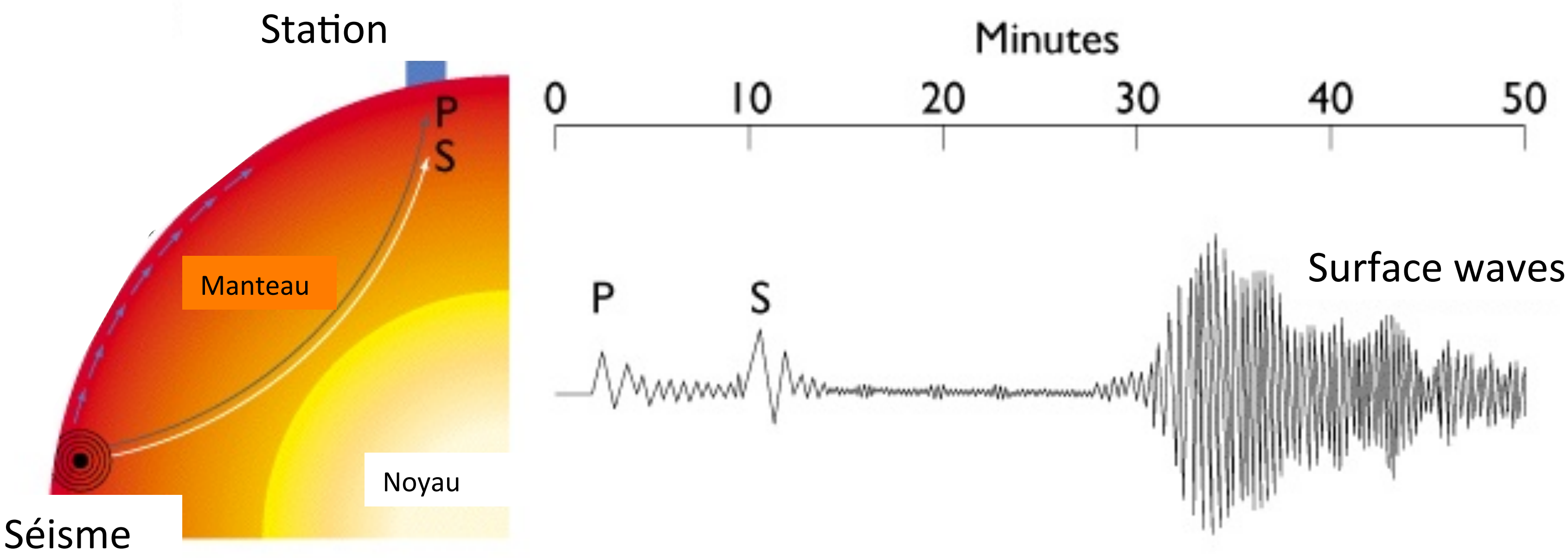
NOTES INTRO 1

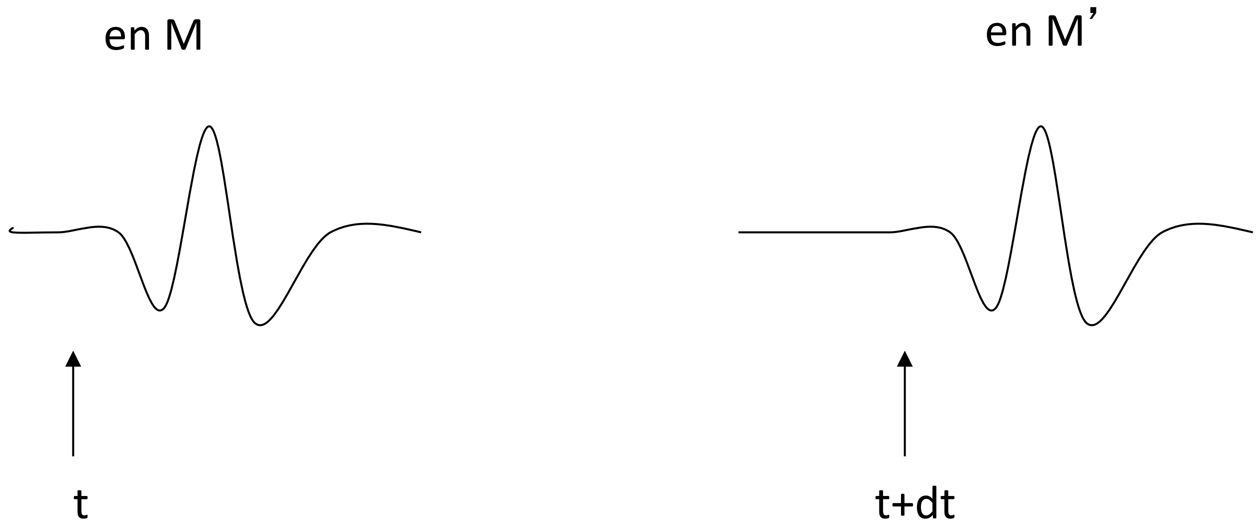


Bulk waves: as rays
P: compression ; S: shear

Surface waves: confined ‘close to’ the surface
R: Rayleigh ; L : Love

Wave?

A perturbation that propagates without changing its form (in an homogeneous body)



dt= MM' /v ; v is the propagation speed

$f(t - x/v)$ for propagation in the direction of x ¿ 0
ou
$f(t + x/v)$ for propagation in the direction of x ¡ 0

$(t \pm x/v)$: 'phase term'

$$\begin{aligned}
\frac{\partial f}{\partial t} &= f'(t - x/v) \\
\frac{\partial^2 f}{\partial t^2} &= f''(t - x/v) \\
\frac{\partial f}{\partial x} &= -\frac{1}{v} f'(t - x/v) \\
\frac{\partial^2 f}{\partial x^2} &= \frac{1}{v^2} f''(t - x/v) \\
\Rightarrow \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2} &= \frac{\partial^2 f}{\partial x^2}
\end{aligned}$$

Wave equation.

Wave equation

$$\frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = \frac{\partial^2 f}{\partial x^2}$$

Second order derivative in time is proportional to second order derivation in space.

3D:

$$\frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = \Delta f$$

Δ : laplacian

3D for a vectorial field:

$$\frac{1}{v^2}\frac{\partial^2 \vec{f}}{\partial t^2} = \vec{\Delta}\vec{f}$$

$\vec{\Delta}$: laplacien vectoriel ($(\vec{\Delta}\vec{f})_x = \frac{\partial^2 \vec{f_x}}{\partial x^2} + \frac{\partial^2 \vec{f_x}}{\partial y^2} + \frac{\partial^2 \vec{f_x}}{\partial z^2}$)

Elasticity in a few words

$\rightarrow$ continuous medium , weak deformations (strains)

$\vec{d}(u, v, w)$ displacement vectorial field

$\rightarrow$ strains : $\star$ normal $\varepsilon_{xx} = \frac{\partial u}{\partial x}$; $\varepsilon_{yy} = \frac{\partial v}{\partial y}$; $\varepsilon_{zz} = \frac{\partial w}{\partial z}$

$$\star \text{ shear } \varepsilon_{xy} = \varepsilon_{yx} = \tfrac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$

$$\varepsilon_{yz} = \varepsilon_{zy} = \tfrac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

$$\varepsilon_{yz} = \varepsilon_{zy} = \ldots$$

Isotropic elasticity $\rightarrow$ Hooke's law : linear relationship between strain and stress

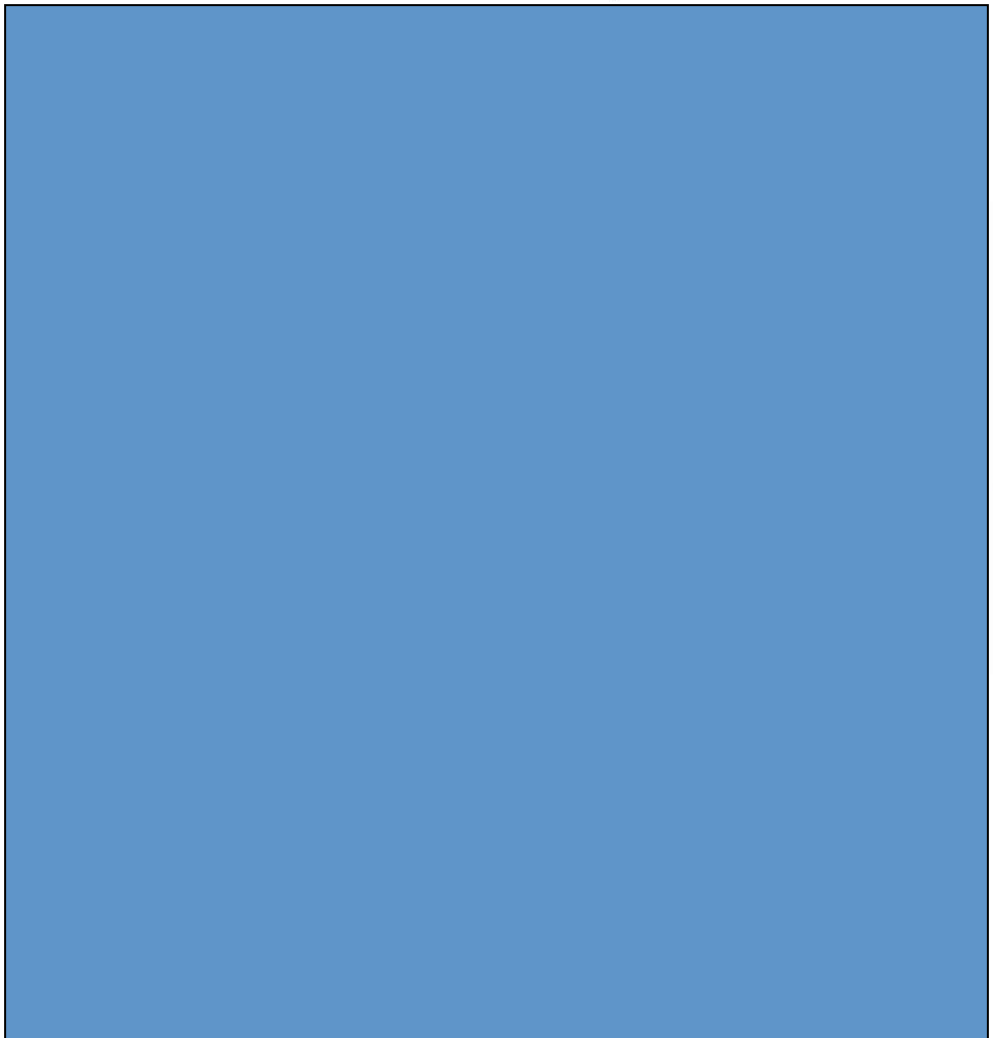
$\sigma_{ii} = \lambda\, D + 2\, \mu\, \varepsilon_{ii}$ $\qquad i = x, y, z$

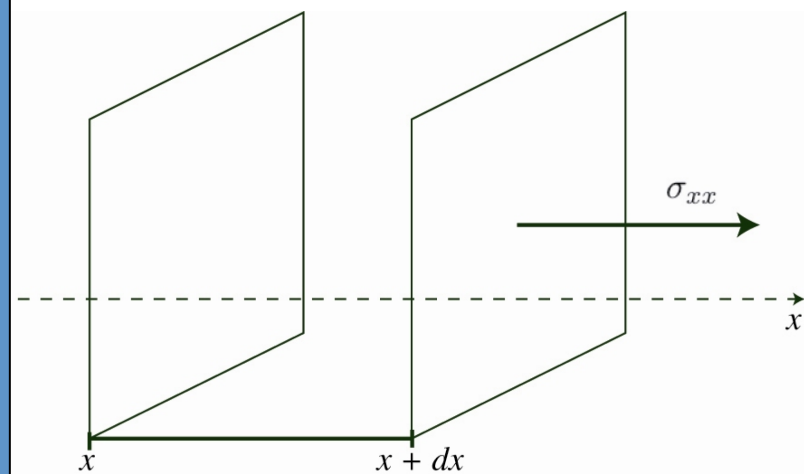
$\sigma_{ij} = 2\, \mu\, \varepsilon_{ij}$ $\qquad i \neq j$

D dilatation : $\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = div\ \vec{d}$

1D propagation

$$\vec{d} = (u, 0, 0) \qquad \frac{\partial \cdot}{\partial y} = 0 \qquad \frac{\partial \cdot}{\partial z} = 0$$





$$\Rightarrow \quad \delta F_x = \frac{\partial \sigma_{xx}}{\partial x} = (\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2}$$

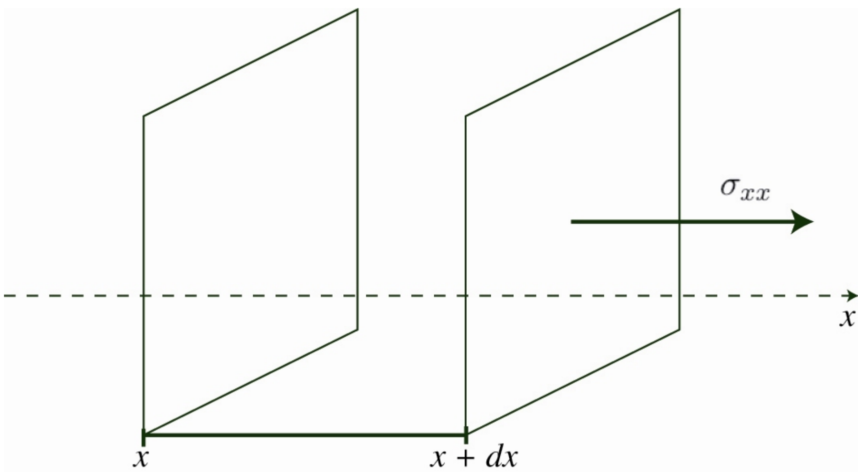
Newton' s law:

$$F_x \; = \; m \, \gamma$$

$$\delta F_x \; dx \; dy \; dz \; = \; \rho \; dx \; dy \; dz \; \frac{\partial^2 u}{\partial t^2}$$

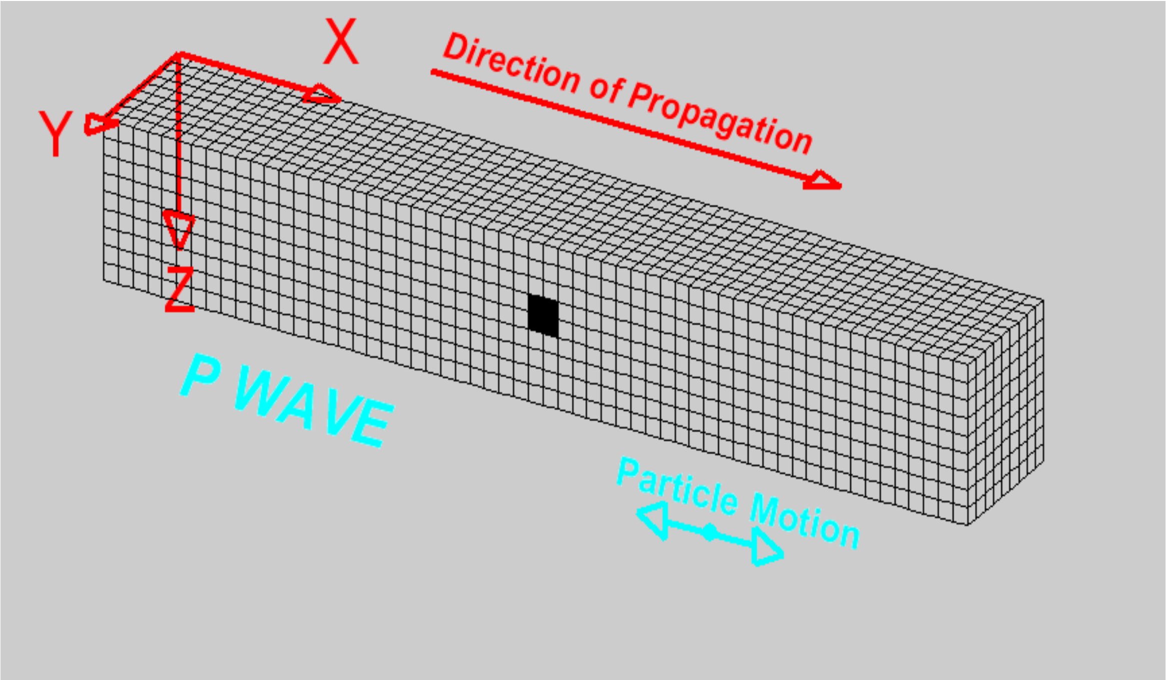
$$\boxed{\Rightarrow \quad \frac{\partial^2 u}{\partial x^2} = \frac{\rho}{(\lambda + 2\mu)} \; \frac{\partial^2 u}{\partial t^2}}$$

Wave equation with velocity $\boxed{\mathrm{C} \; = \; \sqrt{\frac{\lambda+2\mu}{\rho}}}$



Equation of P waves

P waves



1D propagation

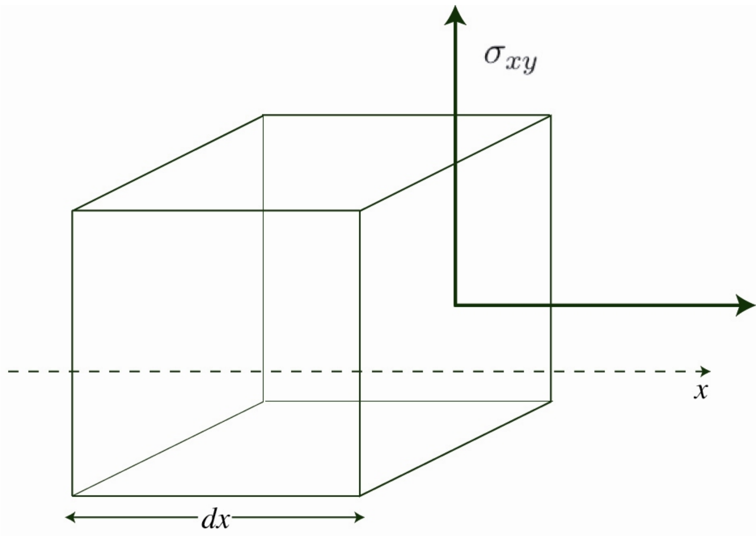
$$\vec{d} = (0, v, 0)$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)$$

dilatation $\quad D = 0 \qquad (div = 0)$

$$\sigma_{xy} = \mu \frac{\partial v}{\partial x}$$

$$dx\ dy\ dz\ \delta F_y = dy\ dz\ (\delta\sigma_{xy}) = dy\ dz\ \frac{\partial \sigma_{xy}}{\partial x}\ dx$$



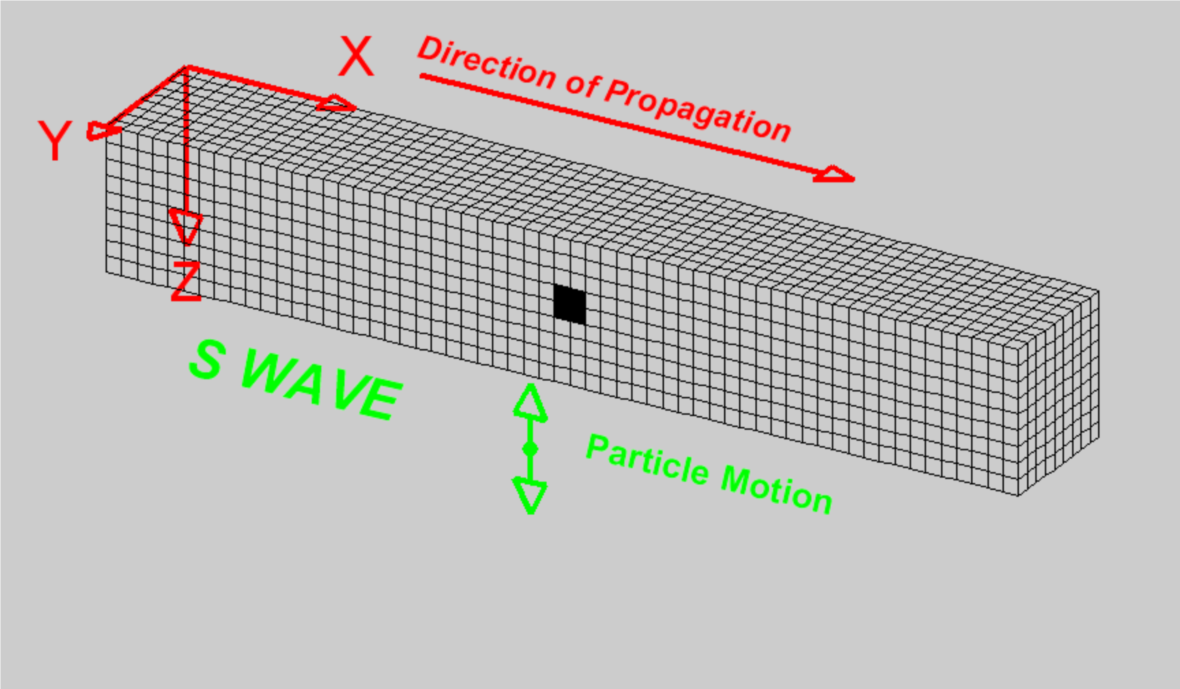
$$\Rightarrow\ \delta F_y = \frac{\partial \sigma_{xy}}{\partial x} = \mu\ \frac{\partial^2 v}{\partial x^2}$$

$$\Rightarrow\ \rho \frac{\partial^2 v}{\partial t^2} = \mu \frac{\partial^2 v}{\partial x^2}$$

$$\frac{\partial^2 v}{\partial x^2} = \frac{1}{w^2}\frac{\partial^2 v}{\partial t^2} \qquad \left(with\ \ w = \sqrt{\frac{\mu}{\rho}}\right)$$

Equation of S waves

S Wave

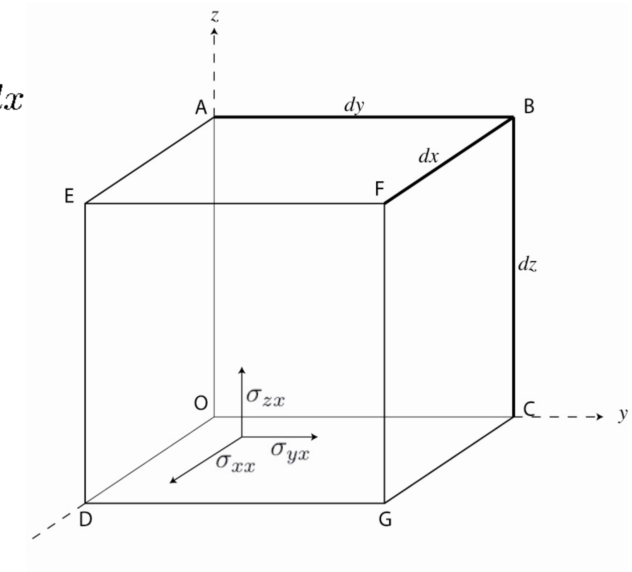


3D elastodynamics

face OABC : F_{OABC} $\sigma_{xx}, \sigma_{yx}, \sigma_{zx}$

$\rightarrow$ Equilibrium $\rightarrow$ $F_{OABC} = -F_{DEFG}$
$\rightarrow$ Disequilibrium $\sigma_{xx}(x,y,z)$; $\sigma_{yx}(x,y,z)$ . . $\sigma_{zx}(x,y,z)$

Sur OABC : $-\sigma_{xx}, -\sigma_{yx}, -\sigma_{zx}$
Sur DEFG : $\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\, dx$; $\sigma_{yx} + \frac{\partial \sigma_{yx}}{\partial x}\, dx$; $\sigma_{zx} + \frac{\partial \sigma_{zx}}{\partial x}\, dx$



surface $dy\, dz$; volume $dx\, dy\, dz$. Resulting forces per volume unit :

$\frac{\partial \sigma_{xx}}{\partial x}$ in the direction x
$\frac{\partial \sigma_{yx}}{\partial x}$ in the direction y
$\frac{\partial \sigma_{zx}}{\partial x}$ in the direction z

$\rightarrow$ similar analysis for the other faces.

Total resulting force in the direction x :

$$Fx = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z}$$

Newton's law :

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z}$$

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z}$$

$$\sigma_{xx} = \lambda D + 2\mu\, \varepsilon_{xx} = \lambda\ div\vec{d} + 2\mu\ (\frac{\partial u}{\partial x})$$

$$\sigma_{yx} = 2\mu \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)$$

$$\sigma_{zx} = 2\mu \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right)$$

$$\Rightarrow\ \rho \frac{\partial^2 u}{\partial t^2} = \lambda \frac{\partial D}{\partial x} + 2\mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2} + \mu \frac{\partial^2 v}{\partial x \partial y} + \mu \frac{\partial^2 w}{\partial x \partial z} + \mu \frac{\partial^2 u}{\partial z^2}$$

$$= \lambda \frac{\partial D}{\partial x} + \mu \left(\frac{\partial}{\partial x} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \right) + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

$$= (\lambda + \mu) \frac{\partial D}{\partial x} + \mu\ \Delta u$$

Identically

$$\rho \frac{\partial^2 v}{\partial t^2} = (\lambda + \mu) \frac{\partial D}{\partial y} + \mu\ \Delta v$$

$$\rho \frac{\partial^2 w}{\partial t^2} = (\lambda + \mu) \frac{\partial D}{\partial z} + \mu\ \Delta w$$

$$\Rightarrow \boxed{\rho \frac{\partial^2 \vec{d}}{\partial t^2} = (\lambda + \mu)\ gr\vec{a}d\ (div\ \vec{d}) + \mu\ \vec{\Delta} \vec{d}}$$

Not really a simple wave equation…….

Helmholtz decomposition

$$\vec{\Delta}\,\vec{V} \;=\; \vec{grad}\;div\vec{V} \;-\; \vec{rot}\;\vec{rot}\;\vec{V} \qquad \forall\,\vec{V}\text{ dérivable}$$

Define $\vec{d} = \vec{\Delta}\,\vec{V}$ $\qquad\qquad$ $\vec{V}$ unknown ; Laplace equation

$$\Rightarrow\; \vec{d} \;=\; \vec{grad}\,\Phi \;+\; \vec{rot}\;\vec{\Psi} \qquad \Phi = div\vec{V}\;;\;\; \vec{\Psi} = -\vec{rot}\;\vec{V}$$

Every field can be decomposed into two parts $\rightarrow$ newtonian
$\rightarrow$ laplacian (or rotationnal)
(Φ no rotation ; Ψ no dilatation)

$\Rightarrow$ 2 wave equations: (with null boundary conditions)

$$\rho\frac{\partial^2\Phi}{\partial t^2} = (\lambda + 2\mu)\;\Delta\;\Phi \quad\Longrightarrow\quad V_P = \sqrt{\frac{\lambda+2\mu}{\rho}} \quad \text{without shear}$$

$$\rho\frac{\partial^2\Psi}{\partial t^2} = \mu\;\vec{\Delta}\;\vec{\Psi} \quad\Longrightarrow\quad V_S = \sqrt{\frac{\mu}{\rho}} \quad \text{without dilatation}$$

For consolidated rocks: $\lambda \approx \mu \rightarrow V_P \approx \sqrt{3}v_S$

Typical velocities

Material	P wave Velocity (m/s)	S wave Velocity (m/s)
Air	332	
Water	1400-1500	
Petroleum	1300-1400	
Steel	6100	3500
Concrete	3600	2000
Granite	5500-5900	2800-3000
Basalt	6400	3200
Sandstone (grès)	1400-4300	700-2800
Limestone (calcaire)	5900-6100	2800-3000
Sand (Unsaturated)	200-1000	80-400
Sand (Saturated)	800-2200	320-880
Clay	1000-2500	400-1000
Glacial Till (Saturated)	1500-2500	600-1000

Typical values for ‘seismic hazard’ problems

Surface hard rock (no weathering)
Vp=5000m/s   Vs=2800m/s

Crust
Vp=6300m/s   Vs=3500m/s

Mantle
Vp=8100m/s Vs=4700m/s

Sediments
Vp=1800m/s   Vs=300m/s

Clay of Mexico
Vp=1500m/s   Vs=40m/s !!

Useful solutions

A point source in an homogeneous space: spherical wave

Wave equation in spherical coordinates:

$$\frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial f}{\partial r})$$

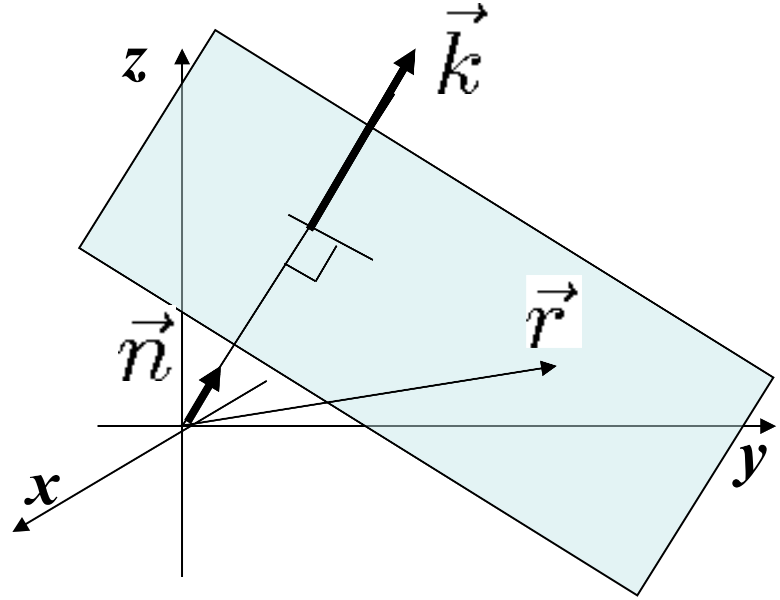
Solutions :

$$F = \frac{1}{r}f(r - vt) + \frac{1}{r}f(r + vt)$$

$\frac{1}{r}$: geometrical expansion (decay).

When $r \to \infty$, the wavefront tends to a plane.

Useful solutions....



Plane waves

Cartesian coordinates:
$\frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = \Delta f$

Plane wave propagating in the direction of unitary vector $\vec{n}$:

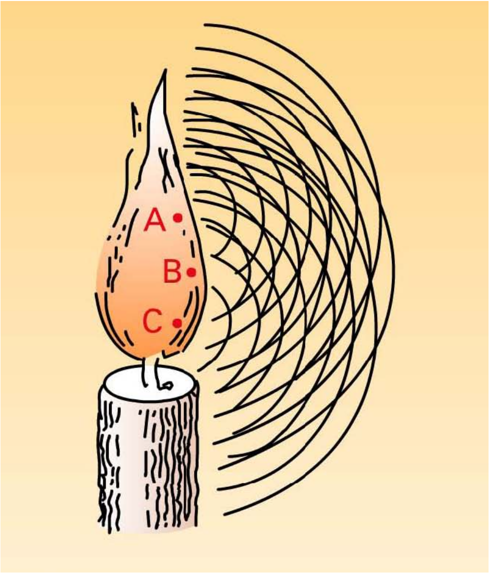
$$F = f(\vec{n}\vec{r} - vt) + g(\vec{n}\vec{r} + vt)$$

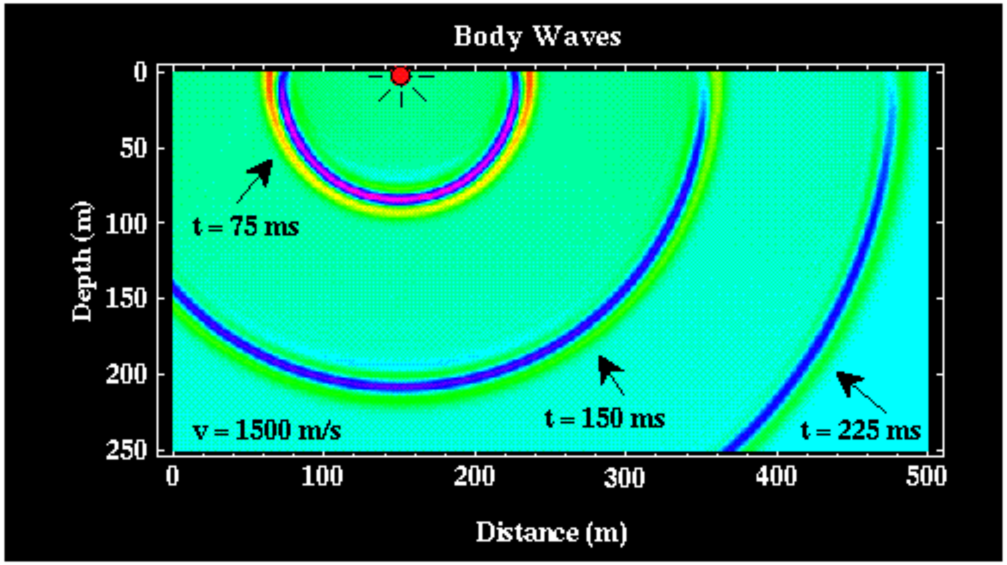
Equiphase surface is a plane defined by [$\vec{n}\vec{r} - vt =$constant]

With harmonic wave of angular frequency $\omega = 2\pi f$

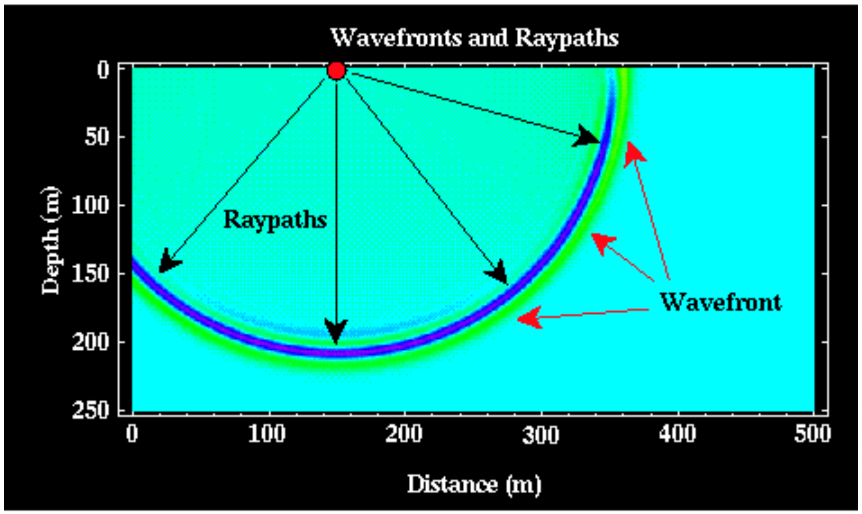
$F = f_0 exp(i(\vec{k}\vec{r} - \omega t)$ avec $\vec{k} = \vec{n}\frac{\omega}{v}$ wave vector.

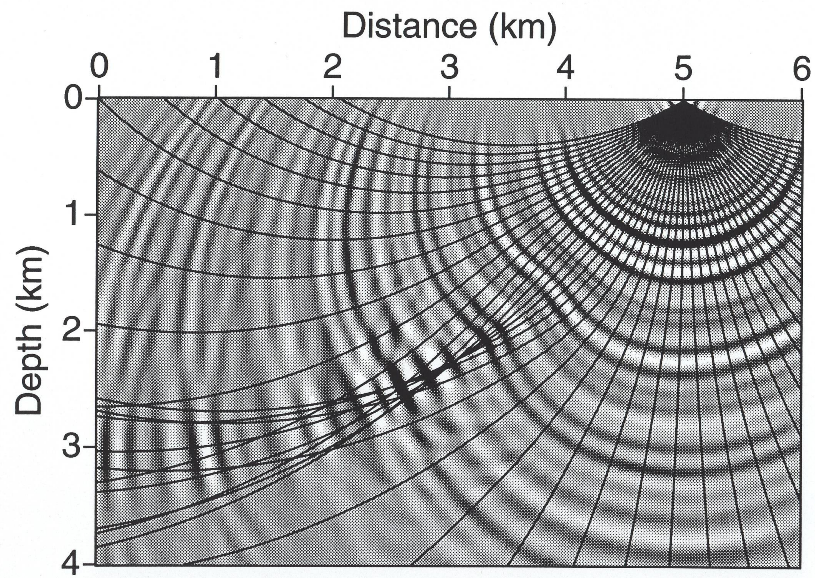
Wave propagation, Huyghens principle





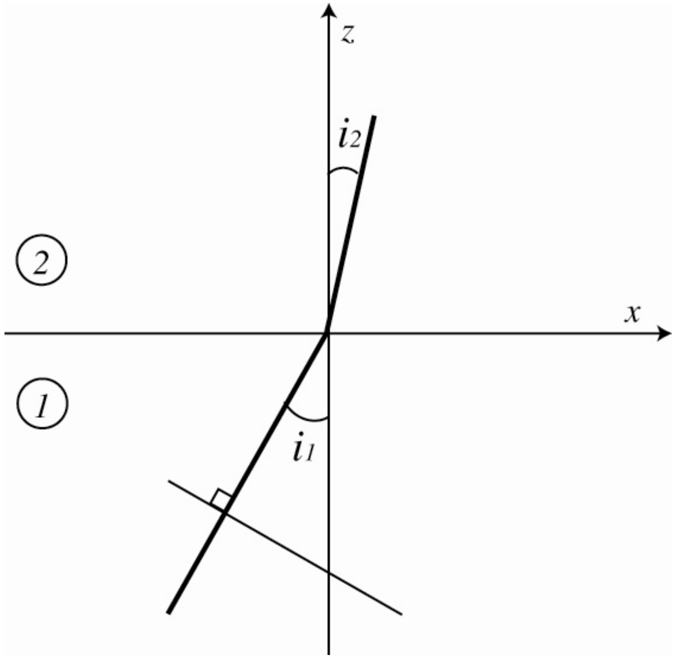
Wavefronts and seismic rays





- Rays are perpendicular to wave fronts in isotropic mediua (high frequency approximation).

Plane wave SH 2D :



$$\vec{u} = (u_1, u_2, u_3) \qquad \text{and } u_2 = v$$

Incident wave

$$v_i = v_0 \exp\, i(\omega t - k_x x - k_z z)$$

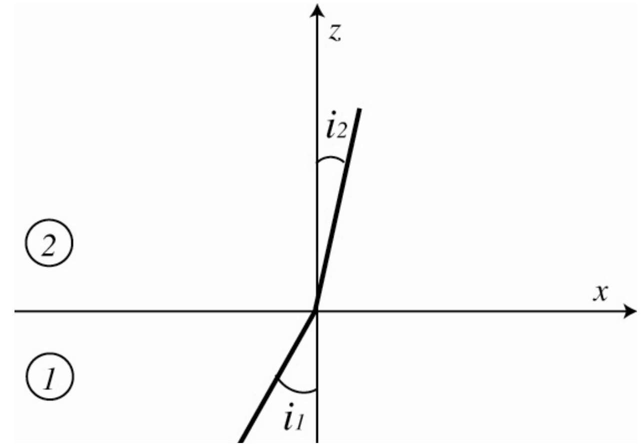
Resulting waves?

reflected : $\quad v_R = v_0\, R \exp i(\omega t - k_x^{'} x + k_z^{'} z)$

Reflection coefficient

transmitted : $\quad v_T = v_0\, T \exp i(\omega t - k_x^{''} x - k_z^{''} z)$

Transmission coefficient



$\Rightarrow$ remark : value of k?

$$\frac{\omega^2}{c^2} = k_x^2 + k_z^2$$

(just wave equation)

geometrical interpretation:

$$k_x = \frac{\omega}{v_1} \sin i_1 \qquad ; k_z = \frac{\omega}{v_1} \cos i_1$$

$\Rightarrow$ Boundary conditions :

for $z = 0 \quad ; \quad \forall x$ $\qquad v_i + v_r = v_t$

$$\tau_i + \tau_r = \tau_t$$

Displacements:

$$\exp(-ik_x x) + R\exp(-ik_x^{'} x) = T\exp(-ik_x^{''} x) \qquad \forall x \Rightarrow k_x = k_x^{'} = k_x^{''}$$

$$k_x = \frac{\omega}{v_1} \sin i_1 \qquad ; \qquad k_x^{''} = \frac{\omega}{v_2} \sin i_2$$

$$\Rightarrow \frac{\sin i_1}{v_1} = \frac{\sin i_2}{v_2}$$

$$1 + R = T$$

Stresses:

$$\tau \;=\; \mu\,\frac{\partial v}{\partial z}$$

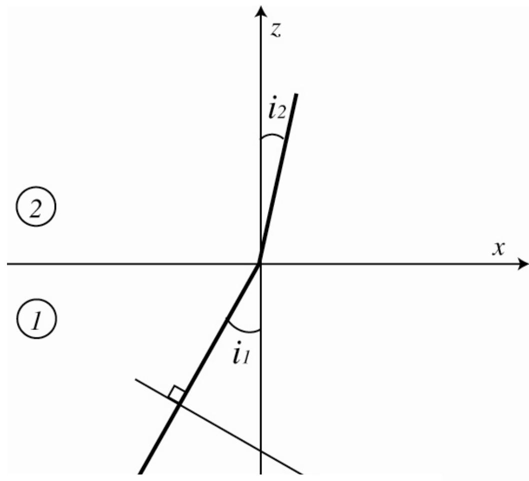
$$\mu_1\,(-ik_z\;+iRk_z)\;=\;\mu_2(-ik_z^{''}T)$$

$$1\;-\;R\;=\;\frac{\mu_2\,k_z^{''}}{\mu_1\,k_z}\,T\;=\;\frac{\mu_2\;\cos i_2\;\frac{\omega}{V_2}}{\mu_1\;\cos i_1\;\frac{\omega}{V_1}}\,T\;=\;\alpha\,T$$

Displacements et stresses:

$$2\;=\;T\,(1\;+\;\alpha)\qquad\Rightarrow\qquad T\;=\;\frac{2}{1\;+\;\alpha}$$

$$2\,R\;=\;T\,(1\;-\;\alpha)\;=\;2\;\frac{1\;-\alpha}{1\;+\alpha}\qquad\Rightarrow\qquad R\;=\;\frac{1\;-\alpha}{1\;+\alpha}$$



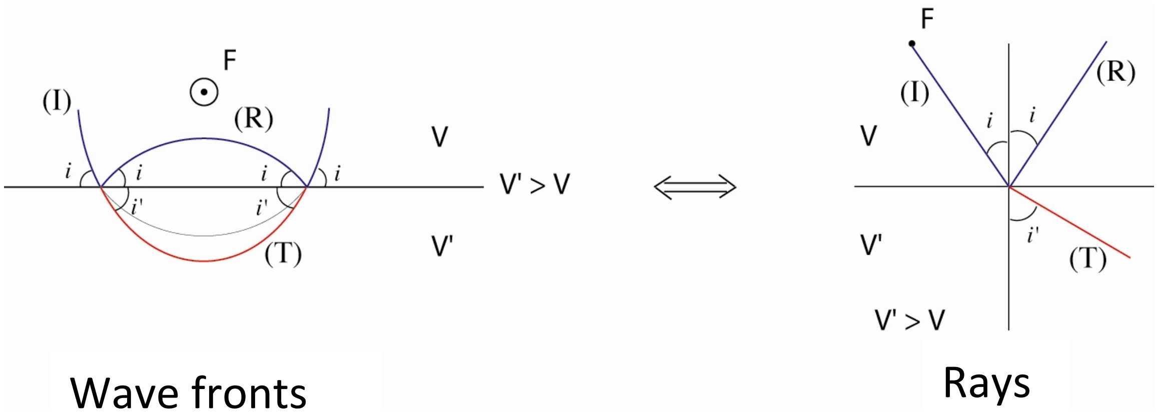
Critical Reflection

$$\frac{\sin i_1}{V_1} = \frac{\sin i_2}{V_2}$$

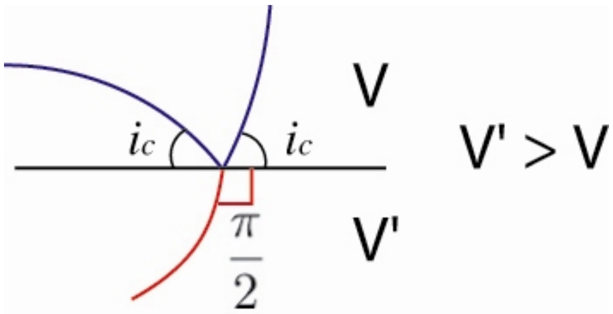
A problem for : $V_1 < V_2$ critical angle i_c ; $\sin i_c = \frac{V_1}{V_2}$

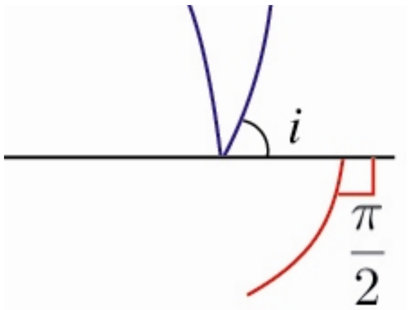
For $i_1 > i_c$ no simple geometrical interpretation($\sin i_2 > 1$!)

Wave fronts at an interface and conical (head) waves :

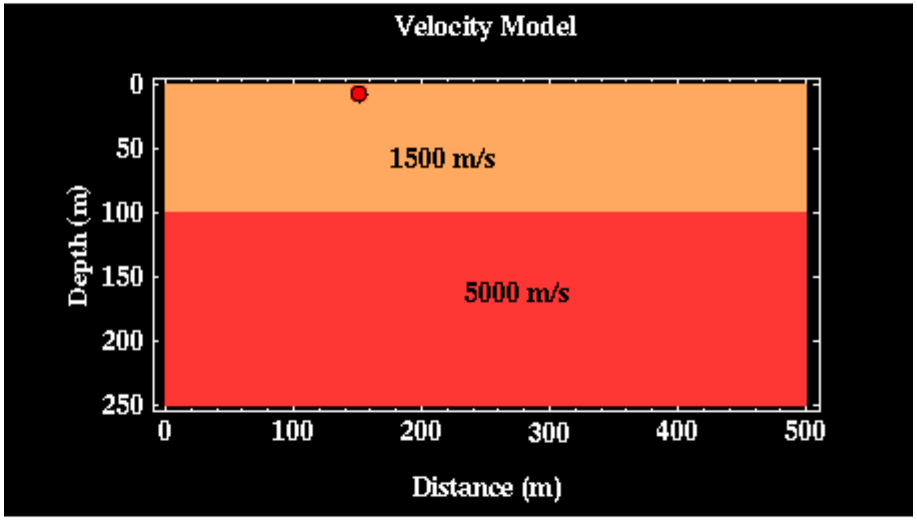


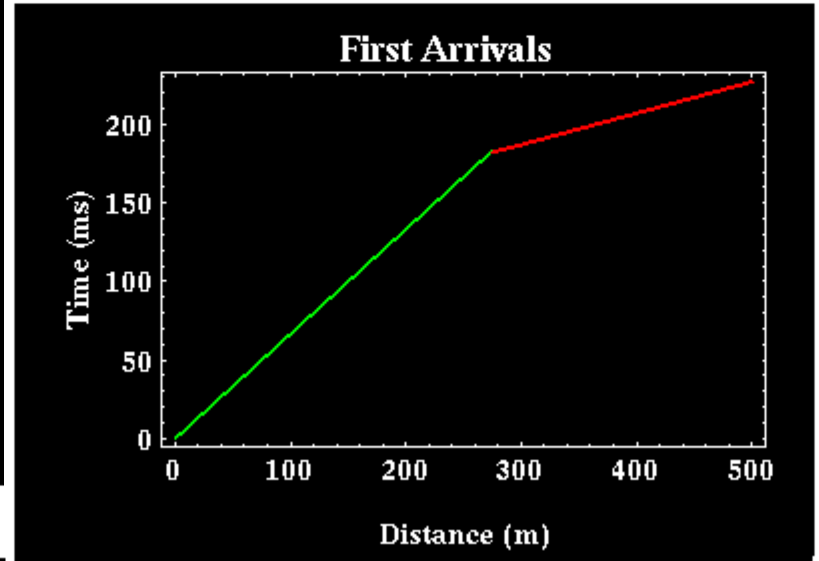
Critical Angle $\rightarrow$ separation of the front

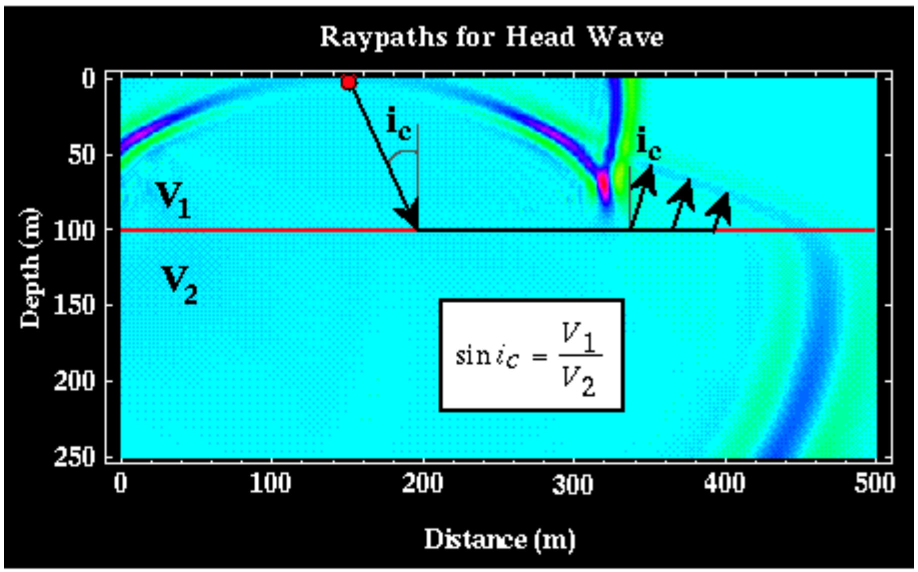


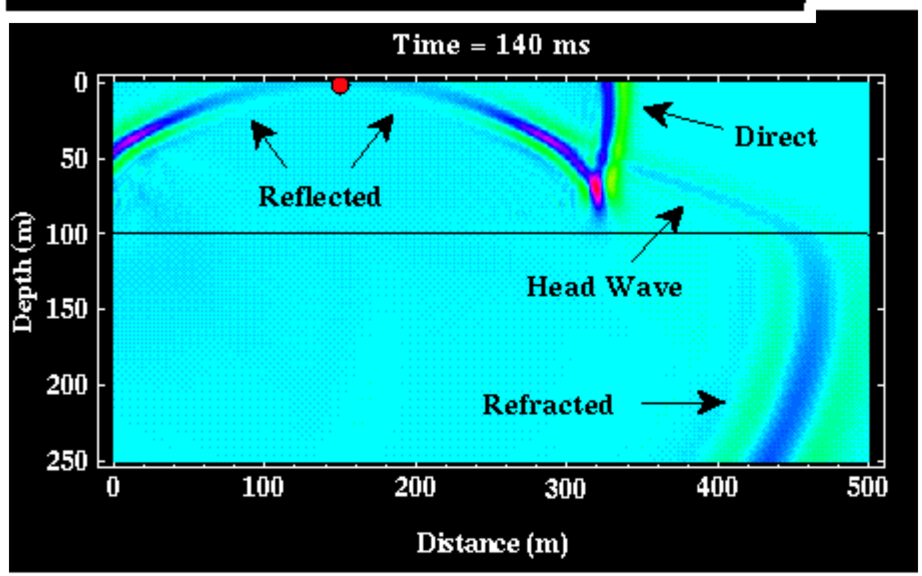


Couche de surface à vitesse faible

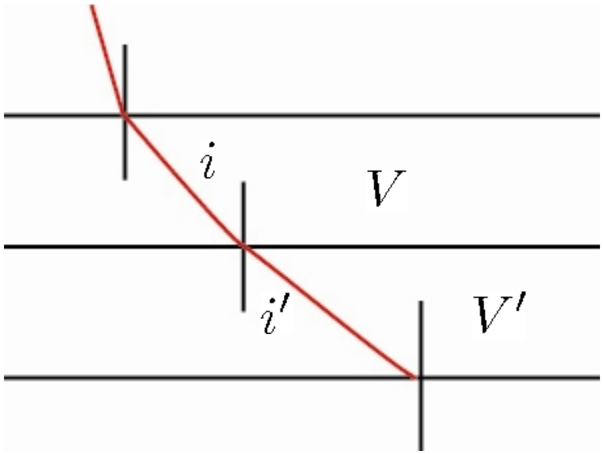








Rays in vertically varying media

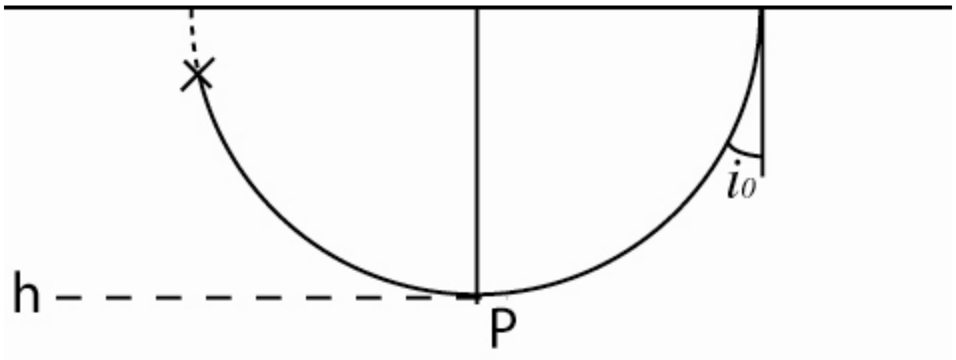


Velocity continuously varying with depth
Snell-Descartes $\rightarrow$ the ray is progressively refracted.
For a given ray :

$$\frac{\sin i}{V} = \frac{\sin i'}{V'} = C^{ste} = p \geq 0$$

$p = \frac{\sin i(z)}{V(z)} =$ ray parameter. It is a constant along a ray.

$\frac{1}{V} = n$, slowness.

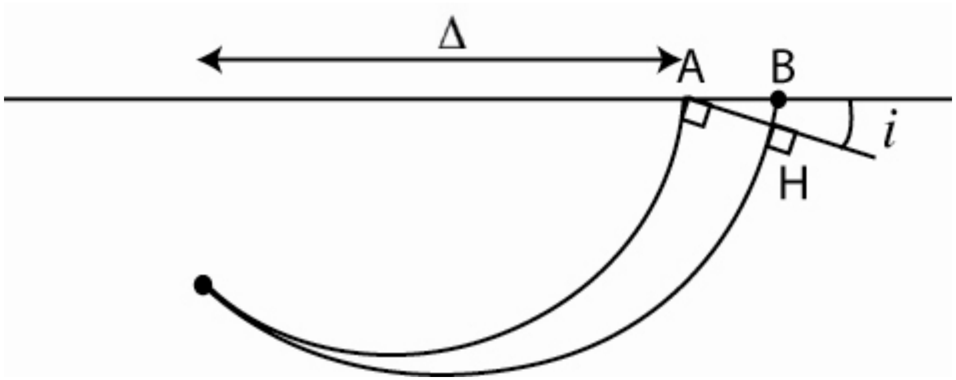


At P, bottoming point, z = h.

$$V = V(h) \qquad i = \frac{\pi}{2} \;\Rightarrow\; \sin i = 1 \qquad and\ p = \frac{1}{V(h)}$$

The ray parameter is equal to the inverse of the velocity at the bottoming point.

Evaluation of ray parameter



2 close rays:
i : incidence angle, Δ epicentral distance, AB = δx ; HB = V δt

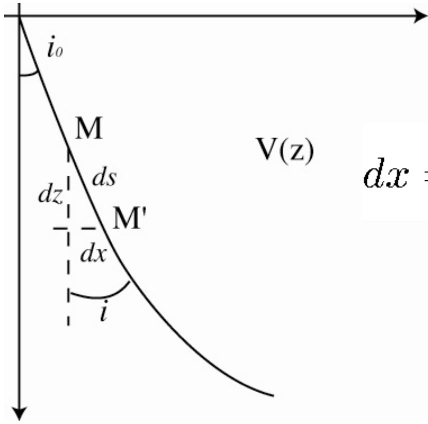
$$\sin i = \frac{HB}{AB} = V\frac{\delta t}{\delta x} \qquad \Longrightarrow \qquad p = \frac{\sin i}{V} = \frac{\delta t}{\delta x}$$

$$p = \frac{dt}{d\Delta}$$

The ray parameter is the slope of the traveltime curve t = t(Δ).

$1/p = d\Delta/dt = V_a$: apparent velocity.

Distance of emergence travel time



$$dx = dz \ \tan i \qquad and \qquad ds = \frac{dz}{\cos i}$$

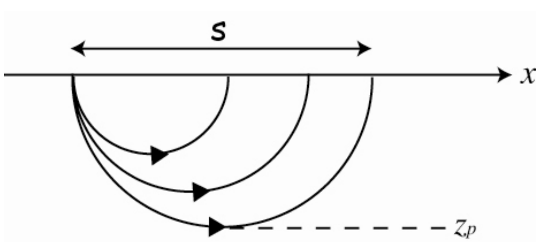
Case of source and receiver at the surface

$$p = \frac{\sin i}{V} \quad \Longrightarrow \quad \tan i = \frac{pV}{\sqrt{1 - p^2 V^2}}$$

$$\Delta = \int_0^{\Delta} dx = 2 \int_0^{z_p} dz \ \tan i = 2 \int_0^{z} \frac{pV(z)}{\sqrt{1 - p^2 V^2(z)}} dz$$

$$t = \int_0^{\Delta} \frac{ds}{V(z)} = 2 \int_0^{z_p} \frac{dz}{V \cos i} = 2 \int_0^{z_p} \frac{dz}{V(z)\sqrt{1 - p^2 V^2(z)}}$$

Change of the integral over z in integral over V .
To z_p corresponds a velocity $V_p = \frac{1}{p}$.

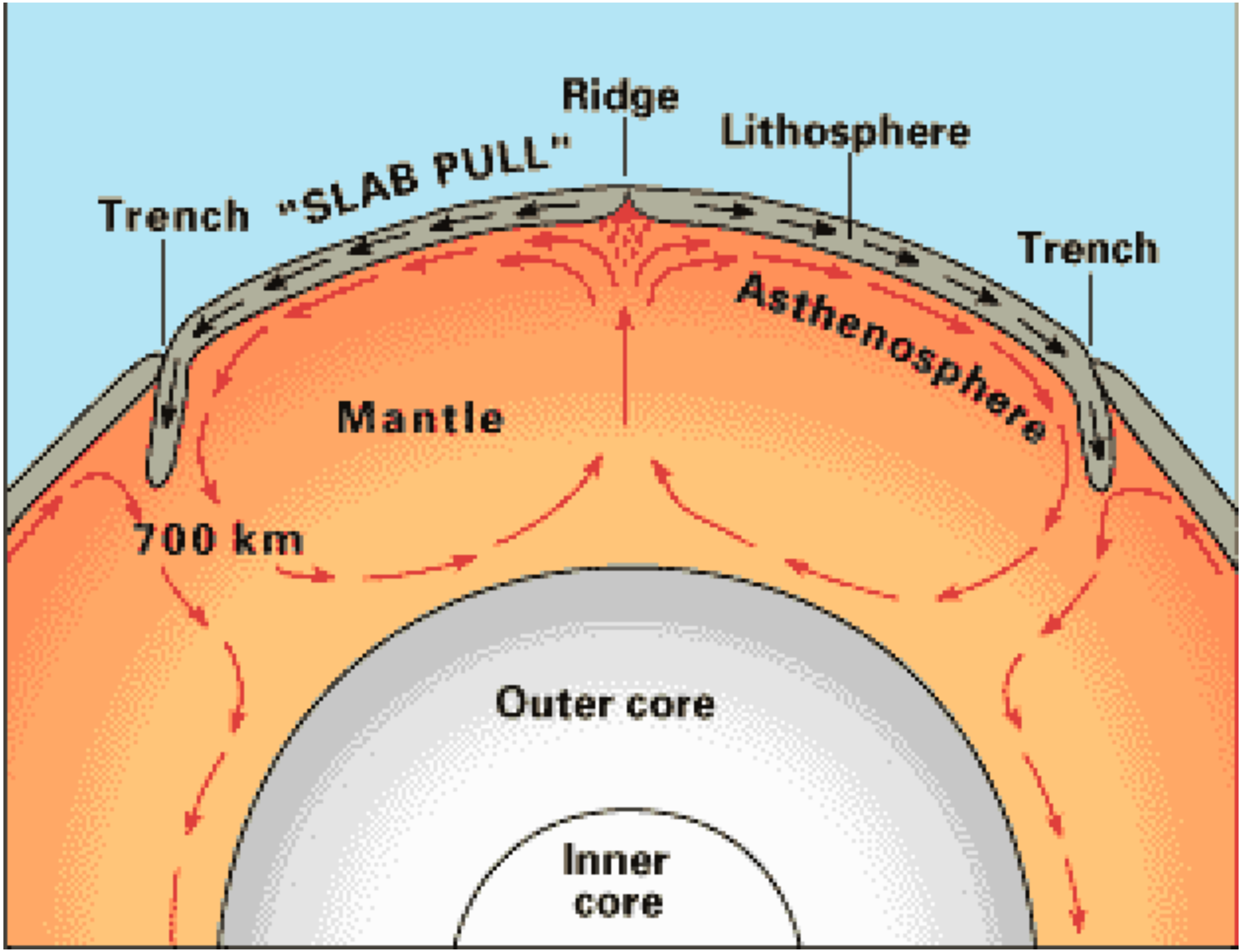


For the case $V = V_0 + kz \quad \Rightarrow \quad dV = k \ dz$

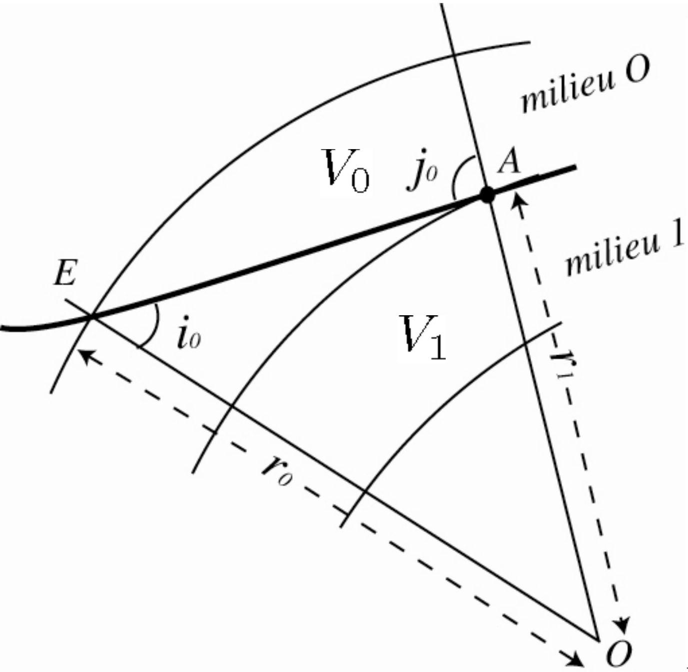
$$\Delta = 2 \int_0^{z_p} \frac{pV}{\sqrt{1 - p^2 V^2}} dz = \frac{2}{k} \int_{V_0}^{\frac{1}{p}} \frac{pV}{\sqrt{1 - p^2 V^2}} dV = \frac{2}{kp} \sqrt{1 - p^2 V_0^2}$$

$$t = 2 \int_0^{z_p} \frac{dz}{V(\sqrt{1 - p^2 V^2})} = \frac{2}{k} \int_{V_0}^{\frac{1}{p}} \frac{dV}{V\sqrt{1 - p^2 V^2}} = \frac{2}{k} \arg ch \frac{1}{pV_0}$$

The rays are circular in this case.



Ridge
Lithosphere
"SLAB PULL"
Trench
Trench
Asthenosphere
Mantle
700 km
Outer core
Inner
core



Radial (spherical) stratification :

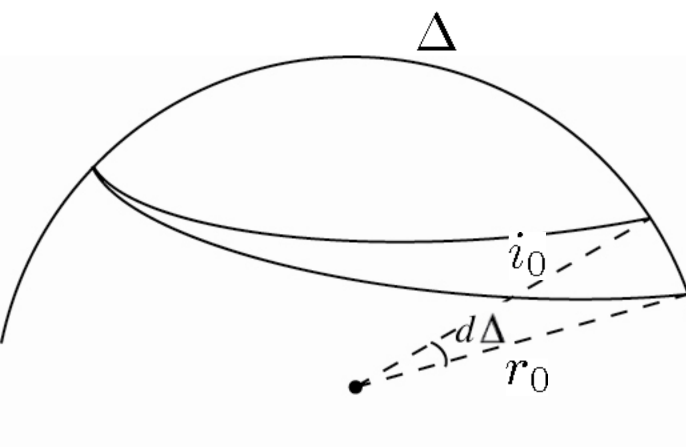
geometry $\Rightarrow \quad r_0 \sin i_0 = r_1 \sin j_0$

at A :

$$\frac{\sin j_0}{V_0} = \frac{\sin i_1}{V_1}$$

$$\Rightarrow \quad \frac{r_0}{r_1} \frac{\sin j_0}{V_0} = \frac{\sin i_1}{V_1}$$

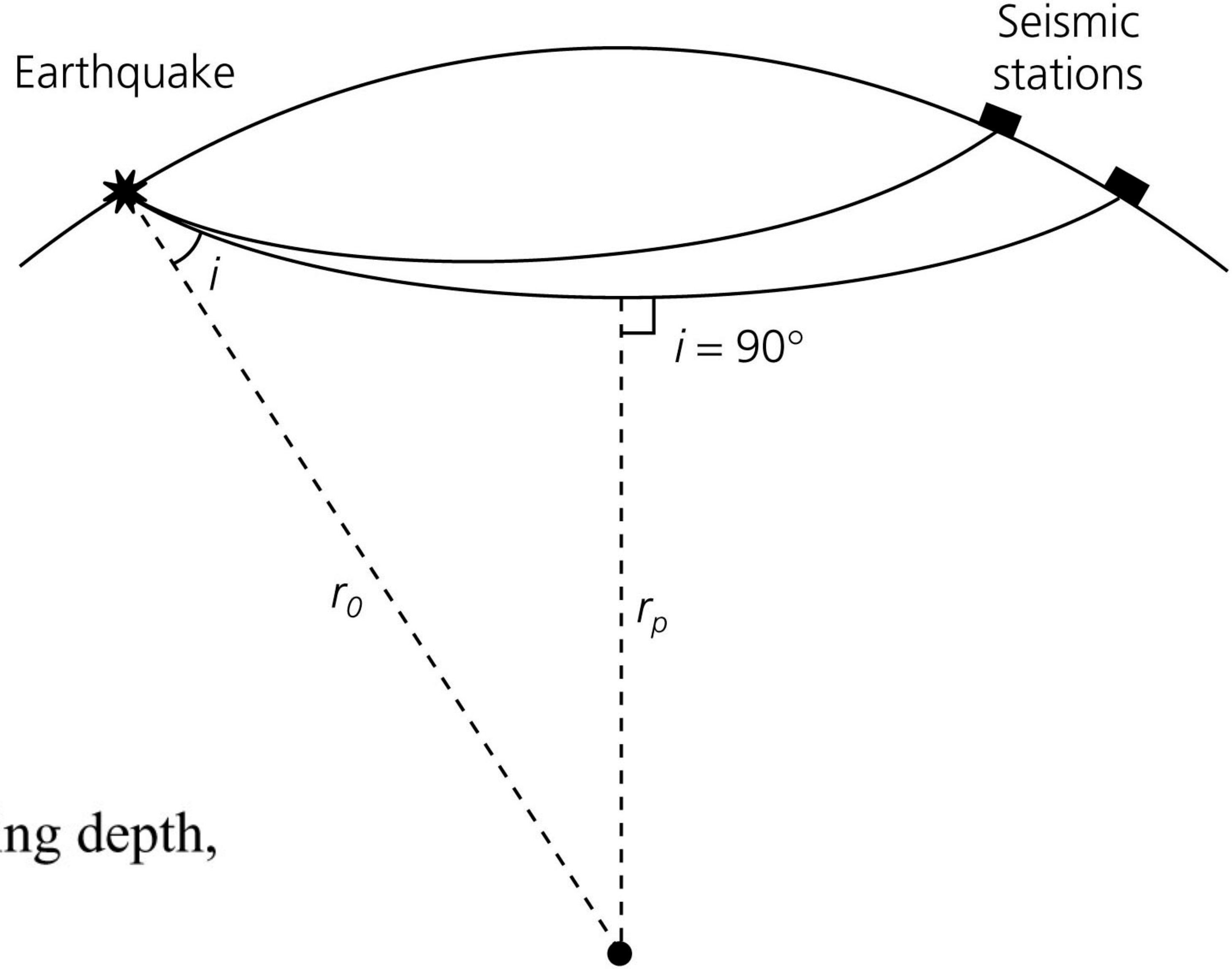
Invariant (ray parameter) $p = \frac{r \sin i(r)}{V(r)}$



$$V_{ap}^{-1} = \frac{dt}{d\Delta} = \frac{r_0 \, d\Delta \, \sin i_0}{V_0} \frac{1}{d\Delta} = \frac{r_0 \sin i_0}{V_0} = p$$

$$p = \frac{dt}{d\Delta} \qquad (s\ /rad)$$

Figure 3.4-2: Geometry of a ray path in a spherical earth.



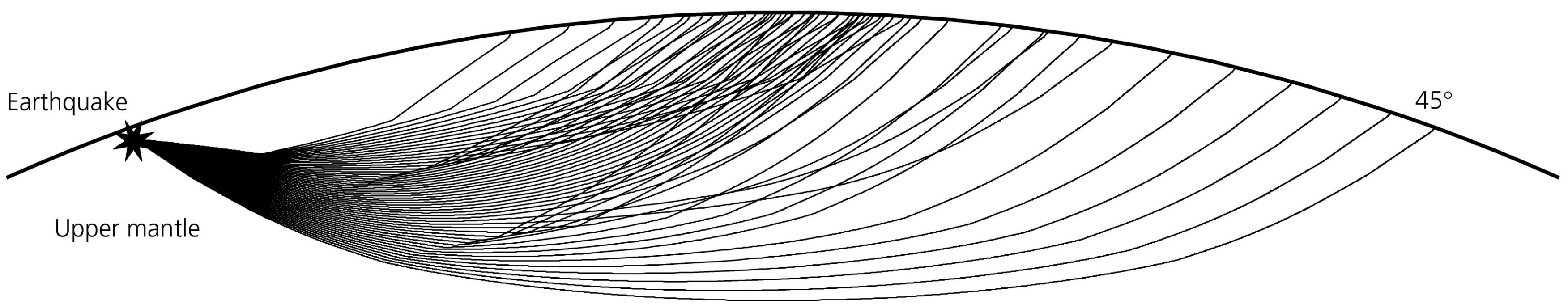
A the surface:

$$p = \frac{r_0 \sin i}{v_0}$$

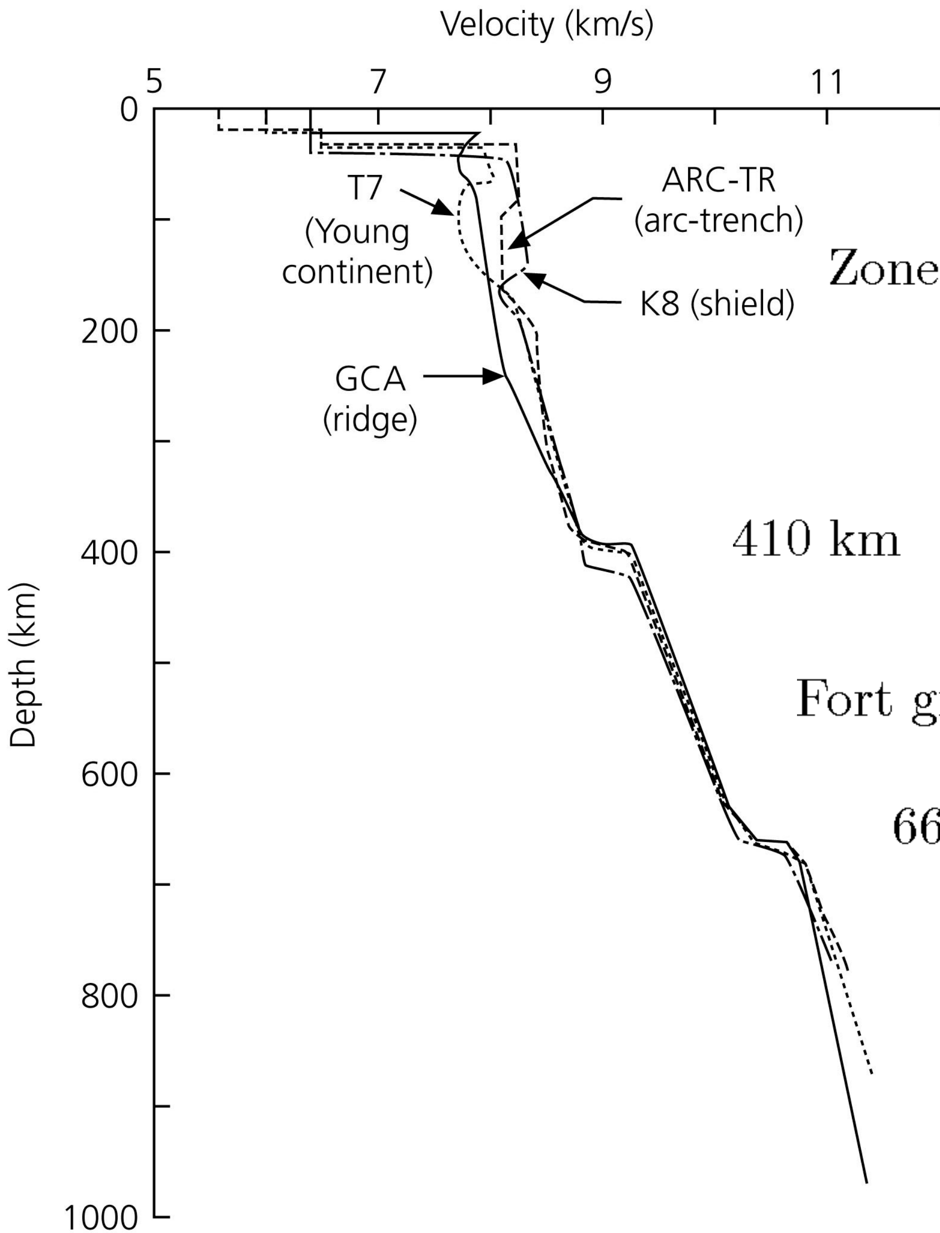
At the bottoming depth, $r = r_p$, and

$$p = \frac{r_p}{v_p}$$

Propagation dans le manteau supérieur



Modèle de vitesse



Zone à moindre vitesse

Manteau supérieur

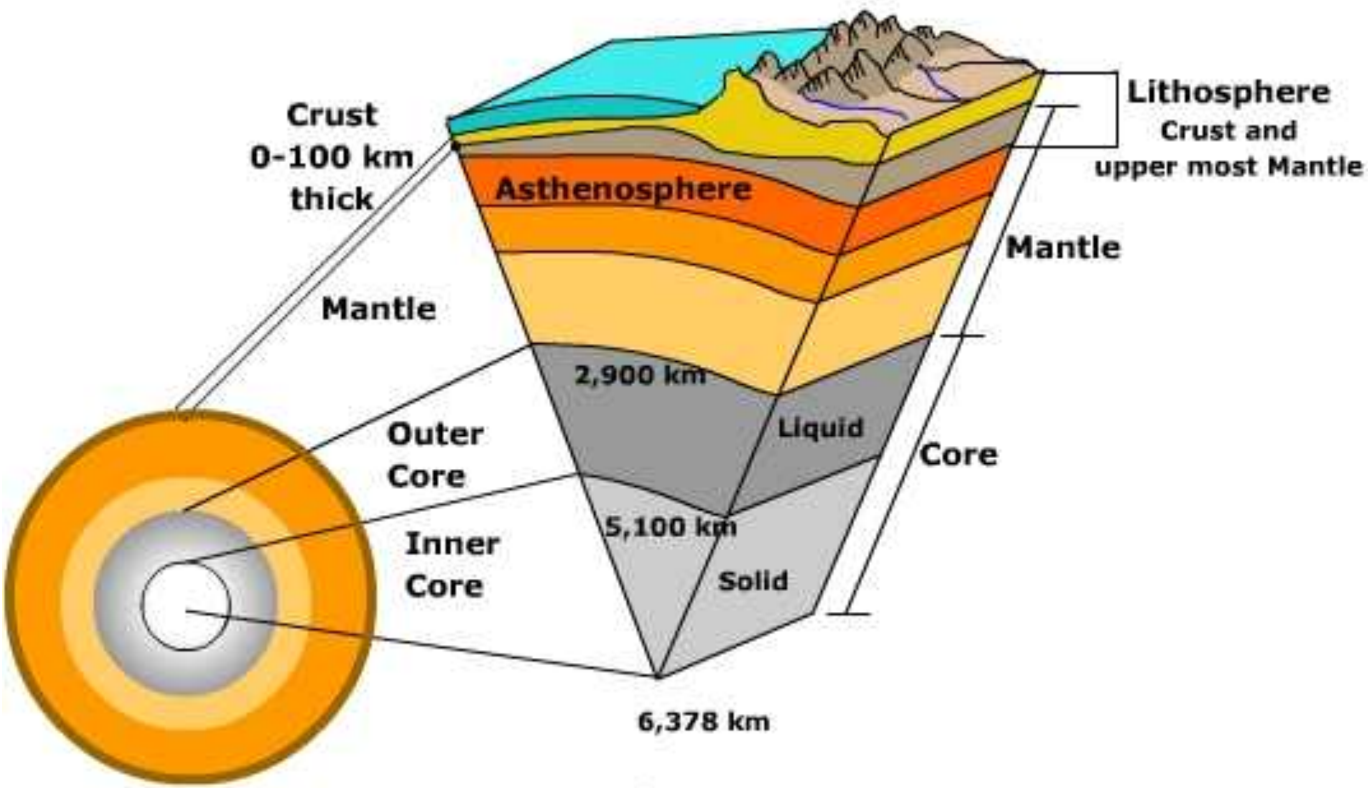
410 km

Zone de transition

Fort gradient

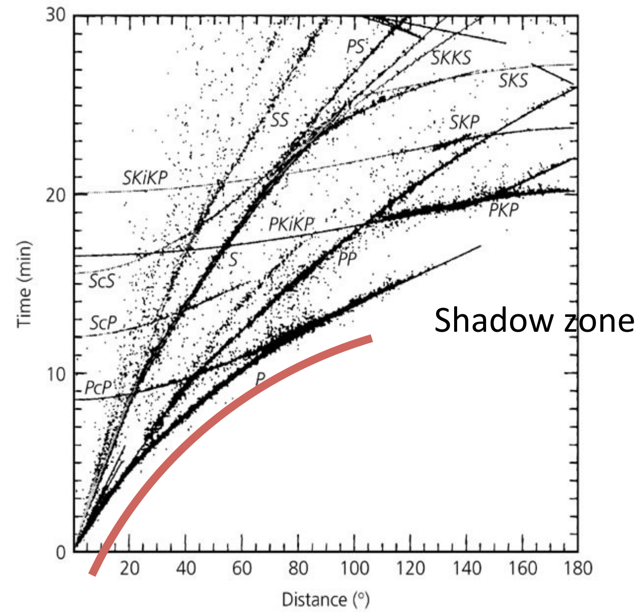
660 km

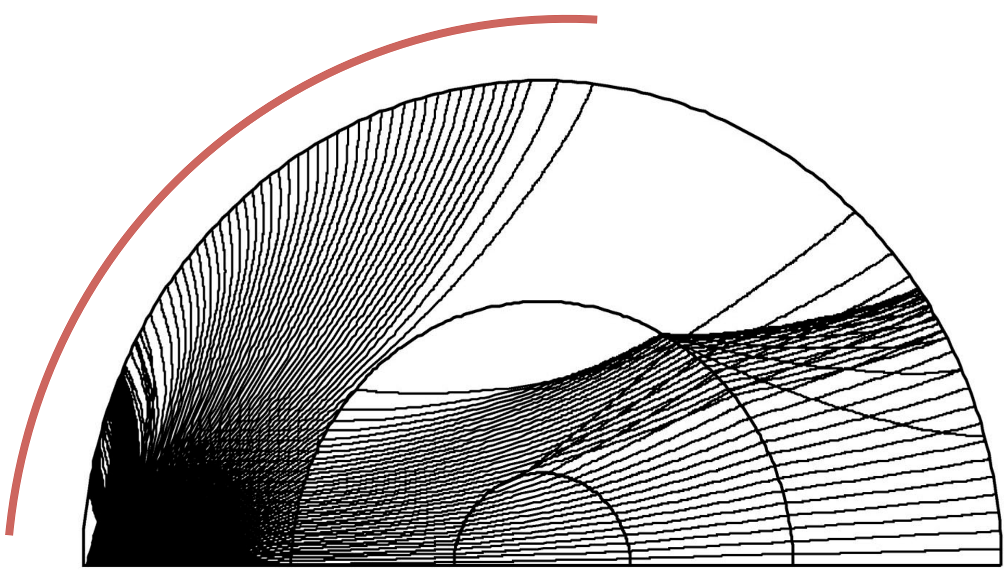
Manteau inférieur



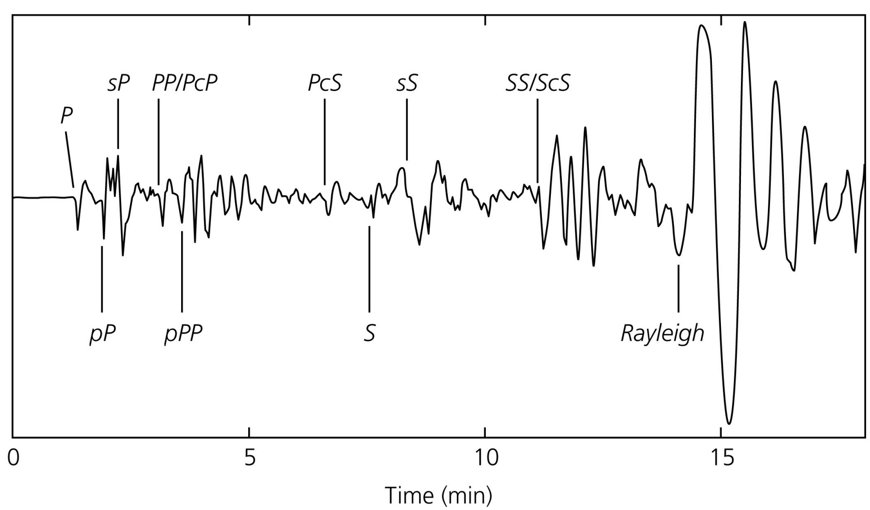
Earth Structure
(Not to Scale)

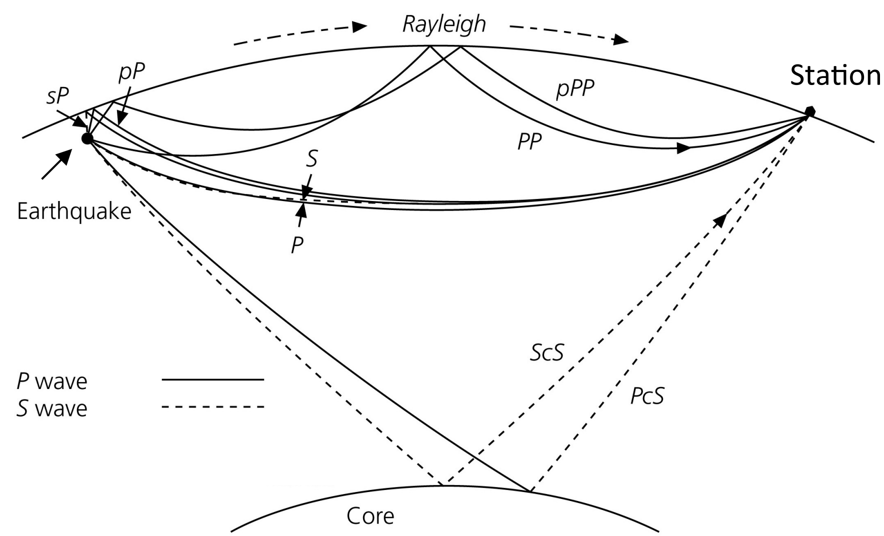
P waves



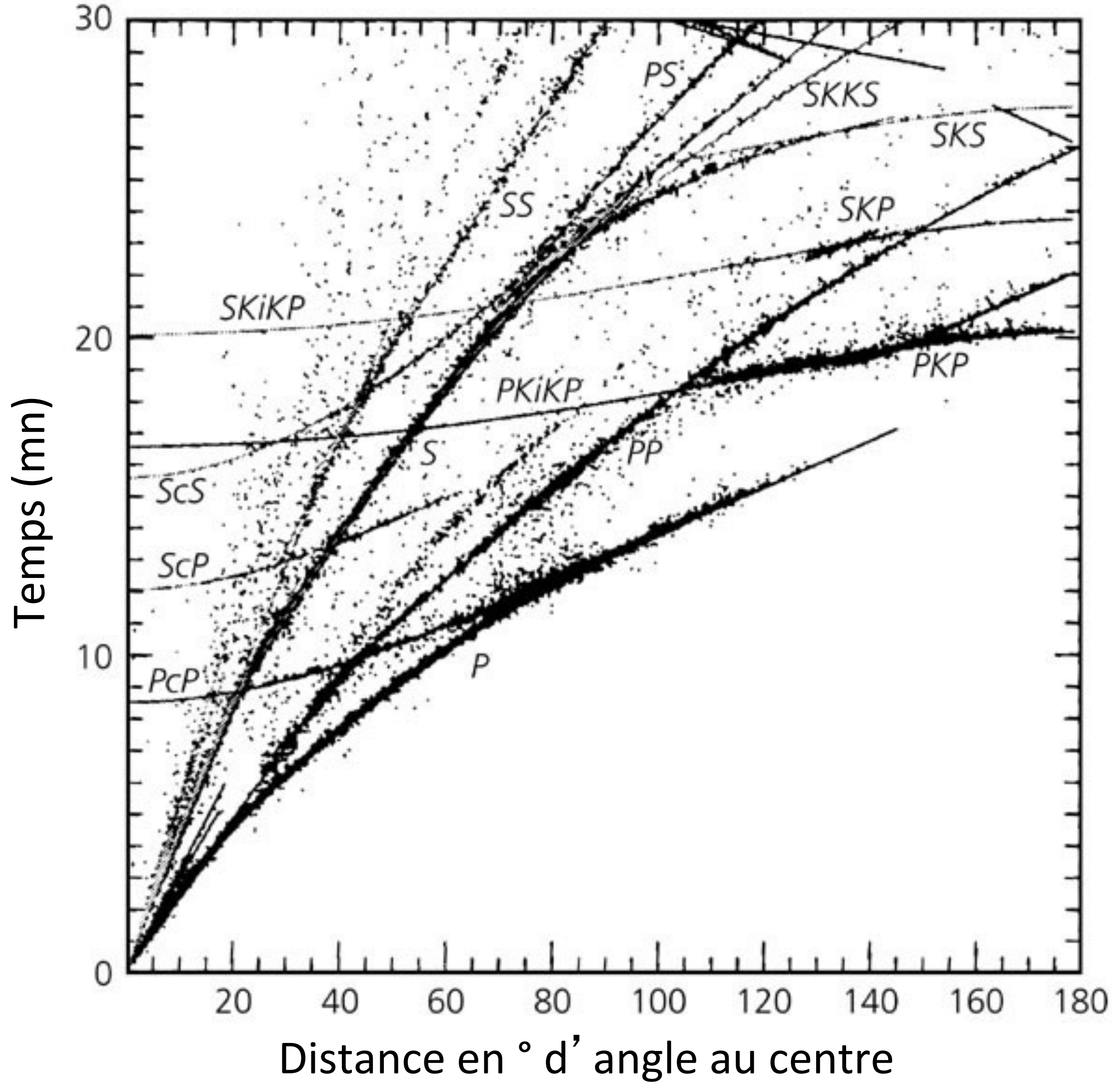


Oldham, 1906: the core!

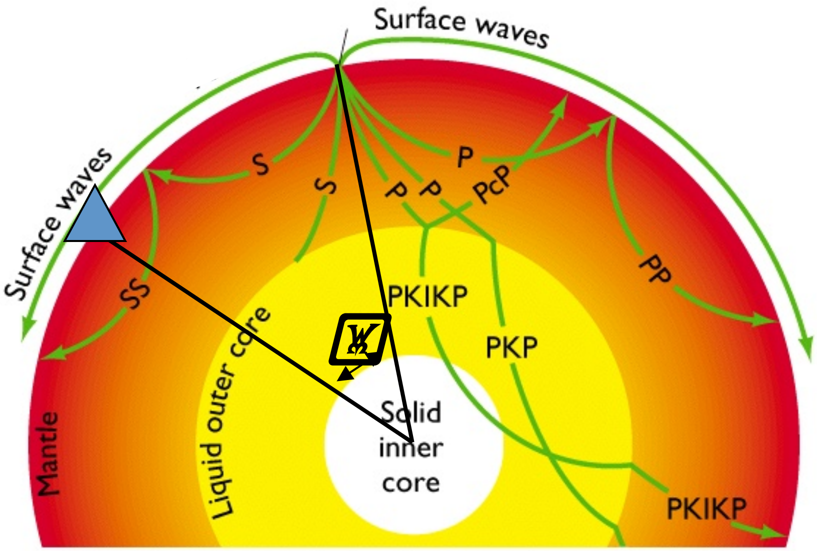




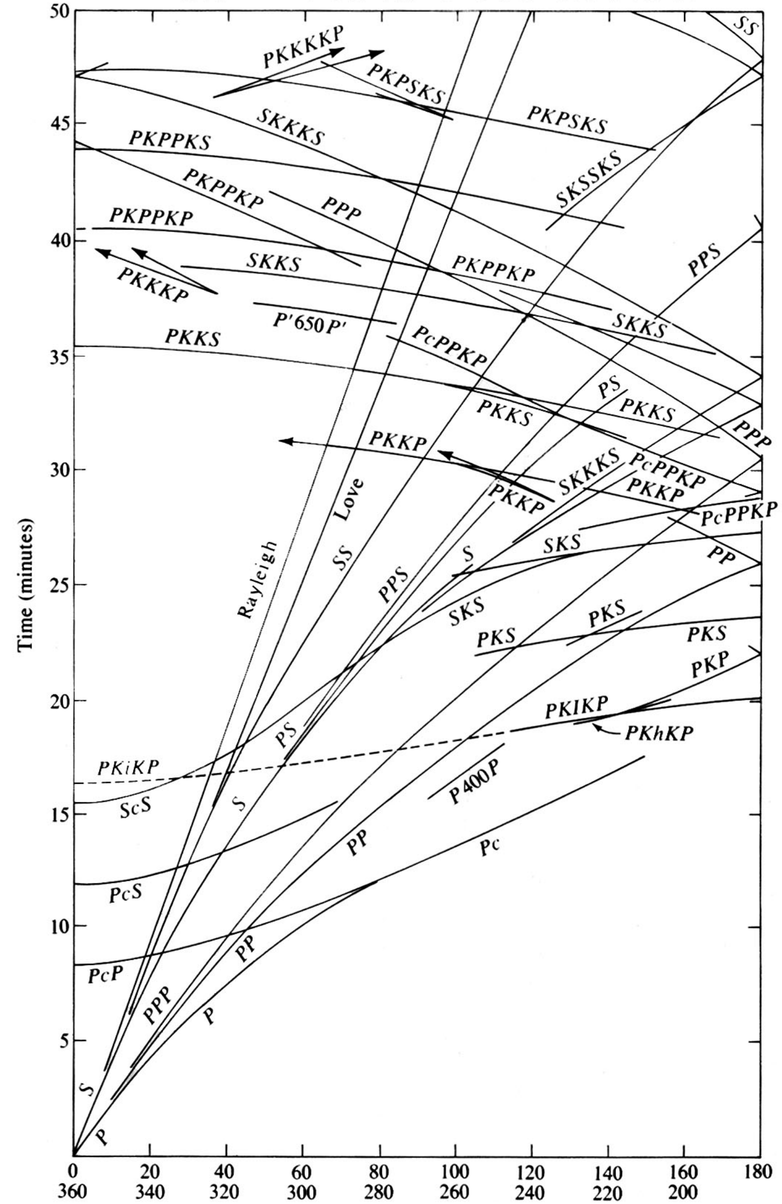
distance=4500km



Temps (mn)
Distance en ° d’angle au centre
0
10
20
30
20
40
60
80
100
120
140
160
180
PS
SKKS
SKS
SS
SKP
SKiKP
PKP
PKiKP
S
PP
ScS
ScP
PcP
P

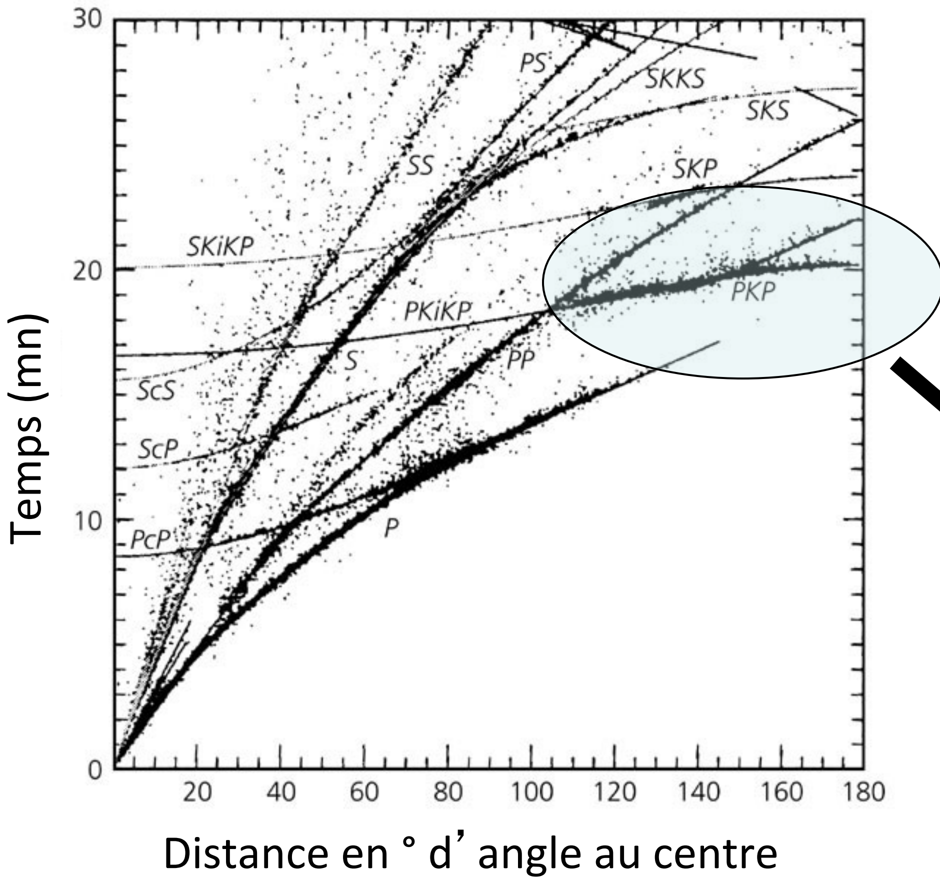


Travel-Time Curves for a Surface Source



Distance angulaire Δ en °

Pointés des temps d'arrivée des ondes de volume: les phases du noyau



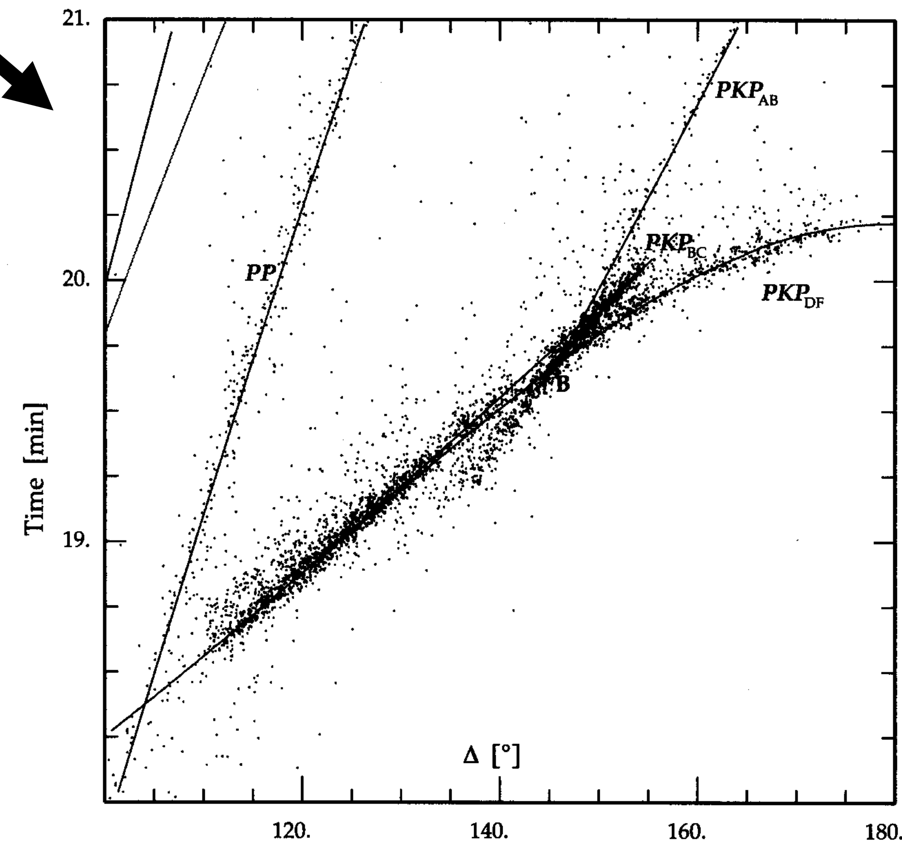


Figure 3.5-7: Ray paths and travel times for major core phases.