Small-scale seismic inversion using surface waves extracted from noise cross correlation

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Abstract: Green's functions can be retrieved between receivers from the correlation of ambient seismic noise or with an appropriate set of randomly distributed sources. This principle is demonstrated in small-scale geophysics using noise sources generated by human steps during a 10-min walk in the alignment of a 14-m-long accelerometer line array. The time-domain correlation of the records yields two surface wave modes extracted from the Green's function between each pair of accelerometers. A frequency–wavenumber Fourier analysis yields each mode contribution and their dispersion curve. These dispersion curves are then inverted to provide the one-dimensional shear velocity of the near surface.

PACS numbers: 43.40.Ph, 43.60.Fg [JM]
Date Received: September 22, 2007 **Date Accepted:** December 10, 2007

1. Introduction

It has been demonstrated both theoretically and experimentally that records of seismic noise, equidistributed in angle, yield the Green's functions between receiver pairs after a time-averaged cross-correlation process. Since 2002, several papers have been published on this noise cross-correlation process in geophysics applied from a hundred of meter scale to a hundred of kilometers scale.[1–4]

One necessary ingredient to extract Green's function from noise correlation is the angle isotropy of seismic noise or, at least, the knowledge of the noise directivity on the seismic network. This strong hypothesis is rarely verified in practical situations when dealing with high frequency seismic noise (above 5 Hz) and small scales (less than 1 km) where noise sources have human origin. A way to get around the noise source distribution ambiguity is to perform the correlation process from a set of user-defined noise sources. When the seismic network is defined as a line of geophones, one way to deal with user-defined noise sources is to spread them in range on 10–50 wavelengths along the receiver alignment.

This technique presents several advantages. First we do not need an active device as a source. The strength of the source is often a problem when working in environments (such as an urban area, for example) where the use of potentially destructive devices such as explosives is prohibited. Here it is the duration of the record that governs the available energy of the signal. Furthermore, no synchronization of source and acquisition is required. However, using a source to perform velocity measurement between two stations is well known and currently used in seismology but it generally leads to apparent velocities which must be corrected according to the source location. Here by using a spatial distribution of sources, whose contributions are eventually averaged, we extract a response between the stations that is independent of the paths source stations, as far as the illumination is sufficient for the Green's function emergence. This condition requires in practice that the source, i.e., the steps, cover a region corresponding to the "end fire lobes" of the receiver line.[5] In this case, no angular correction relying on a speculative velocity model is needed. In conclusion, extracting the Green's function from user-defined noise sources makes the correlation technique a trade-off between an active and a passive method—active since we do not rely on the natural ambient noise in the medium, but passive since the correlation process naturally achieves synchronization and averaging.

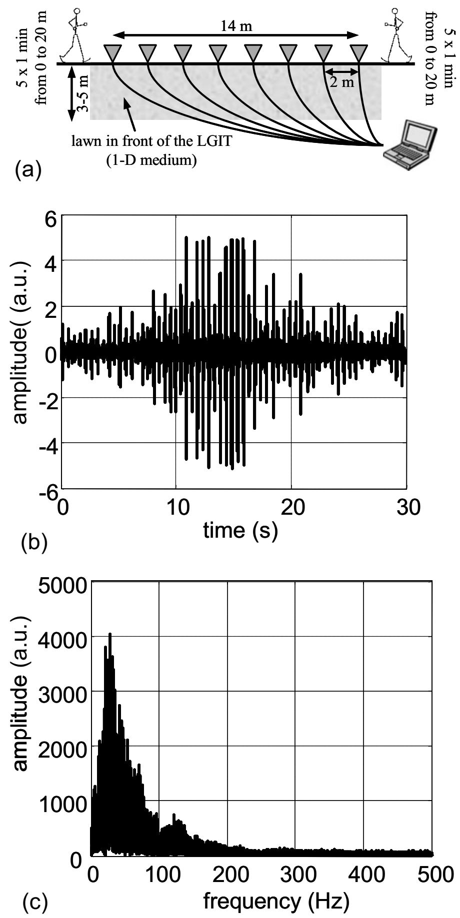

Fig. 1. (a) Experimental setup. The acquisition system is performed from a National Instrument board that gives access to the recordings in real time. The array deployment involved eight geophones but can be adapted to various configurations using up to 16 accelerometers or geophones. (b) Typical 30-s-long time-domain recording and (c) its frequency spectrum. The foot step amplitude depends on the range to the geophone. Frequency spectrum ranges from 4 to about 150 Hz.

The goal of this work is to perform an experimental demonstration of the correlation process of user-defined noise sources at the meter scale using a linear array of geophones. This letter is divided into two parts. In Sec. 2, we present the experimental setup, the recording of user-defined noise sources, and the averaging correlation process that leads to the final seismic section. In Sec. 3, a classical geophysical inversion is performed from the phase velocity dispersion curves of two Rayleigh modes extracted from the seismic section. Finally, the shear velocity profile is inverted over the first 5 m.

2. Retrieving a seismic section from user-defined noise cross correlation

A 14-m-long line of eight evenly spaced vertical geophones has been used to record human steps [Fig. 1(a)]. Two persons walked in the alignment of the accelerometers line, five times 1 min on each side, from 0 to about 30 m away from the geophone array. For a matter of convenience, the studied medium was the lawn in front of the laboratory.

The experimental setup has been designed to be versatile. The array configuration may include up to 16 one-component seismic stations. These seismic sensors could be accelerometers or geophones depending on the expected frequency bandwidth. Similarly, the array length is adjustable to the surface wave wavelength.

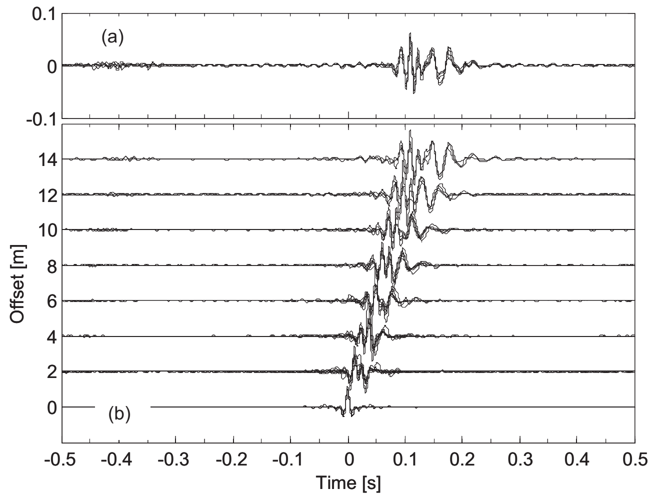

Fig. 2. (a) Superposition of five 1-min-long correlations between a geophone pair separated by 14 m. (b) Superposition of seismic sections obtained from correlation between accelerometer pairs separated by the same range. The signal is normalized at each range.

The main advantage of this system is to be easy and fast to setup. Our ambition was to perform a complete deployment, acquisition, and inversion in approximately 30 min. The system design makes it very convenient for local and near surface measurements. Figure 1(b) shows a typical time-domain recorded signal with its frequency spectrum. Each event in the signal is a step whose amplitude depends on the range to the receiver. The energy spectrum of the 30-s-long record is spread up to 150 Hz. Given the frequency response of the accelerometers and the spatial extension of the array, a frequency interval ranging from 10 to 100 Hz was selected for the analysis.

Since the frequency spectrum of the steps [Fig. 1(c)] is not flat in the frequency interval of interest, and as correlating is mathematically equivalent to a spectrum product, only the most energizing frequencies will emerge in the correlation signal. To enlarge the effective frequency bandwidth, the spectrum of the records will be equalized in the selected frequency interval [10–100 Hz] before the correlation process.

To check the robustness of the correlation process, 5 1-min long records were correlated separately for each receiver pair. Figure 2(a) shows, for a given pair, that the five time-domain correlations superimpose in phase. Thus correlation is robust and does not depend on the person who walked. As those correlations signals superimpose, they are stacked to increase the signal-to-noise Ratio (SNR). The residual signal observed at large lag times in the correlation (for time ~ -0.5 s) corresponds to the correlation between two consecutive steps. This signal is incoherent over the five realizations and vanishes in the 5-min average.

The advantage of the correlation process is to perform an average on the different sources by stacking them with the appropriate time lag without the need for any synchronization between the source and the receivers. The superposition of the correlations of 1-min long signals is thus just a verification of the repeatability of the steps. Stacking the five correlations of 1-min-long records is equivalent to directly correlating a 5-min-long signal.

To obtain a seismic section from the correlation process, each signal recorded at one receiver is correlated with the signal recorded by geophone 1 or 8 located at each extremity of the line array. The seismic section clearly shows wave propagation along the geophone array [Fig. 2(b)]. Taking geophone 1 or 8 as the reference signal does not modify the seismic section. This shows that seismic propagation from left to right is identical to propagation from right to left on the 14-m-long seismic array. The medium can then be assumed as one dimensional (1D) in the frequency bandwidth of the recordings.

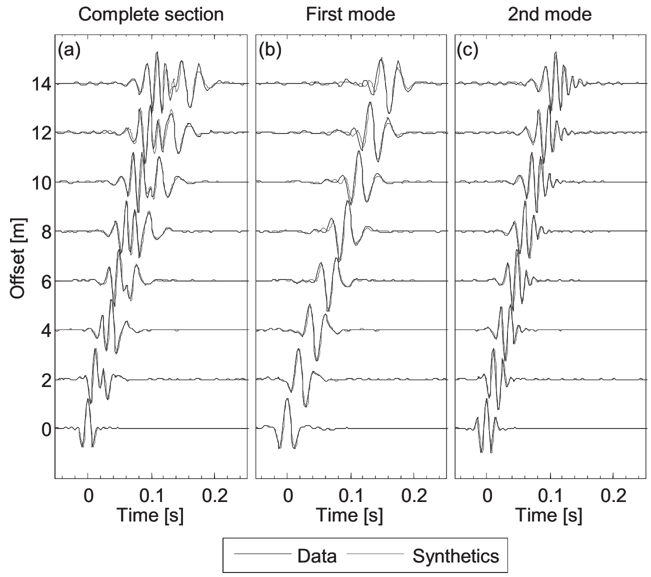

Fig. 3. (a) Seismic section obtained from the correlation process after all averaging operations. The signal-to-noise ratio is above 30 dB for each trace. Phase dispersion is clearly observed for two surface waves with group velocity of about 90 and 120 m/s. (b), (c) The two propagating modes have been extracted from the $F-K$ plot in Fig. 5. Dispersion of each mode is clearly observed. Synthetics obtained from the velocity model in Fig. 6(a) show a good agreement with experimental data.

The 1D argument can be pushed even further. Each receiver pair spaced by the same range should give the same correlation function as the propagation does not depend on the receiver locations but only on the separation between receivers. For each range, there are several equivalent receiver pairs. Figure 2(b) shows the superposition of the time-domain correlations of the equivalent pairs. For shorter ranges, there are more equivalent pairs (7 for 2 m, 6 for 4 m, etc.), which results in an efficient stacking and a better SNR. Figure 4(a) shows the seismic section obtained after all averaging operations. After a 10-min total recording, SNR is above 30 dB. Both phase dispersion and geometrical spreading during propagation are retrieved.

The final seismic section clearly reveals the presence of two surface waves, with mean group velocities of about 90 and 120 m/s. Those low group velocities are good indications of two surface waves identified as Rayleigh modes.

We insist on the fact that this section was obtained from 10 min of unsynchronized human steps only, which makes it nearly a passive method. To get the same result with classical active seismic techniques, much more time would have been needed to synchronize numerous sledgehammer blows. The "passive" method presented here is thus (1) easy to implement, as there is a large flexibility in the array configuration, (2) fast, as it takes only about 30 min to complete the array deployment and the recording, and (3) simple, as there is no synchronization task and processing is performed in real time.

3. Geophysics inversion

Given the high SNR of the seismic section obtained from the correlation process, we completed the analysis by a classical surface wave seismic inversion.[6,7]

A frequency—wave-number ($F-K$) transform was applied to the seismic section [Fig. 3(a)] to measure phase velocity dispersion curves [Fig. 4]. As the geophones are evenly spaced by a 2-m distance, the largest wave number satisfying the aliasing criterion is $2\pi/\mathrm{d}=\pi$. In Fig. 4, this corresponds to the black vertical line. Higher k's are wrapped, and appear as low wave numbers. In this simple case, the wave-number spectrum can be extended by unwrapping the k axis.

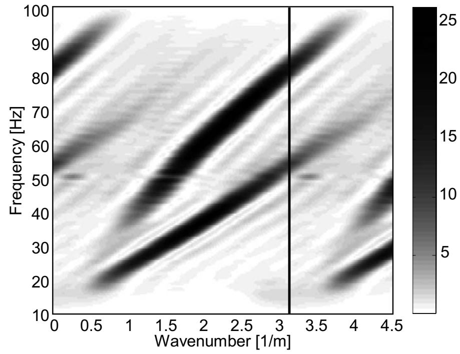

Fig. 4. Frequency–wave-number ($F-K$) transform of the seismic section obtained in Fig. 4(a). The largest measurable wave number according to the aliasing criterion is $2\pi/d$ with $d=2$ m (black vertical line). The aliasing in the $F-K$ diagram is resolved by unwrapping the k axis. The shape of the two modes on the $F-K$ diagram reveals dispersive modes.

In the $F-K$ diagram, modes are separated and extracted by masking a region of the $F-K$ plot and returning to the range/time domain [Fig. 3(b) and 3(c)]. This operation cannot be performed directly in the range/time domain since the two modes are mixed at short ranges.

From Fig. 4, phase velocity curves are measured for each mode separately using the relation $k=2\pi f/c$, where k is the wavelength, f is the frequency, and c is the phase velocity [Fig. 5(b)]. A Monte Carlo inversion is performed simultaneously on the two modes to obtain the most-likely shear velocity profile in the medium[7] [Fig. 5(a)]. The minimum frequency of the surface waves being larger than 20 Hz for a maximal wavelength of 12 m, the model is not constrained for depth deeper than 6 m. Similarly, the *P*-velocity profile and the density were poorly constrained by the surface wave dispersion curves and arbitrarily set to 900 m/s and 1.5 g/cm^3, respectively.

To improve confidence in the model, synthetic sections are computed separately for the two Rayleigh modes [Fig. 3(b) and 3(c)].[8] The computed traces are very close to the extracted modes. To check the relative weight of each mode, their sum is compared to the original seismic section before mode separation [Fig. 3(a)]. As the complete seismic section is well retrieved, the relative weights of the computed and experimental modes are identical, which

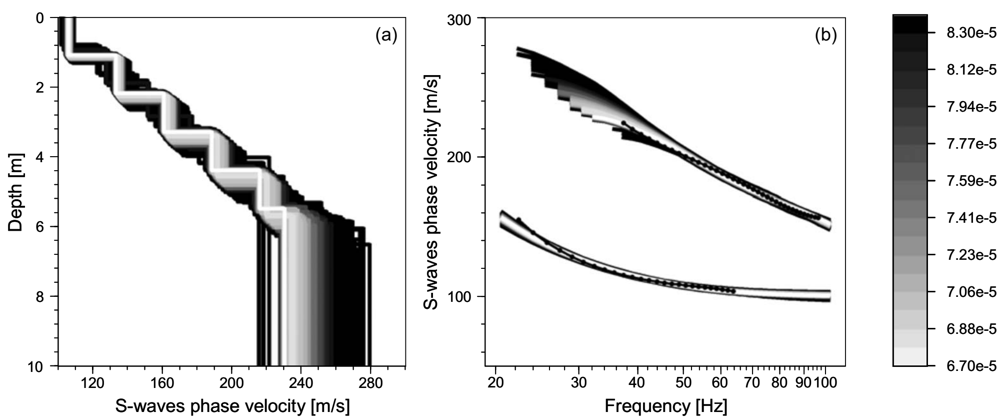

Fig. 5. (a) Distribution of shear velocity profiles obtained from a Monte Carlo inversion using the dispersion curves of the two surface-wave modes. (b) Distribution of phase dispersion curves for the two surface modes in the Monte Carlo inversion. Black spots correspond to the experimental phase-velocity curves obtained from Fig. 5. Colorbar scales correspond to a misfit increase of 24%.

confirms the quality of the inversion model. To increase the precision of the model, a full wave inversion could be performed that would benefit from the high SNR of the seismic section [Fig. 3(a)]. In this case, the inversion kernel should be modified to include both amplitude and time dispersion information.

4. Conclusion

A new method to retrieve surface waves at small scales has been developed. This method, which combines passive processing and active sources is simpler, faster, and easier to implement than classical active seismic techniques. It is also versatile and can be easily adapted to various bottom configurations and expected wavelengths.

A classical seismic inversion was performed using the observed seismic section. The high SNR allows us to get an accurate model for shear velocity in the $(0-7\ \text{m})$ depth.

Future works will investigate the case of two-dimensional (2D) media. In this case the stacking process performed in a 1D case won't be possible, but differences between intercorrelations at the same range but at different places could provide a 2D (range + depth) inversion result.

Acknowledgment

The authors are grateful to Marc Wathelet for his advice in the use of the Monte Carlo inversion software GEOPSY.

References and links

[1]M. Campillo and A. Paul, "Long-range correlations in the diffuse seismic coda," Science **299**, 547–549 (2003).
[2]K. Wapenaar, "Synthesis of an inhomogeneous medium from its acoustic transmission response," Geophysics **68**, 1756–1759, (2003).
[3]R. Snieder, "Extracting the Green's function from the correlation of coda waves: A derivation based on stationary phase," Phys. Rev. E **69**, 046610 (2004).
[4]P. Roux, K. G. Sabra, P. Gerstoft, W. A. Kuperman, and M. C. Fehler, "P-waves from cross-correlation of seismic noise," J. Geophys. Res. **32**, 19303 (2005).
[5]P. Roux, W. A. Kuperman, and the NPAL Group, "Extracting coherent wavefronts from acoustic ambient noise in the ocean," J. Acoust. Soc. Am., **116**, 1995–2003, (2004).
[6]N. M. Shapiro, M. Campillo, L. Stehly, and M. H. Ritzwoller, "High-resolution surface-wave tomography from ambient seismic noise," Science **307**, 1615–1618 (2005).
[7]M. Wathelet, D. Jongmans, and M. Ohrnberger, "Surface wave inversion using a direct search algorithm and its application to ambient vibration measurements," Near Surface Geophysics **2**, 211–221 (2004).
[8]R. B. Herrmann, "Computer programs in seismology," Department of Earth and Atmospheric Sciences, Saint Louis University, http://www.eas.slu.edu/People/RBHerrmann/ComputerPrograms.html.