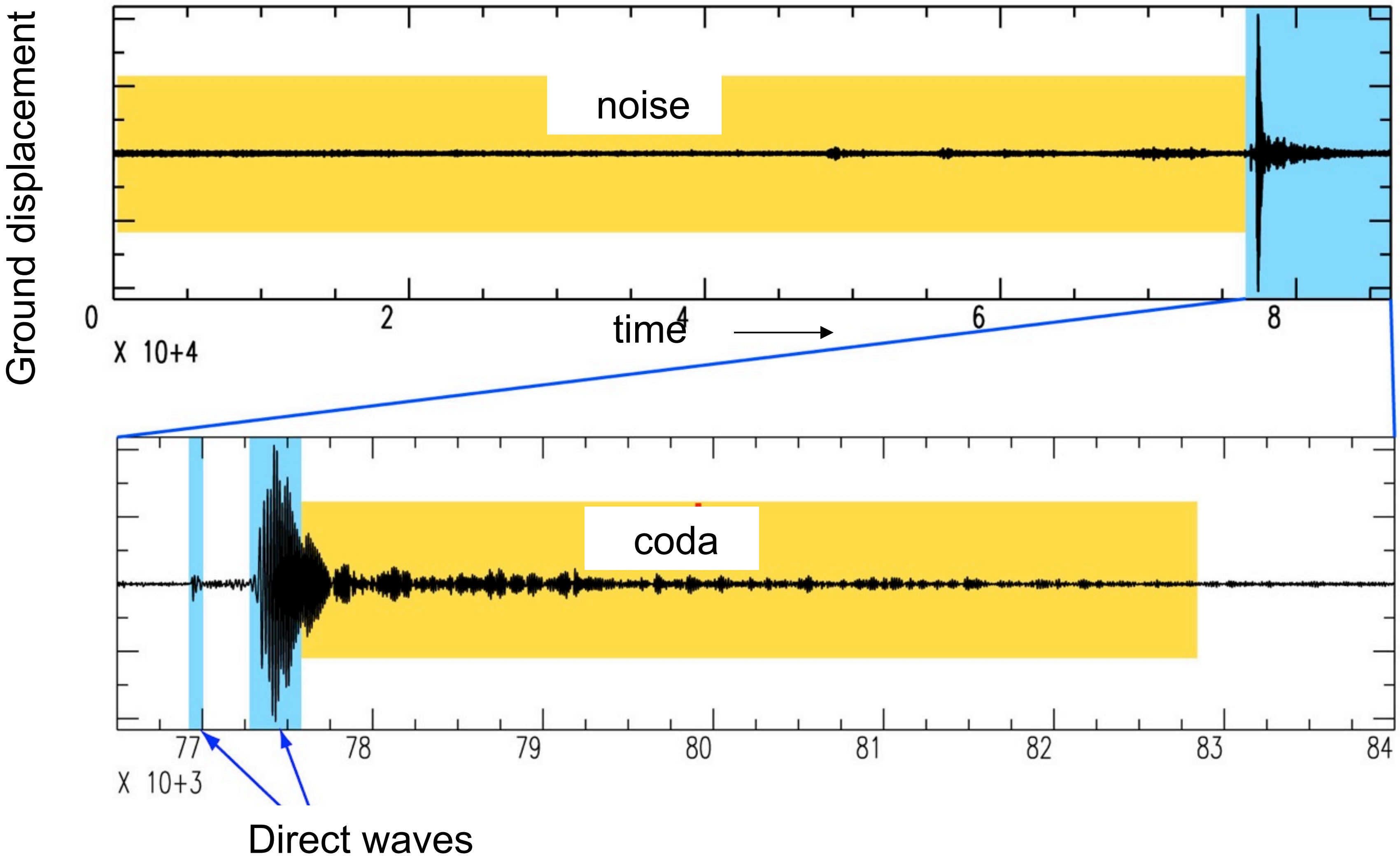
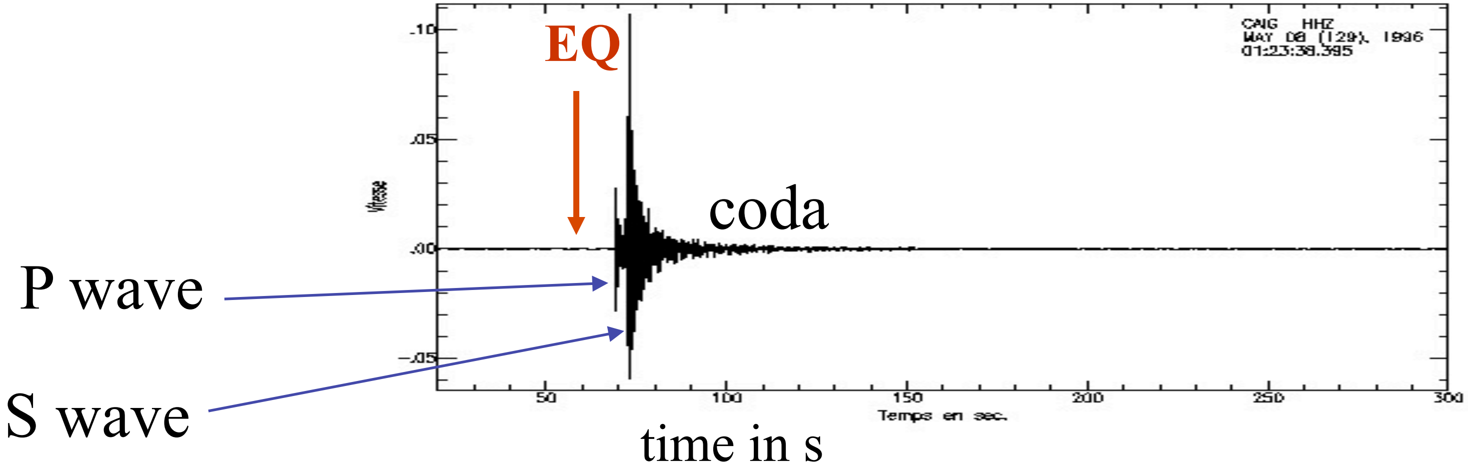
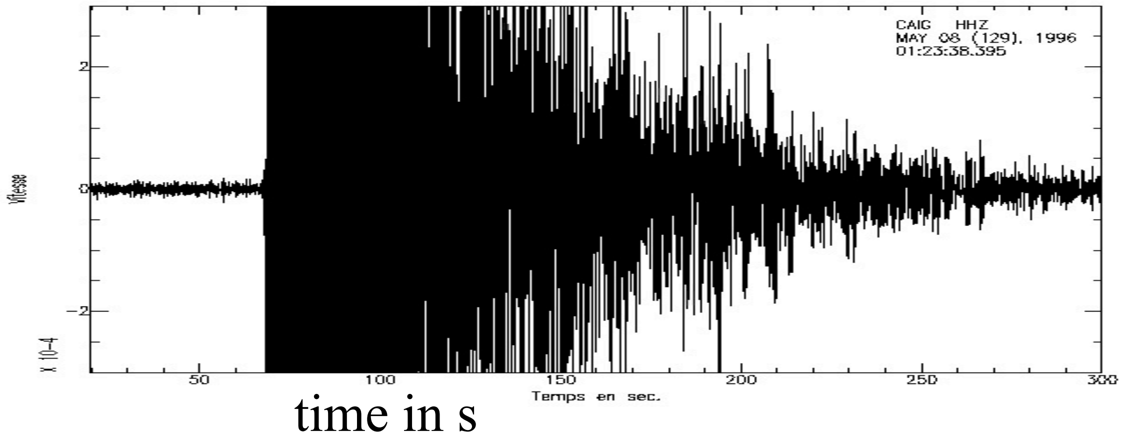


Ballistic waves used for imaging

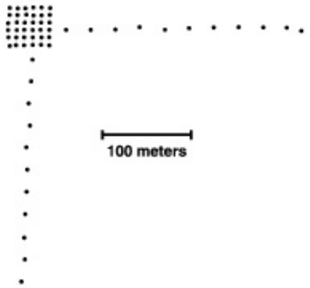
Example of a record of a local earthquake in the band .5-20Hz



scale x 1000:

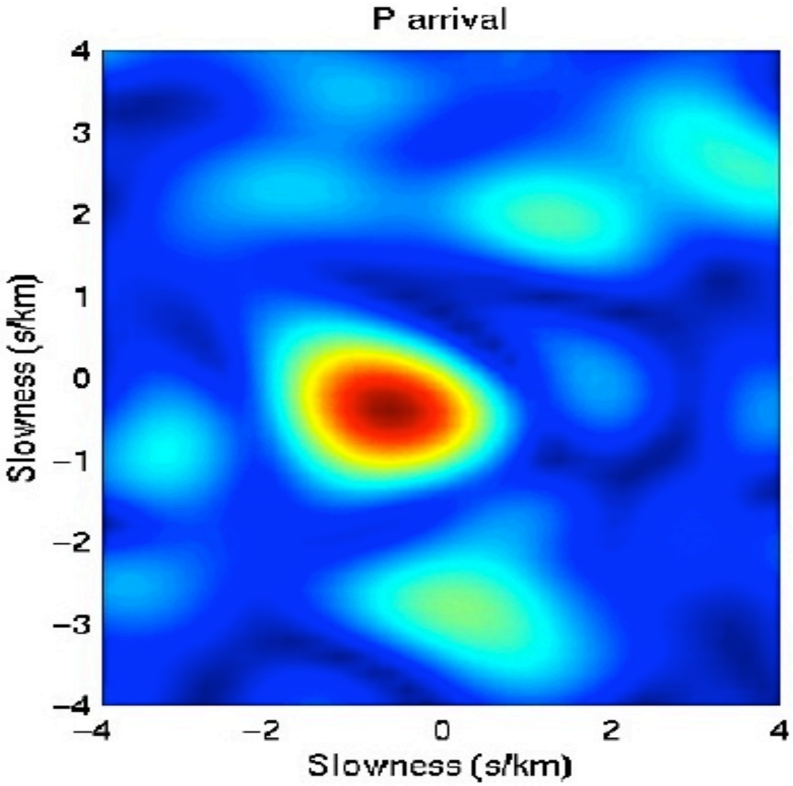


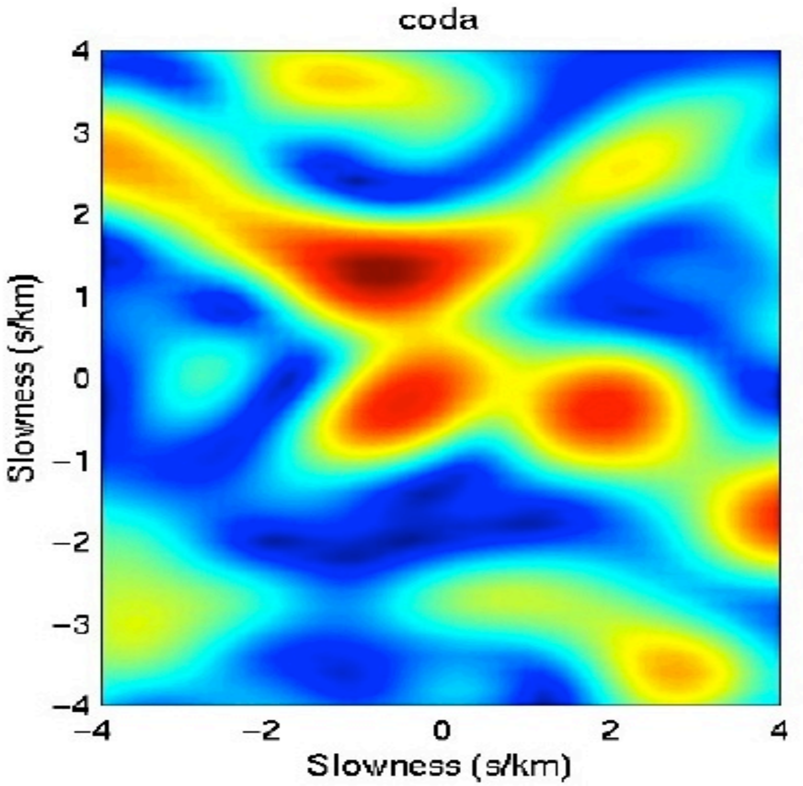
Frequency-wavenumber analysis



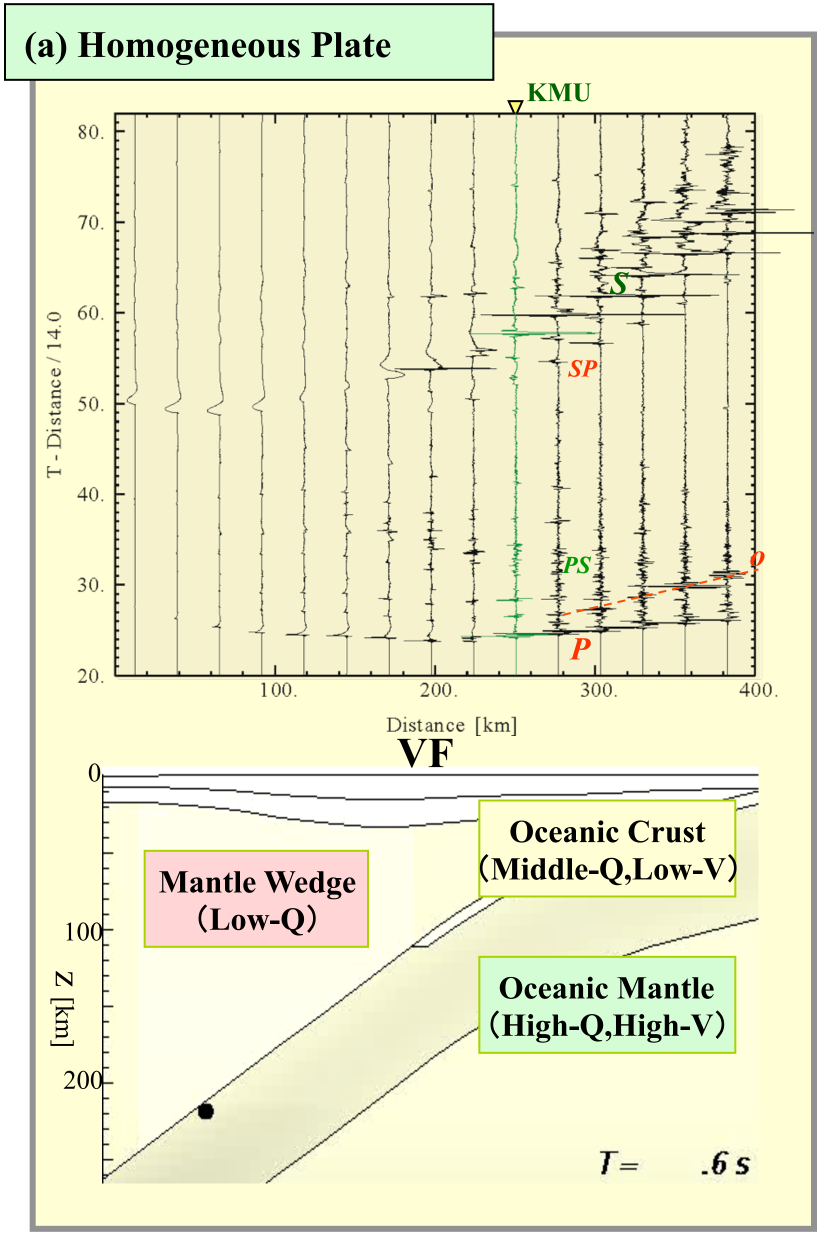
(Pinon Flat Seismometer Array)

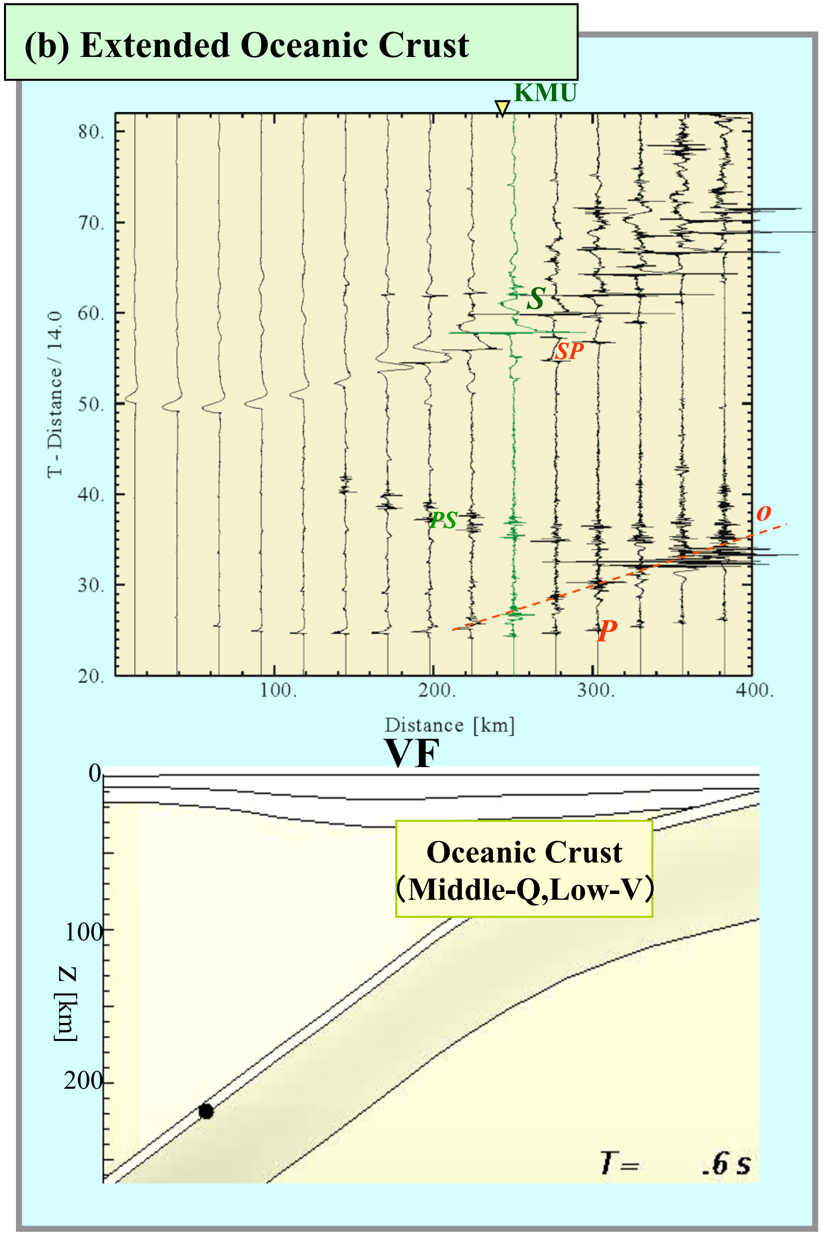
$$u(x,y) \rightarrow u(k_x,k_y)$$

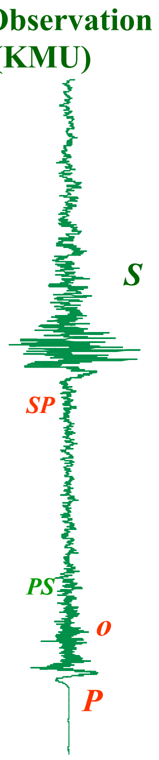




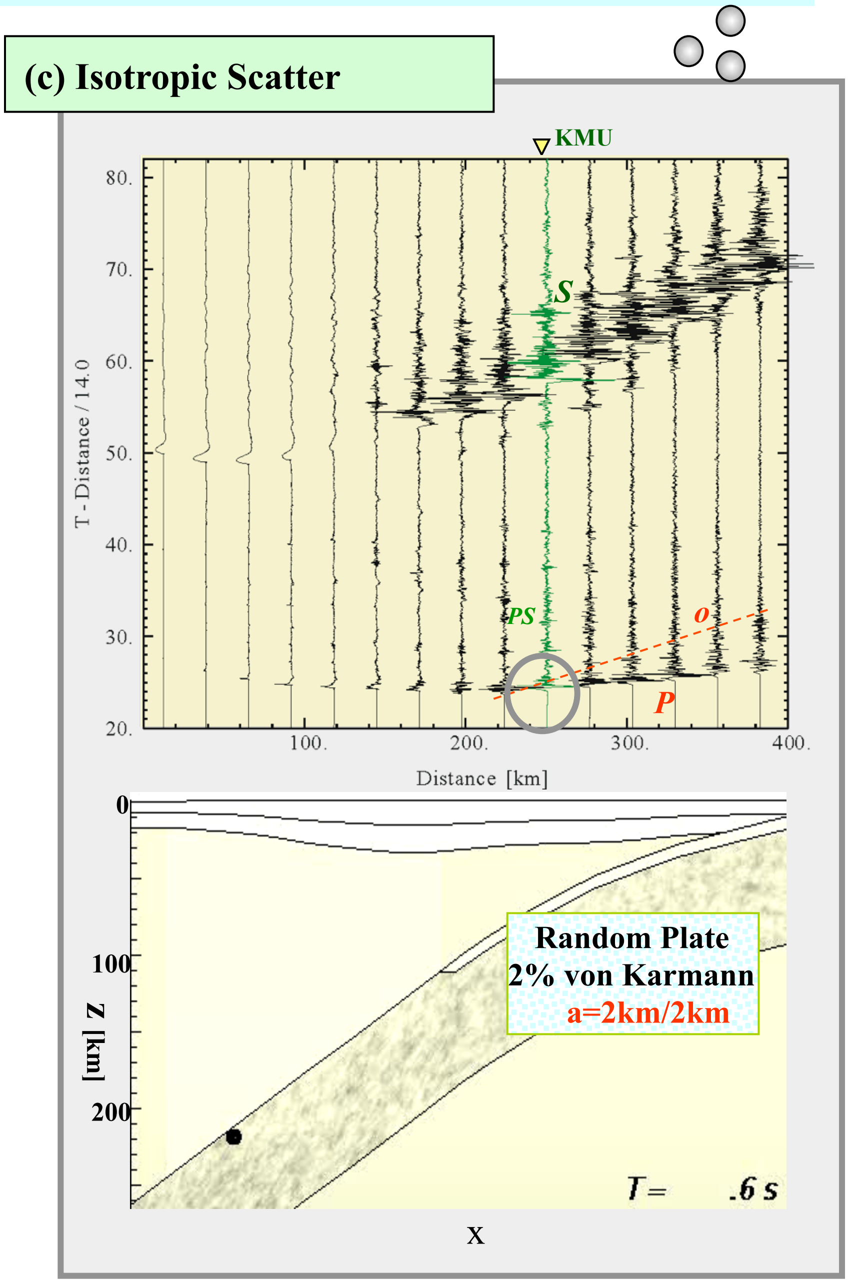
Computer Simulation（1）: Homogeneous Plate (*iasp81)*

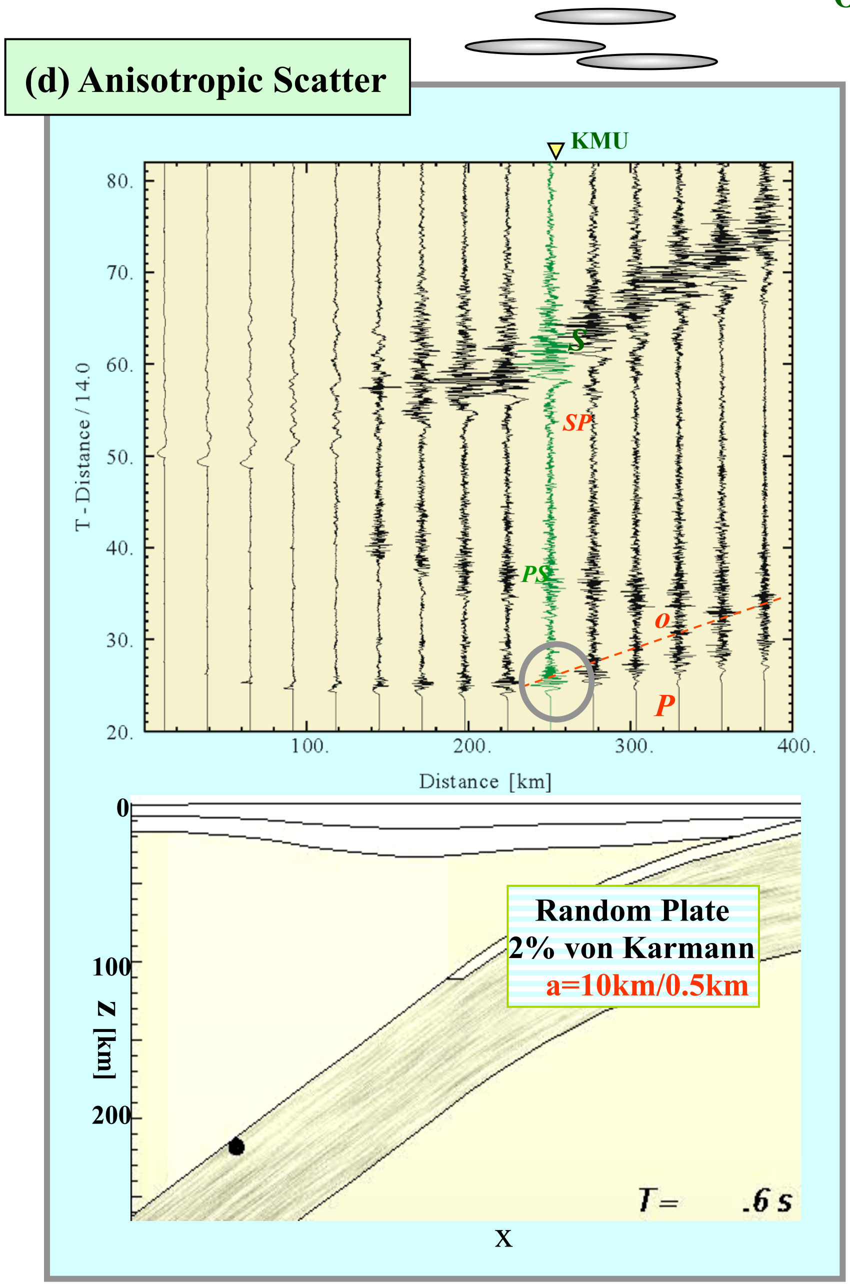


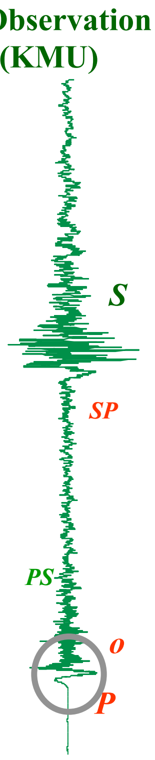




Heterogeneous Plate Model







Propagation in a weakly heterogeneous medium ($Ref.\ Aki\ et\ Richards$ 1980)

Reference medium $(\lambda_0, \mu_0, \rho_0)$
$\vec{u}$ displacement

$$\rho_0\ \ddot{u}_i = (\lambda_0 \vec{\nabla}.\vec{u})_{,i} + [\mu_0\ (u_{i,j} + u_{j,i})\ ]_{,j}$$

(same as: $\rho_0 \frac{\partial^2 \vec{u}}{\partial t^2} = (\lambda_0 + \mu_0)\ \vec{grad}\ (div\ \vec{u})\ + \mu_0 \vec{\Delta}.\vec{u}$)

Perturbed medium
$\rho = \rho_0 + \delta\rho\ (\vec{r});\ \lambda = \lambda_0 + \delta\lambda\ (\vec{r});\ \mu = \mu_0 + \delta\mu\ (\vec{r})$
$\delta\rho\ ,\ \delta\lambda\ ,\ \delta\mu\ \ << \rho\ ,\ \lambda\ ,\ \mu$

Equation of motion:

$$\rho\ \ddot{u}_i = (\lambda \vec{\nabla}.\vec{u})_{,i} + [\mu\ (u_{i,j} + u_{j,i})\ ]_{,j}$$

$$(\rho_0 + \delta\rho)\ddot{u}_i = ((\lambda_0 + \delta\lambda)\vec{\nabla}.\vec{u})_{,i} + [(\mu_0 + \delta\mu)(u_{i,j} + u_{j,i})]_{,j}$$

$$\Rightarrow\ \rho_0\ \ddot{u}_i - \lambda_0(\vec{\nabla}.\vec{u})_{,i} - \mu_0(u_{i,j} + u_{j,i})_{,j} =\ -\ \delta\rho\ \ddot{u}_i\ +\ \delta\lambda(\vec{\nabla}.\vec{u})_{,i}$$

$$+\ \delta\lambda_{,i}\ \vec{\nabla}.\vec{u}\ +\ \delta\mu(u_{i,j} + u_{j,i})_{,j}\ +\ (\delta\mu)_{,j}(u_{i,j} + u_{j,i})$$

with $(u_{i,j} + u_{j,i})]_{,j} = \nabla^2 u_i + (\vec{\nabla}.\vec{u})_i$

$$\Rightarrow\ \rho_0\ \ddot{u}_i - (\lambda_0\ +\ \mu_0)(\vec{\nabla}.\vec{u})_{,i}\ -\ \mu_0\,\nabla^2\,u_i\ =\ -\delta\rho\ \ddot{u}_i\ +\ (\delta\lambda + \delta\mu)(\vec{\nabla}.\vec{u})_{,i}$$

$$+\ \delta\mu\,\nabla^2\,u_i\ +\ (\delta\lambda)_i\vec{\nabla}.\vec{u}\ +\ (\delta\mu)_{,j}\ (u_{i,j} + u_{j,i})$$

$$\Rightarrow \; \rho_0 \, \ddot{u}_i - (\lambda_0 \; + \; \mu_0)(\vec{\nabla}.\vec{u})_{,i} \; - \; \mu_0 \, \nabla^2 \, u_i \; = \; -\delta\rho \, \ddot{u}_i \; + \; (\delta\lambda + \delta\mu)(\vec{\nabla}.\vec{u})_{,i} + \delta\mu \, \nabla^2 \, u_i \; + \; (\delta\lambda)_i \vec{\nabla}.\vec{u} \; + \; (\delta\mu)_{,j} \, (u_{i,j} + u_{j,i})$$

$$u \; = \; u^0 \; + \; u^d$$

$\rightarrow$ u^0 satisfies elastodynamic equation for $(\rho_0, \lambda_0, \mu_0)$

Hypothesis: weak perturbations

$\rightarrow$ neglect terms like $\delta\mu \times (u^d)'$ $(\delta\mu << \mu_0 \; ; \; u^d << u_0)$

(First order Born approximation)

$$\rho_0 \, \ddot{u}_i^d - (\lambda_0 \; + \; \mu_0)(\nabla u^d)_{,i} \; - \; \mu_0 \, \nabla^2 \, u_i^d \; = \; Q_i$$

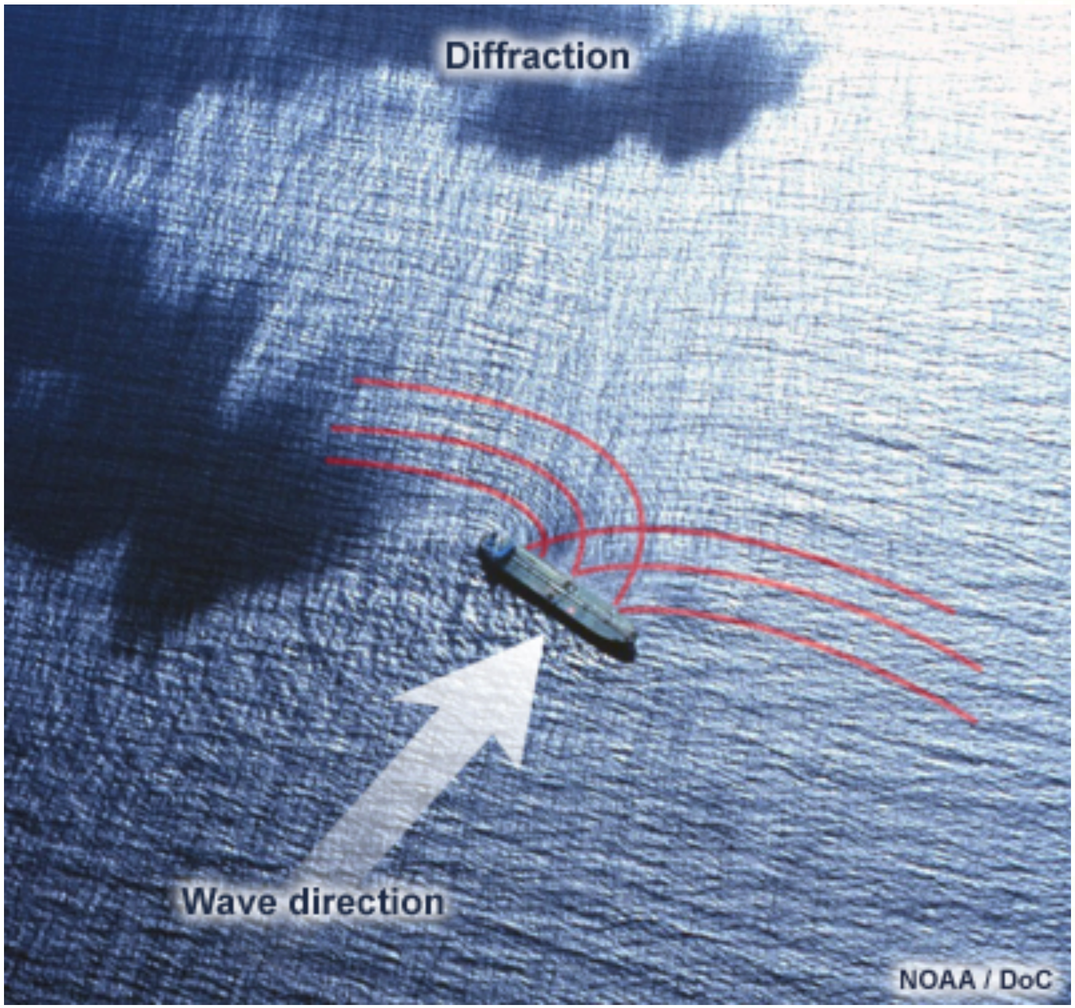
with :

$$Q_i = \; -\delta\rho \, \ddot{u}_i^o \; + \; (\delta\lambda + \delta\mu)(\vec{\nabla}.\vec{u}^o)_{,i} \; + \; \delta\mu \nabla^2 \, u_i^o \; + \; (\delta\lambda)_i \vec{\nabla}.\vec{u}^o \; + \; (\delta\mu)_{,j} \, (u_{i,j}^o + u_{j,i}^o)$$

$\Rightarrow$ extra source terms in the reference model

$\rightarrow$ diffraction = virtual sources

$\rightarrow$ base formula for linearized inverse problem



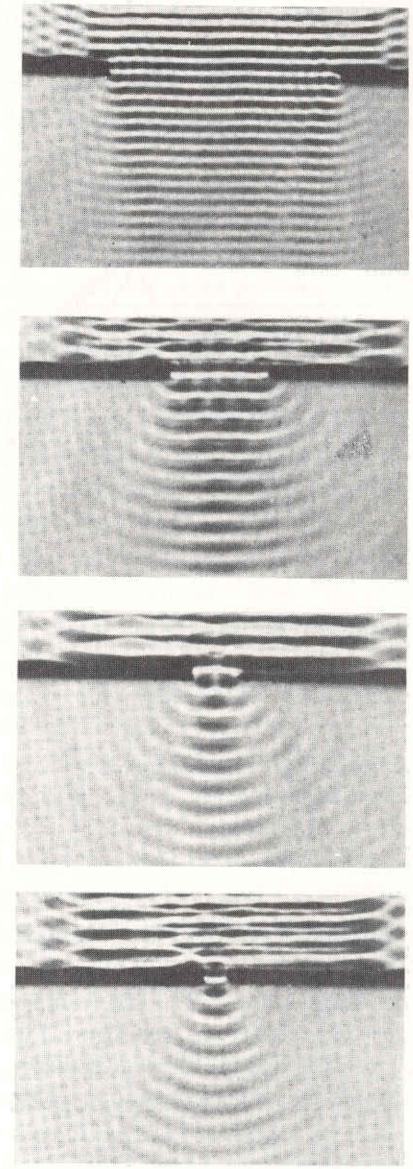
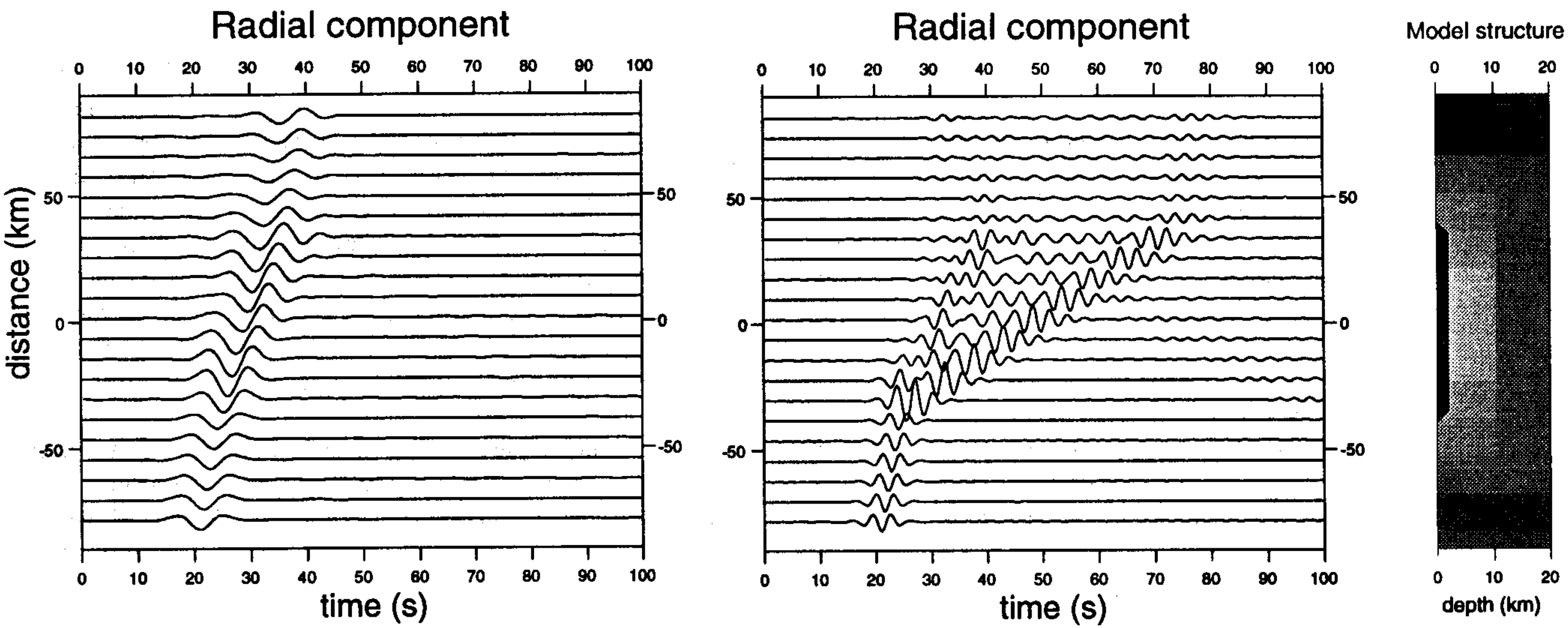


Figure 8.1 Ripples diffracted into quiet water by an opening.

Diffraction of Rayleigh waves by a basin



<0.15 Hz

0.2-1 Hz

DIFFRACTION OF P WAVES :

Primary wave propagating in direction x_1 :

$$u_i^o = \delta_{1i} \ \exp\left(-i \ \omega \ (t - \frac{x_1}{\alpha_0})\right)$$

$$\alpha_0 = \left(\frac{\lambda_0 + 2\mu_0}{\rho_0}\right)^{1/2}$$

$$\Rightarrow$$

$$Q_1 = \left\{\delta\rho \ \omega^2 \ - \ \frac{(\delta\lambda + 2\delta\mu)\omega^2}{\alpha_0^2} \ + \ i \ \frac{\omega}{\alpha_0}(\delta\lambda)_{,1} \ + \ 2i \ \frac{\omega}{\alpha_0}(\delta\mu)_{,1}\right\} \exp(-i\omega(t - \frac{x_1}{\alpha_0})$$

$$Q_2 \ = \ i \ \frac{\omega}{\alpha_0}(\delta\lambda)_{,2} \ \ \exp(-i\omega(t - \frac{x_1}{\alpha_0})$$

$$Q_3 \ = \ i \ \frac{\omega}{\alpha_0}(\delta\lambda)_{,3} \ \ \exp(-i\omega(t - \frac{x_1}{\alpha_0})$$

Identification :

- velocity fluctuation :

$$\alpha = \frac{(\lambda + 2\mu)^{1/2}}{\rho^{1/2}} \qquad \Rightarrow \delta\alpha = \frac{\partial\alpha}{\partial\lambda}\partial\lambda \; + \; \frac{\partial\alpha}{\partial\mu}\partial\mu \; + \; \frac{\partial\alpha}{\partial\rho}\partial\rho$$

$$\delta\alpha = \frac{1}{2}\;\frac{1}{\rho}\;\frac{1}{\alpha}\delta\lambda \; + \; \frac{1}{2}\;\frac{2}{\rho}\;\frac{1}{\alpha}\delta\mu \; - \; \frac{(\lambda + 2\mu)}{\rho^2}\;\frac{1}{2}\;\frac{1}{\alpha}\delta\rho$$

$$\frac{\delta\alpha}{\alpha} = \frac{1}{2}\left(\frac{\delta\lambda + 2\delta\mu}{\lambda + 2\mu} - \frac{\delta\rho}{\rho}\right)$$

→ For Q_1 :

$$\delta\;\rho\;\omega^2 \; - \; \frac{(\delta\lambda + 2\delta\mu)}{\alpha_0}\omega^2 \; = \; -\omega^2\;\rho_0\;\left(2\;\frac{\delta\alpha}{\delta_0}\right)$$

→ force q_1 proportional to the velocity perturbation:

$$q_1 \; = \; -2\;\omega^2\;\rho_0\;\frac{\delta\alpha}{\alpha_0}\;\exp\left(-\omega\;(t \; - \frac{x_1}{\alpha_0})\right)$$

(simple force dans la direction de propagation)

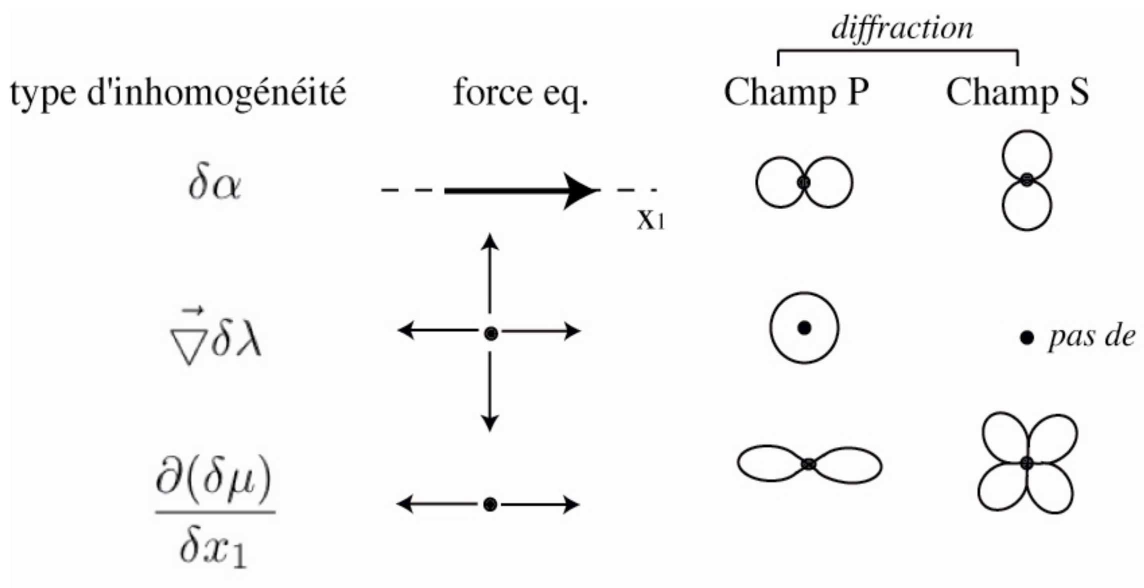
- term proportional to the gradient of λ

$$q' = i \frac{\omega}{\alpha_0} \vec{\nabla}. \delta\lambda$$

- terme proportional to the spatial variation of μ

$$q''_1 = 2i \frac{\omega}{\alpha_0} (\delta\mu)_{,1}$$

For a point perturbation :



- No scattered S wave in the forward direction
- scattered P wave maximum along $x_1 \rightarrow$ backscattering

DIFFRACTION OF S WAVE: $u_i^o = \delta_{2i} \; \exp\left(-i\omega \; (t - \frac{x_1}{\beta_o})\right)$
$\Rightarrow$

- term proportional to the velocity perturbation:

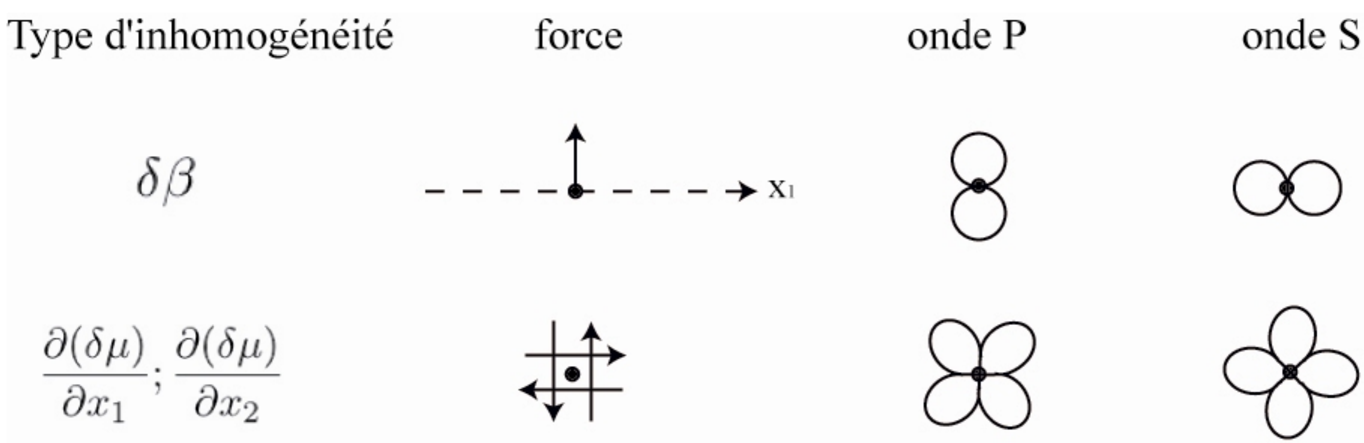
$$q_2 = -2\omega^2 \rho_o \frac{\delta\beta}{\beta_o} \exp\left(-i\omega \; (t - \frac{x_1}{\beta})\right)$$

- term proportional to the variation of $\delta\mu$:

$$q_1' = i \; \frac{\omega}{\beta_o} \; (\delta\mu)_{,2} \; \exp\left(-i\omega \; (t - \frac{x_1}{\beta})\right)$$

$$q_2' = i \; \frac{\omega}{\beta_o} \; (\delta\mu)_{,1} \; \exp\left(-i\omega \; (t - \frac{x_1}{\beta})\right)$$

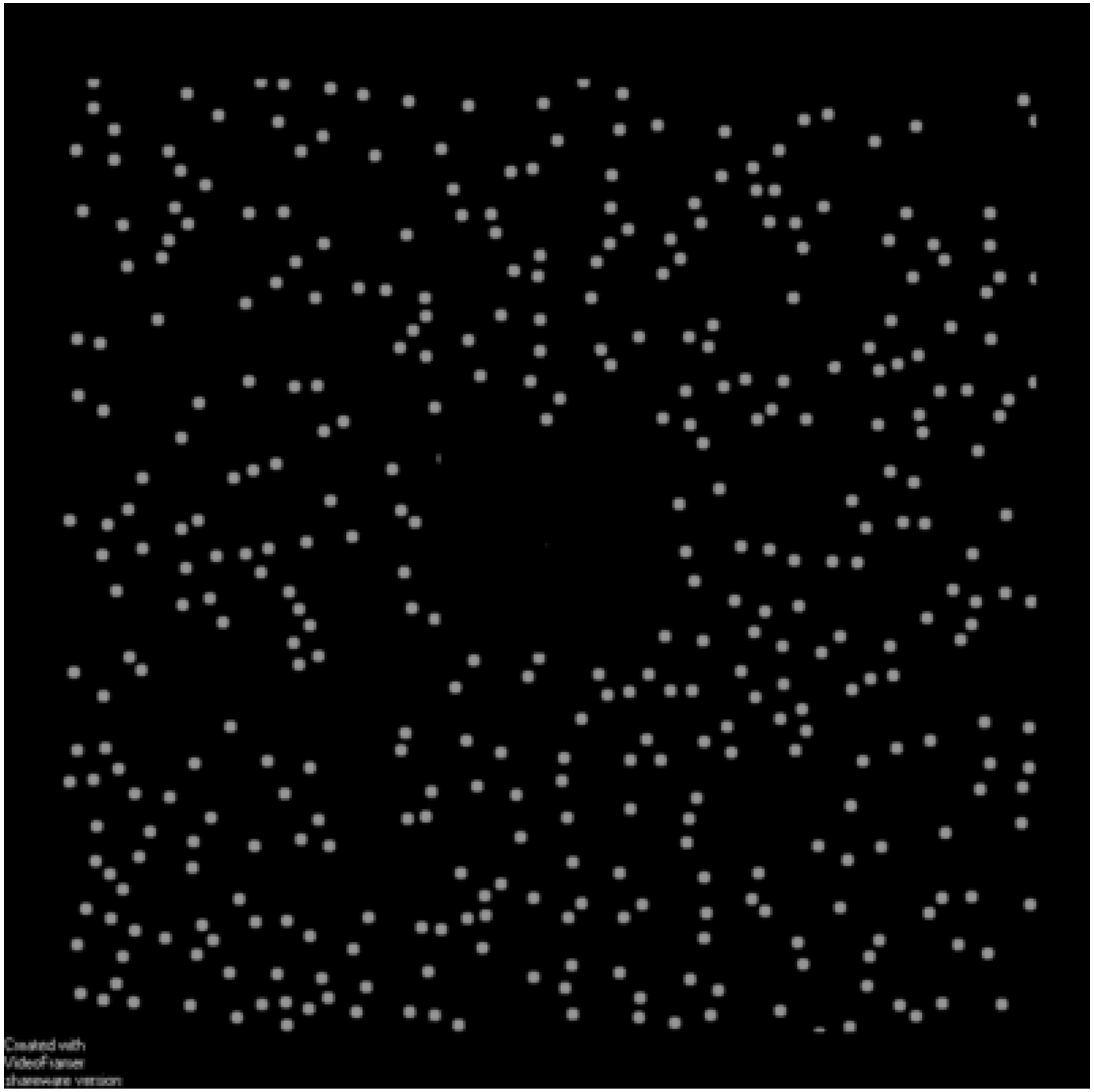
Point perturbation:



- No scattered P along x_1

- P to S coupling via multiple scattering

Transient signals in a complex medium…….



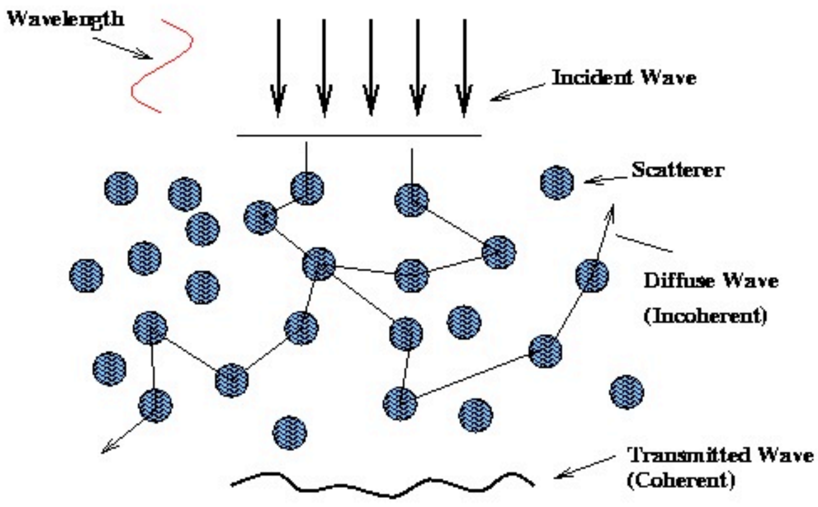
What is the regime relevant to coda waves:

single scattering?

multiple scattering?

diffusion?

Wave Propagation through Random Media



Length Scales: λ, Correlation Length, Propagation Distance

Question: Ensemble Average Response?

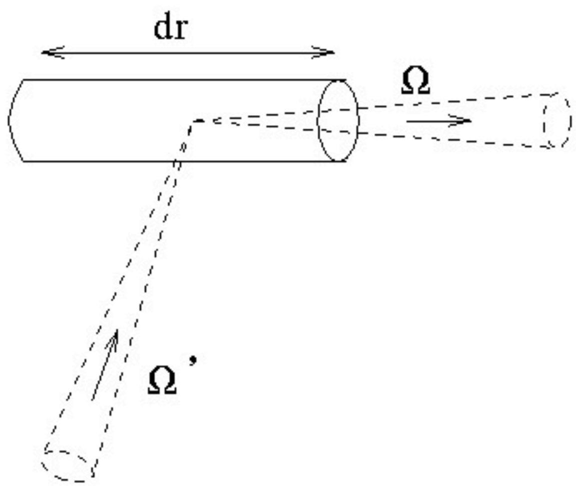
Precise Definition of Coherent and Incoherent Waves

quality factor: $Q = \frac{2\pi}{\lambda} l$

Radiative transfert equation

Heuristic View

Energy balance of a beam of energy propagating a distance dr in the scattering medium



Variation of Intensity

=

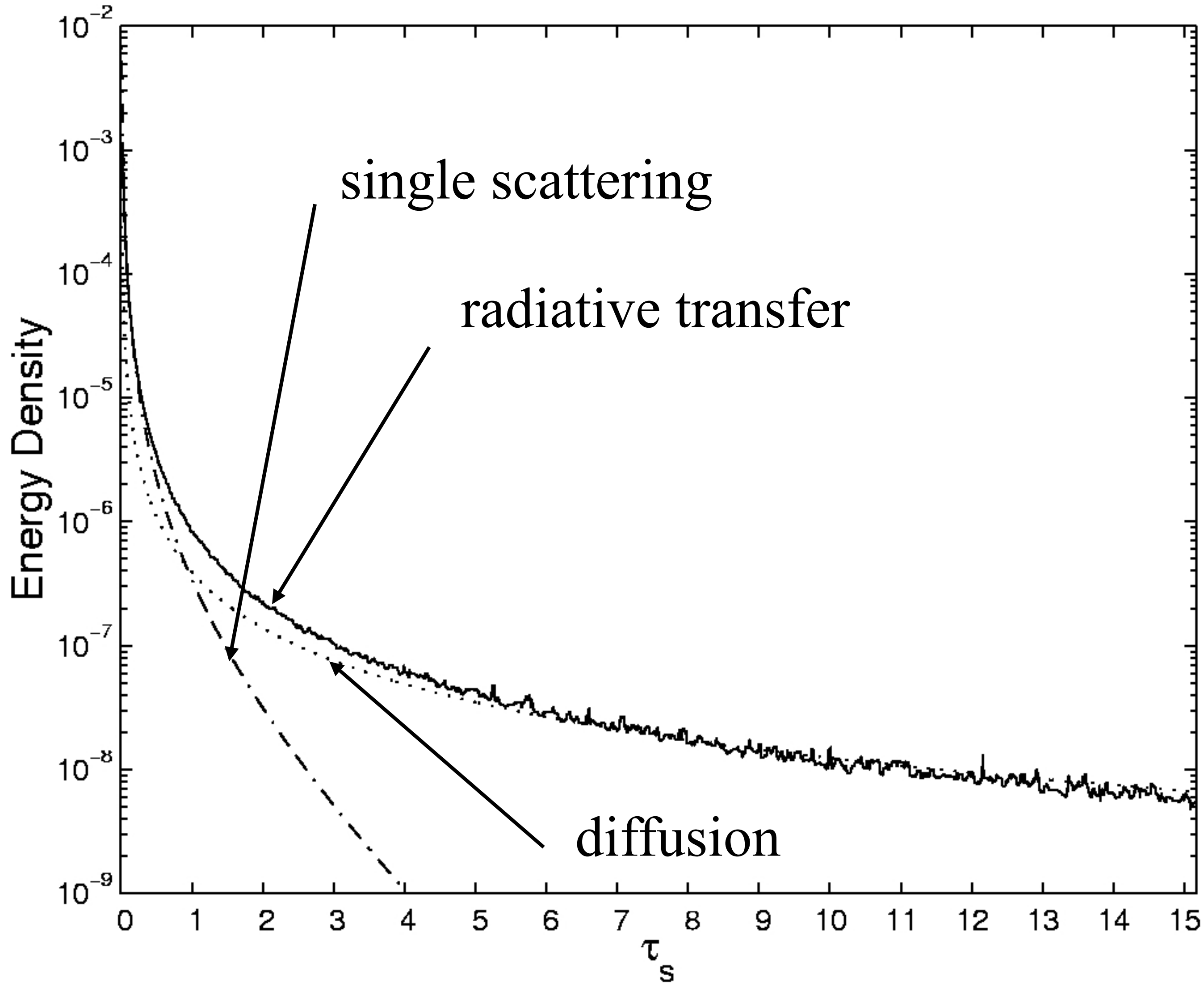
Loss due to scattering into all space directions

+

Gain due to scattering from direction $\vec{\Omega}'$ to direction $\vec{\Omega}$

Elasticity: polarisation + P/S coupling

The approximations of the Radiative Transfer Equation (RTE)



With the diffusion approximation

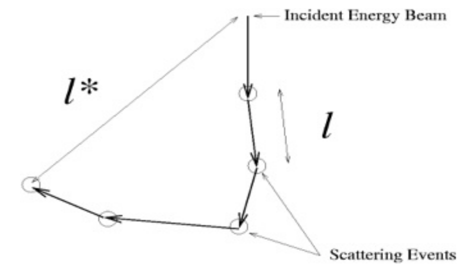
$$\partial_t \rho(\mathbf{R},t) - D\nabla^2\rho(\mathbf{R},t) = \delta(t)\delta(\mathbf{R})$$

ρ is the energy density and **J** the energy current vector

$$\mathbf{J}(\mathbf{R},t) = -D\nabla\rho(\mathbf{R},t)$$

D is the diffusion constant of the waves and is related to the transport mean free time $\tau^\star$ as follows:

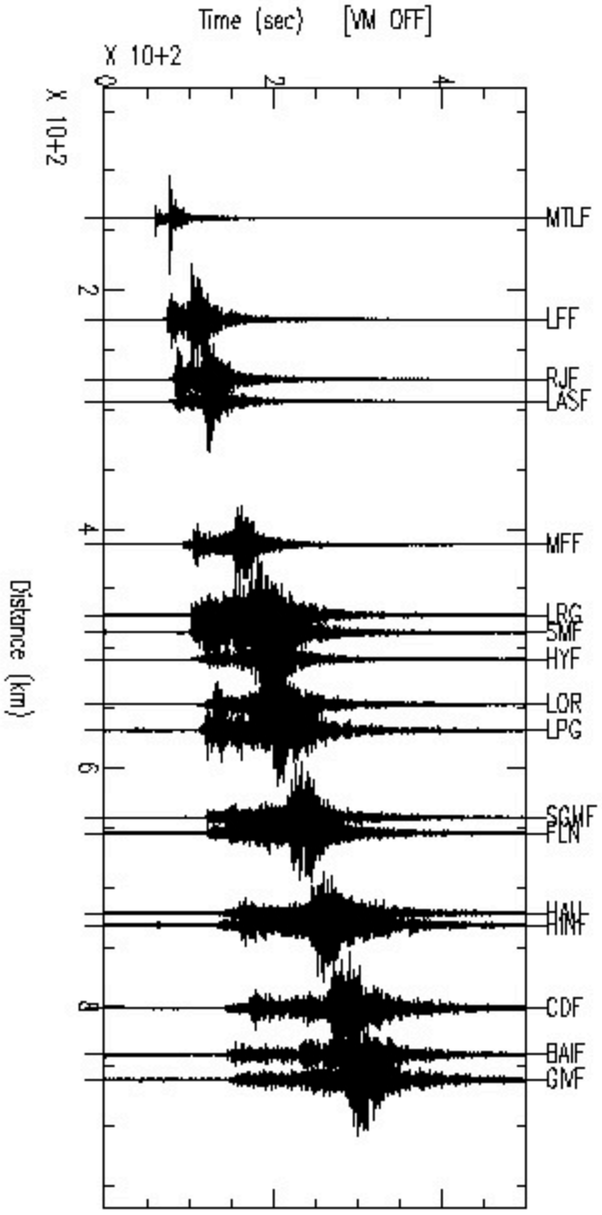
$$D = v^2\tau^\star/3$$



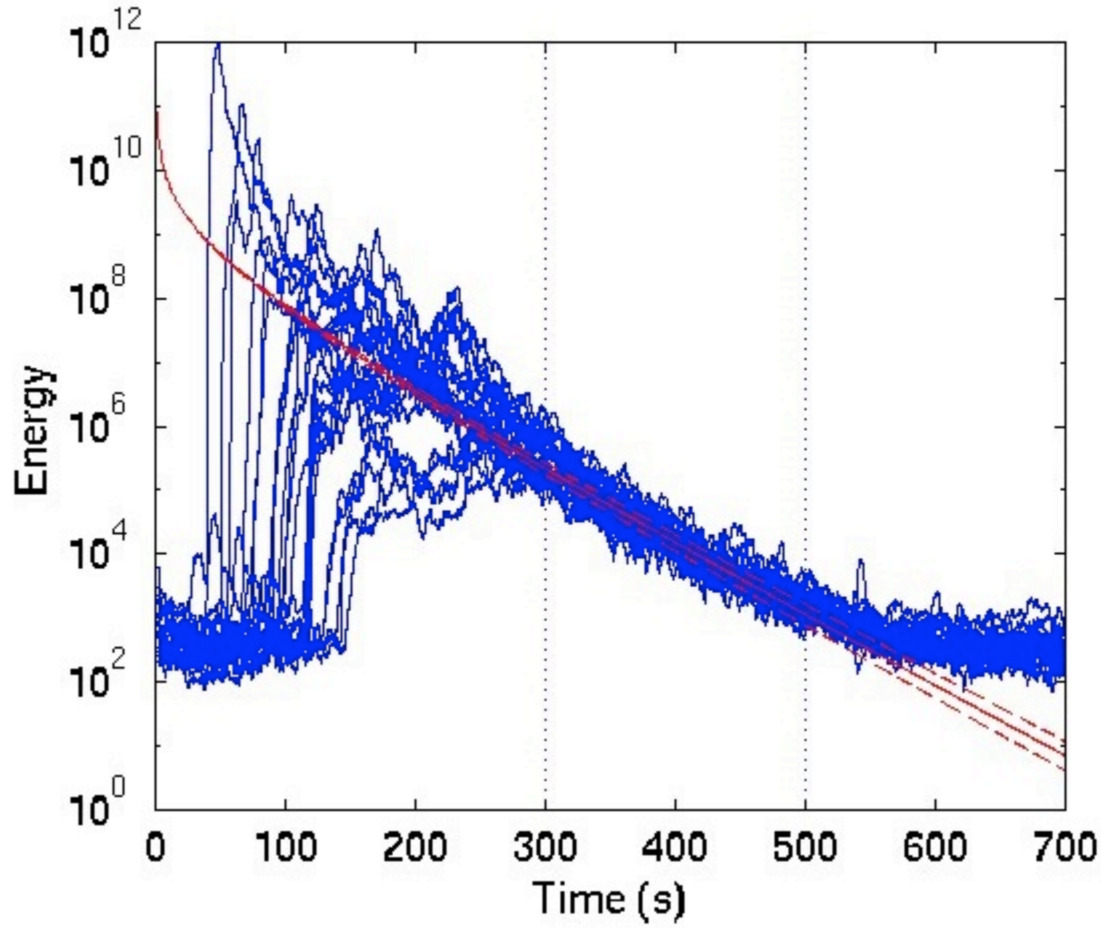
$$\rho(R,t) \approx \exp(-\frac{R^2}{4Dt})\frac{1}{t^{3/2}}$$

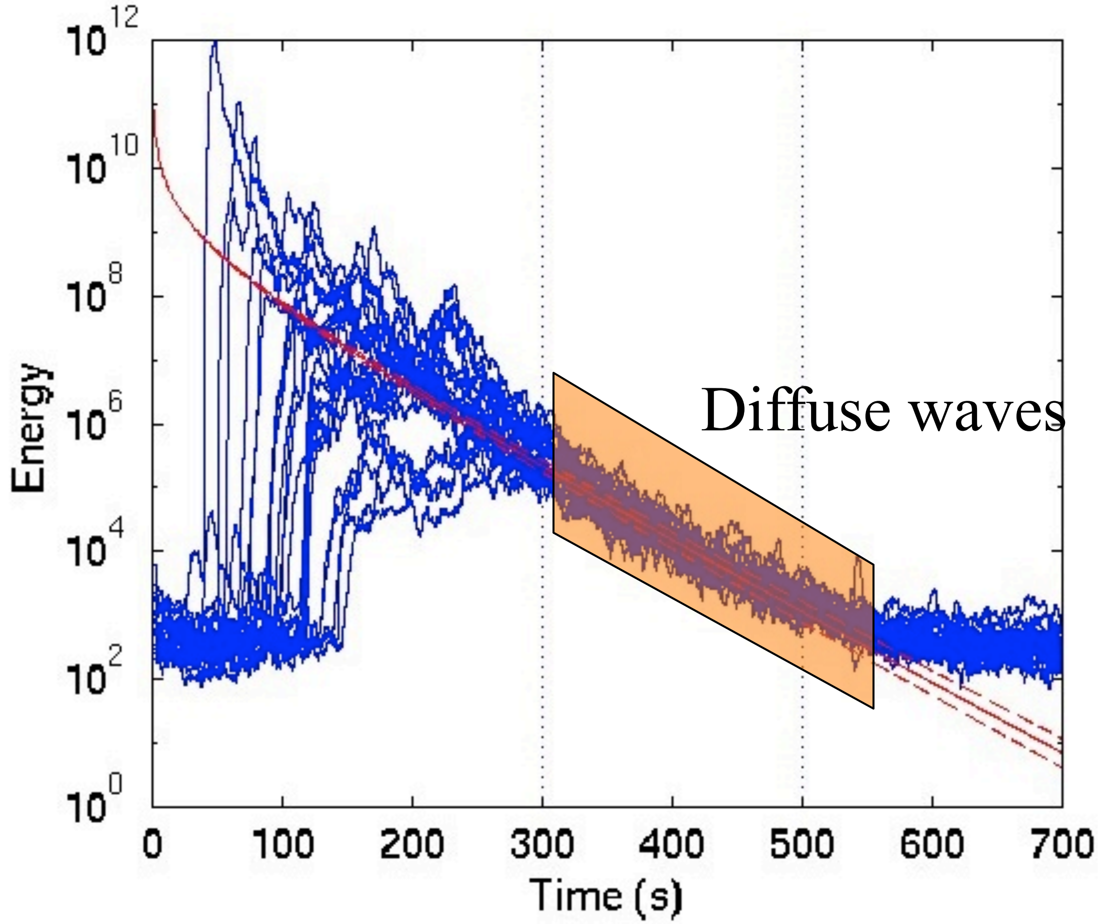
for large lapse times: $\rho(t,\vec{r}) \sim \dfrac{1}{(Dt)^{3/2}}$

Coda of regional seismograms and the separation of scattering and absorption

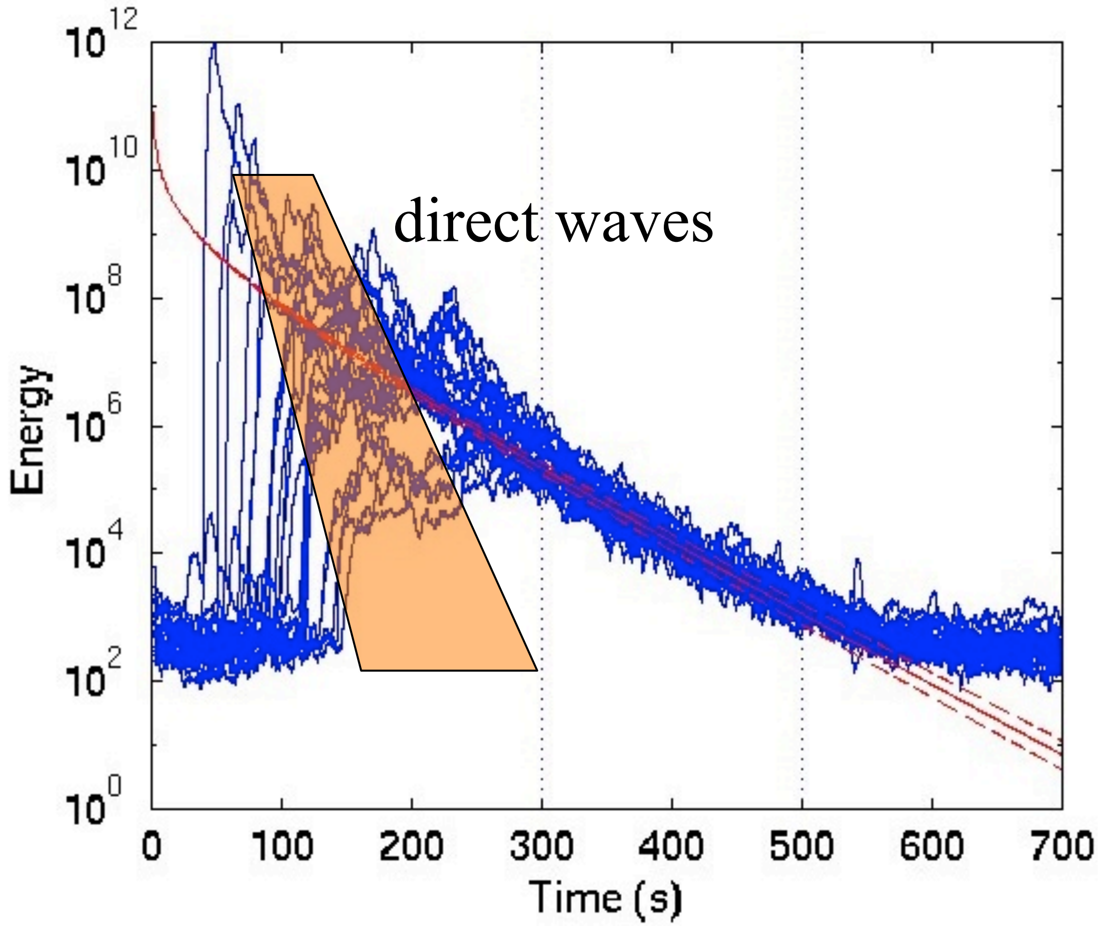


Observations at distances between 150 and 800 km!!

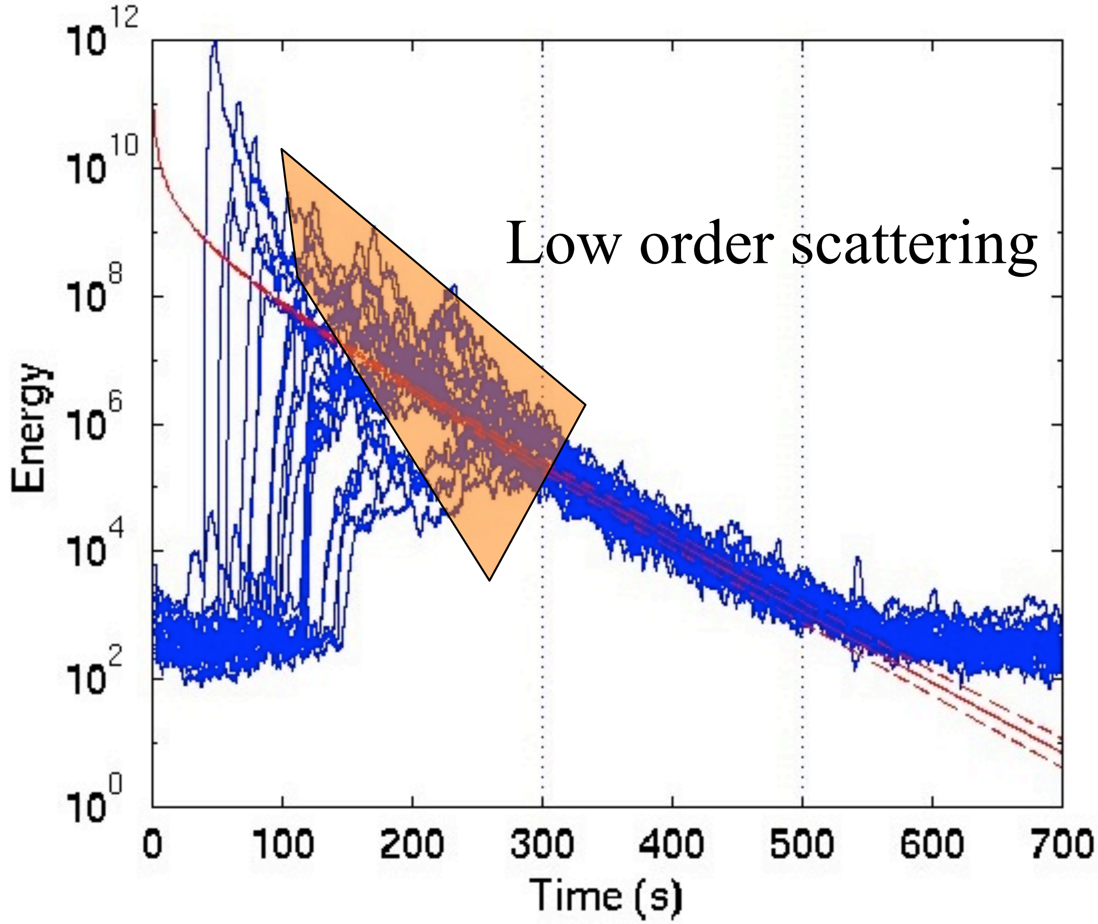




Diffuse waves
Energy
Time (s)

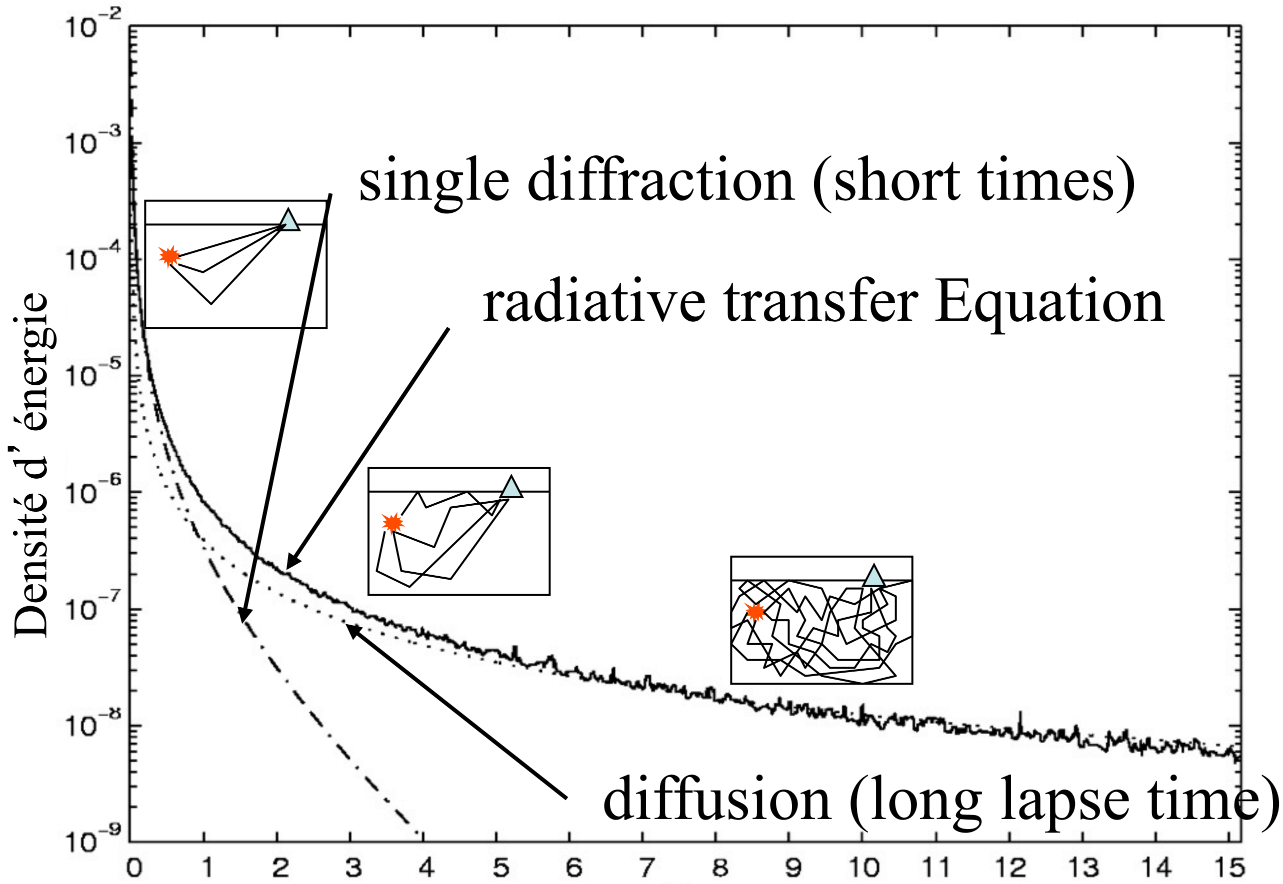


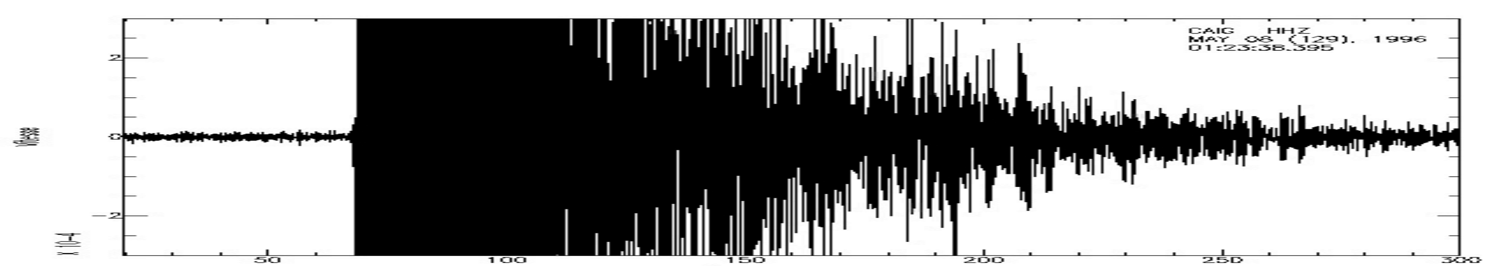
Rapid decay with distance



Low order scattering
Energy
Time (s)

Propagation regimes and energy description





temps

Propagation Regimes

Multiple scattering, diffusion and equipartition

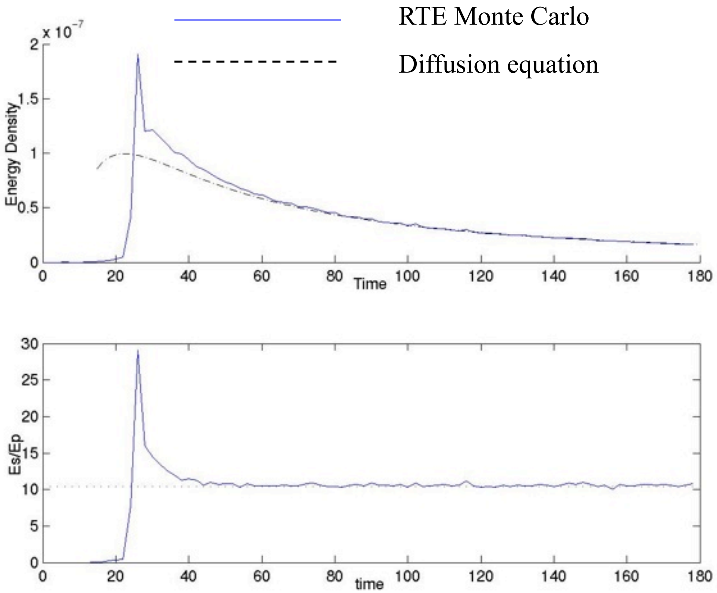
Diffusion, phase and coherent back scattering

Searching for a marker of the regime of scattering...

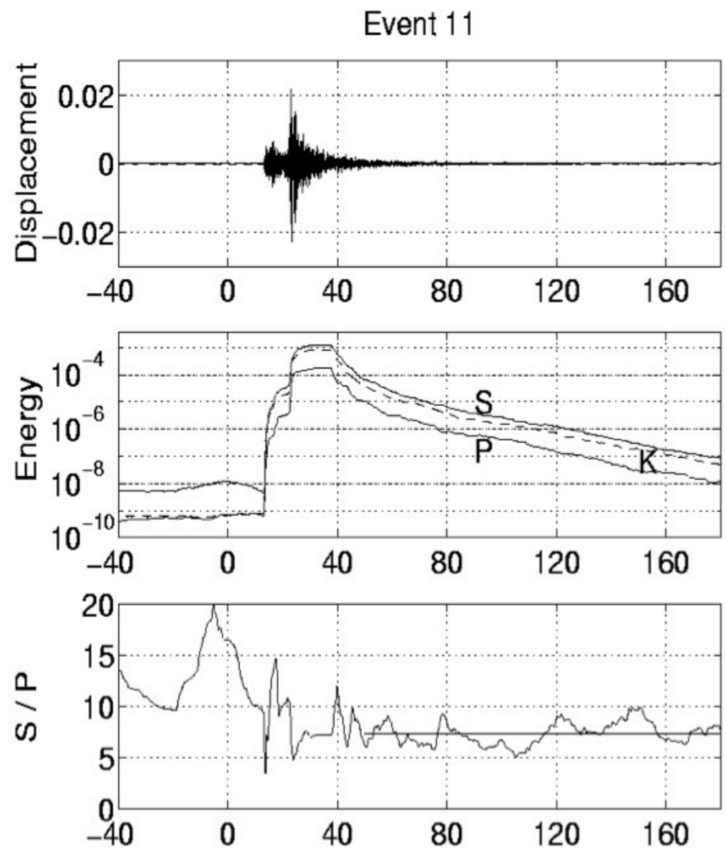
Equipartion principle for a completely randomized (diffuse) wave-field: in average, all the modes of propagation are excited to equal energy.

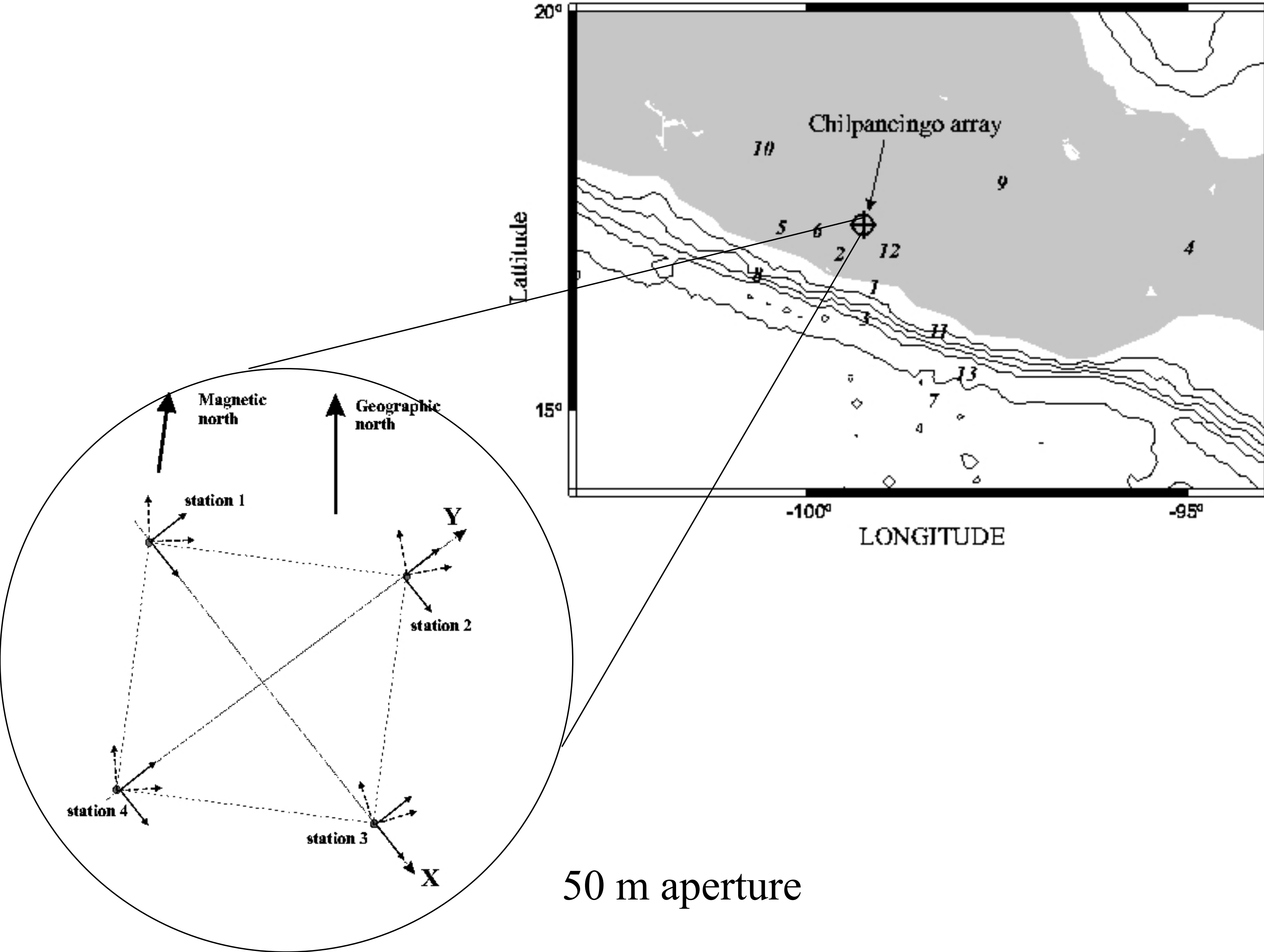
Implication for elastic waves (Weaver, 1982, Ryzhik et al., 1996): P to S energy ratio stabilizes at a value independant of the details of scattering!

Numerical simulation

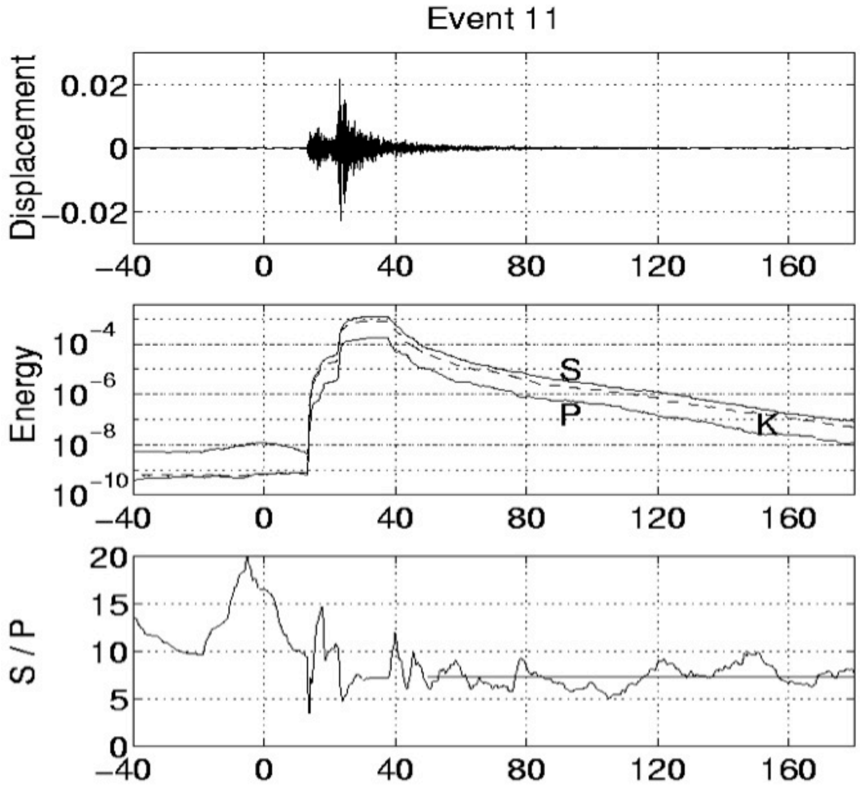


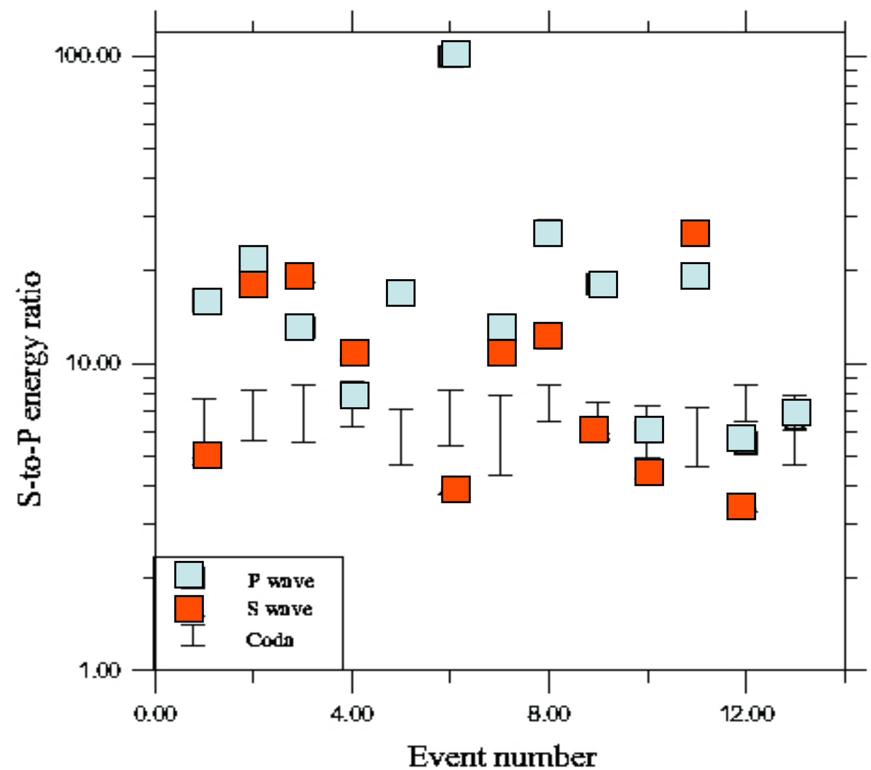
Observations



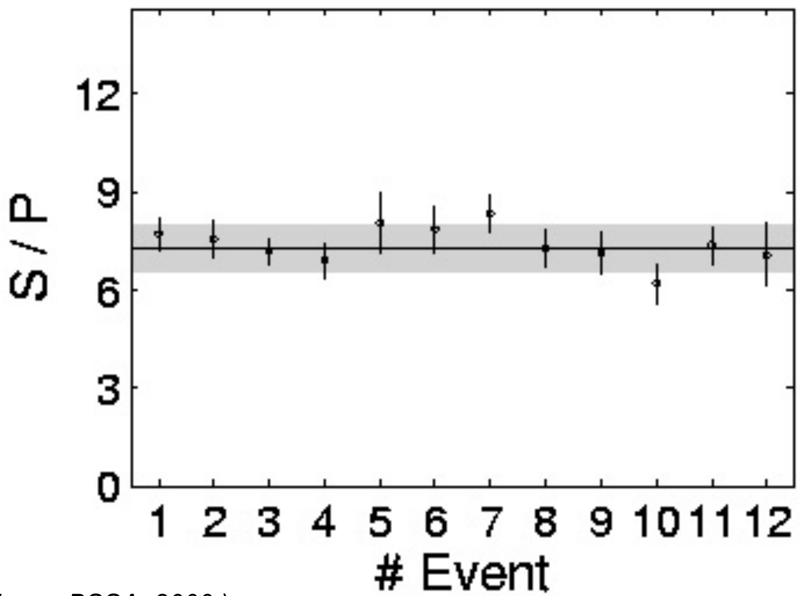


50 m aperture


Event 11
Displacement
0.02
0
−0.02
−40
0
40
80
120
160
Energy
10^{-4}
10^{-6}
10^{-8}
10^{-10}
S
P
K
S / P
20
15
10
5
0


100.00
10.00
1.00
S-to-P energy ratio
0.00
4.00
8.00
12.00
Event number
P wave
S wave
Coda

Values of stabilization for a series of events



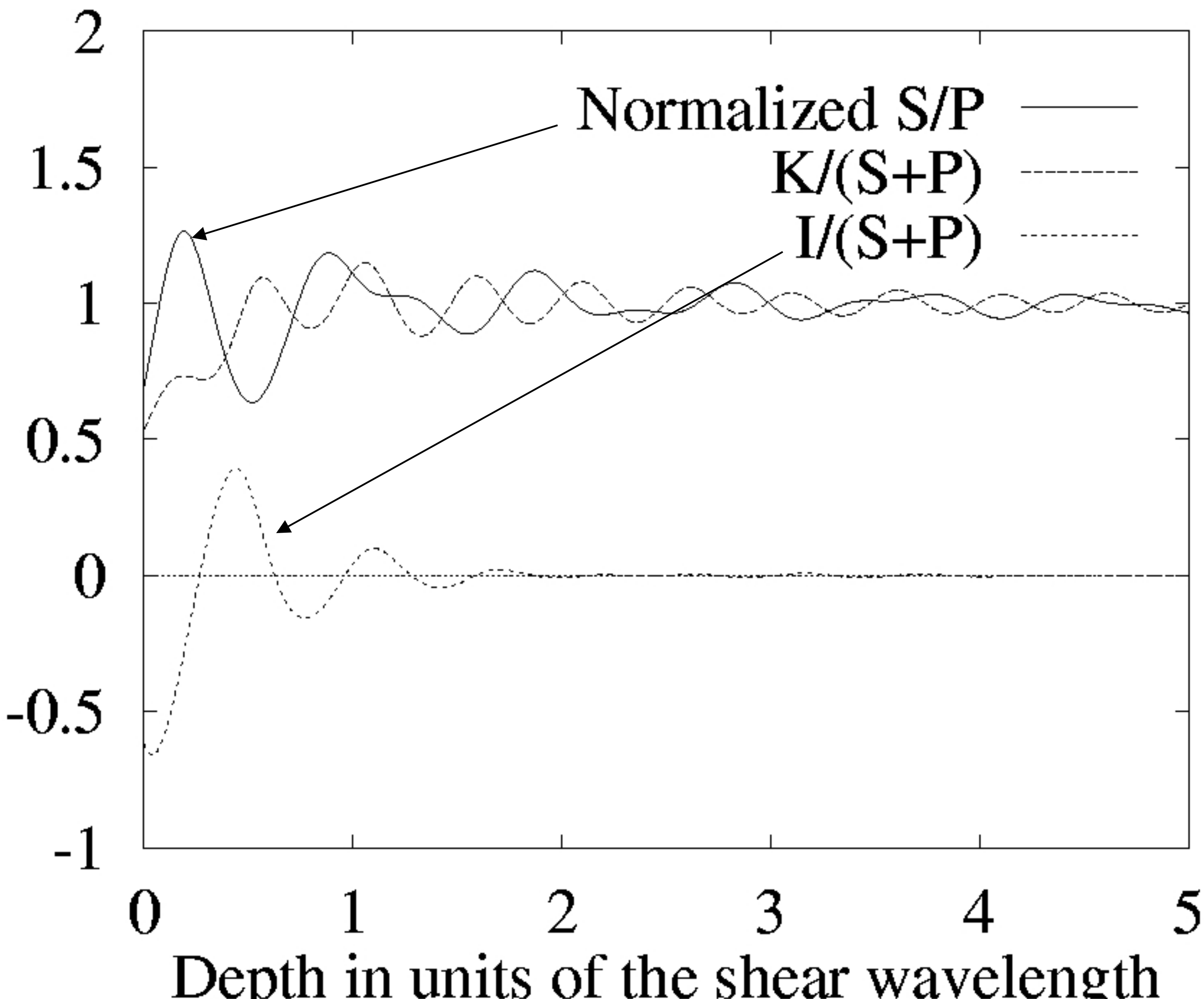
(Shapiro, Campillo, Margerin, Singh, Kostoglodov, Pacheco, BSSA, 2000.)

	OBSERVATIONS	THEORY FULL SPACE	THEORY HALF SPACE (BODY WAVES)	THEORY HALF-SPACE WITH RAYLEIGH
S/P	7.3	10.39	9.76	7.19

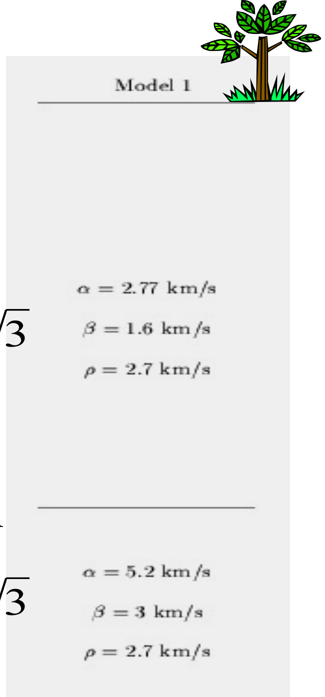
(Hennino et al., PRL, 2001)

Effect of the free surface - A model including Rayleigh waves:

Normal modes of a thick elastic plate



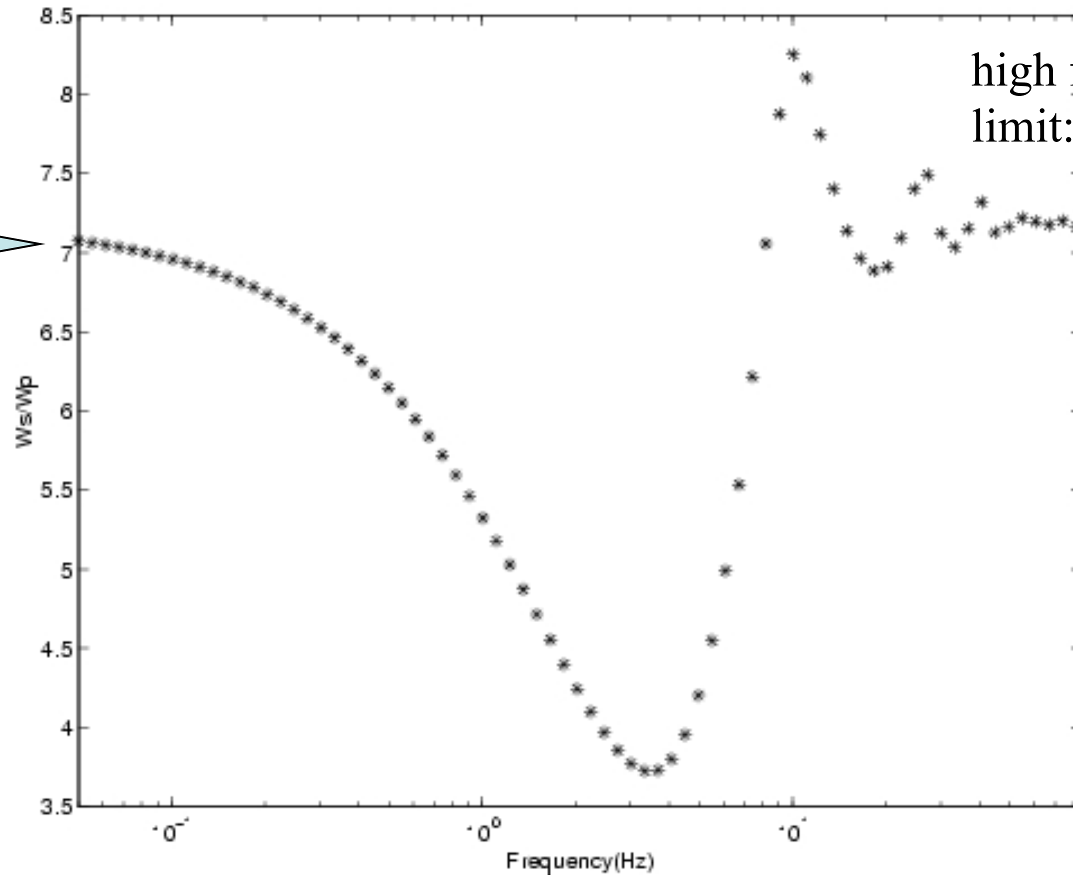
Effect of a ‘thin’ layer: Love and Rayleigh



$\alpha/\beta = \sqrt{3}$

65 m

$\alpha/\beta = \sqrt{3}$



low frequency limit: half space

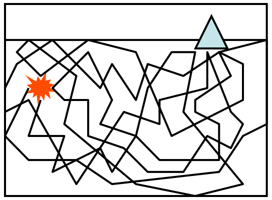
high frequency limit: half space

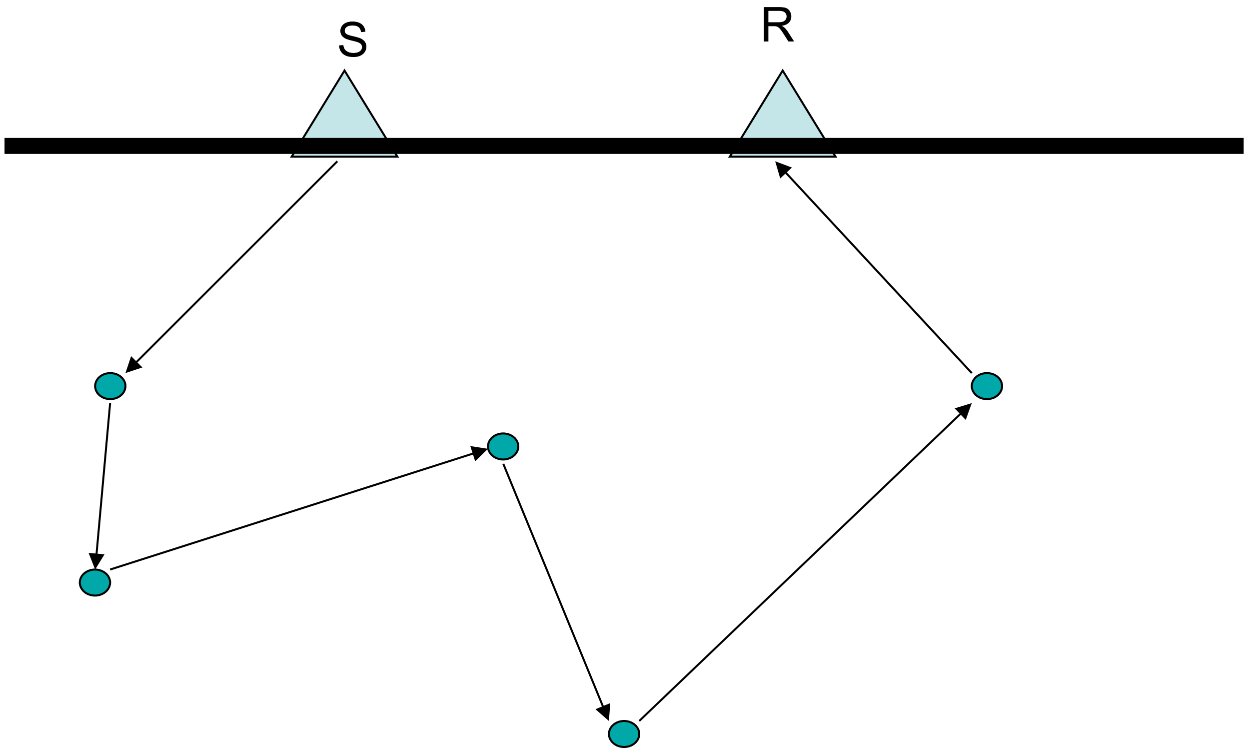
Propagation Regimes

Multiple scattering, diffusion and equipartition

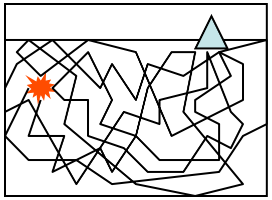
Diffusion, phase and coherent back scattering

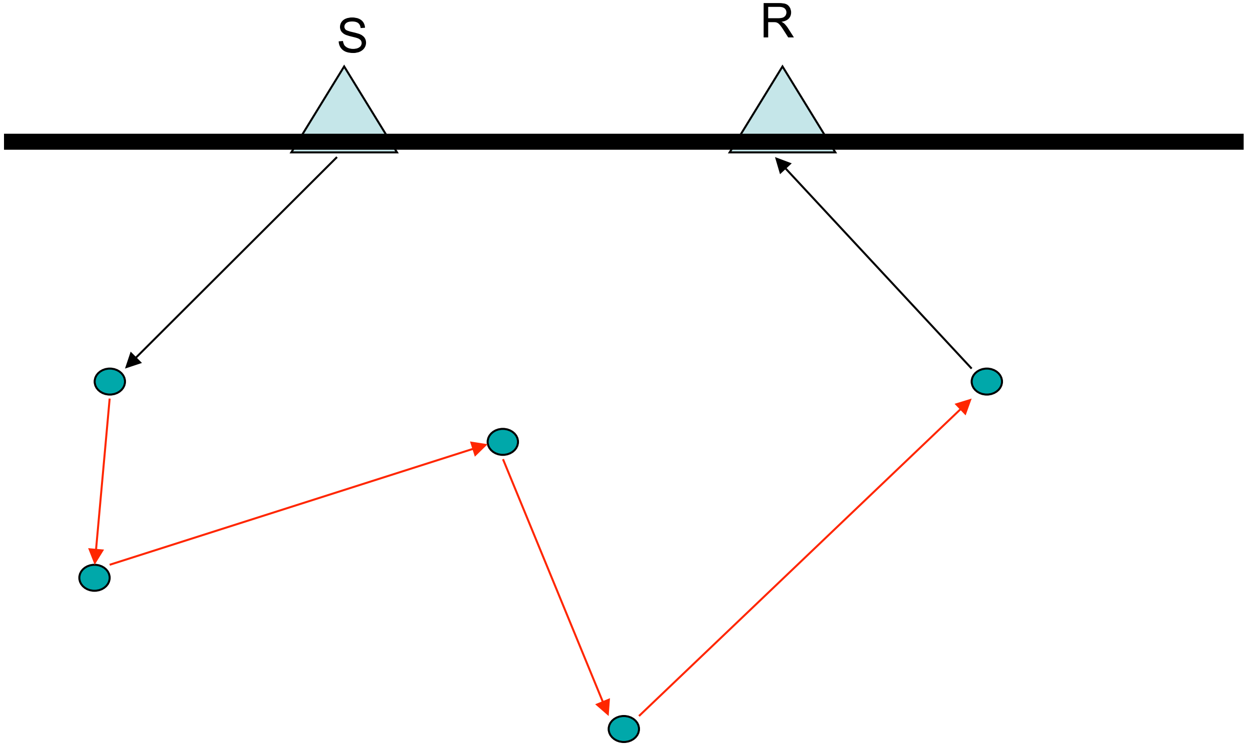
Coherent backscattering: multiple scattering





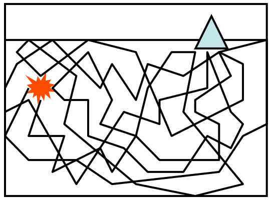
.

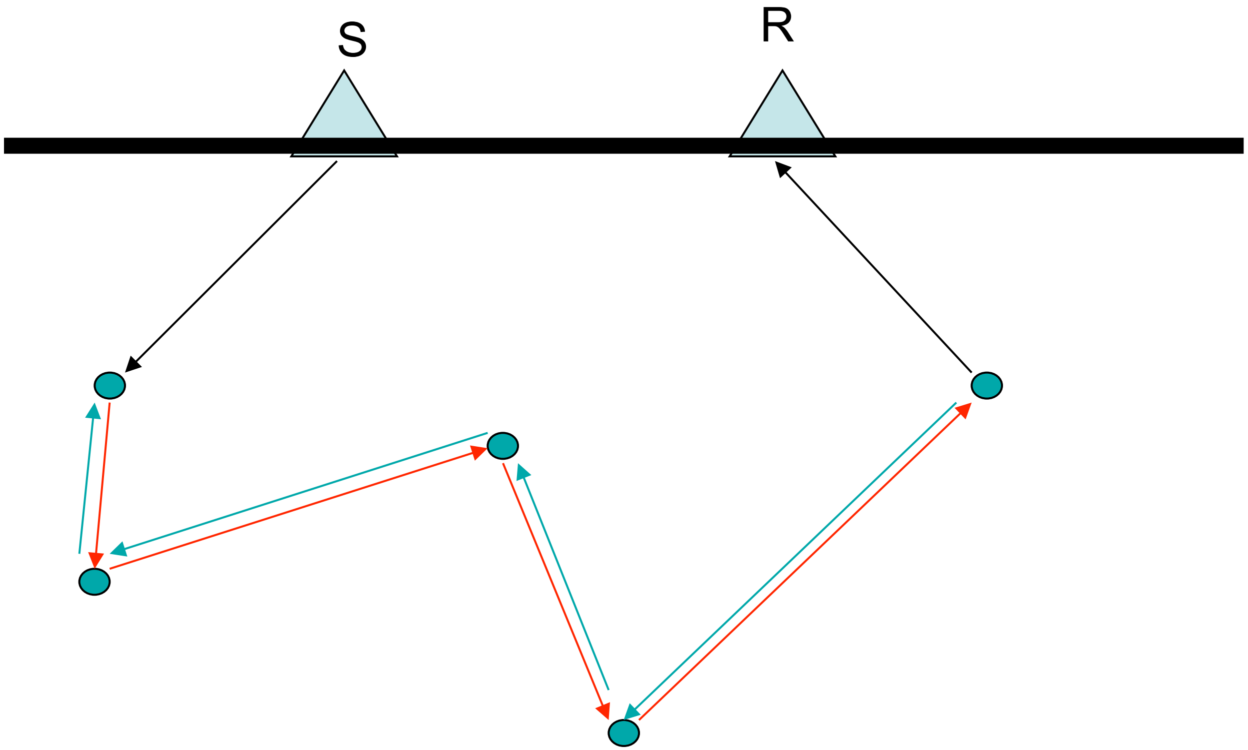




If this path exists..

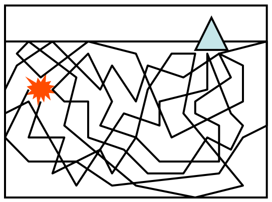
.

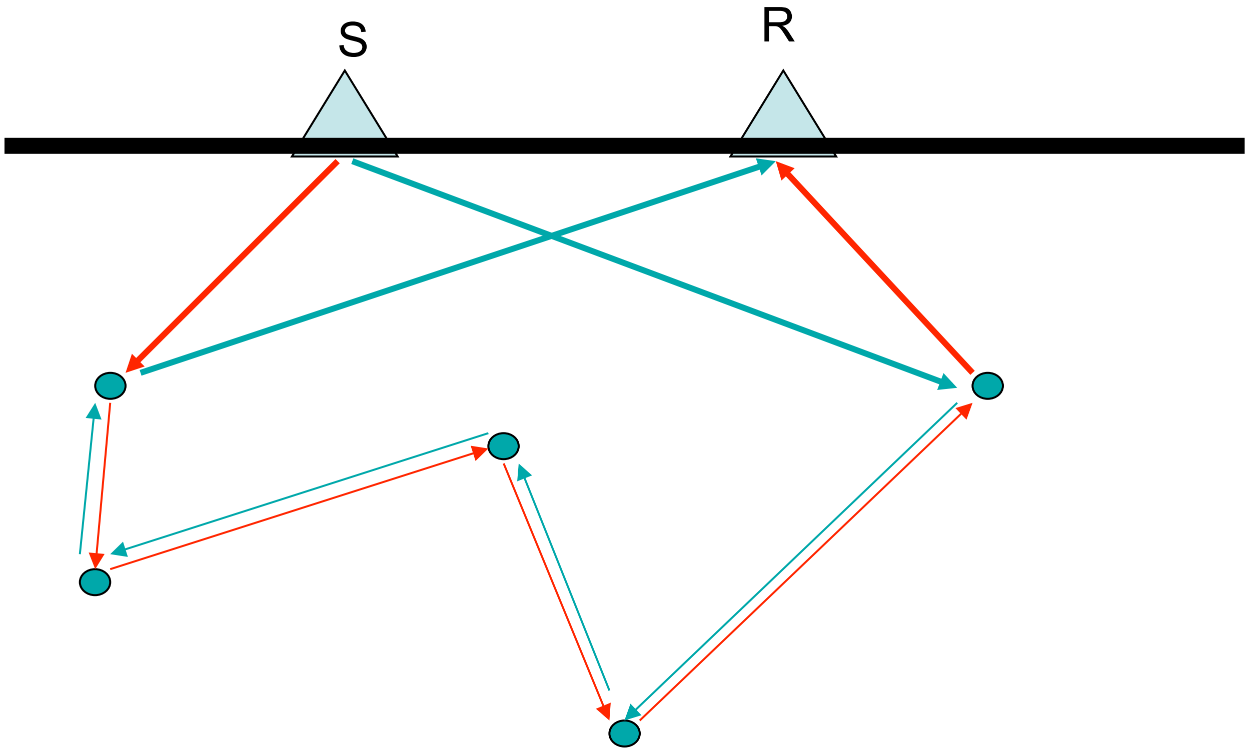




If this path exists, the reciprocal path exists too.

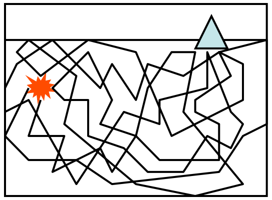
.

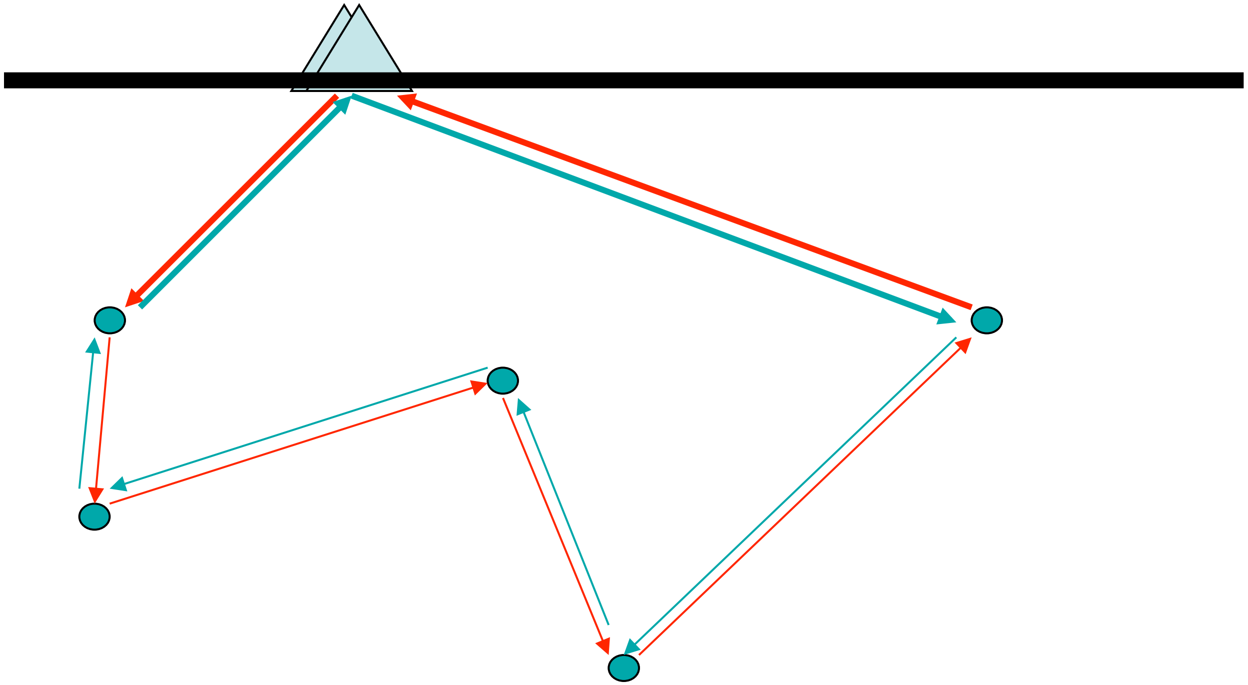




Phase difference: location of the scatterers…

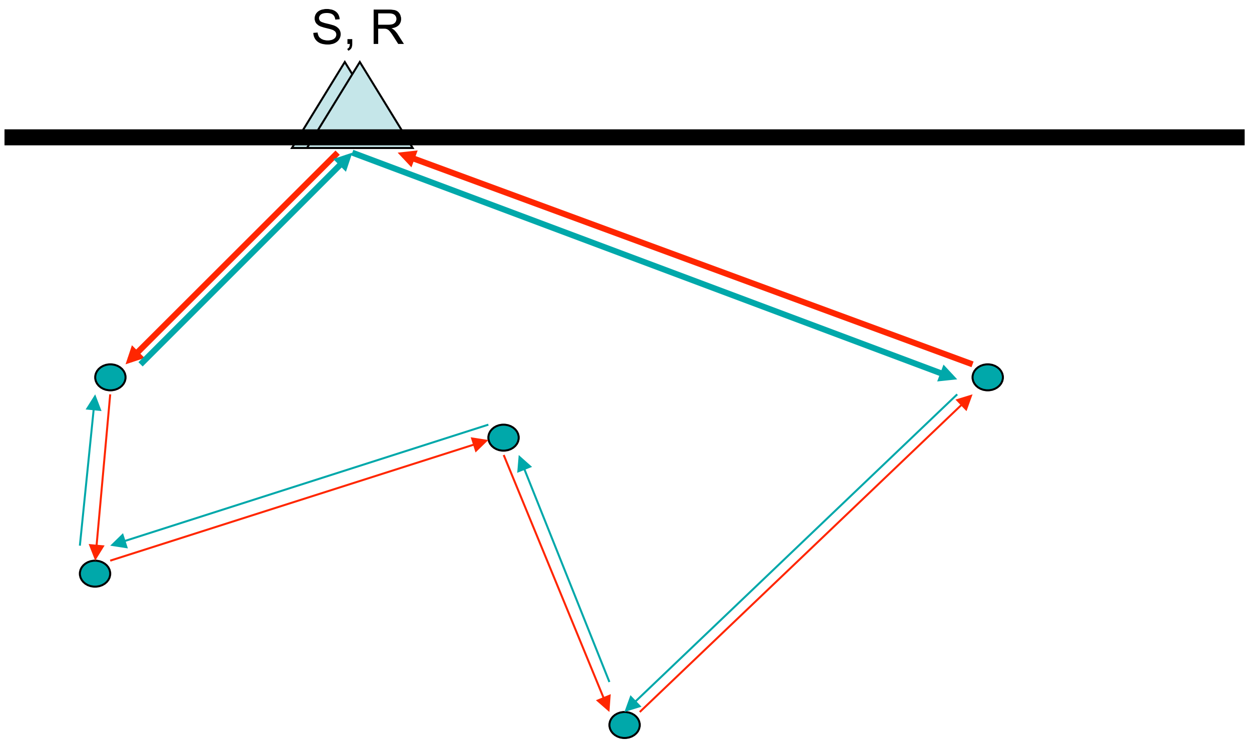
.





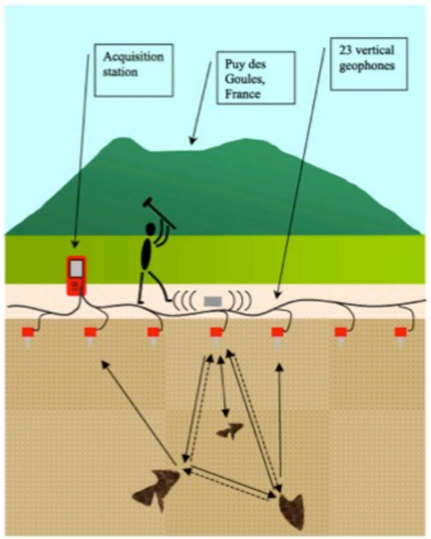
Phase difference: location of the scatterers…
Except if R and S are at the same place

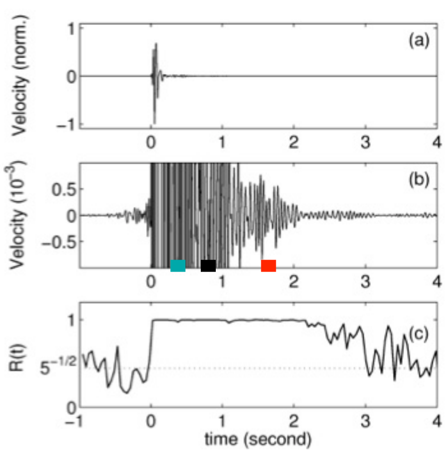
.



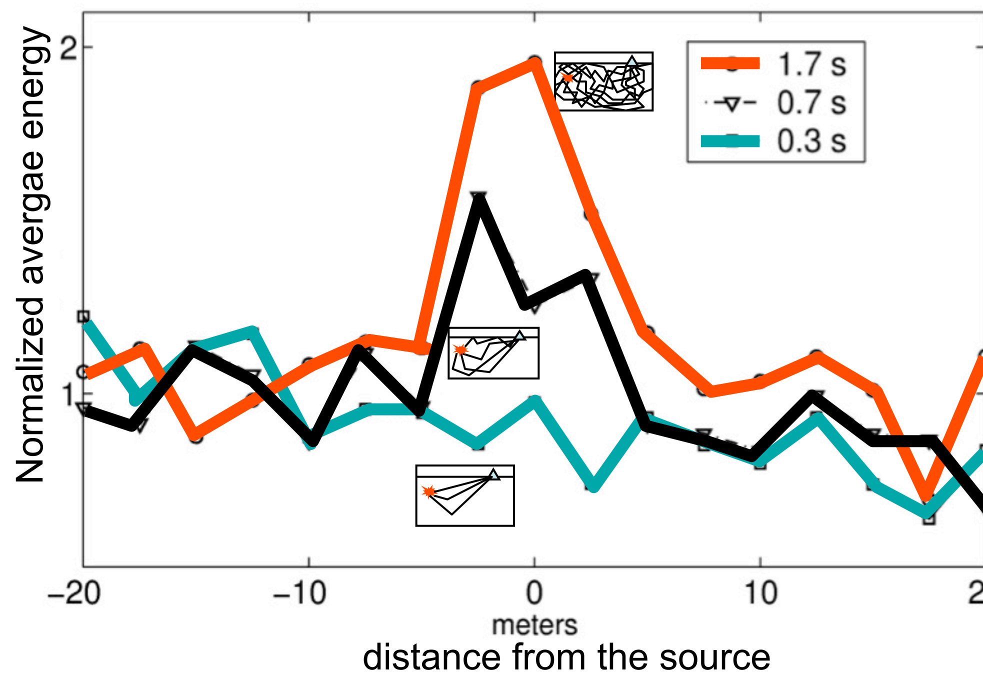
Spot of intensity enhancement: factor 2

Weak localization: coherent backscattering of seismic waves



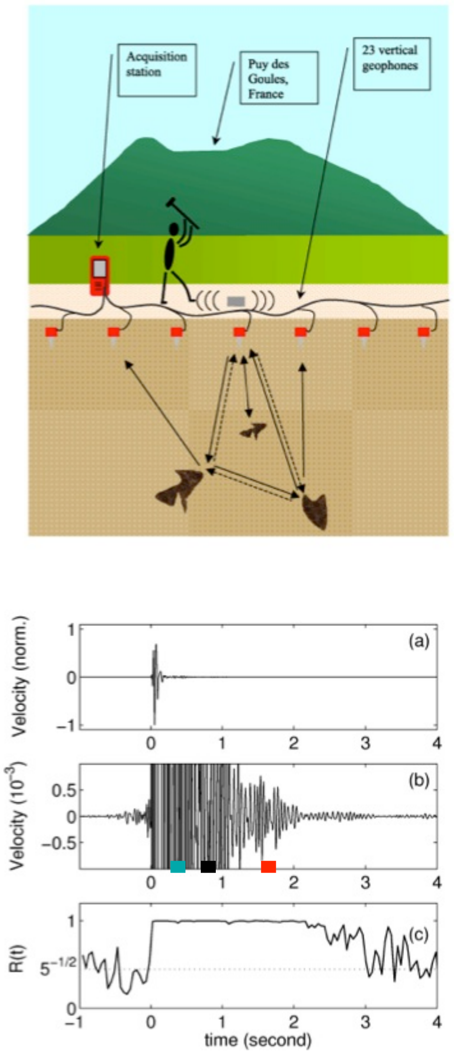


Average spatial energy distribution at different times

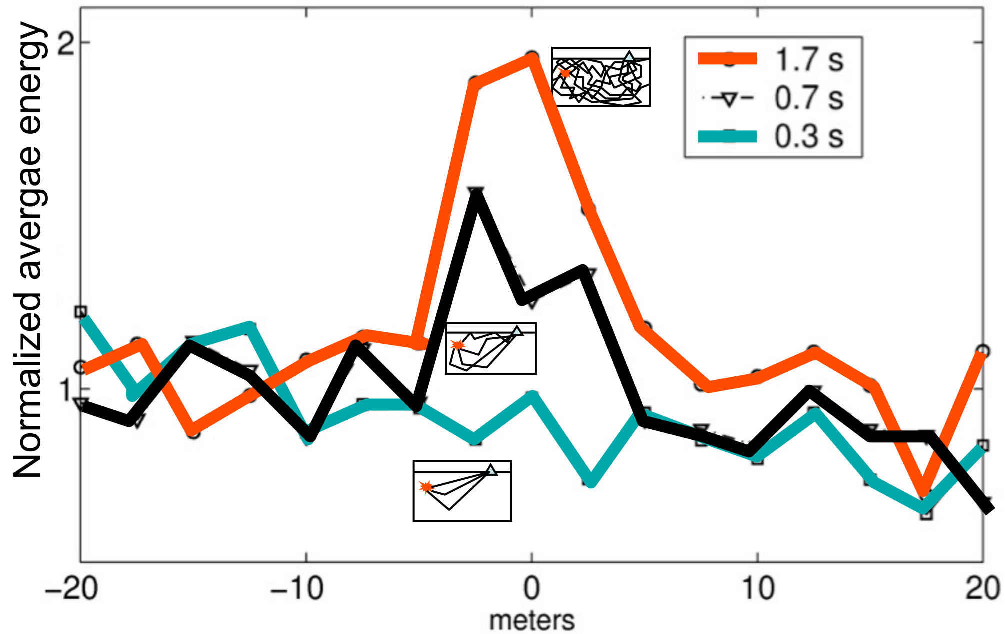


(Larose, Margerin, van Tiggelen and Campillo, PRL 2004)

Weak localization: coherent backscattering of seismic waves



Average spatial energy distribution at different times



(Larose, Margerin, van Tiggelen and Campillo, PRL 2004)

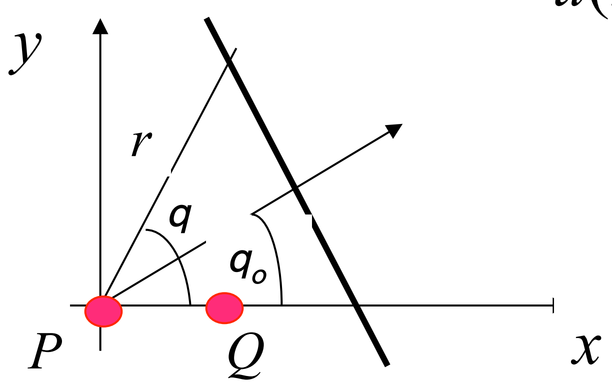
Correlation of diffuse scalar fields and Green function
Open medium

SPAC

In this method one looks for surface waves velocities in order to find the structure. It is assumed that seismic noise is stationary. Spatial seismic arrays are required to made the azimutahl average.

If moreover, noise is isotropic the same result can be obtained with only two stations by means of the cross correlation stackings for a long time.

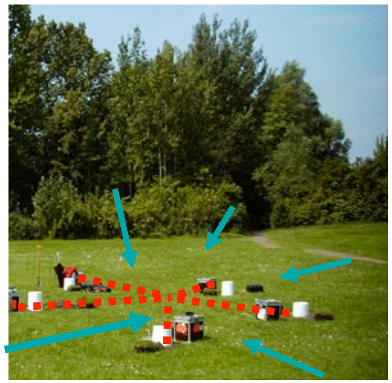
Let's consider the plane wave



$$u(r,\theta,\omega) = F(\omega)\exp(-\mathrm{i}kr\cos(\theta-\theta_0))$$

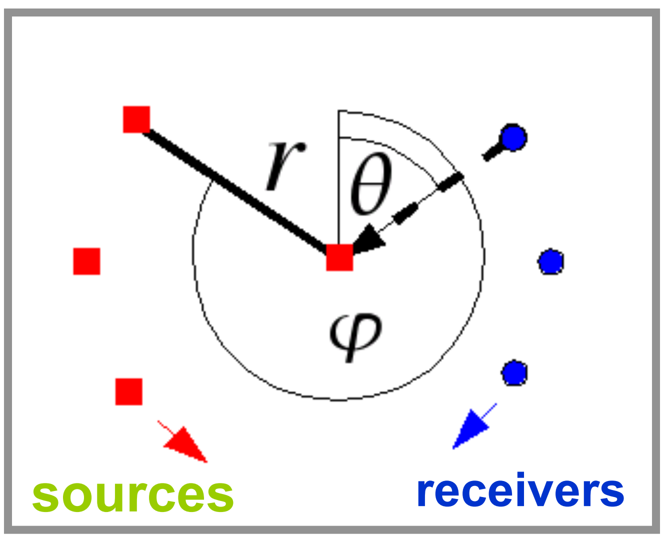
$$\frac{u^P u^{Q^*}}{\left|u^P\right|\left|u^Q\right|} = e^{+ikr\cos\theta_0}$$

Spatial autocorrelation coefficient: Average spatial coefficient (2D circular array)

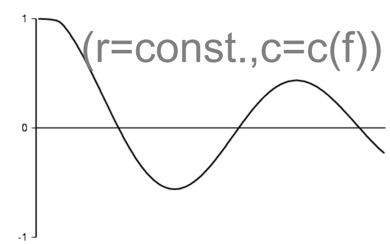


Aki (1957): azimuthal averaged spatial autocorrelation coefficients

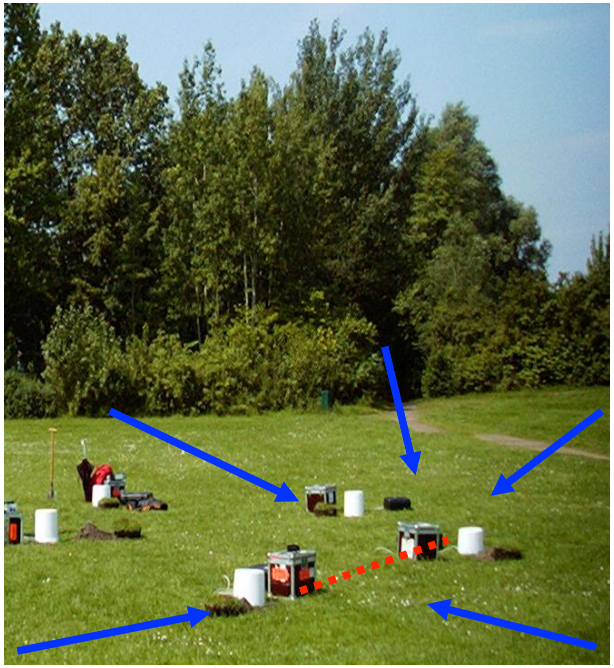
$$\bar{\rho}(\mathbf{r},\omega_0) = \frac{1}{\pi}\int_0^{\pi} \rho(\mathbf{r},\varphi,\omega_0)\,\mathbf{d}(\theta-\varphi)$$

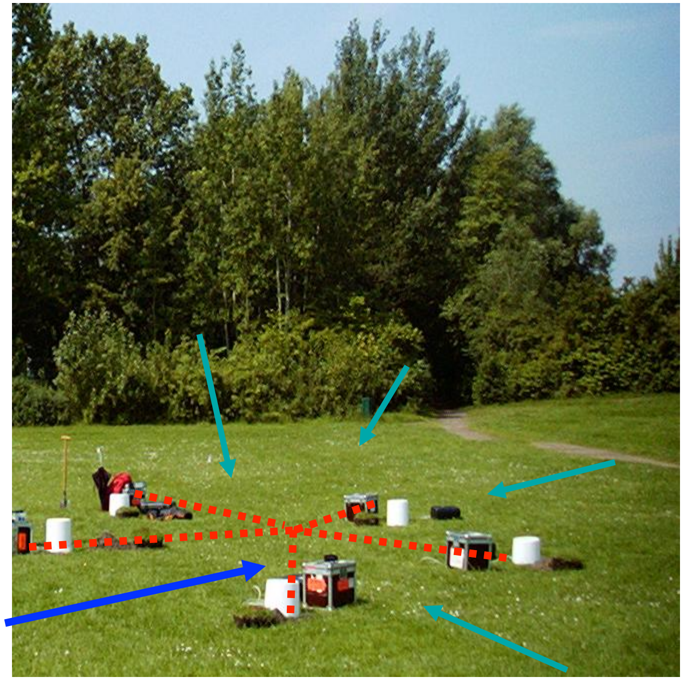


$$\bar{\rho}(r,\omega_0) = J_0\left(\frac{\omega_0 r}{c(\omega_0)}\right)$$



Spatial autocorrelation coefficient: Average spatial coefficient



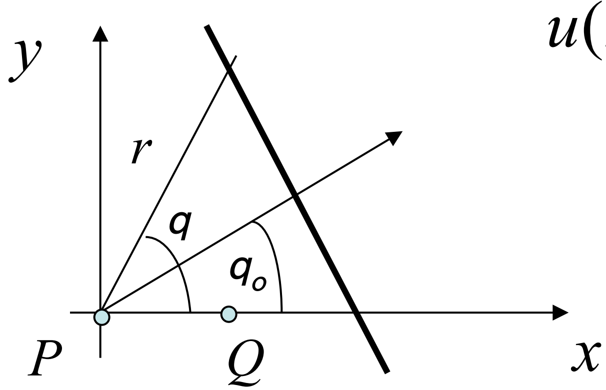


Evaluating k at different frequencies makes it possible to obtain the dispersion curve $C(\omega)$.

The method relies on the hypothesis of the stationnarity of the noise and requires specific array design to perform the azimuthal average.

Another approach consists of using only two points and to rely on long term average to produce the azimuthal average.

Let us consider a plane wave in 2D:



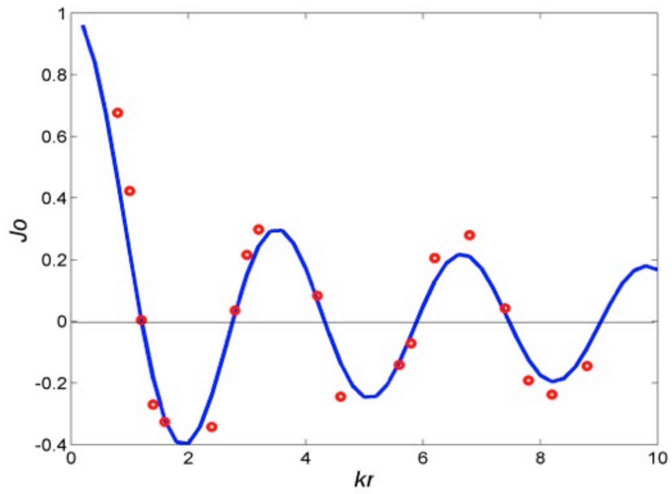
$$u(r,\theta,\omega) = F(\omega)\exp(-\mathrm{i}kr\cos(\theta-\theta_0))$$

$$\frac{u^P u^{Q^*}}{|u^P||u^Q|} = e^{+ikr\cos\theta_0}$$

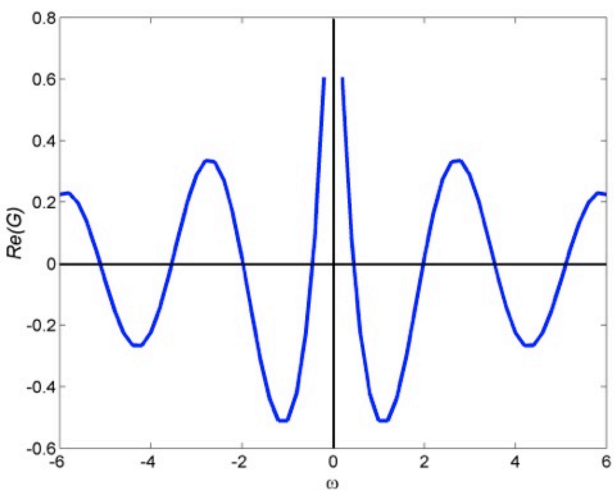
$$\langle \rho(r,\omega) \rangle = \left\langle \frac{u^P u^{Q^*}}{|u^P||u^Q|} \right\rangle = \langle e^{ikr\cos\theta_0} \rangle = \frac{1}{2\pi}\int_0^{2\pi} e^{ikr\cos\theta_0}\, d\theta_0 = J_0(kr)$$

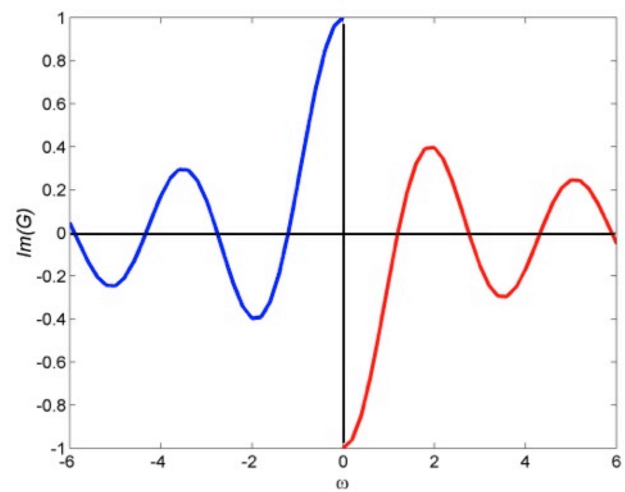
$$\frac{1}{2\pi}\int_0^{2\pi}\left(\sum_{m=0}^{\infty} \varepsilon_m \mathrm{i}^m J_m(kr)\cos m\theta_0\right) d\theta_0$$

azimuthal average of the spatial cross-correlation



$$G = \frac{1}{4\mathrm{i}\mu} H_0^{(2)}\left(\frac{\omega r}{\beta}\right) = \frac{1}{4\mu}\left\{ -Y_0\left(\frac{\omega r}{\beta}\right) - \mathrm{i} J_0\left(\frac{\omega r}{\beta}\right)\right\}$$



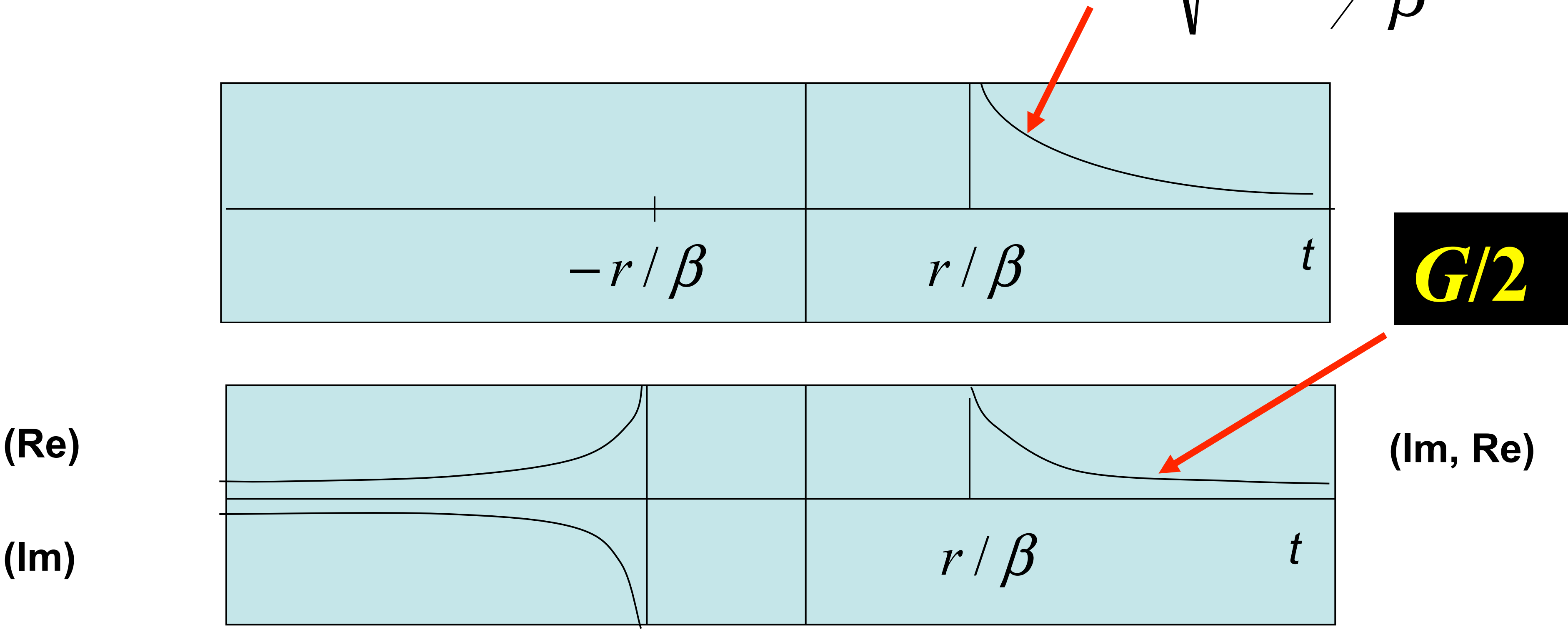


Impossible d'afficher l'image. Votre ordinateur manque peut-être de mémoire pour ouvrir l'image ou l'image est endommagée. Redémarrez l'ordinateur, puis ouvrez à nouveau le fichier. Si le x rouge est toujours affiché, vous devrez peut-être supprimer l'image avant de la réinsérer.

Causality

$$G = \frac{1}{4i\mu} H_0^{(2)}\left(\frac{\omega r}{\beta}\right)$$

$$G = \frac{1}{2\pi\mu} \frac{H\left(t - {r}/{\beta}\right)}{\sqrt{t^2 - {r^2}/{\beta^2}}}$$

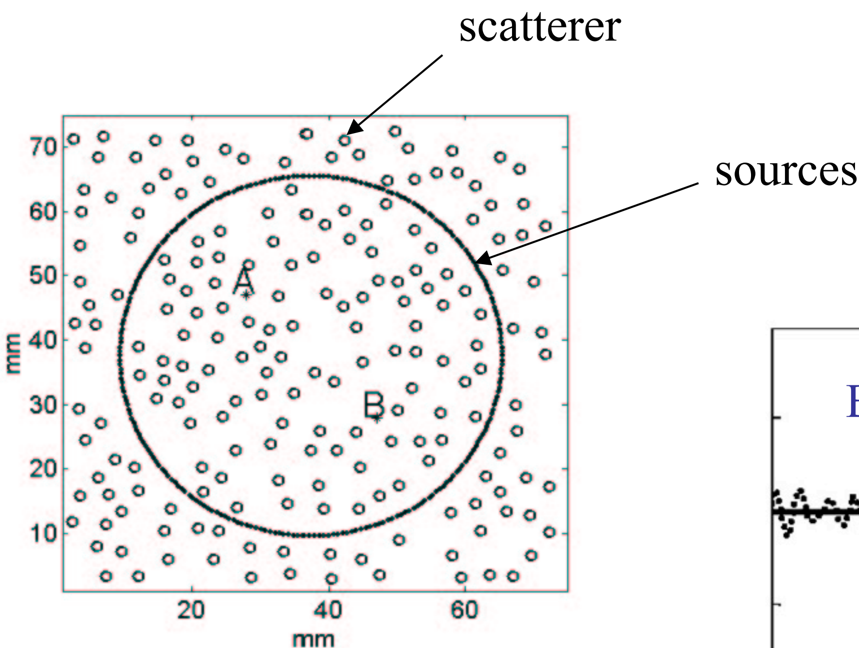


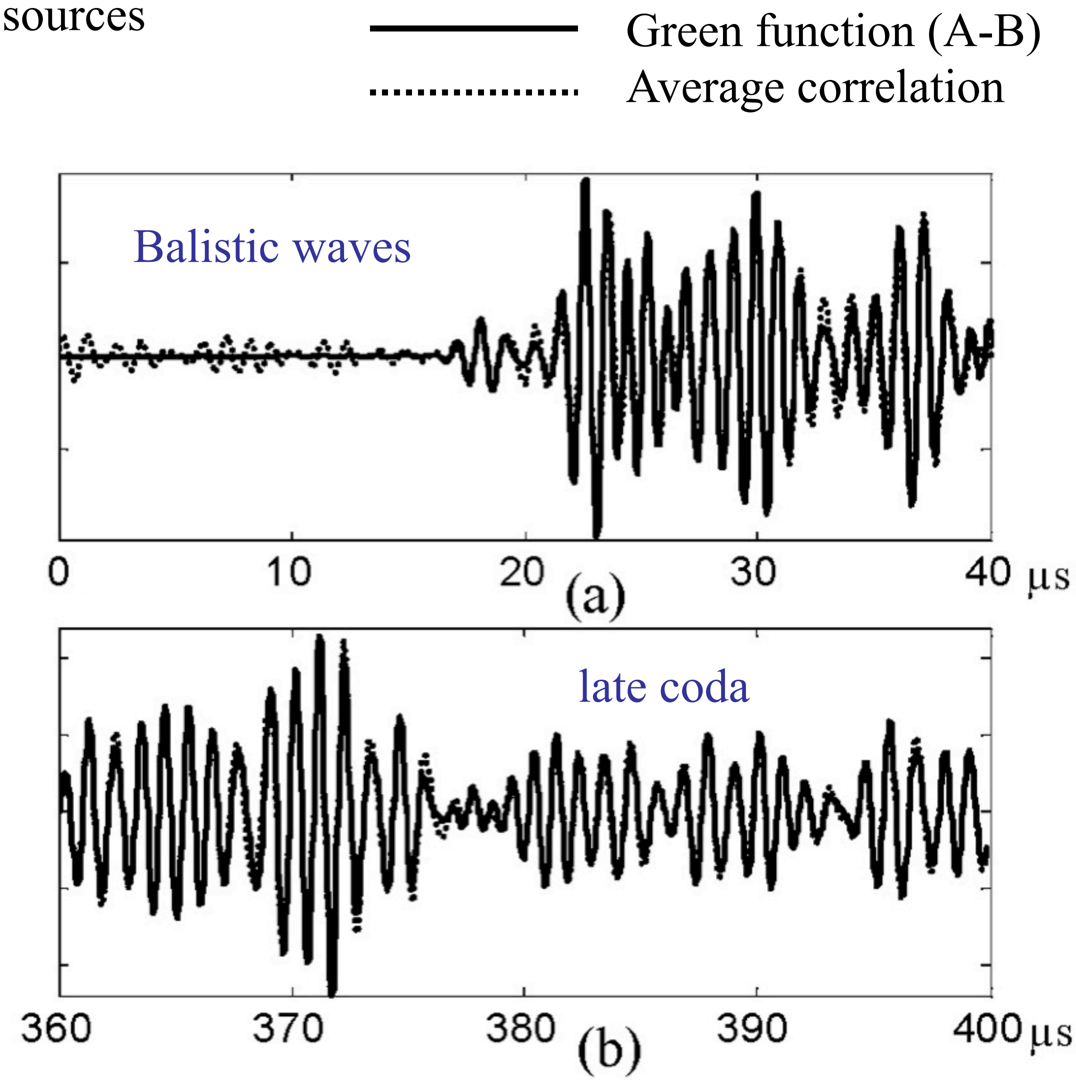
$$G_{22}(r,\omega)=\frac{1}{4\mu}\left\{-Y_0\left(\frac{\omega r}{c}\right)-iJ_0\left(\frac{\omega r}{c}\right)\right\}$$

$$J_0\left(\frac{\omega r}{c(\omega)}\right)=-4\mu\,\mathrm{Im}\big(G_{22}(r,\omega)\big) \qquad r=\left|P,Q\right|$$

$$\boxed{\boxed{\mathrm{Im}\left(G_{22}^{PQ}\right)=\frac{-1}{4\mu}\left\langle\frac{u_2(P)u_2^*(Q)}{\left|u_2(P)\right|\left|u_2(Q)\right|}\right\rangle}}$$

A numerical experiment with an open medium (absorbing boundaries):





Physical interpretations

Time reversal

Rays and stationary phase

Correlation vs Time Reversal

- C source
- A and B receivers

- Correlation :

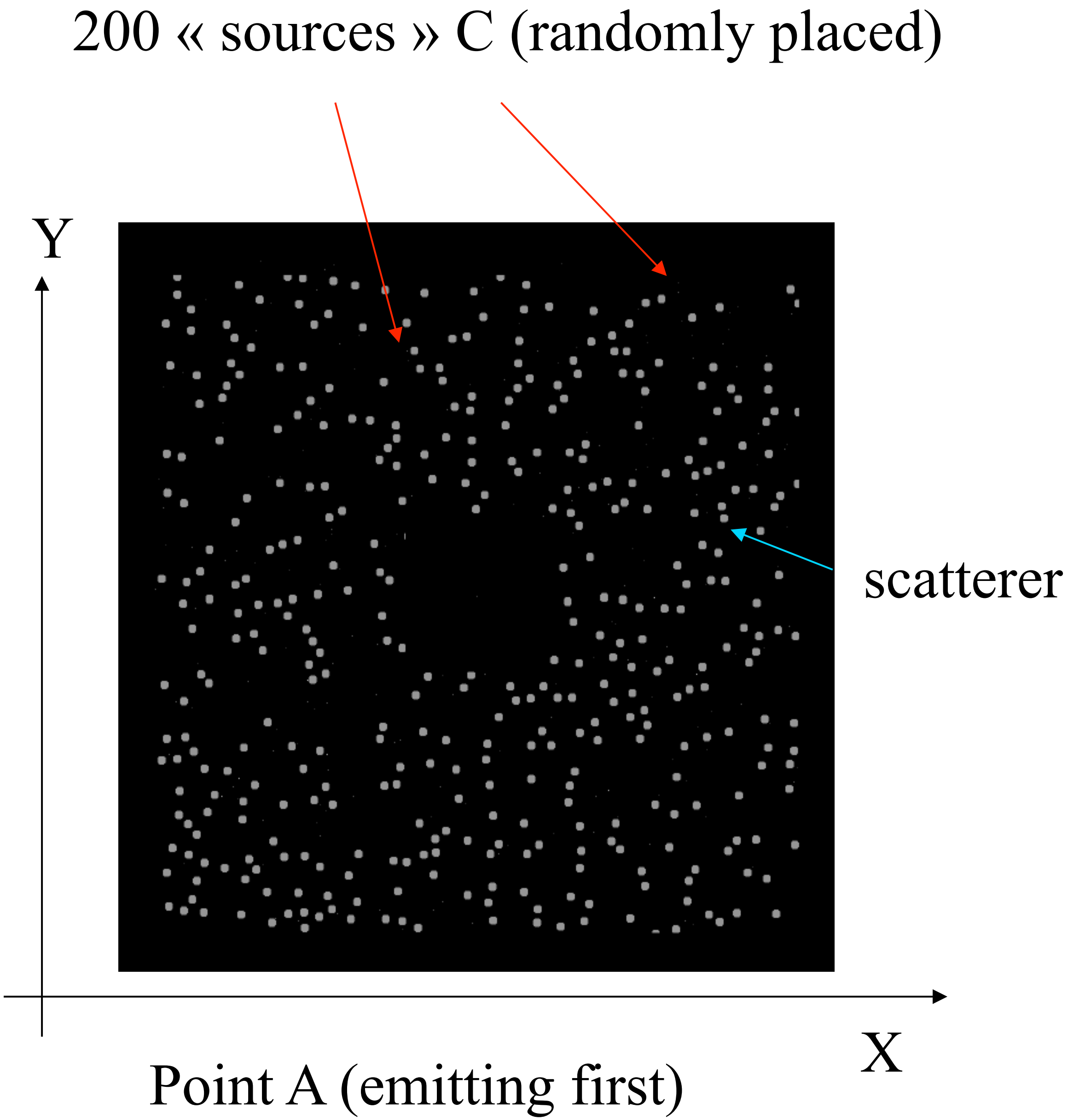
$S_{CA}(t) \times S_{CB}(t)$

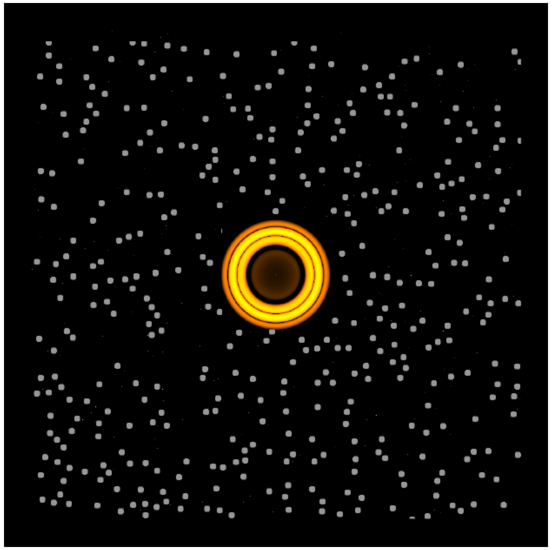
- A source
- C receiver
- C emits time-reversed field
- B receiver

- Convolution :

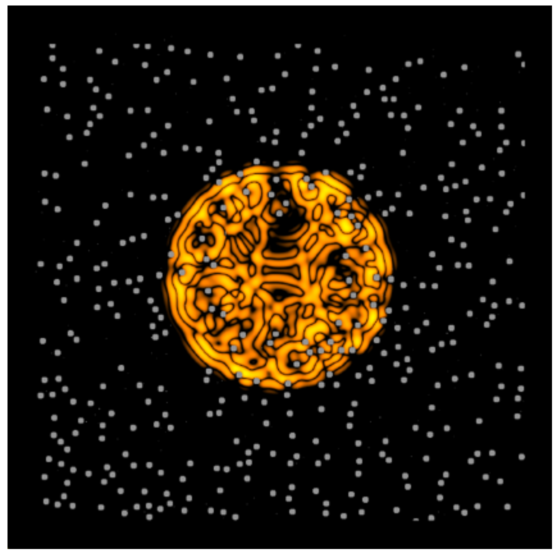
$S_{CA}(t) \otimes S_{CB}(-t)$

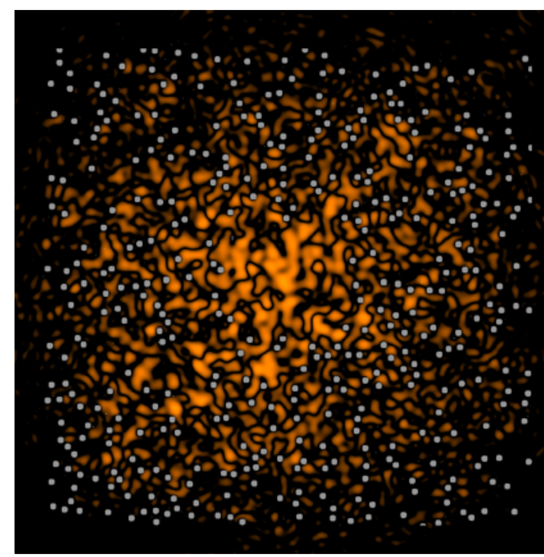
Numerical 2D FD simulation



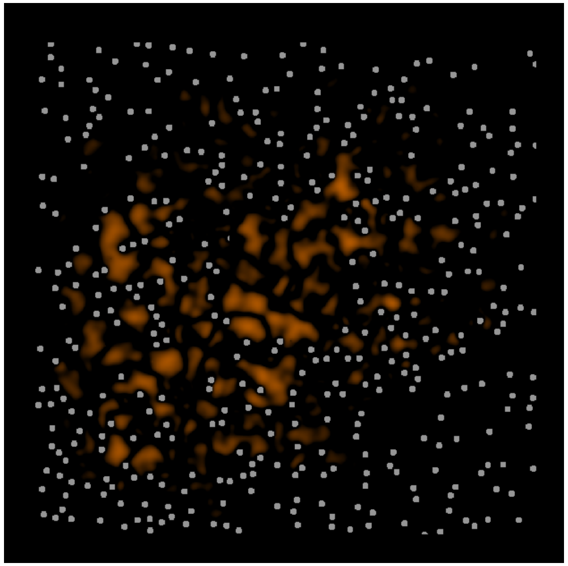


A pulse is emited in A
and recorded at point randomly
distributed

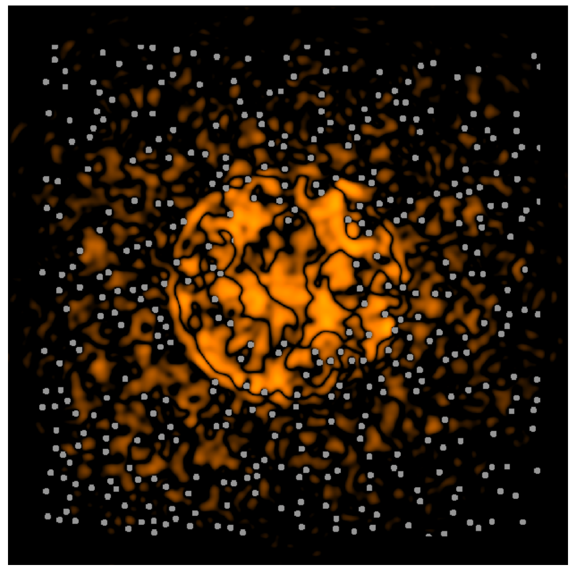




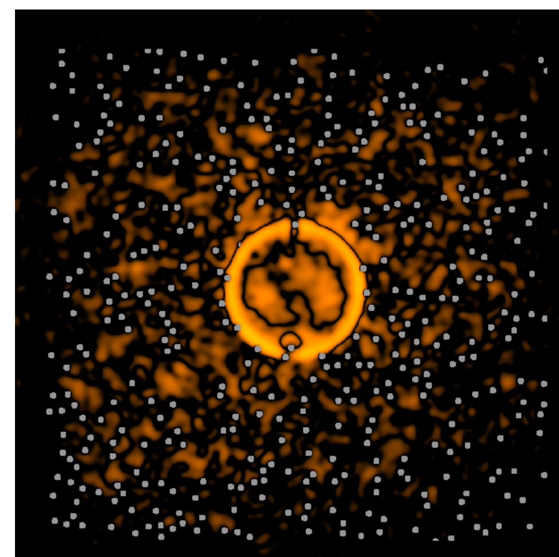
time



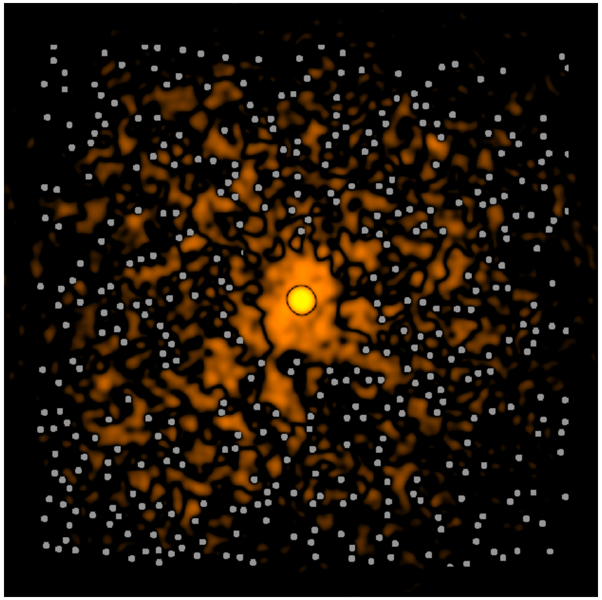
Re-emission from the points ‘C’
of the time-reversed signals
(map of cross-correlations)



Constructive
interferences of time-
reversed field

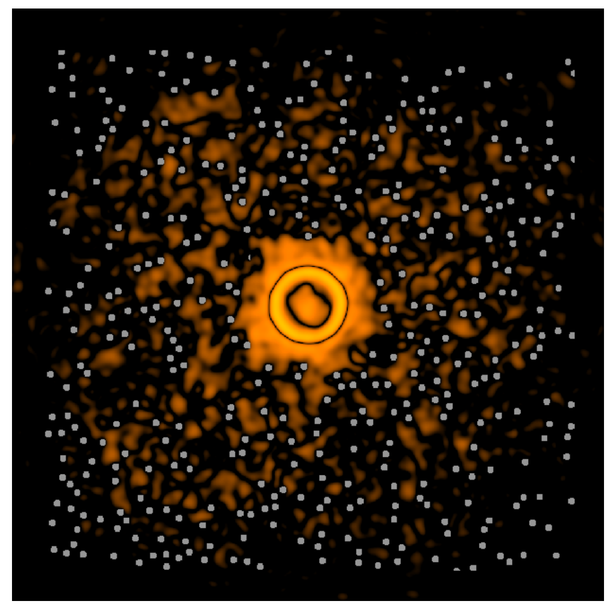


Converging field
: G(-t)

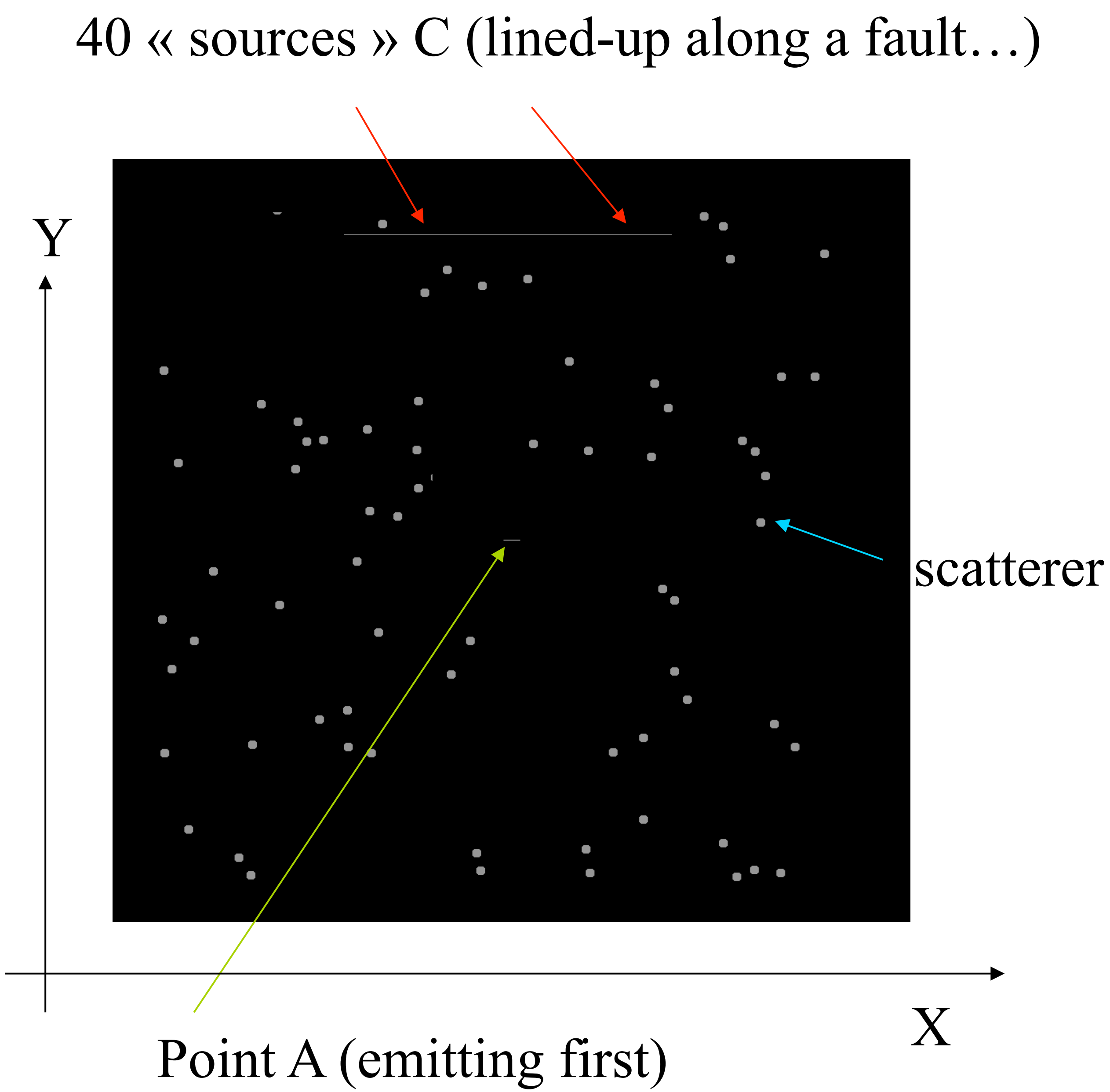


Nearly perfect refocalisation

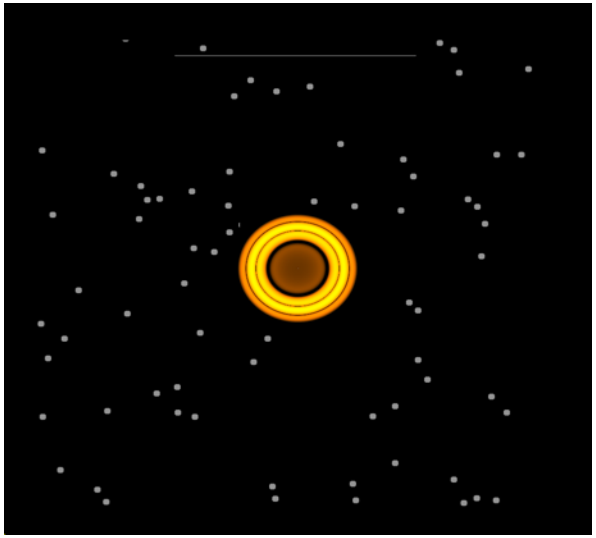
Re-emission from A :
G (t)



A more realistic configuration of sources

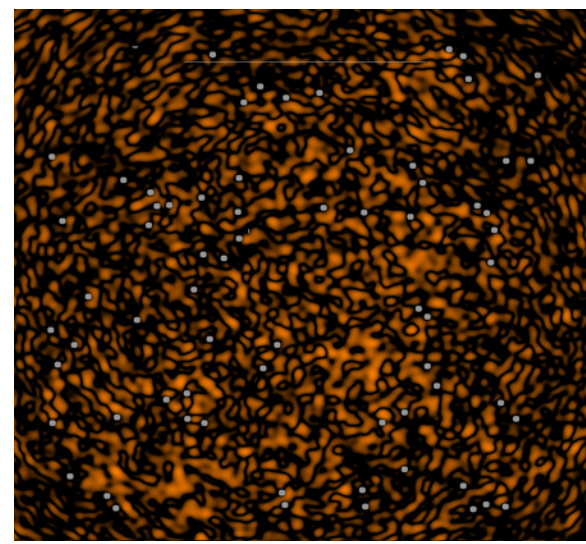


Time reversal experiment

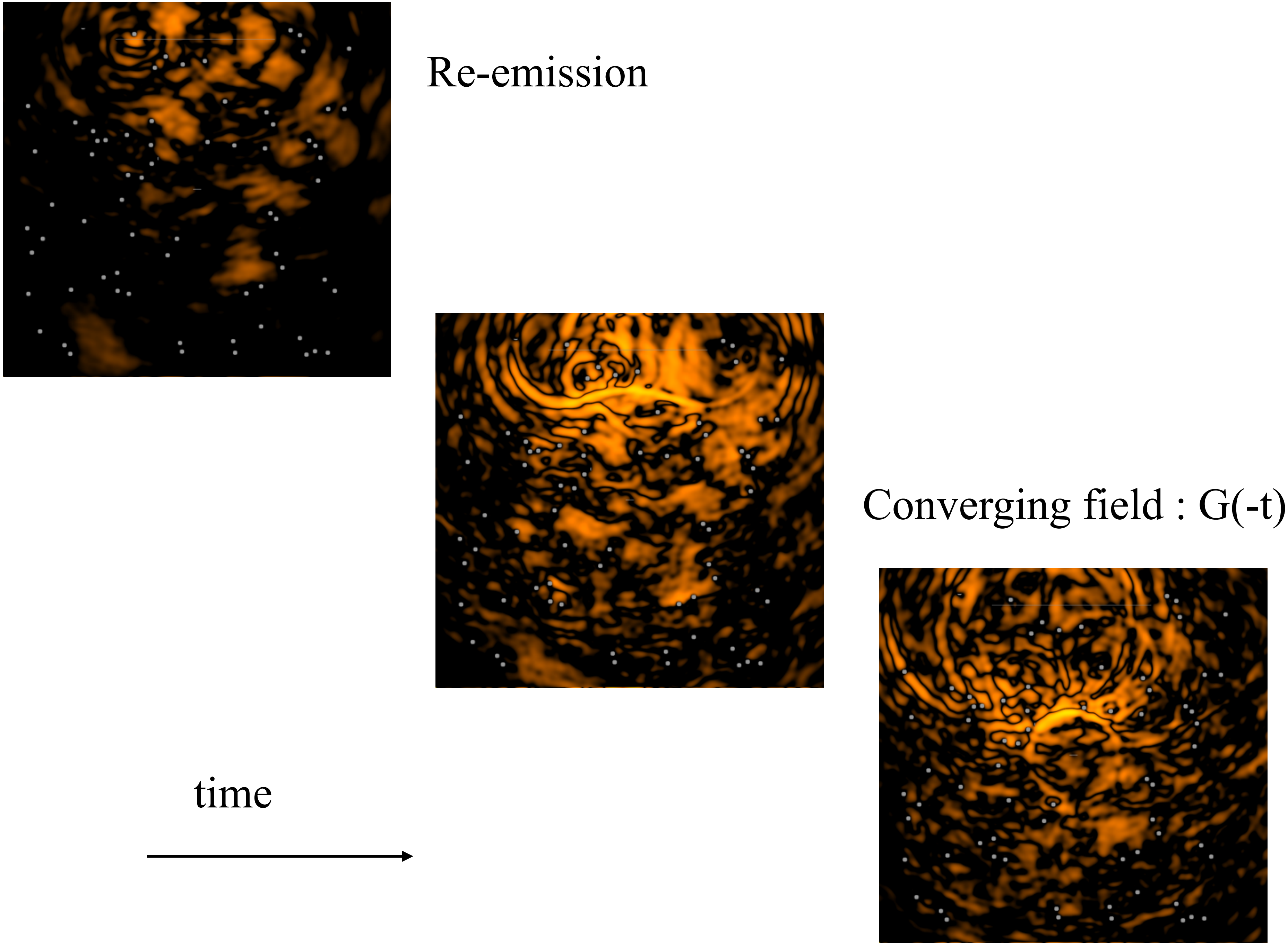


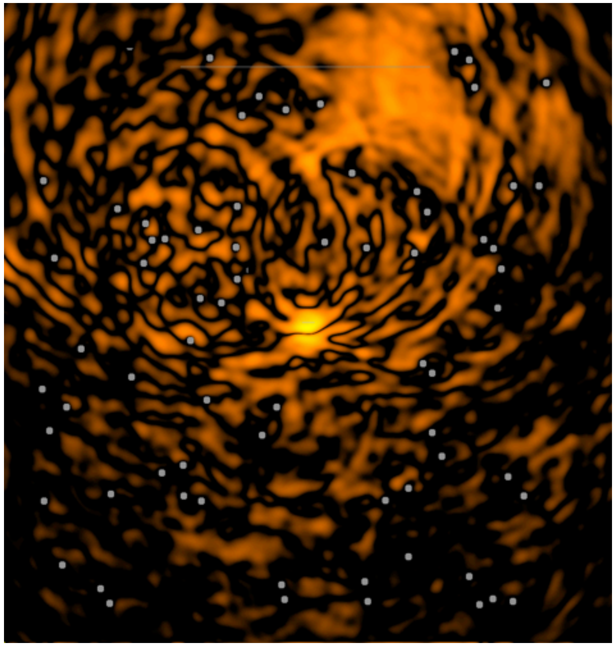
A send a pulse

Scattering effects



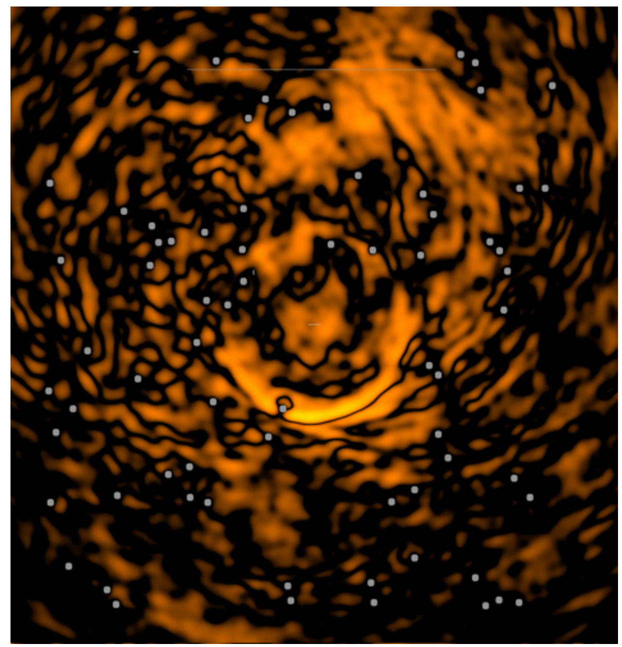
Diffuse field is
also recorded


Re-emission
Converging field : G(-t)
time



Partial focalisation

Diverging field : $h_{AB}(t)$



The symmetry of the Green function is lost!