Pb 1: Consider guided SH waves in a plate of thickness h (ex: an ice sheet).

Use the same approach as for Love waves in the course to calculate the dispersion relation. (you have to select the form of the solutions and to adapt the boundary contions, that are free surface conditions at O and h). Deduce the phase velocity C dispersion forms and the cut off frequencies of the modes. Compute and display the group velocity U .

Some results:

Dispersion relation :

$$C = \frac{2fh\beta}{(4f^2h^2 + n^2\beta^2)^{1/2}}$$

$$U = \frac{\beta^2}{C}$$

Pb 2: Find a relation between C and U using the properties of the Lagrangian.

(write the Lagrangian for a state with a perturbation of angular frequency $\delta\omega$ at the first order, then use the properties of the unpertubated solution)

result:

$$U = \frac{1}{C}\frac{I_2}{I_1}$$

Verify this property in the case of the plate.

Pb 3 Consider the case of an homogeneous half space with a rigid condition at the boundary (i.e. displacements = 0). This would correspond to a 'soft' elastic body in contact with a very rigid body.

Does an equivalent of the Rayleigh wave exist for the rigid boundary?

(Study this problem in the same way as we studied the case of the Rayleigh wave.)