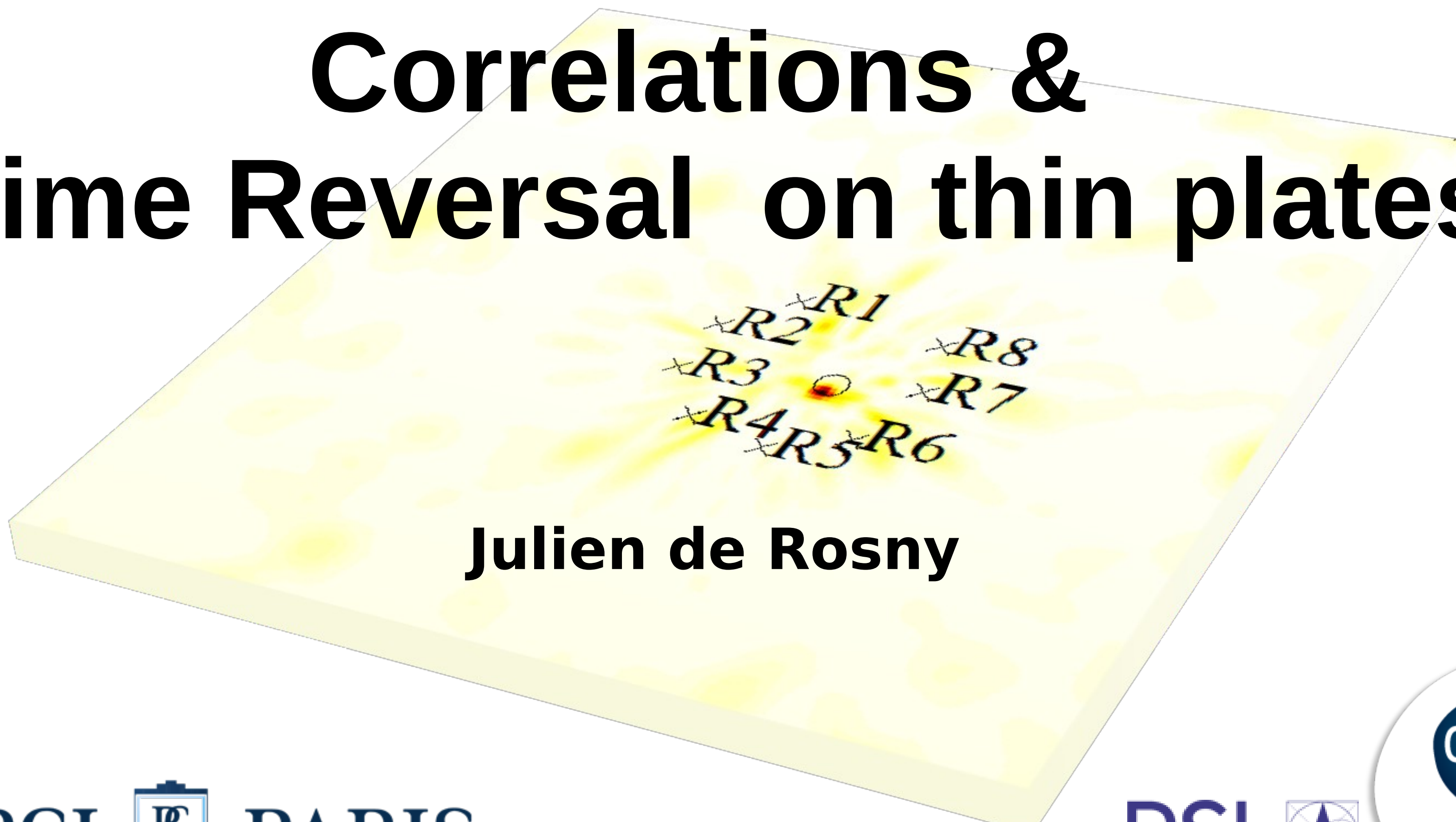
Institut Langevin
ONDES ET IMAGES

Correlations & Time Reversal on thin plates



Julien de Rosny

ESPCI PARIS
ÉCOLE SUPÉRIEURE DE PHYSIQUE ET DE CHIMIE INDUSTRIELLES DE LA VILLE DE PARIS

PSL RESEARCH UNIVERSITY PARIS

cnrs dépasser les frontières

Outline

1) A first « live » demonstration

2) A quick review

3) Relation with time reversal

4) Convergence

5) Application to passive structural heath monitoring

Collaboration

Aida Hejazi
Thèse - ISTP & Langevin

Lynda Chehami
Thèse - IEMN & Langevin

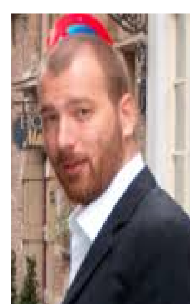
ANR : PASNI

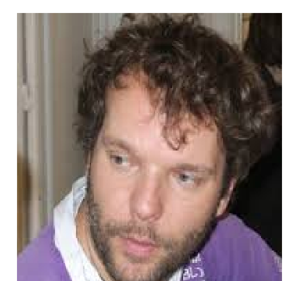
FINANCÉ PAR ANR

European Commission

Lapo Boshi : ISTP – P&M Curie
Philippe Roux : ISTERRE – Univ. Grenoble
Emmanuel Moulin : IEMN – Univ. Valenciennes and Hainaut-Cambresis
Julien de Rosny, Claire Prada : Institut Langevin - ESPCI
Eric Chatelet, Francesco Massi, Giovanna Lacerra :
LaMCoS – INSA LYON



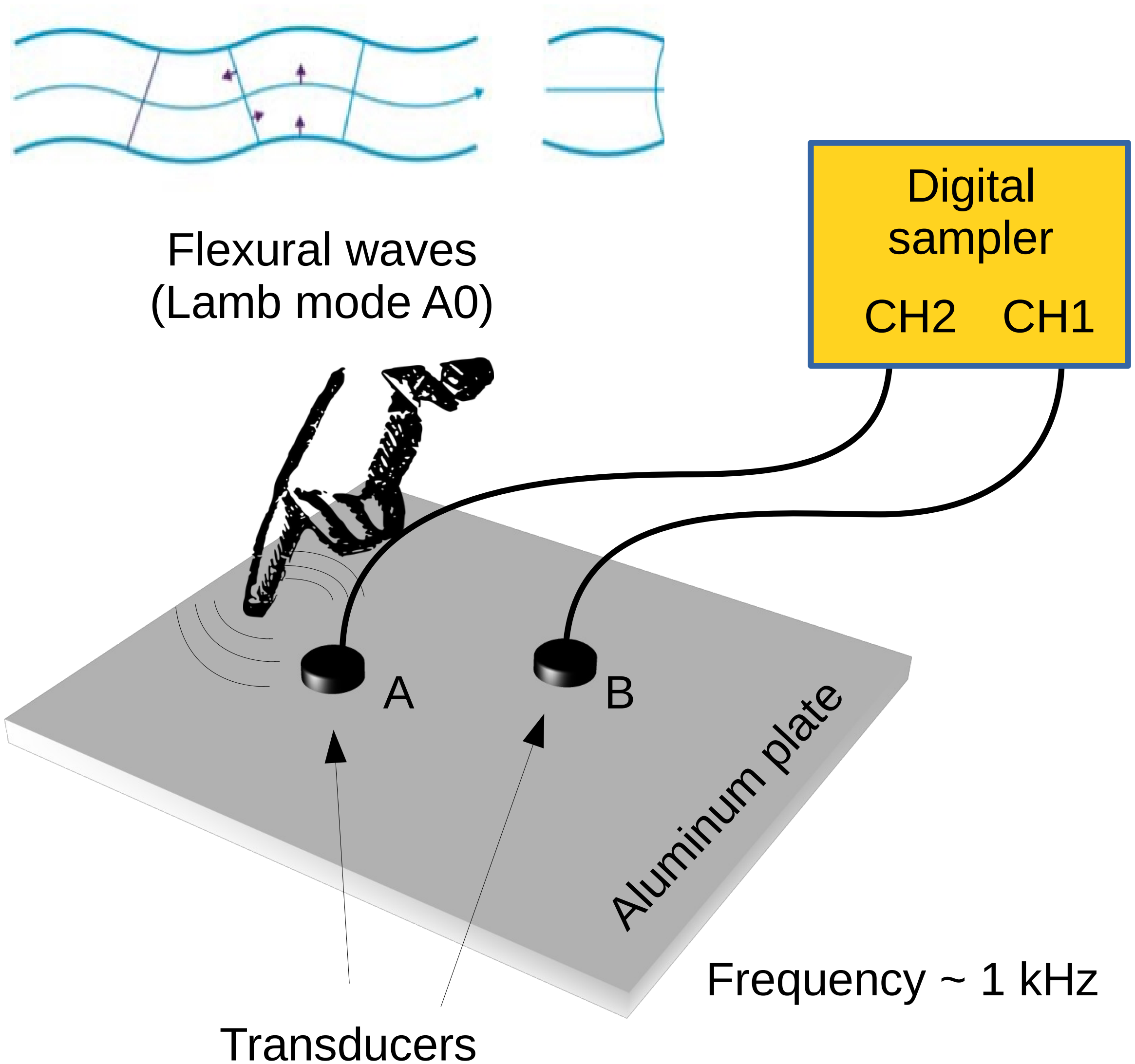



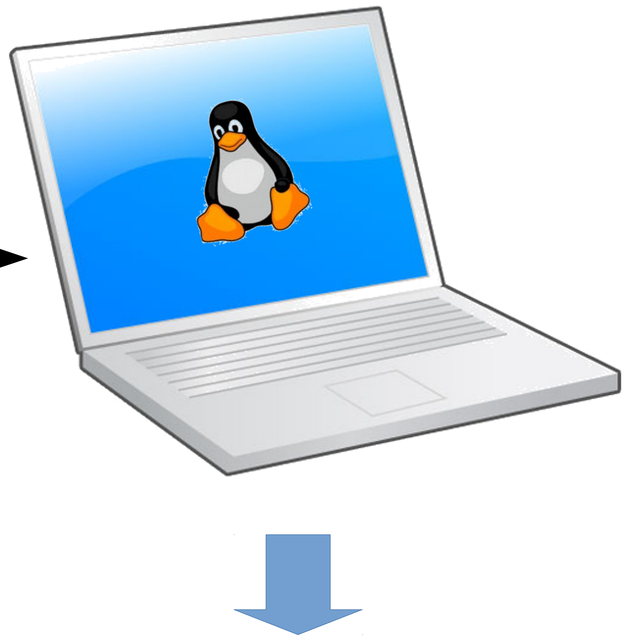


ESPCI PARIS
ÉCOLE SUPÉRIEURE DE PHYSIQUE ET DE CHIMIE INDUSTRIELLES DE LA VILLE DE PARIS

cnrs dépasser les frontières

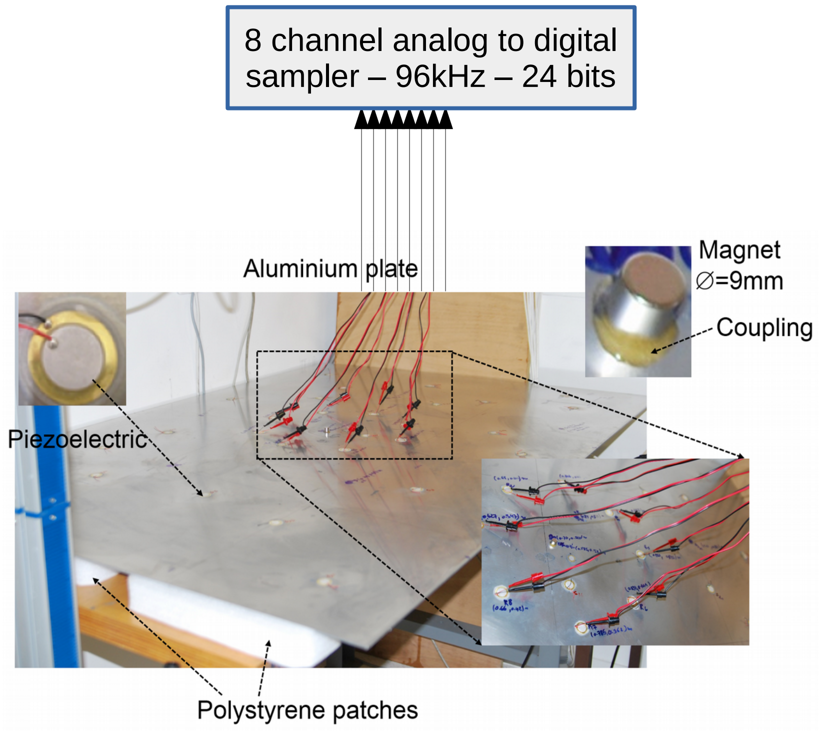
A live demonstration

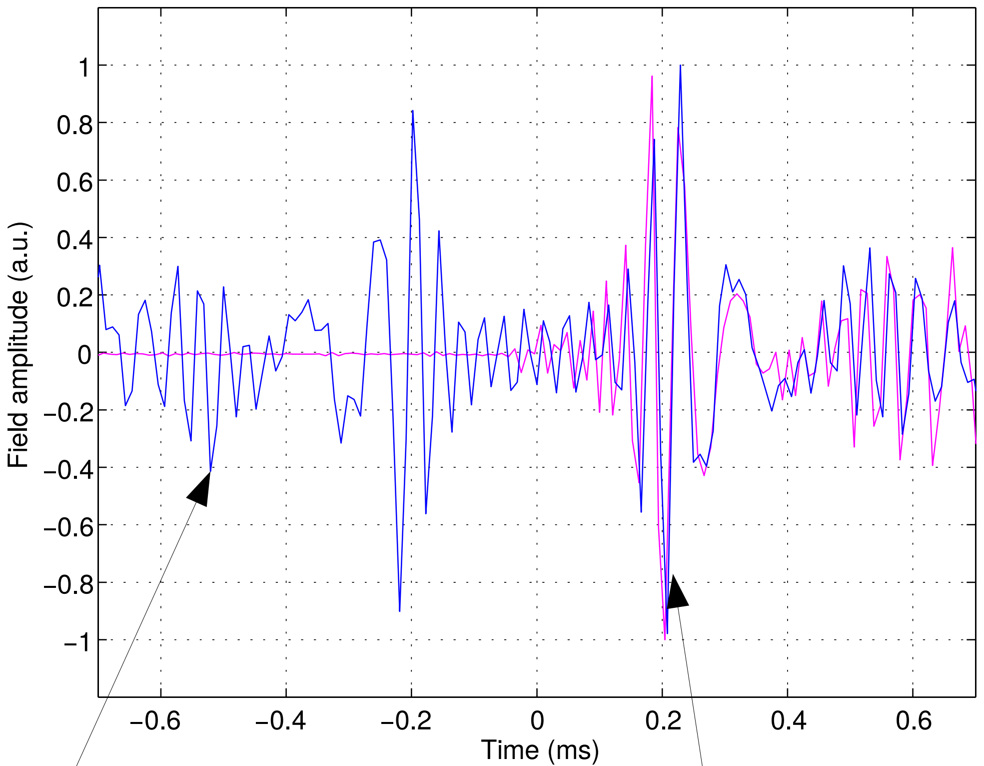




Real time cross-correlation
$C(A,B,t)=S_1(t)\otimes S_2(-t)$
between channel A and B

→ Reconstruction between two points





$$\partial_t C_{AB}(t) \propto [G(A,B,-t) - G(A,B,t)]$$

Noise filtered between 1kHz and 40kHz

Previous works

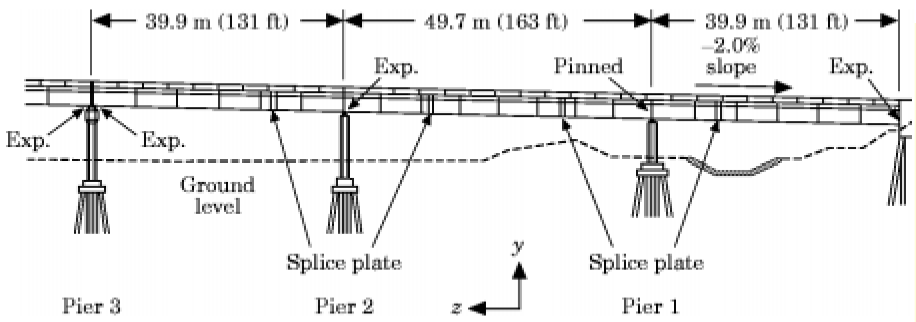
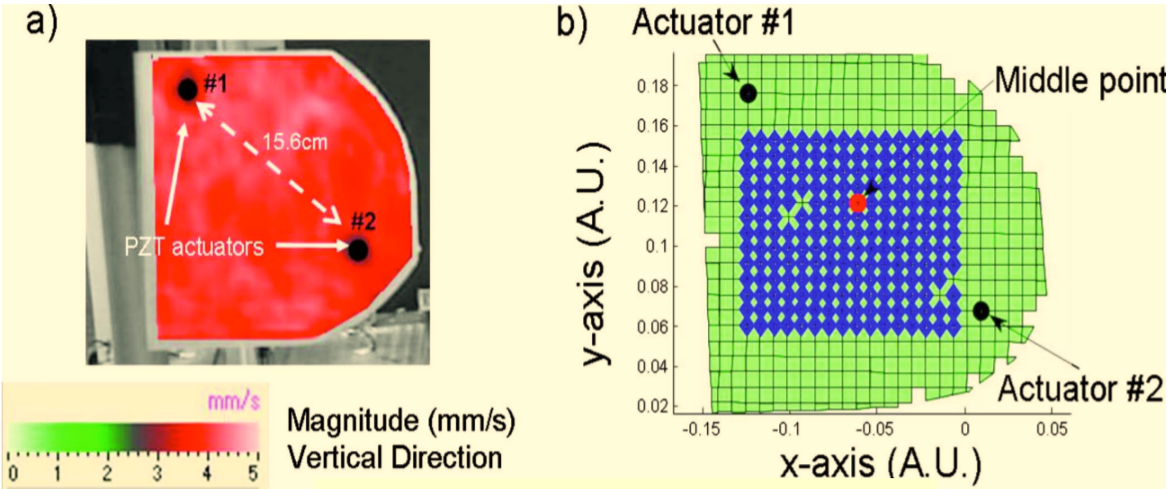
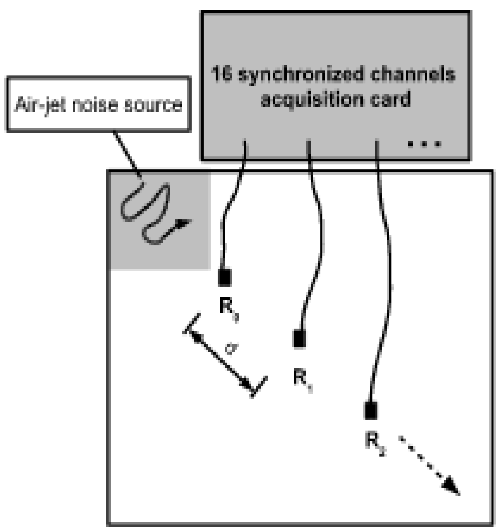


Figure 1. An elevation view of the portion of the eastbound bridge that was tested

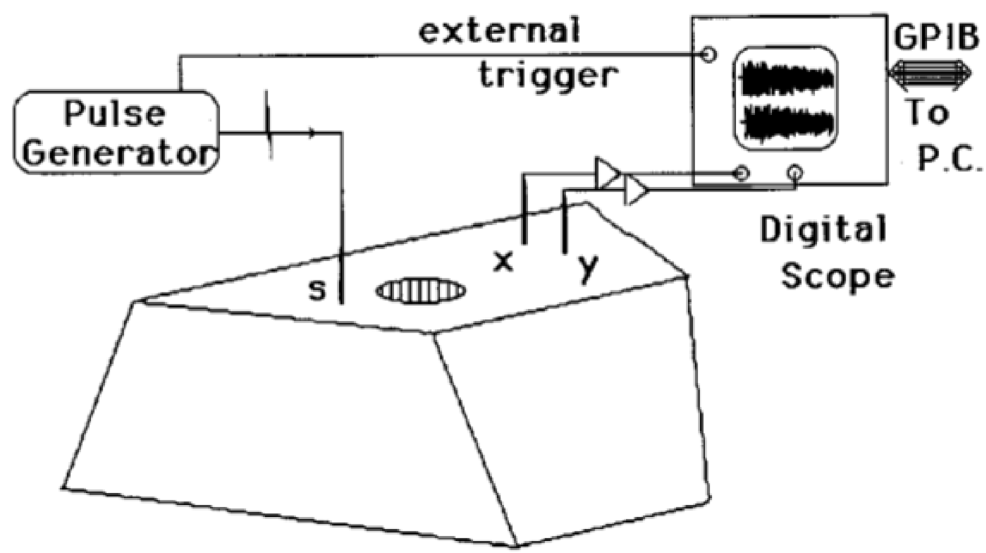
Farrar et al., JSV 1997



Duroux, Sabra et al, JASA 2010

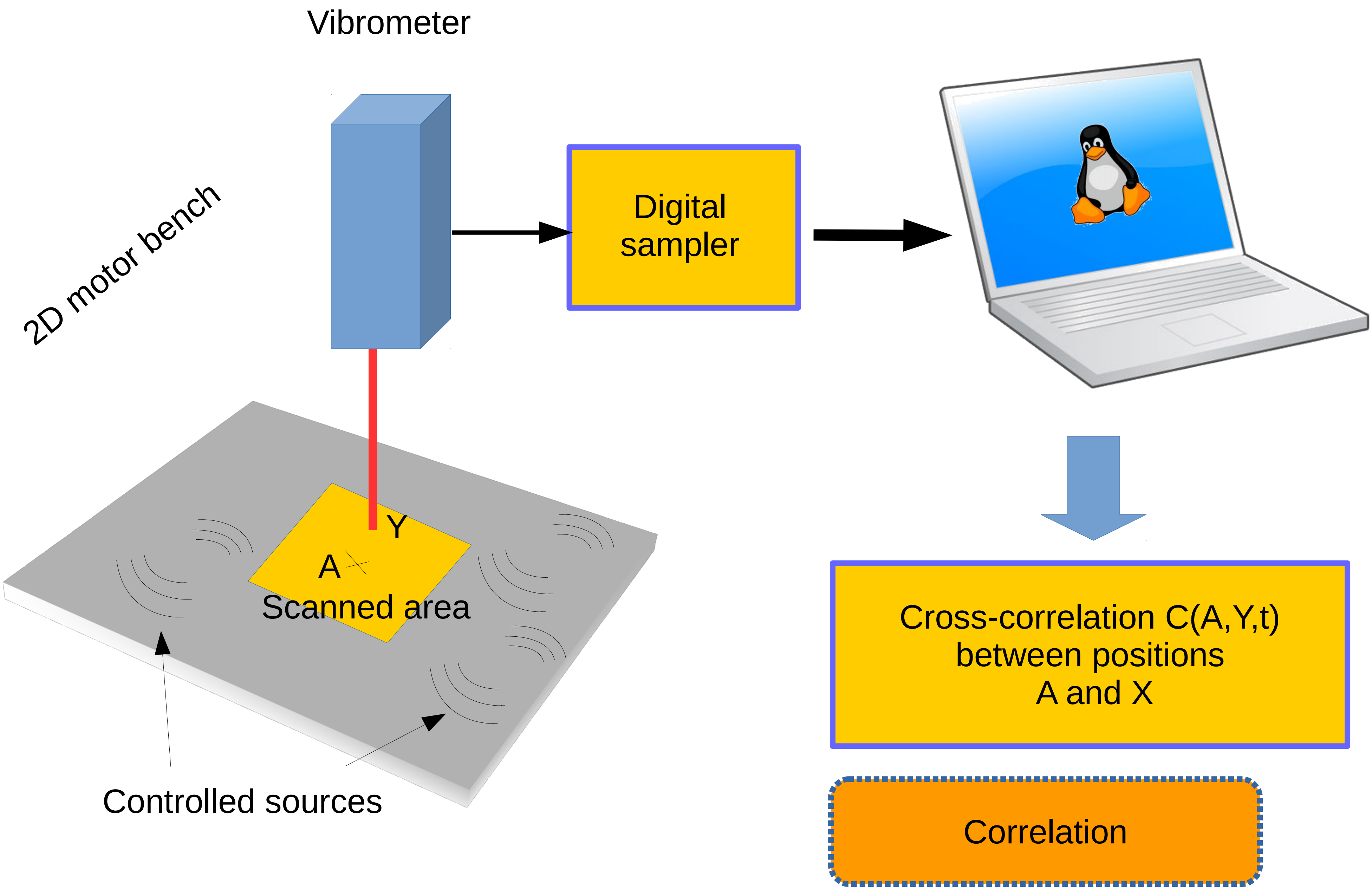


Larose et al., JASA 2009



Lobkis & Weaver., JASA 2001

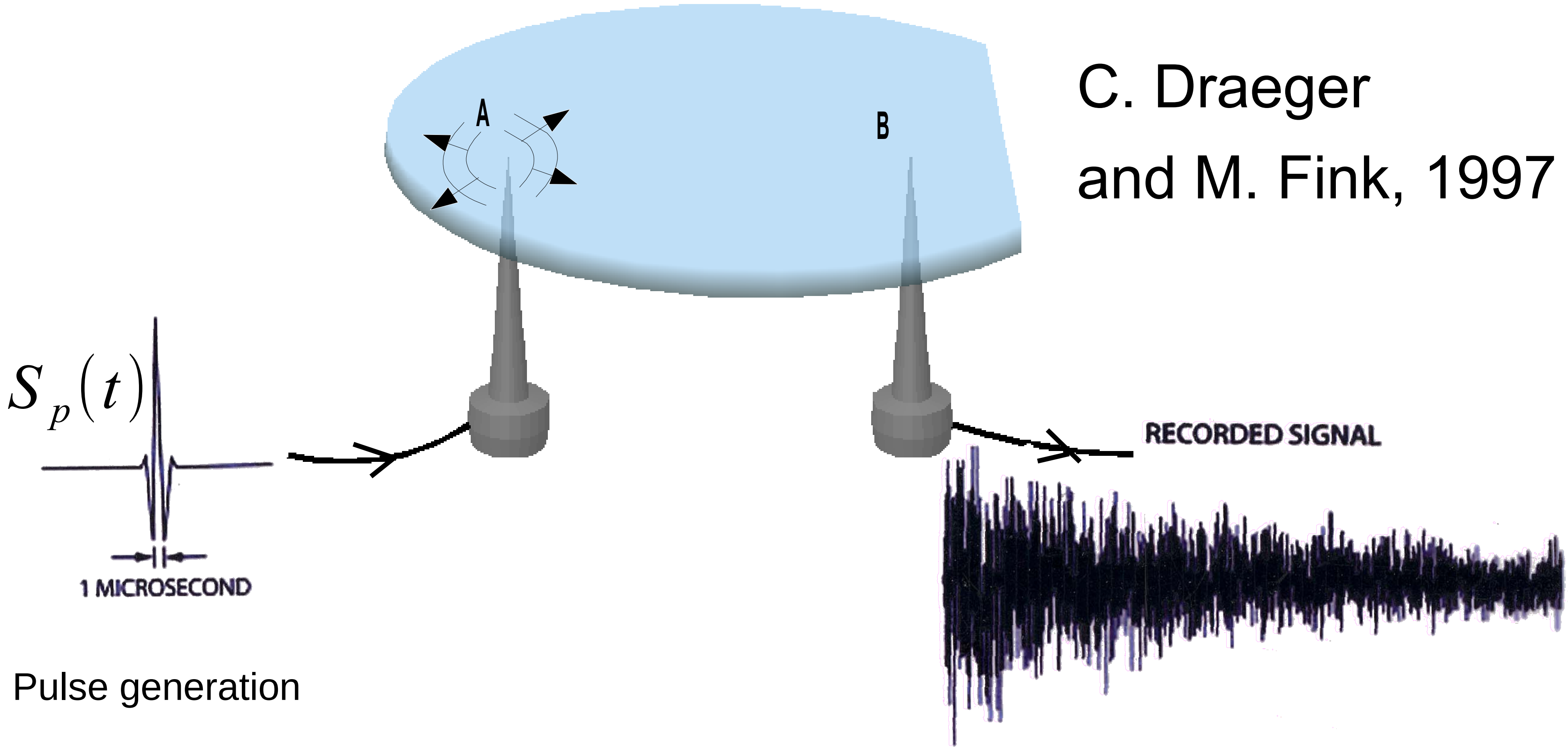
Spatial reconstruction ?



Relationship with time-reversal

Forward step of time reversal process in a cavity

C. Draeger and M. Fink, 1997

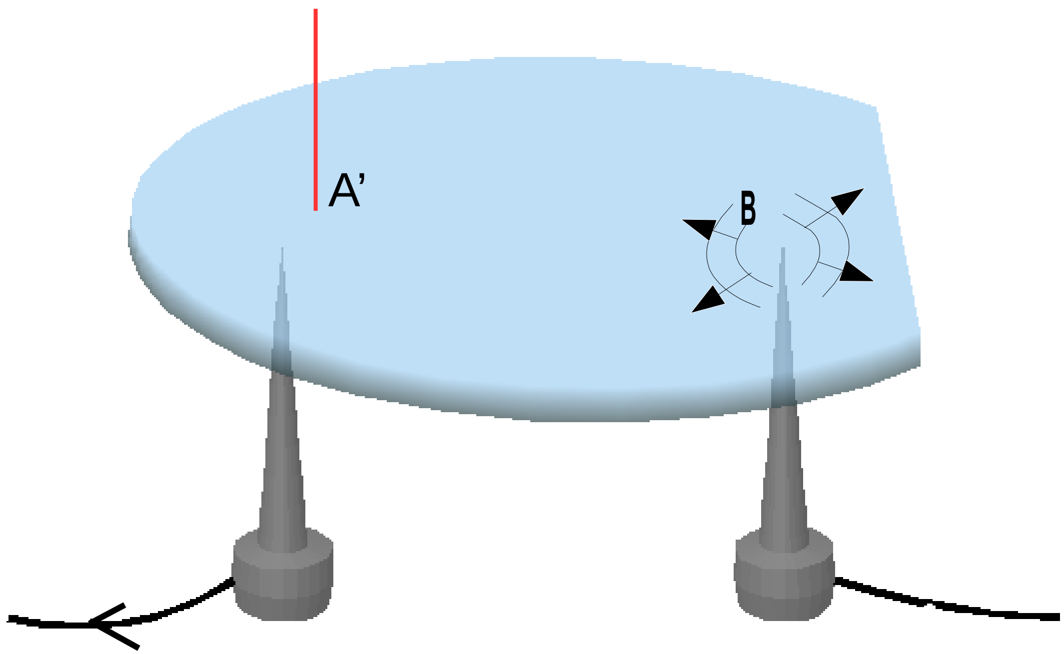


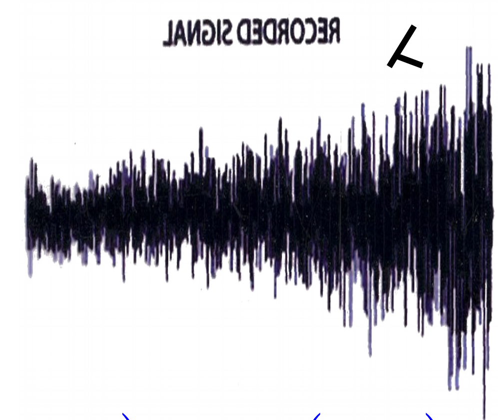
Pulse generation

$$G(r_B, r_A; t) \otimes S_p(t)$$

A taste of linear signal processing

Backward step of time reversal process in a cavity





$$G(r_A', r_B; t) \otimes S_{TR}(-t)$$

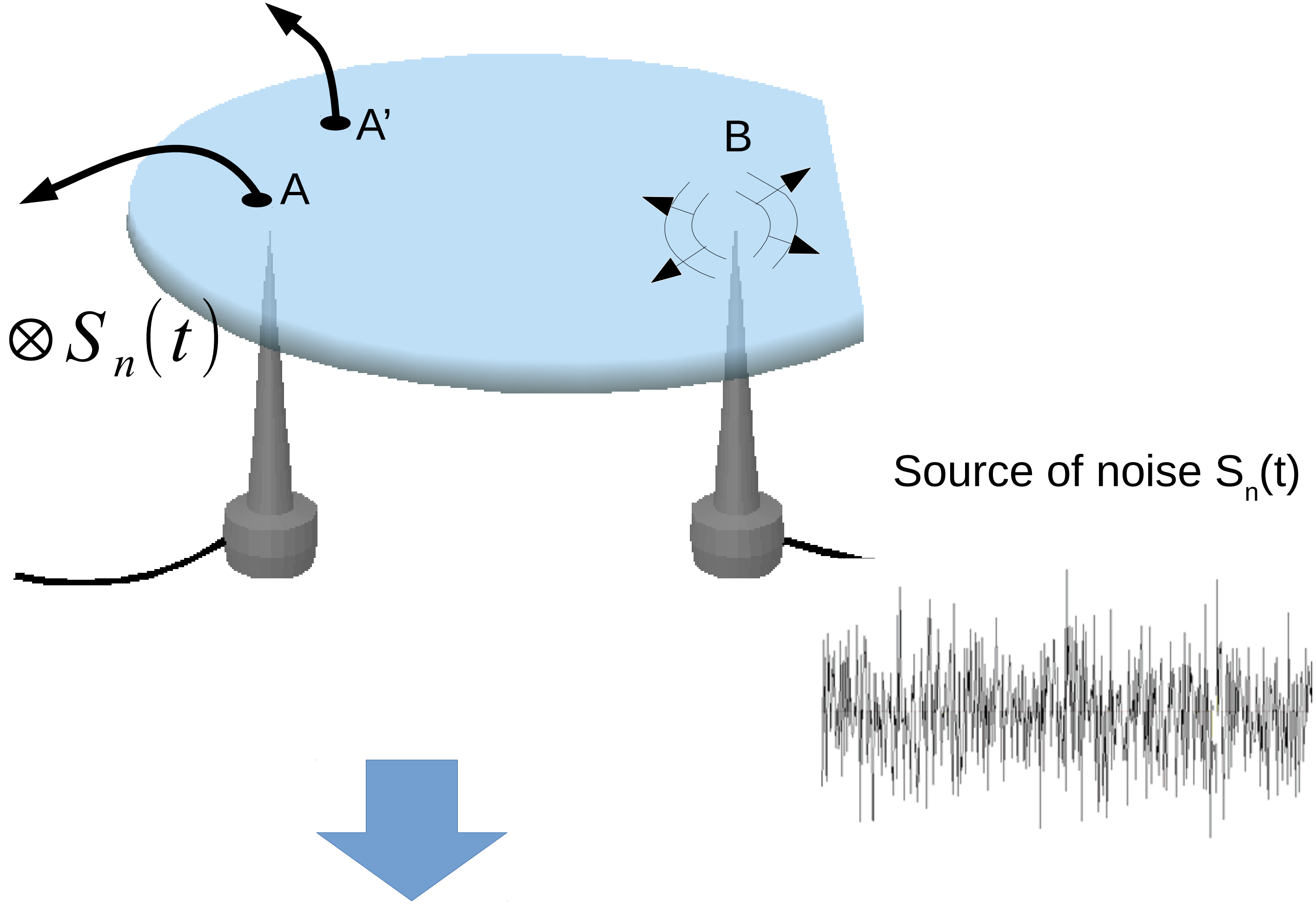
$$S_{TR}(t) = G(r_B, r_A; -t) \otimes S_p(-t)$$

$$\Psi_{TR} = G(r_A', r_B; t) \otimes G(r_B, r_A; -t) \otimes S_p(-t)$$

Noise correlation

$$R_{A'}(t)=G(r_{A'},r_B;t)\otimes S_n(t)$$

Correlation



$$R_A(t)=G(r_A,r_B;t)\otimes S_n(t)$$

$$C(A,A',t)=R_A(t)\otimes R_{A'}(t)$$

$$=G(r_A,r_B;t)\otimes G(r_A',r_B;-t)\otimes S_n(-t)\otimes S_n(t)$$

Time reversal vs Correlations

$$\psi_{RT}(B;t)=\sum_i G(r_{Bi},r_A;-t)\otimes G(r_{A'},r_{Bi};t)\otimes S_p(-t)$$

$$C(A,B;t)=\Delta T\sum_i G(r_A,r_{Bi};-t)\otimes G(r_A',r_{Bi};t)\otimes S_n(t)\otimes S_n(-t)$$

Medium réciprocity

$$G(r_A,r_{Bi};t)=G(r_{Bi},r_A;t)$$

Autocorrelation of pink noise of same bandwidth than $S_p(t)$

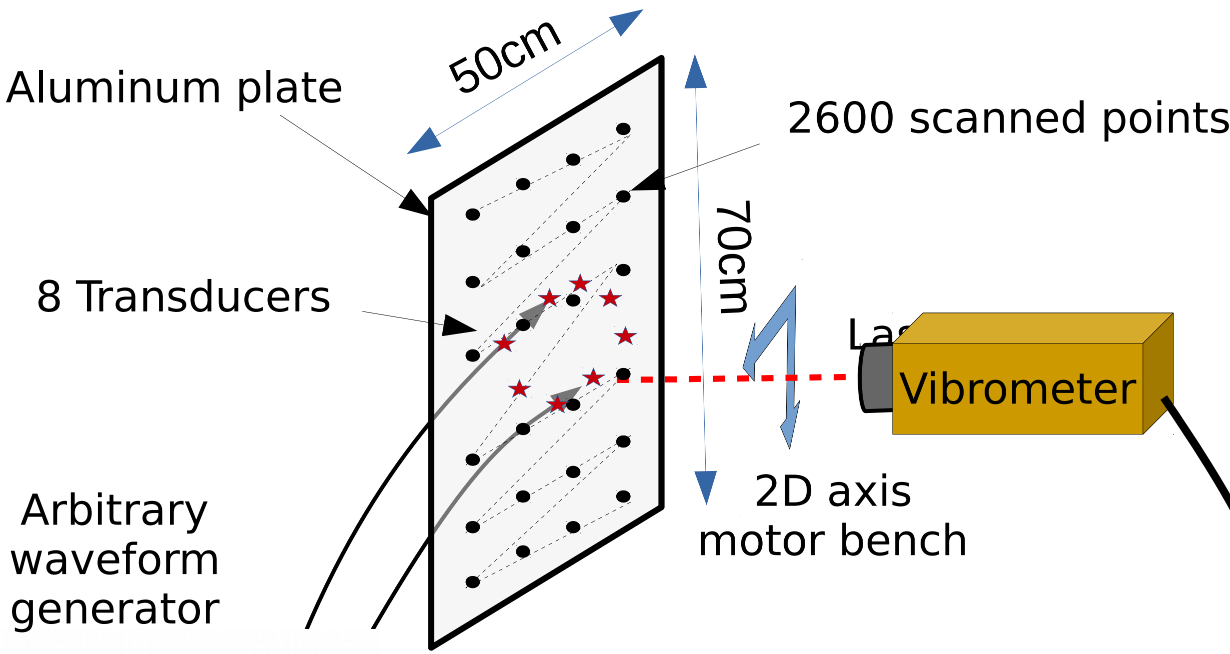
$$S_N(t)\otimes S_N(-t)\propto S_p(t)$$

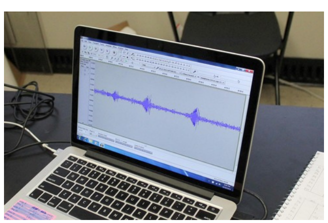
$$\psi_{RT}(A';t)\propto C(A,A',t)$$

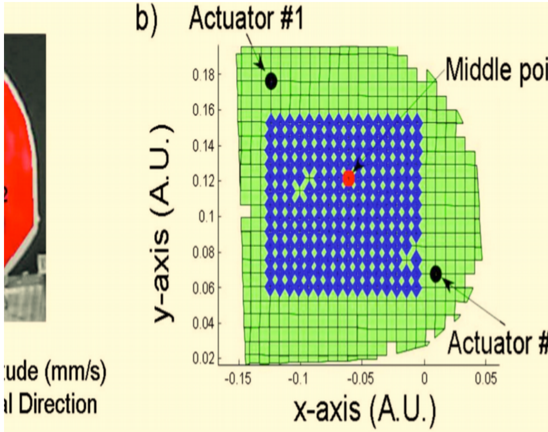
Derode et al., JASA, APL 2003

→ Time Reversal equivalent to correlation

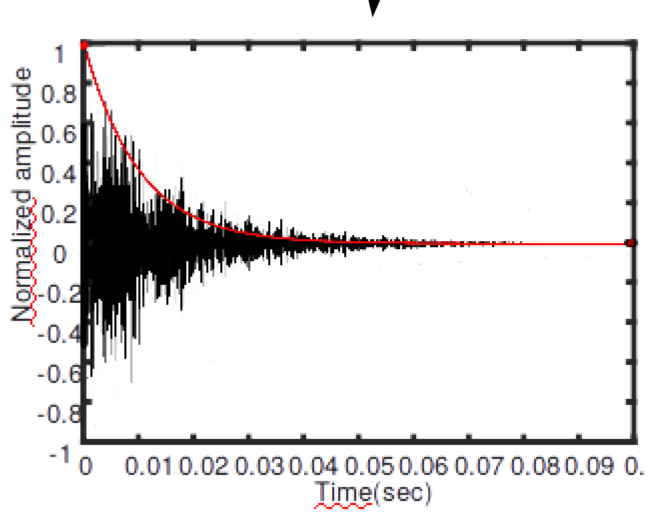
Convergence of the correlation toward Green's function







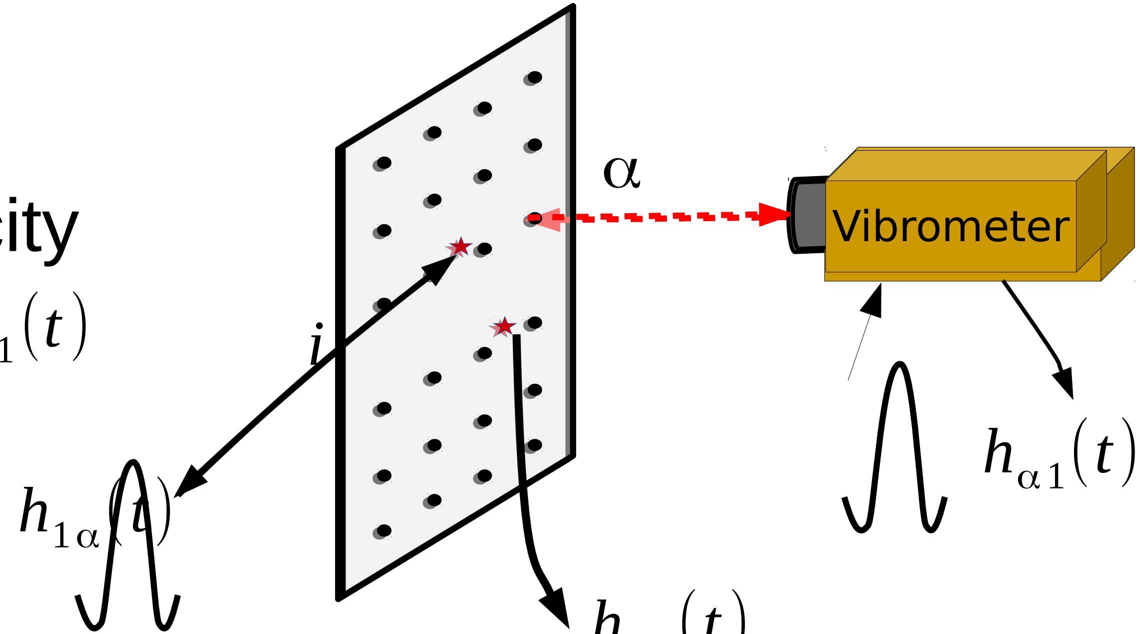
Duroux, Sabra et al.
JASA 2010



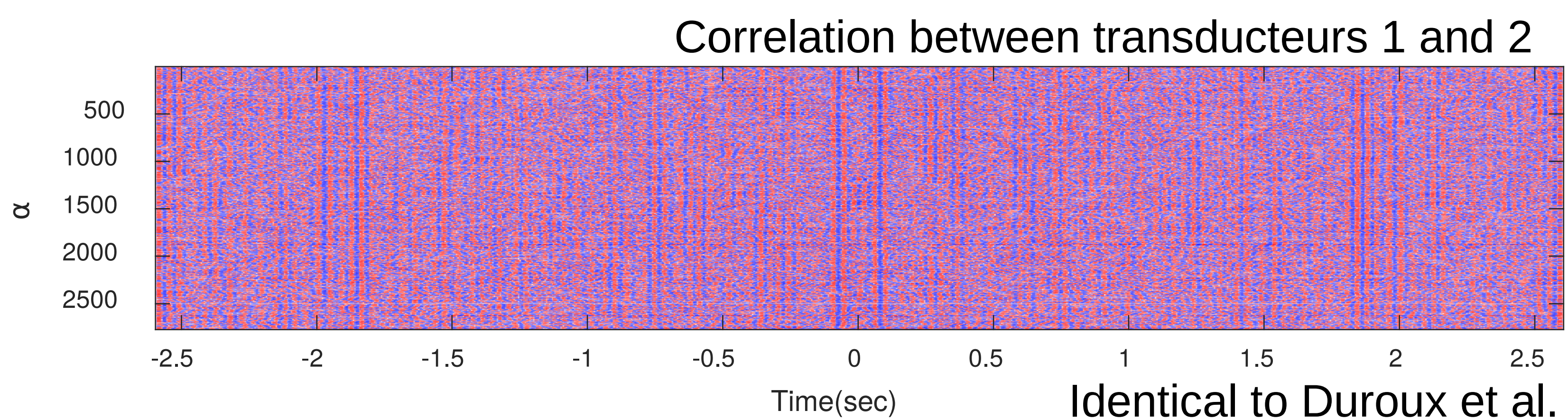
Record of 2600x8 impulses responses

Reciprocity

$$h_{1\alpha}(t) = h_{\alpha 1}(t)$$



$$C_{12}(t) = h_{1\alpha}(t) \otimes h_{2\alpha}(-t)$$

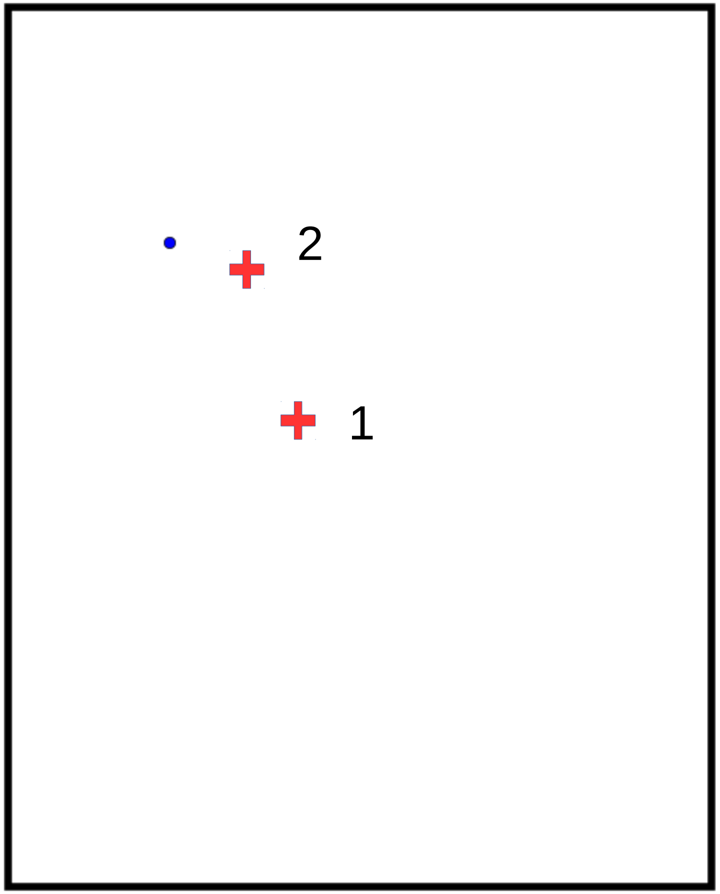


Identical to Duroux et al. 2009

Green's function recovering quality

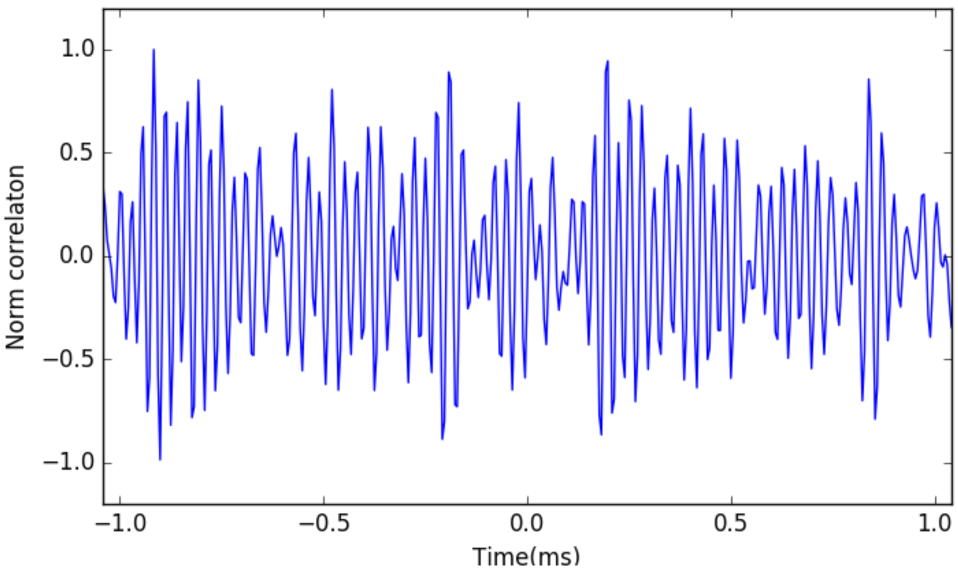
Stacking over noise sources

$$C_{12}^{N}(t)=\sum_{\alpha=1}^{N} h_{1\alpha}(t)\otimes h_{2\alpha}(-t)$$



Number of noise sources : 1

Correlation



Convergence toward a symmetric waveform

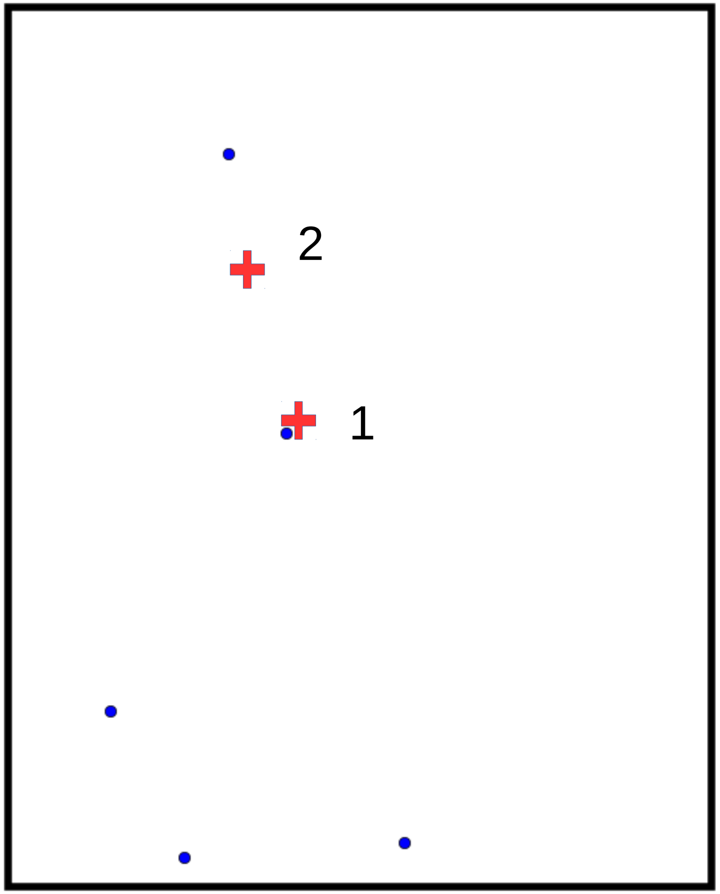
Green's function recovering quality

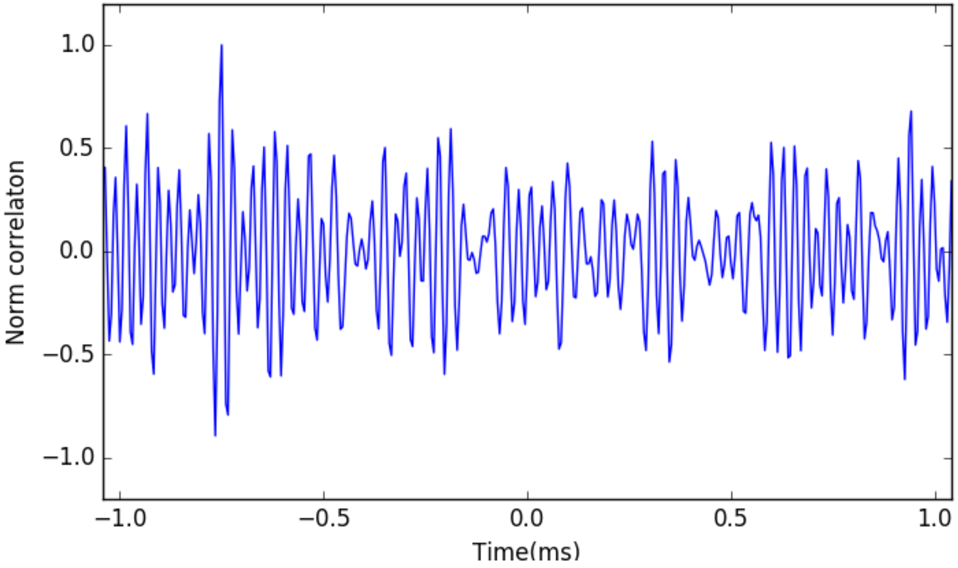
Stacking over noise sources

$$C_{12}^{N}(t)=\sum_{\alpha=1}^{N} h_{1\alpha}(t)\otimes h_{2\alpha}(-t)$$

Number of noise sources : 5

Correlation





Convergence toward a symmetric waveform

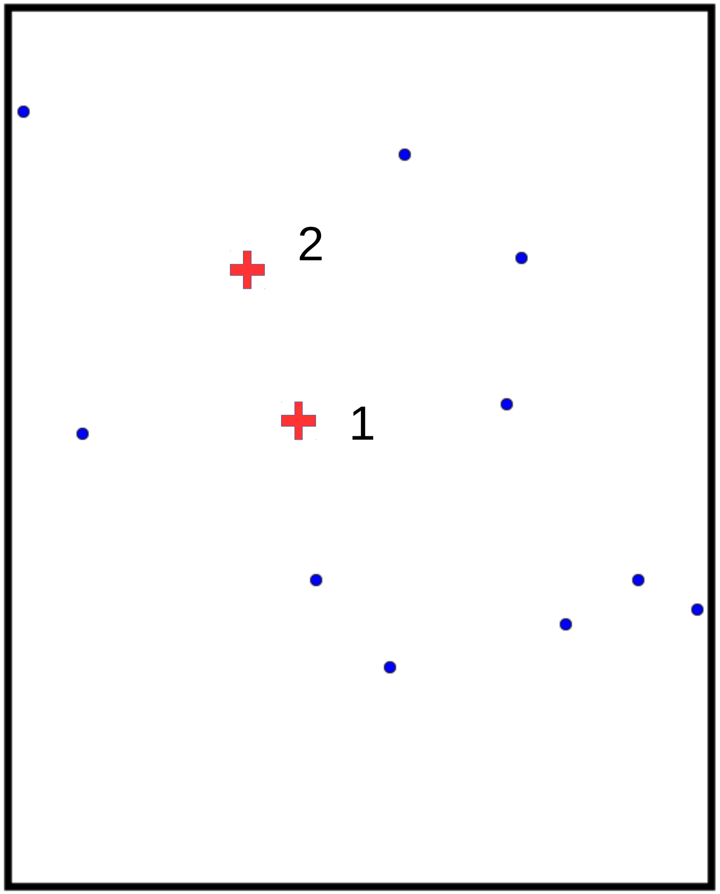
Green's function recovering quality

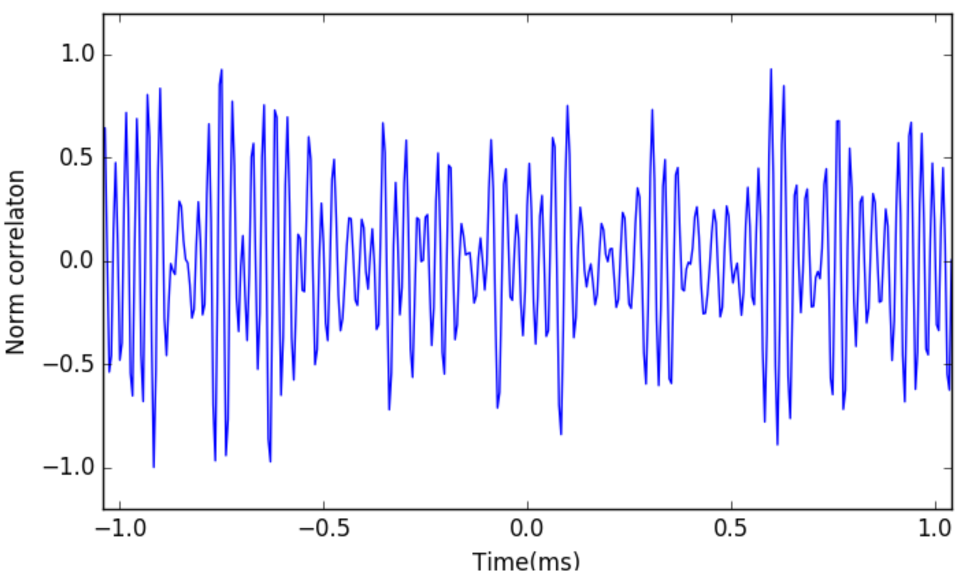
Stacking over noise sources

$$C_{12}^{N}(t)=\sum_{\alpha=1}^{N} h_{1\alpha}(t)\otimes h_{2\alpha}(-t)$$

Number of noise sources : 10

Correlation





Convergence toward a symmetric waveform

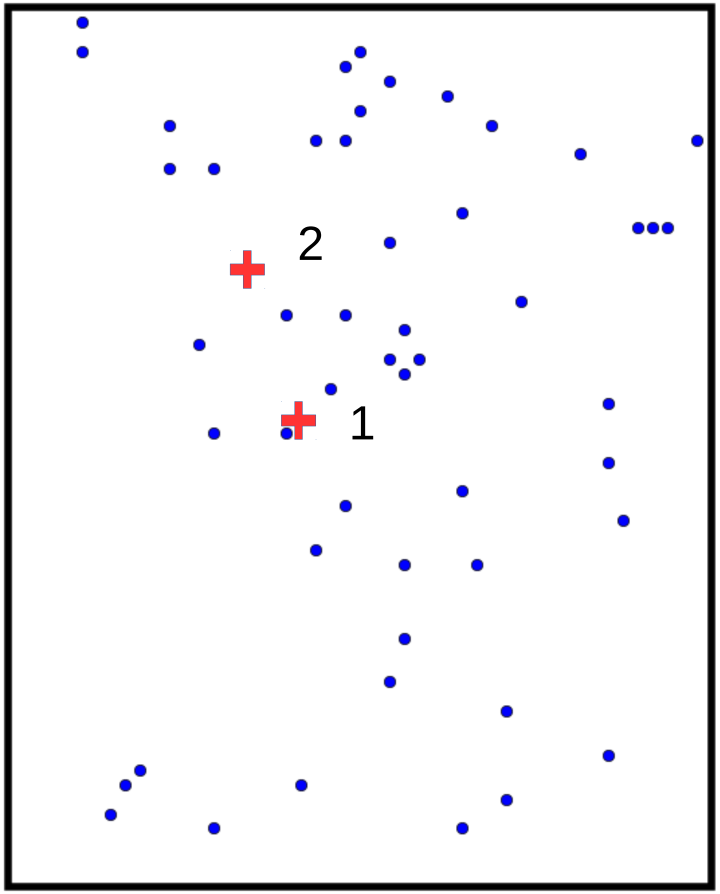
Green's function recovering quality

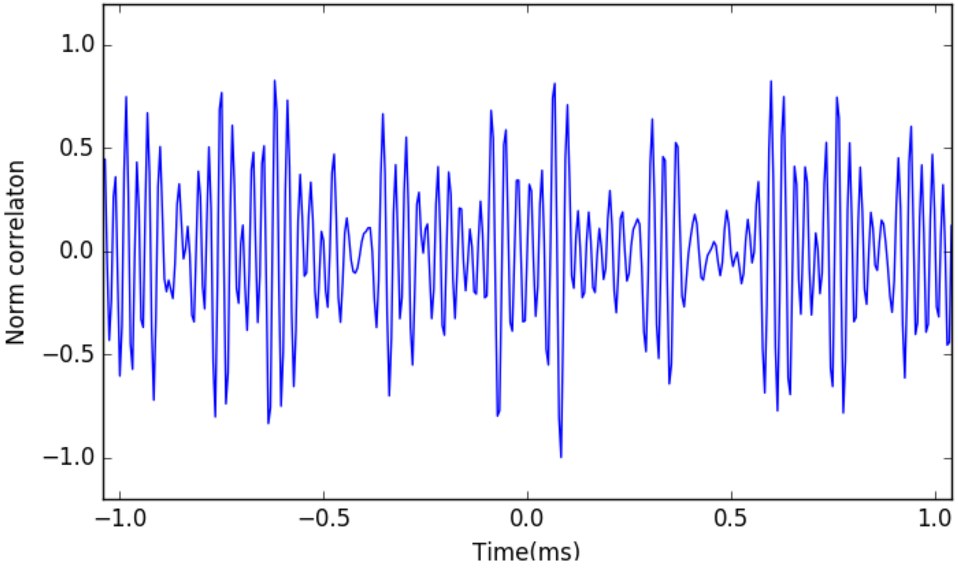
Stacking over noise sources

$$C_{12}^{N}(t)=\sum_{\alpha=1}^{N} h_{1\alpha}(t)\otimes h_{2\alpha}(-t)$$

Number of noise sources : 50

Correlation





Convergence toward a symmetric waveform

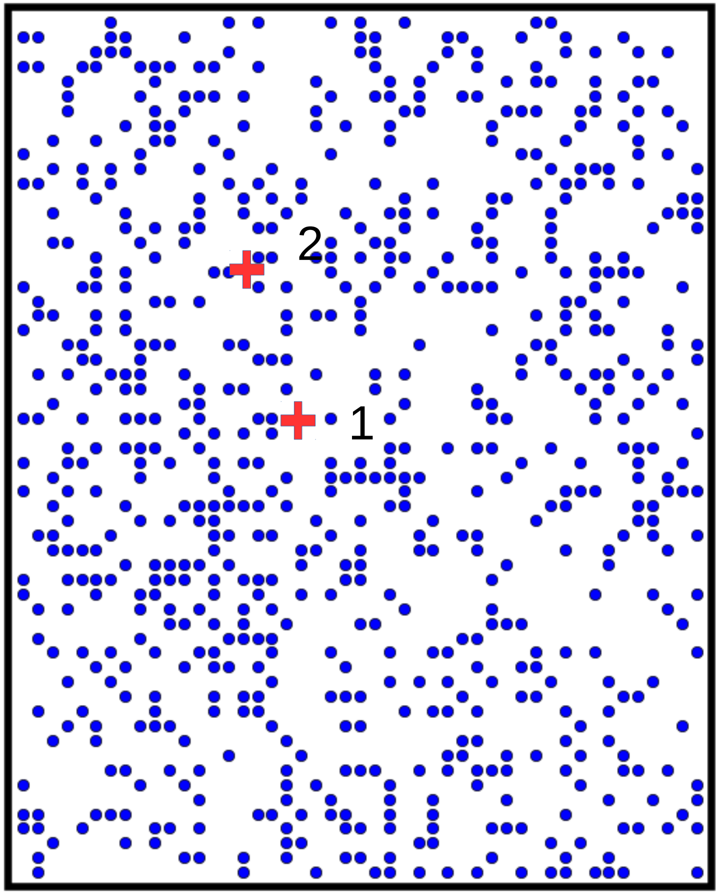
Green's function recovering quality

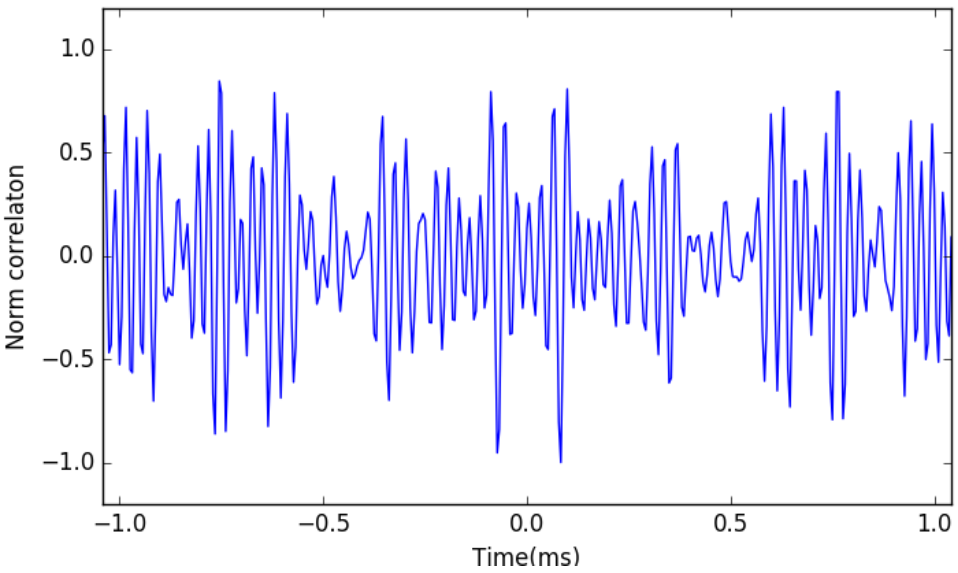
Stacking over noise sources

$$C_{12}^{N}(t)=\sum_{\alpha=1}^{N} h_{1\alpha}(t)\otimes h_{2\alpha}(-t)$$

Number of noise sources : 1000

Correlation





Convergence toward a symmetric waveform

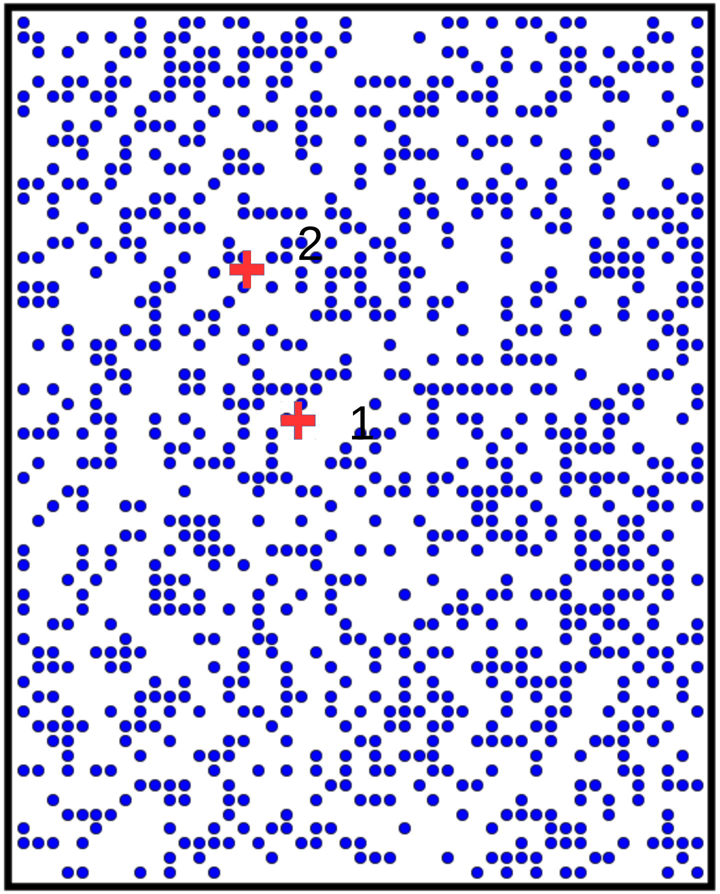
Green's function recovering quality

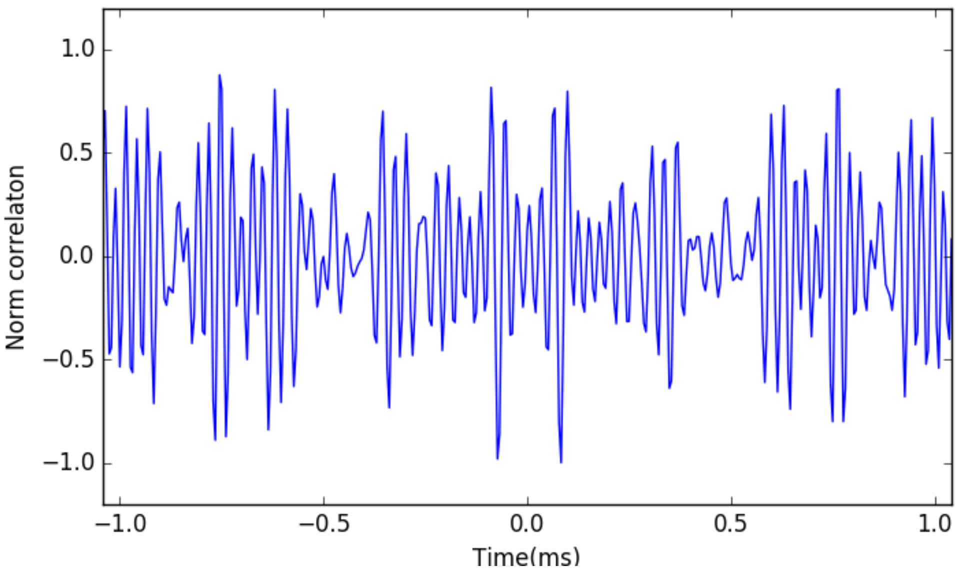
Stacking over noise sources

$$C_{12}^{N}(t)=\sum_{\alpha=1}^{N} h_{1\alpha}(t)\otimes h_{2\alpha}(-t)$$

Number of noise sources : 2700

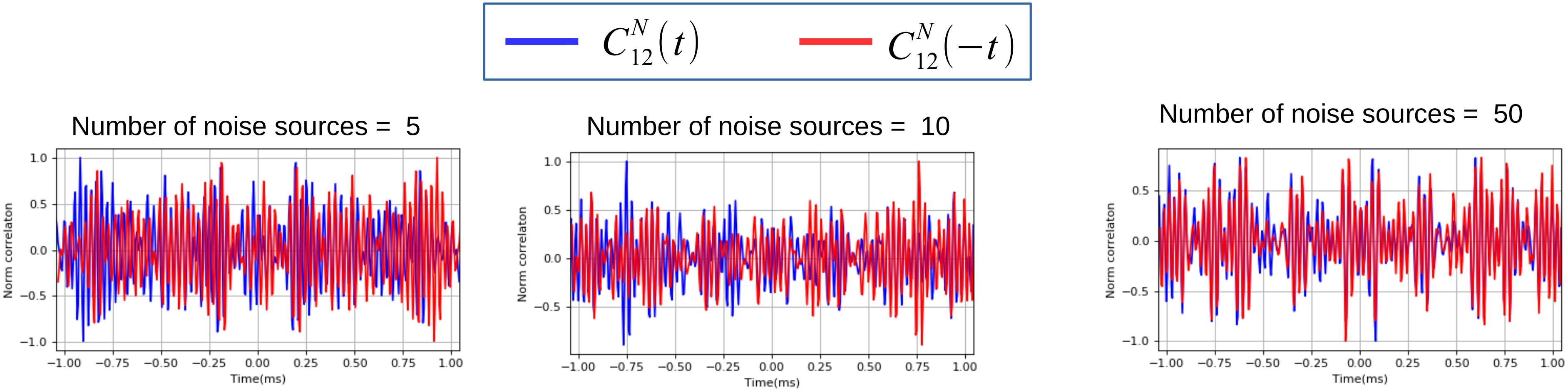
Correlation



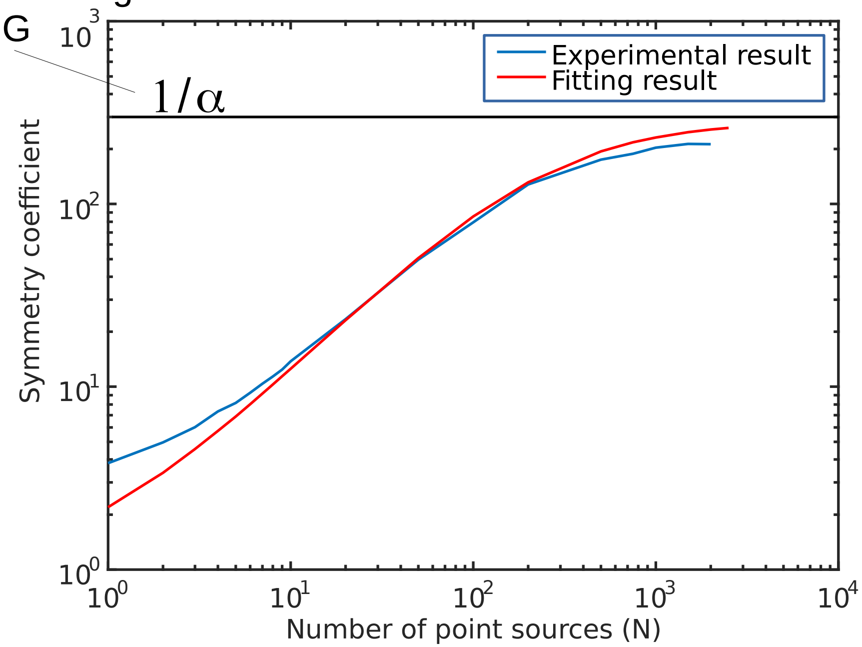


Convergence toward a symmetric waveform

Degree of symmetry



Unperfect convergence toward Im G



Degree of symmetry

$$r=\frac{\int \left(C_{12}^{N}(t)+C_{12}^{N}(-t)\right)^2 dt}{\int \left(C_{12}^{N}(t)-C_{12}^{N}(-t)\right)^2 dt}$$

Random signal $r \rightarrow 1$

Symmetric signal $r \rightarrow \infty$

Simple model

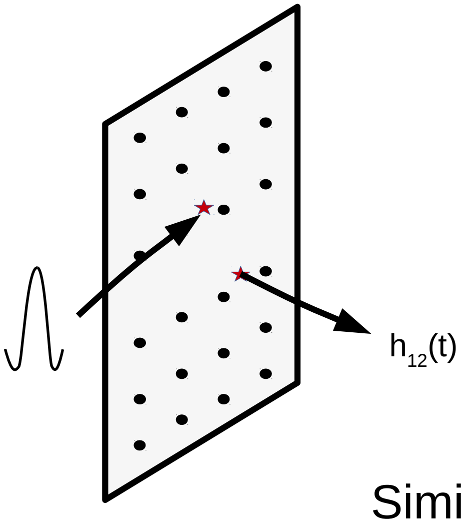
$$C_{12}^{N}(t)=C_{12}^{\infty}(t)+\text{fluctuations}(t)$$

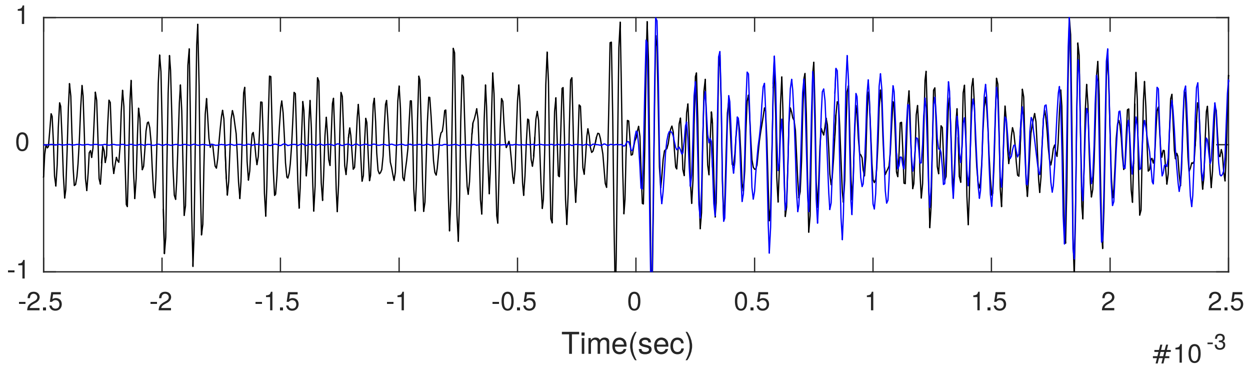
$$r=\frac{2+\frac{\pi n_0}{N\tau_a}}{2\alpha+\frac{\pi n_0}{N\tau_a}}$$

Attenuation time

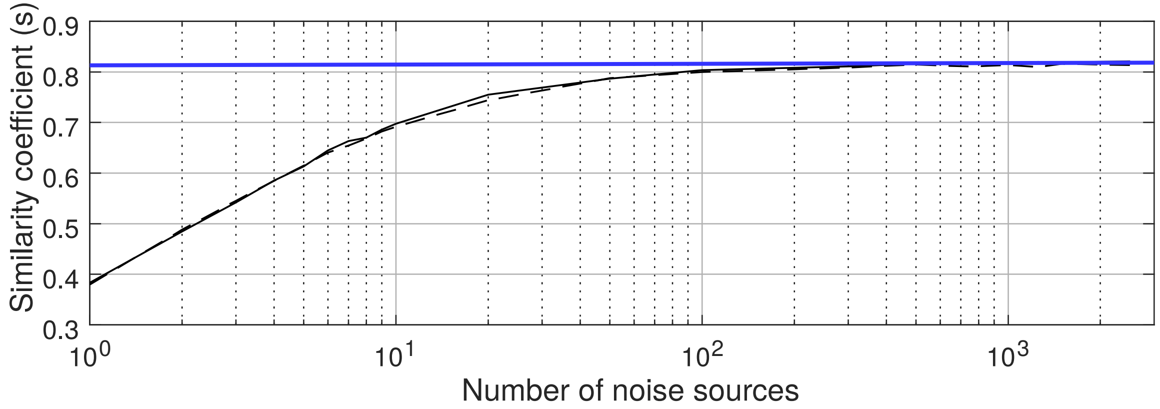
Modal density

Similarity coefficient





Similarity coefficient $s \propto \int h_{12}(t) C_{12}^{N}(t)\, dt$



$s \propto \int_0^{\infty} .dt$

$s \propto \int_0^{\tau_a} .dt$

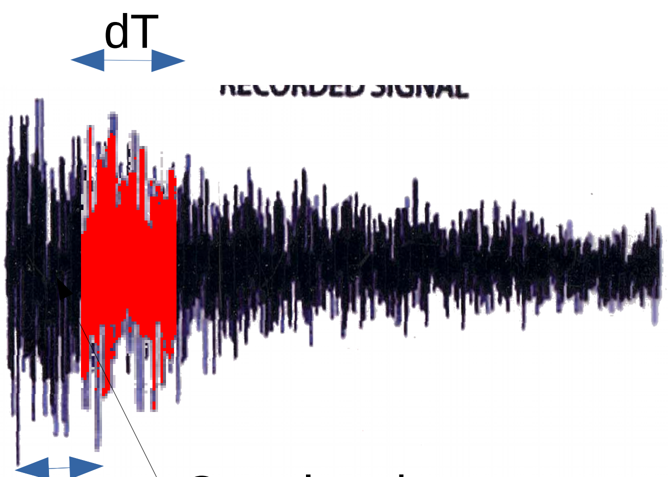
Why this asymptotic plateau ?

$$h_{12}(t) \propto e_1(t) \otimes G_{12}(t) \otimes e_2(t)$$

$$C_{12}(t) \propto e_1(-t) \otimes \Im G_{12}(t) \otimes e_2(t)$$

$e_1(t)$ and $e_2(t)$ electro-acoustical responses of the transducers

Time Windowed correlation



Correlated windows

Modal decomposition as in Lobkis & Weaver, 2001

$$G(r_1, r_2, t) = \frac{1}{\rho} \sum_n \phi_n(\mathbf{r_1}) \phi_n(\mathbf{r_2}) \frac{\exp(-t/\tau)\sin(\omega_n t)}{\omega_n} \qquad t > 0$$

Eigen-modes Attenuation Eigen-pulsation

Similarity coefficient

$$S = \frac{1}{\sqrt{1 + \frac{2}{N}(1+Z)}} \qquad Z = \frac{\pi\kappa}{\tau_a [1 + 2F(\delta r)]} \coth\left(\frac{dT}{\tau_a}\right)$$

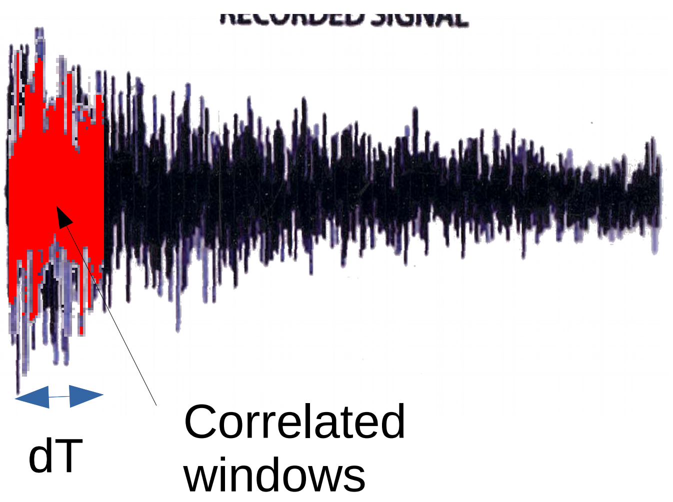
F(δ*r*) :
(J_0(kr) : Spatial correlation function)^2

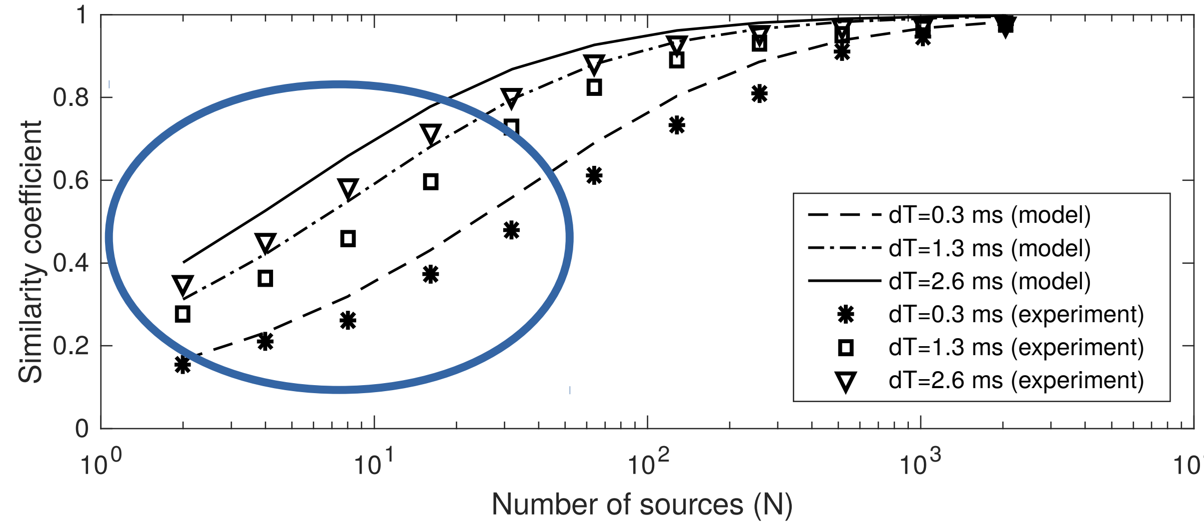
κ is the two-level correlation function :
K ~ n_0

$$\kappa = \frac{\langle n(\omega) n(\omega + \delta\omega) \rangle}{\langle n(\omega) \rangle}$$

- Weakly depends on the starting time of the correlated windows
- When N very large S → 1 even for small windows : Instananeous Time-Reversal (Loshmidt Echo)

Time windowed correlation



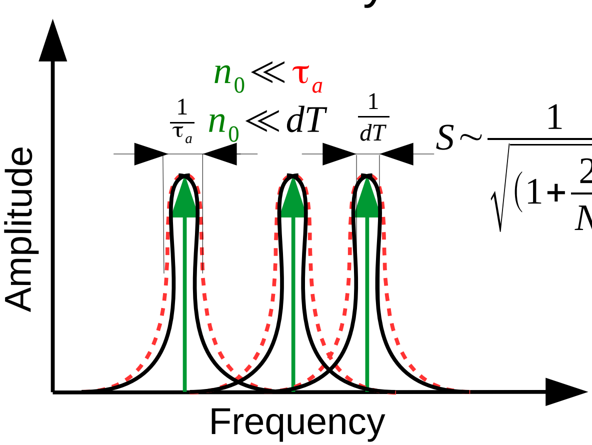


Modal decomposition of correlation (similar to Lobkis & Weaver, 2001)

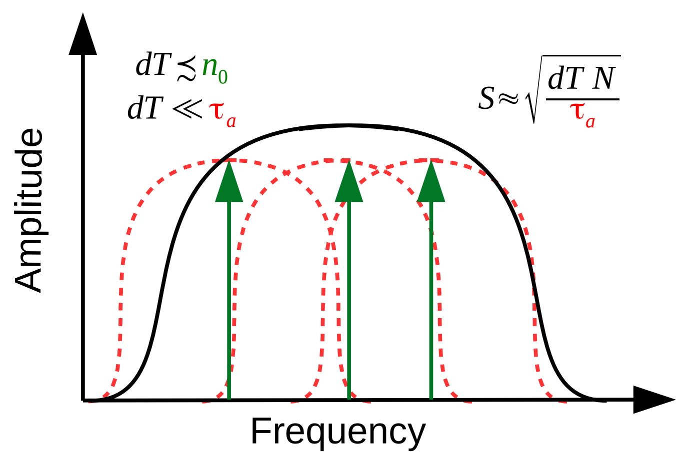
$$S=\frac{1}{\sqrt{1+\frac{2}{N}(1+Z)}}$$

$$Z=\frac{\pi n_0}{\tau_a}\coth\left(\frac{dT}{\tau_a}\right)$$

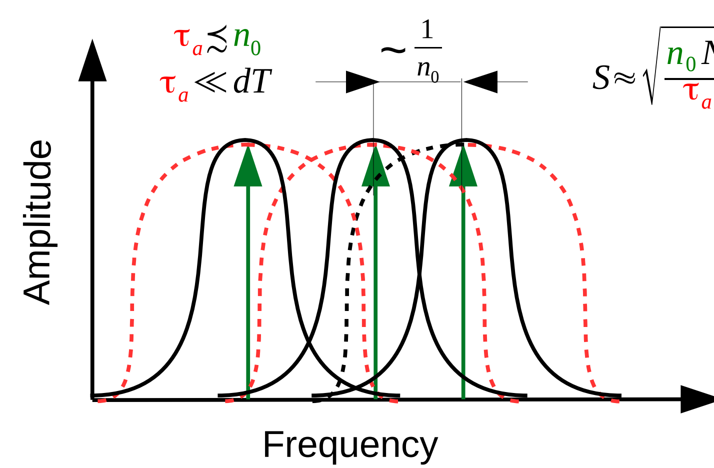
Modal density driven



Window size driven



Attenuation driven



Effect of noise

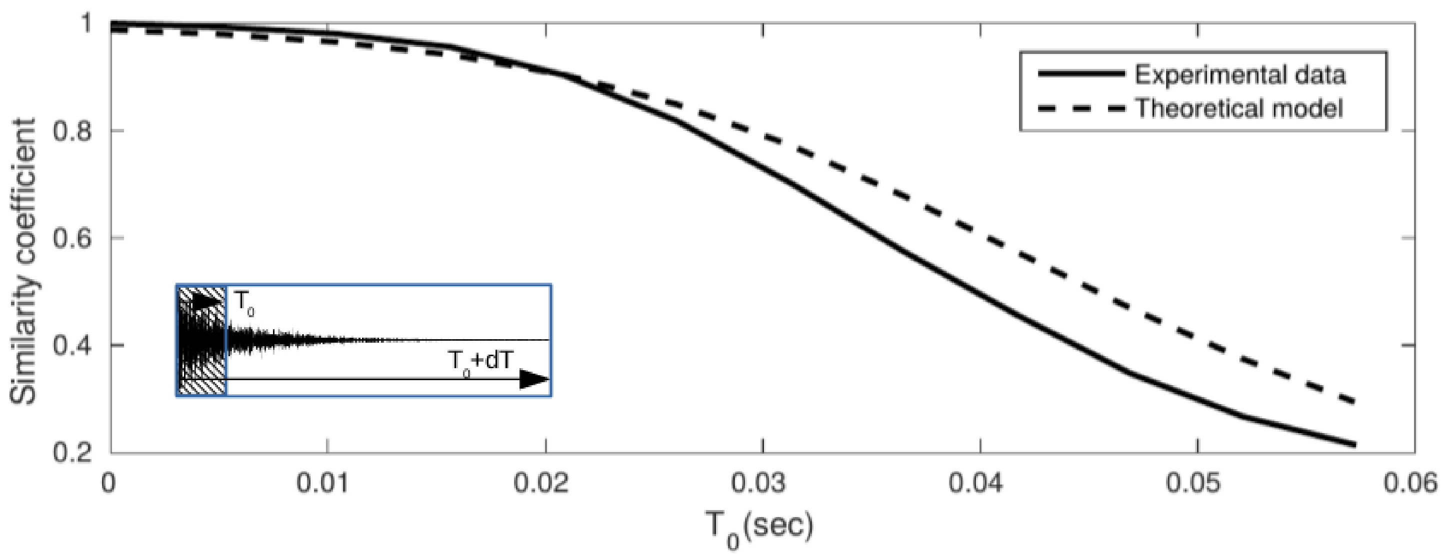
Recorded signal   Transient response   Noise

$$e(t) = h(t) + n(t)$$

Assumes that N and dT are large → S=1 w/o noise

Bandwidth

$$S(C_\infty, C_\infty^{dT}) \approx \left(1 + \frac{\beta B(dT)}{N[e^{-T_0/\tau_a}(1 - e^{-dT/\tau_a})]}\right)^{-0.5}$$



→ Effect of the bandwdith AND the starting position

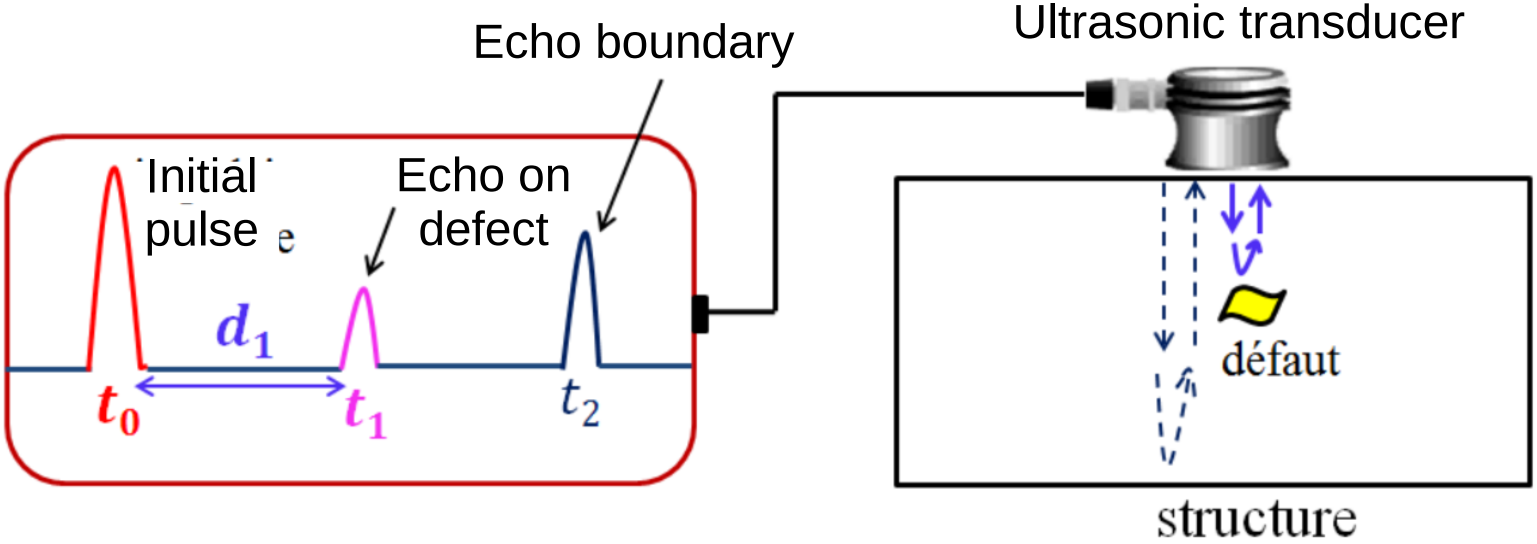
Structural health monitoring

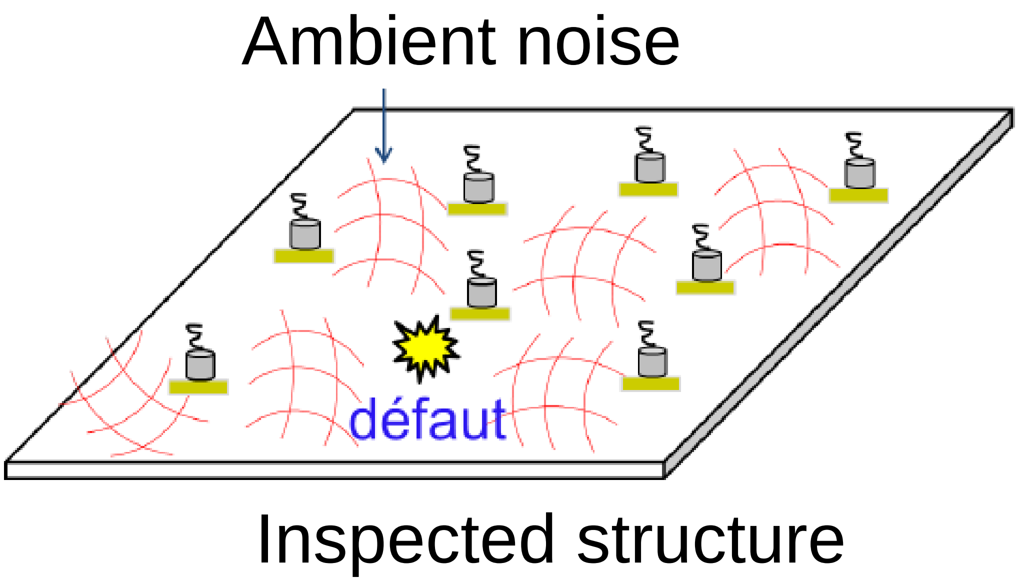
Structural engineering

Nuclear plants

Transports

Conventional active methods

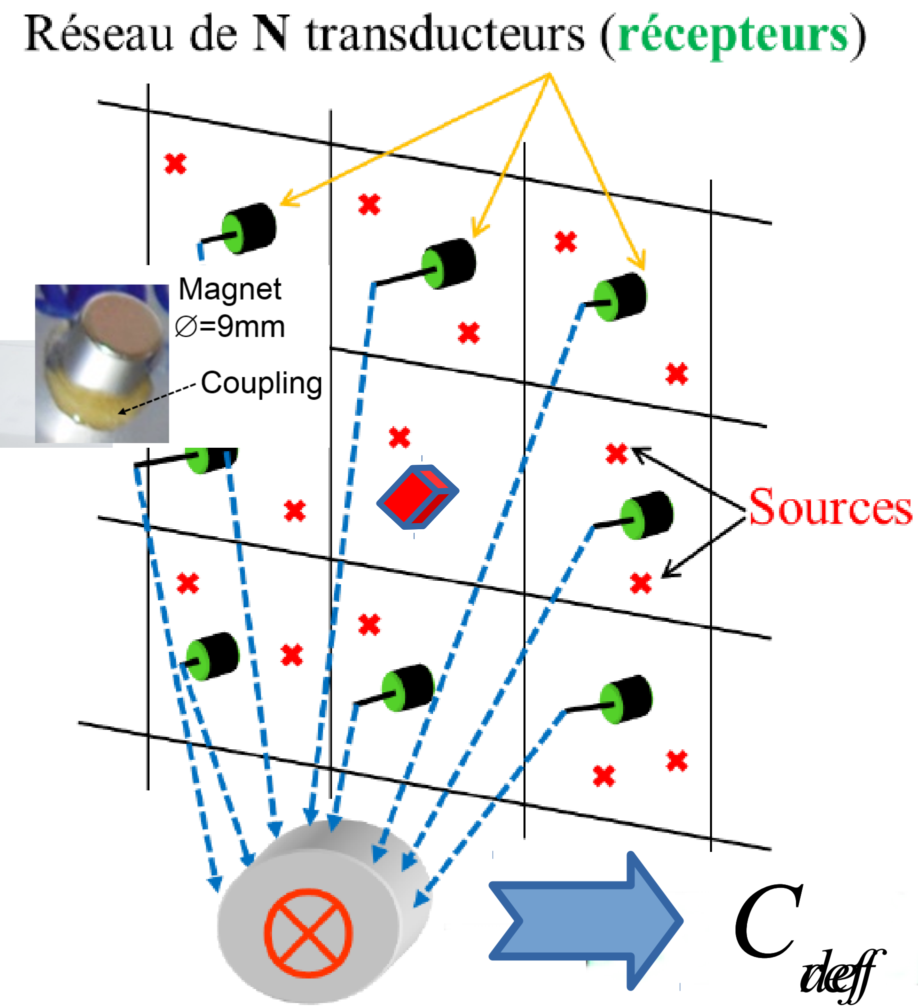




- Detection
- Localization
- Identification

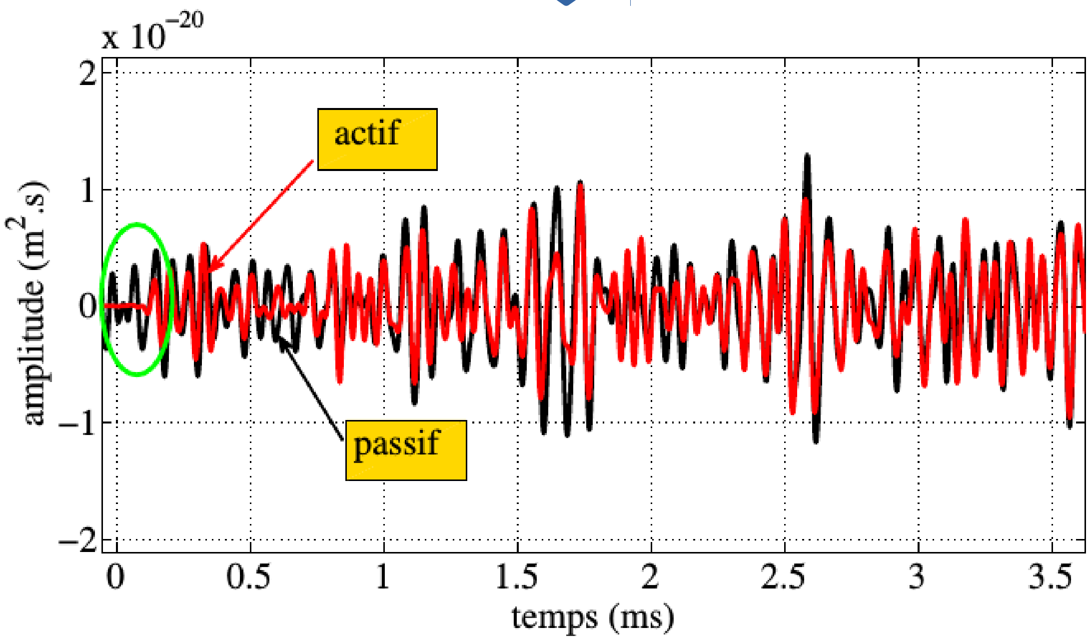
- Low power consumption
- No interferences with other electronic
- Low complexity

Differential detection & localization



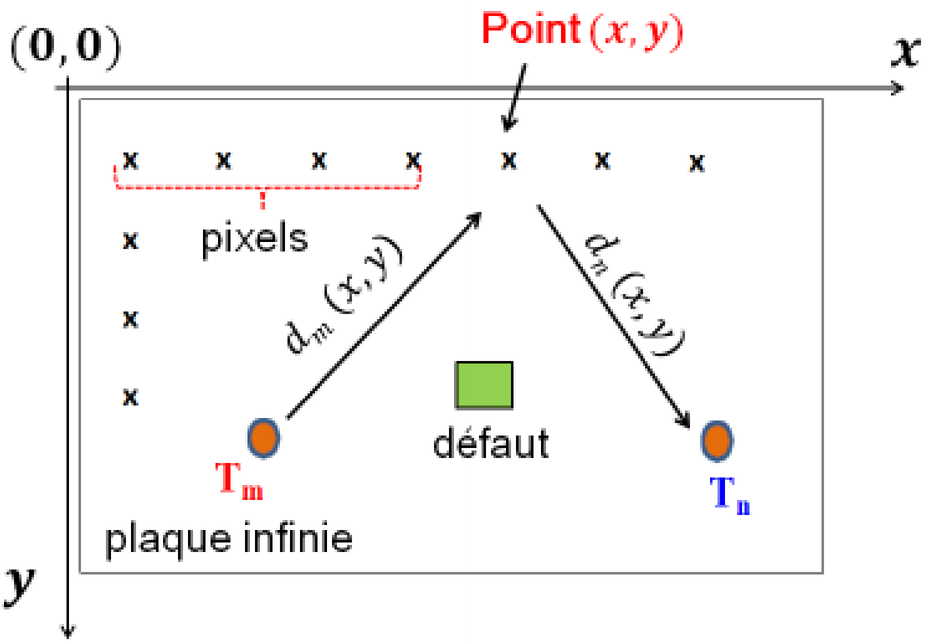
C_{def}

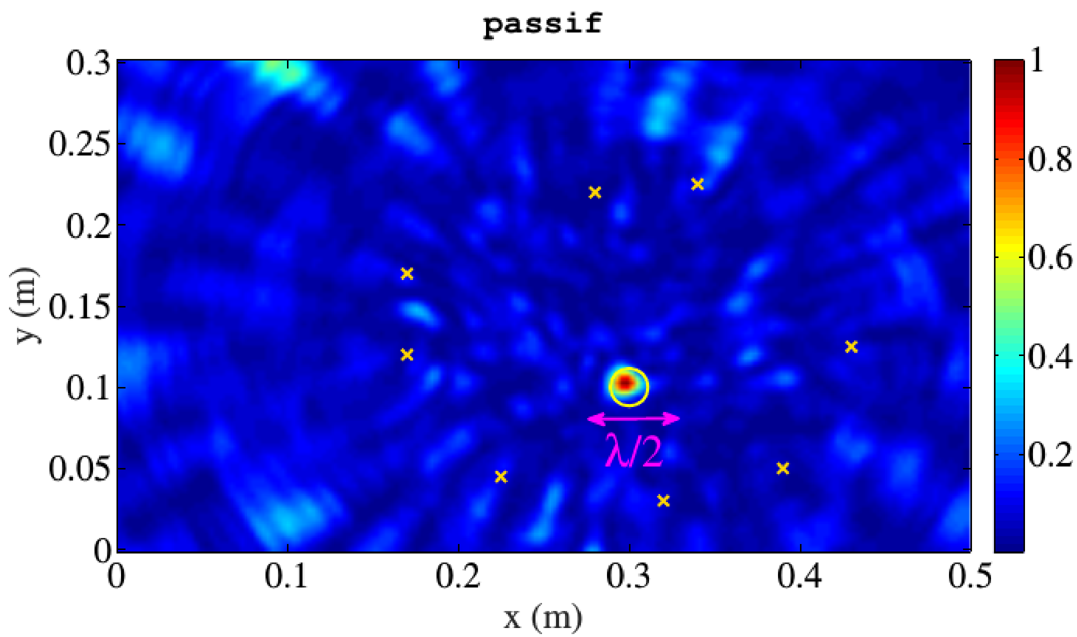
$$\Delta C_{def} = C_{def} - C_{ref}$$

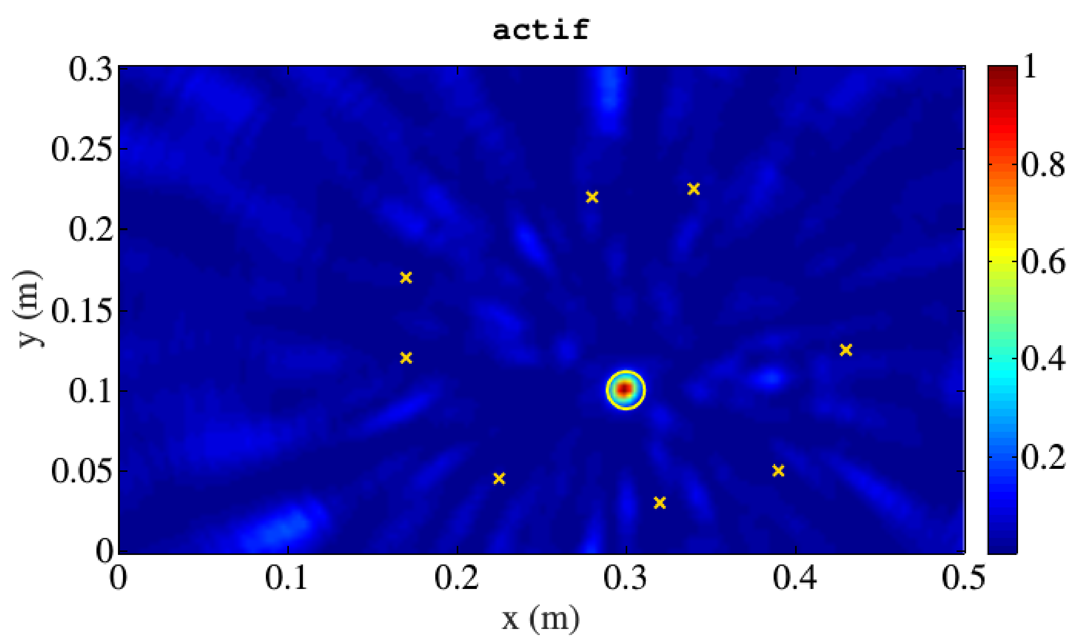


Back-propagation

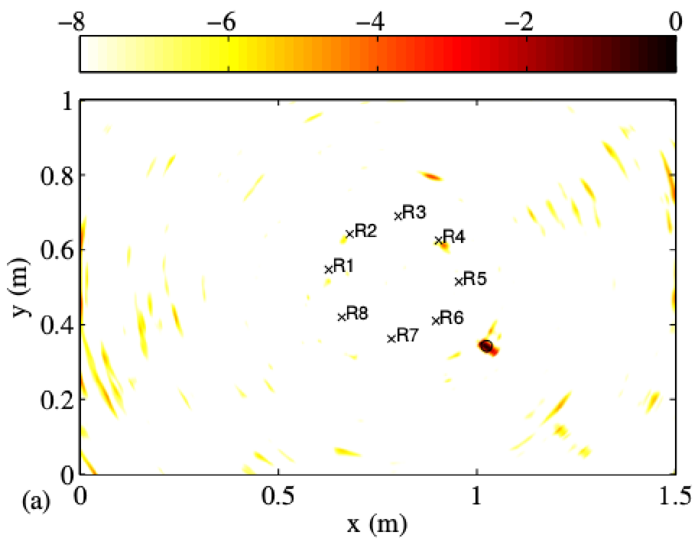
$$bpf(r)=\sum_{m,n}\Delta C_{n,m}(\omega)e^{jk(\omega)[\|r-r_i\|+\|r-r_j\|]}$$

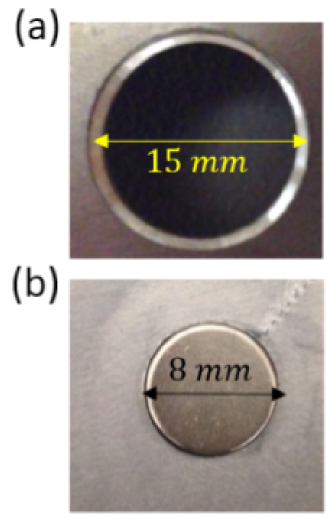


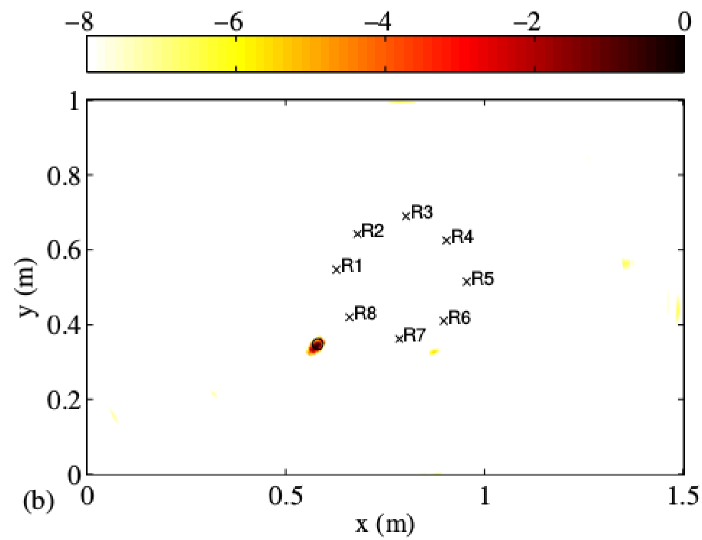


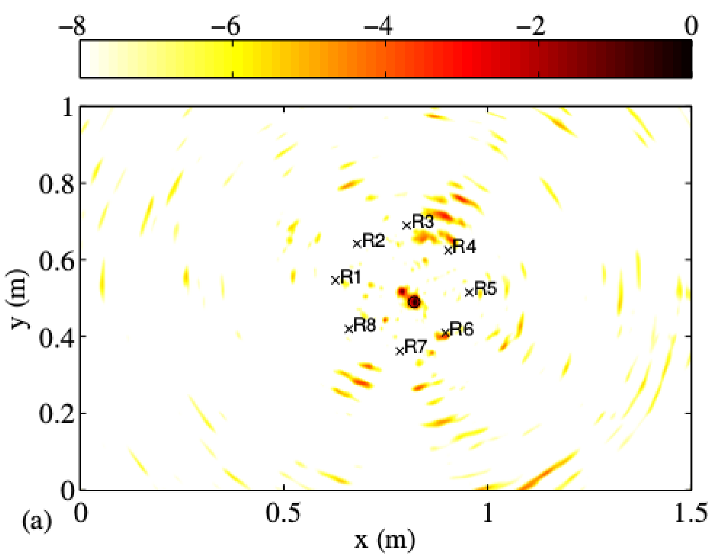


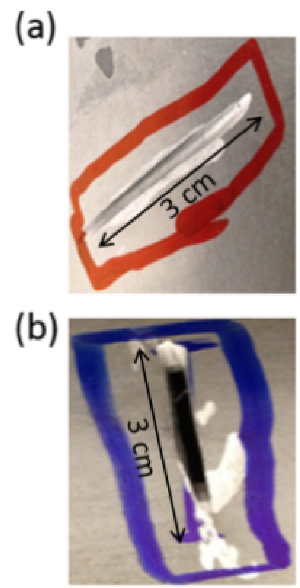
Defects localization

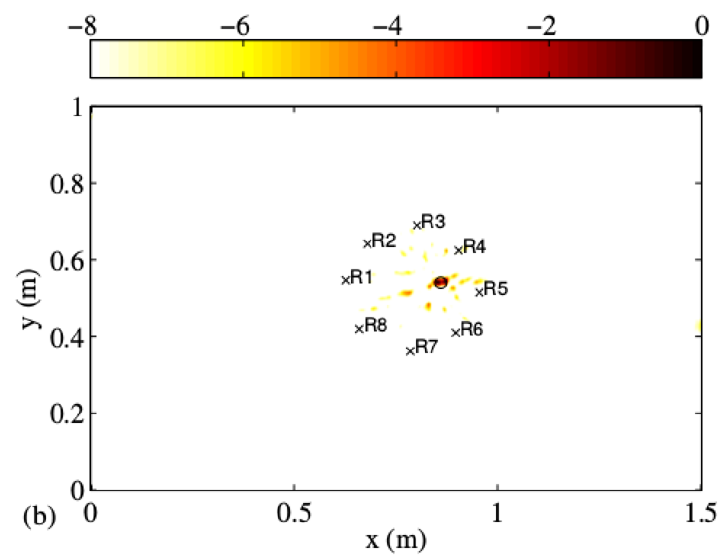






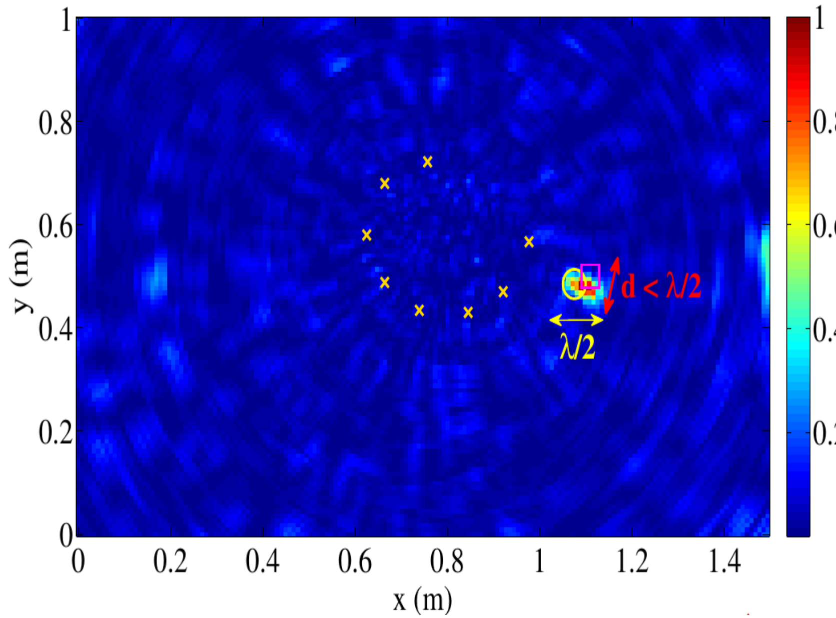






Passive localization for different kind of defects

Resolution



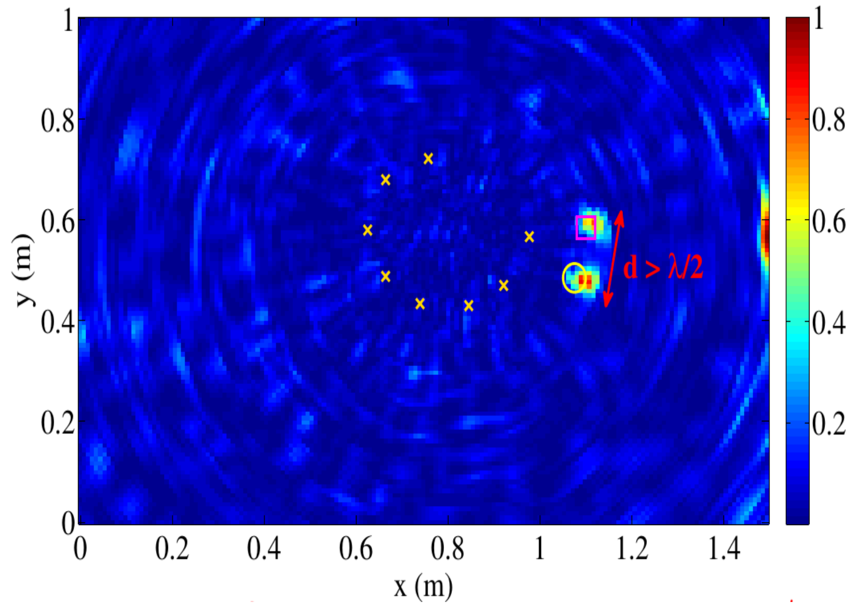
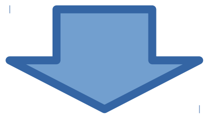
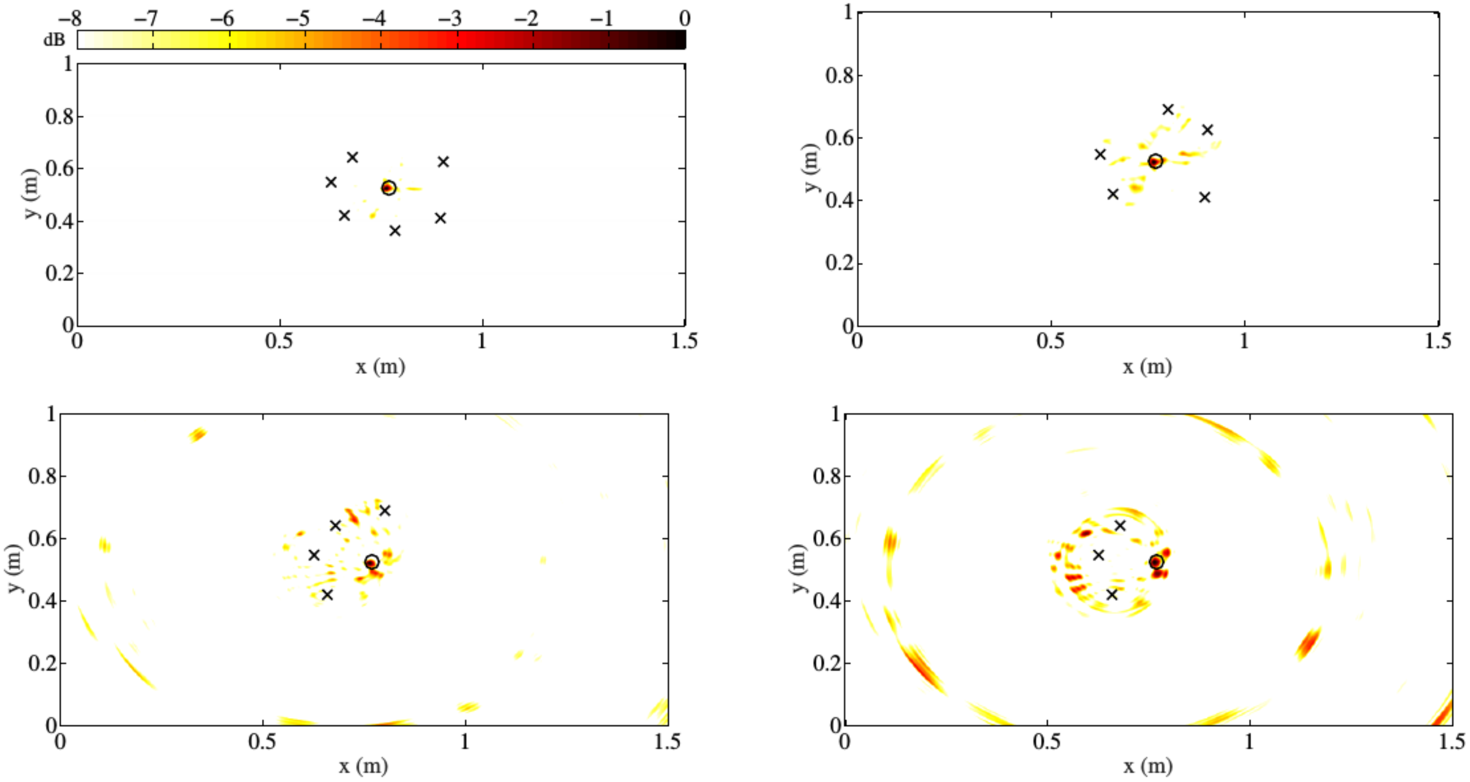


Image performed only from the direct echo



Resolution given by the aperture of the array

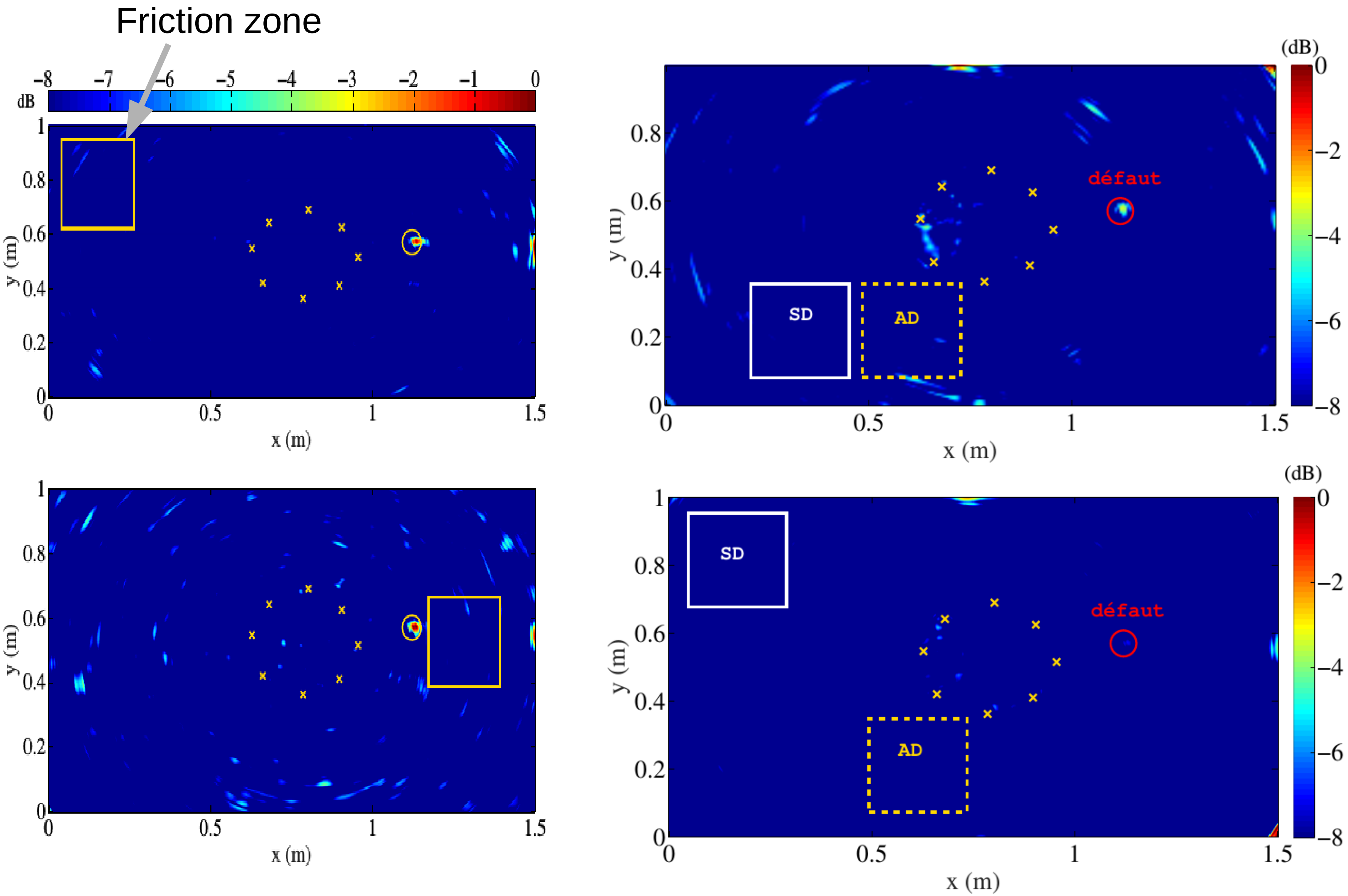
Number of probes



Noise generated by loudspeaker

Detection efficient from 3 receivers

Heterogenous noise



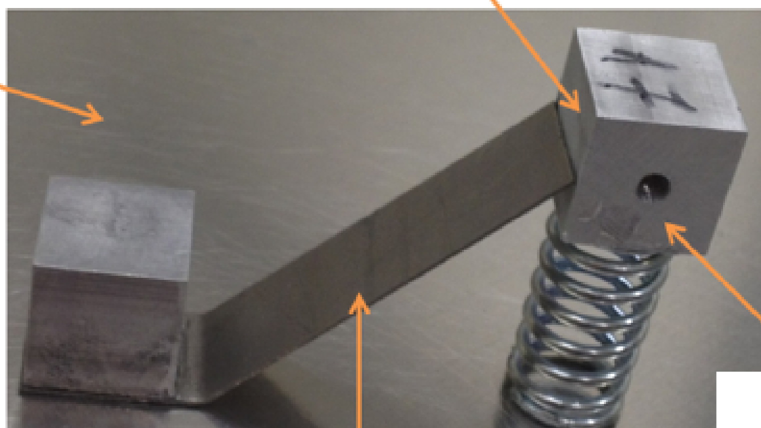
→ Remains efficient provided that the noise is spatially stationnary

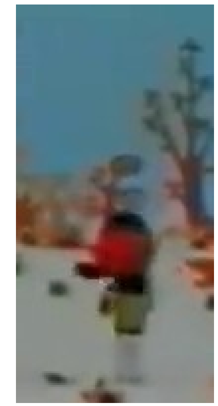
Non-linear noise sources : Zebulon

When ambient noise is not sufficient : use non linear LF to HF converter

Rough surface

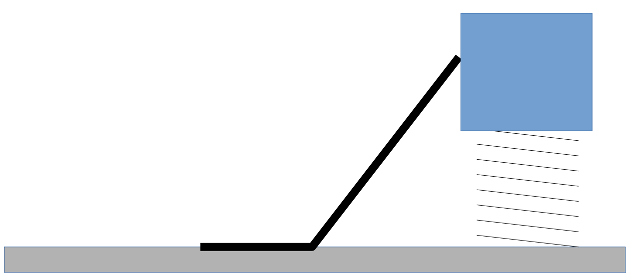
structure d'étude

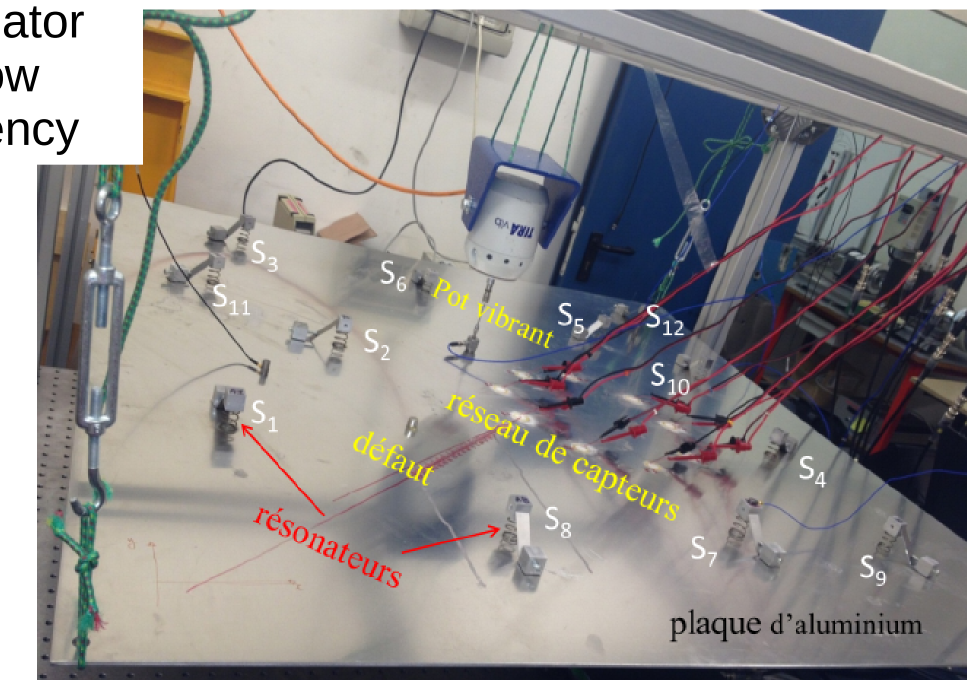




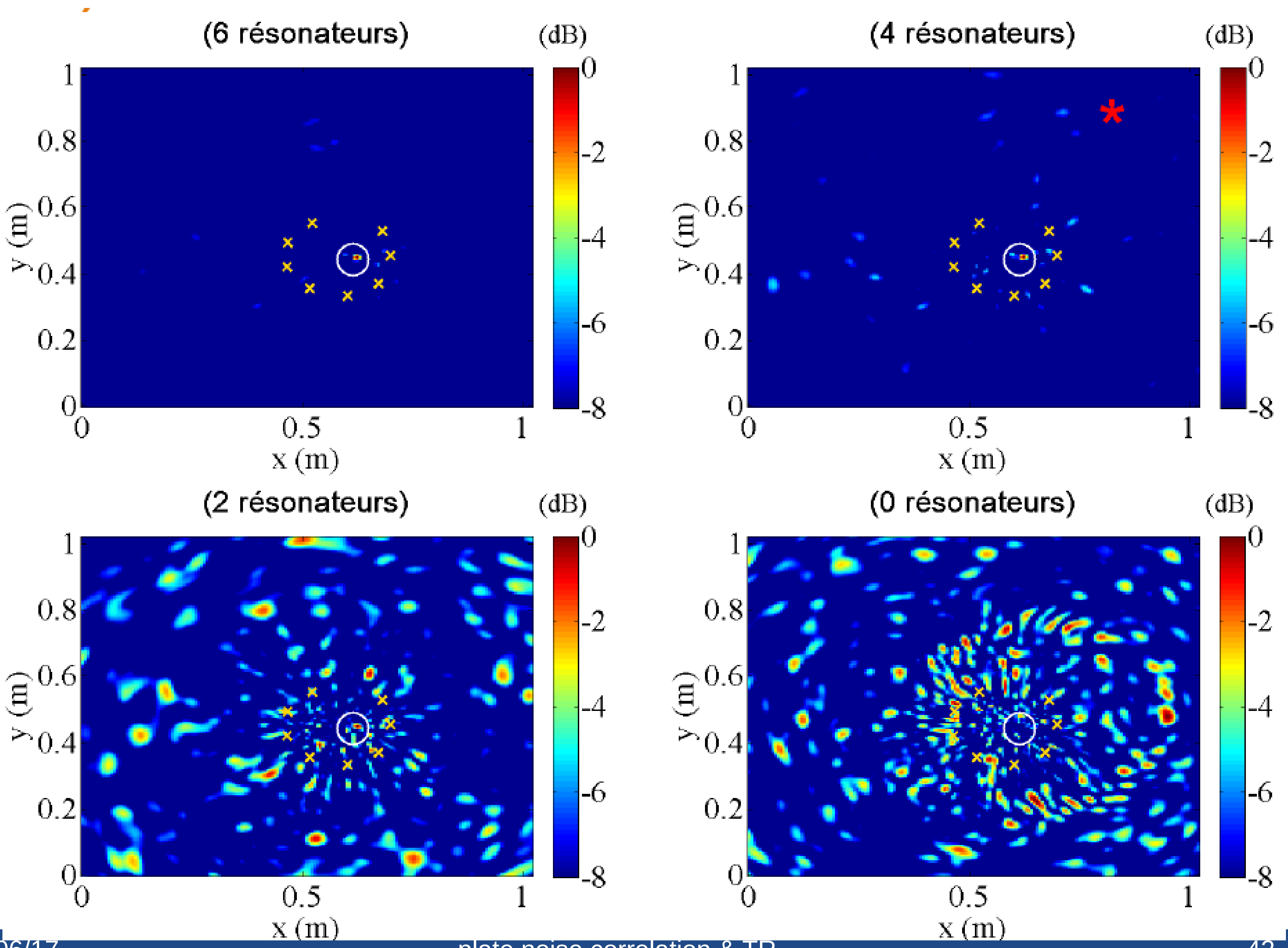
Second resonator at elastic wave frequency

First Resonator At low frequency





Localization vs number of NL



Estimation of the scattering strength

Born approximation

$$\Delta G = f(\theta) G_0 \frac{e^{-jk_0 R}}{\sqrt{R}}$$

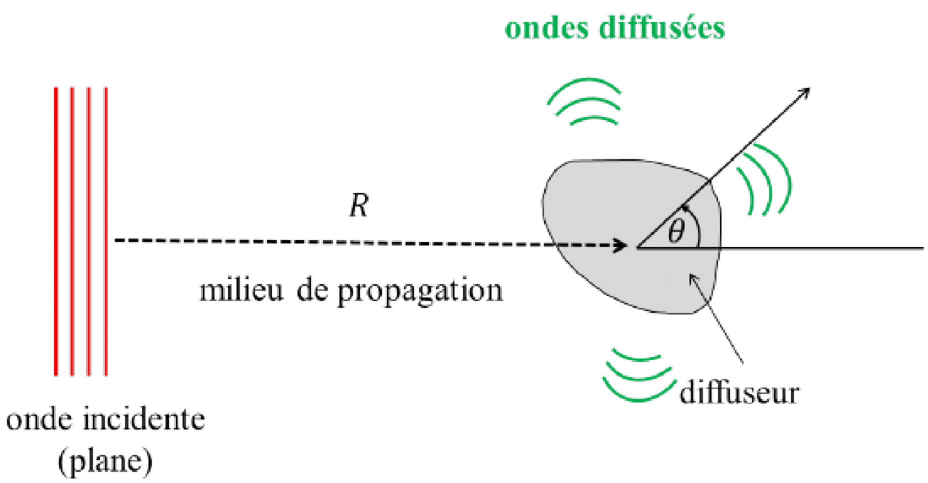
$$\Delta C^{+}_{ll'}(t) = F(\omega)\Delta G$$

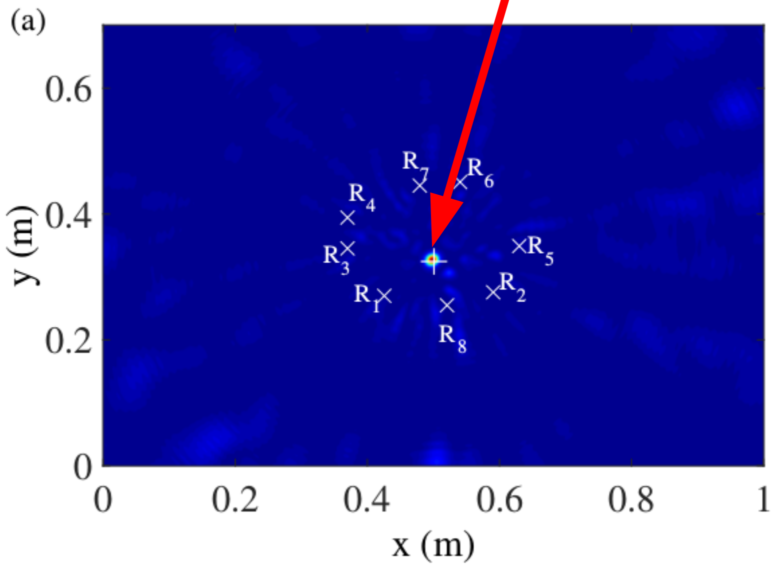
Scattering cross-section

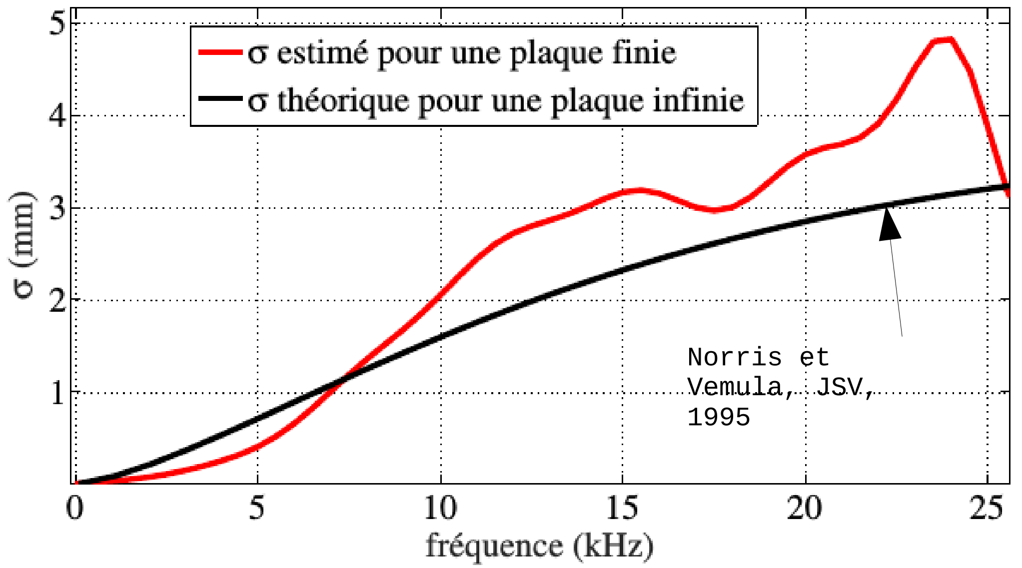
$$\sigma = \int |f(\theta)|^2 d\theta$$

$$\sigma = \frac{4\pi^2 k_0}{\left(\sum_{i,j} r_{id} r_{jd}\right)^2} \frac{|bpf(\boldsymbol{r}_d)|^2}{I_0^2}$$

Mean diffuse intensity





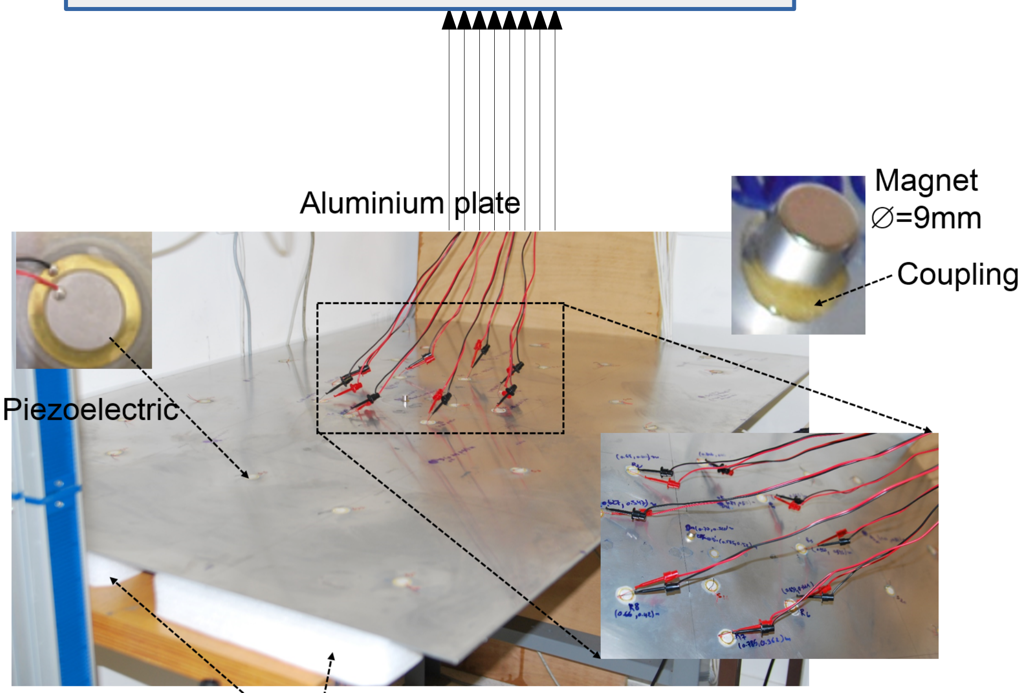


Conclusions

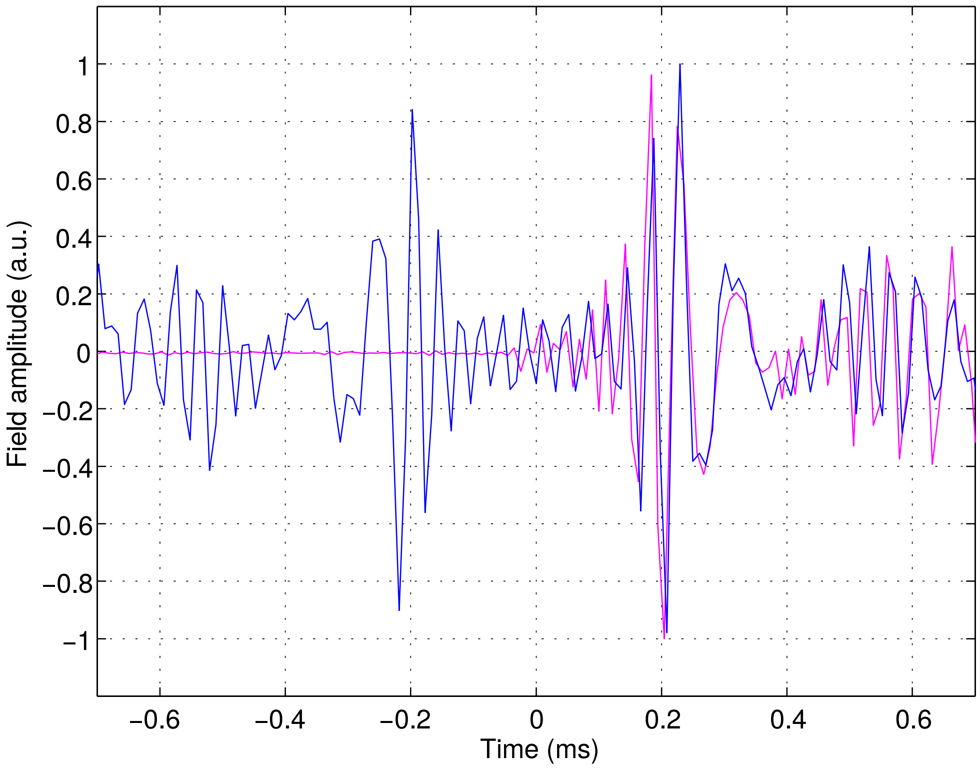
- Plate as reference experiments for studying noise correlation
- Quantitative study of the reconstruction of the Green's function with respect to windowed correlations
  → related to physical quantities
- Robust method for scatterer detection

Thank you

8 channel analog to digital sampler – 96kHz – 24 bits



Noise generated by friction



Noise filtered between 1kHz and 40kHz