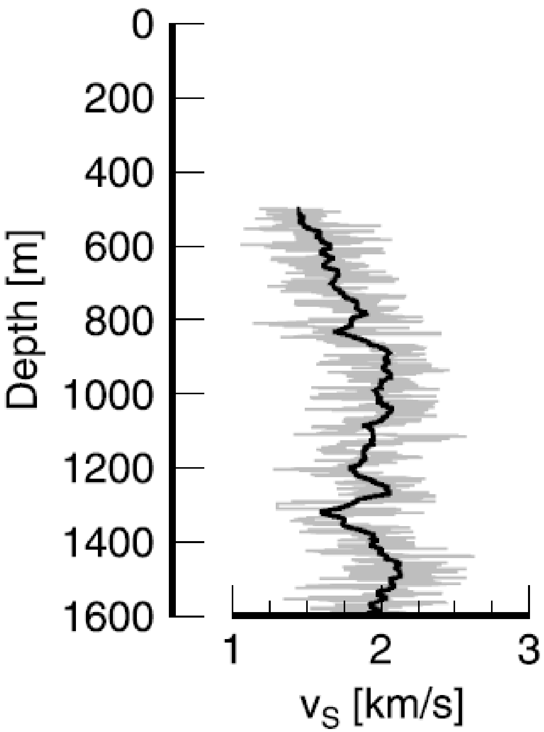
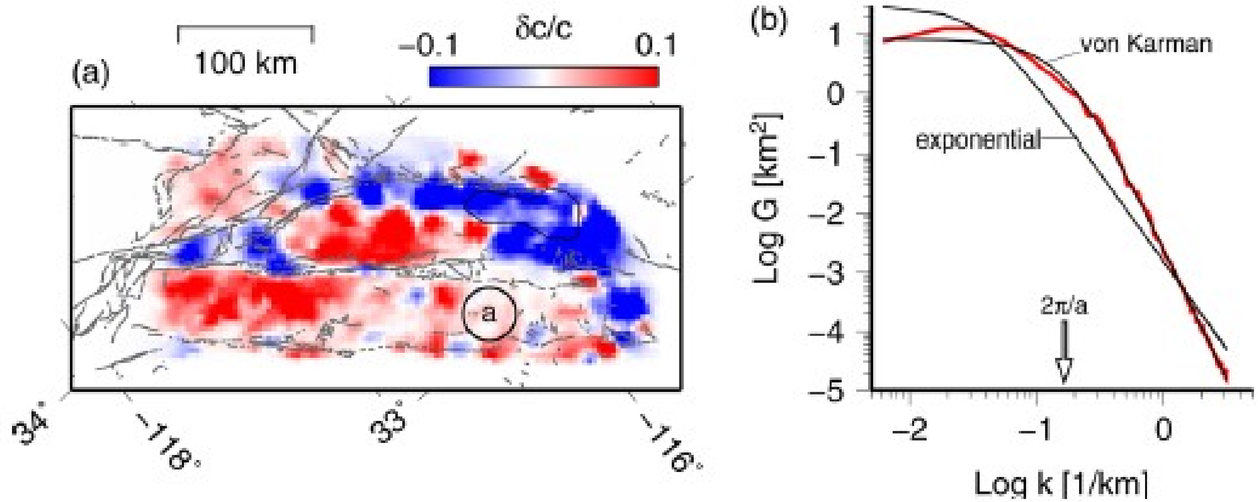
Seismic wave propagation in the heterogeneous crust



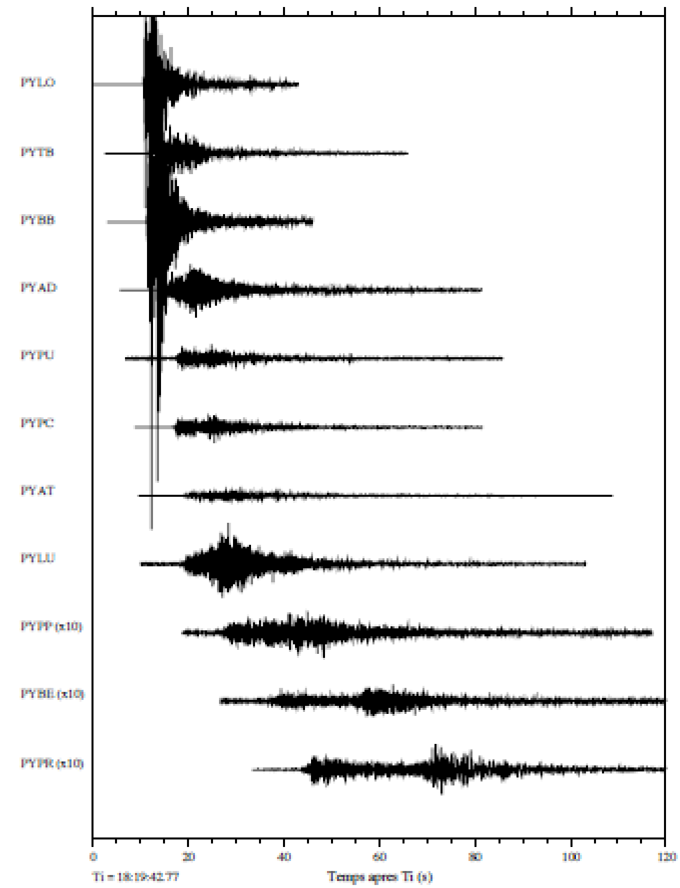


Method of **radiative transfer** or **transport theory** or **radiative transport theory (RTT)** can be applied to the modeling of the space-time distribution of energy released by earthquakes/ propagation of scattered seismic energy

RTT is applicable to systems where disorder or randomness is present

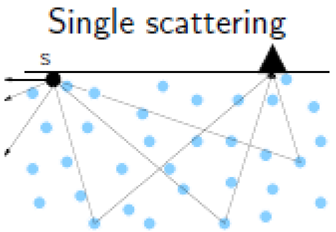
Modeling of energy distribution: Explain the energy envelope of the seismograms only; no complete fitting of waveforms, no information of phase

In disordered media: The phase gets randomized by the scattering events. Hence, the field at a point can be viewed as a sum of waves whose phase and amplitude are independent random variables.



crustal earthquake in the Pyrenees

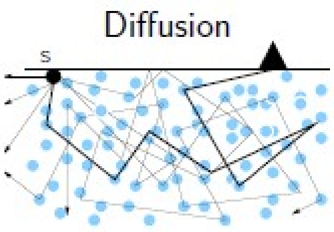
Predominance of scattered waves in short-period seismic data.
Development of two simple theoretical models



[1] single scattering model

(representation of the field as sum of rays is still valid)

Consider rays describing seismic wave propagation in a smooth earth; rays are defined by their initial slowness. Upon interaction with a scatterer the ray slowness can change abruptly and in a 'random' manner.



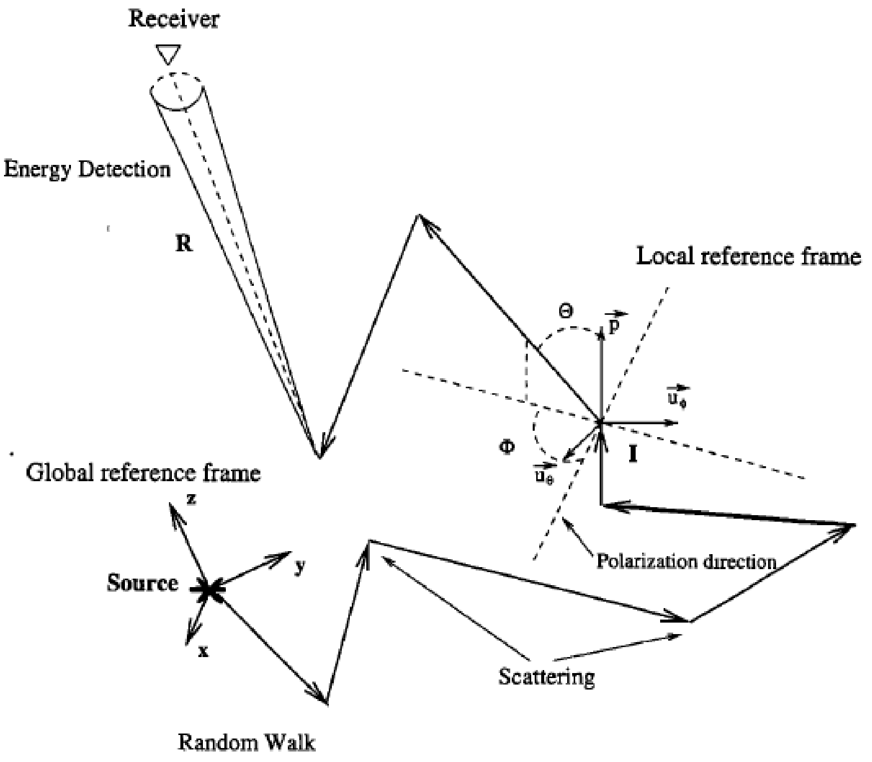
[2] Diffusion model

(wave propagation is described by diffusion equation)

"In between" but also embracing 1 & 2: RTT

Here: phenomenological approach using scalar waves to sketch the main ideas underlying RTT

Connection between wave equation and transfer equation exists (delivers rigorous definition of parameters and physical quantities of RTT; rather technical).

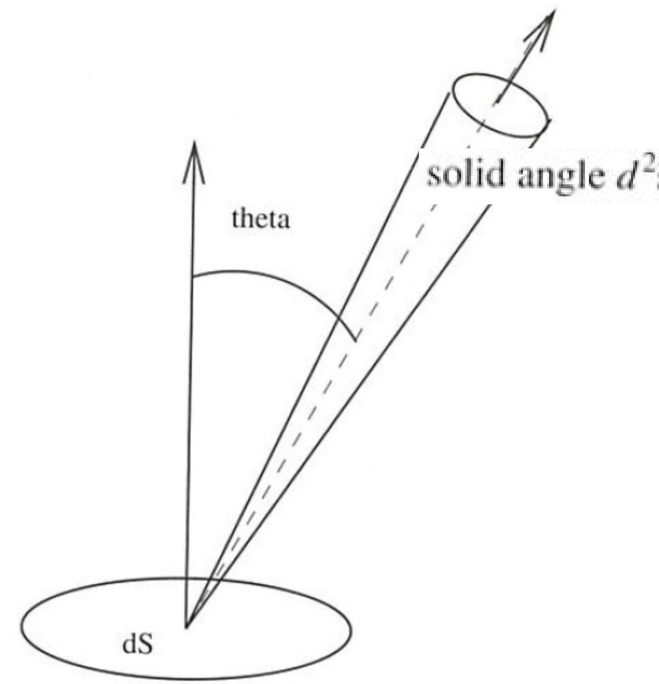


Again: RTT discards the phase information associated with individual rays (=> model the envelope). Focus on transport of energy! That is, keep track of the flux of energy by summing over contributions of individual 'rays'.

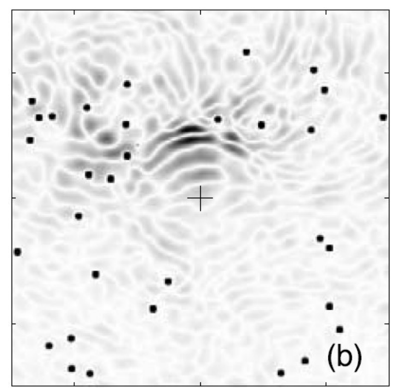
Central quantity in transport theory: **Specific Intensity**
(=> angularly resolved flux)

$$I(\omega, t, \mathbf{r}, \hat{\mathbf{s}}) = \frac{dE}{dS\, d\omega\, dt \cos\theta\, d^2\hat{s}}$$

Intensity: Amount of energy flowing across a surface in a specified direction per unit time, per unit solid angle and per unit surface.



In a scattering medium, energy may flow in all directions and energy may be unevenly distributed among different solid angles (directions in 3D space).



Angular flux is needed to account for scattering anisotropy.

Angular dependence of scattering is a function of ratio between wavelengths and size of scatterer. Therefore, specific intensity is function of frequency.

Idea: Follow beam of energy propagating around direction s during interval dt. (Omit dependence on frequency). Consider, again, surface element dS through which energy flows, which defines cylinder of height vdt.

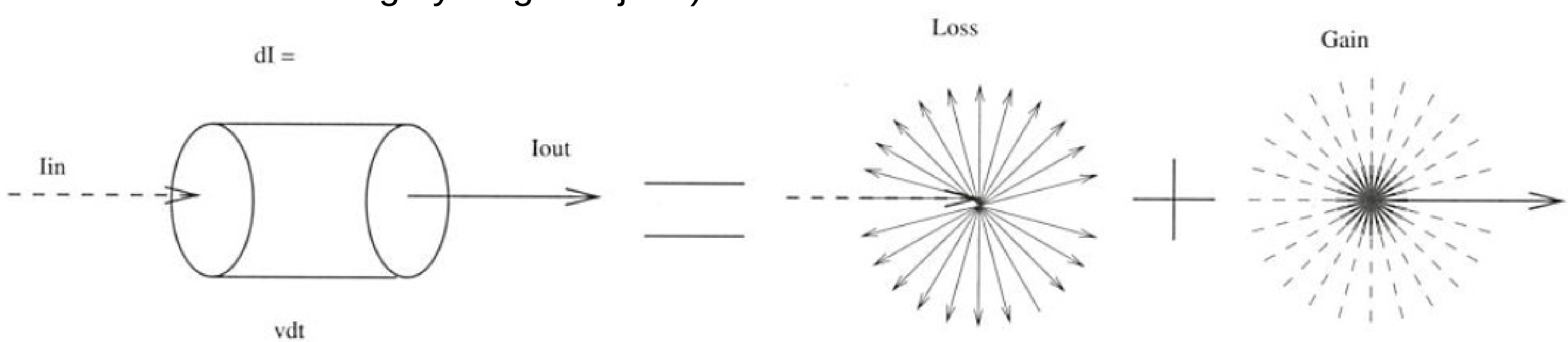
LOSS: Beam energy is scattered in all directions; observations show
(l: scattering mean free path; ok to define for independent scatterers, what about continuously varying properties in real Earth?)

$$\frac{\delta I}{I} = -\frac{v dt}{l}$$

GAIN: Energy comes from all directions; beams with directions s' may transfer part of their energy to reinforce the beam propagating along s.

$$G = \frac{v dt}{l} \int_{4\pi} p(\hat{\mathbf{s}}, \hat{\mathbf{s}}') I(t, \mathbf{r}, \hat{\mathbf{s}}') \, d^2\hat{s}'$$

p: **probability** for a beam propagating along s' to be deflected into direction s. **The role of p: describe quantitatively how scattered energy is redistributed into direction s** taking into account scattering anisotropy (=> cross sections, patterns)
(Think identical scatterers; then p(s,s') can be obtained from solution of scattering by single object.)



Collecting results of loss and gain w/ proper expression for total derivative one finds the standard radiative transfer equation:

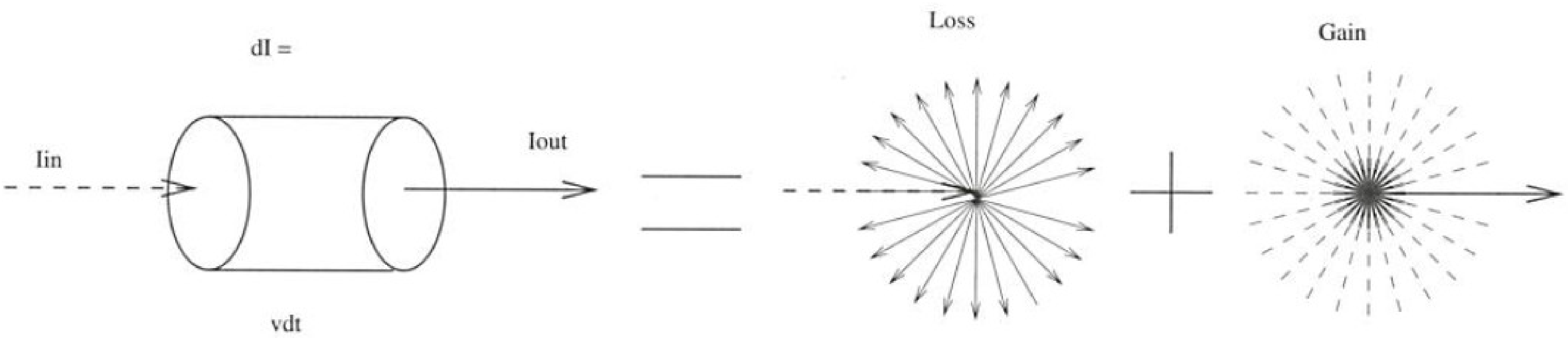
$$\frac{\delta I}{I} = -\frac{vdt}{l}$$

$$G = \frac{vdt}{l} \int_{4\pi} p(\hat{\mathbf{s}},\hat{\mathbf{s}}') I(t,\mathbf{r},\hat{\mathbf{s}}')\, d^2\hat{s}'$$

$$\frac{d}{vdt} = \frac{\partial}{v\partial t} + \hat{\mathbf{s}} \cdot \nabla$$

$$\left(\frac{\partial}{v\partial t} + \hat{\mathbf{s}} \cdot \nabla\right) I(t,\mathbf{r},\hat{\mathbf{s}}) = -\frac{I(t,\mathbf{r},\hat{\mathbf{s}})}{l} + \frac{1}{l}\int_{4\pi} p(\hat{\mathbf{s}},\hat{\mathbf{s}}') I(t,\mathbf{r},\hat{\mathbf{s}}') d^2\hat{s}' + S(t,\mathbf{r},\hat{\mathbf{s}})$$

Equation of Radiative Transfer stems from **local energy balance**



$$\left(\frac{\partial}{v\partial t} + \hat{\mathbf{s}} \cdot \nabla\right) I(t,\mathbf{r},\hat{\mathbf{s}}) = -\frac{I(t,\mathbf{r},\hat{\mathbf{s}})}{l} + \frac{1}{l}\int_{4\pi} p(\hat{\mathbf{s}},\hat{\mathbf{s}}')I(t,\mathbf{r},\hat{\mathbf{s}}')d^2\hat{s}' + S(t,\mathbf{r},\hat{\mathbf{s}})$$

Illustration of transport equation. Consider **stationary case of isotropic point source in non-scattering medium** (fullspace):

$$S(\mathbf{r},\hat{\mathbf{s}}) = \delta(\mathbf{r})/4\pi,\ l \to \infty$$

In this case, the RTT equation can be solved and the solution reads

$$I(\mathbf{r},\hat{\mathbf{s}}) = \frac{\delta\left(\hat{\mathbf{s}} - \frac{\mathbf{r}}{r}\right)}{4\pi r^2}$$

which agrees with standard ray theory (Intensity spreads over surface of sphere).

Hence, the RTT equation is valid in the simple but important case where all rays originate from a single point in the medium. It confirms the validity of the approach (“Ansatz”), i.e. of the local intensity balance)

Other than that, the integro-differential equation is difficult to solve. In most cases one must resort to numerical methods.

Multiple scattering processes uniformize angular dependence of intensity. It is expected that after a sufficiently large number of scattering events the distribution of the intensity is rather isotropic.

This leads to diffusion approximation

The physical idea behind this approximation is to write intensity as sum of two terms:

[1] angular average

[2] term that accounts for slight deviation from isotropy expressed in terms of current vector

$$I(\mathbf{r},\hat{\mathbf{s}},t) = \frac{\rho v}{4\pi}(\mathbf{r},t) + \frac{3}{4\pi}\mathbf{J}(\mathbf{r},t)\cdot\hat{\mathbf{s}} + \cdots$$

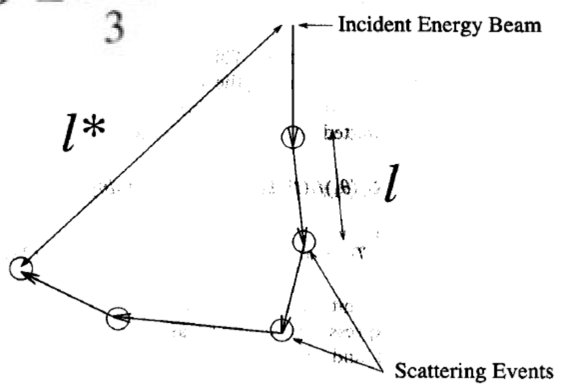
$$\rho(\omega,t,\mathbf{r}) = \frac{1}{v}\int_{4\pi} I(\omega,t,\mathbf{r},\hat{\mathrm{s}})\, d^2\hat{s}$$

$$\mathbf{J}(\omega,t,\mathbf{r}) = \int_{4\pi} I(\omega,t,\mathbf{r},\hat{\mathrm{s}})\, \hat{\mathrm{s}} d^2\hat{s}$$

[…]

$$\frac{\partial\rho(\mathbf{r},t)}{\partial t} - D\nabla^2\rho(\mathbf{r},t) = \delta(t)\delta(\mathbf{r}-\mathbf{r}_0) \qquad D = \frac{vl^*}{3}$$

==> energy density (rho) is solution of diffusion equation



Conservation of the specific intensity along ray bundle

Local energy density

$$\rho(\omega, t, \mathbf{r}) = \frac{1}{v} \int_{4\pi} I(\omega, t, \mathbf{r}, \hat{\mathrm{s}})\, d^2\hat{s}$$

Local energy current density

$$\mathbf{J}(\omega, t, \mathbf{r}) = \int_{4\pi} I(\omega, t, \mathbf{r}, \hat{\mathrm{s}})\, \hat{\mathbf{s}} d^2\hat{s}$$

Meaning of local energy density:

Consider beam of intensity I propagating in a narrow solid angle around direction s.
The amount of energy flowing through an elementary surface dS perpendicular to s, located at at r during time dt is given by I dS dt d^2s. This energy is contained inside small cylinder with volume dV = v dt dS.
Assume that beam energies can be added: The integration over all space directions gives total energy in the vicinity of r. Proper scaling by elementary volume yields expression of the local energy density.

In practice: “Solve” the RTT using numerical methods

Popular tool: Monte – Carlo simulations

$$\left(\frac{\partial}{v\partial t} + \hat{\mathbf{s}} \cdot \nabla\right) I(t, \mathbf{r}, \hat{\mathbf{s}}) = -\frac{I(t, \mathbf{r}, \hat{\mathbf{s}})}{l} + \frac{1}{l}\int_{4\pi} p(\hat{\mathbf{s}}, \hat{\mathbf{s}}') I(t, \mathbf{r}, \hat{\mathbf{s}}') d^2\hat{s}' + S(t, \mathbf{r}, \hat{\mathbf{s}})$$

This approach depends on knowing or having expressions for p

This way, single scattering can be used as building block for multiple scattering (RTT).

Assume that scatterers are spherically symmetric (point scatterer) => scattering properties can be obtained from Born Approximation. Hence, consider P, S wave polarization/ elastic waves (scalar waves >> elastic waves)

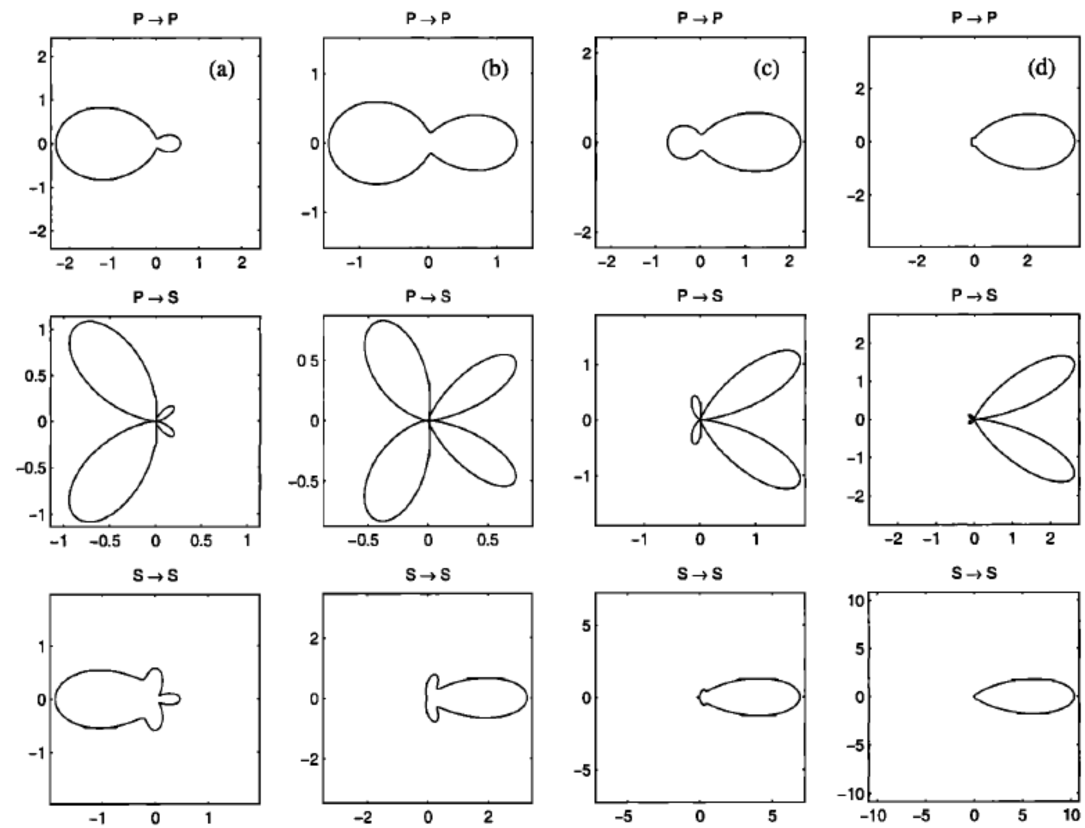
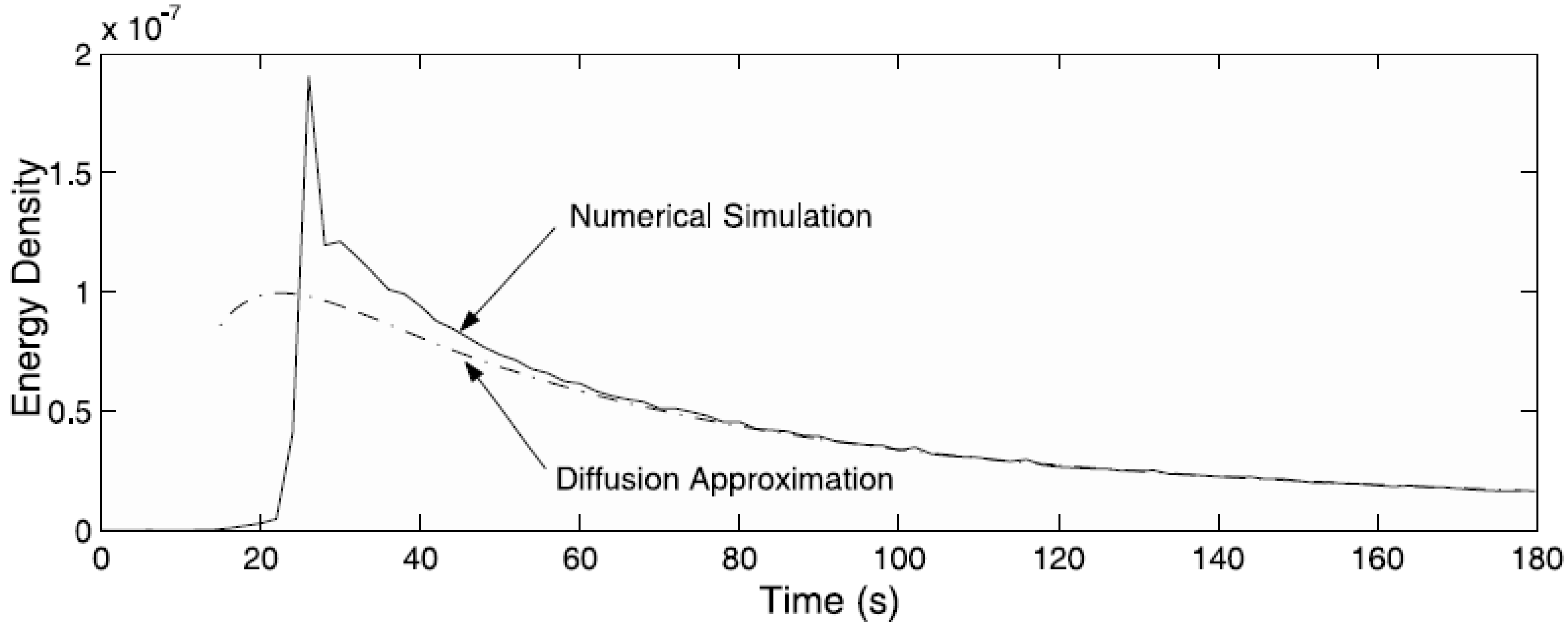
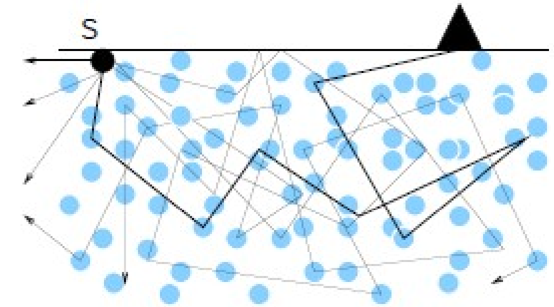


Figure 6. Polar plot of the differential scattering cross sections for the mode conversions *P*-*P*, *P*-*S* and *S*-*S* for (a) Rayleigh regime $k_P a \ll 1$, (b) Rayleigh-Gans (R-G) regime $k_P a = 1.2$, (c) R-G regime $k_P a = 1.6$, and (d) R-G regime $k_P a = 2$



Diffusion



$$\frac{2S}{(4\pi Dt)^{3/2}} e^{-\frac{r^2}{4Dt} - \frac{ct}{l^a}}$$

Again: Monte – Carlo simulations for elastic RTT in 2-D

$$\frac{1}{\alpha_0}\frac{\partial I^p}{\partial t} + \hat{\mathbf{k}}\cdot\nabla I^p = \frac{1}{2\pi}\int g_{pp}(\hat{\mathbf{k}},\hat{\mathbf{k}}')I^p(\hat{\mathbf{k}}')d\hat{\mathbf{k}}' - g_{pp}^0 I^p + \frac{1}{2\pi}\int g_{sp}(\hat{\mathbf{k}},\hat{\mathbf{k}}')I^s(\hat{\mathbf{k}})d\hat{\mathbf{k}}' - g_{ps}^0 I^p$$

Scattering coefficients obtained from
Born approximation:

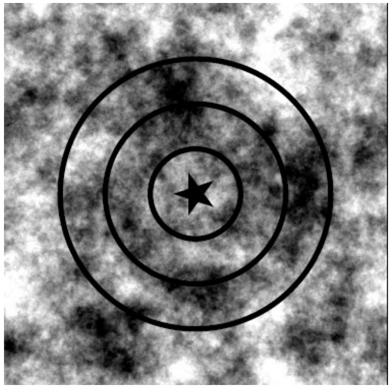
$$g_{pp}(\theta) = \frac{\gamma_0 k_s^3}{8\pi} P\left(\frac{2k_s}{\gamma_0}\sin(\theta/2)\right)|X_r^{pp}|^2,$$

$$g_{ps}(\theta) = \frac{k_s^3}{8\pi} P\left(\frac{k_s}{\gamma_0}\sqrt{1+\gamma_0^2-2\gamma_0\cos\theta}\right)|X_\theta^{ps}|^2,$$

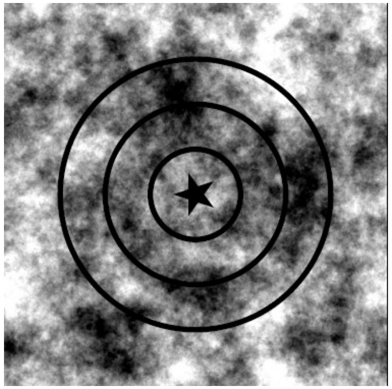
$$g_{sp}(\theta) = \frac{\gamma_0 k_s^3}{8\pi} P\left(\frac{k_s}{\gamma_0}\sqrt{1+\gamma_0^2-2\gamma_0\cos\theta}\right)|X_r^{sp}|^2,$$

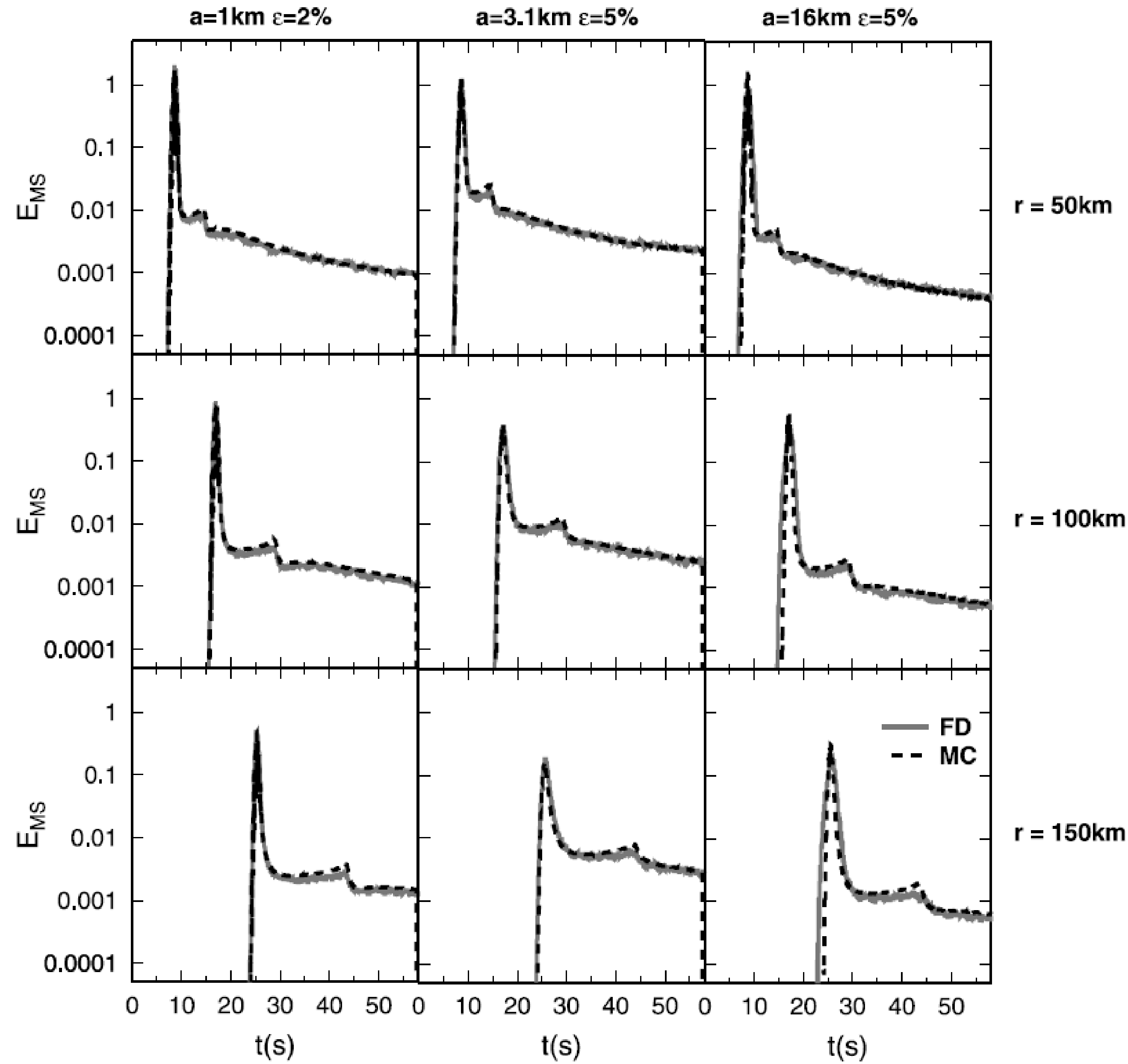
$$g_{ss}(\theta) = \frac{k_s^3}{8\pi} P(2k_s\sin(\theta/2))|X_\theta^{ss}|^2.$$

The random velocity fields enter via

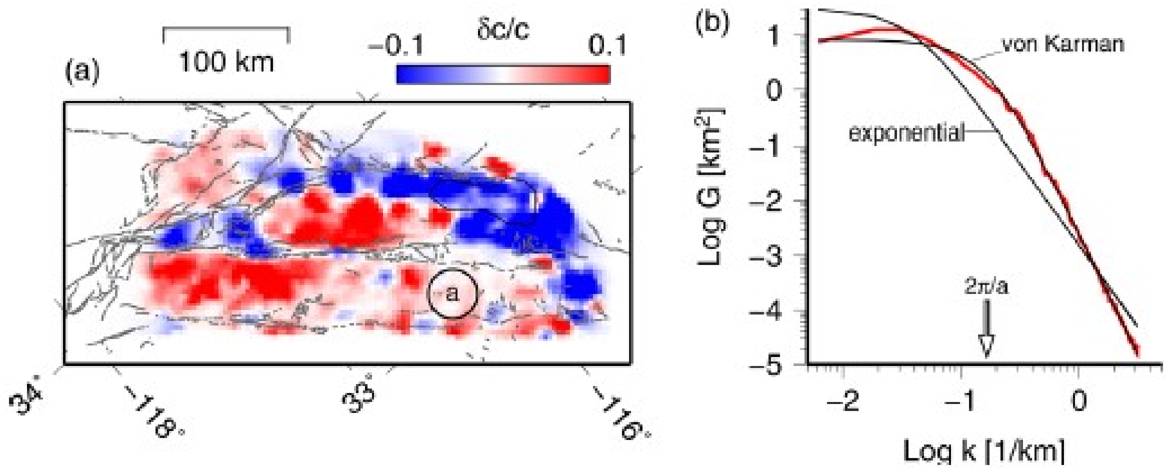


$$P_E(m) = \frac{4\pi\varepsilon^2 a^2}{(1+a^2m^2)^{3/2}}$$





a=1km ε=2%
a=3.1km ε=5%
a=16km ε=5%
E_MS
1
0.1
0.01
0.001
0.0001
r = 50km
r = 100km
r = 150km
FD
MC
0
10
20
30
40
50
t(s)



$$Q_{sc}^{-1}(k) = 2k^2\sigma^2 \int_{\theta_{\min}}^{\pi} P(k_r)\,d\theta$$

$$l = \frac{Q_{sc}(k)c}{\omega} = \frac{Q_{sc}(k)}{k}$$