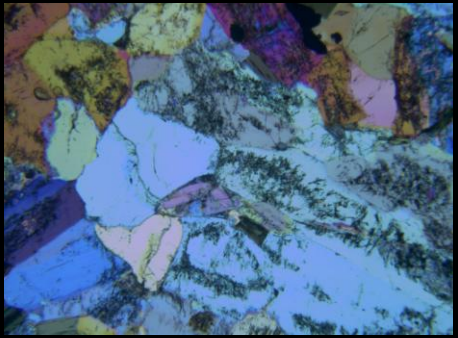
Nonlinear elastic and plastic behavior of dense granular systems

Paul Johnson



Mixed nonlinear dynamics

asssociated information

International Conference on Nonlinear Elasticity in Materials (ICNEM)
Santa Fe, New Mexico USA, July 2-7, 2017
http://icnem.org/index.html

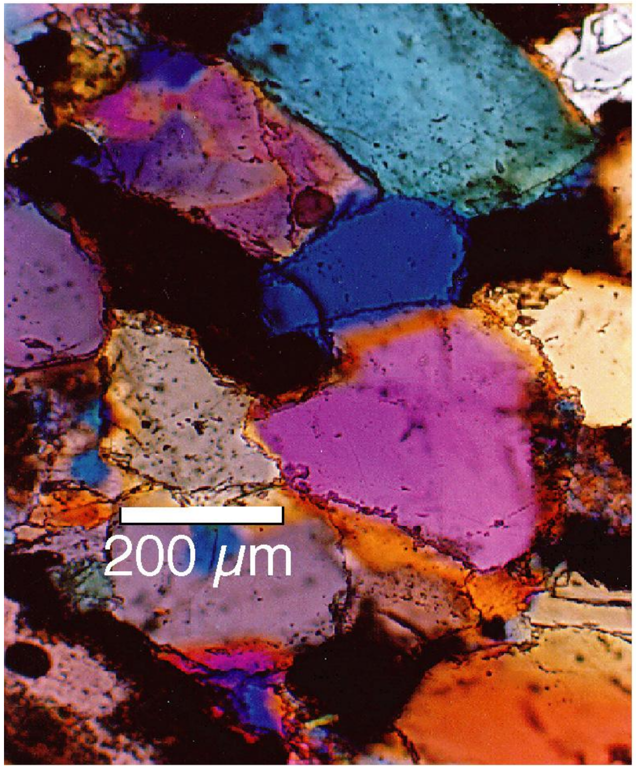
International Symposium on Nonlinear Acoustics (ISNA), Santa Fe, New Mexico July 8-13, 2018

LANL group Home page with links to publications:
http://geophys.lanl.gov/nonlinear/nonlinear.shtml

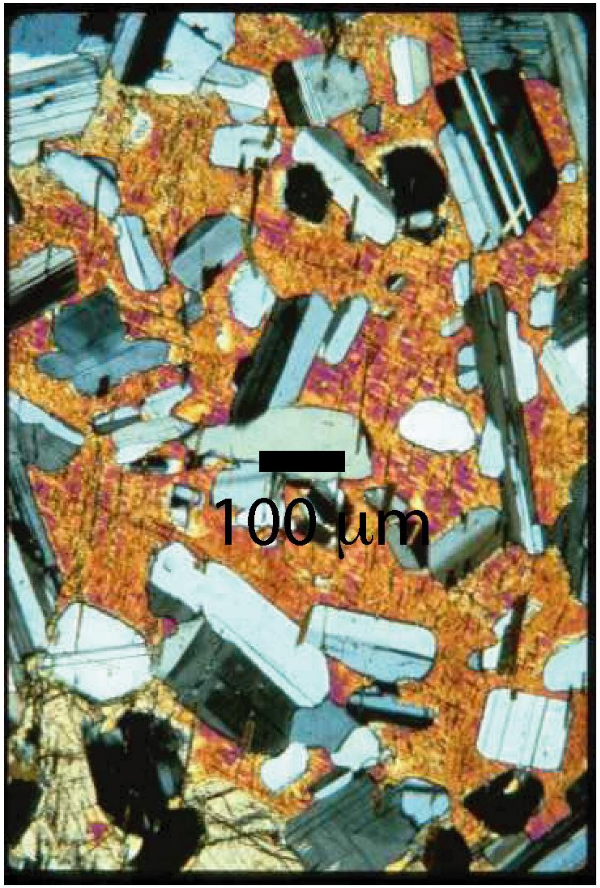
Contents

- Universal concept of nonequilibrium behavior
- Background—quasistatic and dynamic elasicity
- Nonequilibrium behavior in earth materials Simulation—soft modes?
- Theory
- Conclude

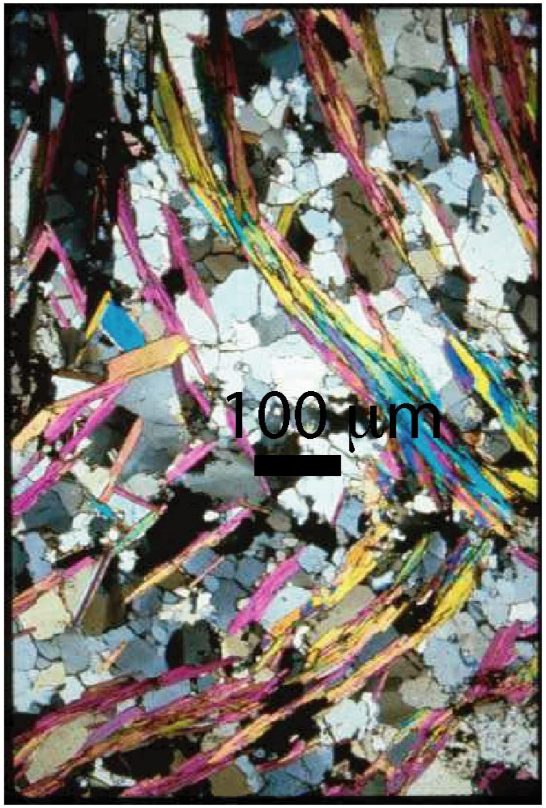
Rocks are complex granular materials, 'sintered' together



Sandstone

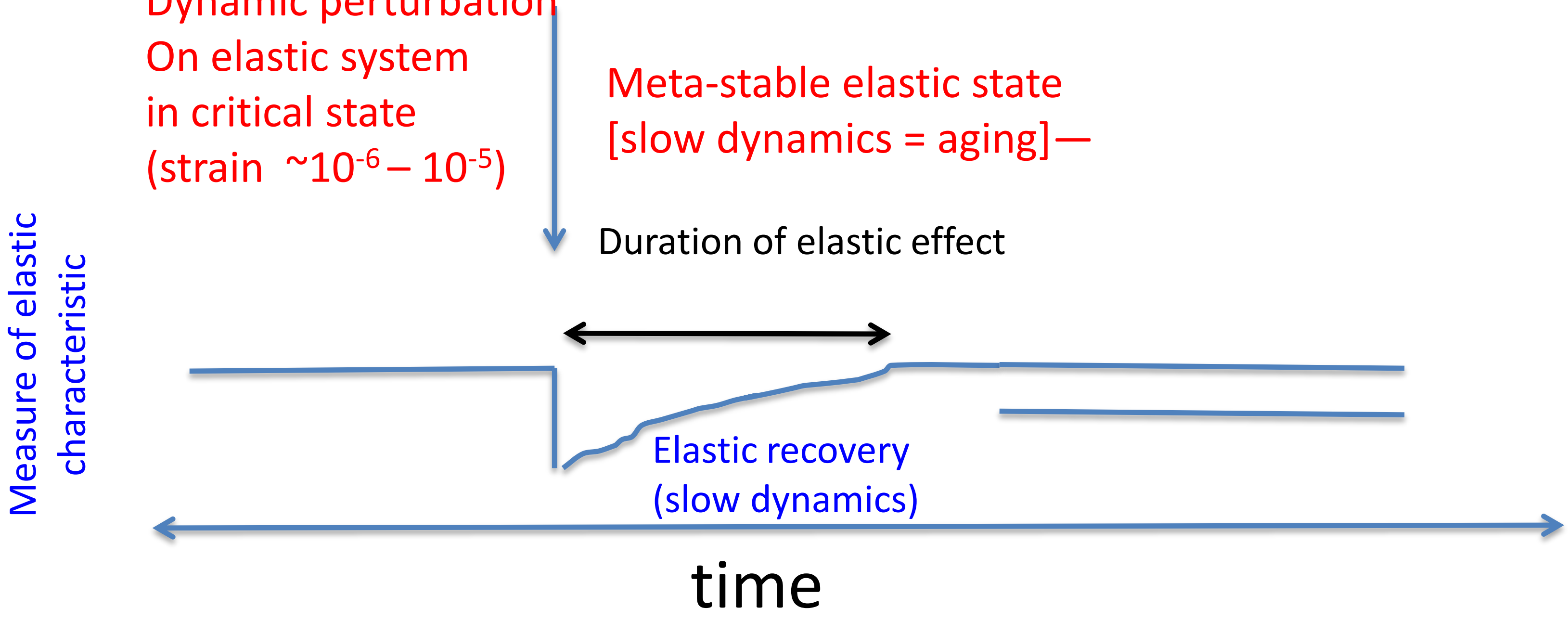


Anorthosite



Quartz-mica schis

All exhibit nonlinear fast and slow dynamics



Dynamic perturbation
On elastic system
in critical state
(strain $\sim 10^{-6} – 10^{-5}$)

Meta-stable elastic state
[slow dynamics = aging]—

Duration of elastic effect

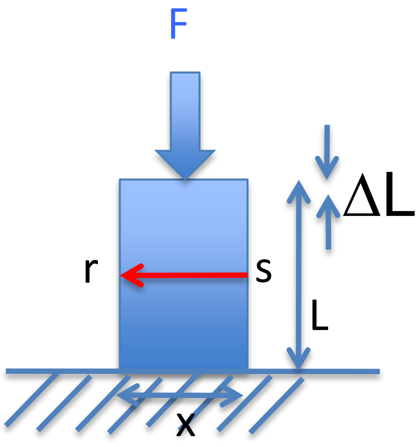
Elastic recovery
(slow dynamics)

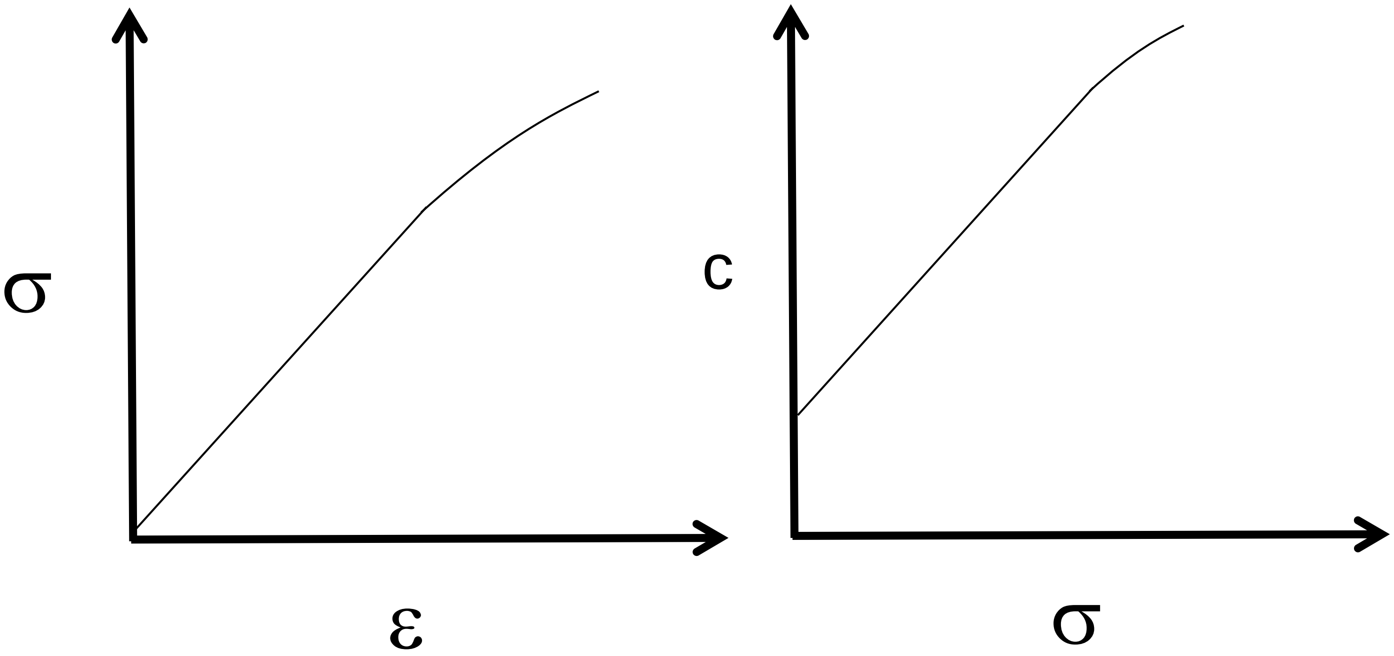
Measure of elastic characteristic

time

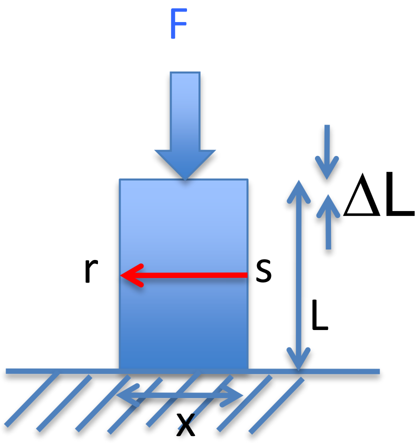
Quasi-static and dynamic nonlinear elasticity in consolidated granular materials (e.g., rock, concrete…)

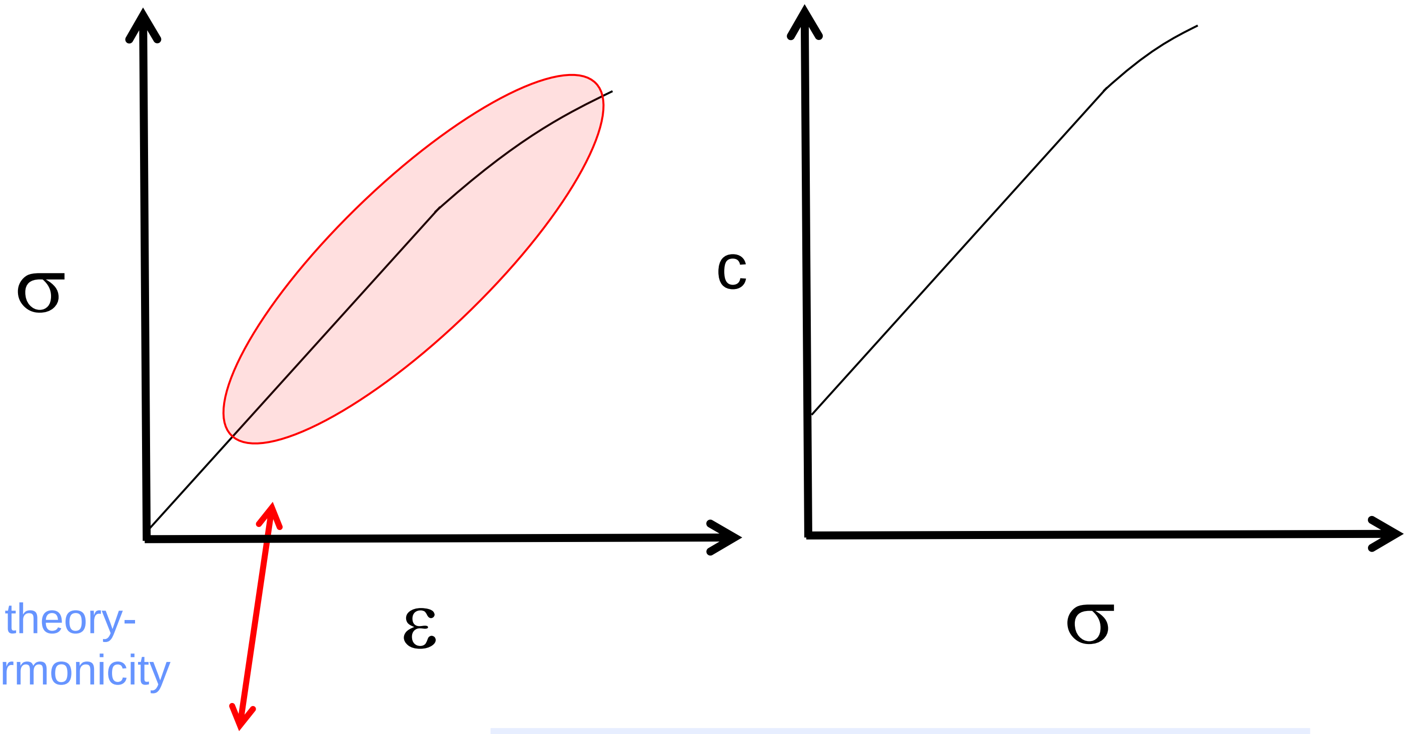
Quasistatic and dynamic elasticity of individuals crystals and many metals





Quasistatic and dynamic elasticity of individuals crystals and many metals





Thermodynamics based theory-
Nonlinearity due to anharmonicity
in the crystalline lattice.

$$s = Me[(1+be)]$$

isotropic

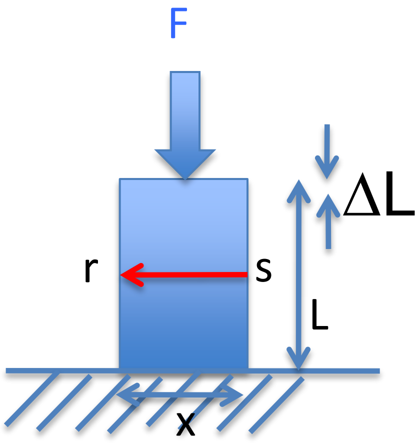
$$b = \frac{3}{4} + \frac{A+B+C}{2rc^2} = \frac{3}{2} + \frac{l+2m}{l+2u}$$

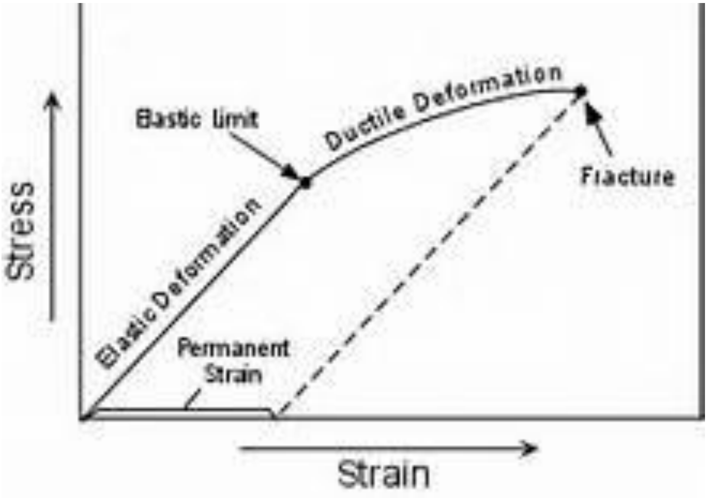
Goldberg

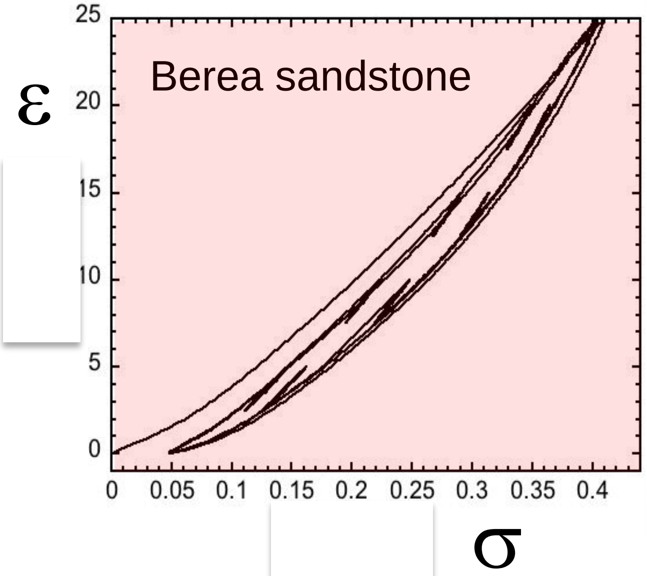
Murnaghan

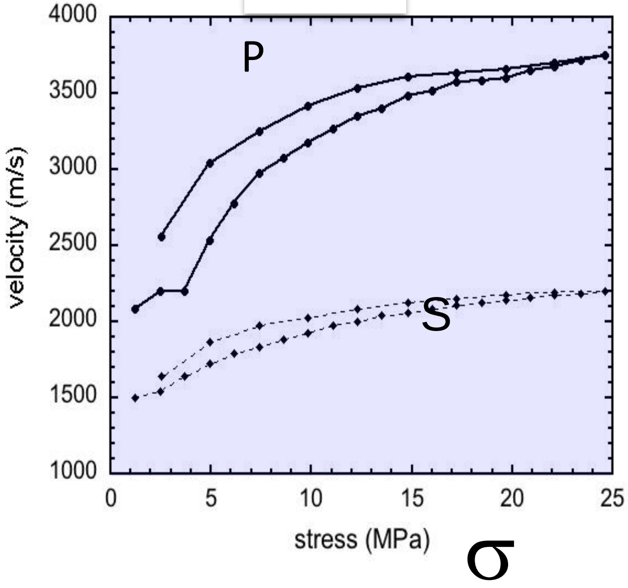
Quasistatic elasticity of rock and ‘damaged’ materials

Nonlinearity much larger than metals and single crystals…..

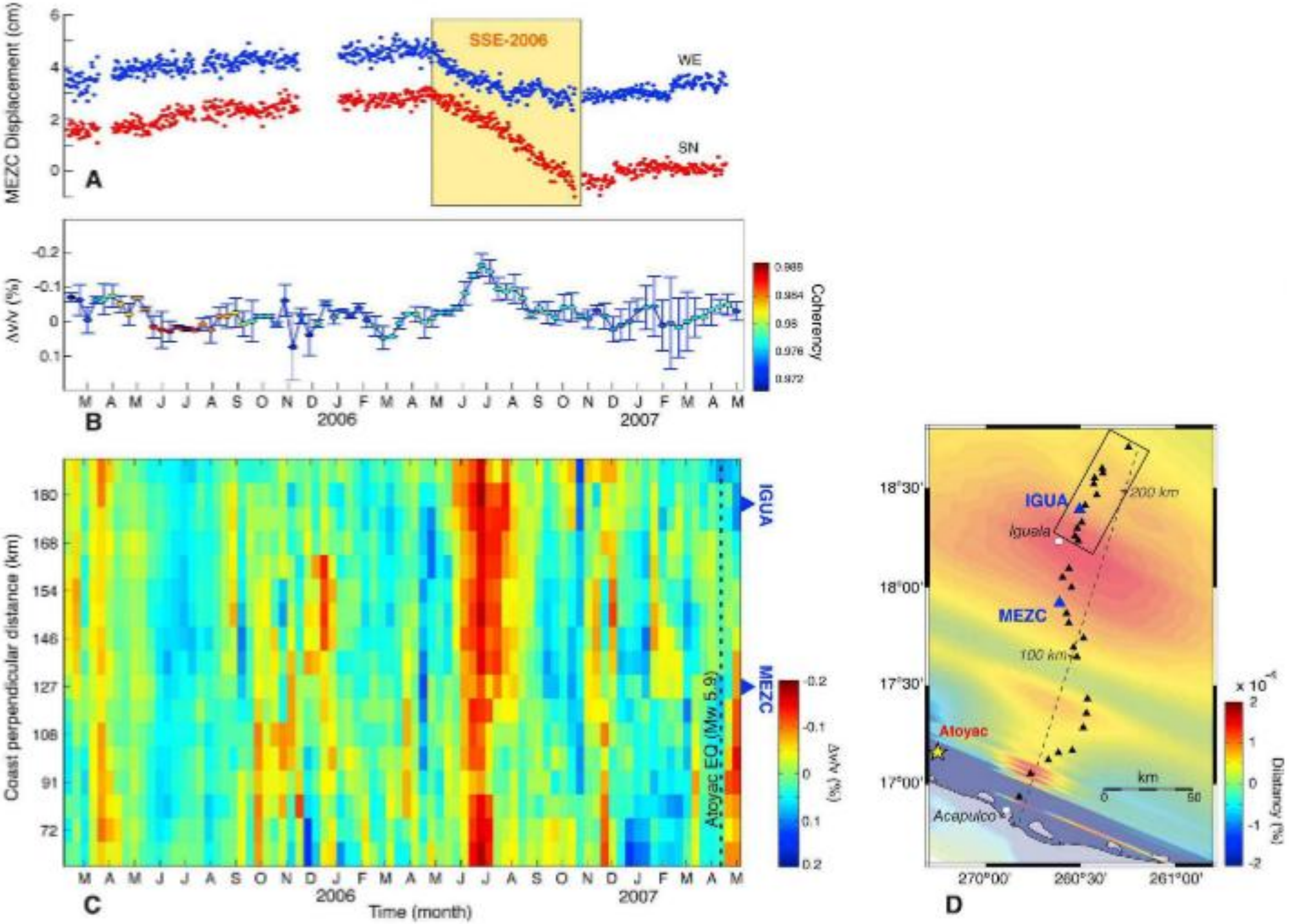






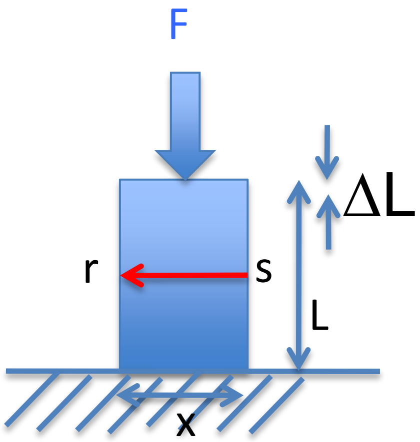


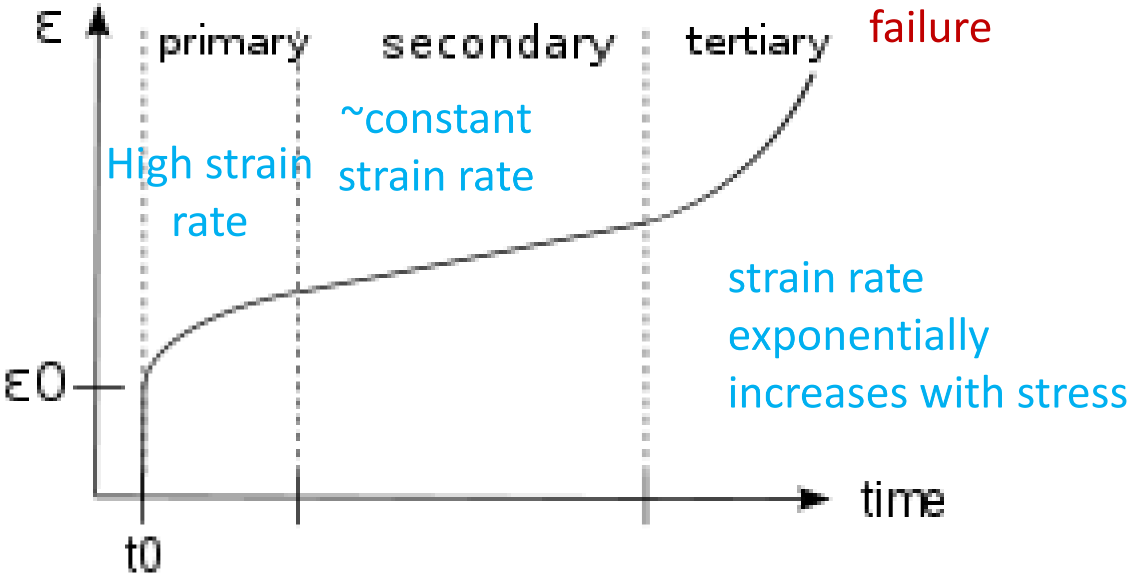
Static forcing of slow slip in Mexico



Rivet et al., GRL 2011

Creep under static loading





failure

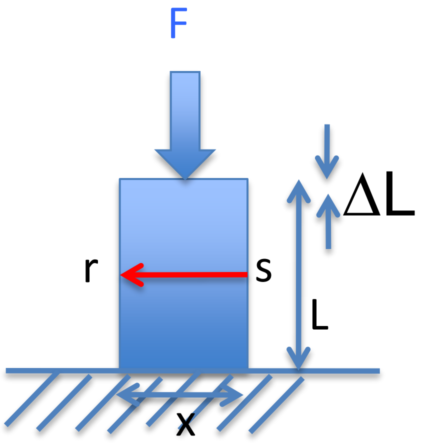
High strain rate

~constant strain rate

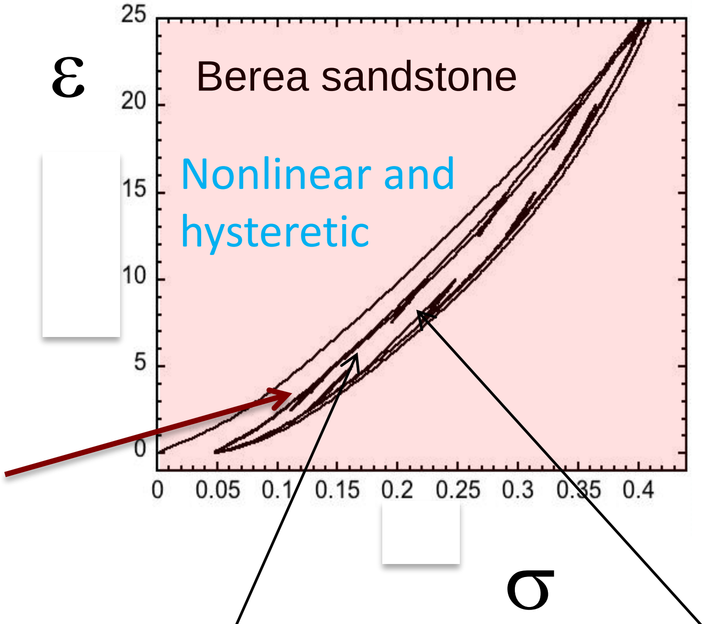
strain rate exponentially increases with stress

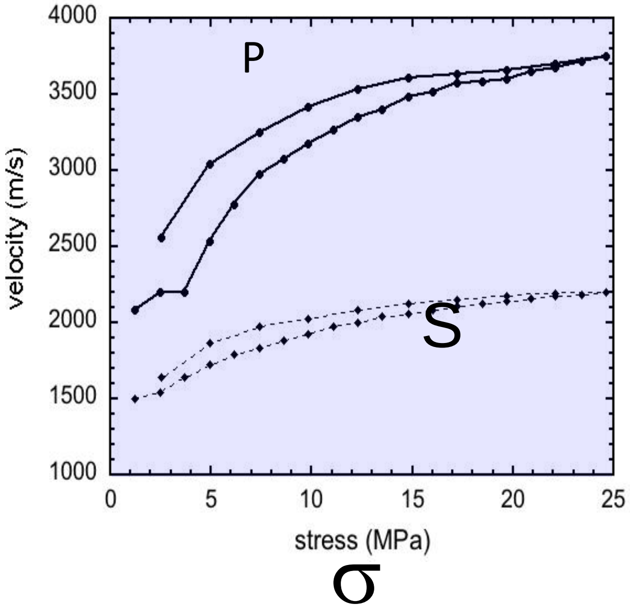
Elasticity of individuals rock and damaged materials

Nonlinearity much larger than metals and single cyrstals…..



End point memory





Phenomenological model of nonlinearity and hysteresis (Preisach-Mayergoyz)

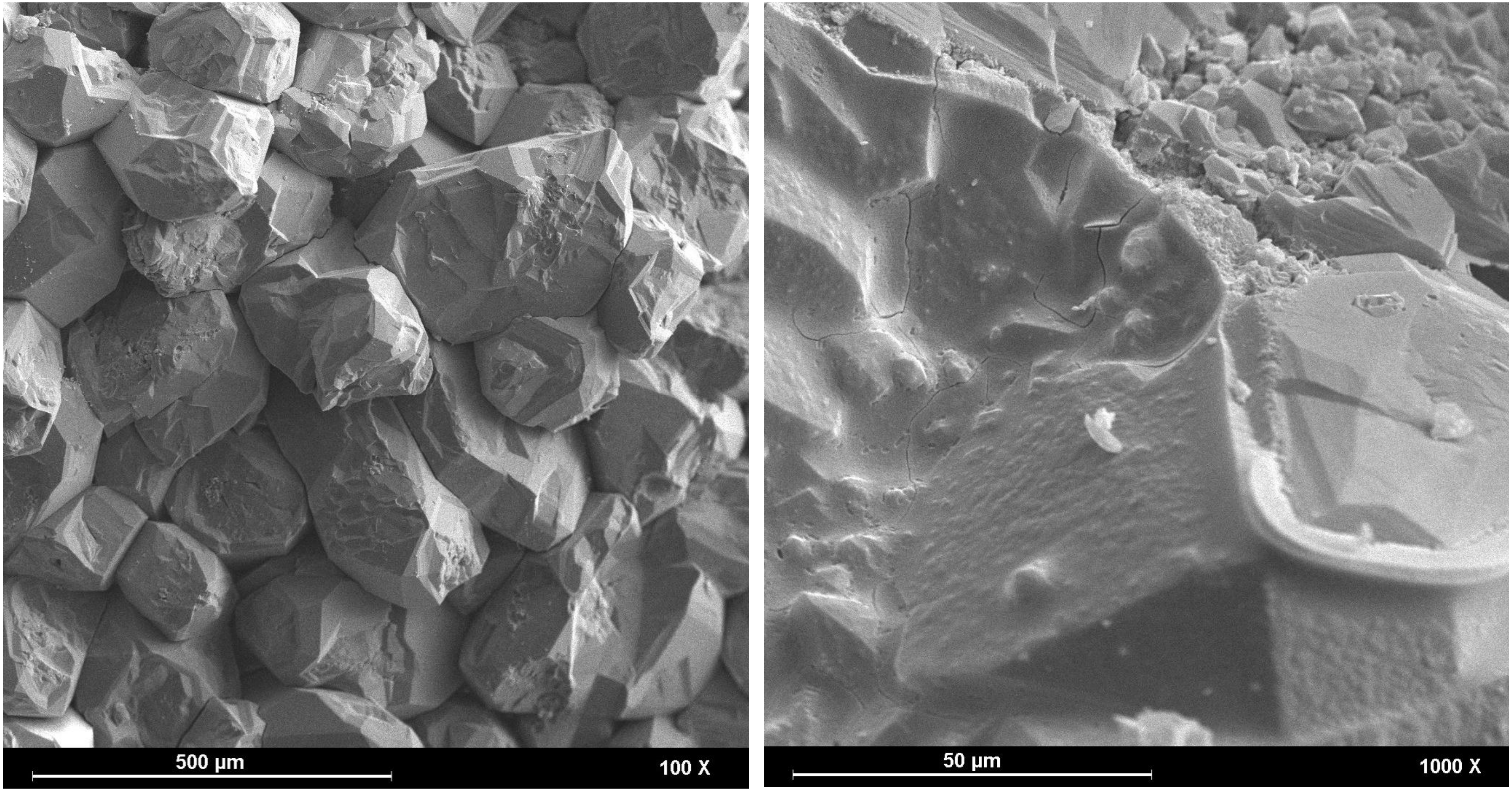
$$s = K_0\left(e + be^2 - a\left[(De)e + sign(\dot{e})\frac{e^2 - (De)^2}{2}\right]\right)$$

$$e = \frac{du}{dx}, b = \frac{3(l+2m) + 2(l+2m)}{2(l+2m)}$$

M=modulus, ε= strain, λ.μ = Lamé coeffs. l, m = 3rd order [Murnaghan] coefficients

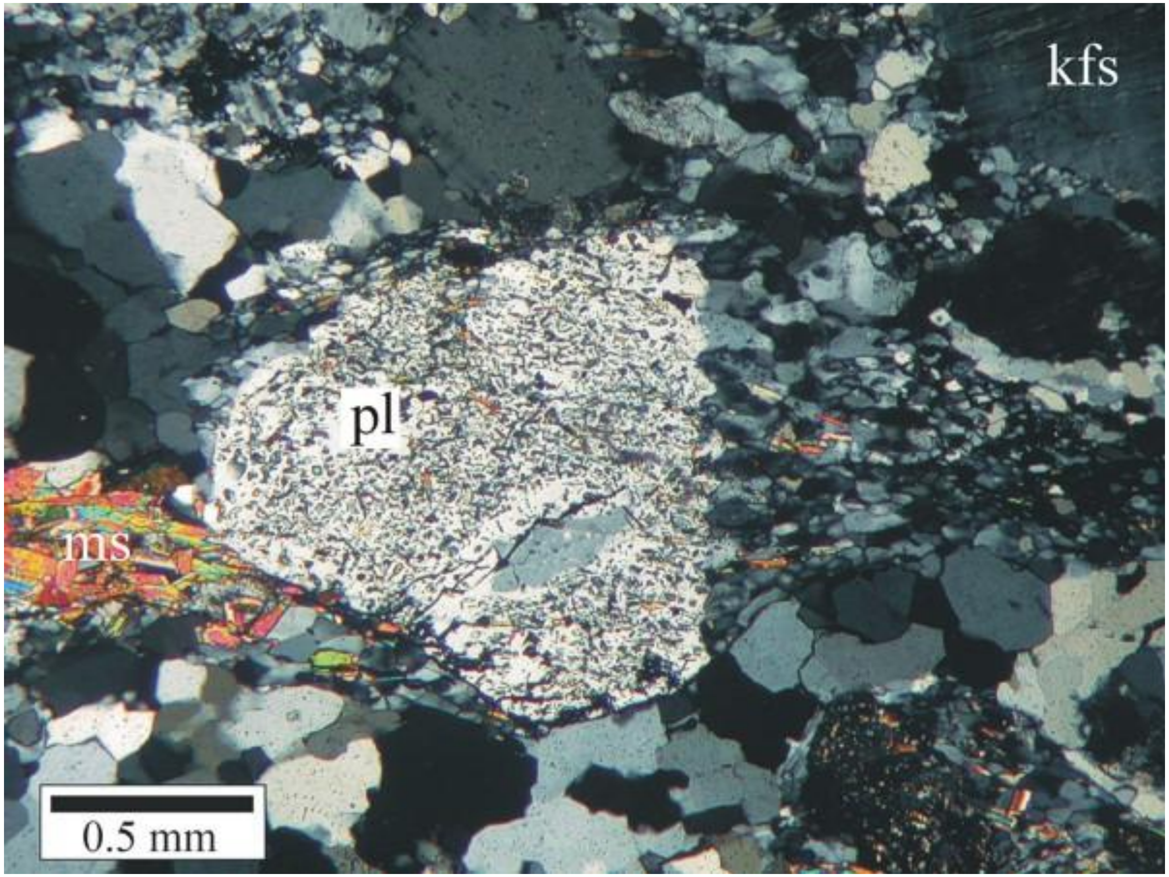
Why the velocity (and dissipation) change with pressure?

Fontainebleau sandstone (SEM)



From Tore Stendahl http://folk.ntnu.no/tores/SiCl4/SEMappendix.html

Why the velocity (and dissipation) change with pressure?



Granite in
Thin-section

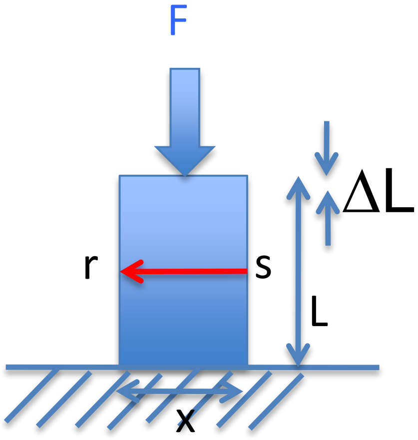
Pressure makes rock
look like ordinary solids,
closing cracks

kfs: K-feldspar
ms: muscovite
pl: plagioclase

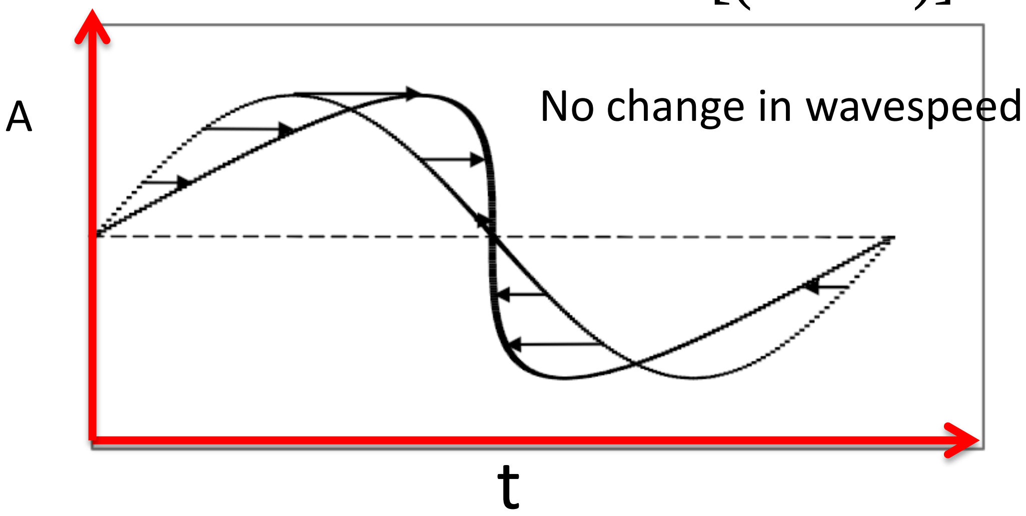
From USGS http://pubs.usgs.gov/of/2003/of03-221/htmldocs/thinsect/95mw0107ts.htm

Dynamic nonlinear elasticity in a traveling wave

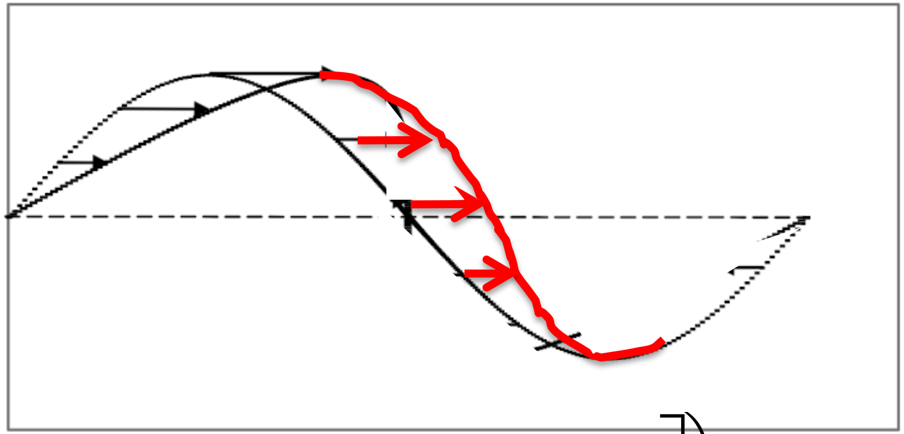
Fix the load and measure elastic wave



classical $\sigma = M\varepsilon[(1+\beta\varepsilon)]$



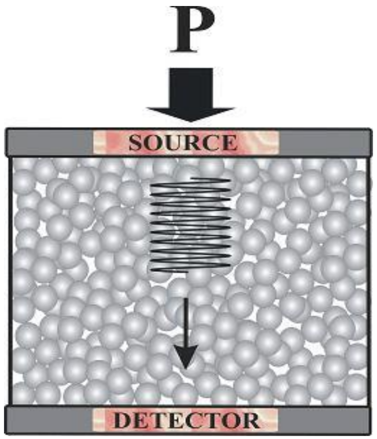
Single crystals, many metals....

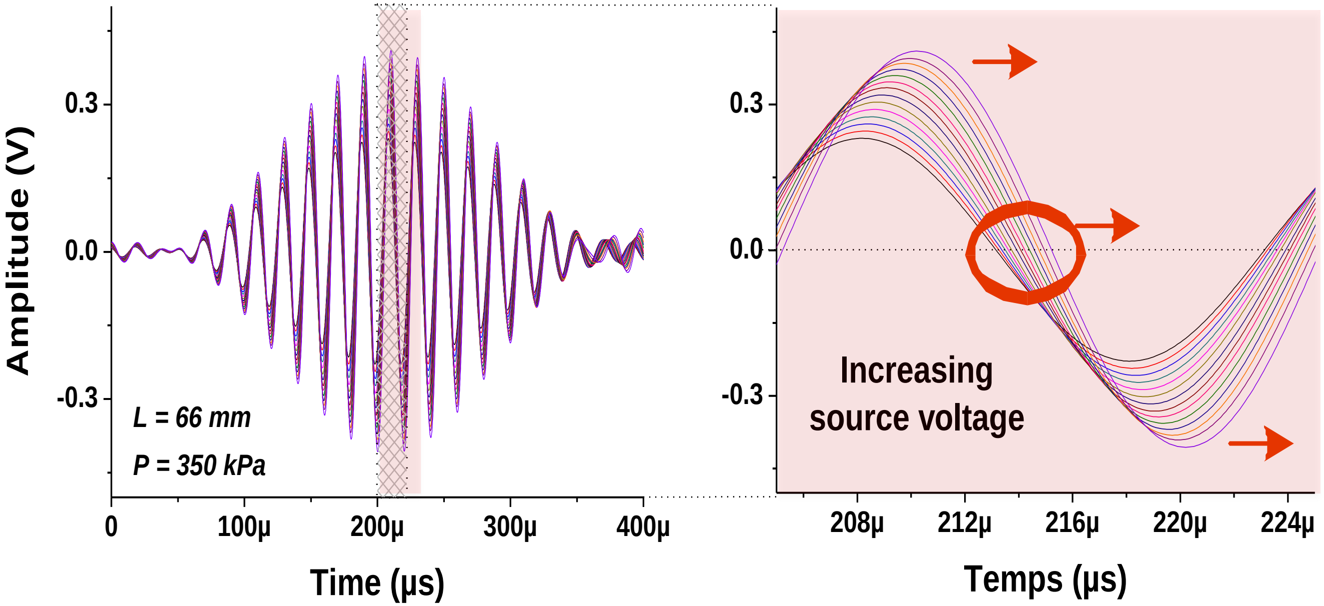


$$\sigma = K_0\left(\varepsilon + \beta\varepsilon^2 - \alpha\left[(\Delta\varepsilon)\varepsilon + \mathrm{sign}(\dot{\varepsilon})\frac{\varepsilon^2 - (\Delta\varepsilon)^2}{2}\right]\right)$$

Nonclassical (rock, GM....)

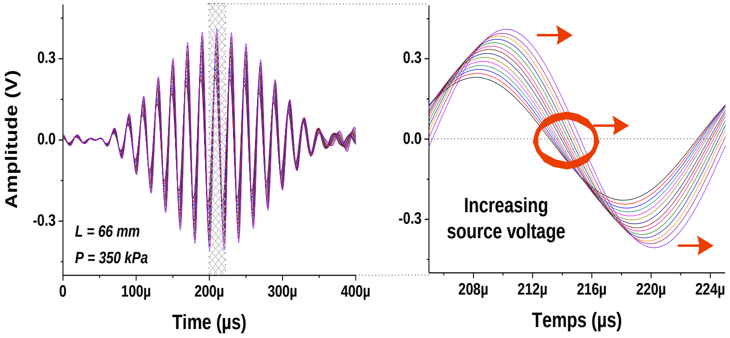
nonlinear acoustics in a traveling wave- example glass bead pack

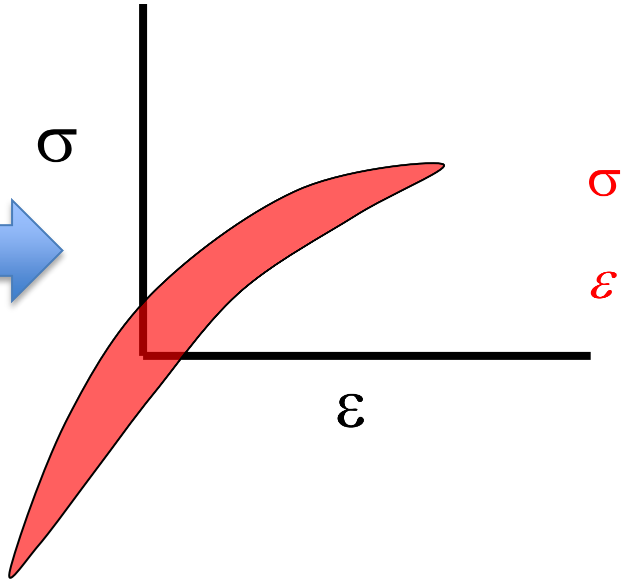




Courtesy X. Jia

Wave dynamics in rock and other earth materials





σ = stress

ε = strain

Like in quasti-static experiments, nonlinearity and hysteresis are present in dynamics when strains exceed ~5x10^{-7} - 10^{-6} [*at low effective pressure*], *Material dependent*

1-D phenomenological wave equation

Normal form:

$$r\frac{\P^2 u}{\P t^2} = \frac{\P s}{\P x} + F$$

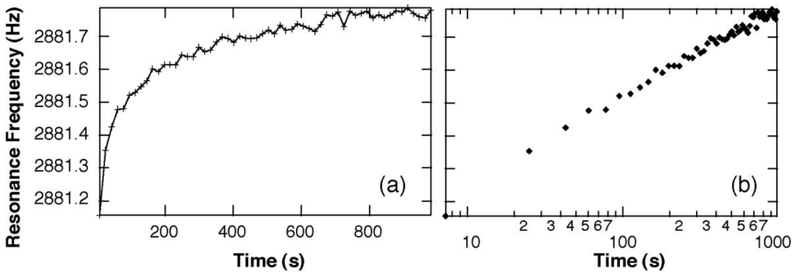
substituting the nonlinear stress relation

$$r\frac{\partial^2 u}{\partial t^2} = \frac{\partial K_0\left(e + be^2 - a\left[(De)e + sign(\dot{e})\frac{e^2 - (De)^2}{2}\right]\right)}{\partial x} + F$$

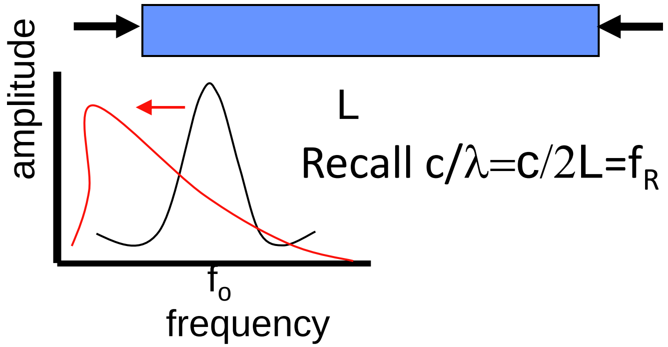
Problems:

i. Phenomenologic

ii. Contains no rate dependence



Exploring regimes of elastic nonlinearity applying wave resonance



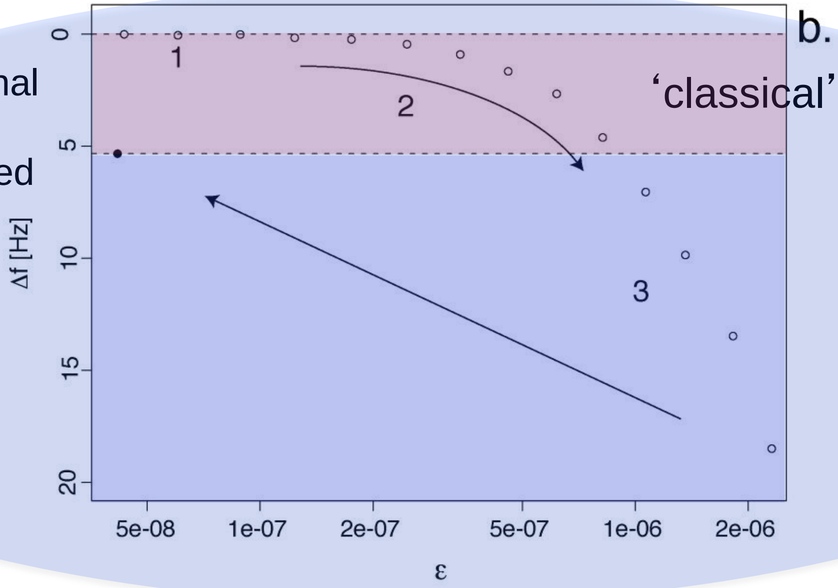
L

Recall $c/\lambda = c/2L = f_R$

Young’s mode experiments in a bar

(sandstone)

Δf proportional to wavespeed



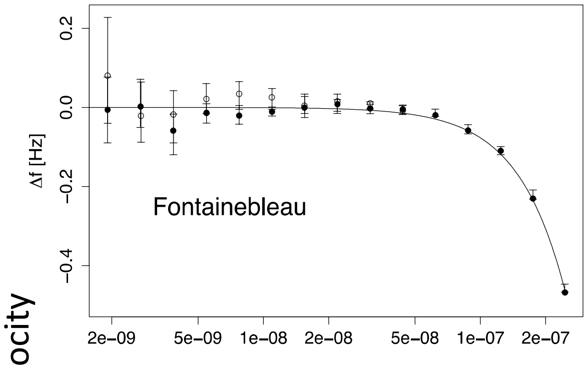
‘classical’ behavior

Nonclassical, nonequilibrium

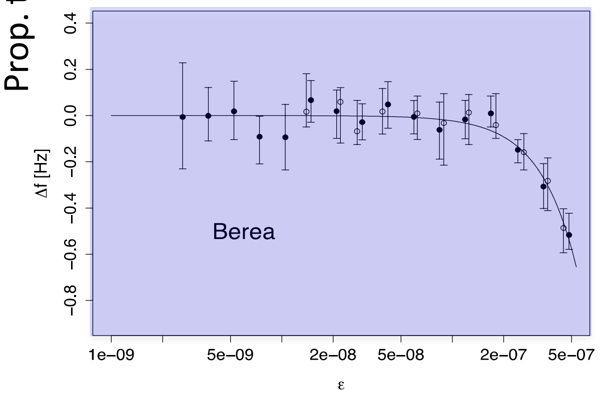
From Pasqualini et al., Jour. Geophys. Res. 2007

Recovery– the slow dynamics (*aging*)

Prop. to velocity

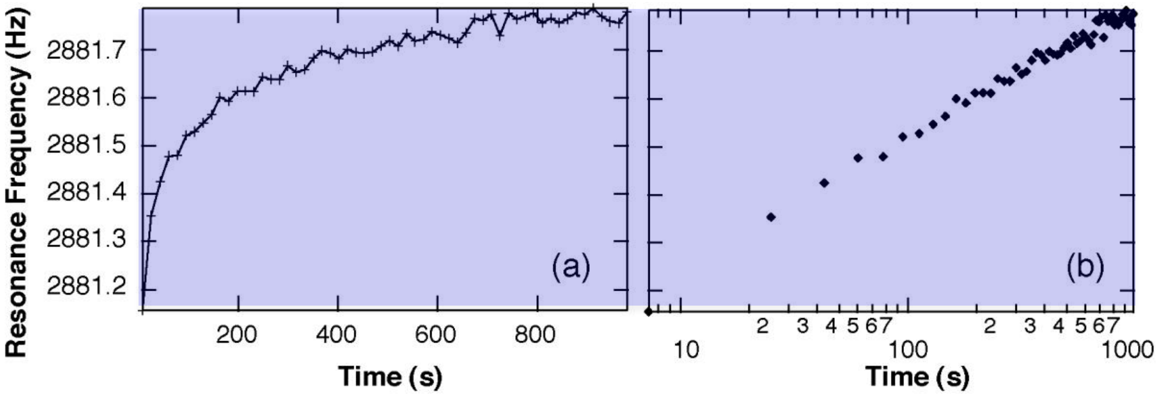


strain



strain

Recovery in Berea sandstone



Linear with the log of time-–

Pasqualini et al. J. Geophys. Res., 2007

Modified from TenCate and Smith, PRL 1997

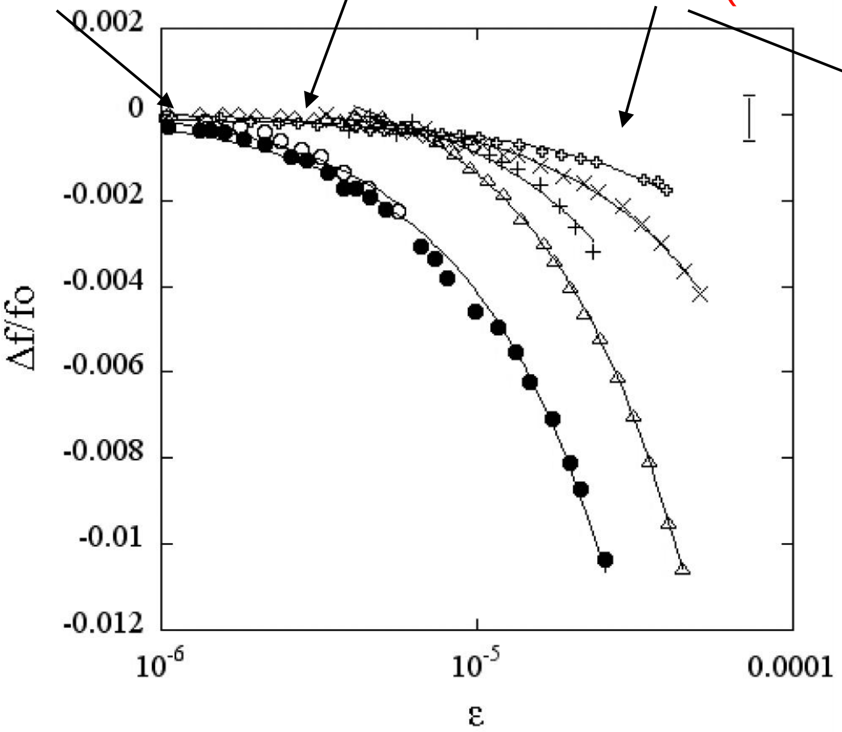
Nonlinear, nonequilibrium behavior in diverse materials

Young’s mode experiments

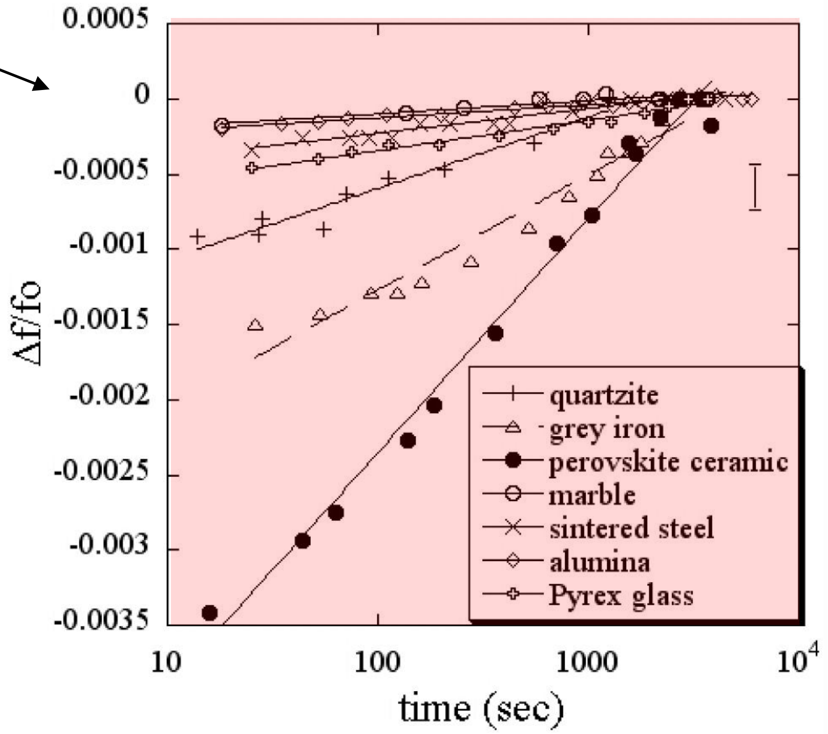
Classical NL behavior

Linear elastic

Nonclassical (nonequilibrium) behavior



Fast dynamics/conditioning



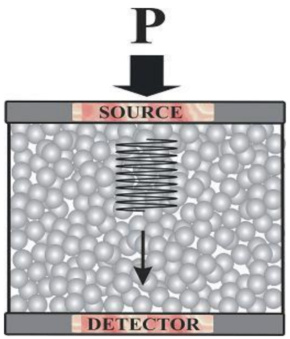
Slow dynamics

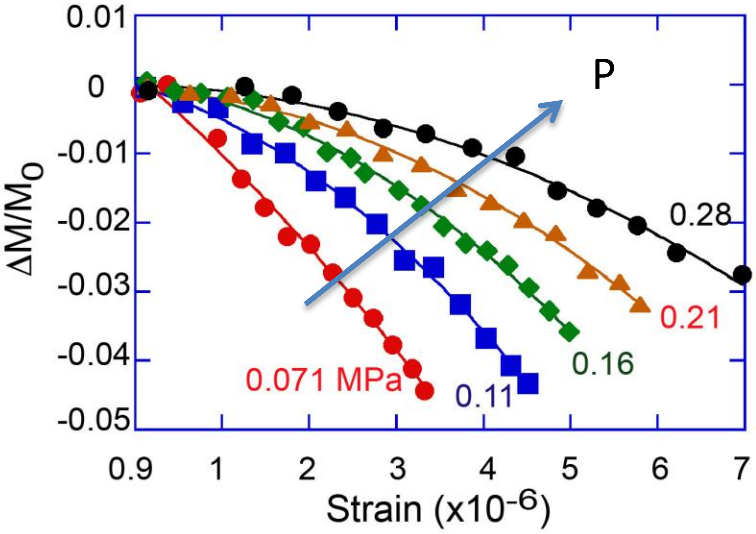
Strong effects on wave attenuation as well.

From Johnson and Sutin, JASA 2005

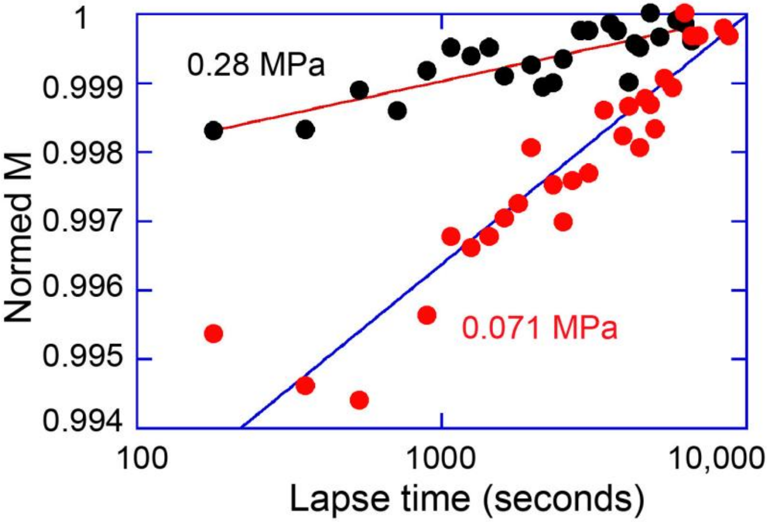
with applied pressure?

Experiments with rock and canisters of glass beads





Decrease in modulus and increase in dissipation with wave amplitude.

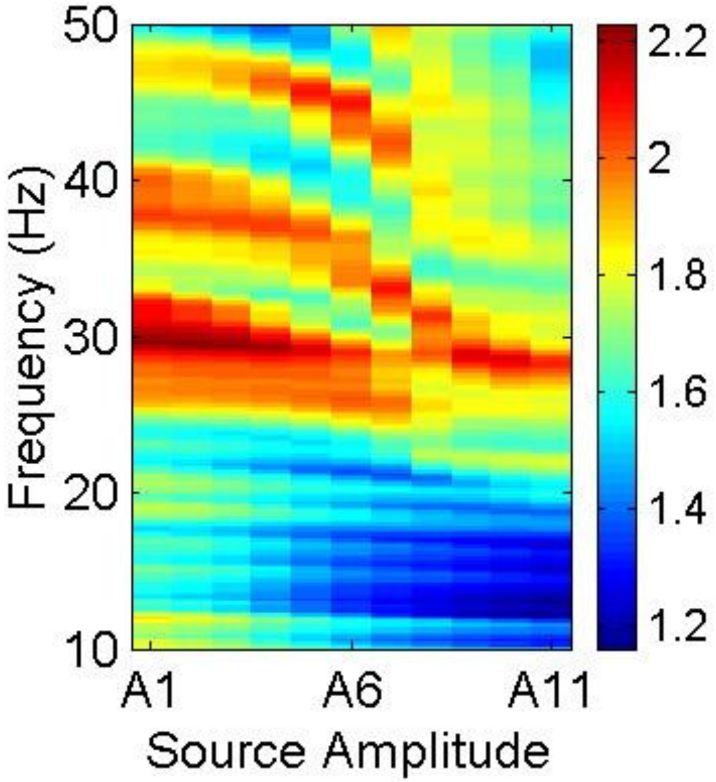


Long recovery to equilibrium (the slow dynamics)

Modified from Johnson and Jia, *Nature*, 2005

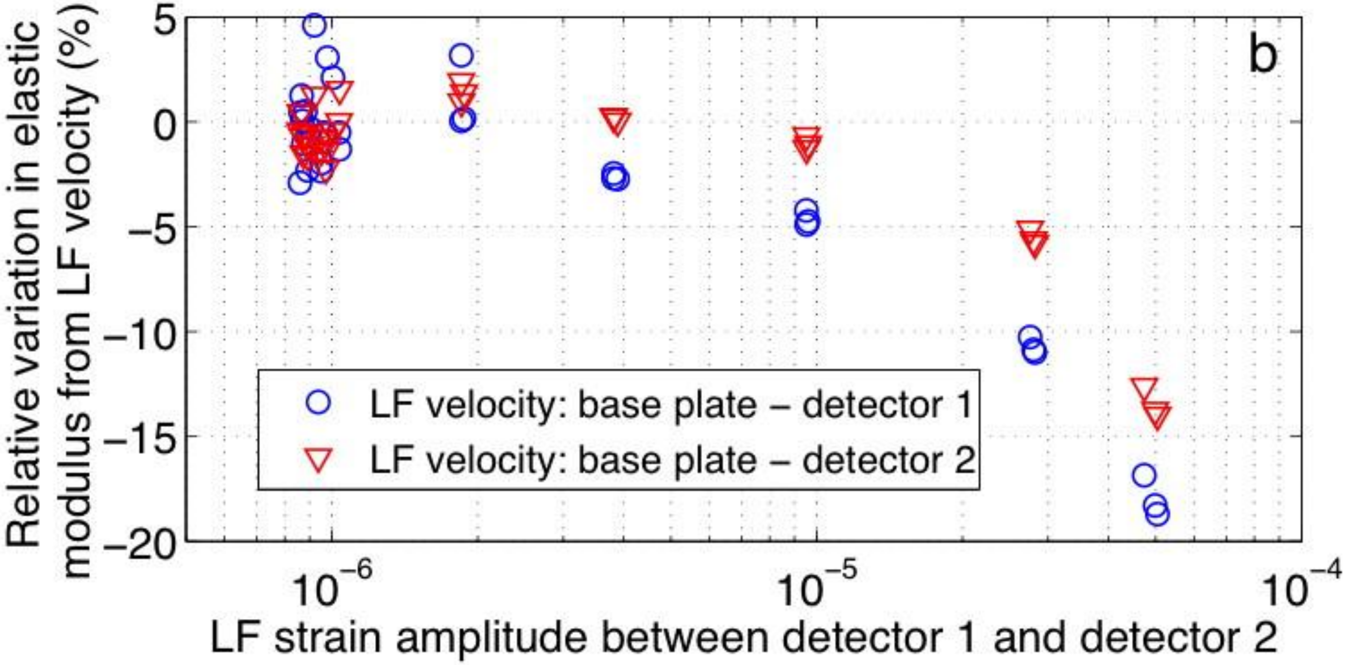
Induced nonlinear behavior with an active source

T-Rex, University of Texas at Austin



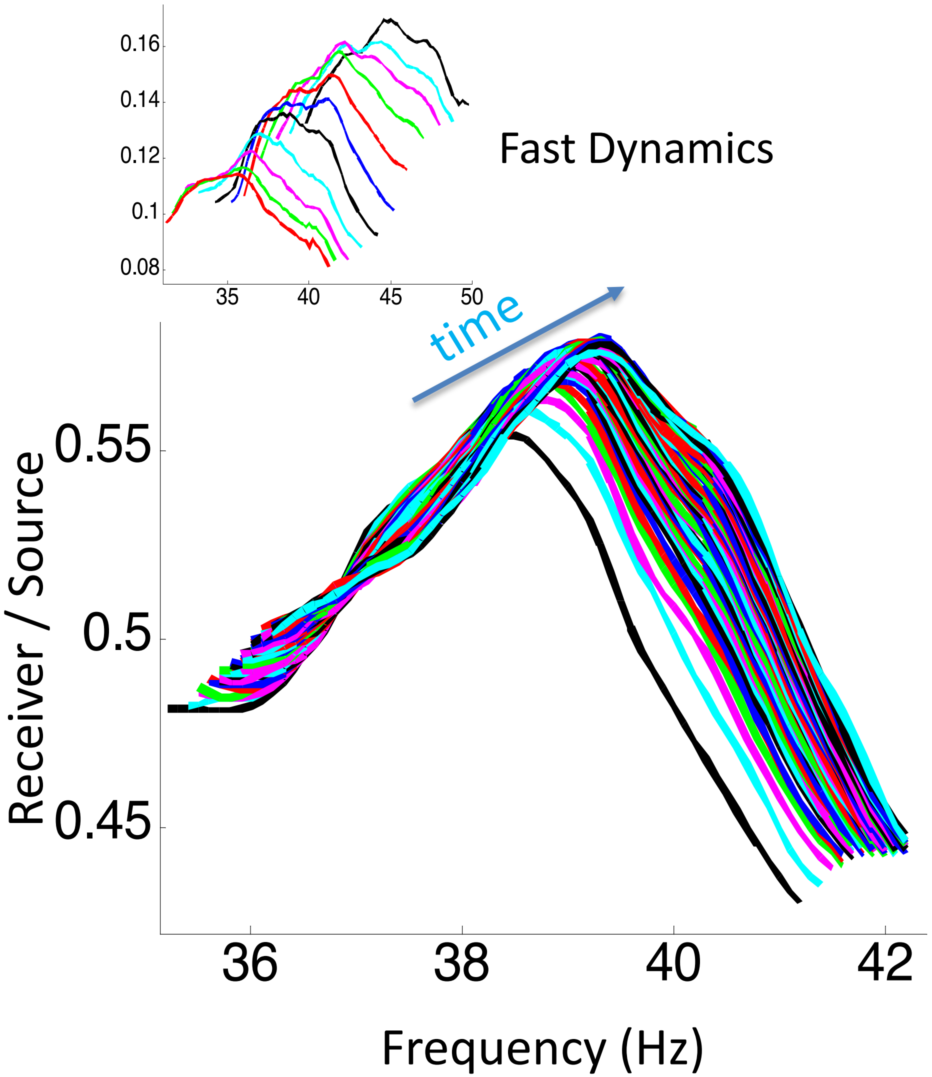
Johnson et al., Journal of Geophysical Research, 2008

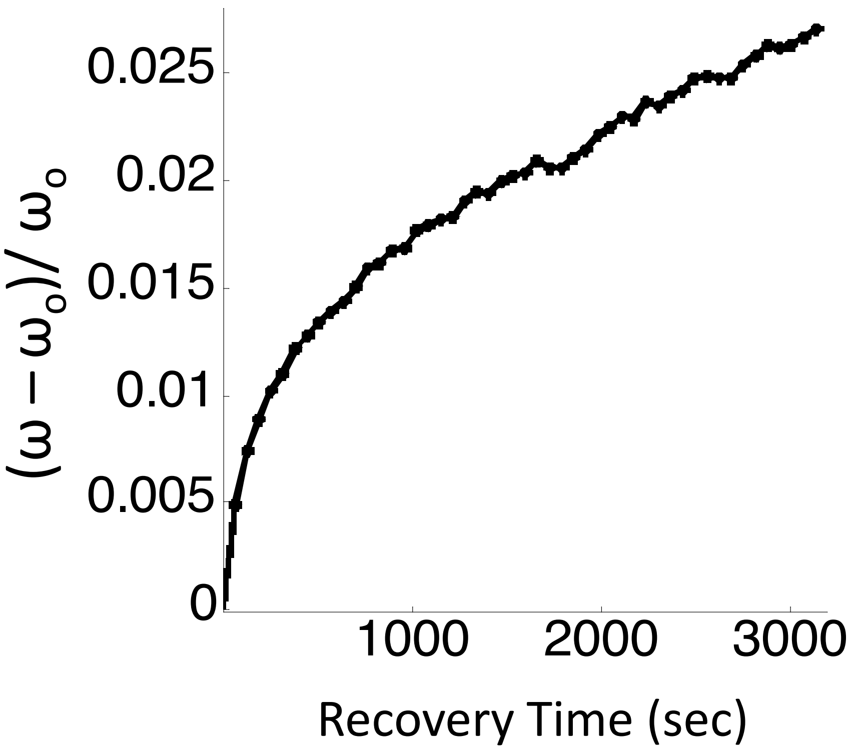
Near surface sediment response from a large vibrator source: site characterization



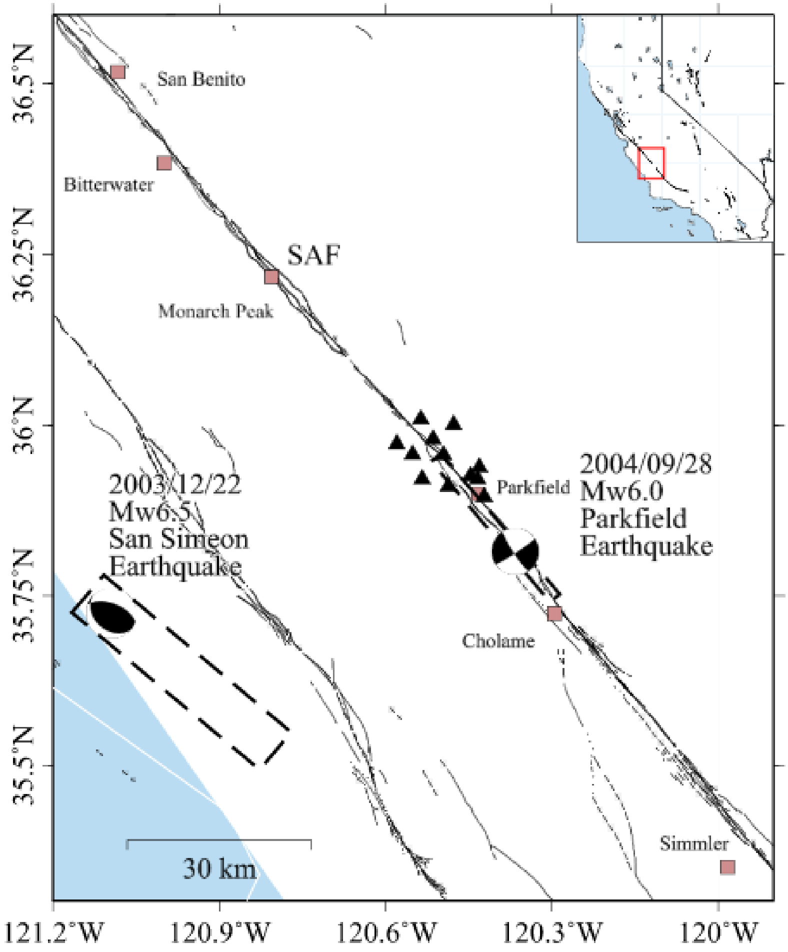
Renaud et al., JGR 2014 in review

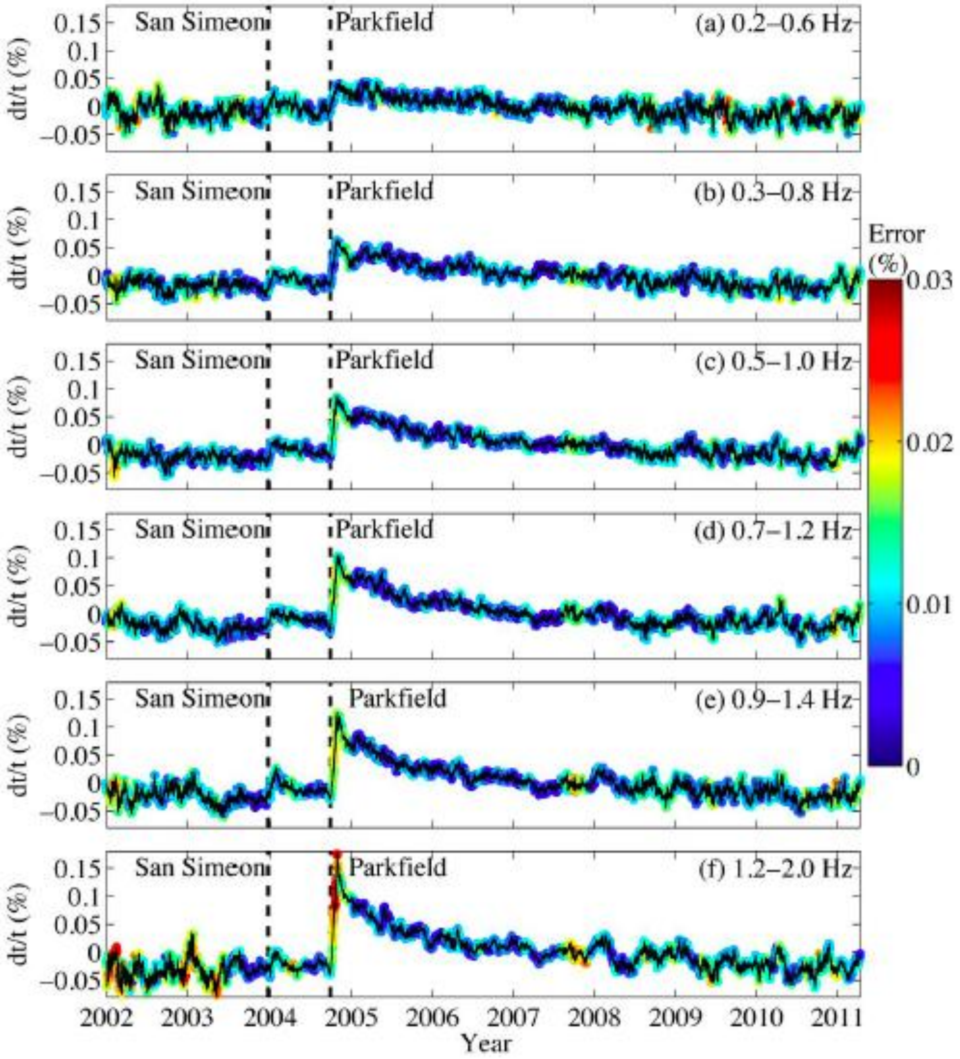
Slow dynamics




Frequency Shift

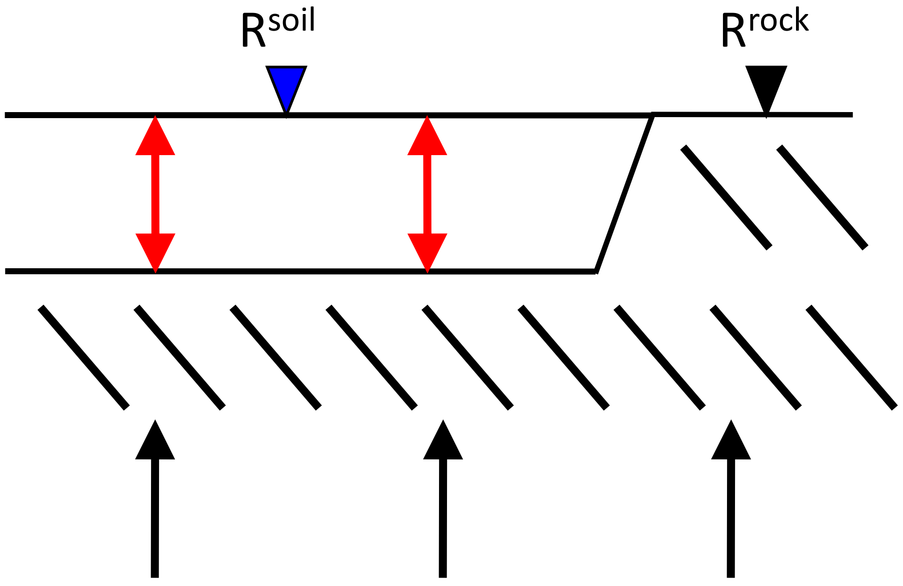
parkfield





Wu et al, GRL 2016

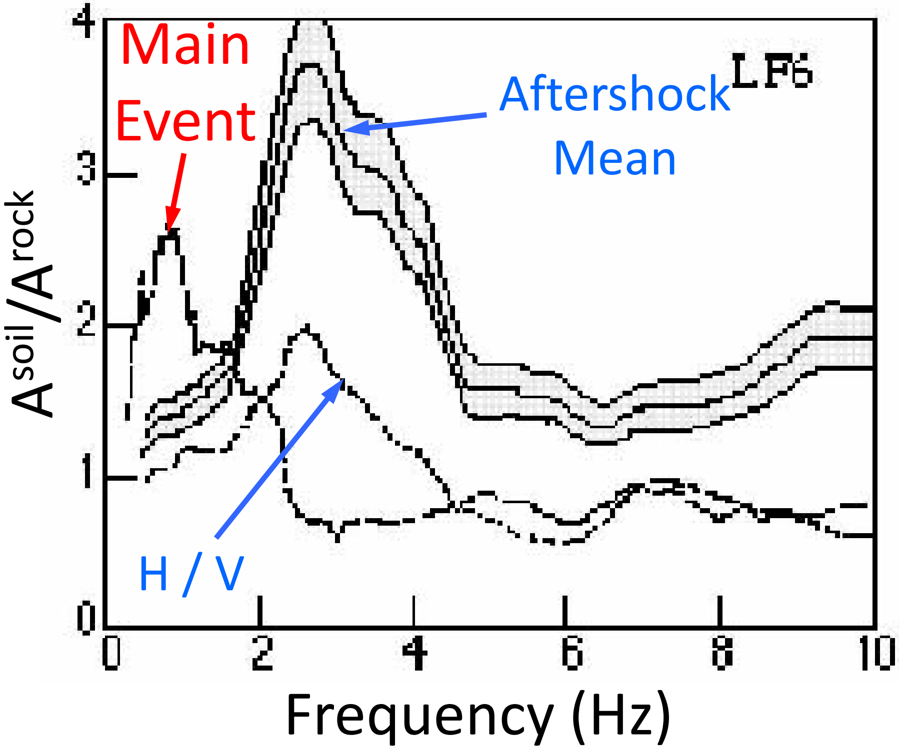
Strong ground motion



Earthquake source at depth

$$\text{Site Response} \approx \frac{A^{soil}(\omega)}{A^{rock}(\omega)}$$

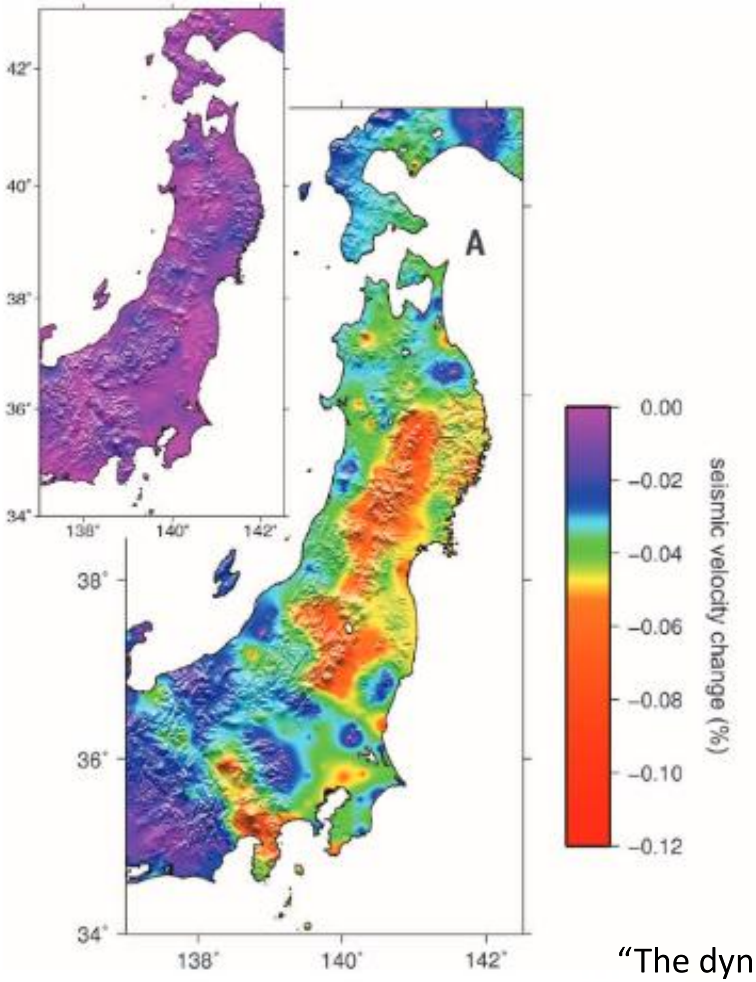
1994 Northridge Earthquake



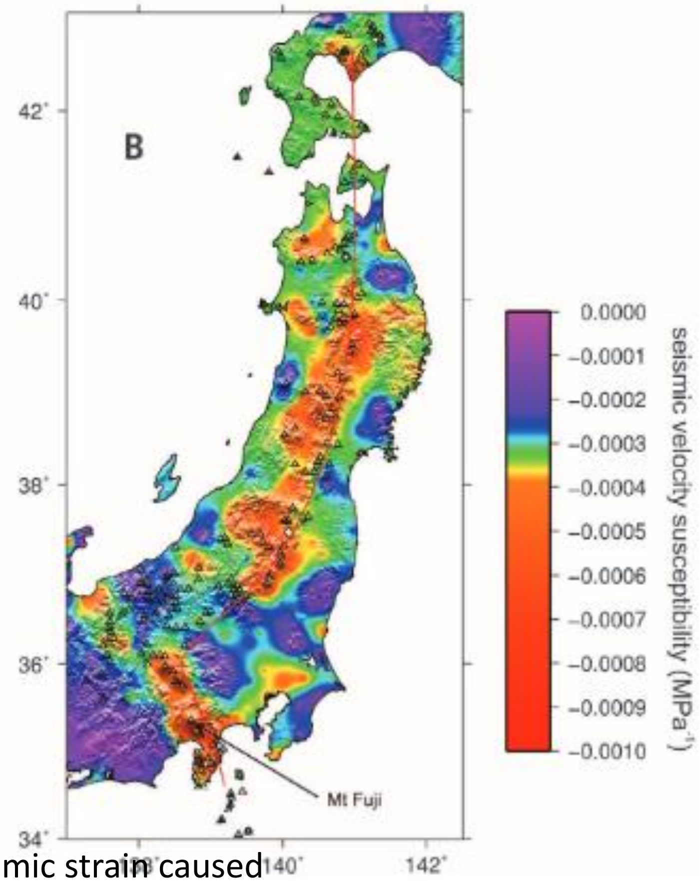
Field et al. Nature, 1998

Quasi-static+ dynamic nonlinear elasticity

Co-seismic velocity change



Velocity suseptibility

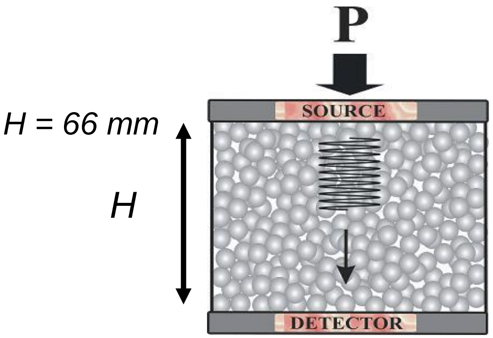


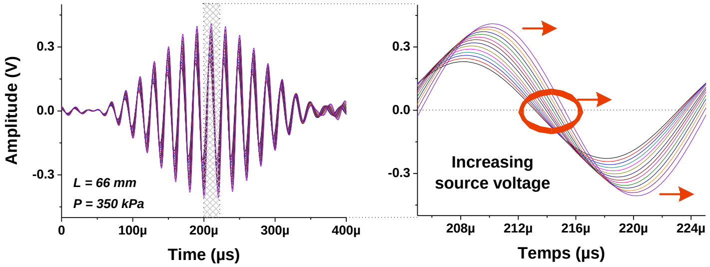
Brenguier et al., Science (2014)

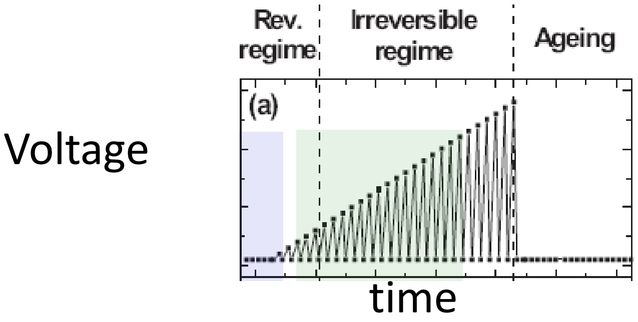
"The dynamic strain caused by the passing of the seismic waves was one to two orders of magnitude higher than the static coseismic strain for Honshu Island. "

Nonlinear elasticity and plasticity in glass beads

Jia, Brunet, Laurent, PRE; 84 (2011); Olson Reichhardt et al., Physical Review E (2015); Limrich et al., in review PRE 2017







Plasticity is not observed in intact rock at typical dynamic strains, but in highly fractured media, it could be (damage zone in a fault)

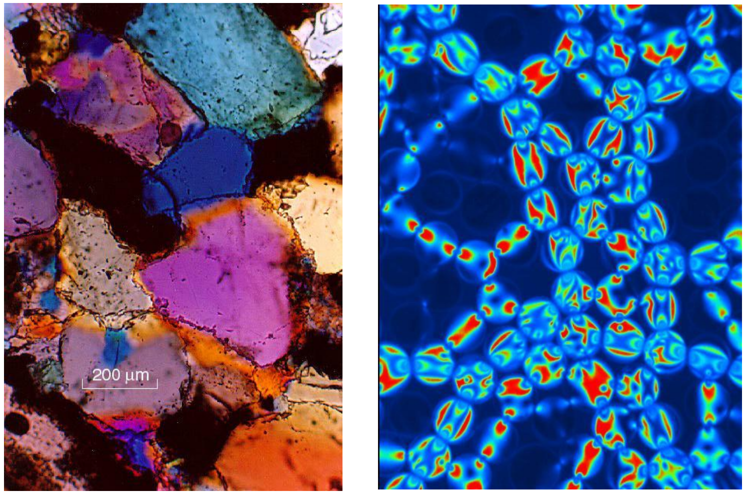
Run time (s)

Class of materials exhibiting rock-like “nonclassical” (hysteretic) nonlinear behavior

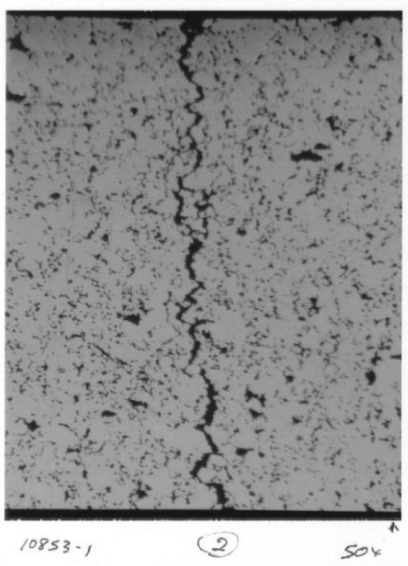
Granular media, some ceramics, all(?) rocks, all damaged materials, sintered metal….

A primary contribution to elastic nonlinearity is mechanical damage.

“Distributed”

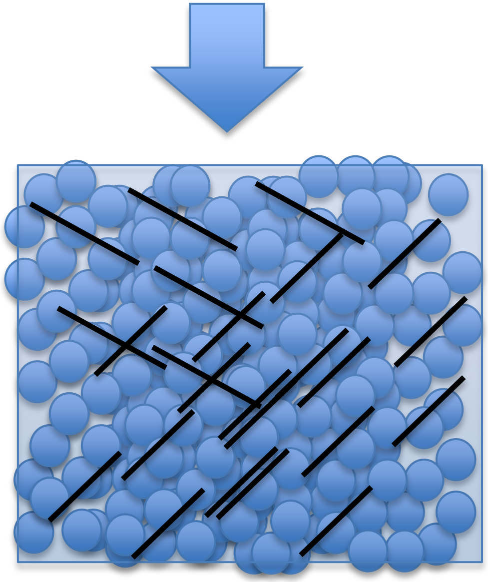


“Localized”



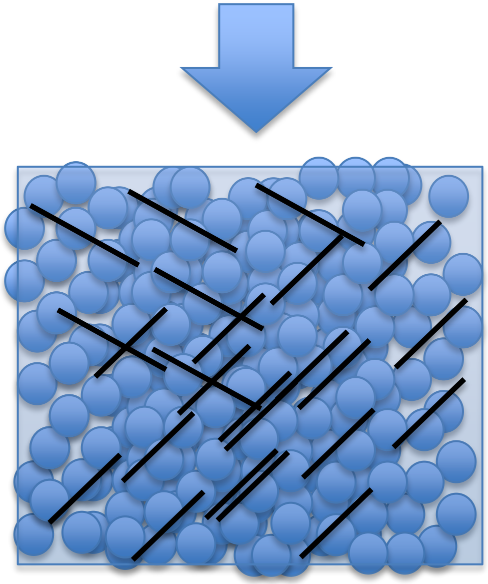
Mechanism?

F cos $(\omega t\text{-}kx)$, $\omega < 2\pi$



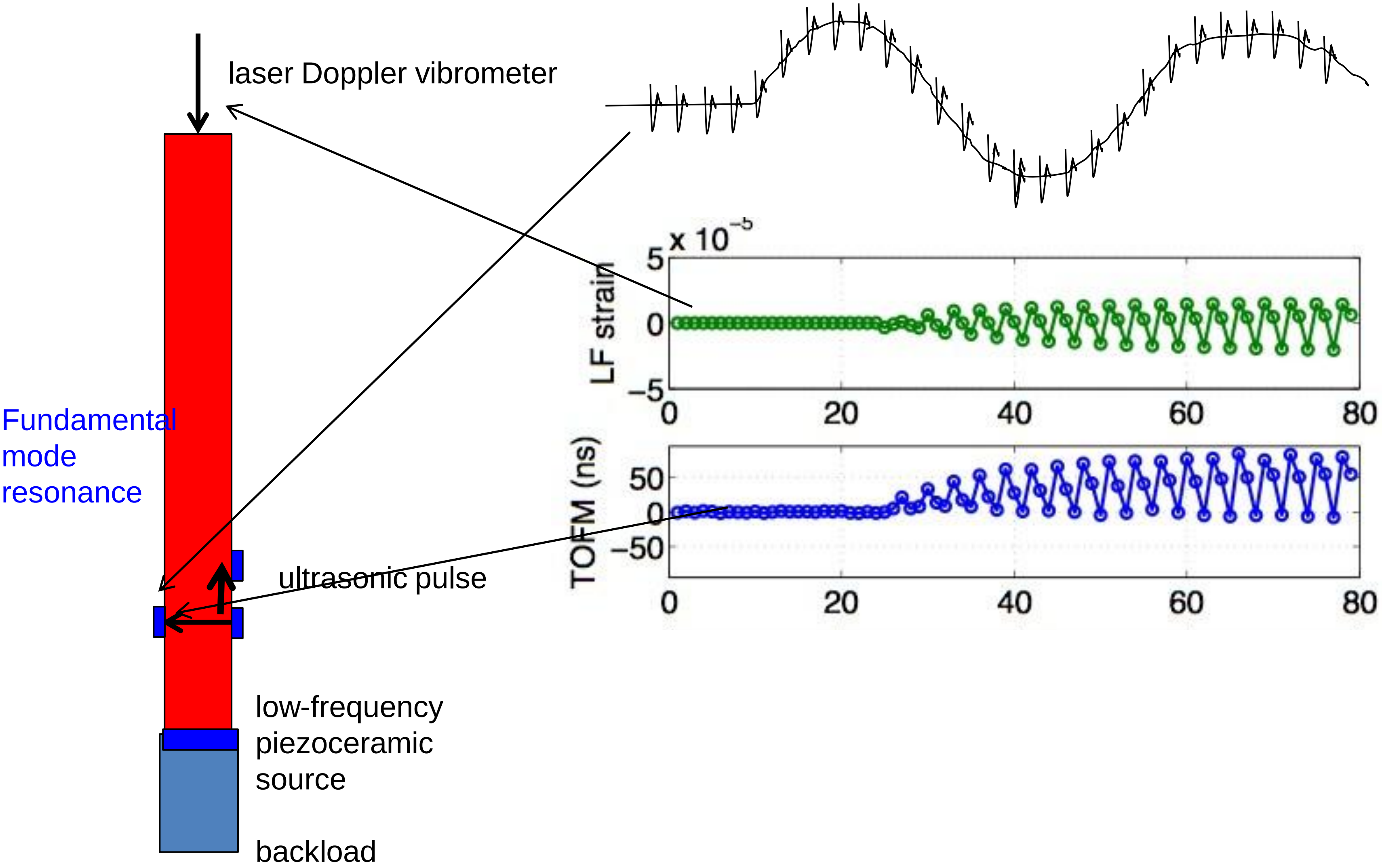
Stress transferred by skeleton. Cracks/bonds/dislocations… are periodically closed by cyclic forcing.   Little mobilization of damage features.

F cos $(\omega t\text{-}kx)$, $\omega >> 2\pi$



Progressively stronger stress mobilization of damage features, primarily by shearing (?)

Probing elastic nonlinearity with dynamic acousto-elasticity



laser Doppler vibrometer

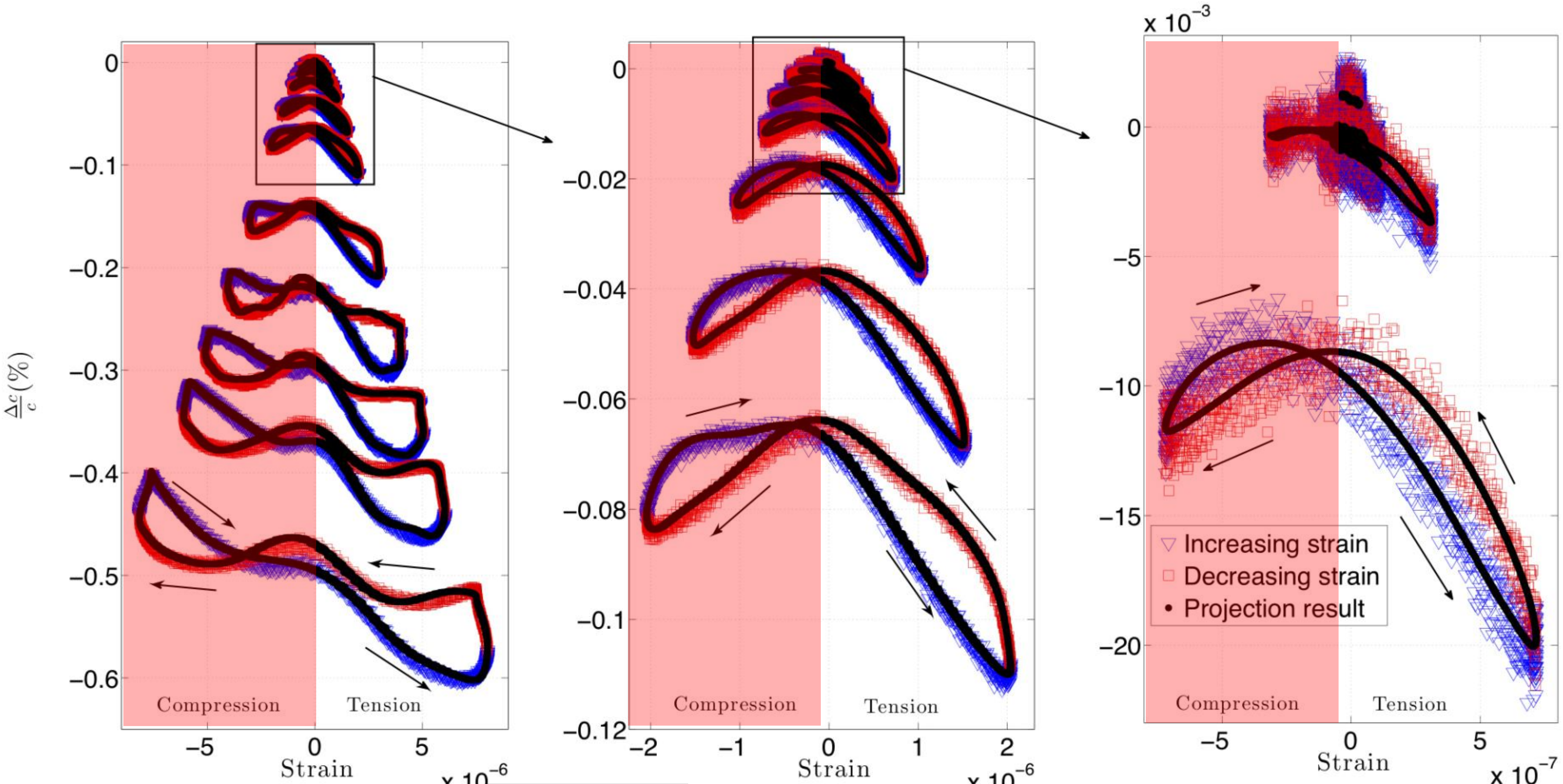
Fundamental mode resonance

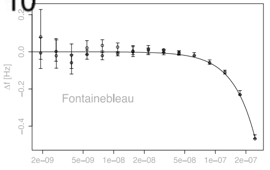
ultrasonic pulse

low-frequency piezoceramic source

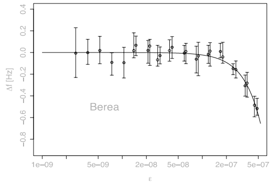
backload

Dynamic acousto-elasticity reveals exotic behavior





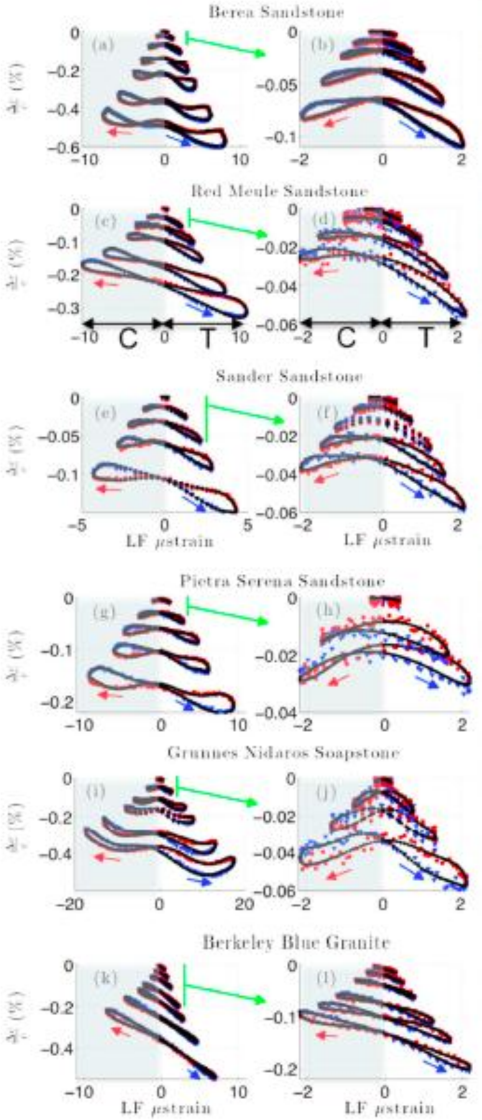
Note progressive offset with amplitude—’conditioning’= resonance peak shift under resonance conditions

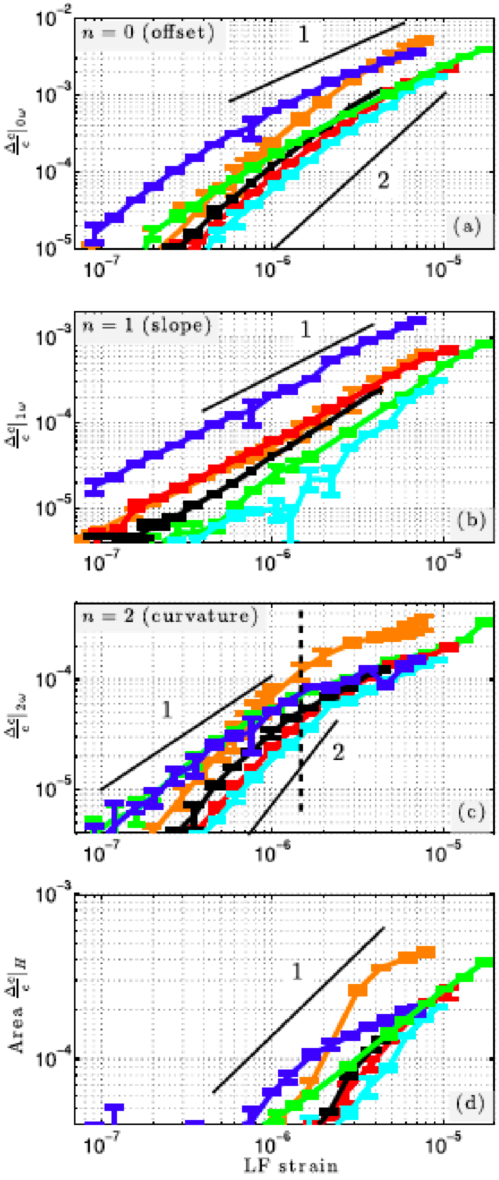


Highly complex behavior that can be described with additional harmonics

Riviere et al., JAP (2013)

Nonlinear parameters from DAE



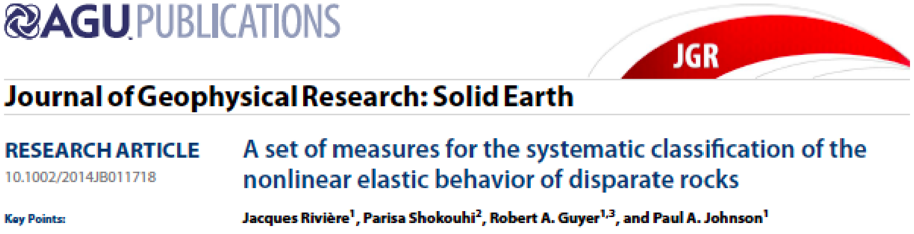


offset

slope

curvature

hysteresis


AGU PUBLICATIONS
JGR
Journal of Geophysical Research: Solid Earth

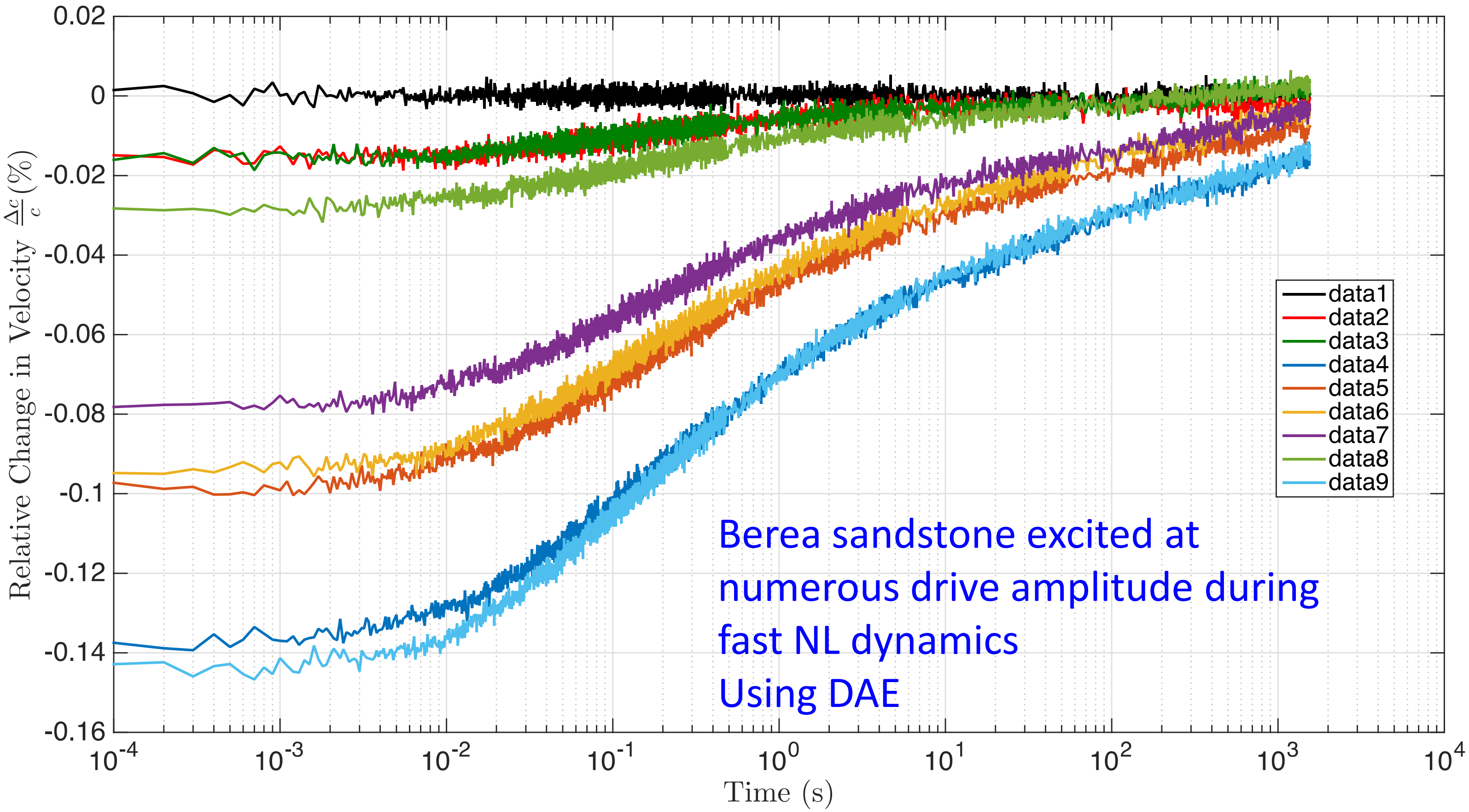
RESEARCH ARTICLE
10.1002/2014JB011718

A set of measures for the systematic classification of the nonlinear elastic behavior of disparate rocks

Key Points:

Jacques Rivière[1], Parisa Shokouhi[2], Robert A. Guyer[1,3], and Paul A. Johnson[1]

Recovery– the slow dynamics: early time



Berea sandstone excited at numerous drive amplitude during fast NL dynamics

Using DAE

Spectrum of barriers…….

From Riviere et al., *in preparation (2017)*

It doesn’t matter how beautiful your theory is, it doesn’t matter how smart you are. If it doesn’t agree with experiment, it is wrong.

Richard P. Feynman

Some Models of non-classical nonlinearity

Applicability

P-M, Ahrennius model

Bentahar et al.,
Phys Rev. B, 2016

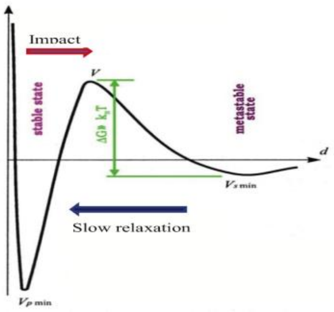
Phenomenologic, basic hystertic units:

$$s = K_0\left(e + be^2 - a\left[(De)e + sign(\dot{e})\frac{e^2 - (De)^2}{2}\right]\right) + \exp(1/KT)$$

All systems

Contact physics model

Lebedev and
Ostrovsky
Acoust. Phys.
2014.



Relaxation of metastable contacts:

$$\frac{dE_s}{dt} = AE_0 \exp\left(-\frac{E_s}{\Lambda}\right) \quad (\Lambda = kT/V)$$

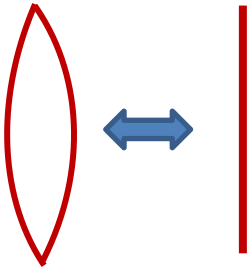
Sound speed relaxation (analytical result)

$$\frac{\Delta c}{c} = \frac{E_s}{2E_0} = \frac{\Lambda}{2E_0}\ln\left[\frac{t}{\tau} + e^{-|E_{s0}/\Lambda|}\left(1 - \frac{t}{\tau}\right)\right]$$

Consolidated and pre-compressed unconsolidated materials.
Models hysteresis and slow time relaxation
Provides a contact scale

Crack healing model

Snieder et al,
Geophys. J. Intern.
Adv. Access, 2016



Superposition of relaxations:

$$R(t) = \int_{\tau_{min}}^{\tau_{max}} \frac{1}{\tau} e^{-t/\tau} d\tau$$

Strain relaxation:

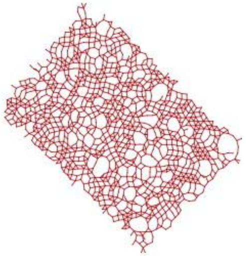
$$\dot{\varepsilon} = -\frac{1}{n\eta}\left(N\sigma - \sum_{i,\ closed} \kappa\varepsilon_i\right)$$

Snider, next

Consolidated materials with microcracks.
Models hysteresis and slow time relaxation

STZ model

Lieu et al,
JGR Solid Earth
2017 (submitted)



Effective temperature

$$\Lambda \equiv \frac{N_+ + N_-}{N} = 2e^{-1/\chi}$$

Linear resonance:

$$\omega_{\text{res}}^2 = \left(\frac{\pi}{H}\right)^2 \frac{M}{\rho G}$$

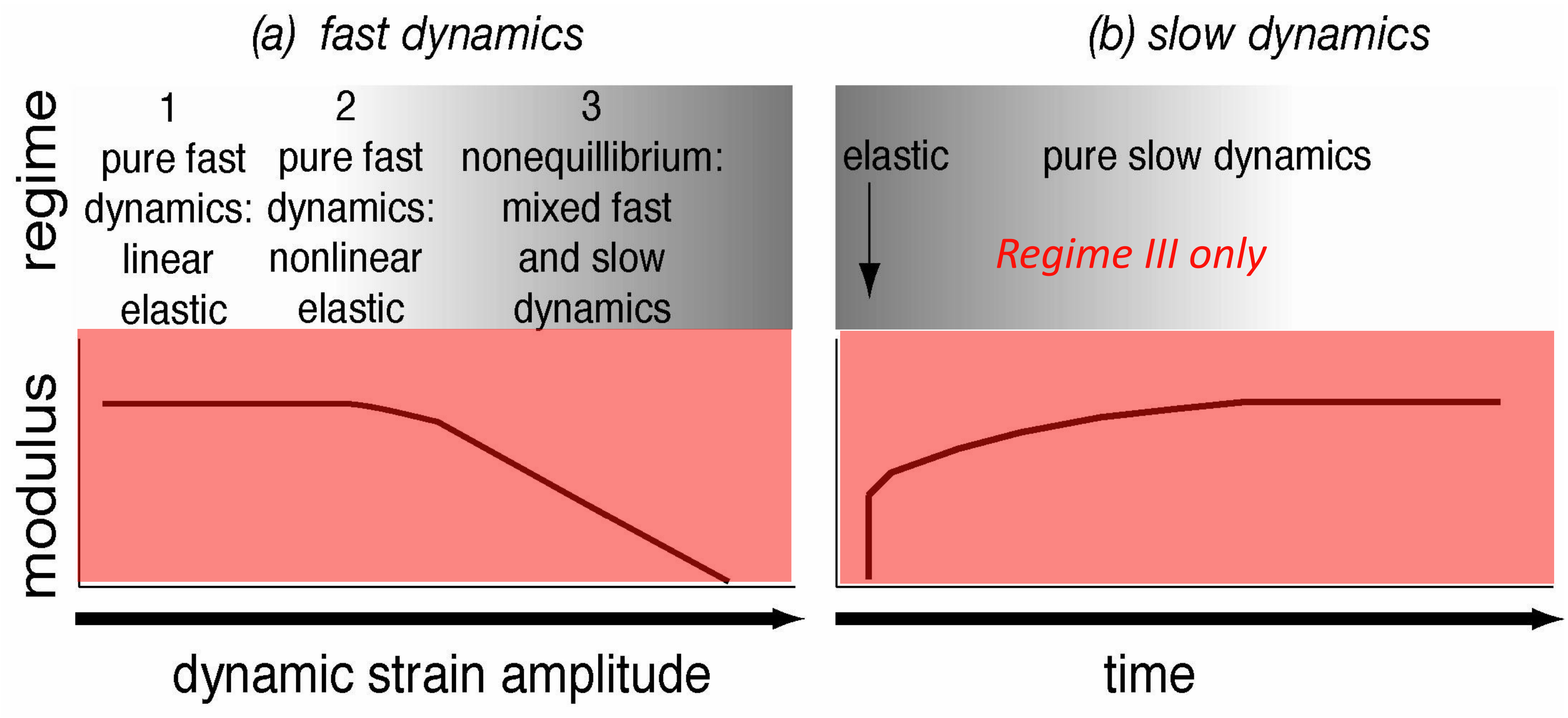
[Daub last Monday]

Non- consolidated materials with grain restructuring.
Models fast time processes
Including nonlinear resonance and losses

”the story is true. I may have skipped over some of the details along the way.”

The Ben Zion, Cargese, 2017

Take away: nonlinear fast and slow dynamics



Regime III only