Let consider a half space with elastic parameters  $\rho$ ,  $\lambda$  and density  $\rho$ . (x,y,z) is a cartesian system with z pointing towards depth.

A propagating harmonic plane wave of frequency 1 Hz is characterized by a wave vector:

$$\vec{k} = (0.00127, 0, 0.00218)$$

The associated displacement is of the form:

$$\vec{u} = (0, v, 0)$$

with amplitude  $v_0$ .

- 1) What is the type of wave? (P, S, SH, SV?)
- 2) Knowing  $\rho=2800kg/m^3$  and  $\mu=1.7510^{10}N/m^2$ , give the angle of incidence of the wave on the horizontal free surface.
- 3) What is the apparent velocity measured between two sensors at the surface?
  - 4) What is the amplitude of the motion observed at the surface?

Exercise 2

We want to study a sedimentary deposit that we will consider as a flat homogenous layer and which lies over a hard homogeneous basement.

We use a line of receivers measuring the horizontal motion in the direction perpendicular to the line. We use as a source an horizontal force acting horizontally and located on the line of receivers.

- 1) What type of waves are you expecting?
- 2) The travel time curve of the first arrivals is in two parts: Between 0 and 1633 m, a straight line, passing by the origin and having a slope of  $10^{-3}s/m$ . At 1633m the slope changes and the slope becomes  $4.5410^{-4}s/m$ . How can you interpret this behavior?
  - 3) What is the thickness of the layer?

- 4) What is the frequency of the fundamental mode of resonance for a S wave incident vertically to the layer?
- 5) What is the amplification expected at a location on the sediments with respect to a hard rock site (assume that the density is  $2000kg/m^3$  for the sediments and  $3000kg/m^3$  for the bedrock?

## Exercise 3

The size of the isoseismal curves (lines of equal value of intensity: see Figure 1 for example) are often used to assess the depth of historical earthquakes. In this exercice we investigate the argument and its limitations.

- 1) Reminding the definition of intensity, what are the type of waves and the frequency range that contribute the most to its value.
- 2) We assume that the source of an earthquake is a point source at depth z in an homogeneous elastic space. We further assume that the far-field approximation is acceptable. Give an expression for the shear wave amplitude as a function of depth z and epicentral distance d.
- 3) Same question neglecting the radiation pattern.
- 4) It was empirically shown by Wald et al. (1999) that the intensity can be related to the peak value of the ground velocity (PGV):

$$I = 3.47 \log(PGV) + 2.35$$

for 5 < I < 9.

Assume that the intensity  $I_0$  is reached at a distance of d=50km. For a source at the surface, at what distance d' the intensity is  $I_0-1$ ? Same question for a source at a depth z=97km

5) Explain why the distance between isoseimal curves (that is d'-d) is regarded as an indication of the depth of the earthquake.

Let us consider now the presence of guided waves (in the crust for example, but it could be another shallow layer).

- 6) What is the range of focal depth for which these waves can be important? What is the expected decay with distance in the purely elastic case?
- 7) Assume that the intensity  $I_0$  is reached at a distance of d = 50km, at what distance d' the intensity is  $I_0 1$ ? Compare the distance d' d with the results of question 4. Discuss the use of isoseimal curves to evaluate the depth of the source of historical events.

## Exercise adv1

We call h the eigenfunction and C the phase velocity of the fundamental Love wave in a stratified model at a period T. We analyze the effect of a slight change of property in a shallow layer at the surface on the phase velocity of a Love wave.  $\Delta z$  is the thickness of the layer in which the rigidity is changing by a constant  $\delta \mu$ .

- 1) Give the relation between the relative change of phase velocity and a perturbation of rigidity .
- 2) You can use the free surface condition to simplify this expression in the case the superficial layer. Assume that  $\delta z$  is much smaller than  $\lambda$  to obtain a very simple expression for  $\delta C/C$  as a function of  $\delta z$ ,  $\delta \mu$  and the energy integral  $I_2$ .
  - 3) Study the behavior of the change of phase velocity for an increasing period.

