*G*´s and the consistency of calculations. A small imaginary part is added to frequency to avoid the Rayleigh pole. The SH part in Chao solution can be computed analytically and shows the integration scheme is adequate. From numerical calculations using equations A3 and A4 the analytical results by Weaver (1985) of Figure 2a for the total contributions. In particular, the imaginary part of Green functions at the free surface is formally the product of the corresponding total partition at the half-space surface (*HSS*) and the imaginary part of the Green function for a single component at full space:

$$\mathrm{Im}\,G_{33}(0,0;\omega) = p_z^{HSS} \times \frac{-\omega}{12\pi\rho\alpha^3}\left(1 + \frac{2\alpha^3}{\beta^3}\right), \text{ and} \tag{A5}$$

$$\mathrm{Im}\,G_{11}(0,0;\omega) = p_x^{HSS} \times \frac{-\omega}{12\pi\rho\alpha^3}\left(1 + \frac{2\alpha^3}{\beta^3}\right). \tag{A6}$$

The minus sign comes from the chosen convention for Fourier transformation.