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Reconstructing the dynamics within the Earth's core

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Contents

Forewords

I present in this dissertation a summary of my research (excluding the work I have carried out during my thesis and my first post-doc in Leeds), together with perspectives associated with it. I currently work in the 'geodynamo' team of ISTerre, where has been developed a culture of the dynamical processes occurring in planetary cores.

Today we benefit from about 15 years of continuous satellite records, and the Swarm mission launched by ESA in November 2013 is planned to last for a decade or so. New observations lead to new questions. In this context, I try to construct a discussion between geophysical (magnetic and geodetic) observations and dynamical constraints. This is achieved through inversion methods; today our community incorporates data assimilation techniques first developed in the context of surface envelopes (ocean, atmosphere).

Over the past 20 years we have witness the success of geodynamo calculations, able to produce some Earth-like behaviors (e.g., reversals of dipole dominated rotating spherical dynamos). They may nevertheless not be suited yet to digest satellite, or even observatory, geomagnetic records. My efforts aim at making the discussion between data and models meaningful, in order to best extract the information contained into observations. This requires defining the area (e.g., in term of time-scales and length-scales) where models are pertinent, and proposing accurate measures of the uncertainties (associated with the accuracy of the data and the simplifications of the models).

Chapter 1

Summary of my research

1.1 A geophysical context

1.1.1 Constraints from geomagnetic observations

We currently live an exciting period for studying the geomagnetic field and the dynamics in the Earth's core. Today, after an era (1999–2013) of single satellite observation (with the Oersted and Champ missions), the Swarm constellation of three satellites offers measures of the field simultaneously at several local times (Friis-Christensen et al, 2006; Olsen et al, 2013). This is a crucial point for better describing external (magnetospheric and ionospheric) sources, which should help separate them from internal (from the core and induced in the mantle) magnetic fields (e.g., Olsen et al, 2010).

I will consider global spherical harmonic models of the internal magnetic field **B**, supposing that it derives from a potential

$$V(r,\theta,\phi,t) = \sum_{n,m} \left(\frac{a}{r}\right)^{n+1} \left[g_n^m(t)\cos m\phi + h_n^m(t)\sin m\phi\right] P_n^m(\cos\theta)\,, \qquad (1.1)$$

with n and m the spherical harmonic degree and order, P_n^m the Schmidt semi-normalized associated Legendre polynomials, and (g_n^m, h_n^m) the Gauss coefficients. State of the art models derived from low-orbiting satellite data converge towards a common description of the secular variation (or SV, the time-derivative of the magnetic field) up to $n \simeq 12$ (see the comparison of models by Finlay et al, 2012). This quantity is of particular interest because it is directly related to the motions in the Earth's core through the induction equation (see below). However, because of the ambiguity between the several sources (and the impossibility, with polar orbiting satellites, to sample the magnetic field simultaneously at all local times), it is not an instantaneous measure of the SV that is provided, but instead a weighted temporal average. I hope that the better description of external fields offered by Swarm will open a window on core magnetic field changes at periods shorter than a few years, the current temporal resolution allowed by single spatial missions and observatories.

Ground based stations (e.g., Matzka et al, 2010) provide complementary information to satellites: continuous vector series covering decadal time-scales. The present records (at about 120 sites) are believed to constrain by themselves the SV up to spherical harmonic degree $n \simeq 8$ (see Holme et al, 2011). The oldest records date back to 1840; they now show a monotonous decrease of the axial dipole field at an average rate of 15 nT/yr for more than 170 years. The persistence of this feature

Table 1.1: Physical parameters estimated for the Earth's core.

	ν	η	Ω	c	ρ	μ
units	$\mathrm{m}^2.\mathrm{s}^{-1}$	$\mathrm{m}^2.\mathrm{s}^{-1}$	$\mathrm{rad.s}^{-1}$	m	$\mathrm{kg.m}^{-3}$	$\mathrm{H.m}^{-1}$
value	$\sim 10^{-5}$	$\sim 10^{0}$	$7.27\,10^{-5}$	$3.485\,10^{6}$	$\sim 10^{4}$	$4\pi\,10^{-7}$

in more ancient epochs is under debate (Gubbins et al, 2006; Finlay, 2008): then no direct absolute intensity measurements are available (historical records only contain directional data prior to 1840, see Jonkers et al, 2003), so that one must rely on less accurate paleo- or archaeo-magnetic records.

Archaeological artifacts and lake sediments are considered to constrain the geomagnetic field over the past millennia (e.g., Constable, 2007). These have been used to produce global models of the geomagnetic field. However, given the uncertainties associated with such records (in term of measurement values and of dating), the harmonic degree up to which those models are constrained by observations is unclear (do we have access to something else than the dipole over millennial time-scales?).

These data are nevertheless important. Indeed, if one wishes to model the core dynamics and the geomagnetic field evolution from recent and accurate records, the reconstruction of the background state (equivalent of the climatic mean in oceanography) is crucial. This is a motivation for the development of data assimilation algorithms from archaeomagnetic data bases (Fournier et al, 2010, 2013)

1.1.2 Dynamics in the presence of magnetic field and rotation

The outer core state (fluid velocity $\mathbf{u}$ and magnetic field $\mathbf{B}$) is governed by the momentum and induction equations,

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\,\mathbf{u} + 2\Omega \mathbf{1}_z \times \mathbf{u} = -\nabla\Pi + \frac{1}{\rho\mu}\nabla\times\mathbf{B}\times\mathbf{B} + \nu\nabla^2\mathbf{u}\,, \quad (1.2)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B}) + \eta\nabla^2\mathbf{B}\,, \quad (1.3)$$

with $\mathbf{1}_z$ the unit vector along the rotation axis, Ω the Earth's rotation rate, Π the reduced pressure, ρ the density, μ the free space permeability, ν the viscosity and η the magnetic diffusivity (I assume homogeneous material properties). It is on purpose that I omit here buoyancy forces. Not that they are negligible, but it is a bet that they act in this instance on so short length-scales that they are not directly probed by observations. This choice contrasts with several studies that will also be discussed in this dissertation.

I wish here to compare several characteristic time-scales of the outer core dynamics. For this I need a typical length-scale ℓ, the intensity B of the magnetic field, and that U of the fluid motions. From these, and fixed physical parameters listed in Table 1.1, one can define

- $\tau_\nu(\ell) = \ell^2/\nu$ the viscous time,

- $\tau_\eta(\ell) = \ell^2/\eta$ the magnetic diffusion time,
- $\tau_u(\ell) = \ell/U$ the vortex turn-over time,
- $\tau_a = c/V_a$ the magnetic Alfvén time (the time it takes for perturbation to cross, as a non-dispersive Alfvén wave of speed $V_a = B/\sqrt{\rho\mu}$, the outer core of radius c), and
- $\tau_\Omega(\ell) = c/(\Omega\ell)$ the inertial time (the time it takes for a perturbation of length-scale ℓ to form a Taylor column, of height $H \sim c$, through the propagation of inertial waves, see Cardin and Olson, 2007).

I have ignored factors of 2π here and there, so that the analysis below should be considered with some freedom. The ratio of these time-scales define several dimensionless numbers, among which

- $P_m = \dfrac{\tau_\eta}{\tau_\nu} = \dfrac{\nu}{\eta}$ the magnetic Prandtl number,
- $E(\ell) = \dfrac{\tau_\Omega}{\tau_\nu} = \dfrac{\nu c}{\Omega\ell^3}$ the Ekman number,
- $R_m(\ell) = \dfrac{\tau_\eta}{\tau_u} = \dfrac{U\ell}{\eta}$ the magnetic Reynolds number,
- $Ro(\ell) = \dfrac{\tau_\Omega}{\tau_u} = \dfrac{Uc}{\Omega\ell^2}$ the Rossby number,
- $A(\ell) = \dfrac{\tau_a}{\tau_u} = \dfrac{\sqrt{\rho\mu}Uc}{B\ell}$ the Alfvén number,
- $S(\ell) = \dfrac{\tau_\eta}{\tau_a} = \dfrac{B\ell^2}{\sqrt{\rho\mu}\eta c}$ the Lundquist number,
- $\lambda(\ell) = \dfrac{\tau_\Omega}{\tau_a} = \dfrac{B}{\sqrt{\rho\mu}\Omega\ell}$ the Lehnert number,

The several time-scales (as well as the quantity U) depend on the considered length-scale, which makes the complete comparison of the many processes occurring in the core complex (Nataf and Schaeffer, 2015). Nevertheless, let's first compare those time-scales in the case of a perturbation of size $\ell \sim c$. Considering $U(c) \sim 10$ km/y, the typical amplitude for the observed westward drift of magnetic patches at the core-mantle boundary (CMB) (e.g. Finlay and Jackson, 2003), and $B \sim 3$ mT (Gillet et al, 2010a), about 10 times the rms field at the CMB (as observed in geodynamo simulations, e.g. Aubert et al, 2009), we obtain in Tables 1.2 and 1.3 typical values for the time-scales and dimensionless numbers for the Earth's core. Time-scales rank as follow:

$$\tau_\nu \gg \tau_\eta \gg \tau_u \gg \tau_a \gg \tau_\Omega\,. \tag{1.4}$$

Table 1.2: Order of magnitude for several time-scales (in years), estimated for the Earth's core.

	$\tau_\nu(c)$	$\tau_\eta(c)$	$\tau_u(c)$	$\tau_a(c)$	$\tau_\Omega(c)$
Earth	$3\,10^{10}$	$3\,10^{5}$	$3\,10^{2}$	3	$5\,10^{-4}$

Table 1.3: Order of magnitude for several dimensionless numbers, estimated for the Earth's core and for numerical simulations of the geodynamo (estimations of the output numbers from Christensen and Aubert, 2006).

	P_m	$E(c)$	$R_m(c)$	$Ro(c)$	$A(c)$	$S(c)$	$\lambda(c)$
Earth	10^{-5}	10^{-14}	10^{3}	10^{-6}	10^{-2}	10^{5}	10^{-4}
simulations	$> 10^{-1}$	$> 10^{-7}$	10^{3}	$< 10^{-1}$	> 1	$< 10^{3}$	10^{-4}

We are interested here in modeling the outer core dynamics from geomagnetic records covering interannual to millennial time-scales, i.e. from a fraction of the Alfvén time to several turn-over times. Geodynamo simulations currently provide Earth-like values for R_m and λ; however, they struggle to generate magnetic fields large enough so that Alfvén waves travel much faster than fluid motions (see Table 1.3). Indeed, if τ_u and τ_a are separated by two orders of magnitudes in the Earth's core, these two time-scales tend to collapse in numerical calculations (Soderlund et al, 2012) – unless artificially low values of η, i.e. large P_m, are considered to crank-up the magnetic energy. This translates into a magnetic energy 10^4 larger than the kinetic energy in the Earth's core at large length-scales, whereas both have about the same magnitude in 3D simulations. This observation asks the question whether one can interpret geomagnetic records with currents 3D models, since it is exactly at these periods between a few years and a few centuries that we have the richest observations.

I present in Section 1.3.1 how the comparison of dimensionless numbers motivates the use of the quasi-geostrophic (QG) approximation. According to the QG hypothesis, fluid motions are assumed to be invariant along the rotation axis, due to the predominant role of rotation forces in the momentum equation (1.2).

1.1.3 Reconstructing the core dynamics

Imaging the state of the Earth's core (magnetic and/or velocity fields, at and/or below the CMB) is written as a forward problem of the form

$$\mathbf{y} = \mathsf{H}(\mathbf{x}) + \mathbf{e}\,, \tag{1.5}$$

with $\mathbf{y}$ an observation vector, $\mathbf{x}$ a vector containing the model parameters to be estimated, H an observation operator, and $\mathbf{e}$ an observation error vector. Obtaining the core state from geophysical records comes down to an optimization problem, where one minimizes a cost function of the form

$$J(\mathbf{x}) = \|\mathbf{y} - \mathsf{H}(\mathbf{x})\|_{\mathsf{C_e}} + \|\mathbf{x} - \mathbf{x}_0\|_{\mathsf{C_x}} \ . \tag{1.6}$$

The first term on the r.h.s. of equation (1.6) is a measure of the misfit between the model predictions and the data, while the second is a measure of the model complexity. I use the notation $\|\mathbf{v}\|_{\mathsf{C}} = \mathbf{v}^T \mathsf{C}^{-1}\mathbf{v}$; $\mathbf{x}_0$ is the background model, $\mathsf{C_e}$ the covariance matrix of observation errors, and $\mathsf{C_x}$ the a priori covariance matrix on the model parameters.

In our situation, we are interested for instance in producing a time-dependent model of the radial magnetic field $B_r(\theta, \phi, t)$ at the CMB. This model will be used in a second step as a surface 'observation' when recovering the dynamics in the Earth's core. It is linked to the core surface flow $\mathbf{u}$ through the radial induction equation at the CMB, which in the frozen-flux approximation ($\eta = 0$) is

$$\frac{\partial B_r}{\partial t} = -\nabla_h \cdot (\mathbf{u} B_r) \ . \tag{1.7}$$

In the kinematic framework (e.g., Holme, 2007), this equation is often translated into a linear problem, where the coefficients defining the secular variation $\partial_t B_r$ enter the data vector $\mathbf{y}$, and the flow coefficients enter the model vector $\mathbf{x}$.

However, only part of the magnetic field is resolved. I note $\langle B_r \rangle$ and B_r' respectively its resolved and unresolved components. Equation (1.7) then becomes

$$\frac{\partial \langle B_r \rangle}{\partial t} = -\nabla_h \cdot (\mathbf{u} \langle B_r \rangle) + \varepsilon^o + \varepsilon^m \, , \tag{1.8}$$

with the SV observation error $\varepsilon^o = \partial_t B_r'$ and the SV model error $\varepsilon^m = -\nabla_h \cdot (\mathbf{u} B_r')$. As in any optimization problem, an unbiased estimate of the core dynamics can be obtained only if $\mathsf{C_x}$ and $\mathsf{C_e}$ properly describe the a priori statistics on $\mathbf{u}$, ε^o and ε^m. This is particularly true if the problem is severely ill-posed, as is the core-flow inverse problem: covariances carry information, and in our situation any source of information is welcome. My attempts at providing accurate estimates for ε^o are presented in Section 1.2.2–1.2.4. My efforts to properly image time changes of core flows, which require (as much as possible) unbiased estimates of ε^m, are presented in Section 1.3.2.

1.2 Stochastic geomagnetic field modeling

1.2.1 Temporal power spectra from geomagnetic series

I discuss here the temporal power spectral density (PSD) of magnetic records. This quantity is of importance because it provides information on the auto-covariance function $\mathcal{C}(\tau)$ of a magnetic series $\varphi(t)$ (τ is the time lag, t the time). Indeed, the two are related through

$$PSD(\varphi) = |\mathcal{F}(\varphi(t))|^2 = |\mathcal{F}(\mathcal{C}(\tau))| \, , \tag{1.9}$$

where $\mathcal{F}$ is the Fourier transform. The shape of the PSD thus gives us some insight about the correlation function, which is a way to constrain the a priori temporal information relative to the geomagnetic field. This latter will enter a priori covariance matrices (see section §1.2.2). I summarize below what comes out of geomagnetic records:

- Paleomagnetic series present a flat PSD at periods longer than 100,000 years (Constable and Johnson, 2005). On shorter time-scales, and down to periods of about a few centuries, the slope of the PSD changes to about -2 (information confirmed by the analysis of individual lake sediments series, see Panovska et al, 2013);
- Observatory series indicate at periods from a few years to a few decades a slope of the PSD close to -4 (Currie, 1968; De Santis et al, 2003), a value retrieved at centennial periods from the longest historical series in London (Pers. comm., C. Finlay, Feb. 2013) ;
- On time-scales shorter than a few years, external variations dominate the signal: its PSD presents a background slope about -1 (Ou et al, 2015), upon which several peaks appear at 1 year, 6 months, 27 days, 1 day, etc. periods, with their harmonics and modulations (e.g., Constable and Constable, 2004; Love and Rigler, 2014).

This last point makes it difficult the inference of core processes at periods shorter than a few years. Note that the ambiguity between internal and external sources is already present on longer time-scales. The best example of this ambiguity is given by the leakage of the modulated 11-years solar cycle into internal coefficients g_1^0 or g_3^0 (e.g., McLeod, 1996).

1.2.2 Stochastic modeling of the magnetic field

It is interesting to discuss spectra and covariances in the framework of stochastic differential equations (SDE). We consider here a process $\varphi(t)$ of order p, governed by an equation of the form (Yaglom, 1962)

$$\alpha_{p-1} d\frac{d^{p-1}}{dt^{p-1}}\varphi + \cdots + \alpha_1 d\varphi + \alpha_0 \varphi dt = d\zeta(t)\,, \tag{1.10}$$

with $\zeta(t)$ a Brownian motion (Wiener) process. In the frequency domain, $d\zeta$ is characterized a white (flat) PSD. It comes from equation (1.10) that a process of order p has a PSD that evolves as f^{-2p} as the frequency f becomes asymptotically large. In the case of the modeling of the geomagnetic field over the observatory era (interannual to decadal periods), it is thus tempting to consider a SDE of order 2, to be consistent with the -4 slope of the PSD found for ground-based series.

We used this framework to derive the a priori information on Gauss coefficients – see equation (1.1). We are aware that observatory series mix the information from all harmonic degrees (so the statistics of coefficients series may well be more complicated). We chose, for the construction of the COV-OBS field model[1] (Gillet et al, 2013), the particular case of the Matérn Auto-Regressive (AR) process of order 2,

$$d\frac{d\varphi}{dt} + \frac{2\sqrt{3}}{\tau_0}d\varphi + \frac{3}{\tau_0^2}\varphi dt = d\zeta(t)\,, \tag{1.11}$$

because it only relies on a single parameter τ_0 that can be analytically derived from the variances $\sigma^2_{g_{nm}}$ and $\sigma^2_{\dot{g}_{nm}}$ of the main field (MF) and SV Gauss coefficients series. For the sake of simplicity, τ_0 is assumed to vary only with the spherical harmonic degree n. Its expression is $\tau_0(n) = \sqrt{3}\tau_{MF}(n)$, where $\tau_{MF}(n)^2 = \sum_m \sigma^2_{g_{nm}} / \sum_m \sigma^2_{\dot{g}_{nm}}$ (Hulot and Le Mouël, 1994). A process defined by equation (1.11) is associated with the covariance function

$$\mathcal{C}(\tau) = \sigma^2 \left(1 + \sqrt{3}\frac{|\tau|}{\tau_0}\right) \exp\left(-\sqrt{3}\frac{|\tau|}{\tau_0}\right)\,. \tag{1.12}$$

This latter replaces the implicit and un-controlled a priori time covariances imposed when penalizing the second or third time-derivative in regularized field models. By using this kind of standard regularization procedures, geomagnetic models actually map a weighted time integral (as opposed to instantaneous) picture of the field changes (see Gillet et al, 2010b; Finlay et al, 2012).

The use of realistic a priori information on the field model allows to use the a posteriori covariance matrix to estimate the errors on the model coefficients. This was the goal, when we decided to produce the COV-OBS field model, to provide an ensemble of plausible models, whose dispersion measures our ignorance. Figure 1.1 illustrates the ensemble of realizations for several SV coefficients over 1840–2010. We see the improvement of the data quality brought by the introduction of proton magnetometers in 1960. We also notice for non-dipole series how the use of realistic a priori information allows sharper changes in comparison with regularized field models such as *gufm1* (Jackson et al, 2000) – the case of internal dipole coefficients is specific, because the external field is accounted for in COV-OBS but not in *gufm1*.

We recently extended the COV-OBS model to 2014.5 (Gillet et al, 2015a) when producing the ISTerre candidate models to the 12th generation of the International Geomagnetic Reference Field (IGRF-12, Thébault et al., 2015). Figure 1.2 illustrates how the use of our stochastic prior allows sharper variations in comparison with regularized models (in particular for small length-scale patterns that are more affected by the damping). We also notice how the presence of satellite data (after 1999) reduces the a posteriori model uncertainties. Such information (the posterior model error) is crucial if one wishes to obtain an unbiased estimate of the core dynamics (e.g., Fournier et al, 2010). Its estimation cannot be obtained from the posterior covariance matrix when standard regularizations are employed. Indeed, by damping in space (time), one a priori assumes that small length-scales (short time-scales) variations of the field are very small, reducing non-uniqueness issues.

[1] http://www.spacecenter.dk/files/magnetic-models/COV-OBS/

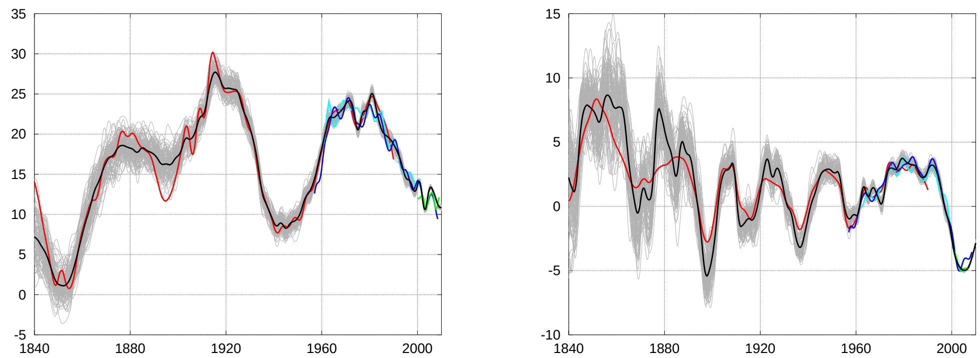

Figure 1.1: Ensemble of realizations of the SV coefficients (in nT/yr) g_1^0 (left) and g_2^1 (right) from the COV-OBS field model (black), compared with several other models (*gufm1* in red). From Gillet et al (2013).

As a consequence the posterior errors on those variations is (unrealistically) very small. This is the reason why Aubert (2014), who defines the flow priors with a free run of a geodynamo simulation, must invent ad hoc measures of the SV errors when imaging core motions from regularized models (i.e. formal errors obtained for regularized models are incompatible with the information contained into geodynamo simulations, contrary to formal errors provided with COV-OBS).

1.2.3 On the secular acceleration (SA)

Interestingly, a process defined by equation (1.11) is only once differentiable ($\mathcal{C}^1$). This means that the first derivative (the SV) is continuous, and the second (the SA) is not defined within the meaning of common functions (its value depends on the rate at which the process is sampled). We would commonly say that the SA presents 'jumps'. This picture, which is only valid in the frequency range where the -4 slope PSD is observed, is coherent with the occurrence of geomagnetic jerks (Courtillot and Le Mouël, 1984). It is thus limited to sampling rates longer than a couple of years (it is the sampled field that is $\mathcal{C}^1$, the magnetic field itself being $\mathcal{C}^\infty$).

Conversely, current geomagnetic field models derived from satellite data enhance rapid changes in the SA, as initiated by Lesur et al (2008). Do satellite records allow a description of field changes on periods shorter than a few years, where the field would show a steeper spectrum? This question is today under debate. I argue that current satellite field models do not probe precisely periods shorter than a year (there are indeed difficulties in modeling accurately the large annual and semi-annual field changes, see Ou et al, 2015). Instead they provide, given the kernels implicitly stated by the regularization process, a weighted integral of the SA; this latter is defined, as is the integral of the white noise in equation (1.11).

I just warn here that interpreting instantaneous pictures of the SA might be misleading, because we are then looking at maps of the field that are blurred in

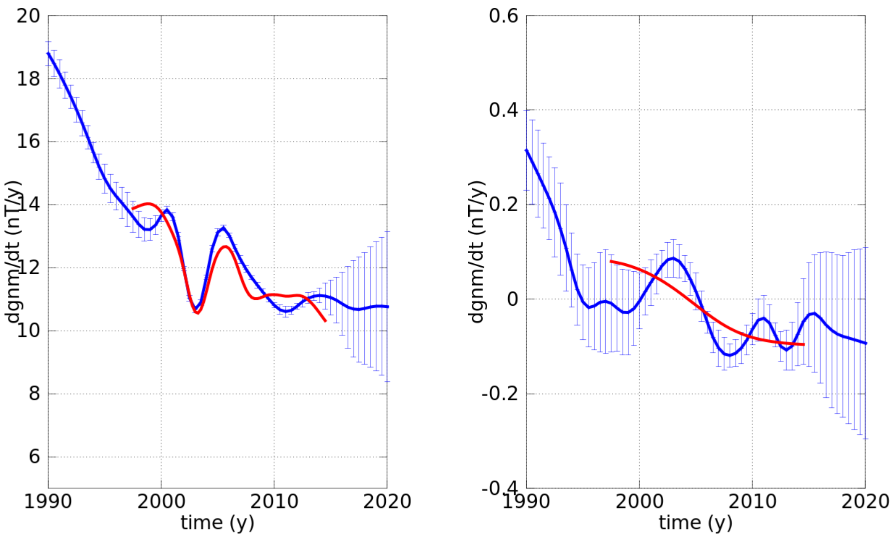

Figure 1.2: SV coefficients g_1^0 (left) and g_8^1 (right) from the COV-OBS.x1 field model (blue, with its one standard deviation errorbars), compared with the CHAOS4plus_v4 model (red) by Olsen et al (2014). From Gillet et al (2015a).

space (the SA would not be resolved by the data above harmonic degree 6 or so, e.g. Lesur et al, 2010), filtered in time (through the regularization), plus looking at the second time-derivative implies applying another filter ! We found that the modeling, using regularization techniques, of synthetic magnetic data derived from the AR-2 process (1.11) provide models with SA spectra very similar to those derived from geophysical observations (Gillet et al, 2013). Furthermore, the prediction of such synthetic AR-2 models at observatories display Earth-like geomagnetic jerks (Brown et al, 2013).

The SA has been further interpreted to define a time-scale of the SV, defined as $\tau_{SV}(n)^2 = \sum_m \sigma^2_{\dot{g}_{nm}} / \sum_m \sigma^2_{\ddot{g}_{nm}}$. Holme et al (2011) observe that τ_{SV} in geomagnetic field models is almost independent of the harmonic degree, with values of about 10 years. This criterion ($\forall n, \tau_{SV}(n) \simeq 10$ years) has been used to assess the geophysical relevance of geodynamo simulations (Christensen et al, 2012). If, as I defend here, field models derived from observations propose a weighted time-integral of the SA, then the comparisons to forward dynamical calculations should be carried with caution. For instance the output of the simulations should be filtered in a way similar to that implied by the damping. One should also wonder how to transpose the numerical time scale into physical units (see Lhuillier et al, 2011). It does not seem obvious to me that the same scaling should be applied to all frequencies (some frequency range may be shrunk or stretched if the parameter regime probed by the simulation is not adequate).

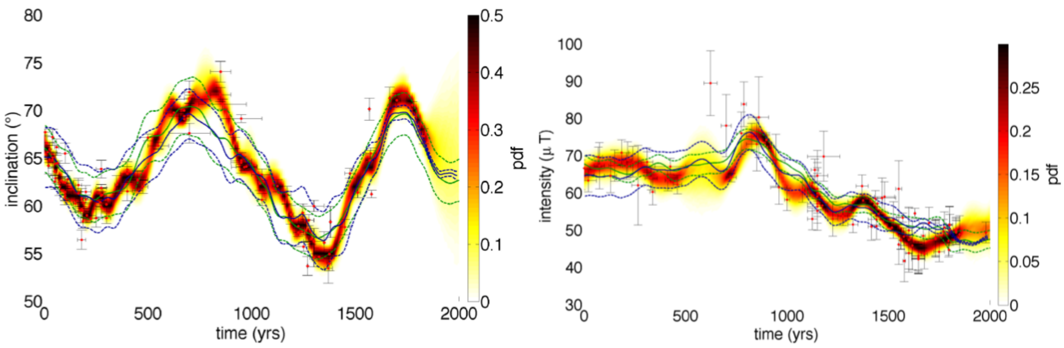

Figure 1.3: Probability density functions of inclination (left) and intensity (right) records from France, compared with the ARCH3k (in blue, Korte et al, 2011) and the A-FM (in green, Licht et al, 2013) models and their associated uncertainties. From Hellio et al (2014).

1.2.4 Stochastic modeling of archaeomagnetic data

During her PhD (defended in January 2015), which I co-supervised with Dominique Jault and Claire Bouligand, Gabrielle Hellio applied the stochastic approach described above to the modeling of archaeomagnetic data. Analyzing more sparse data, of lower quality, spanning a longer time-span, she had to consider alternative SDE, the damped oscillator family,

$$d\frac{d\varphi}{dt} + 2\alpha d\varphi + \omega^2 \varphi dt = d\zeta(t)\,. \tag{1.13}$$

Equation (1.11) corresponds to the case $2\alpha = \omega^2$. With $2\alpha > \omega^2$, one mimics the change from -2 to -4 slopes at longer periods observed for the PSD of paleomagnetic series (Constable and Johnson, 2005), and that of dipole moment series in geodynamo simulations (Olson et al, 2012) – see also Buffett and Matsui (2015). She also developed a method to get rid of the ad hoc choice of splines as temporal support functions: the interpolation in time is then entirely carried by the a priori correlation functions. This has been incorporated in global modeling algorithms, and validated on synthetic data. It presents encouraging results, suggesting (*i*) a non-steady dipole decay over 1500–1840 (as used for instance when constructing the model *gufm1*), (*ii*) the existence of the South Atlantic Anomaly already in 1500, and (*iii*) some constraint from archaeomagnetic data up to harmonic degree $n \simeq 4$. She also notices that her models, together with their uncertainties, are coherent with estimates built from independent and more accurate observatory data.

She also proposed a way to properly account for the dating errors that are specific to these indirect measurements. This has been handled by applying a Monte-Carlo Markov Chains (MCMC) method to efficiently sample the ensemble of possible dates in regional studies (Hellio et al, 2014). Any a priori dating error distribution can be considered. With the approach Gabrielle put forward, accurate measurements do not need to be under-weighted (as is the case with several former modeling strategies). The a posteriori probability density for the model is not necessarily Gaussian, and it presents sharper time variations, in comparison with master curves obtained selecting marginally probable samples of the dates with brute-force bootstrap or jack-knifes methods, and/or employing regularization techniques (see Figure 1.3). Her developments also appear promising for archaeologists who wish to better constrain the dates of the samples.

The application of the MCMC selection of possible dates to global modeling is ongoing, with the difficulty of sampling efficiently a space of dimension 10,000 (versus 100 for regional models).

1.3 Reconstruction of the Earth's core dynamics

1.3.1 Quasi-Geostrophic transient motions

I present here some aspects of my work on the QG modeling of core flows. The use of the QG assumption is first motivated by theoretical considerations. From the time-scales presented in section §1.1.2, viscosity will be neglected ($E \ll 1$). We also discard Reynolds stresses in our analysis of the momentum equation (1.2), due to the small values of the Rossby number. The main source of nonlinearities in this equation will then be the Lorentz force. This point of view is motivated as far as we consider the dynamics at large length-scales.

We will also neglect magnetic diffusion, considering the Earth is in the regime $S \gg 1$. Here again we consider large scale structures. It also means we consider transient motions: as illustrated by Jault (2008) with a two-dimensional problem (in the meridional plane) where $S \gg 1$ (diffusion time much larger than the Alfvén time), it is after a period several times τ_a that the fluid flow geometry reaches the Ferraro's law (isorotation along the magnetic field lines). We now consider perturbations to a background core state. Momentum and induction equations, once linearized around a background field $\mathbf{B_0}$, then simplify to

$$\begin{aligned} \frac{\partial \mathbf{u}}{\partial t} + 2\Omega \mathbf{1}_z \times \mathbf{u} = & \quad -\nabla \Pi + \frac{1}{\rho\mu} \left(\nabla \times \mathbf{b} \times \mathbf{B}_0 + \nabla \times \mathbf{B}_0 \times \mathbf{b} \right) , \qquad (1.14) \\ \frac{\partial \mathbf{b}}{\partial t} = & \quad \nabla \times \left(\mathbf{u} \times \mathbf{B}_0 \right) , \qquad (1.15) \end{aligned}$$

(we ignore the steady components involving $\mathbf{u}_0$ generated by $\nabla \times \mathbf{B}_0 \times \mathbf{B}_0$ and the diffusion of $\mathbf{B}_0$). This system of equations gives rise to inertial waves (when neglecting magnetic forces), Alfvén waves (neglecting rotation forces), and the two branches of the fast and slow Magneto-Coriolis (MC) waves (Malkus, 1967).

Inertial waves, who involve the first three terms in equation (1.14), transport the information concerning the global rotation. These transient 3D motions lead to the columnar flow structure (Taylor columns, or z-invariance). After a few times $\tau_\Omega(\ell)$, a 3D perturbation of size ℓ will generate a column of height $H \sim c$. At large length-scales ($\ell = O(c)$), and for Earth-like parameters, they are much faster than Alfvén waves carried by the magnetic field. Jault (2008) emphasizes that it is the Lehnert number λ, ratio of the propagation time of these two waves, that measures

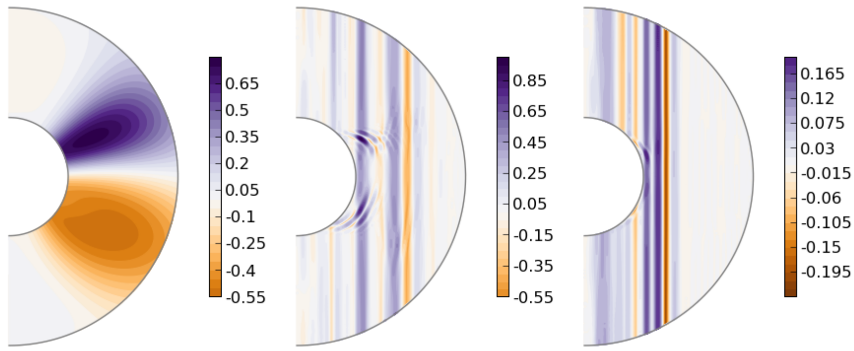

Figure 1.4: Free-decay computations in presence of an imposed magnetic field. Meridional cross-sections of the cylindrical radial component of the imposed field (B_s,left) and of the velocity (u_s) in cases with $\Lambda = 15, \lambda = 9\,10^{-4}$ (middle) and $\Lambda = 2, \lambda = 3\,10^{-4}$ (right). From Gillet et al (2011).

the relative importance of magnetic and Coriolis terms for transient, diffusiveless motions, whereas the often considered Elsasser number $\Lambda = V_a^2/(2\eta\Omega)$ is relevant for slower processes involving diffusion (indeed Λ, ratio between magnetic to rotation forces, is obtained by estimating electrical currents from the diffusive Ohm's law, or $J \sim \sigma UB$). In the Earth's core, given $\lambda(c) \ll 1$, large length-scales transient motions should then tend to be z-invariant.

We illustrated this behavior with 3D computations in presence of an imposed field, extending the above 2D analysis. In figure 1.4 I show the result of free-decay computations in presence of an imposed magnetic field (we initially give a kink to the inner core), once inertial waves have travelled throughout the spherical volume (Gillet et al, 2011). We see that non-axisymmetric transient motions are largely z-invariant, whatever the value of the Elsasser number. In this configuration, the quasi-geostrophic structure of time-dependent motions is due to the small values of the Lehnert number ($\lambda(c) \ll O(1)$ in all cases). We also see some 3D features at small length-scales, in areas where the field is the strongest. This reflects that λ increases towards shorter ℓ (because inertial waves, contrary to Alfvén waves, are dispersive). We have shown that z-invariance is maintained for $\lambda(\ell) < 10^{-2}$. Applied to the Earth's core, it gives the condition $\ell > 50$ km.

This interpretation differs from that of Soderlund et al (2012), who attribute the z-invariant structure observed in geodynamo simulations to the small values found for the dynamic Elsasser number $\Lambda_d = V_a^2/(2\Omega U\ell)$ (Cardin et al, 2002), ratio of magnetic to Coriolis forces when estimating electrical currents from the diffusive-less Ampère's law ($J \sim B/\mu\ell$) . To my opinion, the question of the columnar structure of the flow at periods much longer than τ_a (but much shorter than τ_η) is still open. The uncertainty can be illustrated by rewriting $\Lambda_d = (\lambda/A)(c/\ell)$. Considering that geodynamo simulations reach Earth-like values of λ but unrealistic values of A (see Table 1.3), we have $\Lambda_d(c) \sim 10^{-2}$ in the Earth, against 10^{-4} in simulations... we retrieve here the two decades of time-scales (from τ_a to $100\,\tau_a$), where the influence of the magnetic field should be predominant in the Earth's core, while inertia takes over the force balance in simulations. Furthermore, it is not obvious that a measure of force balance obtained from the ratio of orders of magnitudes is relevant. Indeed, magnetic and velocity fields may organize in order to minimize Lorentz forces. Then, large field values may not necessarily imply large forces; I refer here to the spectral analysis of the forces in geodynamo computations (Schaeffer et al, 2013), or to the interpretation of zonal motions in geodynamo simulations (Aubert, 2005). It is also the case for the super-rotation in magnetized Couette flow experiments (Brito et al, 2011), where the steady flow is explicitly governed by vanishing Lorentz forces. For this reason, I feel safer to consider the ratio of time-scales instead of forces to define dimensionless numbers in section 1.1.2.

1.3.2 Ensemble modeling of time-dependent core flows

The considerations developed in Section §1.3.1 motivate the use of the QG constraint to model core flow changes from geophysical data. In the kinematic approach I followed, it implies imposing the topological constraint (Amit and Pais, 2013)

$$\nabla_h \cdot \left(\mathbf{u} \cos^2\theta\right) = 0\,, \tag{1.16}$$

plus the equatorial symmetry on core surface flows, where θ is the colatitude (with or without special considerations about the cylinder tangent to the inner core). The radial induction equation (1.8) is then translated into an optimization problem to recover the core flow subject to (1.16), as first performed by Pais and Jault (2008). They describe core motions organizing as an eccentric anticyclonic gyre. Surprisingly similar maps have been obtained since then while deriving the a priori information on the flow model from the statistics of a geodynamo simulation free run (Fournier et al, 2011; Aubert et al, 2013; Aubert, 2014). We performed time-dependent inversions without imposing the equatorial symmetry (Gillet et al, 2011), showing that the most accurate (i.e. recent) data tend to favor equatorially symmetric solutions. We also find that less flow parameters are needed to reach a given level of fit to SV data when we a priori impose the symmetry; those evidences encouraged us to use the QG hypothesis.

However, all these studies recover time variable core flows by solving for a series of independent snapshot problems (or linked together only through the projection onto splines, see Jackson, 1997). This is also the case of the recent investigations of the core flow time variability by Pais et al (2014). Ignoring time correlations (either of the SV model errors, or of the flow prior), one will potentially miss information, and then bias the solution obtained for $\mathbf{u}(t)$.

We derived an algorithm to invert for the core flow simultaneously at set of epochs (Gillet et al, 2015b), including a prior temporal correlation for the flow and accounting for the time correlation of SV model errors $\mathcal{E}^m$ in equation (1.8). The

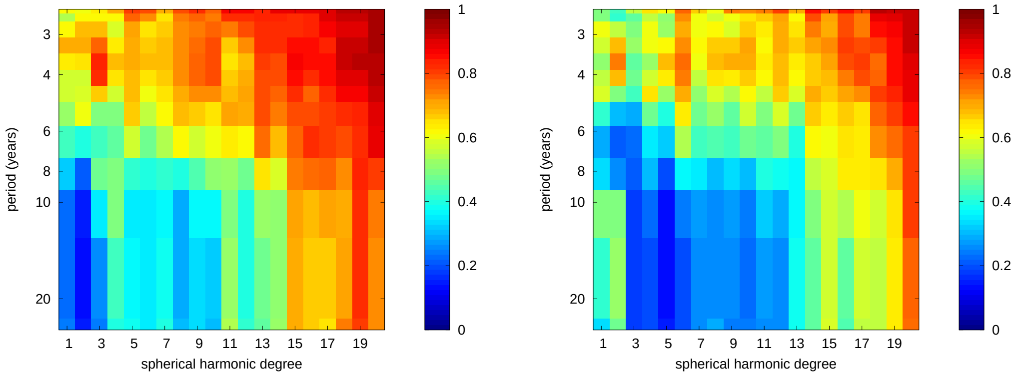

Figure 1.5: Resolution of core motions (blue stand for 100% resolved, red for 100% unresolved) within an ensemble of flow realization, as a function of spherical harmonic degree n and of the period, for the two periods 1940–1975 (left) and 1975–2010 (right). From Gillet et al (2015b).

flow prior is derived from the simplest stochastic process compatible with the -4 slope observed for the PSD of geomagnetic series, i.e. an AR-1 process of the form

$$d\mathbf{u} + \frac{1}{\tau_u}\mathbf{u}dt = d\zeta(t)\,. \tag{1.17}$$

A single τ_u is considered for all components of the flow, not only for the sake of simplicity, but also following considerations in link with Taylor (1963)'s condition (see section §1.3.3), which imply on periods longer than $\tau_a(c)$ a linear relationship between zonal and non-zonal motions.

SV model errors ε^m are correlated in time, because both the flow and the field evolve on decadal or longer time-scales (Gillet et al, 2009). We estimate the statistics of time correlated of SV model errors iteratively using an ensemble method (Evensen, 2009): from an ensemble of realizations of the MF and SV at the core surface (see section 1.2.2), we derive an ensemble of flow models. The dispersions within the ensembles define the unresolved part of the field and the flow. Then we estimate an ensemble of SV model errors from all contributions to the electro-motive force in equation (1.7) involving unresolved components, from which we build the SV model error covariance matrix. Since ε^m depends on the flow, the kinematic core flow inverse problem then becomes nonlinear, requiring an iterative algorithm (Pais and Jault, 2008). The important point to consider here is the correlation in time of SV model errors. It implies technical issues, by requiring to fill and invert for (an ensemble of) dense matrices of size $(N_pN_t)^2$, where N_p is the number of parameters per epoch, and N_t the number of epochs (we end-up with dozens of matrices, each a few Gb big, to fill and invert recursively).

But this price is worth paying. Indeed we show that only by considering time covariances of SV model errors we can map from geomagnetic observations flow changes that fit both the magnetic and the independent geodetic (length-of-day, or LOD) data at both interannual and decadal periods. When ignoring time covariances, decadal LOD predictions from core motions are over-estimated (see Wardinski, 2004; Gillet et al, 2010a), while tuning the flow prior to correct for this issue penalizes too much interannual changes. We also characterize how resolved are flow variations as a function of period and length-scale: we find that satellite observations bring some constraint on decadal flow changes up to degree $n \simeq 15$, with some information at large length-scales down to a few years periods (Figure 1.5). There we quantify how satellite records significantly improve the resolution.

1.3.3 Quasi-Geostrophic MHD modes

Along these lines, the PhD of François Labbé (co-supervised with Dominique Jault, defense planned in September 2015) has been oriented towards the analysis of QG MC modes (Hide, 1966) in presence of a non-axisymmetric magnetic field. His study extends the case of a zonal imposed field previously analyzed by Canet et al (2014), where two families of waves arise: slow magnetic modes and rapid Rossby modes, of periods respectively of the order of a few hundreds of years and dozens of days for Earth-like parameters. Considering a non-zonal field couples together all azimuthal wave numbers m, and also requires accounting for the equation of zonal (geostrophic) motions

$$u_g(s,t) = \oint u_\phi(s,\phi,t)d\phi\,. \tag{1.18}$$

Then, if the imposed field is complex enough (i.e. it contains non-zonal contributions from at least two different orders m) the family of initially uncoupled modes gives birth to a branch of torsional waves on the one hand (dominated by the $m = 0$ component, see section 1.3.4), and several branches of dense (involving all orders m) slow modes on the other hand (see figure 1.6). These can be understood in the framework of Taylor (1963)'s condition

$$\mathbf{1}_\phi \cdot \int_{\Sigma(s)} \nabla \times \mathbf{B} \times \mathbf{B} d\Sigma = 0\,, \tag{1.19}$$

that they satisfy in the limit where their period $\tau_{MC} \gg \tau_a$ ($\Sigma(s)$ are geostrophic cylinders, of height $2H(s)$, and $\mathbf{1}_\phi$ the unit vector in the azimuthal direction).

François finds that the ratio between zonal and non-zonal contributions to the modes scales as the ratio of the zonal to non-zonal imposed field, in agreement with the differential equation obtained by Taylor when considering $\partial_t(1.19)$. The next step is to explore the possibility for such dense MC modes to settle in the decadal period range. If such modes, considered individually, may not be suited to explain by themselves the observed SV, their nonlinear interaction might be relevant to understand the evolution of the geomagnetic field on decadal periods, in which case our description of these elementary bricks may be of importance.

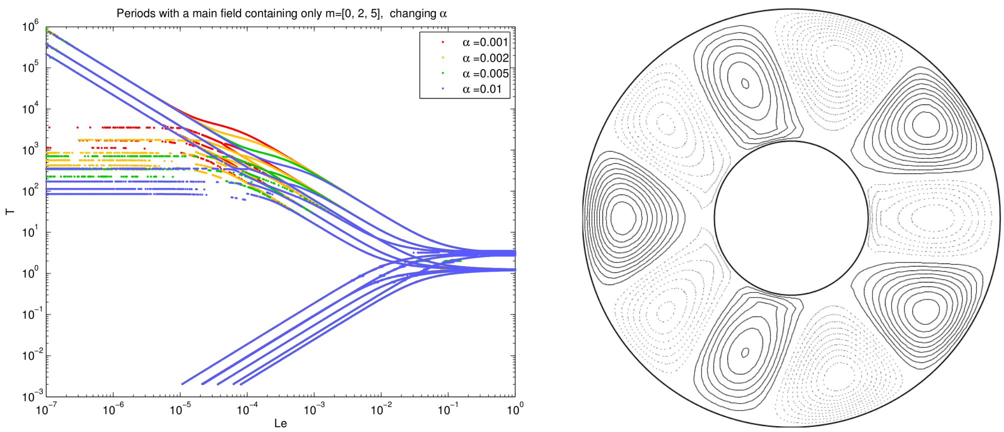

Figure 1.6: Left: Period of the modes as a function of the Lehnert number. They separate between the torsional waves, fast and slow MC modes when the imposed field contains both axisymmetric ($m = 0$) and non-axisymmetric contributions ($m = 2$ and $m = 5$). α measures the relative amplitudes of the non-zonal field, relative to the zonal field. Right: example of slow mode mixing $m = 2$ and $m = 5$ orders, represented with the stream function in the equatorial plane, at $\lambda = 10^{-5}$ (From Labbé et al, 2015).

1.3.4 Torsional waves

We now focus on the specific component of the geostrophic (zonal) QG flows, defined by equation (1.18). These have the particularity of being parallel to the spherical boundary. As a consequence, the projection of the Coriolis force in the momentum equation vanishes for such motions, and their response to a perturbation cannot take the form of a QG MC waves (see above).

Then there exist a family of large length-scales Alfvén waves, called torsional waves (Braginsky, 1970), who consist in the response of the fluid to a perturbation from Taylor's condition (1.19) (neglecting Reynolds stresses and viscosity):

$$\frac{\partial u_g}{\partial t} = \mathbf{1}_\phi \cdot \int_{\Sigma(s)} \nabla \times \mathbf{B} \times \mathbf{B} d\Sigma \,. \tag{1.20}$$

Discarding boundary terms, these motions obey the wave equation

$$\frac{\partial^2}{\partial t^2}\left(\frac{u_g}{s}\right) - \frac{1}{s^3 H}\frac{\partial}{\partial s}\left(s^3 H C^2 \frac{\partial}{\partial s}\left(\frac{u_g}{s}\right)\right) = \int_{\Sigma(s)} \left(\mathcal{H}\left(\mathbf{B}, \mathbf{u}_{NZ}\right) + \mathcal{G}_\eta\left(\mathbf{B}\right)\right) d\Sigma \,. \tag{1.21}$$

This suggests they can be triggered by non-zonal motions $\mathbf{u}_{NZ}$ interacting with the underlying (considered quasi-stationary) magnetic field (Teed et al, 2014). $\mathcal{H}$ and $\mathcal{G}_\eta$ are quadratic functions of the magnetic field, $\mathcal{H}$ depends linearly on the non zonal velocity. The speed of these waves is proportional to the rms value of the cylindrical radial field B_s averaged over geostrophic cylinders:

$$C(s) = \sqrt{\frac{1}{4\pi s H \rho\mu} \int_{\Sigma(s)} B_s(s,\phi,z)^2 d\Sigma} \,. \tag{1.22}$$

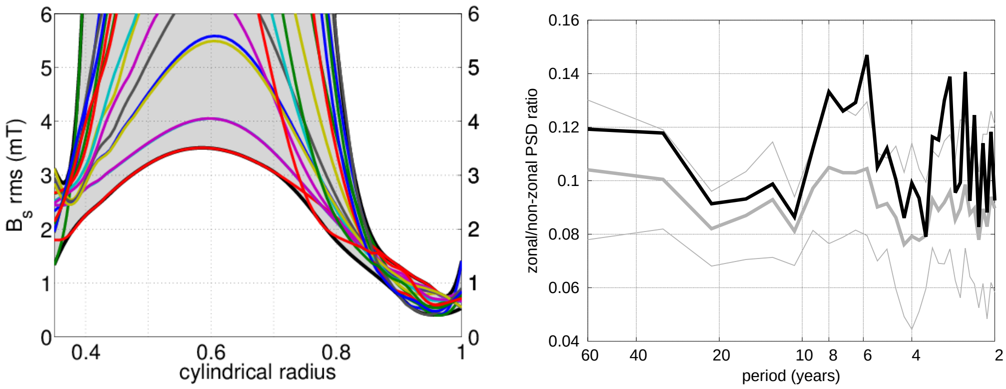

Figure 1.7: Left: profiles of the r.m.s. of the cylindrical radial magnetic field over geostrophic cylinders, inverted from the assimilation of zonal motions in a 1D dynamical model of torsional wave (from Gillet et al, 2010a). Right: ratio of the temporal PSD obtained for zonal and non-zonal flow models, for the ensemble average solution (black) and averaged over the ensemble of solutions (in grey, ± one standard deviation in grey) (From Gillet et al, 2015b).

Such waves do carry angular momentum. As such they have a signature in the independent geodetic observations through LOD changes (Jault et al, 1988; Jackson et al, 1993).

Detecting such waves from geomagnetic data thus offers a window towards the magnetic field inside the core, which is invisible directly from surface observations. As seen with equation (1.21), the sensitivity to the internal field is one-dimensional when associated with the phase speed C, and two-dimensional if extracted from the source term (on the r.h.s. of the wave equation), then depending on our knowledge of the non-zonal flow.

For a long time, torsional waves have been believed to correspond to decadal fluctuations. Oscillation at about 60 years periods in u_g had been imaged from continuous field models, presenting encouraging predictions to observed LOD changes (Bloxham et al, 2002). The drawback of this scenario is essentially that such long periods are associated with a field inside the core (measured through C) of a fraction of mT (Zatman and Bloxham, 1997), weaker than what is observed at the CMB by downward continuation of surface observations.

We performed an analysis of geostrophic motions inverted from the *gufm1* field model of Jackson et al (2000). Focusing then on the period 1955–1975, we inverted for the cylindrical radial profile of the field throughout the outer core, or $C(s)$, using our model of zonal motions, filtered around 6 years periods, as observations in a variational assimilation algorithm (Gillet et al, 2010a). The motivation for the filtering was the signal detected by Abarca del Rio et al (2000), and later confirmed by Chao et al (2014), in LOD series at this particular time-scale. Our flow model provides a good explanation for this independent observation, in a frequency range where external envelopes (atmosphere, oceans) do fail to produce enough angular

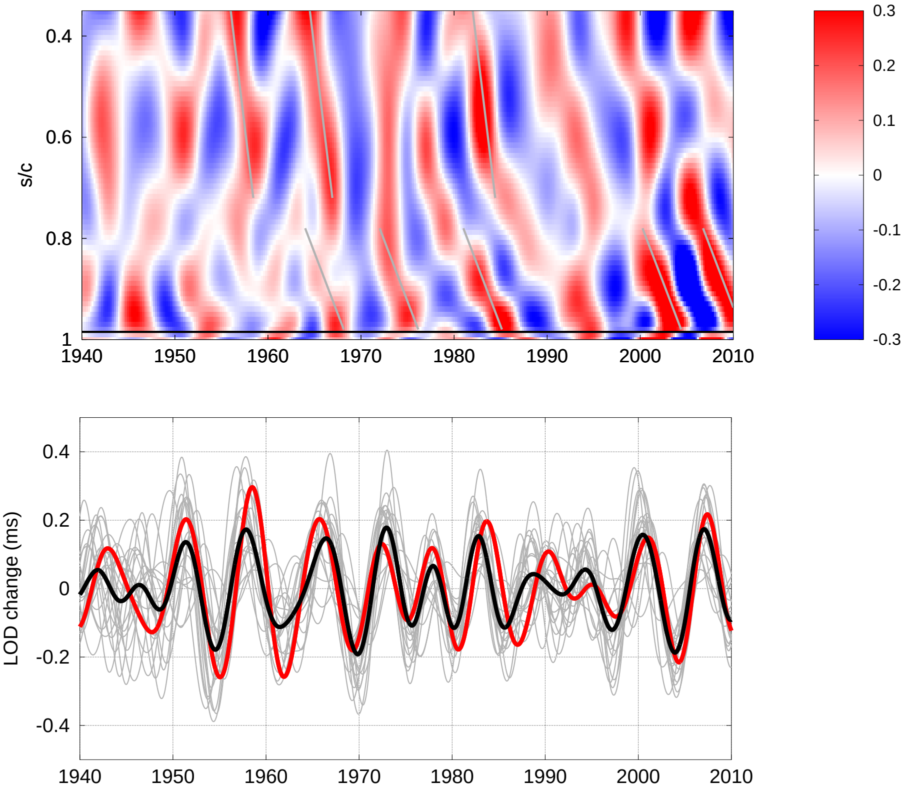

Figure 1.8: Top: Zonal flow (in km.yr^{-1}), band-pass filtered between 4 and 9.5 years. the Y axis is the cylindrical radius; grey lines correspond to phase velocities C of the torsional waves based on a r.m.s. field B_s of 1.9 mT and 0.6 mT (towards respectively the inner core and the equator). The black horizontal line marks the latitude of 10°. Bottom: comparison between LOD predictions (in ms) from all members of our ensemble of flow models (grey), their ensemble average (black) together with the observed LOD changes (red), band-pass filtered between 4 and 9.5 yrs. From Gillet et al (2015b).

momentum changes (Paek and Huang, 2012). We obtained intensities for B_s inside the core of the order of a few mT (see Figure 1.7, left), ten times larger than previously proposed by Zatman and Bloxham (1997). Our analysis reconciles values inferred from geophysical observations with values estimated from the ratio between surface and bulk intensities in geodynamo simulations (e.g., Aubert et al, 2009). It is also coherent with the dissipation of torsional waves associated with core-mantle electro-magnetic coupling (Dumberry and Mound, 2008).

Further investigations of torsional waves were then requiring a continuous field model involving the most recent data together with its associated errorbars. It led to the construction of the COV-OBS field model (see Section 1.2.2). From this model and the methodological developments presented in Section 1.3.2, we extended the analysis of torsional motions over 1940–2010 (Gillet et al, 2015b). We found a peak, at periods around 6 to 8 years, in the ratio between the PSD of the zonal to non-zonal flow components (Figure 1.7, right). This observation supports the possible triggering of torsional waves from the r.h.s. term in equation (1.21), avoiding calling to extra mechanisms (e.g., such as gravitational coupling between the inner core and the mantle, see Mound and Buffett, 2006). We exhibits (see Figure 1.8) a decadal modulation of torsional waves, confirming the good prediction of modulated LOD changes in the frequency range 4–9.5 years. This new analysis also highlights a particularly intense signal at low latitudes since 1995 (to which I come back in Section 2.2).

Chapter 2

Research perspectives

2.1 Deterministic versus stochastic geomagnetic data assimilation

Over the past few years, several groups have started developing geomagnetic data assimilation algorithms, having in mind the forecast and/or the re-analysis of the core state using prognostic models of the core dynamics (e.g., Canet et al, 2009; Fournier et al, 2013; Li et al, 2014; Tangborn and Kuang, 2015). Both variational or sequential methods are considered; our community benefits from the numerous adaptations of the optimal control theory developed in the context of atmospheric and oceanic dynamics (e.g., Kalnay, 2003).

However, it does not appear obvious to me that we currently possess models containing the physics suited to understand the observed geomagnetic changes. As precised in Section 1.3.1, geodynamo simulations may lack two crucial decades of time-scales where magnetic forces might well play a key role. Such simulations can possibly give birth to enough complexity to be able to digest magnetic data; however, this may not be for the right reason if they do not reach the correct asymptotic regime (Wu and Roberts, 2014).

I see another example of the imperfection of the current models, associated with the ghostly behavior of magnetic forces: even though the magnetic energy is very large, Lorentz forces may be unexpectedly weak. Indeed, they could be potentially so strong that the flow and the field self-organize in order to minimize the magnetic induction (a kind of 3D expression of Ferraro's law of isorotation). Such a scenario makes it difficult the construction of a coherent background state $\mathbf{B}^b$ for the study of rapid core dynamics. Indeed, we would then look for a magnetic field creating zero Lorentz forces, thus satisfying a constraint of the form $\mathcal{K}\left(\mathbf{u}(t),\mathbf{B}^b\right)\simeq 0$ (but what does 'nearly zero' mean?). This consideration discards the QG model in its initial expression (Canet et al, 2009, where magnetic forces are explicitly presented as non-zero quadratic quantities of the magnetic field), and requires further developments. This picture has led to the re-orientation of the thesis of François Labbé, which was initially targeted towards the assimilation of SV data using the QG model of Canet et al.

In this context, I felt necessary to concentrate on the estimation of our ignorance. We followed a stochastic avenue with models able to mimic Earth-like behaviors as far as the surface magnetic field (Gillet et al, 2013; Hellio et al, 2014), the core flow (Gillet et al, 2015b) or subgrid nonlinear interactions (Gillet et al,

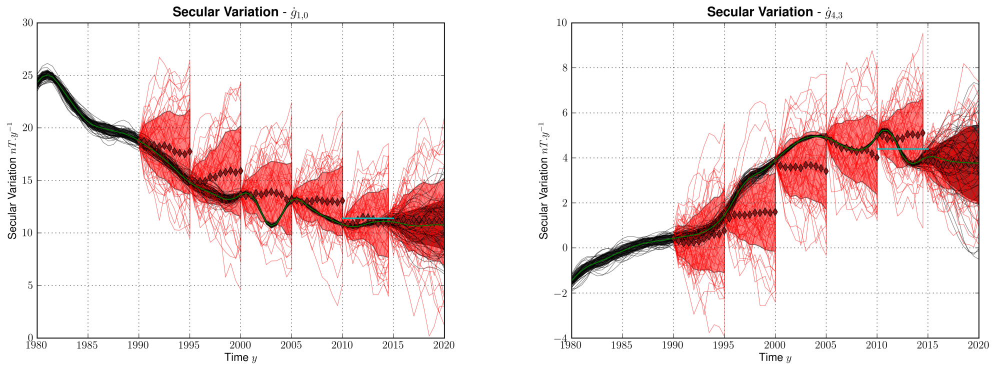

Figure 2.1: SV Prediction with a stochastic flow model implemented in an augmented state Ensemble Kalman filter (red shaded area: $\pm$ one standard deviations; observations in black; average prediction in diamonds). Examples of spherical harmonic coefficients $\dot{g}_1^0$ and $\dot{g}_4^3$, from (Gillet et al, 2015a).

2015a) are concerned. This approach, complementary to the development of deterministic models, is at the heart of the PhD project that Olivier Barrois started last autumn. Olivier stared a benchmark between several algorithms used so far to integrate spatial and/or temporal covariances: for instance a weak formulation versus an augmented stated Kalman Filter to account for time correlated SV model errors, or the impact of the QG assumption in the equatorial area with regard to 3D simulations.

Communities studying surface envelopes (ocean, atmosphere) started their developments concentrating on the deterministic part, and later on introduced stochastic forcings into their prognostic models. But in geomagnetism, much less is known about the physics (do we have the correct equations to describe the proper asymptotic regime?), and much less data are available (a bit as if we were analyzing the state of the atmosphere with only a few days of accurate meteorological records). This makes dynamical uncertainties potentially very important. It is in order to tackle the second issue (i.e., to cover several turn-over times) that we put some efforts in the modeling of the geomagnetic field from archaeological data (Section 1.2.4). Global models that will be produced covering the past millenia (together with their associated errorbars) will help define a background state for the outer core, which may appear crucial to better re-analyze the high quality satellite and observatory data with dynamical models.

However, producing models only capable of producing an envelope of solution with no physics is not a goal in itself. It should be considered as a first step that can be used to produce a 'state zero' of the ability to predict the evolution of the geomagnetic field – cf Figure 2.1 and the prototype augmented state ensemble Kalman filter approach we followed when building the ISTerre 'test SV candidate model' to IGRF-12 (Gillet et al, 2015a). Then the potential prediction power of deterministic models could be tested against this 'zero' measure, by answering to questions such

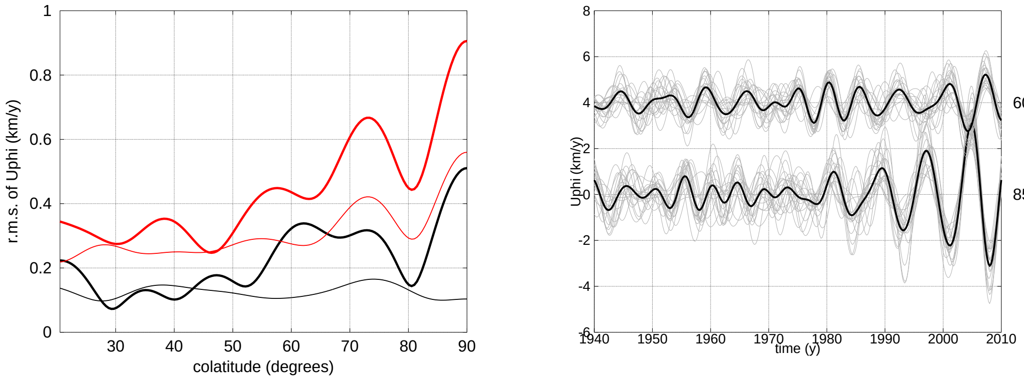

Figure 2.2: Left: As a function of colatitude, profile of the r.m.s. azimuthal velocity, filtered at sub-decadal periods: zonal in black, non-zonal in red, for periods 1965–1975 (thin) and 2000–2010 (thick). Right: Ensemble of non-zonal azimuthal flow models at the equator for two different longitudes as a function of time, filtered at sub-decadal periods. From Gillet et al (2015b).

as 'does this particular prognostic physics produce a better forecast in comparison with knowing only silly statistical information?' or 'over which period do we forecast better the field than a stochastic model?'. One can think of many candidates: geodynamo simulations (Aubert et al, 2013), magnetic waves in a stratified layer at the top of the core (Buffett, 2014), 3D Taylor state models (Li et al, 2014), QG models, etc.

2.2 Focus on the equatorial dynamics

With the methodological developments presented in Section 1.3.2, we put some emphasis on the equatorial dynamics. This region is specific in more than one respect. First, the Coriolis force is singular in this region where the outer core surface is parallel to the rotation axis. Second, it represents a wide area of the CMB, and as such it should be relatively well constrained by surface observations (in term of azimuthal wave number for instance). It is also the place for the most intense field changes at decadal and longer periods (Finlay and Jackson, 2003).

We detected there a striking geometry of azimuthal motions in our QG flow models: they present a minimum in amplitude at 10° away from the equator, on both zonal and non-zonal components, and this whatever the studied period range (cf Figure 2.2). One may wonder if this observation is (*i*) a signature of a physics specific to this region; (*ii*) a consequence of the actual resolution of core flows; (*iii*) a topological consequence of the QG constraints we imposed. Indeed, alternative dynamical hypotheses can be defended (for instance a stratified layer at the top of the core, see Braginsky, 1999), which may lead to such a projection when imposing the QG constraints. Although it is certainly a limiting factor, I would discard (*ii*) alone since we also observe the minimum on the zonal component (see Figure 1.8).

Note that geostrophic motions are specific in the context of a possible stratified layer, as they would not cross the radial density gradient (thus they cannot be directly distorted by gravity waves).

Particularly intense SV variations have been detected recently from satellite observations (Olsen and Mandea, 2007; Chulliat and Maus, 2014), which settle primarily in the equatorial region. My current point of view is that such constraints from magnetic records do not require the presence of a stratified layer, since we find no difficulty to account for these rapid SV changes in the QG framework (we see no need for meridional flows crossing the equator at the CMB). However, if we find that intense, non-zonal flow variations are responsible for the observed SV (see Figure 2.2), wee lack a physical understanding for such features. This requires further theoretical and numerical studies, together with a more focused analysis of magnetic data. Testing several topological constraints over the past 20 years where the signal is the most pronounced would definitely help. Many possible physical hypotheses may be visited: instabilities involving a magnetic field gradient, a velocity shear, topography, mixing of equatorially trapped Rossby waves, etc. Swarm observations will help make progress on this recent issue.

2.3 Imaging the core dynamics: towards shorter periods

I show in Section 1.3.2 that we are currently limited to periods longer than a few years when imaging core flow changes. The Swarm trio of satellites offers a unprecedented opportunity for looking at variations occurring on shorter time-scales, by helping better separate internal and external sources. There is currently no evidence for a strong screening of the magnetic signal by the mantle conductivity (Jault, 2015). Furthermore, the -4 slope observed for the PSD of geomagnetic series, once cleaned from external contributions using comprehensive models, seem to extend down to periods of a few months (Finlay et al, 2013).

These arguments motivate up-coming studies of the core dynamics at periods potentially shorter than a couple of year from magnetic observations. I first think of better resolving torsional motions, since the currently accessible period range only marginally includes the first harmonic of the fundamental period at 6 to 8 years. Furthermore, our current phenomenological description of these wave-like patterns, though motivated by the good fit to LOD series, is insufficient to properly separate geostrophic waves from zonal motions obeying Taylor's condition (cf Section 1.3.3). We need to study numerically the response (propagation, reflection, quality factor, etc.) of torsional motions to random forcings in the volume, in order to provide a better physical understanding of the reconstructed motions. A better description of rapid non-zonal flow changes will also help characterize the potential source I advocate for in Section 1.3.4.

Considering even shorter periods (of the order of one month), there may also be a possibility for detecting QG Rossby waves. Rossby Waves can be described analytically or numerically (e.g., Zhang et al, 2001, 2004), so that we potentially know both the period and the waveform for these modes. The knowledge of their spatio-temporal structure might help detect them in magnetic series 'polluted' by external signals, extending the preliminary synthetic study by Vidal (2013). There are several difficulties. First, QG Rossby waves are particular: the larger their length-scale, the faster their propagation. Since we have access neither to large wave-numbers, nor to high frequencies, there would be only a window in space and time where they could be detected. Second, they are also sensitive to the possible presence of a stratified layer (Helffrich and Kaneshima, 2010), which would only allow large length-scales modes to reach the CMB (Takehiro and Lister, 2001; Vidal and Schaeffer, 2015). Finally, the presence of a finite conductivity of the lowermost mantle might require associating electrical currents in the mantle to the core dynamics at monthly periods (Jault, 2015). The difficulties expressed above make this study exciting. Indeed, studying Rossby waves thus have implications on wider geophysical issues in link with the heat flux at the CMB (Pozzo et al, 2012, e.g.,), or thermo-chemical structures at the base of the mantle.

2.4 Fundings and collaborations

Most of the work presented in the above sections have been supported by the ANR grants 'VSQG' (2006–2009) and 'AVS-geomag' (2011-2016). This latter, for which I am co-PI with Alexandre Fournier, is a collaboration with IPGP. My research in link with the analysis of satellite data is supported by the French Spatial Agency CNES (I am co-PI with Dominique Jault of a response to annual calls from CNES). I am also part of a team supported by the International Space Science Institute in Bern (PI: Chris Finlay).

I will continue responding to annual call from the CNES to support my future research in link with Swarm, in particular concerning the rapid dynamics. The work on the triggering of torsional waves is already initiated, performed together with Elisabeth Canet (ETH Zurich). The collaboration with IPGP will carry on, in particular with Julien Aubert through the thesis of Olivier Barrois. We envision a future common ANR proposal on geomagnetic data assimilation from millennial to interannual periods. A few years ago I started collaborating with Jiaming Ou, whose stays in Grenoble were supported by IGGCAS (Beijin) in the general context of a future Chinese satellite mission. We will try to co-fund a post-doc position for him at ISTerre, on the topic of the separation between internal and external sources from satellite and observatory data. Finally, my research involves a tight collaboration with Chris Finlay in Copenhagen, with whom I share algorithms to deal with data analysis and core flow computations. We plan to continue these exchanges, which are at the heart of our work where we try to construct a discussion between magnetic data and the core dynamics.

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Appendix A

CV, communications, student supervision...

A.1 Curriculum Vitae

Nicolas Gillet
37 year old, 2 children
CNRS Research fellow (CR1)

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Research

- Since 2009: CNRS research fellow at ISTerre (Grenoble) – reconstruction and the understanding of the dynamics in the Earth's outer core, from archeomagnetic (past millenia), historical (past centuries) and satellite (past decades) measurements of the geomagnetic field.
  - co-PI of the project 'rapid dynamics in the Earth's core as inferred from satellite data' (annual calls for fundings from the French Spatial Agency CNES, annual budget ∼15,000 €).
  - co-PI of the geomagnetic data assimilation project 'AVS-geomag' (ANR funding 2011-2016, collaboration with IPG Paris, http://avsgeomag.ipgp.fr/, annual budget ∼15,000 €).
  - member of the ISSI team working on the project 'Rapid dynamics in the Earth's core: Assimilation of satellite observations into MHD models'
- 2007-2009: Post-doc fellow at ISTerre in the 'quasi-geostrophic secular variation' project (ANR grant, coll. Dominique Jault and Alexandre Fournier).
- 2004-2007: Post-doc fellow at the Institute of Geophysics and Tectonics (University of Leeds, UK) on the geomagnetic inverse problem (NERC grant, coll. Andy Jackson).
- 2001-2004: PhD Thesis at the Laboratoire de Géophysique Interne et Tectonophysique (University of Grenoble), on 'magnetoconvection in a rapidly rotating sphere: numerical and experimental models of convection in planetary cores' (Supervision: Dominique Jault and Daniel Brito).

Education, Honors and Awards

- 2012: Zatman memorial Lecture, SEDI meeting, Leeds, UK
- 2008: Doornbos memorial prize, SEDI meeting, Kunming, China
- 2004: PhD in geophysics with 1st class honors, University of Grenoble, France
- 2000: M. Sc. in 'Oceanography, Meteorology and Environment', University of Paris 6, France
- 1997-2000: Ecole Nationale Supérieure des Techniques Avancées, Paris, France

Services, synergistic activities

- Correspondent for communication at ISTerre (2012–2014)
- Animation of monthly seminars on the inverse problem (ISTerre, 2009-2012)
- Organisation of weekly seminars (IGT, Leeds, 2006-2007)
- Reviewer for *Geophys. J. Int.*, *J. Fluid Mech.*, *Nature*, *Nature Geosciences*, *Proceedings of the National accademy of Sciences*, *Phys. Earth Planet. Int.*, *Earth Planets and Space*, *G-cubed*, *Earth Planet. Sc. Lett.*, *Journal of Atmospheric and Solar-Terrestrial Physics*, *Space Science Review*, and for grant proposals for the INSU and ETH.
- Co-conveener for sessions at AGU (2011) and IAGA (2013)

Teaching

- Master lectures on 'Geomagnetism and the dynamics in the outer core' (15 h/year, 2010–2014, University of Grenoble/ENS Lyon)
- Popularized undergrad online lectures on 'Magnetic stories in the Universe' (25h, 2011-2012, University of Grenoble)
- PhD lectures on the 'Physics of the geomagnetic field' (12h, 2010, University of Grenoble).

A.2 Peer-reviewed publications

On March the 9th, 2015, the 15 published papers listed below resulted in 362 citations (source: Google Scholar), with an h index of 11 (number h of publications cited at least h times).

18 Thébault, E., et al., International Geomagnetic Reference Field: the twelfth generation, *Earth Planets and Space*, in revision.

17 **Gillet, N.**, Barrois, O. and C. C. Finlay, Stochastic forecasting of the geomagnetic field from the COV-OBS.x1 geomagnetic field model, candidate for the IGRF-12, *Earth Planets and Space*, in revision.

16 **Gillet, N.**, Jault, D. and C. C. Finlay, Planetary gyre, time-dependent eddies, torsional waves and equatorial jets at the Earth's core surface, *J. Geophys. Res.*, in revision.

15 Ou, J., **N. Gillet** and A. Du, Geomagnetic observatory monthly series, 1930-2010 : empirical analysis and unmodeled signals estimation, *Earth, Planets and Space*, 67(1), 1–20.

14 Hellio, G., **N. Gillet**, C. Bouligand and D. Jault, Stochastic modeling of regional archeomagnetic series, *Geophys. J. Int.*, 199(2), 931-943 (2014).

13 **Gillet, N.**, D. Jault, C. C. Finlay and N. Olsen, Stochastic modeling of the Earth's magnetic field : Inversion for covariances over the observatory era, *Geochem., Geophys. Geosyst.*, doi:10.1029/2012GC004355 (2013).

12 Finlay, C. C., A. Jackson, **N. Gillet** and N. Olsen, Core Surface Magnetic Field Evolution 2000-2010, *Geophys. J. Int.*, DOI : 10.1111/j.1365-246X.2012.05395.x (2012).

11 **Gillet, N.**, N. Schaeffer and D. Jault, Rationale and geophysical evidence for quasi-geostrophic dynamics within the Earth's outer core, *Phys. Earth Planet. Int.*, 187 (3-4), 380-390 (2011).

10 Fournier, A., G. Hulot, D. Jault, W. Kuang, A. Tangborn, **N. Gillet**, E. Canet, J. Aubert and F. Lhuillier, An introduction to data assimilation and predictability in geomagnetism, *Space Sci. Rev.*, 155 (1), 247-291, DOI : 10.1007/s11214-010-9669-4 (2010).

9 **Gillet, N.**, D. Jault, E. Canet and A. Fournier, Fast torsional waves and strong magnetic field within the Earth's core, *Nature*, 465, 74-77 (2010).

8 **Gillet, N.**, V. Lesur and N. Olsen, Geomagnetic core field secular variation models, *Space Sci. Rev.*, 155 (1), 129-145, DOI : 10.1007/s11214-009-9586-6 (2010).

7 **Gillet, N.**, M. A. Pais and D. Jault, Ensemble inversion of time-dependent core flow models, *Geochem. Geophys. Geosyst.*, 10, Q06004, doi:10.1029/2008GC002290 (2009).

6 **Gillet, N.**, A. Jackson and C. C. Finlay, Maximum entropy regularistion of time-dependent geomagnetic field models, *Geophys. J. Int.*, 171, 1005–1016. (2007).

5 Jackson, A., C. Constable and **N. Gillet**, Maximum entropy regularisation of the geomagnetic core field inverse problem, *Geophys. J. Int.*, 171, 995–1004. (2007).

4 **Gillet, N.**, D. Brito, D. Jault and H.-C. Nataf, Experimental and numerical studies of convection in a rapidly rotating spherical shell, *J. Fluid Mech.*, 580, 83-121 (2007).

3 **Gillet, N.**, D. Brito, D. Jault and H.-C. Nataf, Experimental and numerical studies of magnetoconvection in a rapidly rotating spherical shell, *J. Fluid Mech.*, 580, 123-143 (2007).

2 **Gillet, N.** and C. A. Jones, The quasi-geostrophic model for rapidly rotating spherical convection outside the tangent cylinder, *J. Fluid Mech.*, 554, 343-369 (2006)

1 Aubert J., **N. Gillet** and P. Cardin, Quasigeostrophic models of convection in rotating spherical shells, *Geochem. Geophys. Geosys.*, 4, 1029/2002GC000456 (2003).

A.3 Invited communications in meetings

- Torsional waves and quasi-geostrophic equatorial jets at the Earth's core surface, *IUGG meeting*, Prague (Czech Republic), June 2015.
- Imagerie de la dynamique dans le noyau terrestre : les apports de l'assimilation de données, *'Assimilation de données en géosciences'*, IGN, Saint-Mandé (France), May 2015.
- Stochastic versus deterministic reconstructions of the core dynamics over the observatory era, *Zatman lecture, SEDI meeting*, Leeds (UK), July 2012.
- Reconstruction the magnetic field over the observatory era with a Bayesian approach, *AGU Fall meeting*, San Francisco (USA), December 2011.
- Magnetic field models derived using a Bayesian approach, *I Magnet Brazil*, Buzios (Brazil), June 2011.
- Fast torsional waves and strong magnetic field within the Earth's core, *AGU Fall meeting*, San Francisco (USA), December 2009.

- Models of the present and historical field, *ISSI Workshop on Terrestrial Magnetism*, Bern (Switzerland), March 2009.
- Geomagnetic data and the Earth's core dynamics, *SEDI meeting*, Kunming (China), July 2008.
- Experimental and numerical studies of nonlinear (magneto)convection in a rapidly rotating sphere, IUGG *meeting*, Peruggia (Italy), July 2007.

A.4 Student supervision

PhD thesis

- Gabrielle Hellio (2011–2015): Modèles stochastiques de mesures archéomagnétiques (co-supervision with D. Jault and C. Bouligand)
- François Labbé (2011–): Modes quasi-géostrophiques dans une sphère en présence d'un champ magnétique (co-supervision with D. Jault)
- Olivier Barrois (2014–): La dynamique dans le noyau de la Terre vue au travers des observations satellitaires

MSc, Undergrads

- Olivier Barrois (M2R, 2014): Prédictions stochastiques du champ magnétique terrestre
- Jérémie Vidal (M1, 2013): Ondes inertielles quasi-géostrophiques dans les séries magnétiques d'observatoires
- Thomas Chauve (M1, 2011): Problème inverse sur l'état du noyau à partir des données géomagnétiques
- Elsa Yobregat (L3, 2010): Analyse en composantes principales d'un ensemble de modèles d'écoulements dans le noyau
- Grégory Fanjat (M2R, 2009): Inversion directe de l'état du noyau à partir des données géomagnétiques
- Eloïse Kiefer (M1, 2004): Etude expérimentale de la magnéto-convection dans une sphère en rotation

A.5 Popularization of sciences

- Conférences grand public:
  - Le magnétisme du Soleil, *Université Inter-Ages du Dauphiné*, Grenoble, Février 2013

  - Dernières nouvelles du noyau terrestre, *Université Inter-Ages du Dauphiné*, Grenoble, Février 2011;
  - Dernières nouvelles du noyau terrestre, *Fête de la Science*, Bourg-en-Bresse, Octobre 2009

- Expériences publiques de convection rotation, *Fête de la Science*, Pont-de-Claix, Octobre 2013

- Notre article de 2010 dans *Nature* a fait l'objet d'un communiqué de presse de la part du CNRS/INSU, et d'articles dans *Sciences et avenir* ('Double éclairage sur le noyau', Juillet 2010), *New Scientist* ('Earth's twisted heart changes the length of the day', Mai 2010), *Physics World* ('Earth's magnetic field gathers momentum', Mai 2010).

Appendix B

Selection of publications

In this chapter are selected the following papers :

- Planetary gyre, time-dependent eddies, torsional waves and equatorial jets at the Earth's core surface (Gillet, Jault & Finlay, *J. Geophys. Res.*, in revision)
- Stochastic forecasting of the geomagnetic field from the COV-OBS.x1 geomagnetic field model, candidate for the IGRF-12 (Gillet, Barrois & Finlay, *Earth Planets and Space*, in revision)
- Stochastic modeling of regional archeomagnetic series (Hellio, Gillet, Bouligand & Jault, *Geophys. J. Int.*, 2014).
- Stochastic modeling of the Earth's magnetic field : Inversion for covariances over the observatory era (Gillet, Jault, Finlay & Olsen, *Geochem., Geophys. Geosyst.*, 2013).
- Rationale and geophysical evidence for quasi-geostrophic dynamics within the Earth's outer core (Gillet, Schaeffer & Jault, *Phys. Earth Planet. Int.*, 2011).
- Fast torsional waves and strong magnetic field within the Earth's core (Gillet, Jault, Canet and Fournier, *Nature*, 2010).