Sensitivity functions for static and dynamic mapping of Earth's heterogeneity

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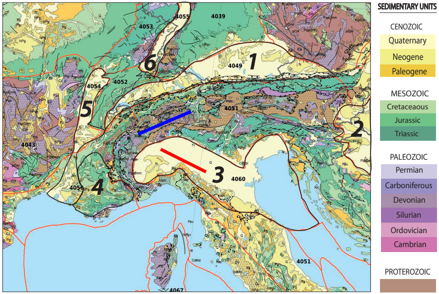
Passive imaging from seismology to ultrasound

Content

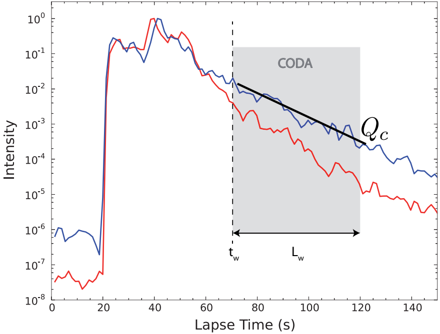
Static imaging of small-scale heterogeneity

Spatial variation of attenuation in the Alps

Geological Map



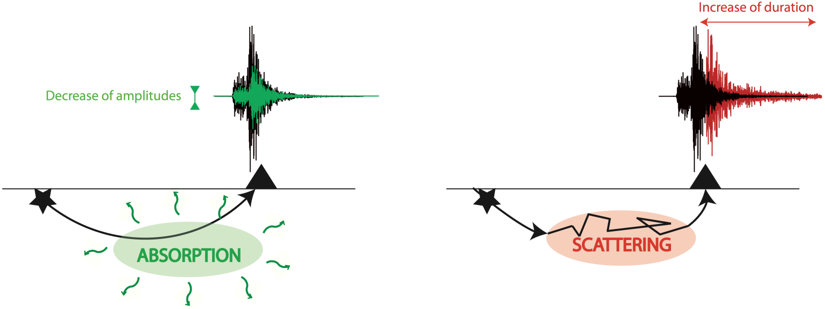
Energy Decay at 6 Hz



- $Q_c \approx 740$ in crystalline massifs
- $Q_c \approx 430$ in the Pô Plain
- Clear lateral variations of propagation properties at 100kms scale

Basic mechanisms of attenuation

$$\text{Attenuation } Q^{-1} = \text{Absorption } Q_i^{-1} + \text{Scattering } Q_{sc}^{-1}$$



- Absorption = **True** Loss

- Scattering = **Apparent** Loss
- Attenuation of coherent wave
- Generation of coda waves

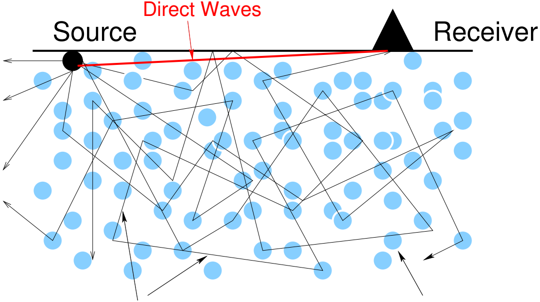
Goal: Retrieve distribution of Q_{sc}^{-1}, Q_i^{-1} from envelopes characteristics

Needed: Quantify impact of scattering/absorption anomalies on energy envelopes?

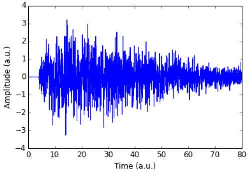
Dynamic imaging: coda wave interferometry

- Repeatable source
- Heterogeneous medium

Initial State



Waveform



Dynamic imaging: coda wave interferometry

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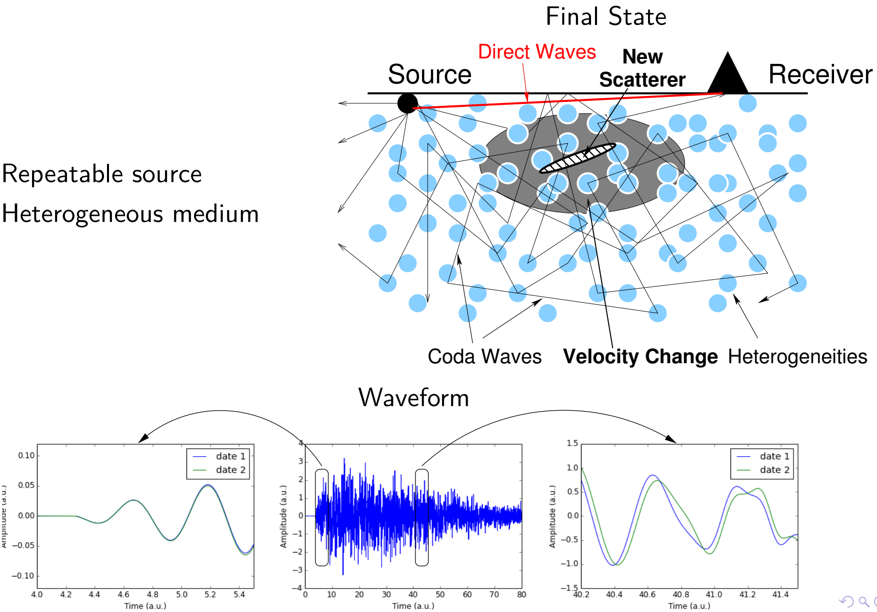
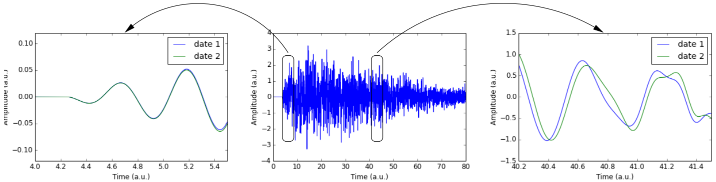


Quantification and interpretation of waveform changes



Travel time perturbation (Poupinet et al., 1984; Snieder et al., 2002, 2004)

- Delay time as a function of lapse-time t : $\delta t(t)$
- Weak velocity changes in the medium $\frac{\delta c}{c}$

Distortion of waveforms (Larose et al., 2010; Obermann et al., 2013; Planès et al., 2014)

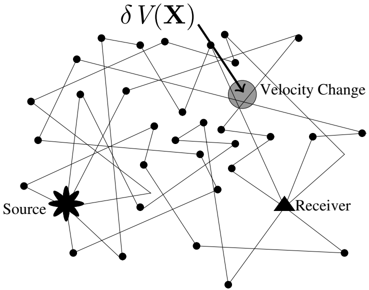
- Decorrelation coefficient : $dc(t) = 1 - \frac{\langle u_1 u_2 \rangle_t}{\sqrt{\langle u_1^2 \rangle_t \langle u_2^2 \rangle_t}}$
- Addition of new scatterer

Goal of this talk:

Present sensitivity kernels for each observable valid in a broad range of regimes

The random walk approach to travel-time and absorption sensitivity functions

Idea: the sensitivity at time t in the coda is proportional to the time spent by the waves in the volume where the change occurs. (Pacheco & Snieder, 2005)



Velocity perturbation

$$\underbrace{\delta t(t)}_{\text{Delay in the coda}} = -\frac{\delta c}{c}(\mathbf{X}) \times \underbrace{T(\delta V(\mathbf{X}), t)}_{\text{Time spent in } \delta V}$$

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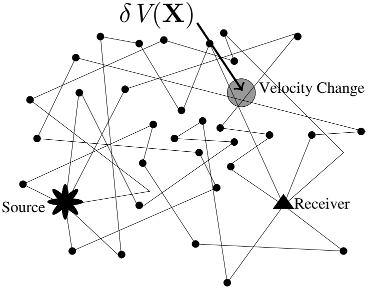


Velocity perturbation

$$\underbrace{\delta t(t)}_{\text{Delay in the coda}} = -\frac{\delta c}{c}(\mathbf{X}) \times \underbrace{T(\delta V(\mathbf{X}), t)}_{\text{Time spent in } \delta V}$$

Absorption perturbation

$$\underbrace{\frac{\delta I}{I}}_{\text{Intensity P}^\circ} = -\underbrace{\omega}_{\text{Freq.}} \delta Q_i^{-1}(\omega, \mathbf{X})\, T(\delta V(\mathbf{X}), t)$$

$$T(\delta V(\mathbf{X}), t) = \underbrace{K_{tt}(\mathbf{X}, t)}_{\text{Sensitivity Kernel}} \times \delta V(\mathbf{X})$$

Calculation of the sensitivity kernel

Application of Bayes Formula:

$$K_{tt}(\mathbf{X},t) = \int_0^t \frac{\overbrace{P(\mathbf{R},\mathbf{X},t-t')}^{\text{Probability to go from Change to Receiver}} \quad \overbrace{P(\mathbf{X},\mathbf{S},t')}^{\text{Probability to go from Source to Change}}}{\underbrace{P(\mathbf{R},\mathbf{S},t)}_{\text{Probability to go from Source to Receiver}}} dt'$$

Fundamental Property of the Kernel: $\displaystyle\int_{\text{Full Space}} K_{tt}(\mathbf{X},t)\, dV(\mathbf{X}) = t$

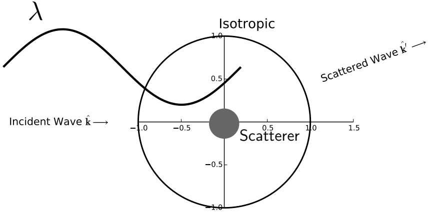
Diffusion Model for the Probability:

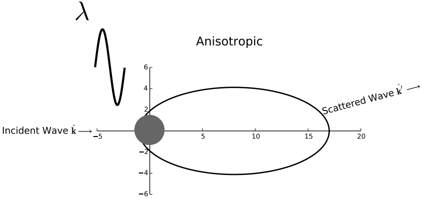
$$\frac{\partial P(\mathbf{X},\mathbf{S},t)}{\partial t} - D\nabla^2 P(\mathbf{X},\mathbf{S},t) = \delta(\mathbf{X}-\mathbf{S})\delta(t)$$

$$D = \frac{cl}{d} = \frac{\text{velocity} \times \text{mean free path}}{\text{space dimension}}$$

Limitations of the diffusion approximation

- Only diffuse waves. No ballistic waves.
- Only valid for MFP $\ll$ propagation distance, i.e; $l \ll |\mathbf{SX}|$
- Poor job at modeling scattering anisotropy





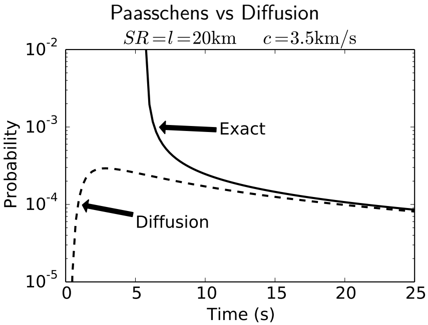
$$l \rightarrow \overbrace{l^*}^{\text{Transport MFP}} = \frac{l}{1 - \underbrace{\langle \widehat{\mathbf{k}} \cdot \widehat{\mathbf{k}'} \rangle}_{\text{mean cosine of scattering angle}}}$$

Can we extend the Pacheco & Snieder theory beyond diffusion?

The Paasschen's solution (Paasschens, 1997; Shang & Gao, 1988)

An *exact* solution of the isotropic random walk problem in 2-D

$$P(\mathbf{X},\mathbf{S},t) = \overbrace{\frac{e^{-ct/l}}{2\pi SX}\delta(ct - SX)}^{\text{Ballistic Propagation}} + \overbrace{\frac{\overbrace{\theta(ct - SX)}^{\text{Causality}}\, e^{\sqrt{c^2t^2 - SX^2}/l - ct/l}}{2\pi l\sqrt{c^2t^2 - SX^2}}}^{\text{Diffuse Propagation}} \quad (1)$$



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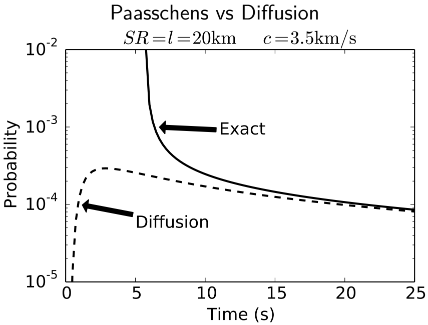
What if we substitute (1) into Pacheco & Snieder Formula?



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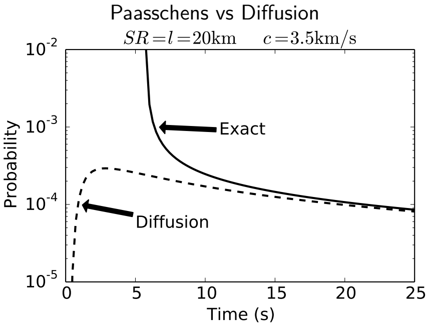
What if we substitute (1) into Pacheco & Snieder Formula?

!!! $$\int_{\text{Full Space}} K_{tt}(\mathbf{X},t)\,dV(\mathbf{X}) \neq t$$ **!!!**



How to fix this problem?

The radiative transfer approach to the travel time kernel

$$\overbrace{\left(\frac{\partial}{\partial t} + c\widehat{\mathbf{k}} \cdot \nabla_{\mathbf{X}} + \frac{c}{l}\right)}^{\text{Coherent Propagation}} P(\mathbf{X}, \widehat{\mathbf{k}}, \mathbf{S}, t) = \frac{c}{l} \overbrace{\oint \overbrace{p(\widehat{\mathbf{k}}, \widehat{\mathbf{k}}')}^{\text{Scattering anisotropy}} P(\mathbf{X}, \widehat{\mathbf{k}}', \mathbf{S}, t)\, d^2\hat{k}'}^{\text{Multiple Scattering Term}} + \overbrace{\delta(\mathbf{X} - \mathbf{S})\delta(t)}^{\text{Source term}}$$

$P(\mathbf{X}, \widehat{\mathbf{k}}, \mathbf{S}, t)$: Probability that a random walker emitted at source $\mathbf{S}$ at time $t = 0$ be at position $\mathbf{X}$ *and direction* $\widehat{\mathbf{k}}$ at time t

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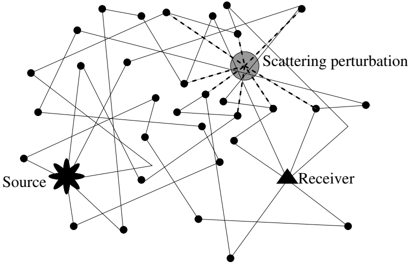
$$K_{tt}(\mathbf{X}, t) = \int_0^t \oint \frac{\overbrace{P(\mathbf{X}, -\widehat{\mathbf{k}}, \mathbf{R}, t - t')}^{\substack{\text{Probability to go} \\ \text{from Change to Receiver}}} \quad \overbrace{P(\mathbf{X}, \widehat{\mathbf{k}}, \mathbf{S}, t')}^{\substack{\text{Probability to go} \\ \text{from Source to Change}}}}{\underbrace{\oint P(\mathbf{R}, \widehat{\mathbf{k}}, \mathbf{S}, t)\, d\hat{k}}_{\text{Probability to go from Source to Receiver}}} d^2\hat{k}\, dt'$$

- Diffusion Limit: $l \ll |\mathbf{SX}| \longrightarrow$ Pacheco & Snieder, 2005
- Single-scattering Limit: $l \gg |\mathbf{SX}| \longrightarrow$ Pacheco & Snieder 2006

Decorrelation and intensity scattering kernel

Idea: consider the new scattering paths created by the object



- Addition of an isotropic scatterer σ at $\mathbf{X}$
- Local perturbation $\delta Q_{sc}^{-1}(\mathbf{X})$

Decorrelation

$$dc \approx \frac{\overbrace{\langle (u_2 - u_1)^2 \rangle}^{\substack{\text{Intensity of the field difference} \\ = \text{Intensity of scattered field}}}}{2I_1}$$

$dc \propto$ Extra-scattered intensity

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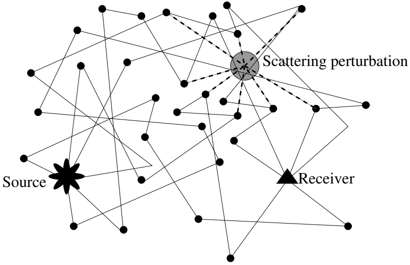

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$dc \propto$ Extra-scattered intensity

Intensity

$$\frac{\delta I}{I} \approx \frac{\overbrace{\langle I_2 - I_1 \rangle}^{\text{Difference of the Intensity fields}}}{I_1}$$

$$\frac{\delta I}{I} \propto \text{Extra-scattered Intensity} - \text{Extra scattering attenuation}$$

Decorrelation and intensity scattering kernel

- DC due to addition of an isotropic scatterer σ at $\mathbf{X}$
- Change of I caused by δQ_{sc}^{-1} in $\delta V(\mathbf{X})$

Decorrelation

$$dc(t) = \frac{c\sigma}{2} K_{dc}(\mathbf{X}, t)$$

Intensity

$$\frac{\delta I}{I}(t) = \omega \delta Q_{sc}^{-1}(\mathbf{X}, \omega) \left(K_{dc}(\mathbf{X}, t) - K_{tt}(\mathbf{X}, t) \right) \delta V$$

$$K_{dc}(\mathbf{X}, t) = \int_0^t \overbrace{\frac{P(\mathbf{X}, \mathbf{R}, t-t')P(\mathbf{X}, \mathbf{S}, t')\,dt'}{P(\mathbf{R}, \mathbf{S}, t)}}^{\text{The probabilities are integrated over all } \widehat{\mathbf{k}} \text{ for an isotropic scatterer}}$$

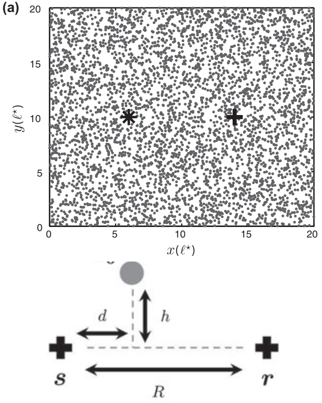
Formally Identical to Pacheco & Snieder !!!

To be compared with:

$$K_{tt}(\mathbf{X}, t) = \int_0^t \oint \frac{P(\mathbf{X}, -\widehat{\mathbf{k}}, \mathbf{R}, t-t')P(\mathbf{X}, \widehat{\mathbf{k}}, \mathbf{S}, t')}{P(\mathbf{R}, \mathbf{S}, t)} d^2\hat{k}\,dt'$$

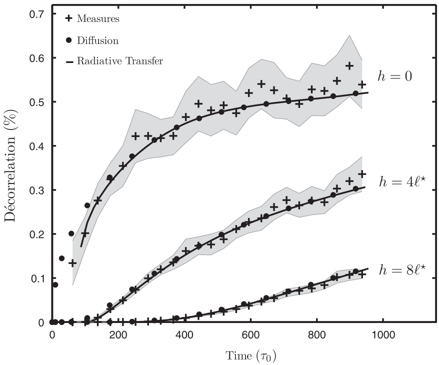
Validation of the radiative transfer approach for K_{dc}

Numerical Simulation



Addition of a *single* isotropic scatterer

Numerics vs Theory



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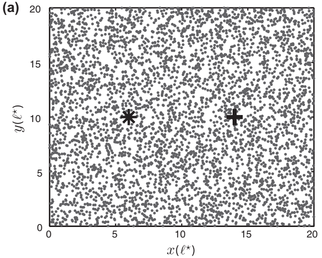

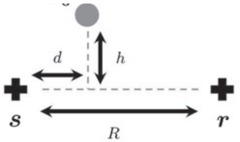

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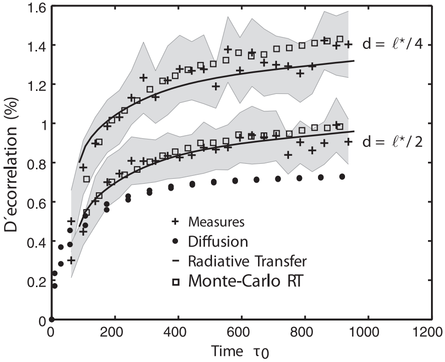


Good agreement between radiative transfer theory and wavefield simulations

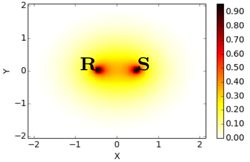
(Planès et al., 2014)

Travel-time, decorrelation, and intensity kernels

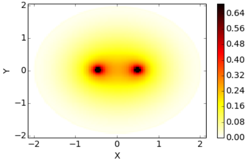
2-D calculation $l = 35$km, $SR = l$, $t = 40$s

Isotropic $\langle \widehat{\mathbf{k}} \cdot \widehat{\mathbf{k}}' \rangle = 0$

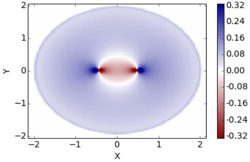
Traveltime



Decorrelation



Intensity



Travel-time, decorrelation, and intensity kernels

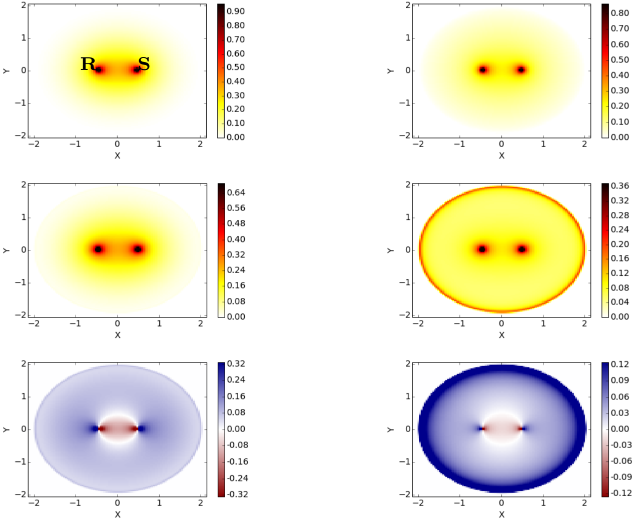
2-D calculation $l = 35$km, $SR = l$, $t = 40$s

Isotropic $\langle \widehat{\mathbf{k}} \cdot \widehat{\mathbf{k}'} \rangle = 0$ | Anisotropic $\langle \widehat{\mathbf{k}} \cdot \widehat{\mathbf{k}'} \rangle = 2/3$

Traveltime

Decorrelation

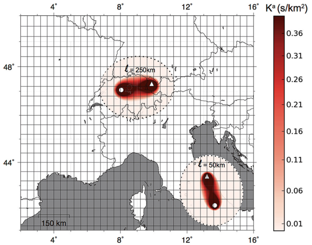
Intensity



Principle of the mapping:

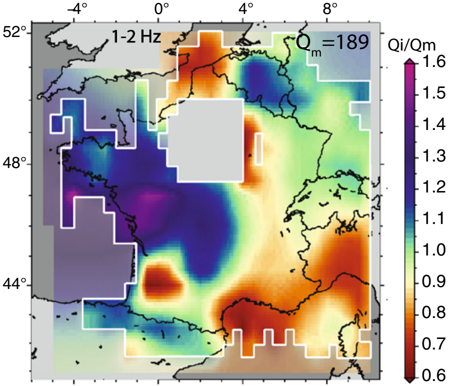
$$Q_c^{-1}(\mathbf{R},\mathbf{S},t) \approx t^{-1}\int Q_i^{-1}(\mathbf{X})K_{tt}(\mathbf{R},\mathbf{X},\mathbf{S};t)dV(\mathbf{X})$$

- Discretization of the kernel on a grid:



- Sensitivity on pixels crossed by the ray

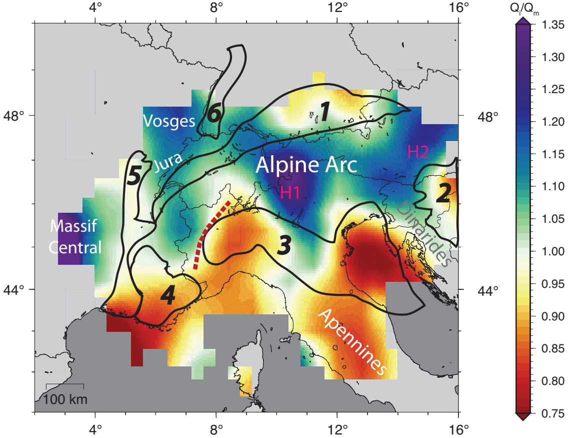
88000 Waveforms



(Mayor et al., Bull..Earthquake Eng, 2017)

Absorption quality factor mapping at 1.5Hz in the Alps

- 40000 waveforms
- Sensitivity ∼ Direct ray
- $Q_m \approx 200$
- Correlation with surface geology
- 1: Molasse Basin; 2: Pannonian Basin; 3: Pô Plain; 4: Southeast France Basin; 5: Rhône Valley; 6: Rhine Graben
- H1: Adamello intrusive complex; H2: Pohorje Pluton



(Mayor et al., E.P.S.L., 2016)

Conclusions

$$
\begin{aligned}
K_{dc}(\mathbf{X},t) &\neq K_{tt}(\mathbf{X},t) \\
K_{abs}(\mathbf{X},t) &= K_{tt}(\mathbf{X},t) \\
K_{sc}(\mathbf{X},t) &= K_{dc}(\mathbf{X},t) - K_{tt}(\mathbf{X},t)
\end{aligned}
$$

- Anisotropic scattering can strongly influence the sensitivity
- Implications for monitoring remain to be determined
- Attenuation structure is important for seismic hazard assessment
- Future works: extension to 3-D, elastic, sensitivity with depth (ongoing)
- Further details:

Planès et al., Waves in random and complex media, 24, 99-125, 2014.

Validation of the DC sensitivity kernel with full waveform simulations

Margerin et al., Geophysical Journal International, 204, 650-666, 2016.

Numerical calculation of the kernels

Mayor et al., Earth and Planetary Science Letters, 439, 71-80, 2016;

Attenuation structure of the Alps