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Supplementary Material for “Dynamic regimes in planetary cores: τ - ℓ diagrams”

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This article is a draft (not yet accepted!)

S1. τ - ℓ Python package

In the attached tau-ell_programs.zip archive, we provide short Python scripts used to draw our article's figures.

- read_parameters.py: document the parameters of the natural object, or numerical simulation, or Lab experiment, for which one wishes to draw τ - ℓ regime diagrams (or templates). The objects can be fluid full spheres (as in the article) or fluid spherical shells.
- plot_object.py: plot τ - ℓ diagram of the chosen object. You select a number of options, including the force balance you want to test, and the program calls plot_template and plot_scenario.
- plot_template.py: plot the template for the chosen object (common to natural objects, DNS and experiments).
- plot_scenario.py: overlay the τ - ℓ regime diagram produced by scenarios built upon various force balances (CIA, QG-CIA, QG-VAC, MAC, QG-MAC, QG-MAC_JA, IMAC). More scenarios can be added.
- plot_DNS.py: plot τ - ℓ diagrams of numerical simulations, given their spherical harmonic degree n -spectra.
- plot_experiment.py: plot τ - ℓ diagrams of Lab experiments, given their wavenumber k -spectra.
- plot_Kolmogorov.py: plot τ - ℓ diagram of Kolmogorov's universal turbulence, as an example.
- plot_convection_onset.py: plot dynamo onset parameters for rotating and non-rotating convection (full sphere).
- tau_ell_lib.py: library gathering τ - ℓ conversion rules from spectra, graphical functions, and other common functions.

The numerical simulation and Lab experiment data used for the examples shown in our article are available in folders:

- DNS_Guervilly: u and ρ spectra of Guervilly et al. [2019]'s 3D rotating convection simulation at $Ek = 10^{-8}$.

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- DNS_Schaeffer: u , b , and ρ average spectra of Schaeffer et al. [2017]’s S2 numerical geodynamo simulation.
- DNS_Dormy: u , b , and ρ average spectra of weak and strong dynamo numerical simulations proposed by Dormy [2016].
- experiment_DTS: u and b wavenumber spectra of a composite run of the DTS liquid sodium experiment [e.g., Brito et al., 2011].

This package is also available at https://gricad-gitlab.univ-grenoble-alpes.fr/natafh/shell_tau-ell_programs.

S2. τ - ℓ regime diagram of the DTS liquid sodium experiment

We think that laboratory experiments can also provide a better perspective when translated into τ - ℓ regime diagrams. As an illustration, we present in Figure S1 the composite τ - ℓ diagram of a representative run of the DTS experiment. The DTS experiment is a magnetized spherical Couette experiment. Fifty liters of liquid sodium are enclosed in a spherical container ($R_o = 0.21$ m) that can rotate around a vertical axis. A central inner sphere ($R_i = 0.074$ m) can rotate independently around the same axis, and hosts a strong permanent magnet producing an axial dipolar magnetic field. See Nataf et al. [2008], Brito et al. [2011] for more details.

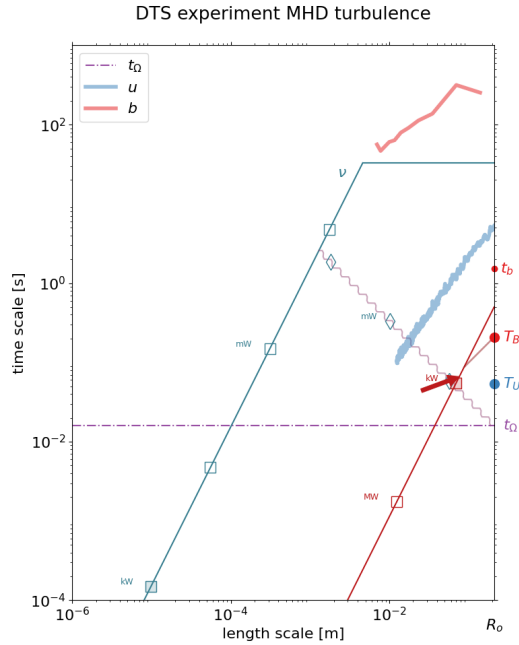


Figure S1. Composite representative τ - ℓ diagram of DTS magnetized spherical Couette laboratory experiment. Outer shell rotation frequency is 10 Hz, while inner sphere spins at 20 Hz. Time T_B (large red dot) is obtained from the energy of the dipolar magnetic field applied by the inner sphere magnet, while time t_b (small red dot) represents the time-averaged induced magnetic field. Time T_U (blue dot) is deduced from the time-averaged kinetic energy. Red horizontal dotted line marks dissipated power (700 W). Thick red line $\tau_b(\ell)$ is obtained from a k -spectrum of magnetic fluctuations measured at the outer shell surface. Thick blue line $\tau_u(\ell)$ is obtained from a frequency-spectrum of flow velocity fluctuations measured at mid-depth in the fluid using Ultrasound Doppler Velocimetry.

We draw τ_ν and τ_η lines from properties of liquid sodium at 130 C ($\nu = 6.5 \times 10^{-7} \text{ m}^2\text{s}^{-1}$, $\eta = 8.8 \times 10^{-2} \text{ m}^2\text{s}^{-1}$). Power markers along these lines are deduced from liquid volume and density ($\rho = 930 \text{ kg m}^{-3}$). From the energy of applied dipolar magnetic field, we deduce time T_B and draw the $\tau_{Alfven}(\ell)$ red wavy line of Alfvén wave propagation. We pick a run with outer shell rotation rate $f_o = 10 \text{ Hz}$, which yields horizontal line t_Ω and wavy line τ_{Rossby} .

We consider an inner sphere differential rotation rate $\Delta f = 10 \text{ Hz}$, for which a power dissipation $\mathcal{P}_{diss} \simeq 700 \text{ W}$ is measured. Cabanes et al. [2014] reconstructed time-averaged flow and induced magnetic field by a joint inversion of a comprehensive set of flow velocity profiles and magnetic field measurements. Their Table III provides the energies of these fields, which we convert into vortex overturn time at integral scale T_U , and the corresponding time for induced magnetic field t_b . Note that their analysis is for $f_o = 0$, but should approximatively apply to our case.

The three components of the induced magnetic vector are measured every 6° along a meridian between latitudes -57° and $+57^\circ$. We thus computed a k -spectrum of magnetic energy density from a set of 60s-long records, which was converted into line $\tau_b(\ell)$ according to equation (42) of the article's Appendix A. No velocity measurement was available for that run, but a nice profile of angular velocity was measured using Ultrasound Doppler Velocimetry for another run with $f_o = 5 \text{ Hz}$ and $\Delta f = 10 \text{ Hz}$. Using equation (61) of Appendix A, we extract the frequency power spectral density of a 40s-long record at fluid mid-depth, and convert it to a kinetic energy density k -spectrum using equation (62) with U deduced from the same profile, yielding line $\tau_u(\ell)$ drawn in Figure S1. Note that this spectrum might be contaminated by instrumental noise. The resulting τ - ℓ diagram suggests that velocity fluctuations are mostly quasi-geostrophic because line τ_u plots above Rossby line. Magnetic energy fluctuations are in a strongly dissipative region, and are almost three orders of magnitude smaller than kinetic energy fluctuations, in agreement with observations of Figueroa et al. [2013]. Under the combined influence of strong rotation and strong imposed magnetic field, energy fluctuations of both types are two to three orders of magnitude smaller than time-averaged energies (tagged by T_U and t_b in Figure S1), as noted by Nataf and Gagnière [2008], and Kaplan et al. [2018].

The short red wavy line $\tau_{Alfven}(\ell)$ indicates that geostrophic Alfvén waves can propagate but are severely damped, as analyzed by Tigrine et al. [2019]. Dissipation is dominated by Ohmic dissipation of the time-averaged flow.

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