Institut Langevin
ONDES ET IMAGES

Time Manipulations of Waves

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The duality between Space and Time variables in Wave Physics

$$\left\{\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\frac{1}{c(\vec{r})^2}\frac{\partial^2}{\partial t^2}\right\}\varphi(\vec{r},t)=0$$

2^{d} order Linear PDE
Space-Time (4D)

Physicists want to determine the solutions in a « hypervolume (4D) » if one knows the field on its boundary (a « hypersurface (3D) »)

We may define two types of Cauchy conditions that contain enough information to predict the field everywhere at any time (past or future) :

1 – Cauchy (spatial) boundary conditions **(BC)** prescribe both

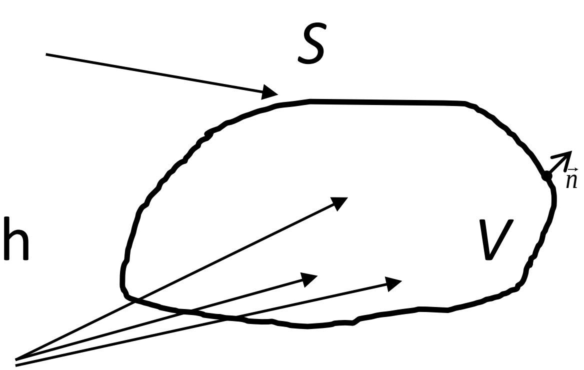
$$\left\{\varphi(\vec{r},t),\partial_{\mathrm{n}}\varphi(\vec{r},t)\right\} \quad \text{for } \vec{r}\in S, \quad \text{for all } t$$

2 spatial and 1 temporal dimensions

2 – Cauchy Initial conditions **(IC)** prescribe both

$$\left\{\varphi(\vec{r},t=t_i),\partial_t\varphi(\vec{r},t=t_i)\right\} \text{ for all } \vec{r} \in V$$

3 spatial dimensions

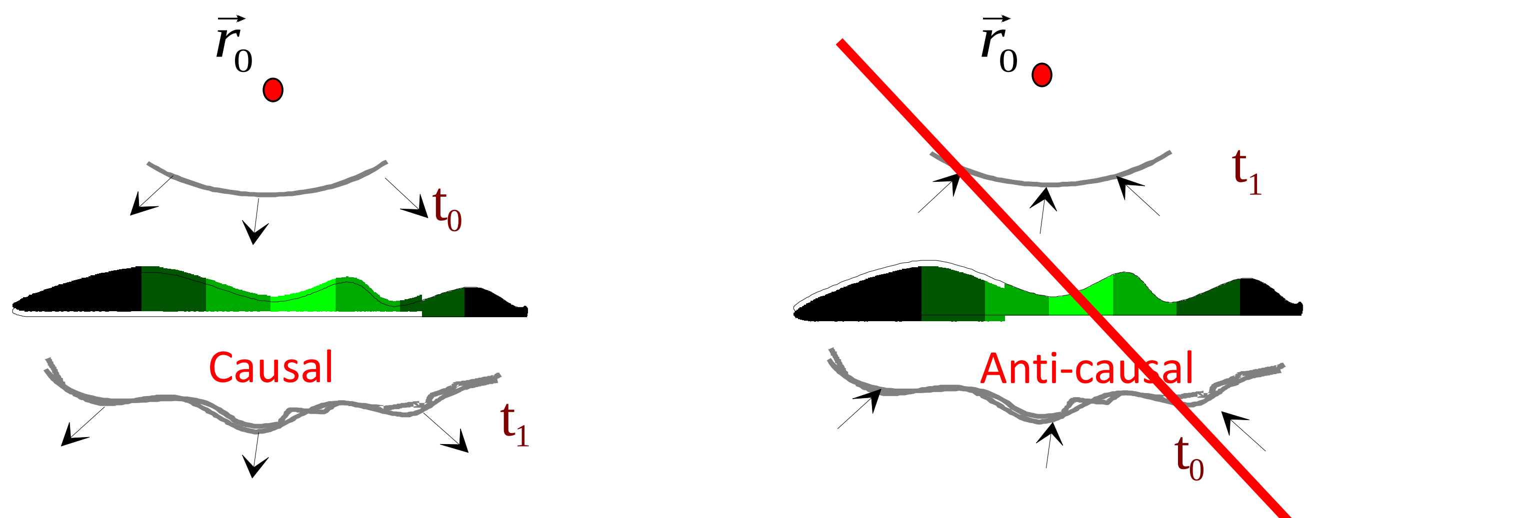


Causality

Non dissipative heterogeneous medium with a source

$$\left\{\Delta - \frac{1}{c^2(\vec{r})}\frac{\partial^2}{\partial t^2}\right\}\varphi(\vec{r},t) = s(\vec{r},t)$$

Dual Solutions - Time-Reversal Invariance



$\varphi_{caus}(\vec{r},t)$ $\qquad$ $\varphi_{caus}(\vec{r},-t) = \varphi_{anticausal}(\vec{r},t)$

To build a Time Machine for Waves : 2 approaches

1- The Loschmidt approach (IC) : instantaneous TR

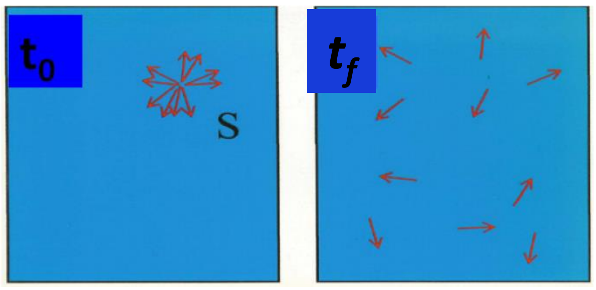
2- TR on the boundary (BC) : the time reversal mirror

I – Manipulating initial conditions (IC).
The Instantaneous Time Mirror (ITM) “à la Loschmidt”

- record on the <u>whole volume V</u> the final conditions at time t_f

$$\left\{\varphi(\vec{r}\,',t_f);\partial_t\varphi(\vec{r}\,',t_f)\right\}$$

Analogy with Trajectory Reversal of N particles

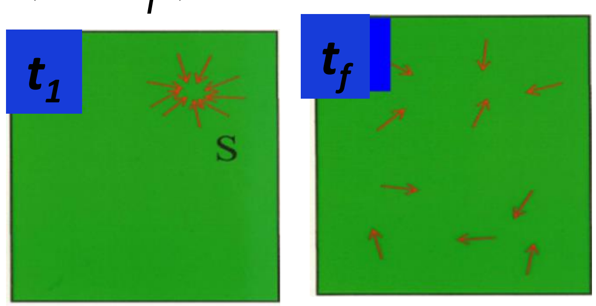


- prepare new initial conditions:

$$\varphi(\vec{r}\,',t_i)=\varphi(\vec{r}\,',t_f)\ \text{ and }\ \partial_t\varphi(\vec{r}\,',t_i)=-\partial_t\varphi(\vec{r}\,',t_f)$$

$$\left\{\varphi(\vec{r}\,',t_f);-\partial_t\varphi(\vec{r}\,',t_f)\right\}$$

The Loschmidt Daemon

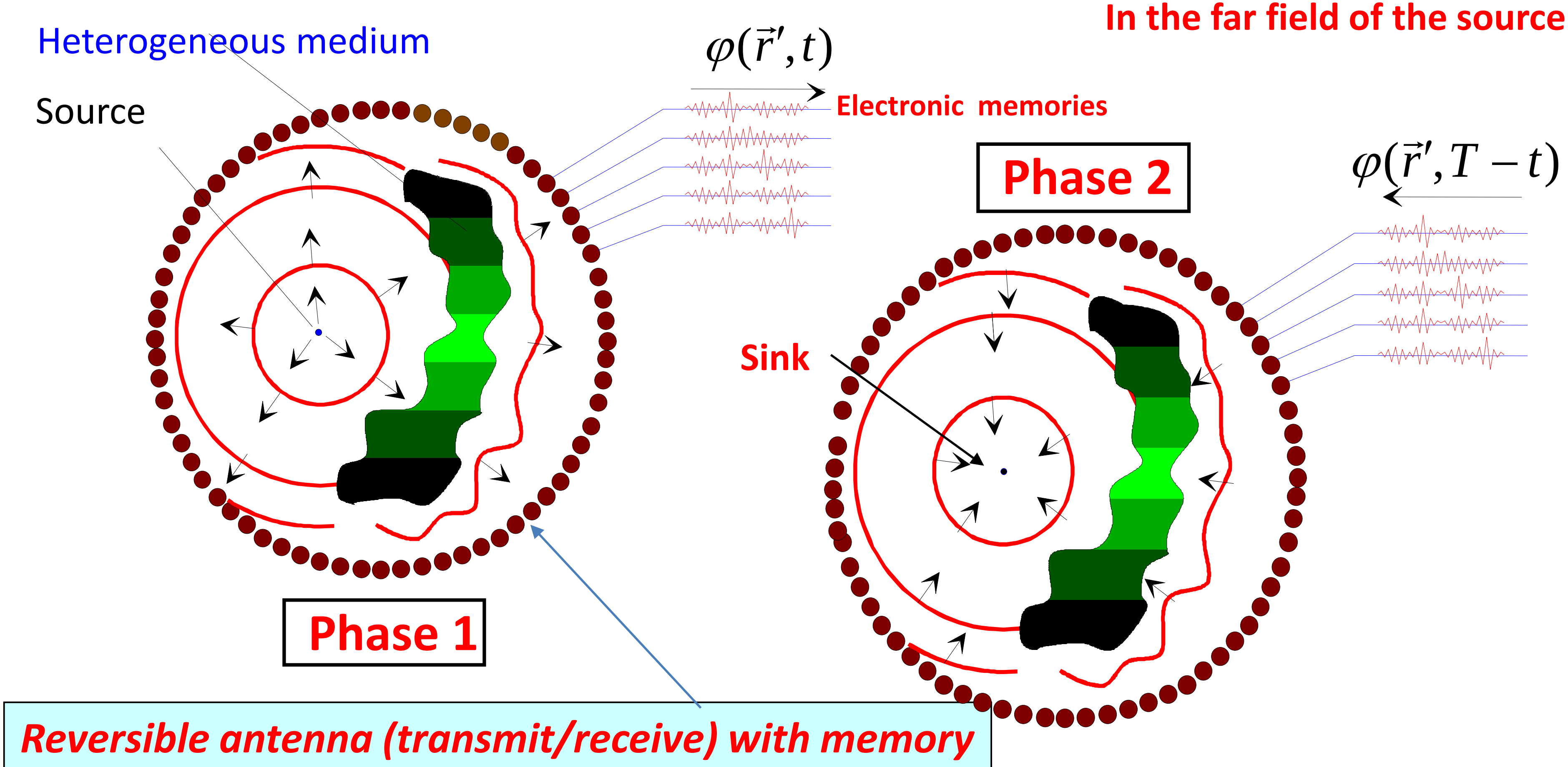


How can you change the relation between the wave field and its temporal derivative ? The concept of time boundary!!

II – Manipulating (spatial) boundary conditions (BC) : <u>the Time-Reversal Mirror approach</u>

- record on the boundary $\varphi(\vec{r}\,',t); \sout{\partial_n \varphi(\vec{r}\,',t)}$
- transmit from the boundary $\varphi(\vec{r}\,',T-t); \sout{\partial_n \varphi(\vec{r}\,',T-t)}$

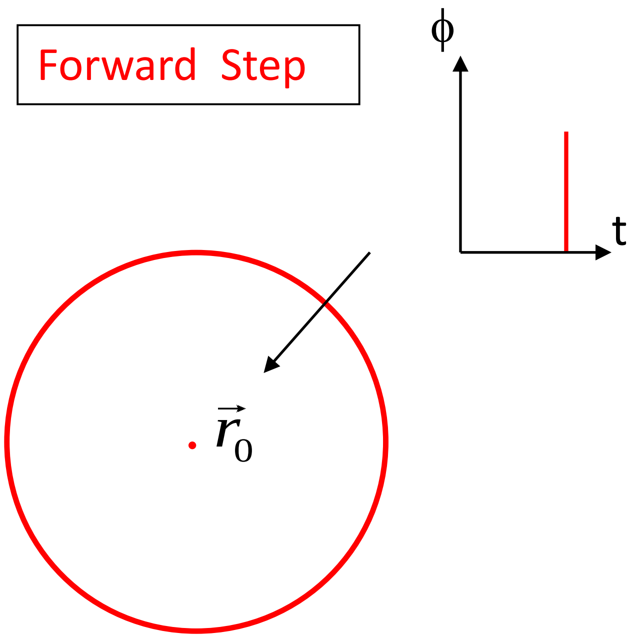
In the far field of the source



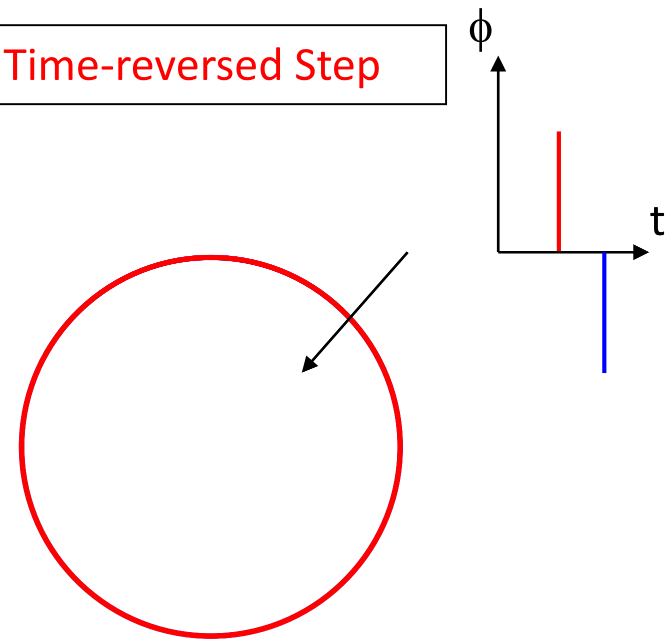
Origin of Diffraction Limits in Wave Physics

Pulsed mode – the homogeneous medium

Forward Step



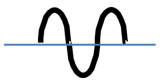
Time-reversed Step

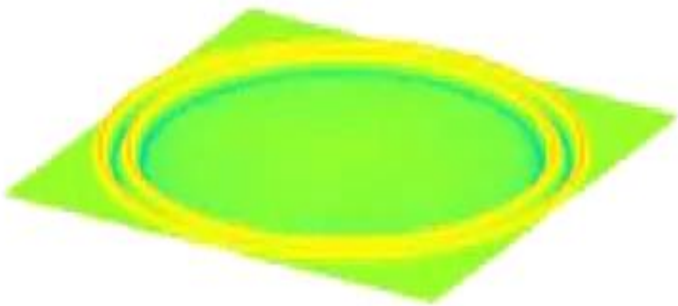


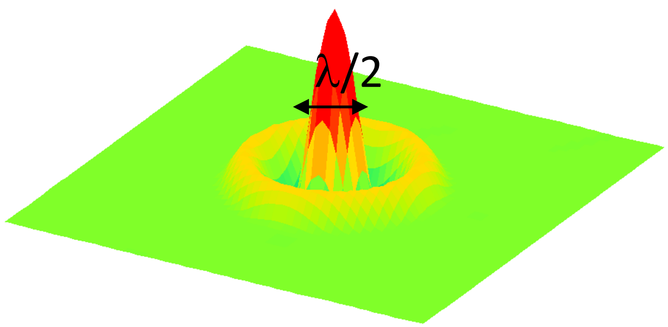
$$G^0_{caus}\left(\vec{r},\vec{r}_0;t\right)$$

$$\varphi_{tr}\left(\vec{r},t\right)=G^0_{caus}\left(\vec{r},\vec{r}_0;-t\right)-G^0_{caus}\left(\vec{r},\vec{r}_0;t\right)$$

Origin of Diffraction Limits in Wave Physics

wavelength λ 

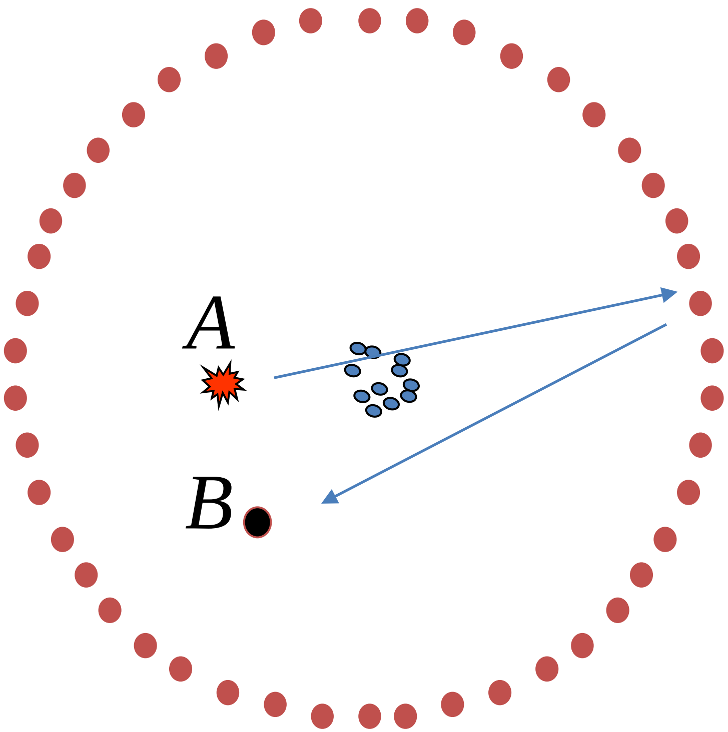




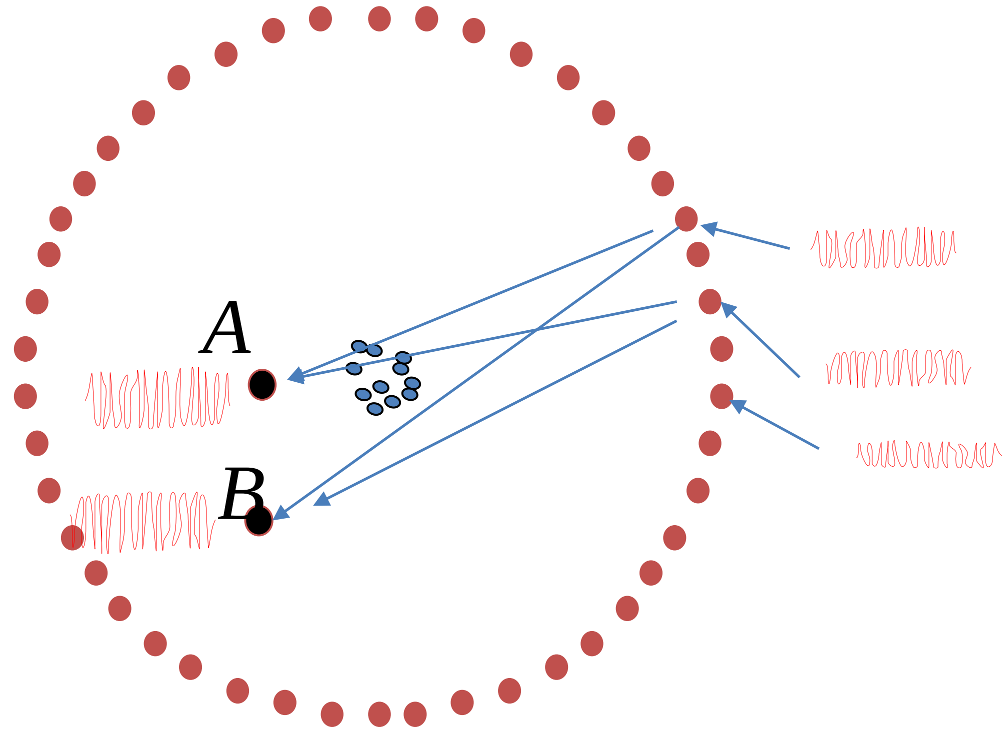
Diffraction limit is only due to the fact that we live in a **Causal World**

Analogy bewteen a TR experiment and spatial correlation in white noise

Closed Time-Reversal Mirror



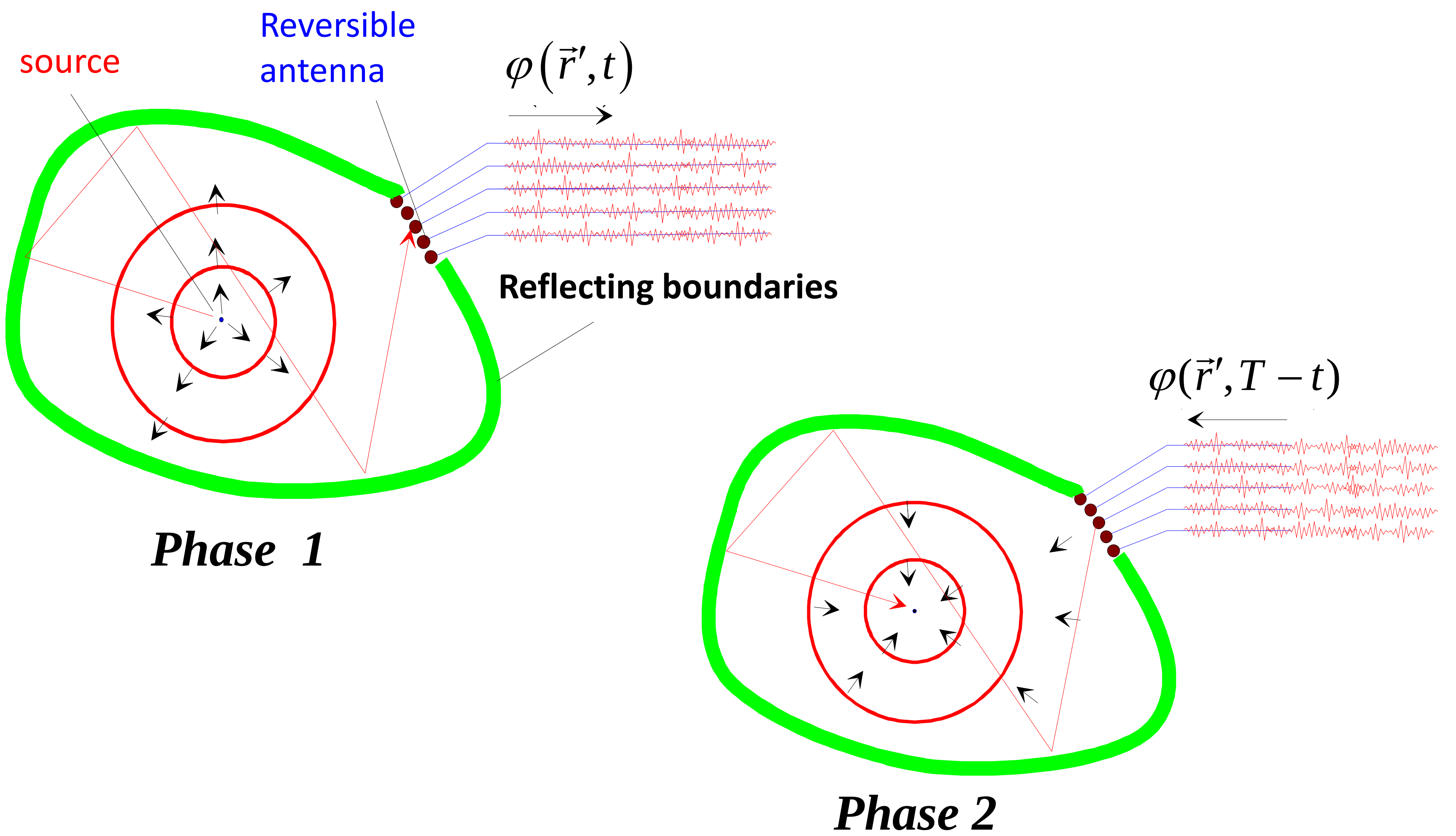
Isotropic white noise



$$\varphi_{TR-mirror}\left(B,t\right)=G_{caus}\left(A,B;-t\right)-G_{caus}\left(A,B;t\right)$$

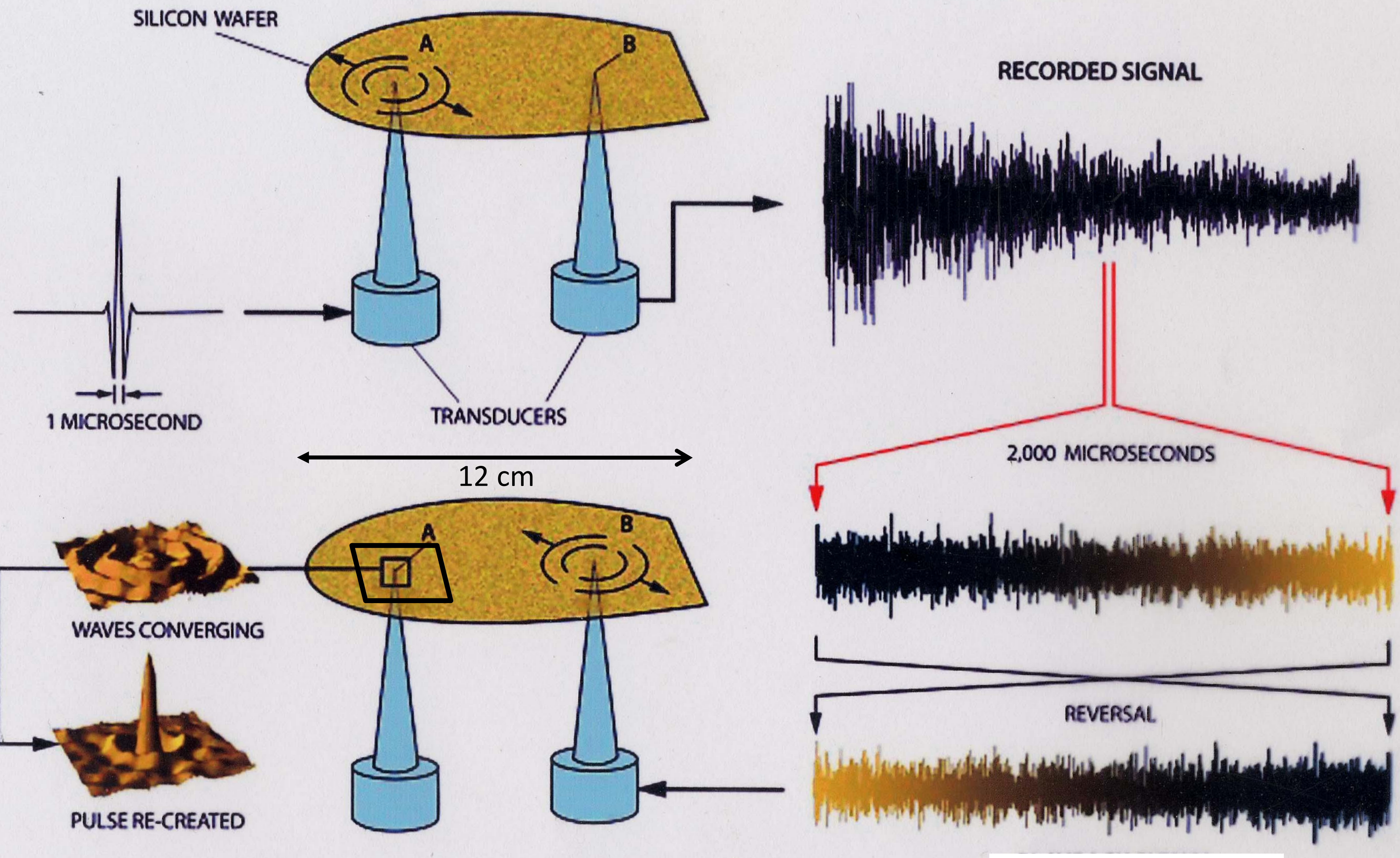
$$\partial_t C\left(A,B,t\right)\prec G_{caus}\left(A,B;-t\right)-G_{caus}\left(A,B;t\right)$$

Time-reversal mirror in a reverberating medium



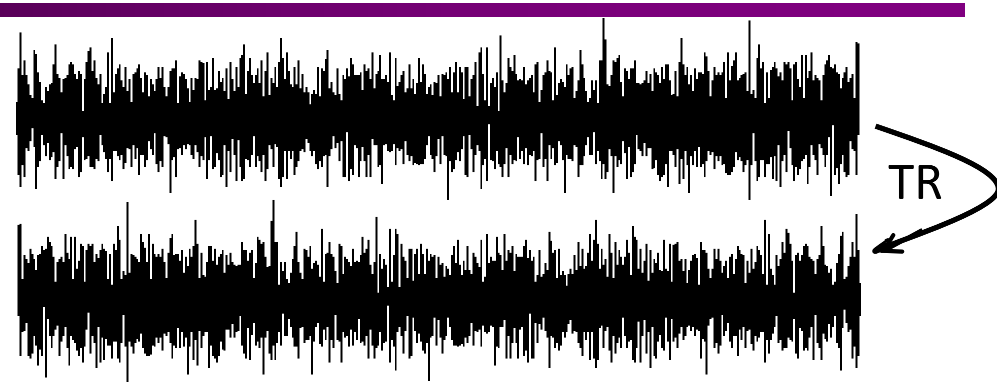
A small number of antenna with large memory is enough !!!

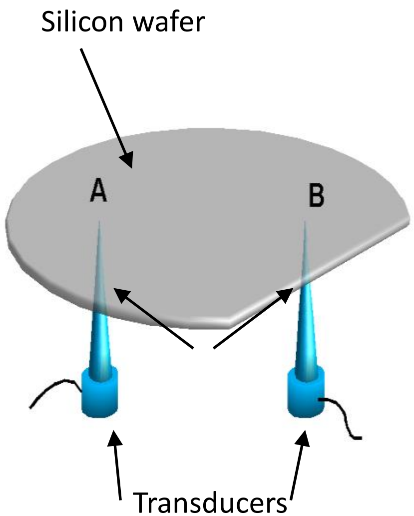
A one transducer time-reversal mirror



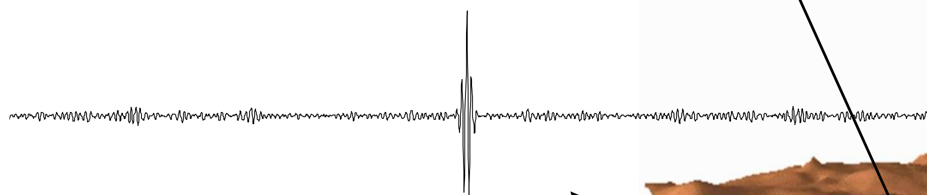
The time-reversed wave opticaly detected

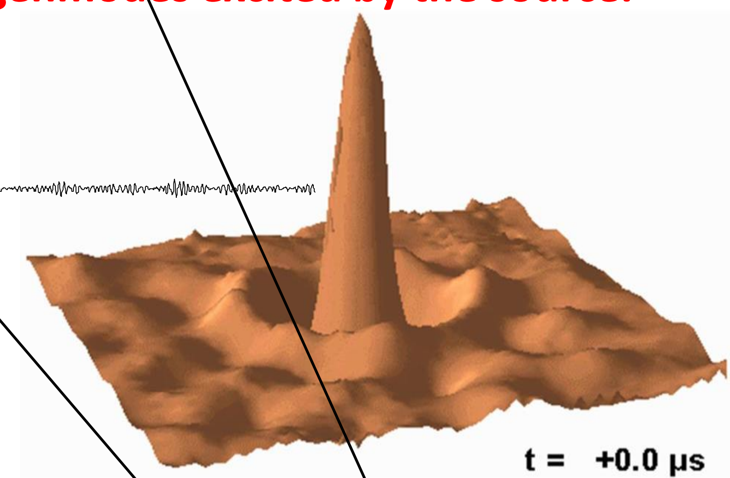
A 2 ms duration signal transmitted by point B :

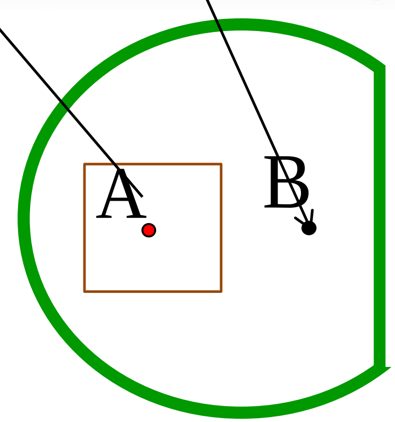




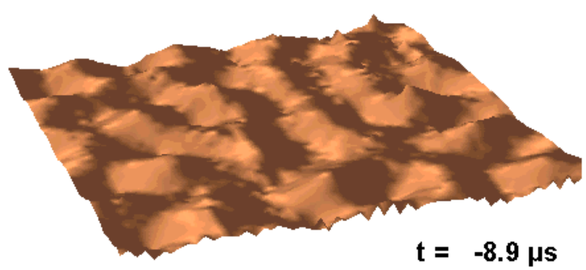
The key: number of eigenmodes excited by the source.
Frequency diversity







Displacement field recorded on a square 15 x15 mm^2



II- Time-Reversal « à la Loschmidt » Manipulating time boundaries.

The Instantaneous Time Mirror

A water wave experiment

Revisiting Loschmidt point of view

- record on the <u>whole volume V</u> the final conditions at time t_f

$$\varphi(\vec{r},t_f);\partial_t\varphi(\vec{r},t_f)$$

- prepare new initial conditions : changing the relation between the wave field and its temporal derivative

$$\{\varphi(\vec{r},t_i);\partial_t\varphi(\vec{r},t_i)\} = \{\varphi(\vec{r},t_f);-\partial_t\varphi(\vec{r},t_f)\}$$

A first alternative : Canceling the time derivative : $\partial_t\varphi(\vec{r},t_i)=0$

$$\{\varphi(\vec{r},t_i);\partial_t\varphi(\vec{r},t_i)\} = \{\varphi(\vec{r},t_f);0\}$$

« À la Neumann »

$$\{1/2\,\varphi(\vec{r},t_f);1/2\,\partial_t\varphi(\vec{r},t_f)\}+\{1/2\,\varphi(\vec{r},t_f);-1/2\,\partial_t\varphi(\vec{r},t_f)\}$$

1/2 the forward wave + 1/2 the time-reversed wave

A second alternative : Canceling the field : $\varphi(\vec{r},t_i)=0$

$$\{\varphi(\vec{r},t_i);\partial_t\varphi(\vec{r},t_i)\} = \{0;\partial_t\varphi(\vec{r},t_i)\}$$

« À la Dirichlet »

$$\{1/2\,\varphi(\vec{r},t_f);1/2\,\partial_t\varphi(\vec{r},t_f)\}-\{1/2\,\varphi(\vec{r},t_f);-1/2\,\partial_t\varphi(\vec{r},t_f)\}$$

1/2 the forward wave − 1/2 the time-reversed wave

How to change suddendly the relation between the wavefield and its temporal derivative ?
It depends on the wave velocity.

Imagine that you can **change instantaneously the wave velocity** in the whole space ?

Let us look **Water Waves** as a first example

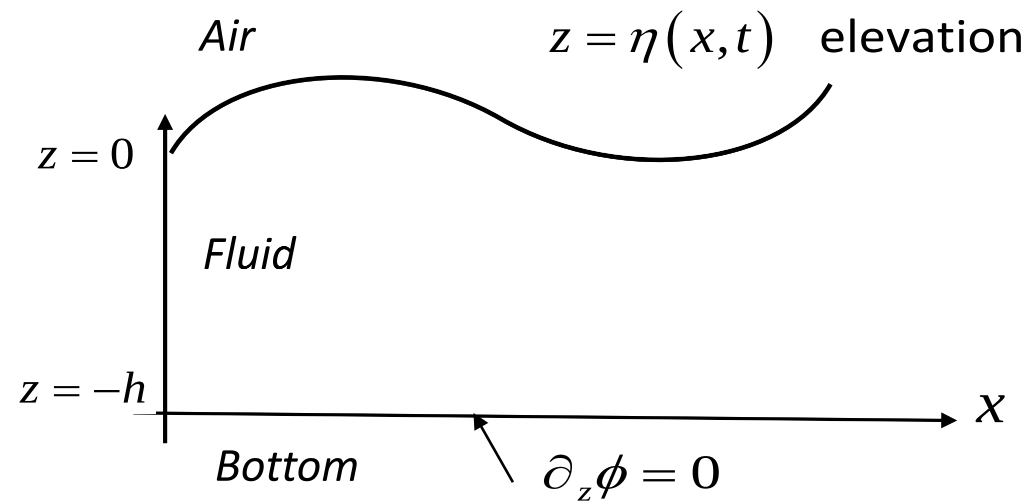
The case of Water Waves .

The main restoring force is Gravity : Gravity Waves

inviscid and incompressible flow

$$\vec{v}(\vec{r},t) = \vec{\nabla}\phi$$

$$\Delta\phi = 0$$



Linearization

$\partial_t \eta = \partial_z \phi$ at $z = 0$

$\partial_t \phi + g\eta = 0$ at $z = 0$

$\phi(x,z,t) = Z(z)\exp(kx - \omega t) \Rightarrow$ Dispersion relation $\omega^2 = gk \tanh(kh)$

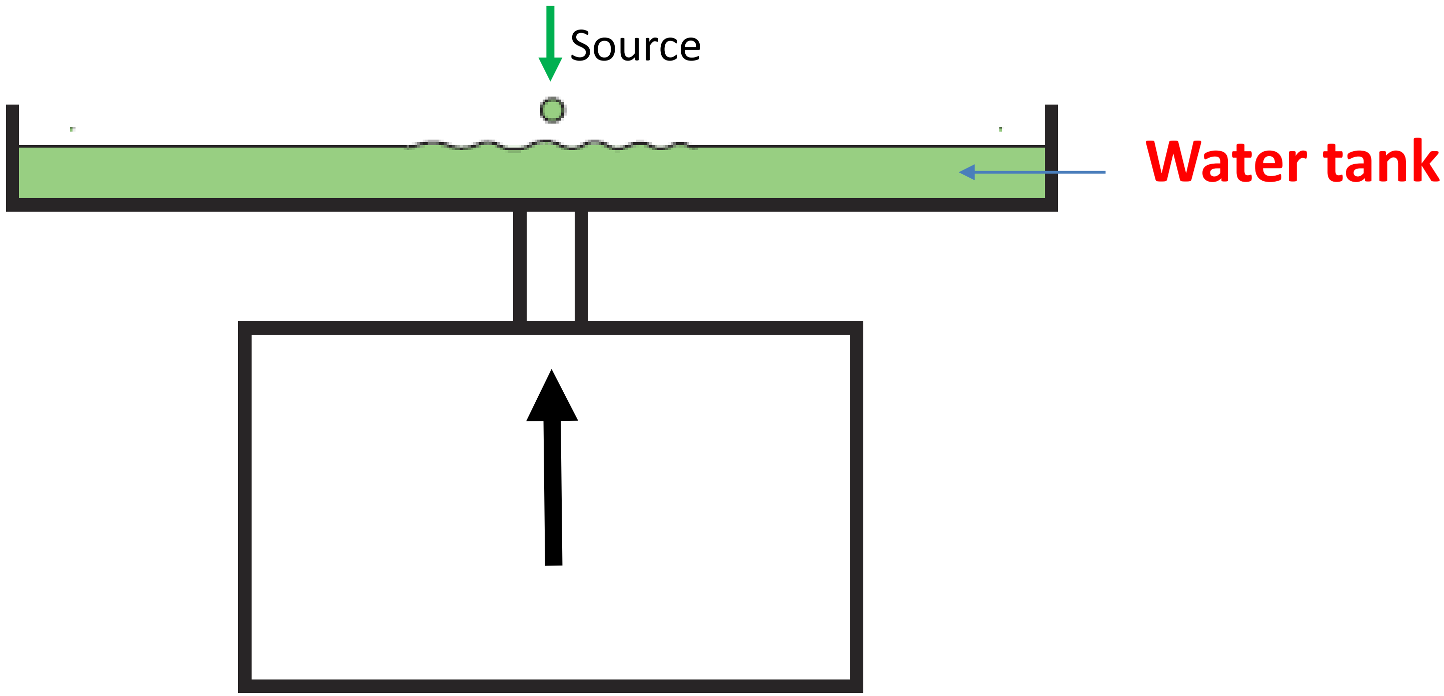
For shallow water $c_0 = \sqrt{gh}$

How to change wave velocity ?

change gravity !!!!!!

Let us try a very brief vertical acceleration of a water tank !!

Transient observation of water wave radiated by an impulsive source

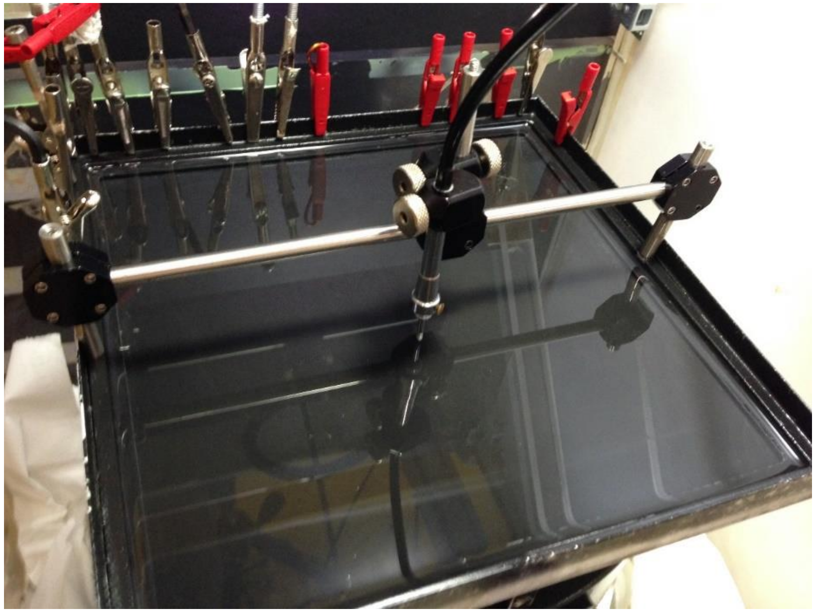


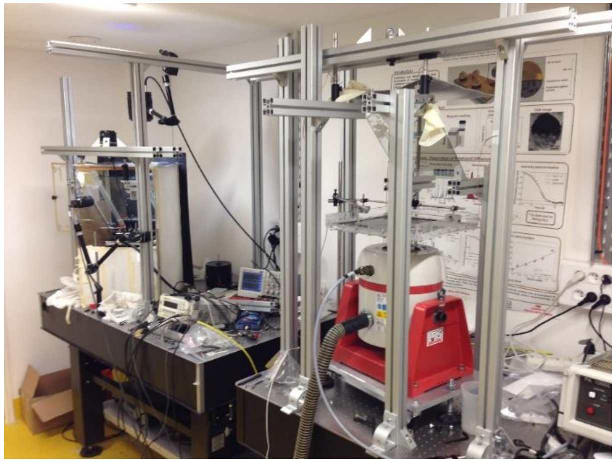
Here some time after the wave is radiated, we suddendly shake vertically the water tank. The gravity is modulated $g \Rightarrow g+\gamma(t)$, therefore the wave velocity is suddendly changed with respect to c_0^2 to $c_0^2\left(1+\gamma(t)/g\right)$ with $\gamma(t)/g \sim \beta\delta\left(t-t_f\right)$

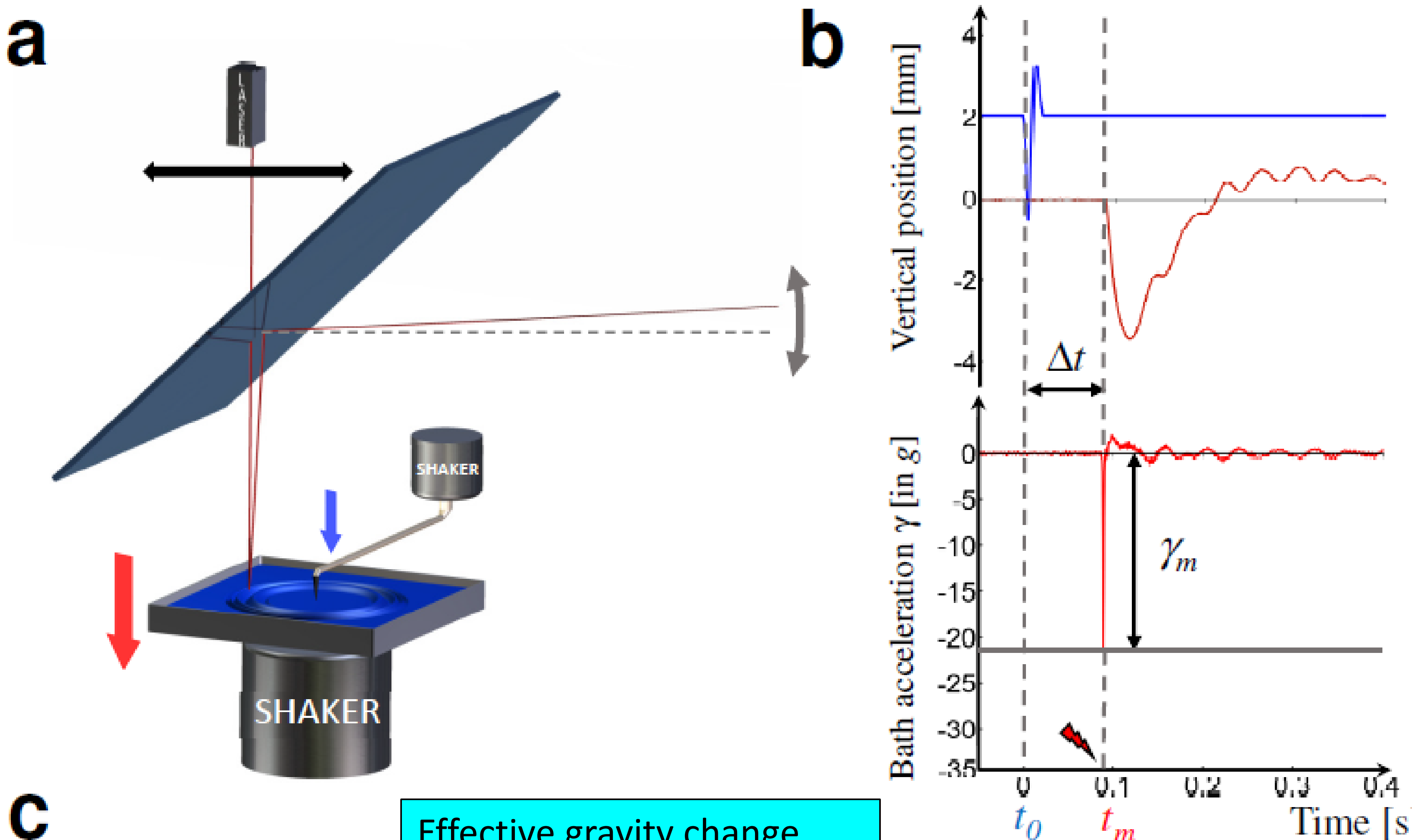
One creates a time discontinuity in the water tank by changing suddendly the wave velocity

V. Bacot, M. Labousse, A. Eddy, M. Fink , E. Fort.
« Time reversal and holography with spacetime transformations »,, *Nature Physics* (Oct 2016)

The setup



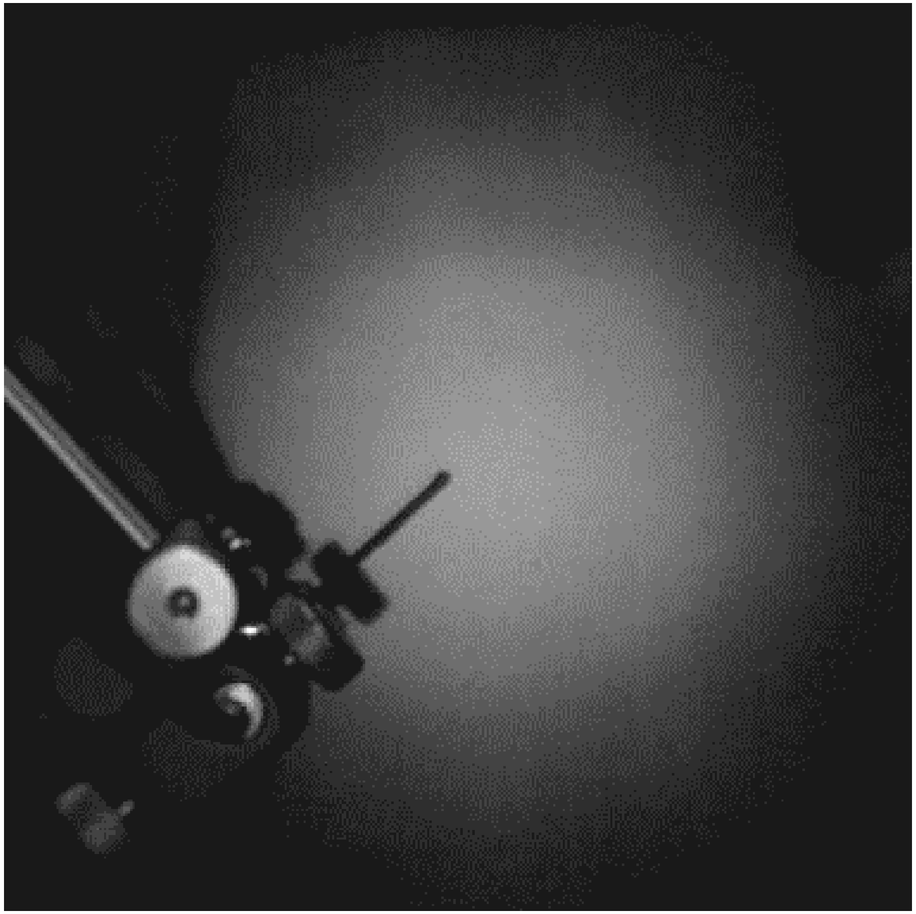




Effective gravity change from g to 20 g during 1 ms

V. Bacot, M. Labousse, A. Eddy, M. Fink , E. Fort.
« Time reversal and holography with spacetime transformations »,, *Nature Physics* (2016)

The instantaneous time mirror (ITM) experiment



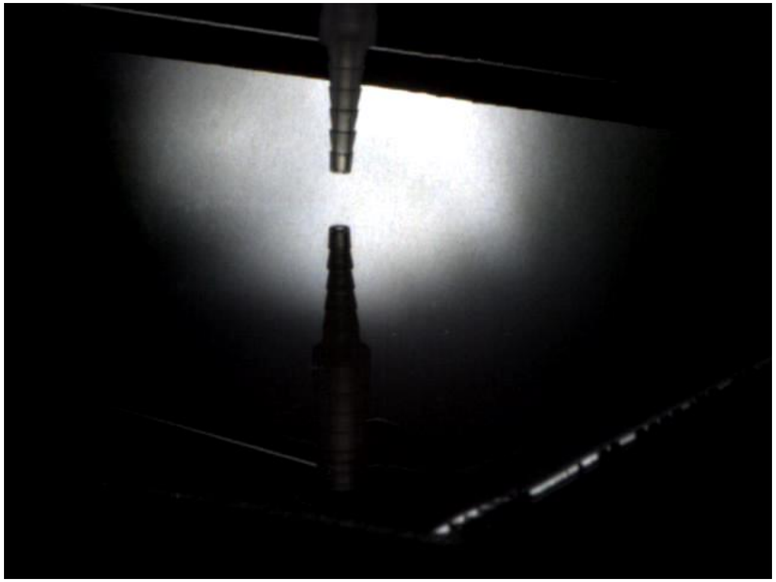
From above, slowed down 27 times

Instantaneous emission of a backpropagating wave ...

... from the whole space

In situ time reversal : instantaneous, no need to record the phase

The instantaneous time mirror (ITM) experiment



From the side, slowed down 50 times

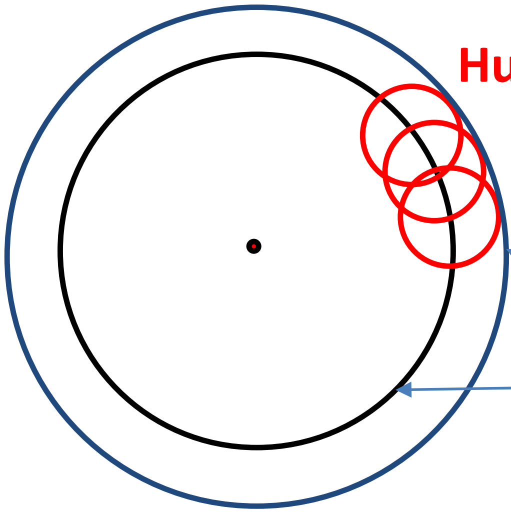
Apparition of reversed wave
at instant of jolt

Why ?

Huygens Principle revisited.
The Cauchy Problem

The Huygens Intuition

The case of shallow water (no dispersion)



$$\left\{\Delta - \frac{1}{c_0^{\ 2}}\partial_{tt}\right\}\varphi(\vec{r},t) = \delta\left(\vec{r},t\right)$$

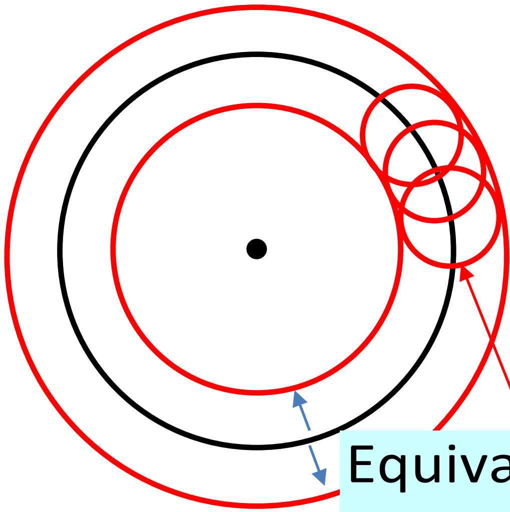
The causal Green's Function :

The wavefront at any instant conforms to the upper envelope of spherical wavelets emanating from every point on the wavefront at the prior instant

Why does an expanding spherical wave continue to expand outward from its source, rather than re-converging inward back toward the source ?

Later Fresnel and Kirchoff introduced the concept of interference and shows that in order to build a self-consistent solution the wavelets are to be monopole and dipole with obliquity factors

Huygens revisited by a sudden shake



$$\left\{\Delta - \frac{1}{c_0{}^2\left(1+\gamma(t)/g\right)}\partial_{tt}\right\}\varphi(\vec{r},t) = 0$$

Energy injection

at time t_f

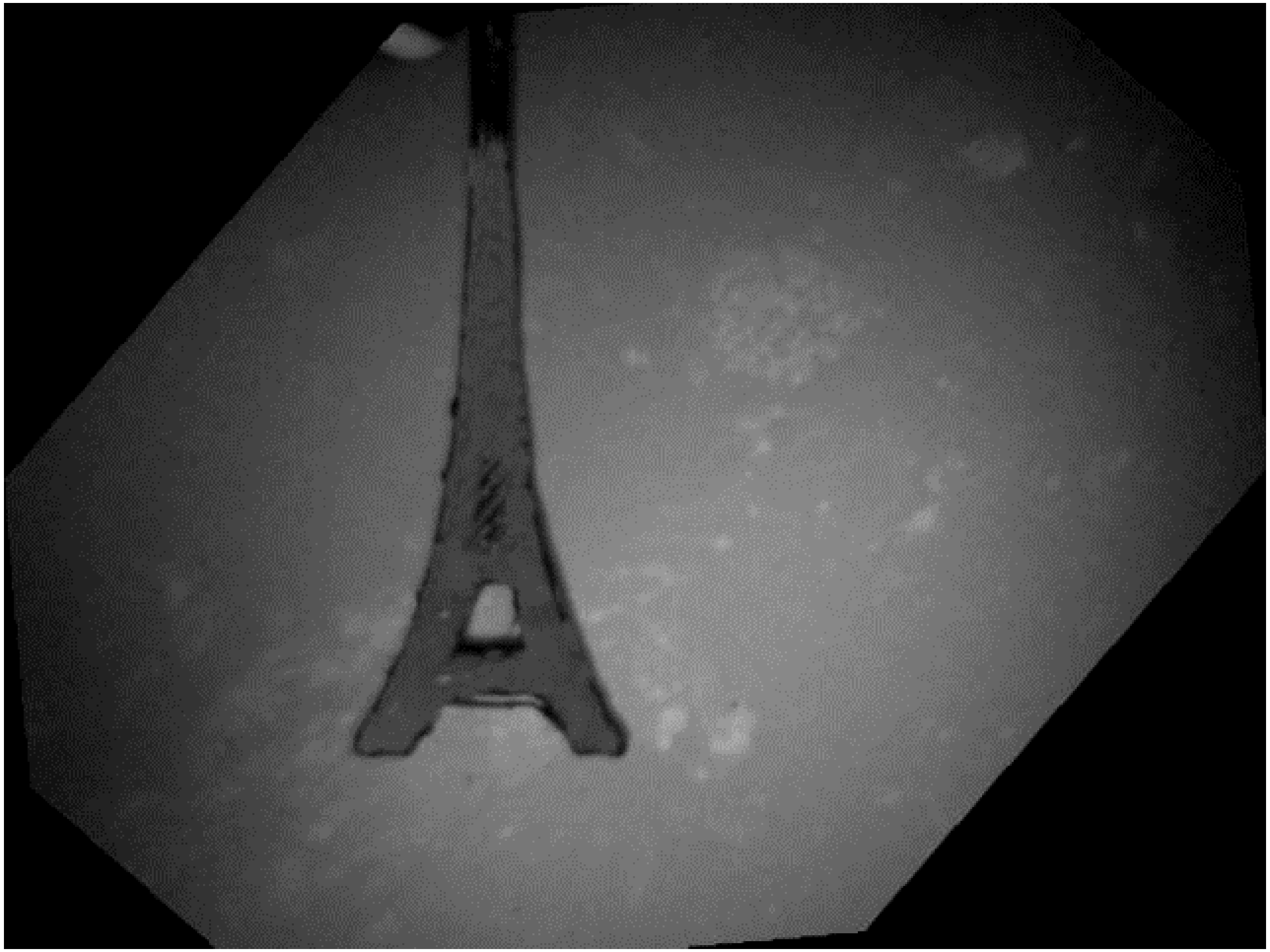
Equivalent to a wave equation with a source term

$$\left\{\Delta - \frac{1}{c_0{}^2}\partial_{tt}\right\}\varphi(\vec{r},t) = s(\vec{r},t)$$

where $s(\vec{r},t) \simeq -\frac{\gamma(t)}{c_0{}^2 g}\partial_{tt}\varphi(\vec{r},t)$ with $\gamma(t)/g \approx \beta\delta(t-t_f)$

monopoles

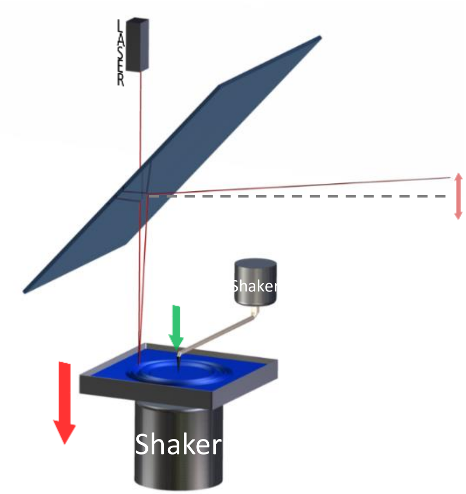
$$\varphi(\vec{r},t) = \iiint\left[G(\vec{r},\vec{r}';t-t_i)\,s(\vec{r}',t_f)\right]\mathrm{d}^3\vec{r}' \quad \text{with } s(\vec{r}',t_f) = -\frac{\beta}{c_0{}^2}\partial_{tt}\varphi(\vec{r}',t=t_f)$$

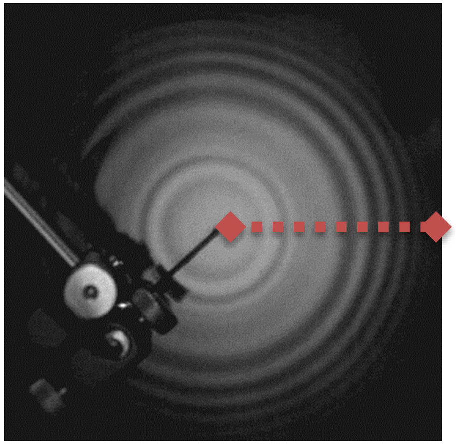


V. Bacot, M. Labousse, A. Eddy, M. Fink , E. Fort.
« Time reversal and holography with spacetime transformations »,, *Nature Physics* (2016)

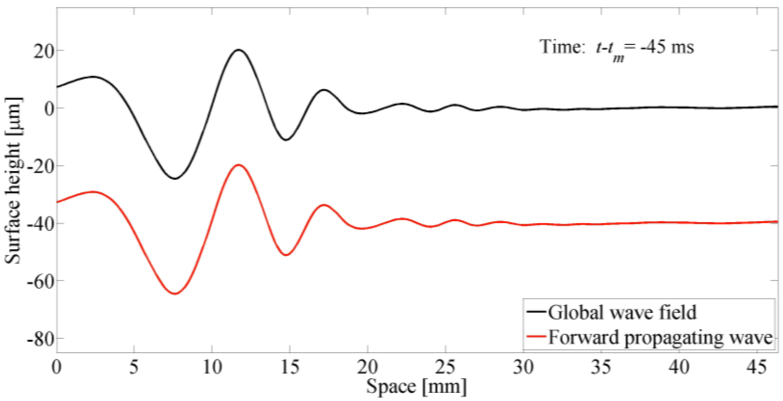
An instantaneous hologram that is played back with a time discontinuity.
It creates a real image of the object !!!!!!!!!!!!!!!!!!

The effect of dispersion: Capillary-Gravity waves





Mesurement of the elevation along a radius

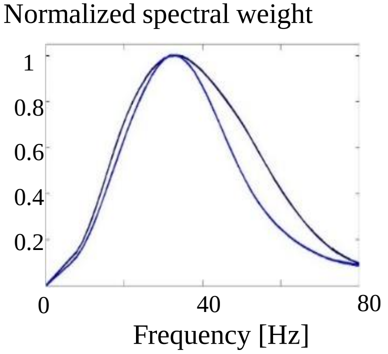


Blue curve : time-reversed wave
Dispersion effect is canceled

with γ surface tension

$$\omega^2(k) = gk + \frac{\gamma}{\rho}k^3$$

Broad band
Instantaneous
Time Reversal



Conservation Laws

Conservation law for spatial discontinuity *c(z)*

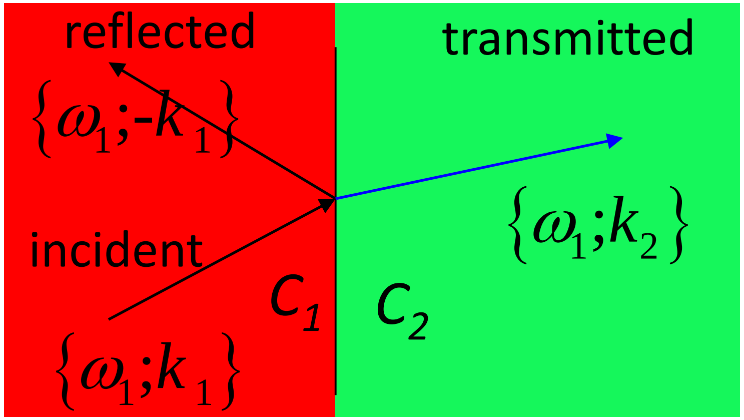
Phase matching conditions $\forall t$

Energy conservation

$$\omega_1 = \omega_t = \omega_r$$

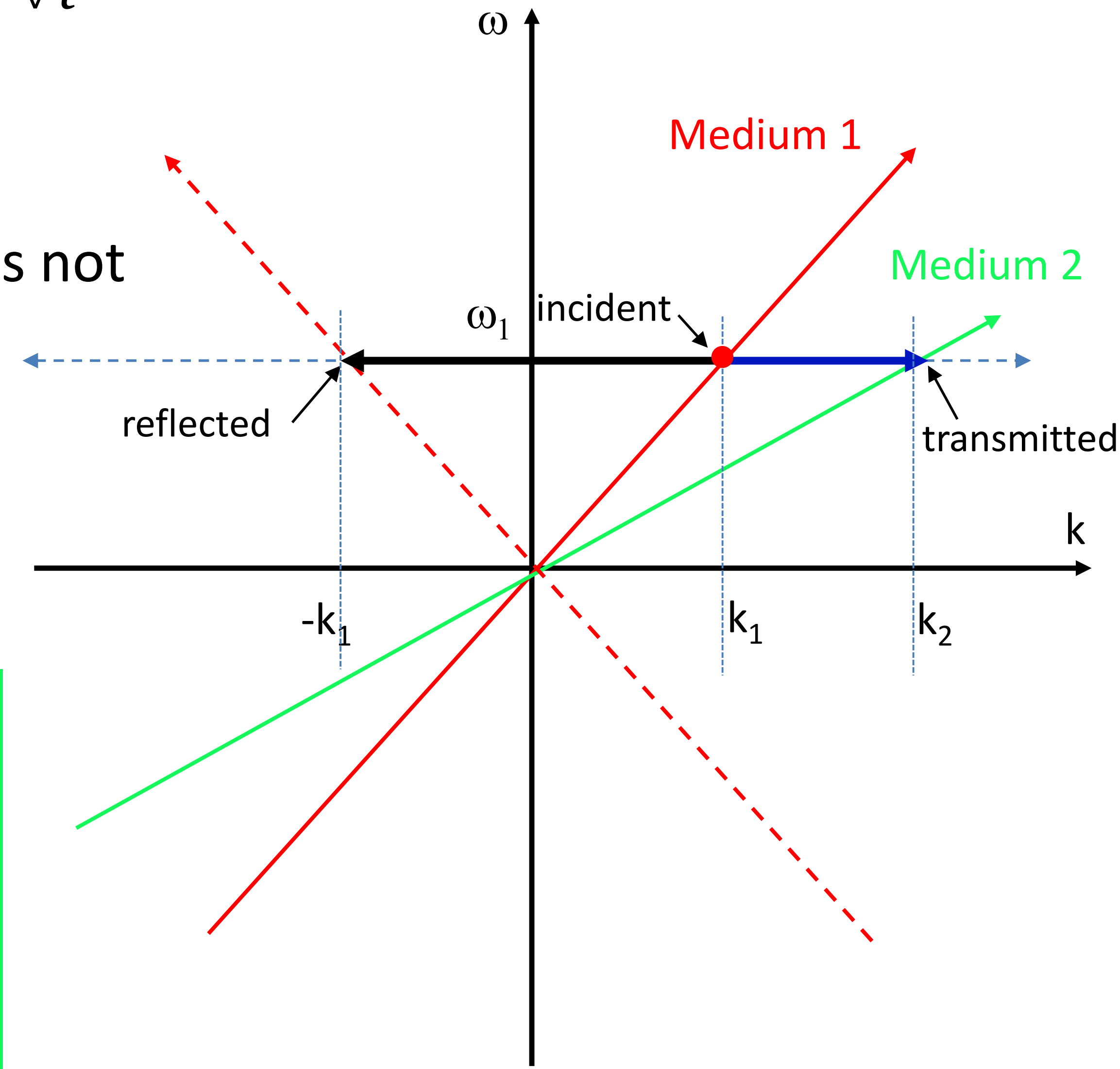
Wave vector (momentum) is not conserved

A plane wave approach



Medium without dispersion

Dispersion relation



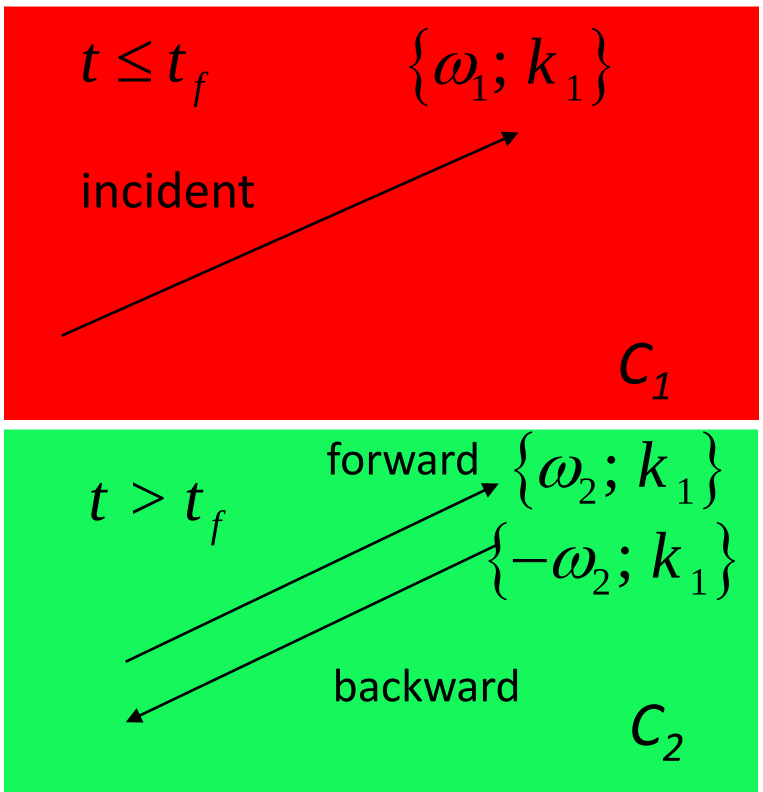
Conservation law for temporal discontinuity *c(t)*

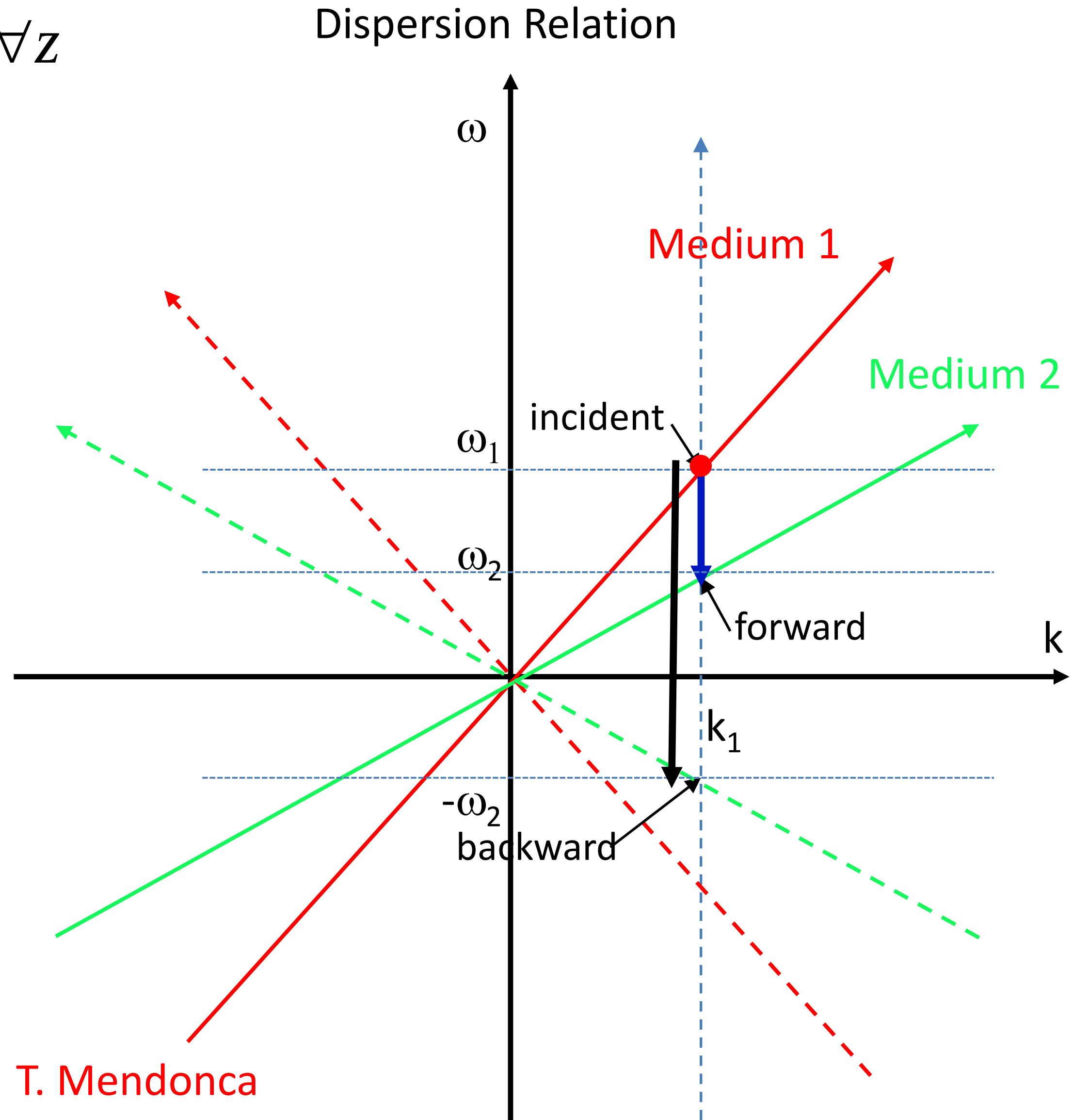
Phase matching conditions $\forall z$

Energy (frequency) is not conserved $\omega_2 \neq \omega_1$

Wave vector conservation

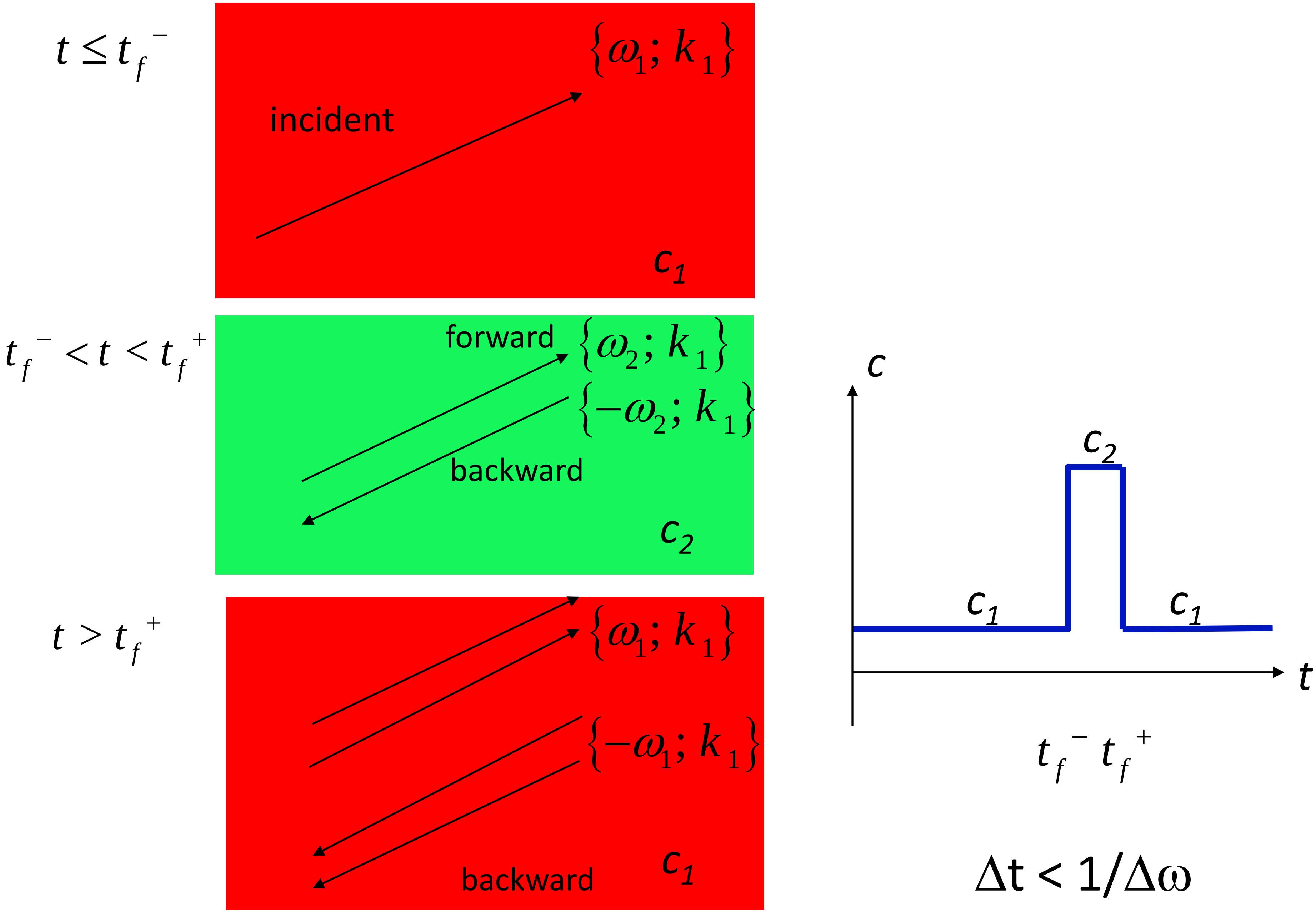
$k_1 = k_f = k_b$ momentum



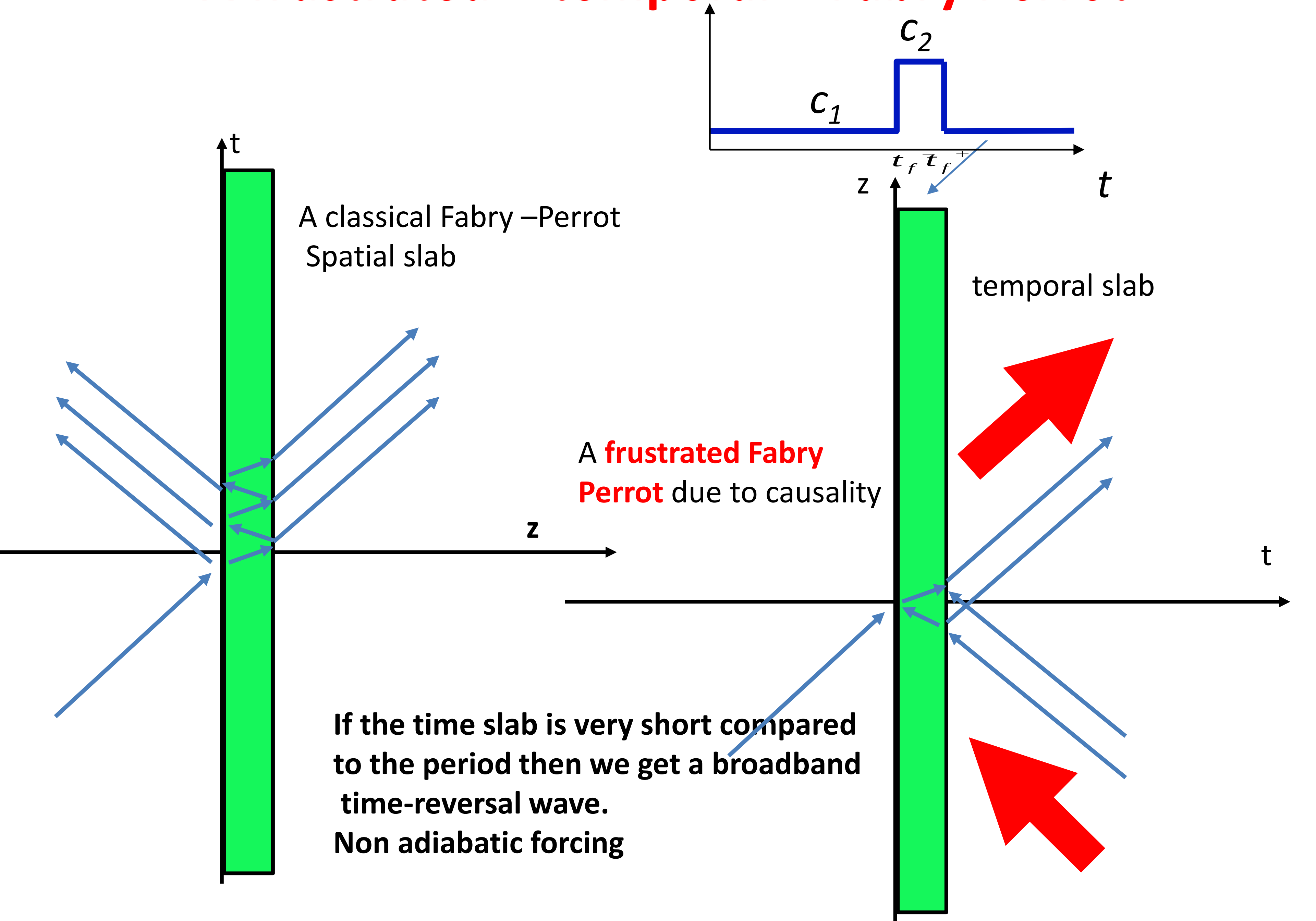


Note that the backward and forward waves oscillate at frequency different from the one of the incident wave !

To recover the initial spectrum : 2 successive discontinuities

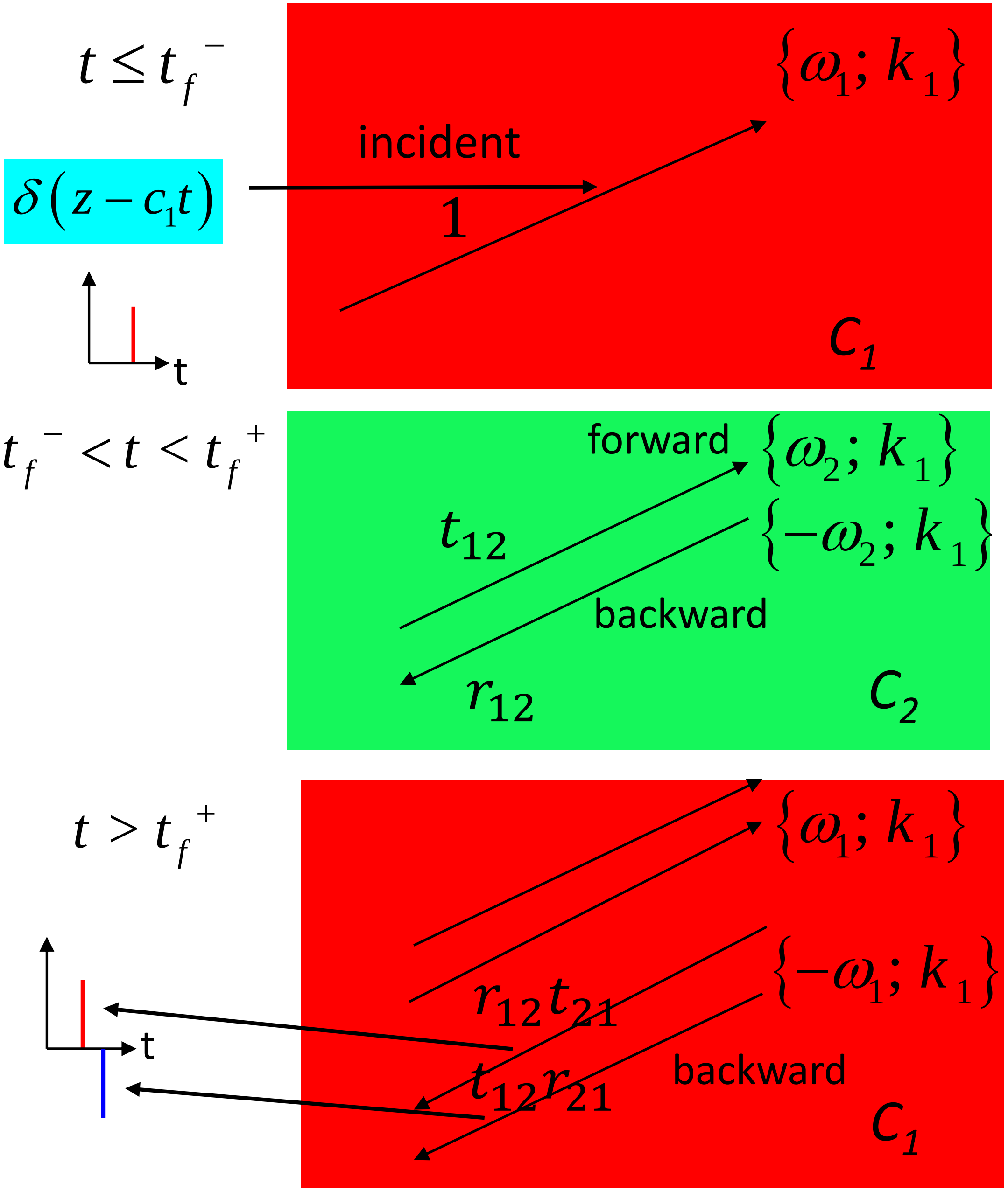


A frustrated « temporal » Fabry Perrot



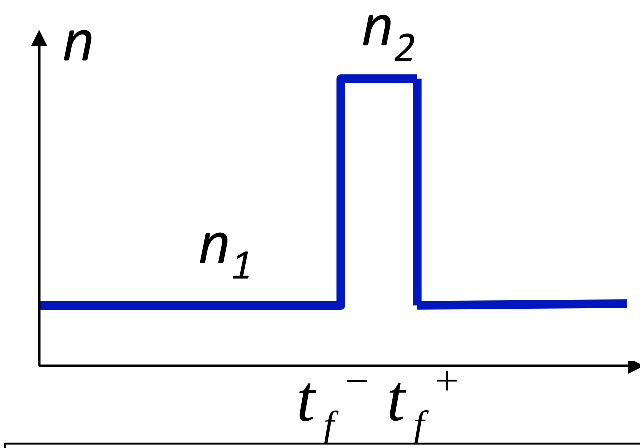
If the time slab is very short compared to the period then we get a broadband time-reversal wave.

Non adiabatic forcing



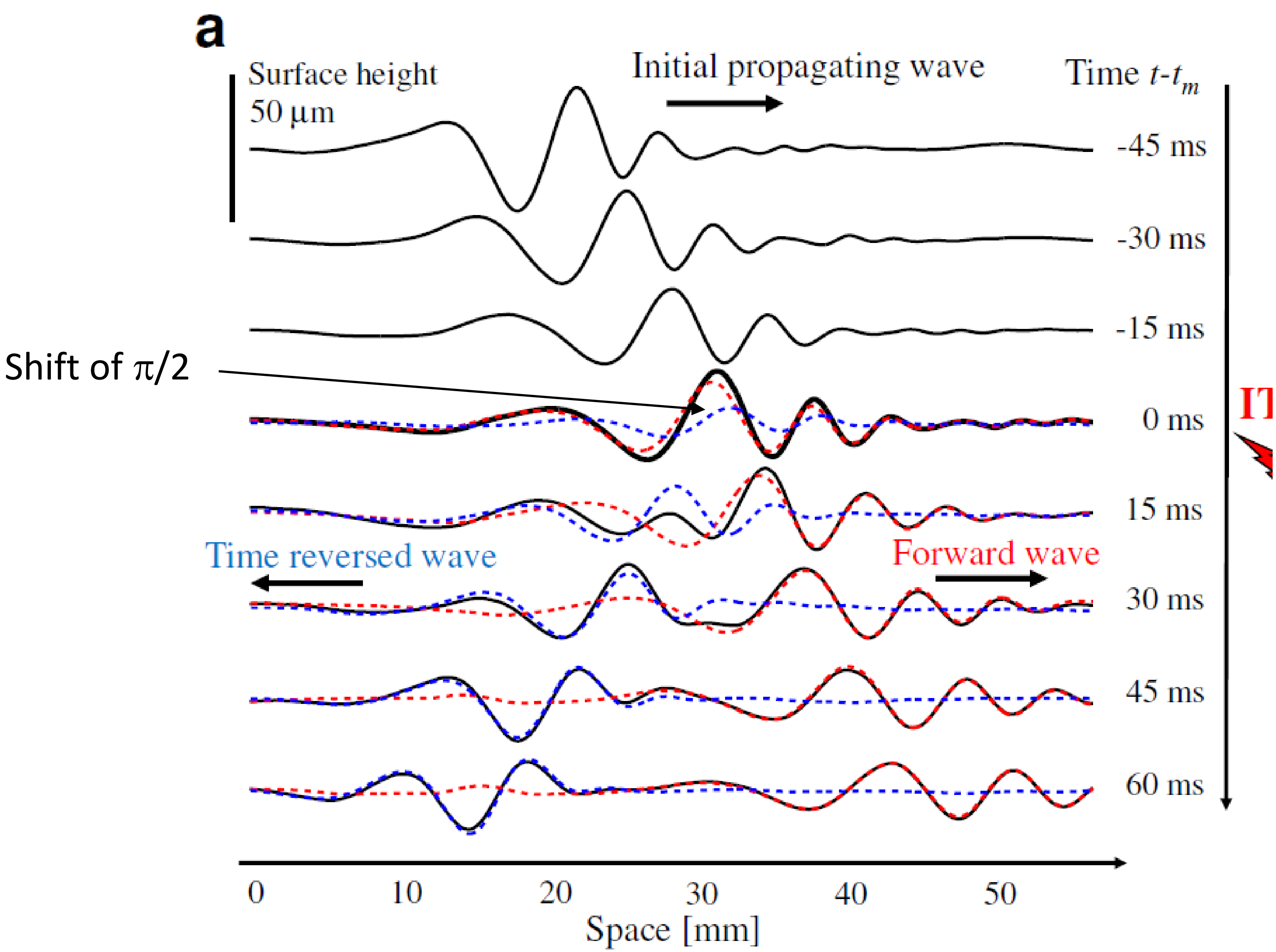
$$t_{12} = \frac{n_1 + n_2}{2n_1} \qquad r_{12} = \frac{n_1 - n_2}{2n_1}$$

$$t_{21} = \frac{n_1 + n_2}{2n_2} \qquad r_{21} = \frac{n_2 - n_1}{2n_2}$$



Note that the duration of the Temporal Fabry Perrot has to be shorter than the inverse of incident spectrum : **non adiabatic process**

When $t_f^+ - t_f^-$ tends to 0, one gets for the total backward wave $\delta'(z + c_1 t)$


a
Surface height
50 µm
Initial propagating wave
Time t-t_m
-45 ms
-30 ms
-15 ms
Shift of π/2
0 ms
IT
15 ms
Time reversed wave
Forward wave
30 ms
45 ms
60 ms
0
10
20
30
40
50
Space [mm]

How to describe our Experiment in term of Initial Conditions Transformation ?

Our initial goal was (The Loschmidt Daemon) :

$$\left\{\varphi(\vec{r},t_f);\partial_t\varphi(\vec{r},t_f)\right\} \Rightarrow \left\{\varphi(\vec{r},t_f);-\partial_t\varphi(\vec{r},t_f)\right\}$$

Or more modestly :

$$\left\{\varphi(\vec{r},t_f);\partial_t\varphi(\vec{r},t_f)\right\} \Rightarrow \left\{0;\partial_t\varphi(\vec{r},t_f)\right\}$$

In fact we are doing only :

$$\left\{\varphi(\vec{r},t_f);\partial_t\varphi(\vec{r},t_f)\right\} \Rightarrow \left\{\varphi(\vec{r},t_f);\partial_t\varphi(\vec{r},t_f)\right\} + \left\{0;\frac{\beta}{c^2}\partial_{tt}\varphi(\vec{r},t_f\right\}$$