Evaluate a level of groundmotion requires to know:

-the probability of occurrence of an earthquake at any place

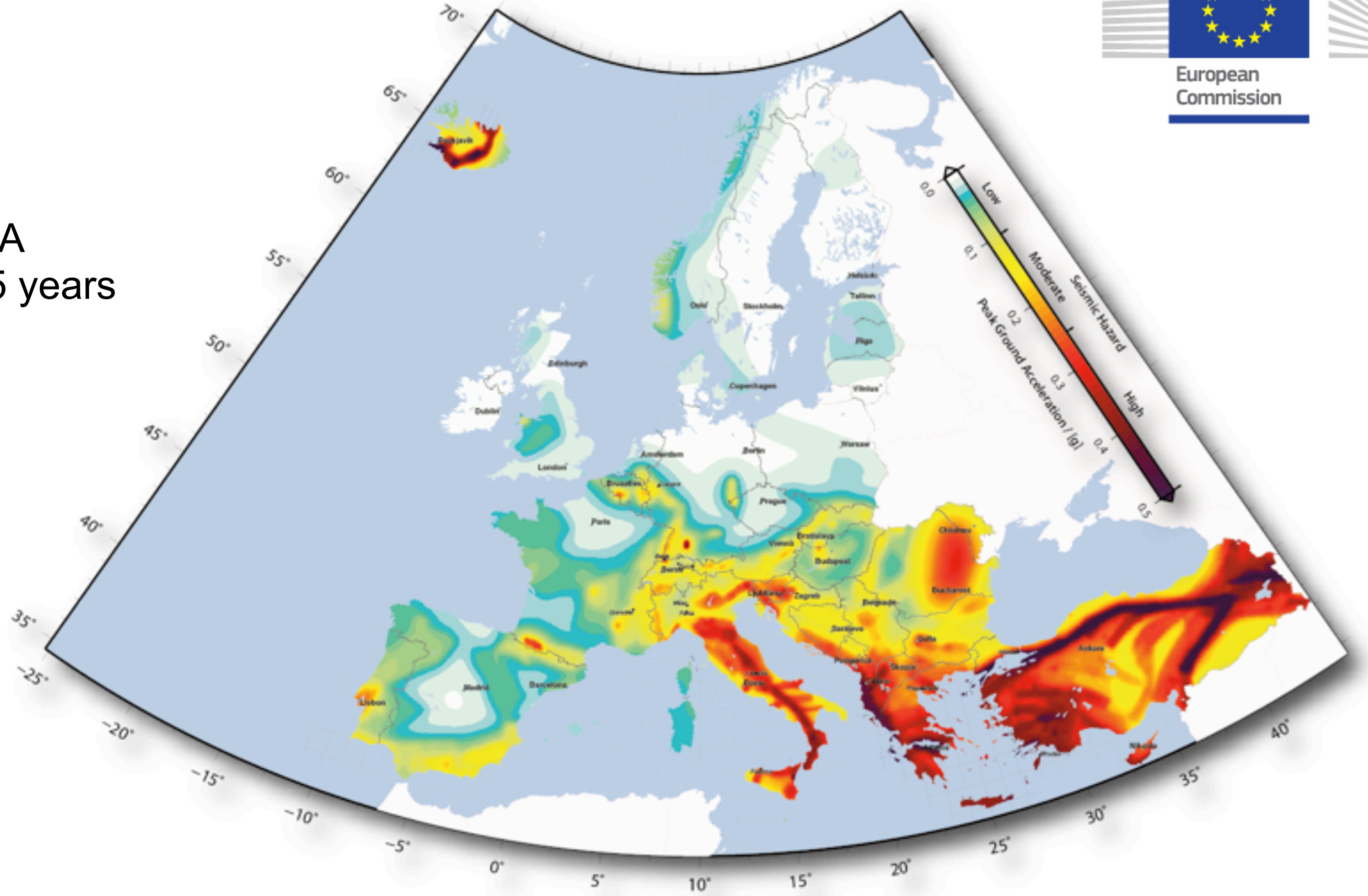
-the ground motion associated with this earthquake at a given distance and the associated variability (emprical information+simple functional forms)

Example share: peak acceleration associated for a probability of 2% on a period of 50 years

50 years: lifetime of a building..
2%: acceptability (subjective)

SHARE-GEM hazard map released in June

PGA
475 years

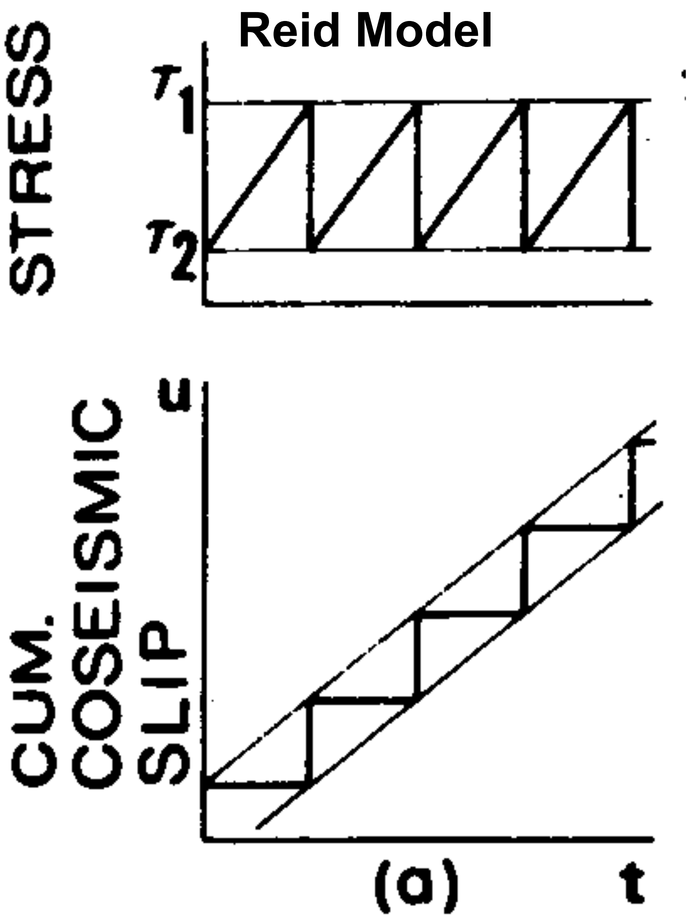


D. Giardini, J. Woessner, L. Danciu, H. Crowley, F. Cotton, G. Gruenthal, R. Pinho, G. Valensise, S. Akkar, R. Arvidsson, R. Basili, T. Cameelbeck, A. Campos-Costa, J. Douglas, M. B. Demircioglu, M. Erdik, J. Fonseca, B. Glavatovic, C. Lindholm, K. Makropoulos, F. Meletti, R. Musson, K. Pitilakis, K. Sesetyan, D. Stromeyer, M. Stucchi, A. Rovida, Seismic Hazard Harmonization in Europe (SHARE): Online Data Resource, doi: 10.12686/SED-00000001-SHARE, 2013.

Earthquake statistics

Earthquake statistics and Probabilistic Seismic Hazard

1. Seismic gap theory and earthquake probability (with and without memory)
2. Frequency-magnitude relationships « Gutenberg Richter law » and earthquakes probabilities (without memory).
3. Aftershocks
4. Probabilistic Seismic Hazard Assessment
5. Research needed ?
6. Exercices

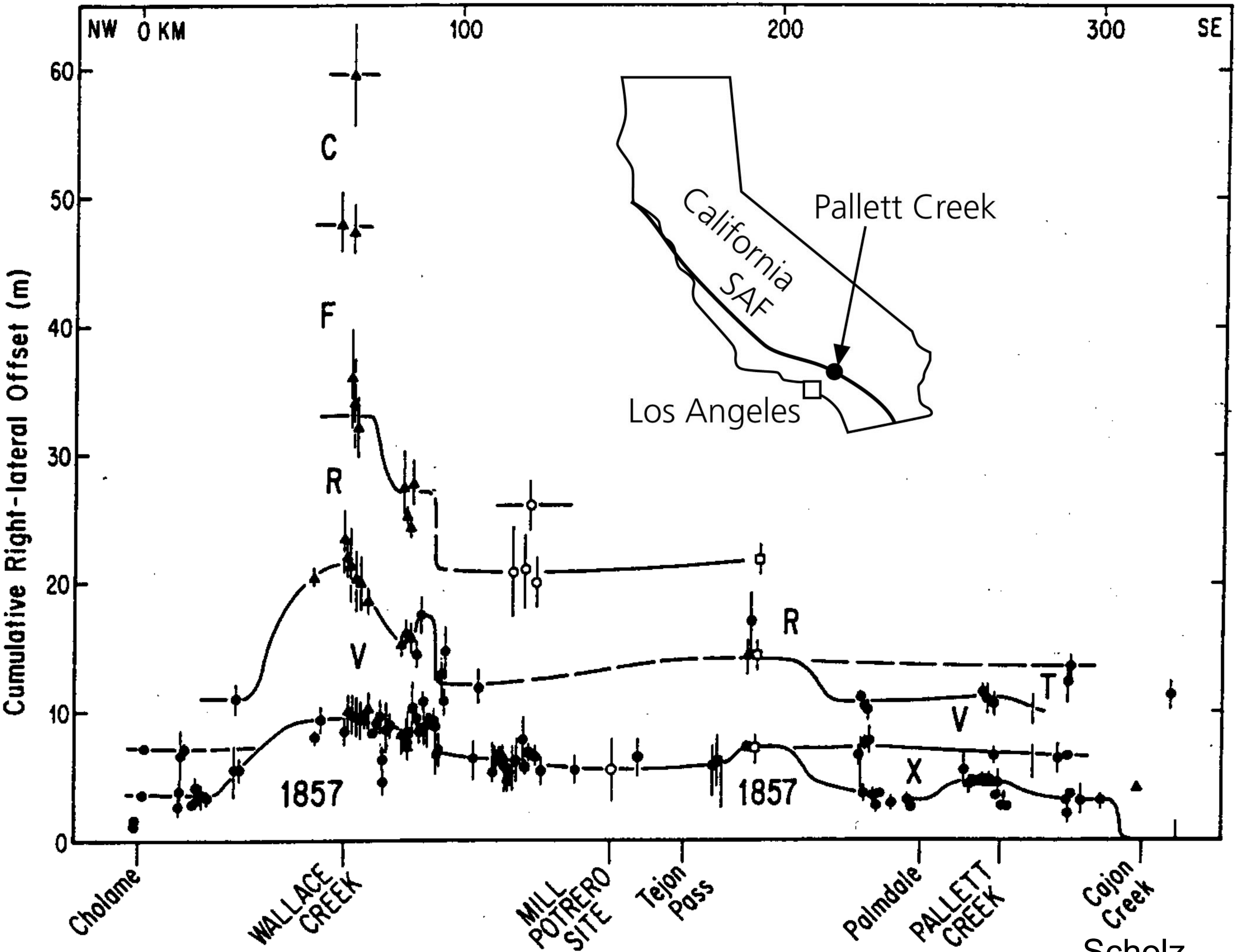


Is the reality so simple ?

RI : recurrence interval

Scholz, 1989

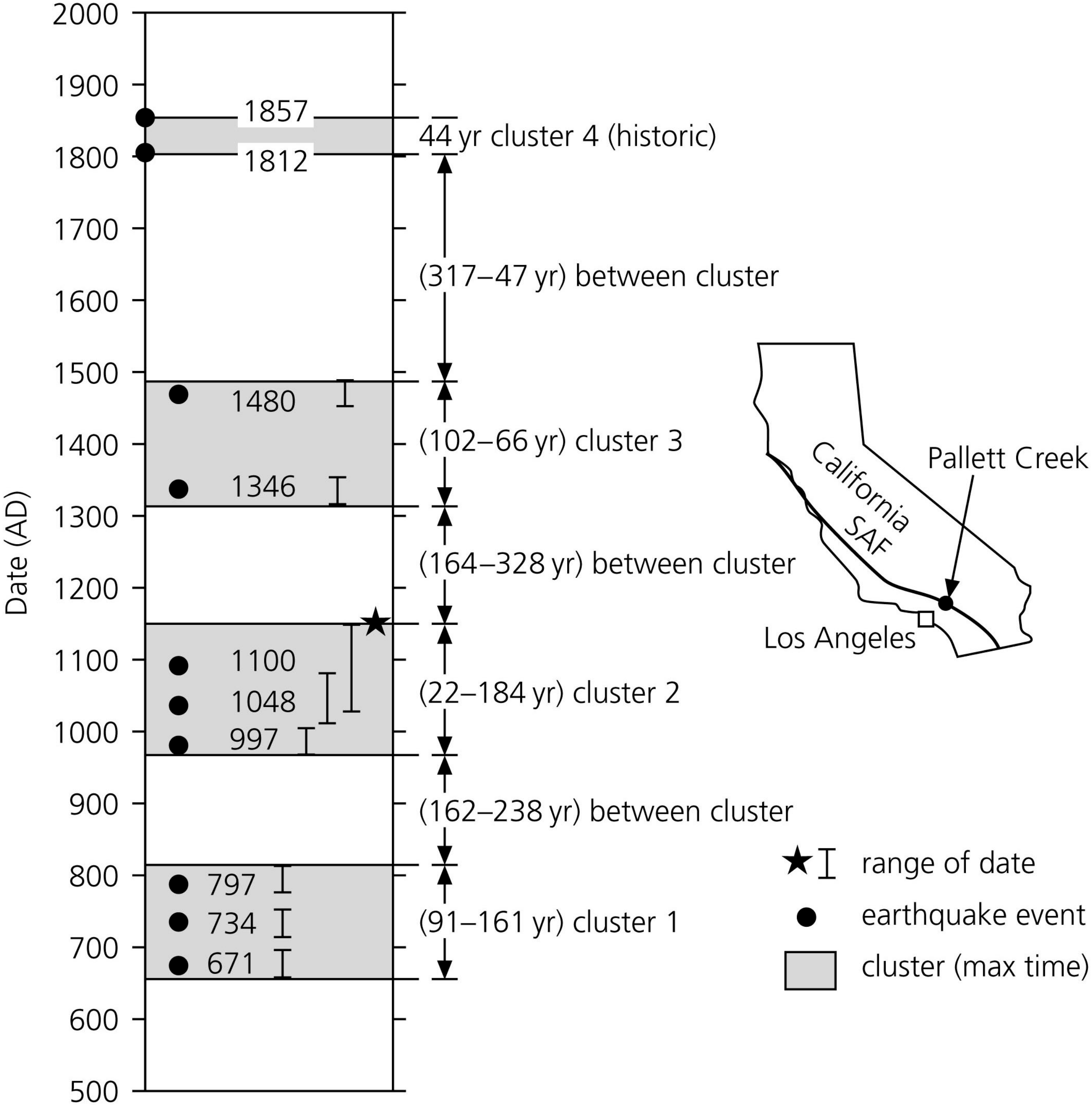
Source : Iris



NW
0 KM
100
200
300
SE
Cumulative Right-lateral Offset (m)
60
50
40
30
20
10
0
C
F
R
V
R
T
V
X
1857
1857
California
SAF
Pallett Creek
Los Angeles
Cholame
WALLACE CREEK
MILL POTRERO SITE
Tejon Pass
Palmdale
PALLETT CREEK
Cajon Creek

Scholz

Figure 1.2-15: Paleoseismic time series for the San Andreas near Pallett Creek.



SW

Gaussian (normal), log normal and Poisson statistics

C.7.2 Normal Distribution

The most commonly used probability distribution in statistics is the *normal distribution* (or *Gaussian distribution*). Its PDF, which plots as the familiar bell-shaped curve of Figure C.6a, describes sets of data produced by a wide variety of physical processes. The normal distribution is completely defined by two parameters: the mean and standard deviation. Mathematically, the PDF of a normally distributed random variable X with mean $\bar{x}$ and standard deviation σ_x is given by

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left[-\frac{1}{2}\left(\frac{x-\bar{x}}{\sigma_x}\right)^2\right] \qquad \text{(C.18)}$$

The PDF and CDF for a normal distribution are illustrated in Figure C.6. Examples of normal pdf's for random variables with different means and standard deviations are shown in Figure C.7.

Integration of the PDF of the normal distribution does not produce a simple expression for the CDF, so values of the normal CDF are usually expressed in tabular form. The

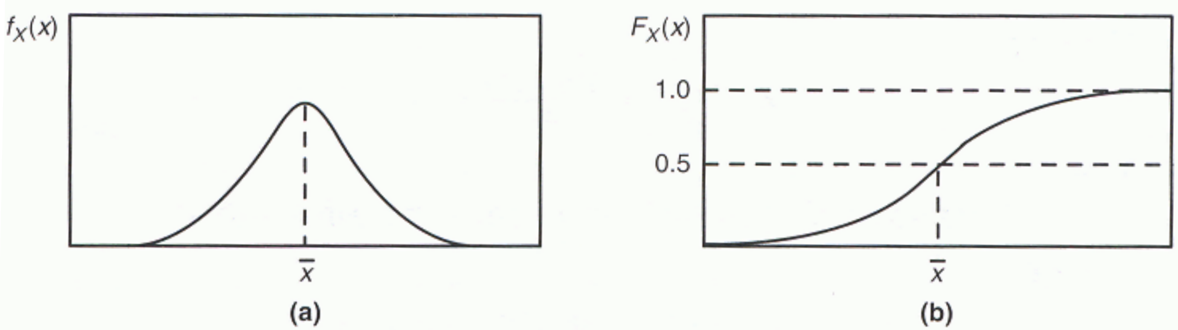


Figure C.6 Normal distribution: (a) probability density function; (b) cumulative distribution function.

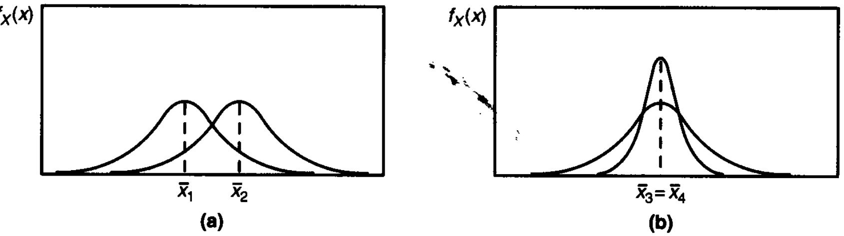


Figure C.7 Normal distributions for (a) two random variables, X_1 and X_2, with different means but the same standard deviation, and (b) two random variables, X_3 and X_4, with the same mean but different standard deviations.

normal CDF is most efficiently expressed in terms of the *standard normal variable*, Z, which can be computed for any random variable, X, using the transformation

$$Z = \frac{X - \bar{x}}{\sigma_x} \tag{C.19}$$

Whenever X has a value, x, the corresponding value of Z is $z = (x - \bar{x})/\sigma_x$. Thus, the mean value of Z is $\bar{z} = 0$ and the standard deviation is $\sigma_z = 1$. Tabulated values of the standard normal CDF are presented in Table C-1.

Example C.5

Given a normally distributed random variable, X, with $\bar{x} = 270$ and $\sigma_x = 40$, compute the probability that (a) $X < 300$, (b) $X > 350$, and (c) $200 < X < 240$.

Solution (a) For $X = 300$,

$$Z = \frac{X - \bar{x}}{\sigma_x} = \frac{300 - 270}{40} = 0.75$$

Then

$$P[X < 300] = P[Z < 0.75] = F_z(0.75) = 1 - F_z(-0.75) = 1 - 0.2266 = 0.7734$$

(b) For $X = 350$,

$$Z = \frac{X - \bar{x}}{\sigma_x} = \frac{350 - 270}{40} = 2.0$$

Then

$$P[X > 350] = P[Z > 2.0] = 1 - F_z(2.0) = F_z(-2.0) = 0.0228$$

(c) For $X = 200$,

$$Z = \frac{X - \bar{x}}{\sigma_x} = \frac{200 - 270}{40} = -1.75$$

For $X = 240$,

$$Z = \frac{X - \bar{x}}{\sigma_x} = \frac{240 - 270}{40} = -0.75$$

Then

$$P[200 < X < 240] = P[-1.75 < Z < -0.75] = F_z(-0.75) - F_z(-1.75)$$
$$= 0.2266 - 0.0401 = 0.1865$$

TABLE C-1 Values of the CDF of the standard normal distribution, $F_Z(z) = 1 - F_Z(-z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0304	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0859	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4365	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Kramer, 1996

4.4.3.1 Poisson Model

The temporal occurrence of earthquakes is most commonly described by a Poisson model. The Poisson model provides a simple framework for evaluating probabilities of events that follow a *Poisson process*, one that yields values of a random variable describing the number of occurrences of a particular event during a given time interval or in a specified spatial region. Since PSHAs deal with temporal uncertainty, the spatial applications of the Poisson model will not be considered further. Poisson processes possess the following properties:

1. The number of occurrences in one time interval are independent of the number that occur in any other time interval.
2. The probability of occurrence during a very short time interval is proportional to the length of the time interval.
3. The probability of more than one occurrence during a very short time interval is negligible.

These properties indicate that the events of a Poisson process occur randomly, with no "memory" of the time, size, or location of any preceding event.

Kramer, p128

For a Poisson process, the probability of a random variable N, representing the number of occurrences of a particular event during a given time interval is given by

$$P[N = n] = \frac{\mu^n e^{-\mu}}{n!} \tag{4.14}$$

where μ is the average number of occurrences of the event in that time interval. The time between events in a Poisson process can be shown to be exponentially distributed. To characterize the temporal distribution of earthquake recurrence for PSHA purposes, the Poisson probability is usually expressed as

$$P[N = n] = \frac{(\lambda t)^n e^{-\lambda t}}{n!} \tag{4.15}$$

where λ is the average rate of occurrence of the event and t is the time period of interest. Note that the probability of occurrence of at least one event in a period of time t is given by

$$\begin{aligned} P[N \geq 1] &= P[N = 1] + P[N = 2] + P[N = 3] + \ldots \\ &\quad + P[N = \infty] = 1 - P[N = 0] = 1 - e^{-\lambda t} \end{aligned} \tag{4.16}$$

When the event of interest is the exceedance of a particular earthquake magnitude, the Poisson model can be combined with a suitable recurrence law to predict the probability of at least one exceedance in a period of t years by the expression

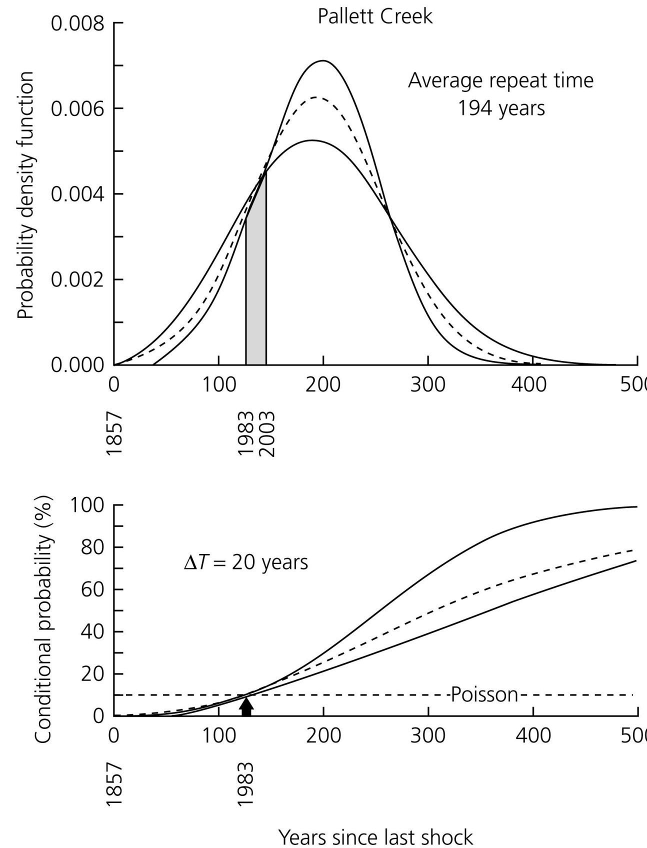
$$P[N \geq 1] = 1 - e^{-\lambda_m t} \tag{4.17}$$

Using Bayes' theorem:

Conditonal probability $C(T,T_0)$: earthquake occuring between T and T_0.

$C(T,T_0)=(P(T)-P(T_0))/(1-P(T_0))$

Figure 4.7-9: Earthquake probability estimate for the Pallett Creek segment of the San Andreas fault.



Gaussian vs Poissonian

SW

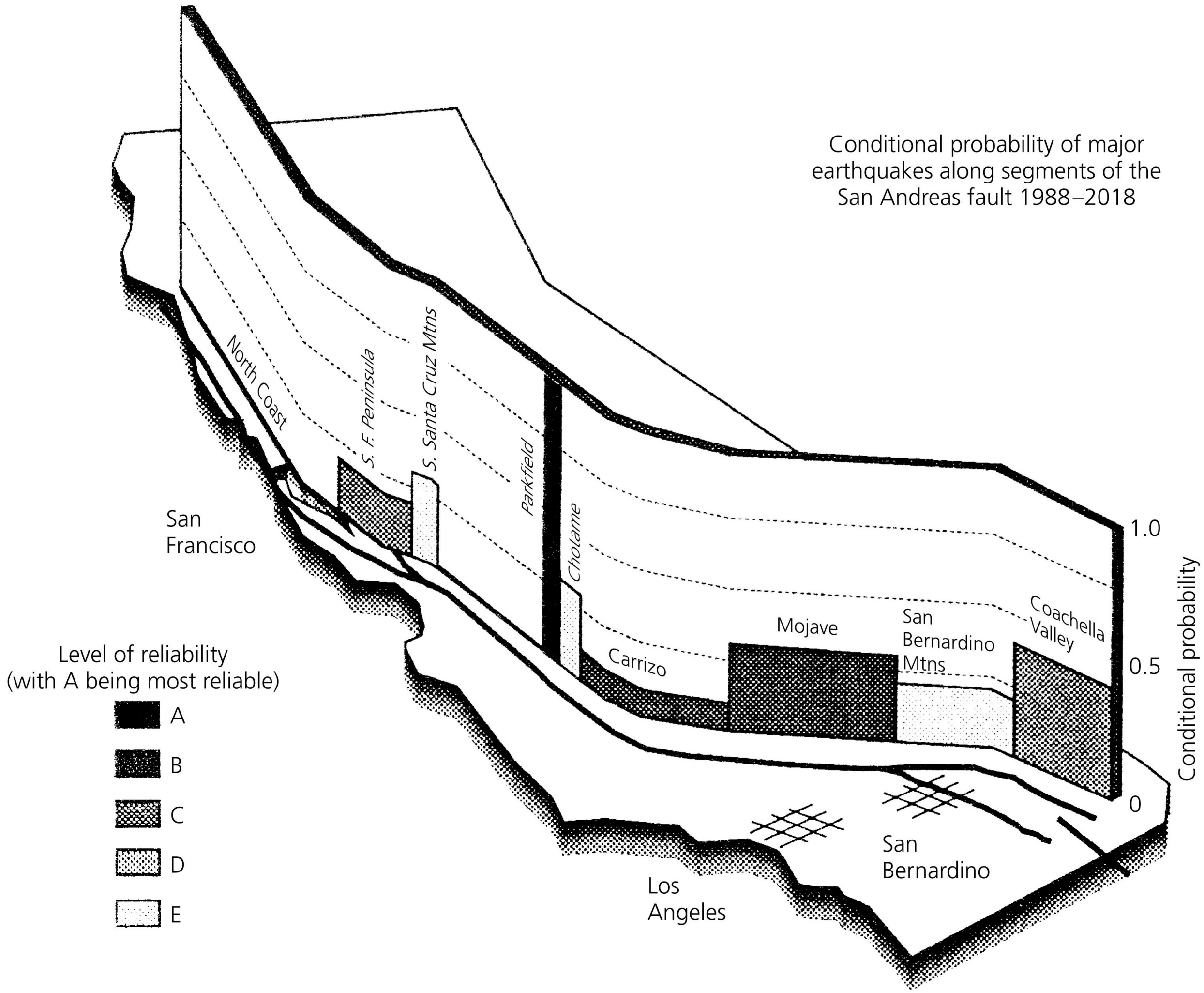
Gap theory

Provides a quantitative method to assess the relative hazard of different major fault segments.

This is near the state of the art in earthquake prediction.

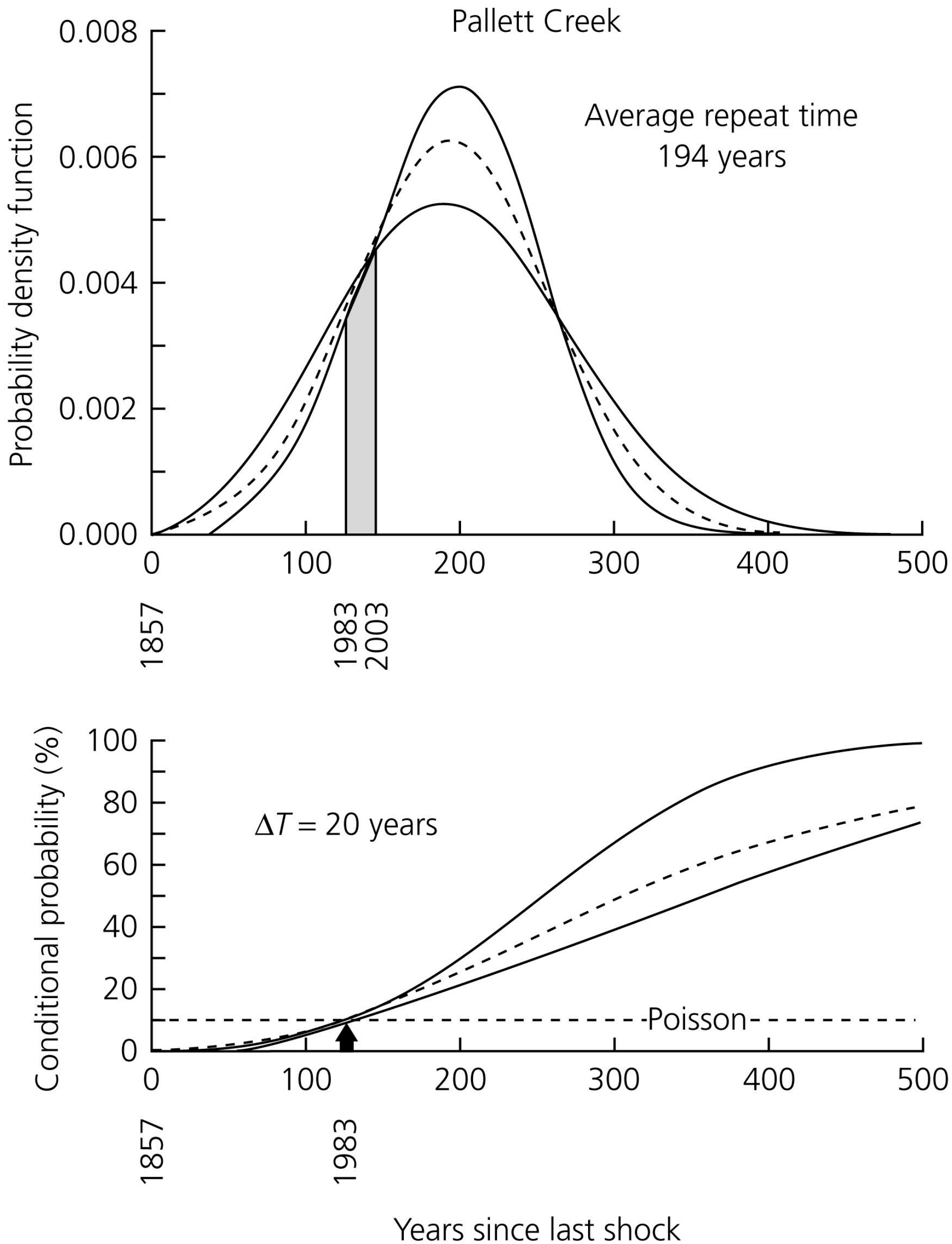
Uncertainties are high at present.

Figure 4.7-12: Conditional probabilities for various San Andreas fault segments.



Stein and Wysession, chapter 4

Figure 4.7-9: Earthquake probability estimate for the Pallett Creek segment of the San Andreas fault.



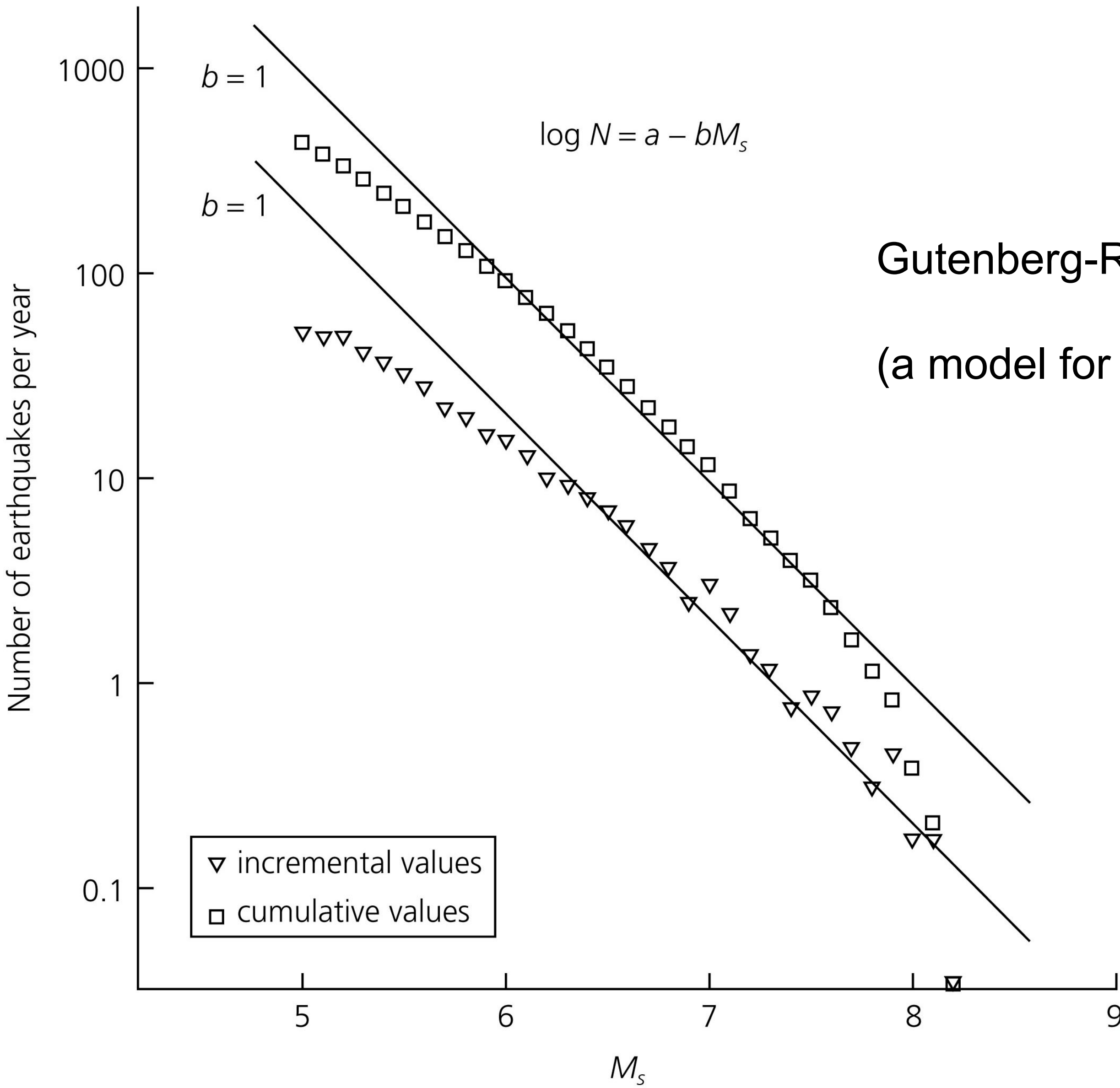
SW

EQ catalogues

- Time is in Grenwich Mean Time (GMT). This is also called Universal Time. Since earthquakes are recorded across many time zones, it is essential for seismologists to select a worldwide common time standard.
- Convention is that north latitude is positive, east longitude is positive.

Courtesy of F. Scherbaum

Figure 4.7-1: Frequency-magnitude plot for earthquakes during 1968-1997.



Gutenberg-Richter law

(a model for the data : not a law)

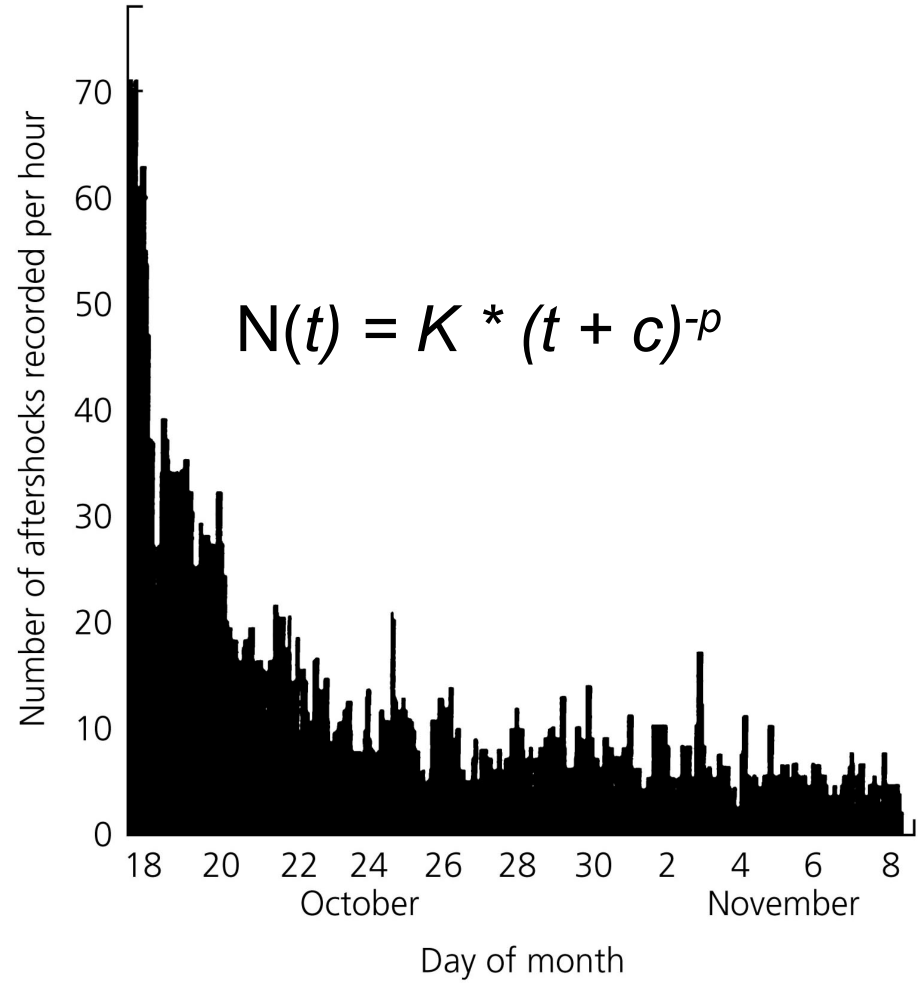
SW

Seismicity

- Main shock - largest earthquake in a sequence.
- Foreshocks - smaller earthquakes before the main shock (but there is no reliable method to determine if an event is a foreshock !!)
- Aftershocks - smaller earthquakes that follow the main shock
- Swarm - sequence of earthquakes in which several of the largest events are about the same size.

Figure 4.7-8: Aftershocks following the 1989 Loma Prieta earthquake.

Magnitude	Number	Effect
5	2	Damaging
4	20	Strong
3	65	Perceptible
2	384	Not felt
1	1855	Not felt
<1	2434	Not felt
Total	4760	



$$N(t) = K * (t + c)^{-p}$$

4760 aftershocks of the Loma Prieta earthquake had been recorded by noon on November 7, 1989. The diminishing number of aftershocks with time is typical for large California earthquakes.

SW

Generally, use declustered catalogs

- Raw seismic catalog is highly clustered.

Bulletin of the
Seismological Society of America

Vol. 64 | October 1974 | No. 5

IS THE SEQUENCE OF EARTHQUAKES IN SOUTHERN CALIFORNIA, WITH AFTERSHOCKS REMOVED, POISSONIAN?

BY J. K. GARDNER and L. KNOPOFF

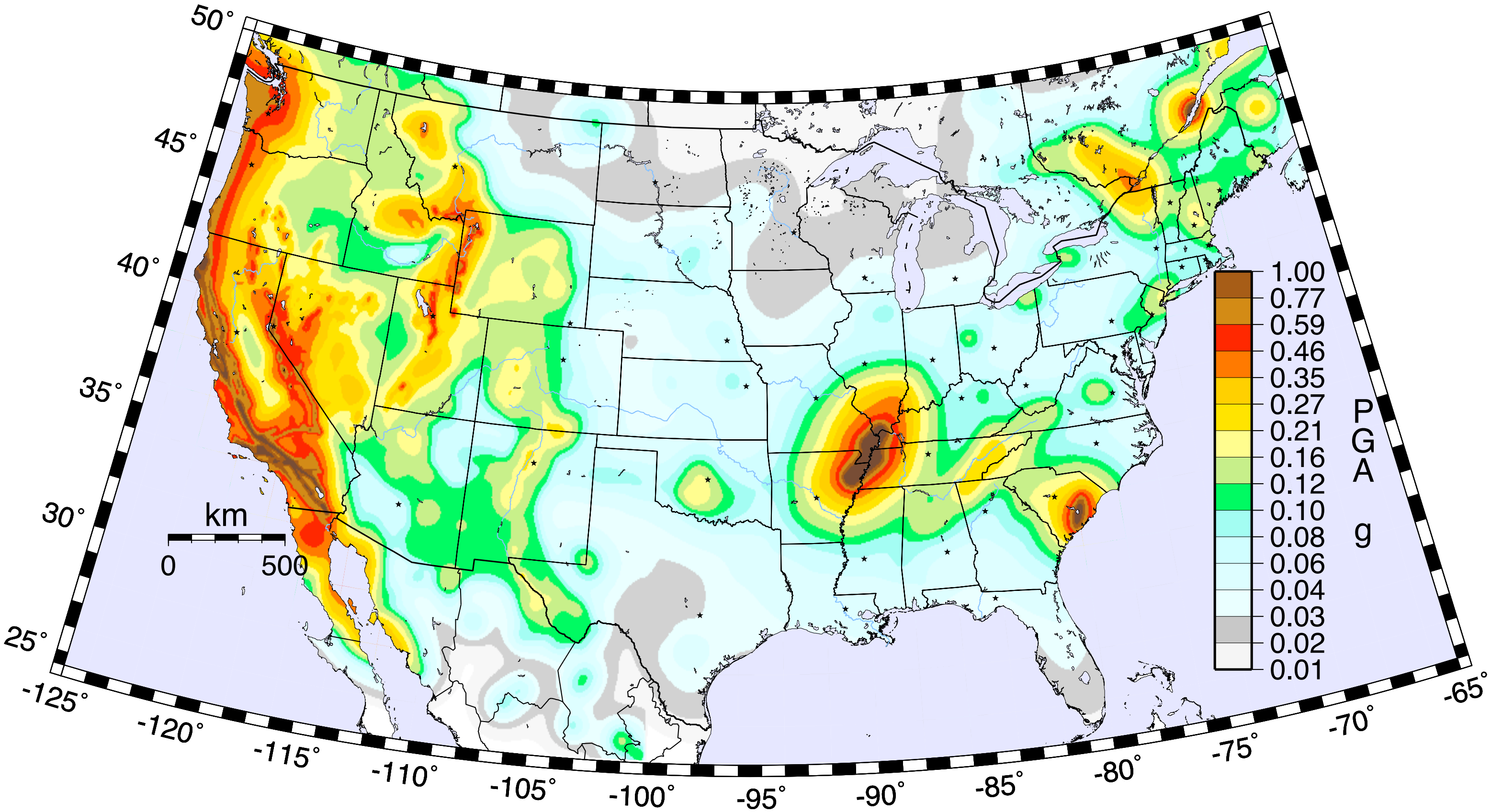
ABSTRACT

Yes.

Probabilistic Seismic Hazard Analysis (PSHA)

- Can be described as a combination of many scenarios
- Procedure follows from elementary probability theory

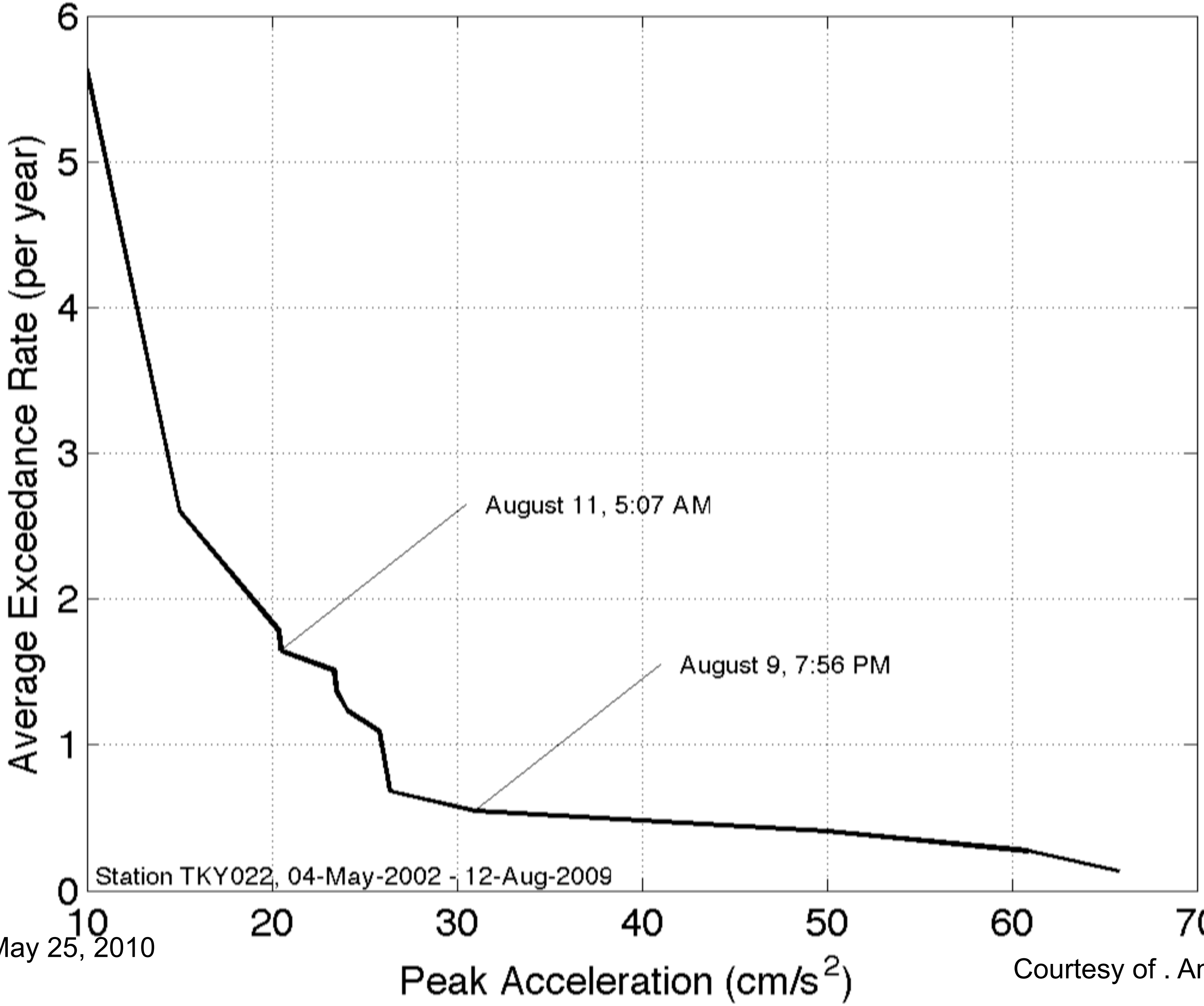
PGA with 2% in 50 year PE. BC rock. 2008 USGS



2% in 50 years means a recurrence time of 475 years for a poissonian process

PSHA through experiment

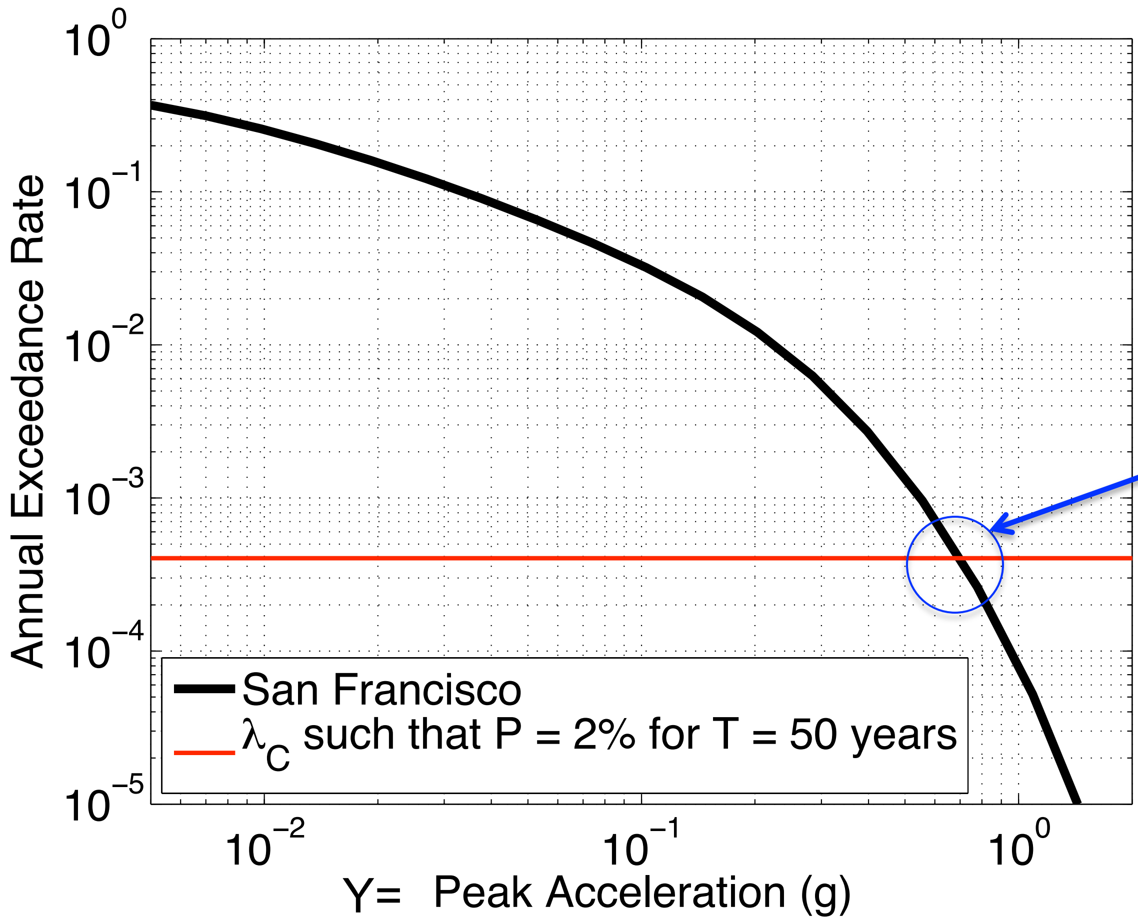
- Suppose one were to run a strong motion accelerograph at a site for 10,000 years.
- From that data, we could determine the average rate that any peak acceleration is equaled or exceeded (provided it occurs at least once in 10,000 years).
- The result is called a hazard curve.
- A probabilistic seismic hazard analysis tries to predict the outcome of this experiment.



Courtesy of . Anderson

PSHA Methodology

Annual Exceedance Rate $\lambda_C(Y)$



Contour these points to generate a hazard map.

Source: USGS web site

May 25, 2010, John Anderson lecture

PSHA Methodology

General integral to calculate $\lambda_C(Y)$:

$$\lambda_C(Y) = \iint n(M, r_{flt}) \Phi(y \geq Y \mid \hat{Y}(M, r_{flt}), \sigma_T) \, dM \, dr_{flt}$$

$n(M, r_{flt})$ Seismicity model

$\Phi(y \geq Y \mid \hat{Y}(M, r_{flt}), \sigma_T)$ Ground motion prediction eqn.

Courtesy of . Anderson

General integral to calculate $\lambda_C(Y)$:

$$\lambda_C(Y) = \iint n(M, r_{flt})\Phi(y \geq Y \mid \hat{Y}(M, r_{flt}), \sigma_T)\, dM\, dr_{flt}$$

$\lambda_C\left(y \geq Y\right)$

- The expected (or mean) number of events per year in which the amplitude of a measure of the ground motion *y* exceeds a given threshold *Y*.

Courtesy of . Anderson

Defining equation for PSHA

$$\lambda_C(Y) = \iint \underbrace{n(M, r_{flt})}_{\text{Seismicity Model}} \Phi(y \geq Y \mid \hat{Y}(M, r_{flt}), \sigma_T)\, dM\, dr_{flt}$$

- The seismicity model gives the number of events per year, of magnitude M, and in a location **x**. Note that $r_{flt}=|\mathbf{x}-\mathbf{x}_{site}|$, where the hazard curve is for the location $\mathbf{x}_{site}$.
- Models range from simple to complex.
- Only include main shocks in the model.

PSHA Methodology

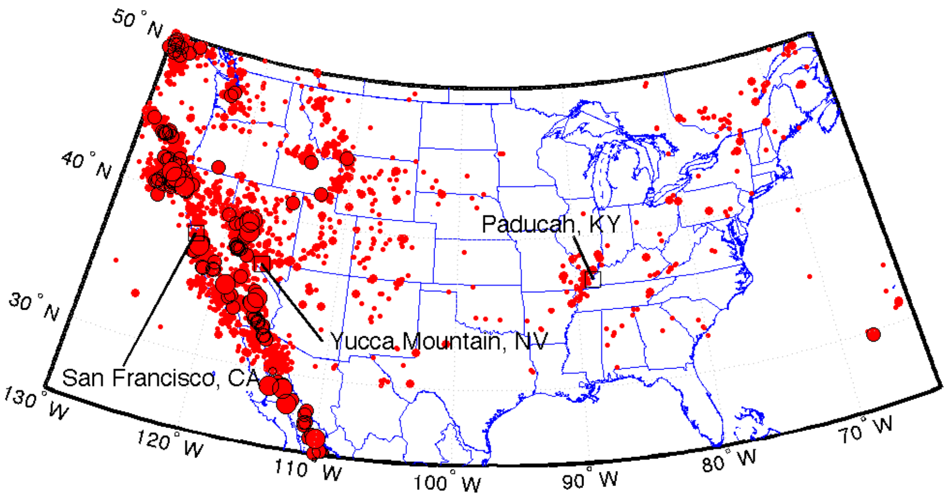
May 25, 2010, John Anderson lecture

$$n(M, r_{flt})$$

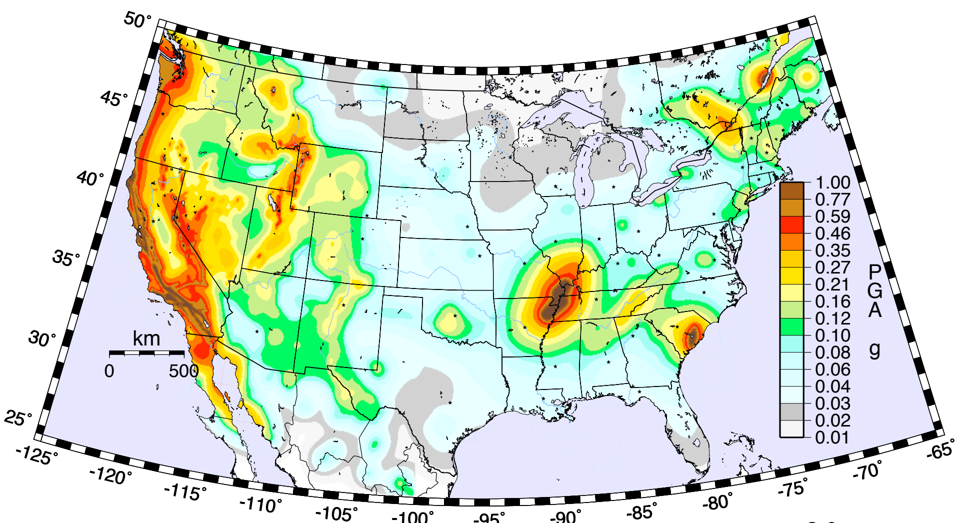
Seismicity model

Large scale –
- should look about like the seismicity map.

Fine scale –
- depends on details of fault locations, magnitudes, activity rates,
- can be very difficult to develop



PGA with 2% in 50 year PE. BC rock. 2008 USGS

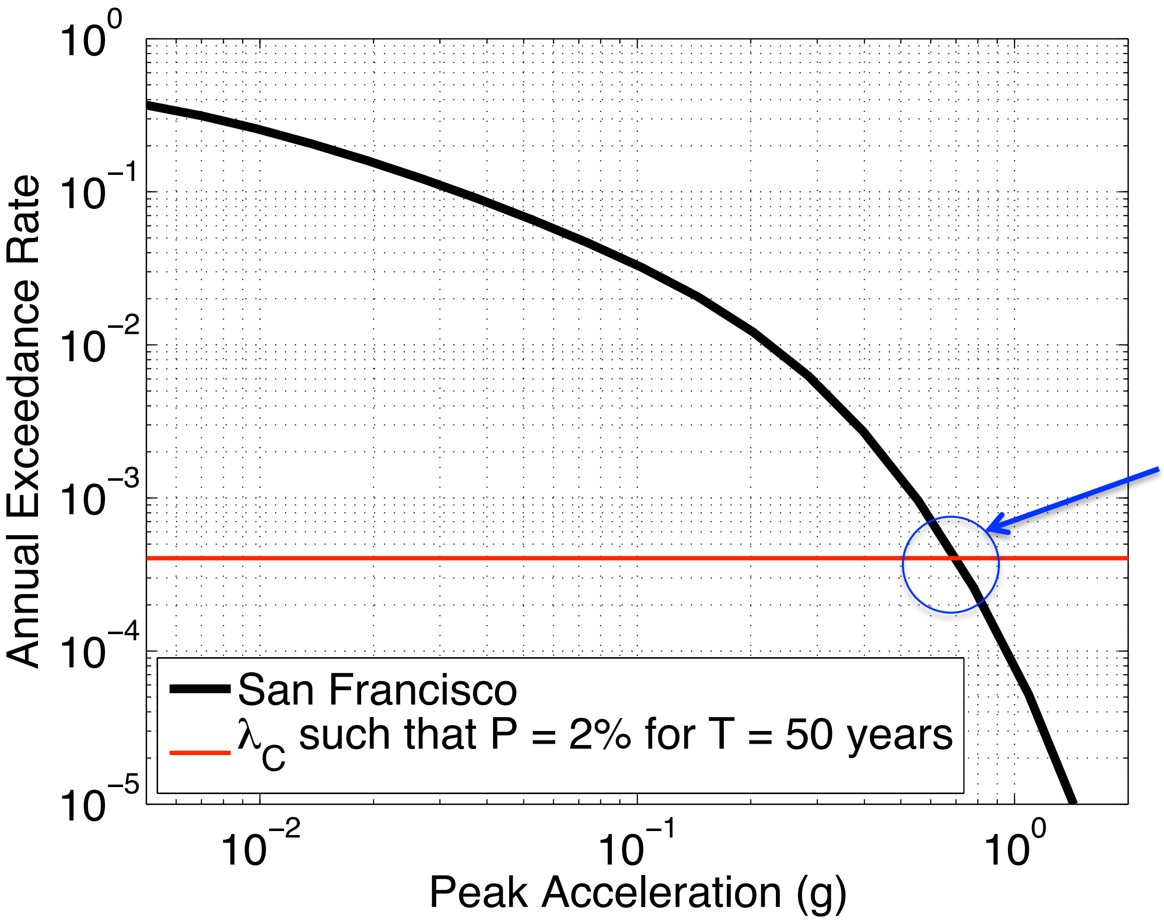


Courtesy of . Anderson

PSHA Methodology

Annual Exceedance Rate $\lambda_C(Y)$

Hazard curve $P(Y,T) = 1 - \exp(-\lambda_C(Y)T)$



Y: amplitude of ground motion (acceleration in this case)
T: time interval

Contour these points to generate a hazard map.

Quality of hazard map depends on quality of $\lambda_C(Y)$

Source: USGS web site

Defining equation for PSHA

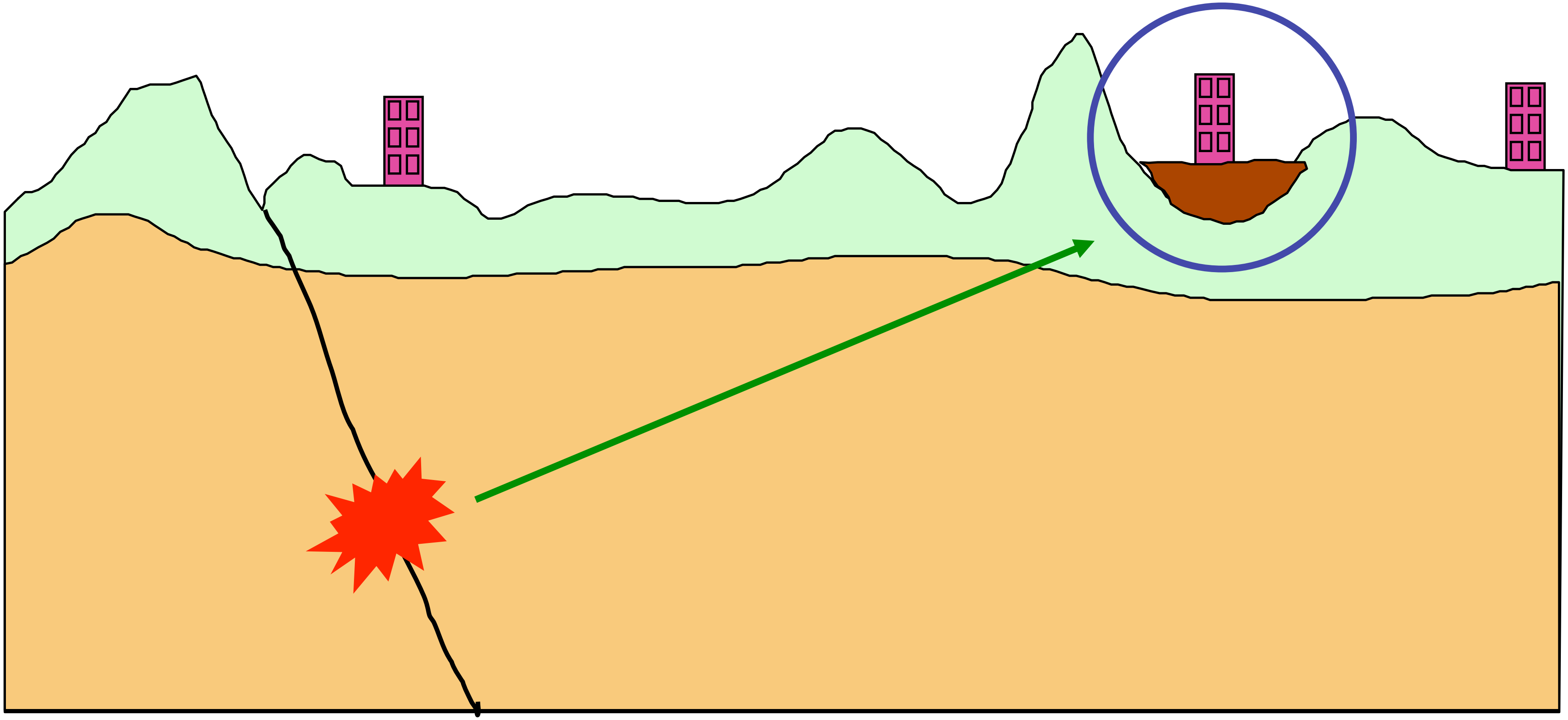
$$\lambda_C(Y) = \iint n(M, r_{flt}) \underbrace{\Phi(y \geq Y \mid \hat{Y}(M, r_{flt}), \sigma_T)}_{\text{Ground motion prediction equation}} dM dr_{flt}$$

- r_{flt} is the distance from the source to the station.
- $\Phi(\bullet)$ gives a probability of exceeding *Y* conditional on *M* and *r*. In other words, if the ground motion prediction equation predicts a smaller ground motion, but still has a dispersion about the mean prediction, then this must calculate the probability of exceeding *Y* considering that dispersion.

Courtesy of . Anderson

Ground motion evaluation

Source + Path + Site



$$M_L = \log\left(\frac{A}{A_o}\right)$$

magnitude

$$A = A_o 10^{M_L}$$

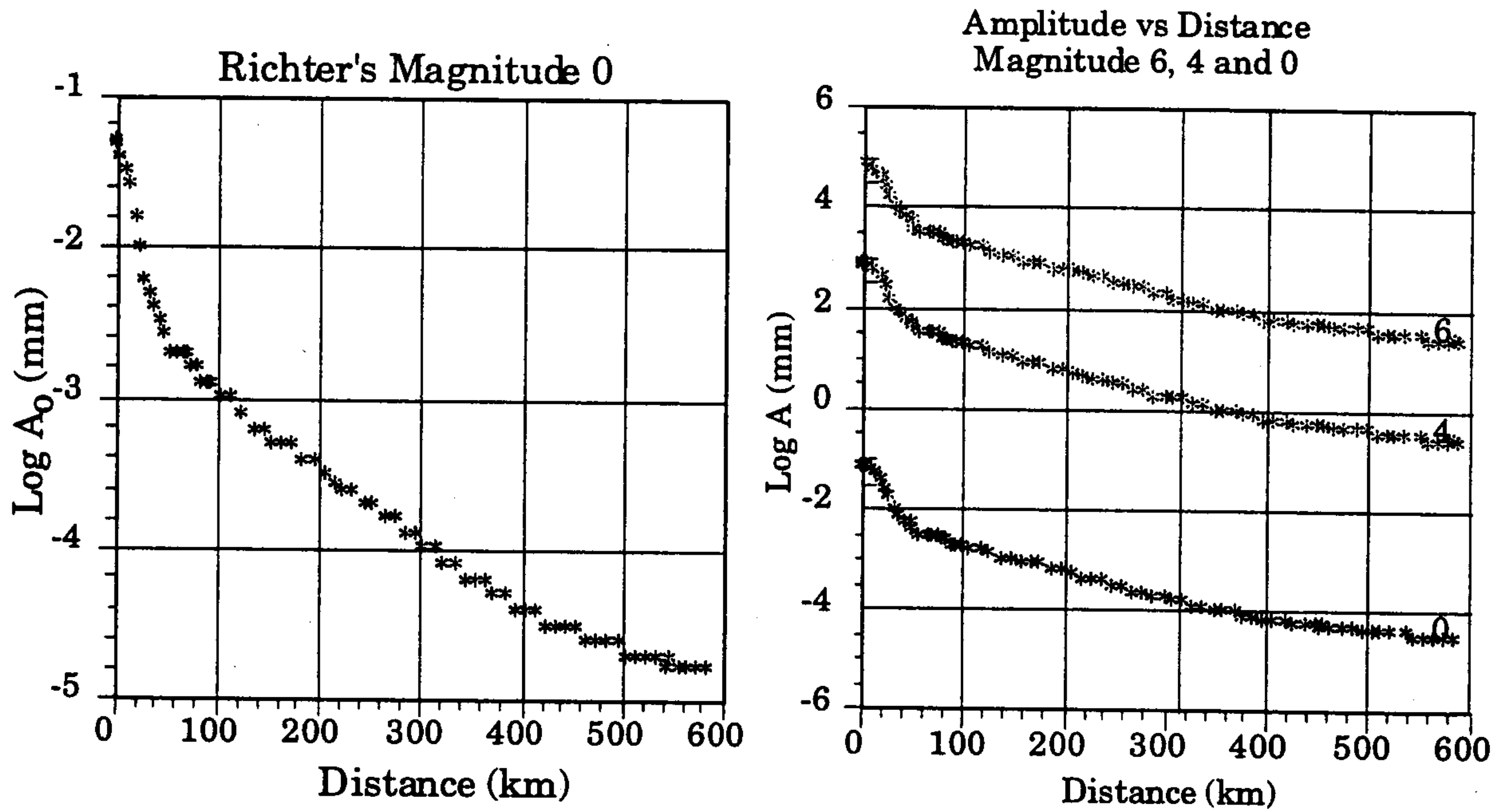


Figure 2-12. Amplitude versus distance for a M_L 0 earthquake (left) and for M_L 0, 4 and 6 earthquakes on the right. Note that magnitude is given on a logarithmic scale. A change in magnitude is simply a shift of the M_L 0 curve.

$$A = A_o\left(\frac{R_o}{R}\right)$$

Geometrical spreading

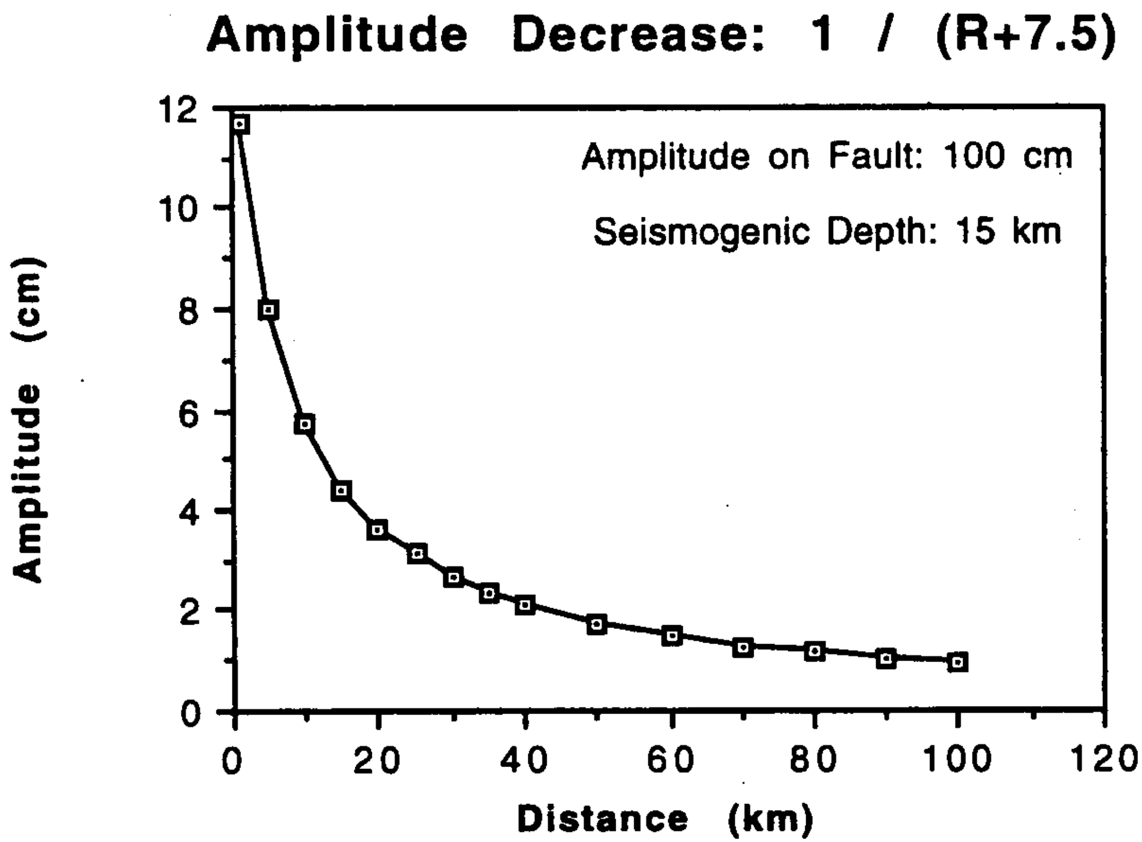


Figure 2-25. Amplitude decay with distance of a body wave.

$$A(x,t) = A_o e^{-\pi f R / v_s Q(f)}$$

Anélastic attenuation

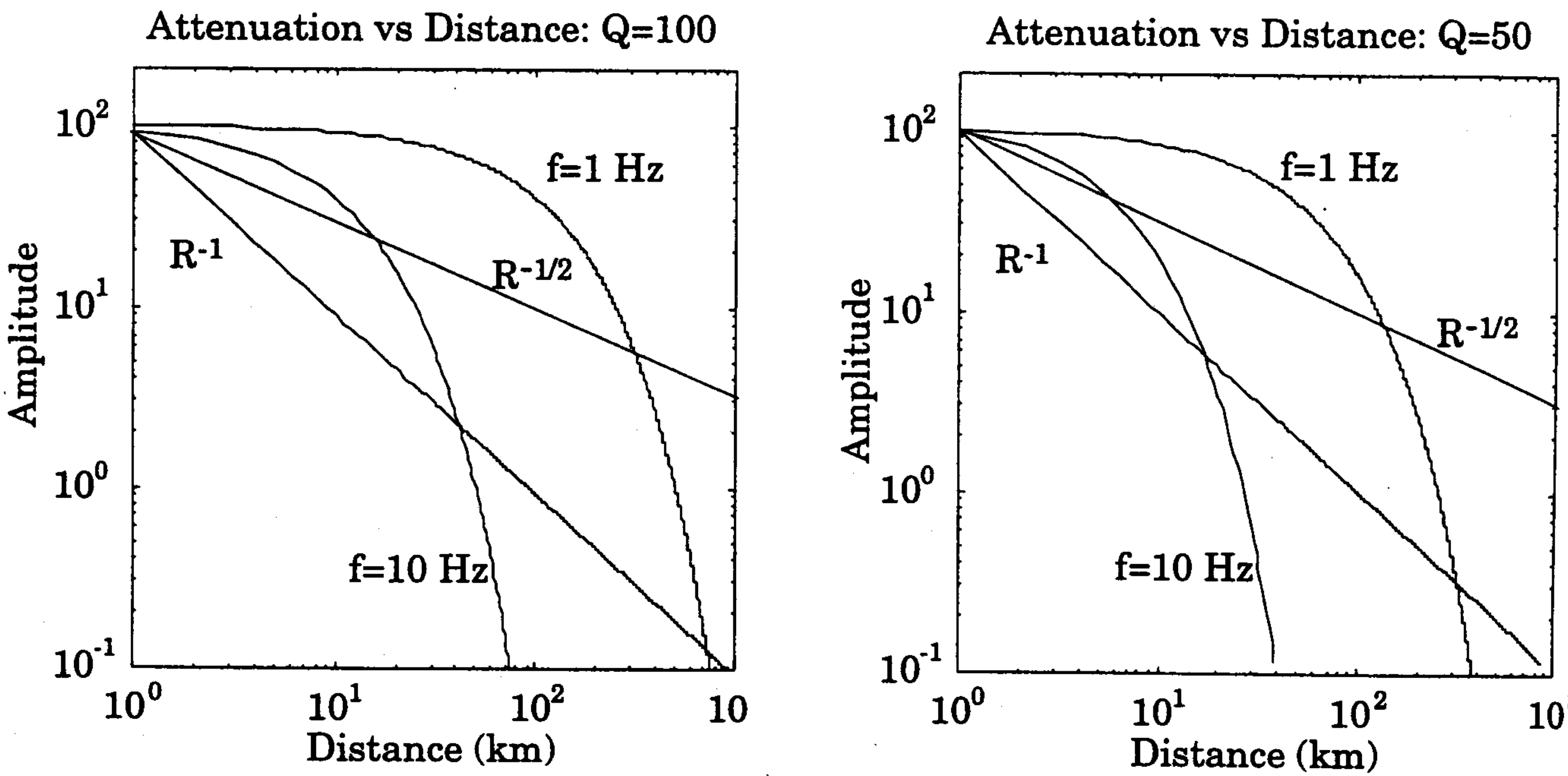
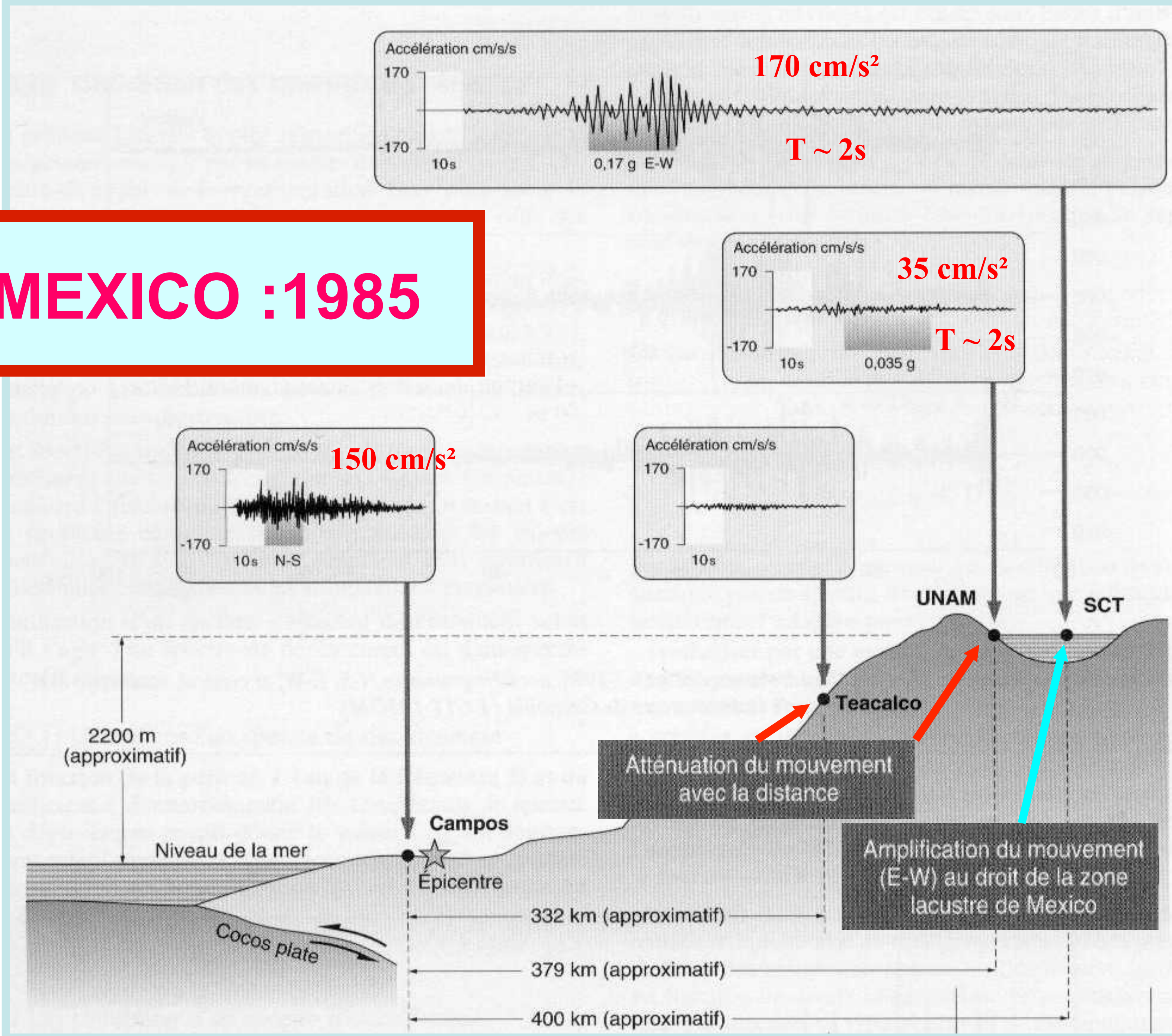


Figure 2-26. Attenuation due to geometrical spreading and intrinsic attenuation for two frequencies and for two values of Q. Geometrical attenuation is dominant for distances less than 10 km for moderate values of Q. Attenuation is strongly frequency dependent.

Effet de site : séisme de Mexico (1985)

MEXICO :1985



Accélération cm/s/s
170
-170
10s
0,17 g E-W
170 cm/s²
T ~ 2s
Accélération cm/s/s
170
-170
10s
0,035 g
35 cm/s²
T ~ 2s
Accélération cm/s/s
170
-170
10s
N-S
150 cm/s²
Accélération cm/s/s
170
-170
10s
UNAM
SCT
Teacalco
Atténuation du mouvement avec la distance
Amplification du mouvement (E-W) au droit de la zone lacustre de Mexico
2200 m (approximatif)
Campos
Niveau de la mer
Épicentre
Cocos plate
332 km (approximatif)
379 km (approximatif)
400 km (approximatif)

log(Y) = f(M, F, R, S) : simplest form ?

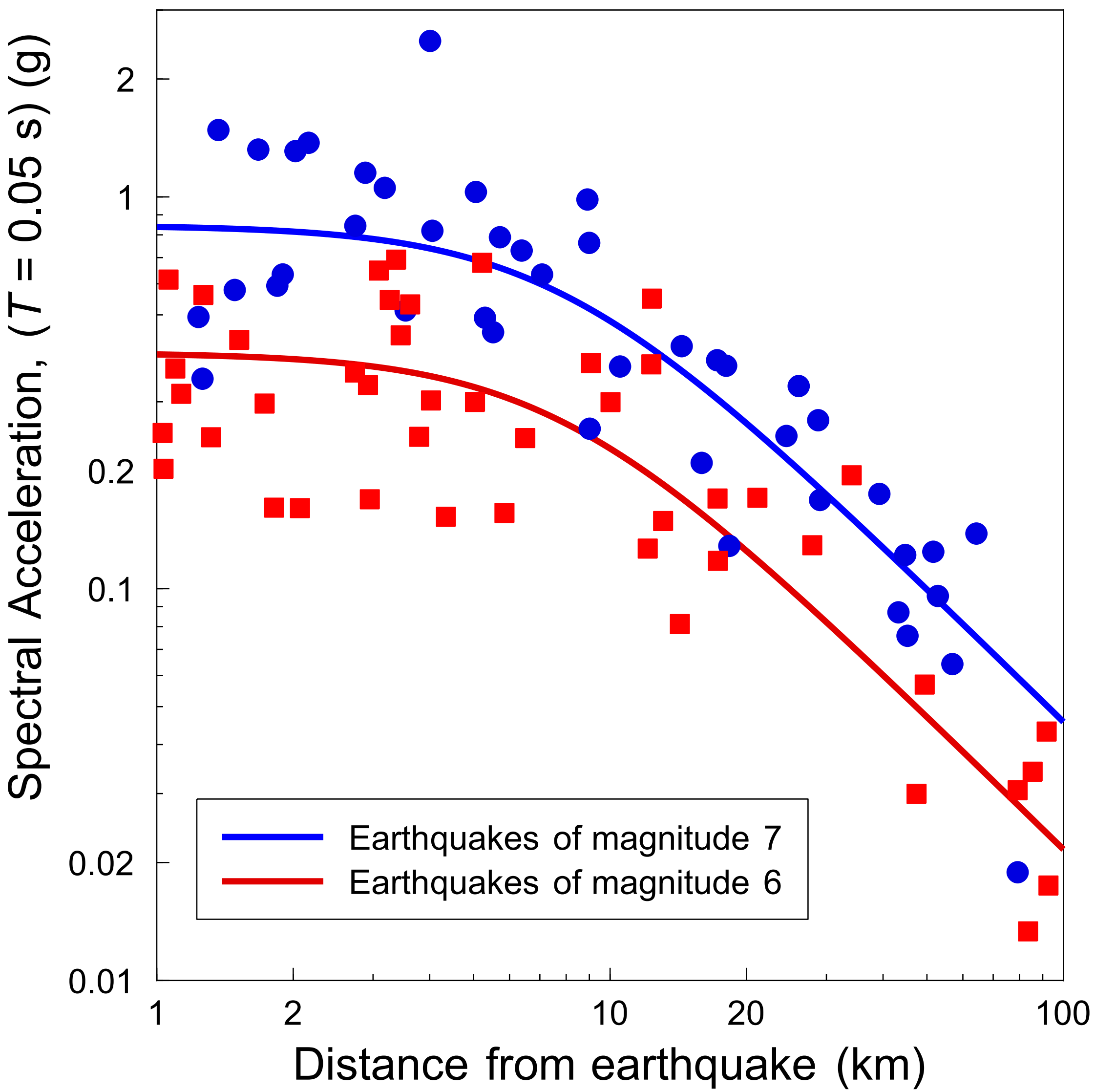
$$\log\big(PSA(f)\big) = \boxed{a(f)\cdot M} + \boxed{b(f)\cdot R - \log(R)} + \boxed{c(i,f)}$$

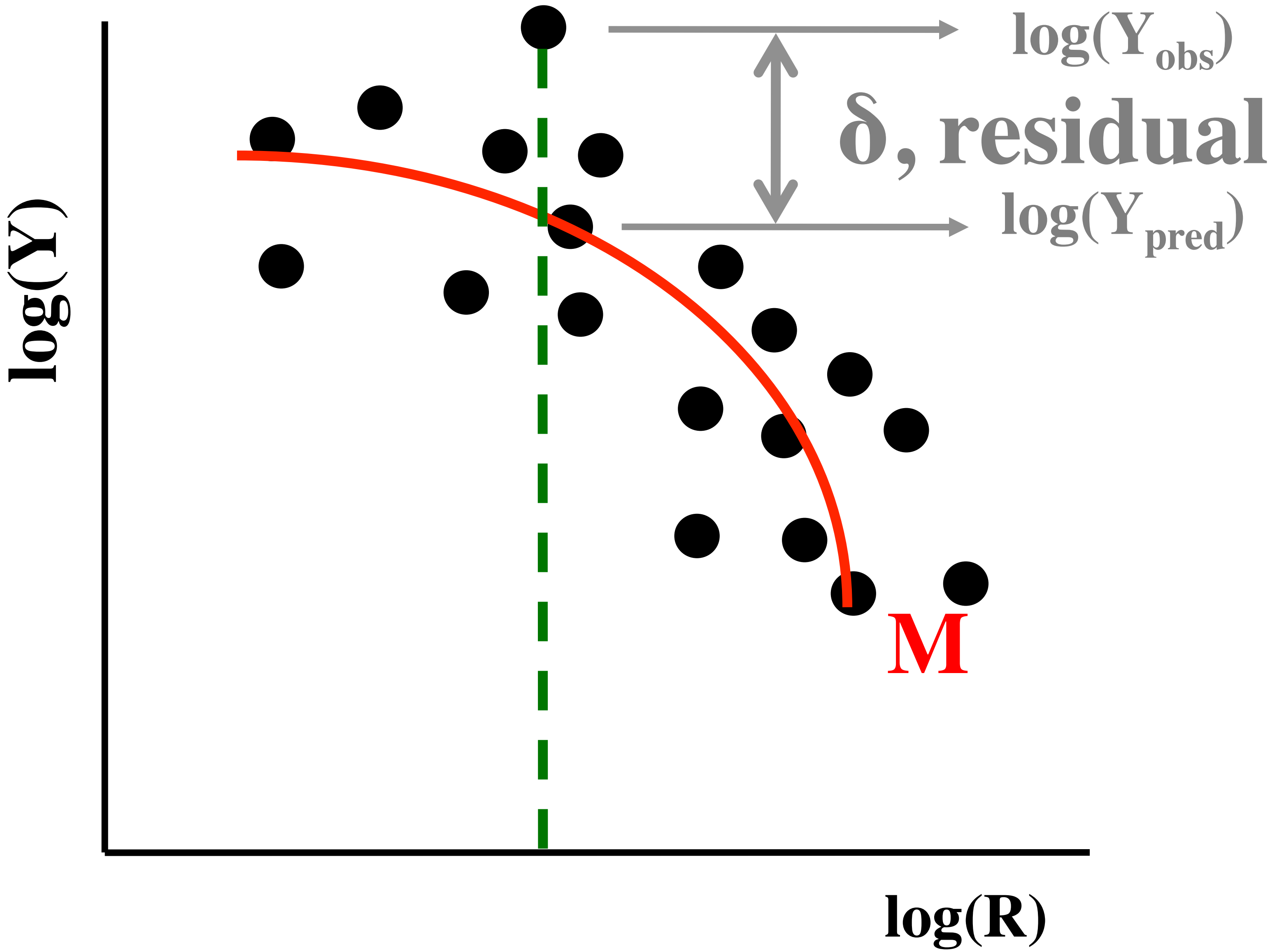
Source	Magnitude
Path	Distance
Site	Vs30 (S wave velocity in the last 30 meters)

How are empirical models derived ?

- **Choice of a functional form (choice of the equation which describes the distance and magnitude dependence of ground motion)**
- Choice of a database
- « Regression » : regression analysis is the mathematical process used to determine the coeffient in the equation in order to fit the data

GMPE coefficients determined by regression analysis on recorded data

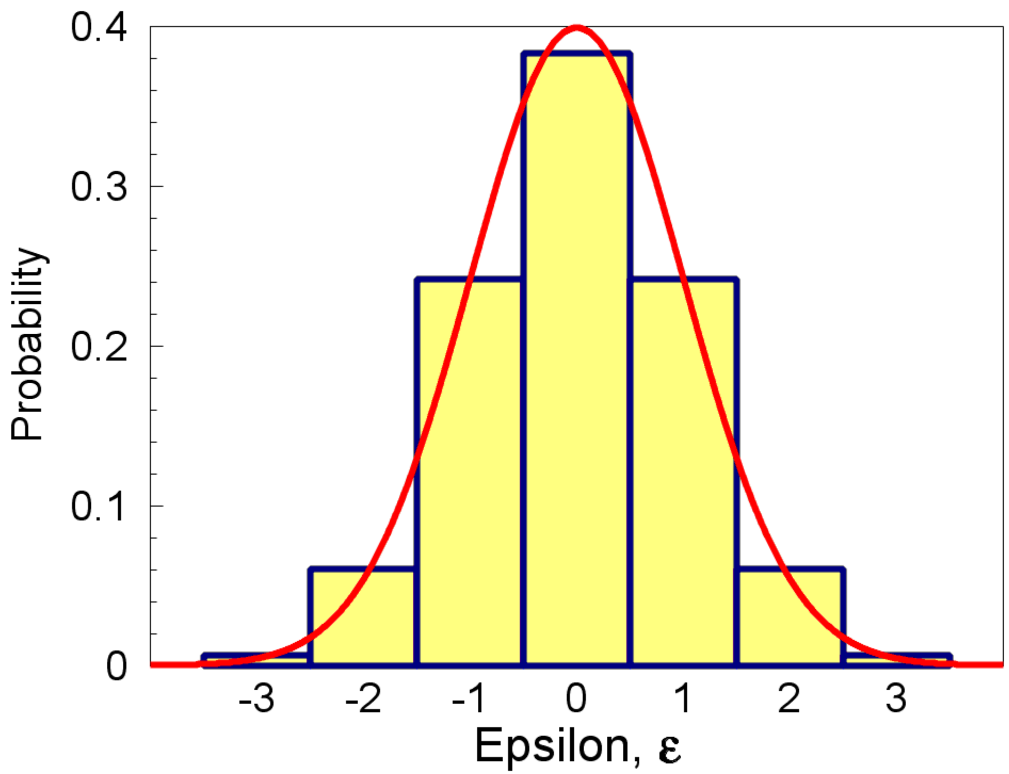




log(Y)
log(Y_obs)
δ, residual
log(Y_pred)
M
log(R)

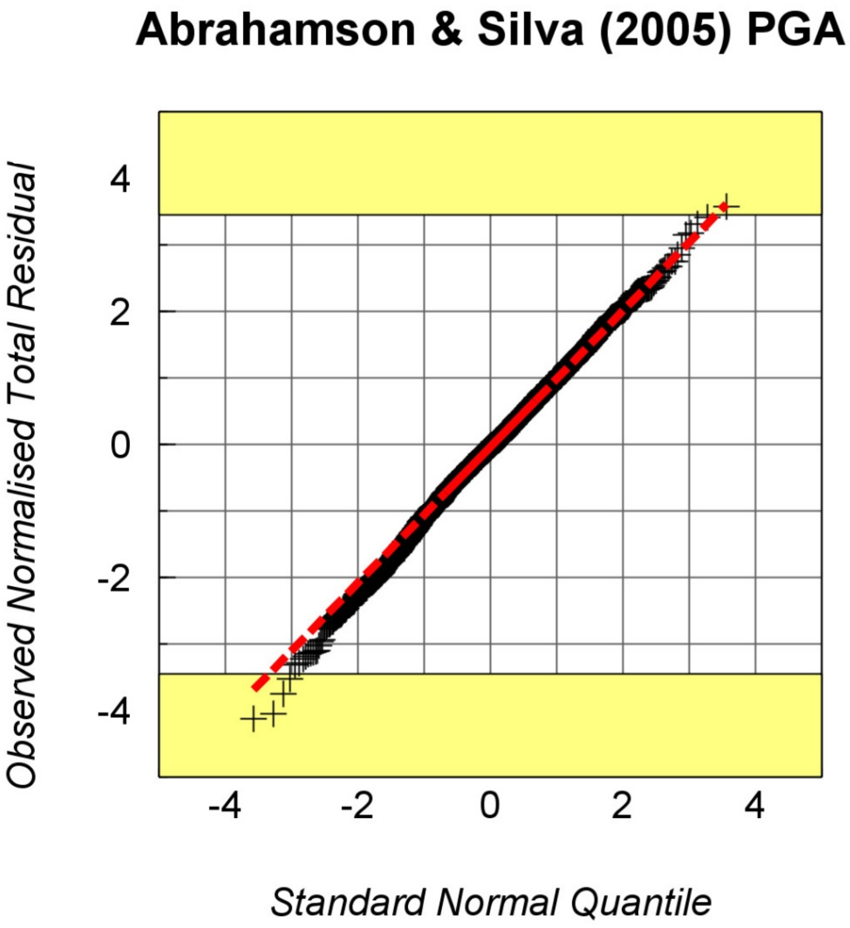
$$\delta = \log(Y_{obs}) - \log(Y_{pred}) = \log(Y_{obs}) - f(M, R)$$

We introduce ε, to represent the residuals normalized by the standard deviation, $\varepsilon = \delta/\sigma$



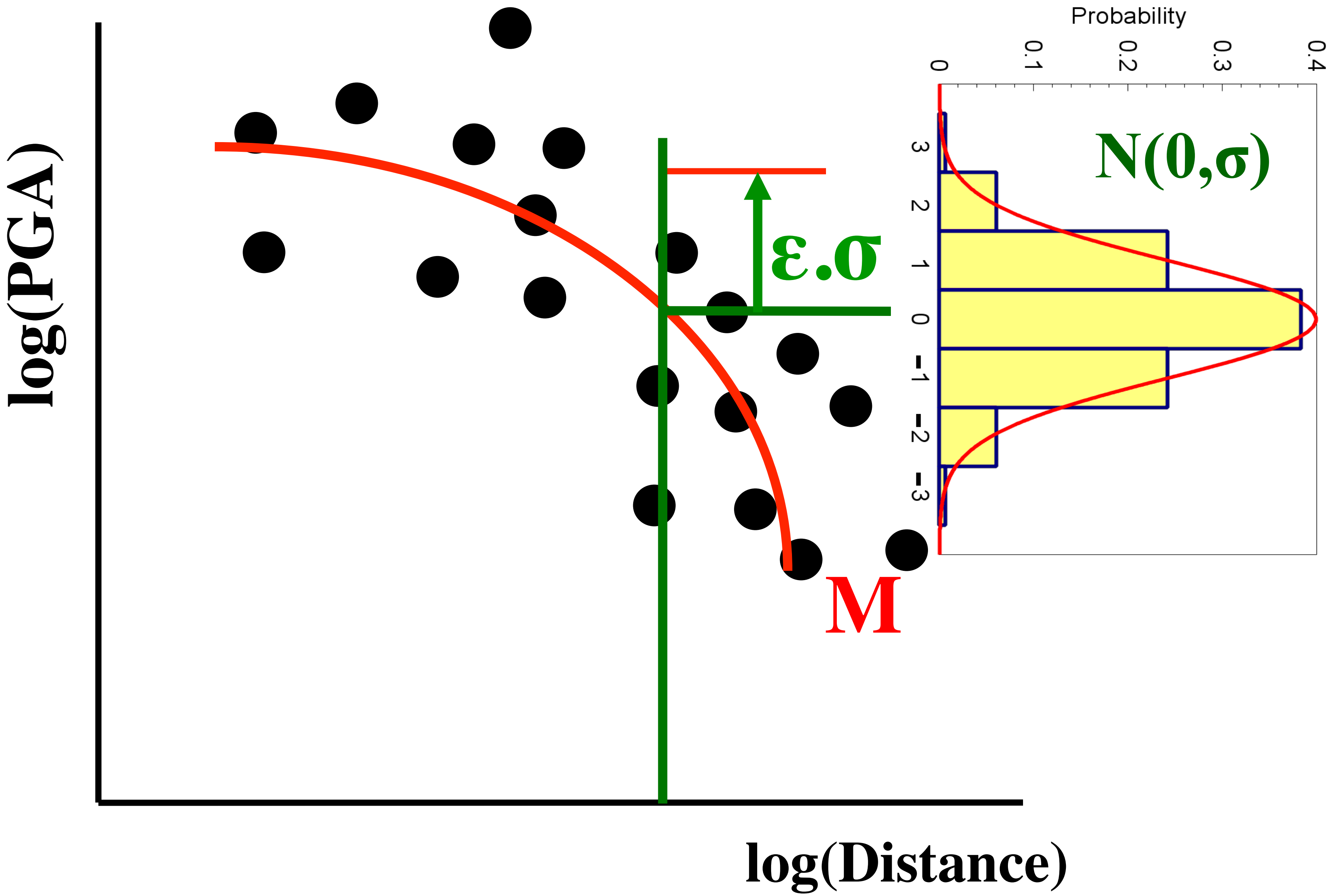
ε	Probability of Exceedance
0	50% (median)
1	16%
-1	84%
2	2.3%
3	0.1%

The logarithmic residuals are generally found to conform to a normal (Gaussian) distribution with mean 0 and standard deviation σ

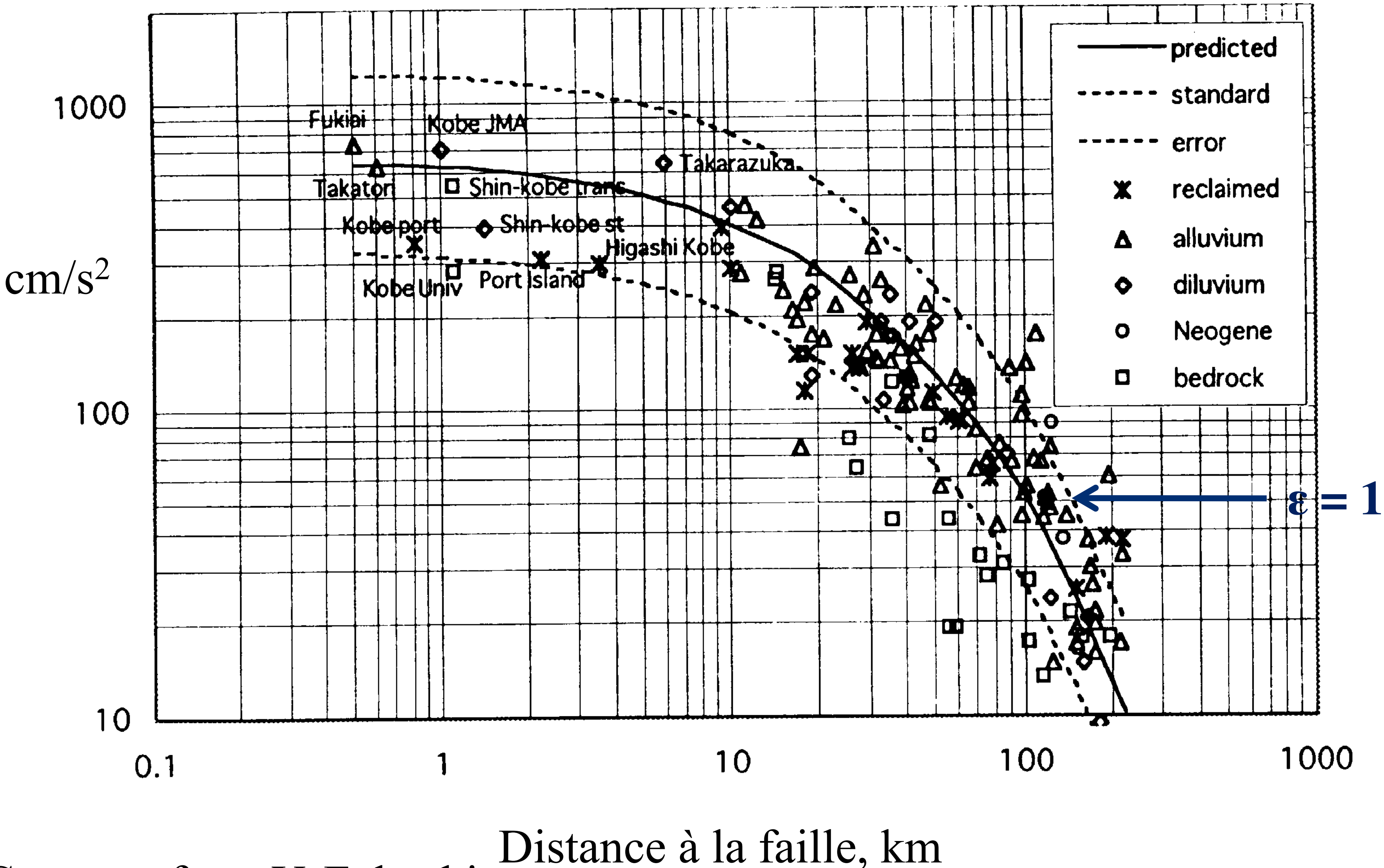


The distribution of the ground-motion residuals can therefore be completely characterized by the logarithmic standard deviation, σ

$$\log(Y) = f(M, F, R, S) + \delta = f(M, F, R, S) + \varepsilon.\sigma$$



Recordings of 1995 Kobe earthquake compared to the Tanaka and Fukushima (1990) GMPE

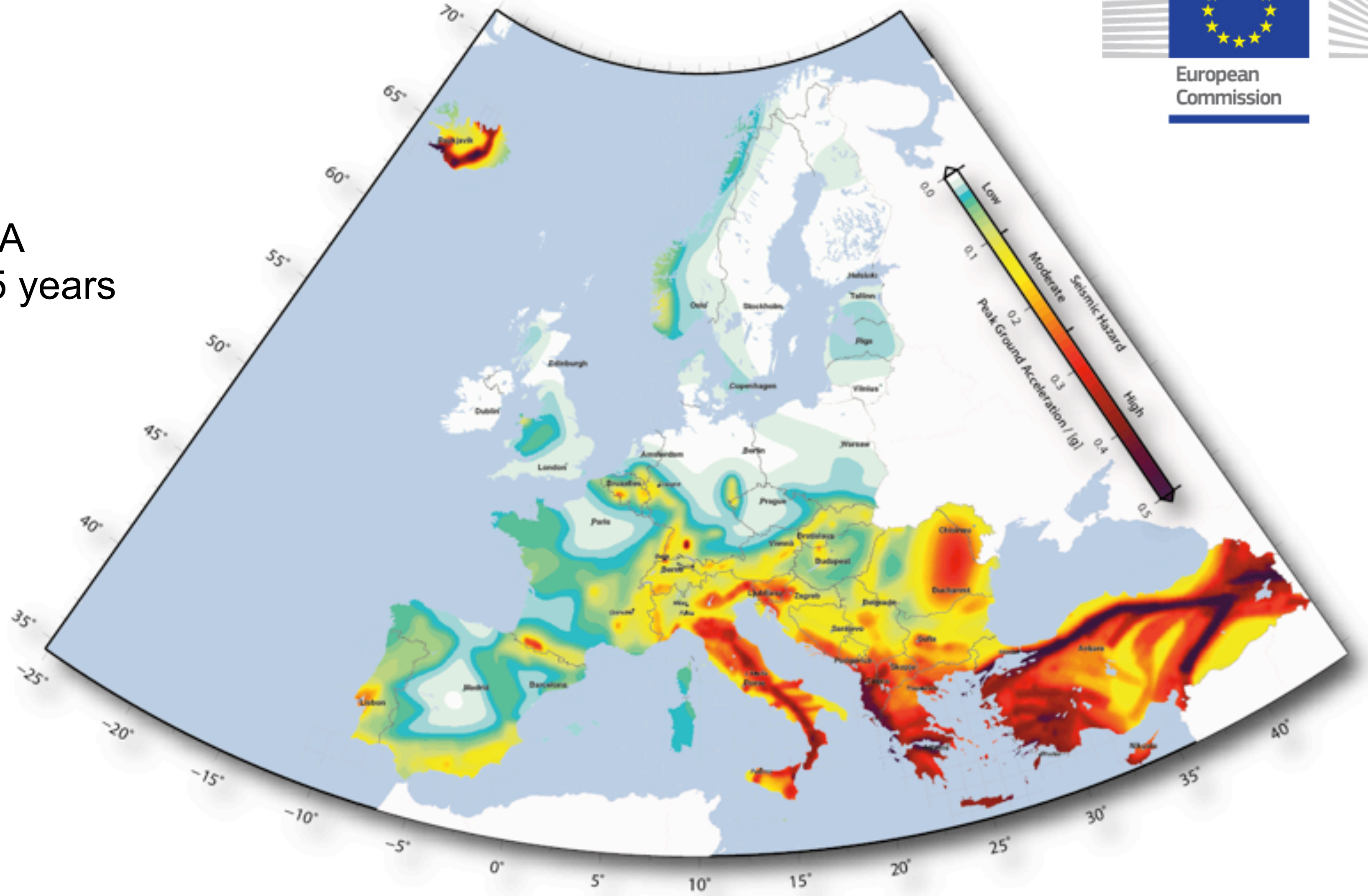


Courtesy from Y. Fukushima

SHARE-GEM hazard map released in June

European Commission

PGA
475 years



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