Mapping b-values in France using two different magnitude ranges: Possible non power-law behavior

Céline Beauval[1,2] and Oona Scotti[1]

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[1] The first step in probabilistic seismic hazard assessment is the characterization of seismic sources. The fundamental assumption is that the Gutenberg-Richter power law can be applied and is valid at all scales. In this study, we found that the power-law model may not be verified in the southeastern regions of France. Frequency-magnitude distributions are mapped using 2 different magnitude ranges: (1) [3.0–4.4] using only homogeneous instrumental data, (2) [3.5-M_{max}(observed)] using instrumental and historical data. b-values estimated on these two magnitude ranges are similar in the Pyrenees and the Rhine Basin. However, they differ significantly in the Southern Alps: the slopes estimated on magnitude range [3.0–4.4] are much steeper ($b > 1.4$) than the slopes estimated on magnitudes above 3.5 ($0.9 < b < 1.1$). Until a clear identification of the underlying processes is made, a conservative option (i.e., lowest b-values) should be considered for probabilistic estimation of hazard in the eastern part of France. *INDEX TERMS:* 7223 Seismology: Seismic hazard assessment and prediction; 7230 Seismology: Seismicity and seismotectonics; 8160 Tectonophysics: Rheology—general. **Citation:** Beauval, C., and O. Scotti, Mapping b-values in France using two different magnitude ranges: Possible non power-law behavior, *Geophys. Res. Lett.*, *30*(17), 1892, doi:10.1029/2003GL017576, 2003.

1. Introduction

[2] Probabilistic seismic hazard analyses (PSHA) based on the method of *Cornell* [1968] rely on three key elements: (1) identifying source zones, (2) estimating the occurrence of earthquakes in these zones, and (3) modeling the ground movement that these earthquakes will induce at a site. This paper does not consider seismic source zones nor attenuation models. Discussion is focused on the parameter estimation for the occurrence model of earthquakes as first proposed by *Gutenberg and Richter* [1944]: logN = a-bM, where earthquakes exponentially decay in number (N) as a function of their magnitude (M). Seismicity rate 10^a, and the slope of the G-R curve b, which is indicative of the relative distribution of small and large earthquakes, are the parameters that have to be calculated.

[3] Numerous methods have been proposed in the literature for the computation of the b-value (*Utsu* [1965]; *Aki* [1965]; *Page* [1968]; *Bender* [1983]). The method used in this study is Weichert's method [1980], which is a generalization of Aki and Utsu's maximum likelihood methods able to handle annual rates calculated for different periods of time (thus maximizing the number of events used).

[4] The stability analysis of G-R parameters estimations performed in numerous studies (see *Wiemer and Wyss* [1997], for a complete description) indicated that these parameters can vary both spatially and temporally at all scales. The physical interpretations that have been proposed in the literature for the spatial variability of the b-value range from the degree of material heterogeneities [*Mogi*, 1962; *Mori and Abercrombie*, 1997]), to stress variations in the crust [*Sholz*, 1968; *Wyss*, 1973], and differences in the mechanical behavior of the faults [*Amelung and King*, 1997]. Temporal variations of the b-value, on the other hand, have often been related to the preparation process for natural earthquakes and rock fracture in the laboratory [*Kanamori*, 1981; *Urbancic et al.*, 1992].

[5] The present study addresses the spatial variation of the b-value at regional scales in France and shows that the b-value can be locally extremely dependent on the magnitude range used to calculate it.

2. Data

[6] Several difficulties have to be dealt with before computing G-R parameters: the catalogues should contain homogeneous magnitude determinations, declustering should be performed, completeness periods should be determined and a geographic window has to be chosen.

[7] Magnitudes. The Laboratoire de Détection Géophysique (LDG), Bruyères le Châtel, France, installed a national seismic network in 1962. The LDG has been measuring local magnitudes homogeneously since then [*Nicolas et al.*, 1998]. The LDG instrumental catalog used in this study, covering the 1962–1999 period, shows (Figure 1) that France is a country of moderate seismicity (5 events with $M_L > 5.0$ every 10 years, Mmax = 5.8). Nevertheless, great earthquakes have occurred in the past and have been documented in the historical record. The SISFRANCE historical database covers the last thousand years. The historical catalog we use, an extraction from this base, contains 10 events with intensity MSK equal or greater than VII every 50y, for the last 5 centuries. Historical magnitudes were calculated using an intensity-magnitude correlation deduced from macroseismic observations for 73 events reported in both the historical database SisFrance and the instrumental catalogue LDG [*Levret et al.*, 1994] located all over France. The reference magnitude for this study is therefore the M_L of the LDG catalog. Due to the lack of data, it is difficult to compute regional correlations.

[8] Declustering. According to our declustering tests, the seismicity of the instrumental catalogue is only weakly clustered. Moreover, we found that results based on the

[1]Institut de Radioprotection et Sûreté Nucléaire, Fontenay-aux-Roses, France.
[2]LGIT, Grenoble, France.

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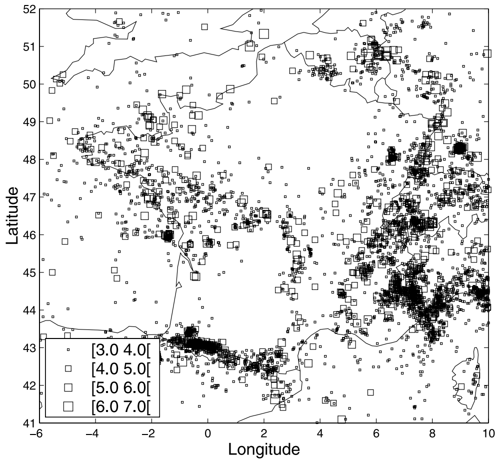

Figure 1. Instrumental and historical seismicity in France, covering the period [1500–1999].

declustered catalog were not significantly different from results based on the original catalog. The catalogues used in this study are therefore not declustered.

[9] Completeness. Studies were performed using all earthquakes in the window [−6; 10] in longitude and [41; 52] in latitude. Using cumulative number of earthquakes versus time plots and assuming stationary seismic rates, it can be shown that the period of completeness is 17 years (1983–1999) for magnitudes 3.0–3.1 and 38 years (1962–1999) for magnitudes equal or above 3.2. Completeness periods calculated for the combined catalog (historical appended to instrumental) are reported in the Table 1. Magnitudes are binned in intervals of 0.5 because 0.5 is the average uncertainty in historical magnitudes.

3. Mapping the b-Values

[10] A grid of 0.25° × 0.25° is defined over the study region and for each grid point, the b-value is computed using all earthquakes falling inside a circle with a fixed radius. The procedure is similar to that of *Wiemer and Wyss* [1997], however the size of the sampled area is larger. The radius of the circle is fixed at 100 km in order to explore the variability in the b-values for sizes of source zones similar to those used in PSHA studies in France. b-values are only shown at grid points where $\sigma(\beta) < 0.4$ (σ is the standard deviation of $\beta = b * \ln(10)$, b is the b-value). It is found that this criterion insures that a sufficient number of events are taken into account in the computation. The number of earthquakes used for the computations thus varies from grid point to grid point between 50 and 600. In order to check on the stability of the GR parameters, two magnitude intervals are tested: [3.0–4.4] is representative of the instrumental period and [3.5-M_{max}(observed)] is representative of the instrumental and historical period (hereafter called M3.5+).

[11] The first interval has a minimum threshold imposed by the completeness period, which varies spatially for magnitudes lower than 3.0. As will be discussed later, dealing with magnitudes greater than 3.0 also reduces potential problems related to artificial seismicity or clusters. The higher limit of this first interval is due to the fact that rates for magnitudes higher than 4.5 cannot be estimated within 100 km-radius circle: the 38-y period is no longer representative for such an area (the time period being too short).

[12] As for the second interval, the minimum threshold of 3.5 is chosen to allow the historical data to contribute to the b-value estimations, minimizing the contribution of the instrumental catalog. In Weichert's method [1980], the higher the number of earthquakes that contributes to the calculation of the annual rate, the higher their weight in the estimation of the slope. The minimum threshold cannot be increased to 4.0 because the number of data within 100 km-radius circle would be too small to estimate the b-values with confidence.

4. Results

[13] Two b-value maps are obtained, Figure 2a displays b-values estimated on the instrumental catalog (38y), and Figure 2b displays b-values estimated on the combined catalog (historical and instrumental, 500y). Given the severity of the filtering criterion ($\sigma(b) < 0.18$), only the higher seismic rate zones are highlighted: the Pyrenees to the South, the Rhine Basin and the Alps to the East and the southern part of Brittany to the West. Values calculated at each grid point are extended to a 0.25° × 0.25° area in order to create the map.

[14] Based on the instrumental data only (Figure 2a), b takes values over a wide range between 0.6 and 1.8. The thick black contour in the South-East of France corresponds to iso-values of b = 1.7. Previous studies worldwide have shown that the exponent b usually takes values between 0.75 and 1.25 [*Okal and Romanowicz*, 1994]. Such values are found in the Pyrenees and in the North-East of France (Rhine Basin). Values above 1.5 have previously been reported only in volcanic areas [*Wiemer et al.*, 1998]. Such high values are surprisingly found in the Southern Alps, at the boarder between France and Italy.

[15] b-values calculated on the combined catalog (historical and instrumental, $M \geq 3.5$) are mainly contained in the interval [0.6–1.25]. Results are consistent in the Pyrenees and in North-East of France between the two maps, whereas they differ significantly in the Alps. *Utsu* [1992] proposed a method to test the significance of b value differences, based on the number of events used. The results of this test are plotted on Figure 2c, to highlight zones where both slopes are statistically different with a minimum of 99% probability. Grid points where slopes are considered significantly different are colored in dark gray; points that did not pass the test are colored light gray. Points that passed the test correspond approximately to b-value differences greater than 0.4. Discrepancies are clearly localized over the Alps.

[16] In order to emphasize the extreme variability in b-value estimate that can result from the choice of the

Table 1. Completeness Periods[a]

Magnitude interval	Completeness period	Time interval (year)
[3.0–3.1]	1983–1999	17
[3.2–4.4]	1962–1999	38
[4.5–4.9]	1900–1999	100
[5.0–5.4]	1870–1999	130
[5.5–5.9]	1800–1999	200
[6.0–6.9]	1500–1999	500

[a]A completeness period is a time interval over which all events in the magnitude bin have been recorded.

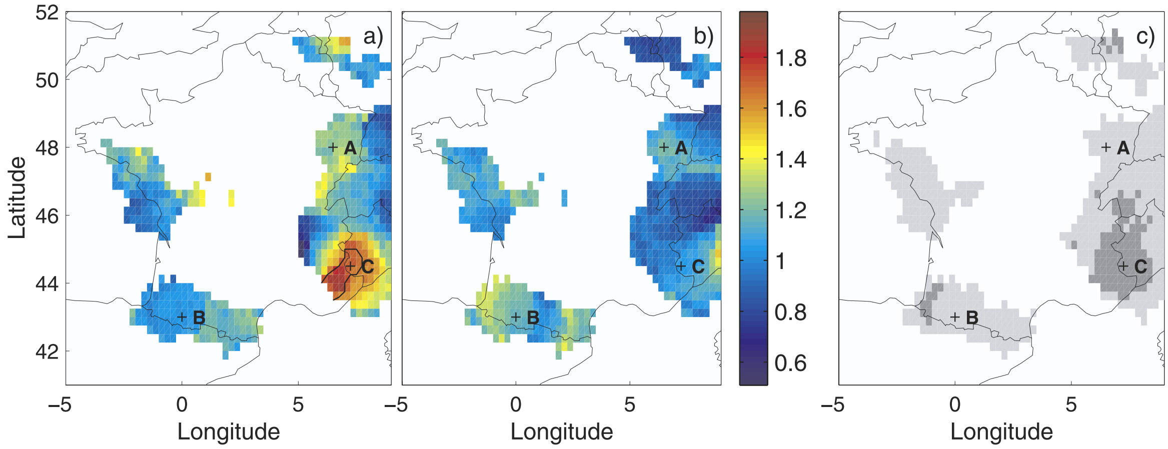

Figure 2. b-values map, estimated with Weichert's method. (a) On the magnitude interval [3.0–4.4], black thick line corresponds to the 1.7-isochrone, number of earthquakes used varies between 50 and 600. (b) On magnitude interval M3.5+, number of earthquakes varies between 25 and 250. Standard deviations of b are lower than 0.18. (c) Results of applying at each grid point the method of *Utsu* [1992] to test the significance of b value differences at the 99% confidence limit. The individual b-value sets are displayed in panels (a) and (b); dark gray cells passed the test, light gray ones failed.

magnitude range used, three examples of the calculated Gutenberg-Richter are shown for three different locations (Figure 3, Table 2). At location A (Rhine Basin) and B (central Pyrenees), b-values are similar: 1.26/1.20 and 1.03/1.17 (cf. Table 2). At point C however, b-values are extremely different: 1.71 versus 1.06. Notice that the computations on the instrumental catalog and on the combined catalog use the same events in the [3.5–4.4] magnitude range, since the completeness period for these magnitudes is of 38 years (the instrumental period). However the resulting estimated slopes can be different: the ones estimated on the instrumental catalog are largely controlled by the magnitude bins between 3.0 and 3.5, whereas those estimated on the combined catalog are largely controlled by the magnitude bins between 3.5 and 4.5.

5. Discussion and Conclusion

[17] The remarkable difference observed in the Alps in the b-value estimates between these two magnitude intervals raises two fundamental questions: are these slopes both correct or is there a bias that is introduced in one or both catalogues? If they are both correct, what is the underlying physical mechanisms that can account for such a non-linear behavior of the b-value as a function of the magnitude range?

[18] b-values estimated in the interval [3.0–4.4] could be biased for two reasons: the presence in the instrumental catalog of aftershocks and/or induced seismicity. The declustering of the LDG catalog with Reasenberg's algorithm [1985] gave very similar results. Since only $M \geq 3$ are used in the calculations, only a few clusters were detected by the algorithm and their presence has a minor impact on the estimation of seismicity rates over a 17- or a 38-year period. The second source of biased estimation may be the presence in the catalogue of artificial seismicity. A histogram of the number of events as a function of the hour of the day can help to identify induced seismicity. In the East of France, bins corresponding to working hours between 9 am and 4 pm indeed contain an abnormal number of earthquakes, but this artificial seismicity is important only in the magnitude interval [2.5–3.0], (which is not used in the calculations). A rough estimate shows that it represent less than 15% of the total seismicity for the range of magnitudes studied [3.0–3.2]. A reduction of 15% in the number of events in the magnitude interval [3.0–3.2] only slightly modifies the b-value estimations. The highest discrepancies located in the Southern Alps remain and another explanation must be sought.

[19] b-values estimated in the interval M3.5+ are within the usual expected range [0.75–1.25]. However they could

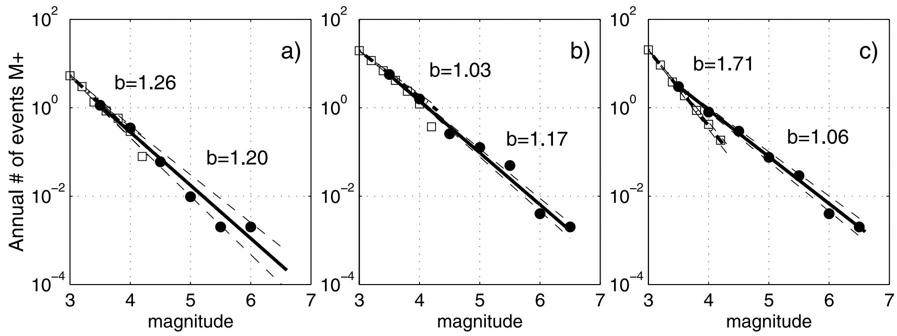

Figure 3. Cumulative Gutenberg-Richter plots for 2 different magnitude intervals, superimposed to cumulated observed annual rates, (a) at location A (6.5;48); (b) at location B (0;43); (c) at location C (7.25;44.5).

Table 2. b-Value Estimation at Three Different Locations; Subscript I Corresponds to Modeling on Instrumental Catalog and Interval [3.0–4.4]; Subscript C Corresponds to Modeling on Combined Catalog and Interval M3.5+

Location	b_I	σ_I	N_I	b_C	σ_C	N_C
A (6.5; 48)	1.26	0.12	153	1.20	0.12	48
B (0; 43)	1.03	0.06	576	1.17	0.05	239
C (7.25; 44.5)	1.71	0.08	540	1.06	0.06	138

N is number of events used.

also be biased by the choice of the magnitude-intensity correlation used to convert the historical intensities into magnitudes. We are working on a new correlation based on the 10 most reliable events for which we have both a good macroseismic description and an instrumental magnitude. This second correlation estimates higher magnitudes than the Levret correlation (0.4 magnitude degree on average). b-values estimated on this alternative historical catalog (appended to the instrumental one) are only slightly modified (the decrease in the b-values is lower than 0.2).

[20] Assuming that both slopes, estimated separately on intervals [3.0–4.4] and M3.5+, are correct, then a physical mechanism must be sought to explain the high discrepancies (up to 0.8 difference) observed in the Alps. Both intervals of magnitude might be linked to different physical processes. The magnitude interval M3.5+ could reflect regional tectonics whereas smaller magnitudes might be linked to a local phenomenon. The presence of the identified Ivrea Body in this zone (inclusion of mantle at crustal levels [*Paul et al.*, 2001] could be responsible for high b-values in the [3.0–4.4] interval. Accurate hypocenters localizations [*Paul et al.*, 2001] show that events are located at the interface between the Ivrea Body and the crust. We attribute the high instrumental b-values in the magnitude interval [3.0–4.4] to a creeping behavior at the interface between crust and mantle inclusion. The stretching of the high b-value zone to the southwest of point C is due to the smoothing and the lack of seismicity to the west. Indeed, for points along the borders of the seismic zones, earthquakes are mainly located in one side of the circles. The eastern limit of the high b-value region is not affected by this border effect, since there is seismicity to the east.

[21] This study shows that the modeling of earthquake recurrence curves by a unique exponential decrease is questionable in the eastern regions of France and likely incorrect over the Alps. Using a minimum threshold lower than 3.5 when using the combined catalog would have yielded average slopes that could not satisfactorily fit neither the small nor the moderate range of magnitudes. Estimating b-values on both magnitude intervals inside source zones for PSHA calculations leads to a difference of 50% in PGA (Peak Ground Acceleration) estimates in the Alps. Until a clear identification of the processes is made, a conservative option (estimation of b-values in the magnitude range M3.5+) should be considered for the probabilistic estimation of hazard in this region.

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C. Beauval and O. Scotti, Institut de Radioprotection et Sûreté Nucléaire, Fontenay-aux-Roses, France. (celine.beauval@irsn.fr)