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Westward drift, core motions and exchanges of angular momentum between core and mantle

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The westward drift is one of the most well known features of the geomagnetic field. In this paper we come back to the apparent drift of the main field as seen at the Earth's surface, and show that there is no evidence for a body drift but rather clear evidence for a latitude-dependent drift. Computing the fluid flow at the core mantle boundary (CMB), in the frozen flux and geostrophic hypothesis, we also obtain, among other components, a clear differential zonal rotation of the fluid, symmetrical with respect to the Equator. This differential rotation changes with time, with a time constant of the order of 10 years. Dynamic considerations lead us to think that, during such a time interval, the change in the zonal toroidal flow inside the core consists of motions in which axial cylindrical annuli rotate rigidly about the Earth's axis of rotation. Then, for the 1969–85 time-span, the change in the core angular momentum is shown to balance the change in the mantle angular momentum.

Bullard *et al.*[1] stated that the westward drift u of the geomagnetic field presents no clear variation with latitude; this point has been reasserted[2,3] since then. Bullard *et al.* compared two charts of the main field, one relative to 1907.5, the other one relative to 1945 and estimated the drift of the three elements X, Y, Z between 1907.5 and 1945, on nine different parallels. Their results are illustrated by Fig. 1*a*, based on their Table 8. Whereas the curves relative to Y and Z (u_Y and u_Z) are similar (presenting higher values around the Equator), the one relative to X (u_X) displays two peaks, at the colatitudes 50° and 130°. Considering an average of u_X, u_Y, u_Z, Bullard and his co-authors inferred that no variation of u with latitude could be significantly determined.

In the present study the drift rate $u_E(\theta)$ of element $E(E = X, Y, Z)$ for colatitude θ has been computed by minimizing the quantity

$$d^2 = \int_0^{2\pi} \left(\frac{\partial E}{\partial t} - u_E(\theta)\frac{\partial E}{\partial \phi}\right)^2 \mathrm{d}\phi \qquad (1)$$

We have used the models of Vestine *et al.*[4] of the main and secular variation (SV) fields up to 1955, then IGRF models[5-7]. The representative curves (Fig. 1*b, c, d*) look like the Bullard *et al.* curves (Fig. 1*a*). For each element, the general trend is quite coherent from one curve to another, and suggests a differential rotation. We can make a guess about the different behaviour of X on one hand, Y and Z on the other hand. A simple differential rotation $u(\theta)$ of the geomagnetic potential about the polar axis would indeed imply

$$\frac{\partial V}{\partial t} - u\frac{\partial V}{\partial \phi} = \frac{\partial Y}{\partial t} - u\frac{\partial Y}{\partial \phi} = \frac{\partial Z}{\partial t} - u\frac{\partial Z}{\partial \phi} = 0$$

but

$$\frac{\partial X}{\partial t} - u\frac{\partial X}{\partial \phi} = \frac{1}{a}\frac{\partial u}{\partial \theta}\frac{\partial V}{\partial \phi}$$

where a is the Earth's radius and r, θ, ϕ the usual spherical coordinates. Then if the latitude distribution of u were represented by a square window function, then u_X would display two peaks at the edges of the window, as observed in Fig. 1.

The body drift of the main field, as seen by Bullard *et al.*, was tentatively explained by a body rotation of the fluid of the core relative to the mantle at the CMB ($r = b$) (a true motion or a wave; see refs 1 and 8). Now from the observation of a latitude-dependent drift of the field, we rather expect a rotation of the fluid of the core at the CMB varying with latitude. Therefore we shall consider the zonal part of the toroidal ingredient of the flow computed at the CMB by inverting SV data in the frozen flux[9,10] and geostrophic approximations[11-13]. The horizontal flow velocity $\mathbf{v}$ at the CMB is written in the form

$$\mathbf{v} = \mathbf{t} + \mathbf{s} = \mathbf{n} \wedge \nabla T + \nabla S$$

T and S are the toroidal and consoidal scalars, $\mathbf{n}$ is the unit outward radial vector. In our computations the expansions of S and T in spherical harmonics are limited to degree 10. We have computed the flow at the CMB from 1969 to 1987, using the SV models listed in Table 1[6,14-23]. We have then extracted from $\mathbf{v}$ its toroidal axisymmetric part $\mathbf{t}_{\mathrm{ax}}$

$$\mathbf{t}_{\mathrm{ax}}(\theta) = -\mathbf{n} \wedge \nabla\left(\sum_{n=1}^{\infty} d_n P_n(\cos\theta)\right)$$

$$= \sum_{n=1}^{\infty} \Theta_n^0 = t_{\mathrm{ax}}(\theta)\hat{\phi} \qquad (2)$$

P_n is the Legendre polynomial of degree n, d_n are the coefficients (dimension = length^2/time) of the expansion of the zonal part of T, $\hat{\phi}$ the unit azimutal vector.

Figure 2 illustrates the distribution versus latitude of the angular velocity $w(\theta)$ ($w = t_{\mathrm{ax}}/b \sin\theta$). $w(\theta)$ appears to be quite symmetrical about the Equator, larger at low latitudes (so are the apparent drift rates of the elements Y and Z). Its distribution may be represented by a rectangular window centred on the Equator (this distribution accounting for the two peaks noticed on the distribution of $u_X(\theta)$). The amplitude of the so-obtained zonal core motions is weaker (by at least a factor two) than the estimates of the apparent drift rate at the Earth's surface (ref 1); this point is reminiscent of the results of Kahle *et al.*[24]. Indeed most of the secular variation must be attributed to non-zonal motions, whereas the estimates of the drift given by (1) try to represent the SV only by zonal motions.

Is it possible to infer some knowledge of the flow inside the core from the distribution of the zonal toroidal component $\mathbf{t}_{\mathrm{ax}}$ of the so-computed surface flow at the CMB? In an incompressible fluid enclosed within a sphere, the system of equations governing the slow motions ($|\partial\mathbf{u}/\partial t| \ll |2\mathbf{\Omega} \times \mathbf{u}|$) is

$$2\rho\mathbf{\Omega} \times \mathbf{u} = \mathbf{F} - \nabla \mathrm{p}, \qquad \mathrm{div}\,\mathbf{u} = 0 \qquad (3)$$

(where $\mathbf{u}$ is the flow in the frame rotating at angular velocity Ω, $\mathbf{F}$ the sum of all the forces, except the pressure contribution, applied to each element of fluid). Taylor[25] showed that the system (3) has a solution $\mathbf{u}$, $\mathbf{F}$ being given, if and only if

$$\Gamma = \mathbf{k} \cdot \int_{s=s_0} \mathbf{r} \times \mathbf{F}\, \mathrm{d}S = 0 \qquad (4)$$

(s is here a cylindrical radius, $\mathbf{k}$ the unit vector of the axis of rotation). In other terms, there is no mean torque Γ on any co-axial annular cylinder. Rigid rotations $\mathbf{t}_s$ of axial annular cylinders, about the axis of rotation appear as arbitrary motions (integration constant of the system (3)) (see ref 26). When this condition is not fulfilled ($\Gamma \neq 0$), it becomes necessary to introduce a relative acceleration. One can show, as Braginsky did[27], that only the relative accelerations t_s of the cylindrical annuli have to be taken into account. These accelerations respond to the average torque. Coriolis acceleration responds to all other forces through the system (3) slightly modified. In fact if $\Gamma \neq 0$, we can introduce $\mathbf{F}_0$ such that

$$\mathbf{F}_0 = F_0\hat{\phi}, \qquad \frac{\partial F_0}{\partial z} = 0, \qquad \Gamma = \int_{s=s_0} (\mathbf{r} \times \mathbf{F}_0) \cdot \mathbf{k}\, \mathrm{d}S$$

and the system (3) becomes

$$2\rho\mathbf{\Omega} \times \mathbf{u} = \mathbf{F} - \mathbf{F}_0 - \nabla \mathrm{p} \qquad \mathrm{div}\,\mathbf{u} = 0 \qquad \rho\partial \mathrm{t}_s/\partial t = F_0$$

Roberts[28] conjectured that if this kind of flow is responsible

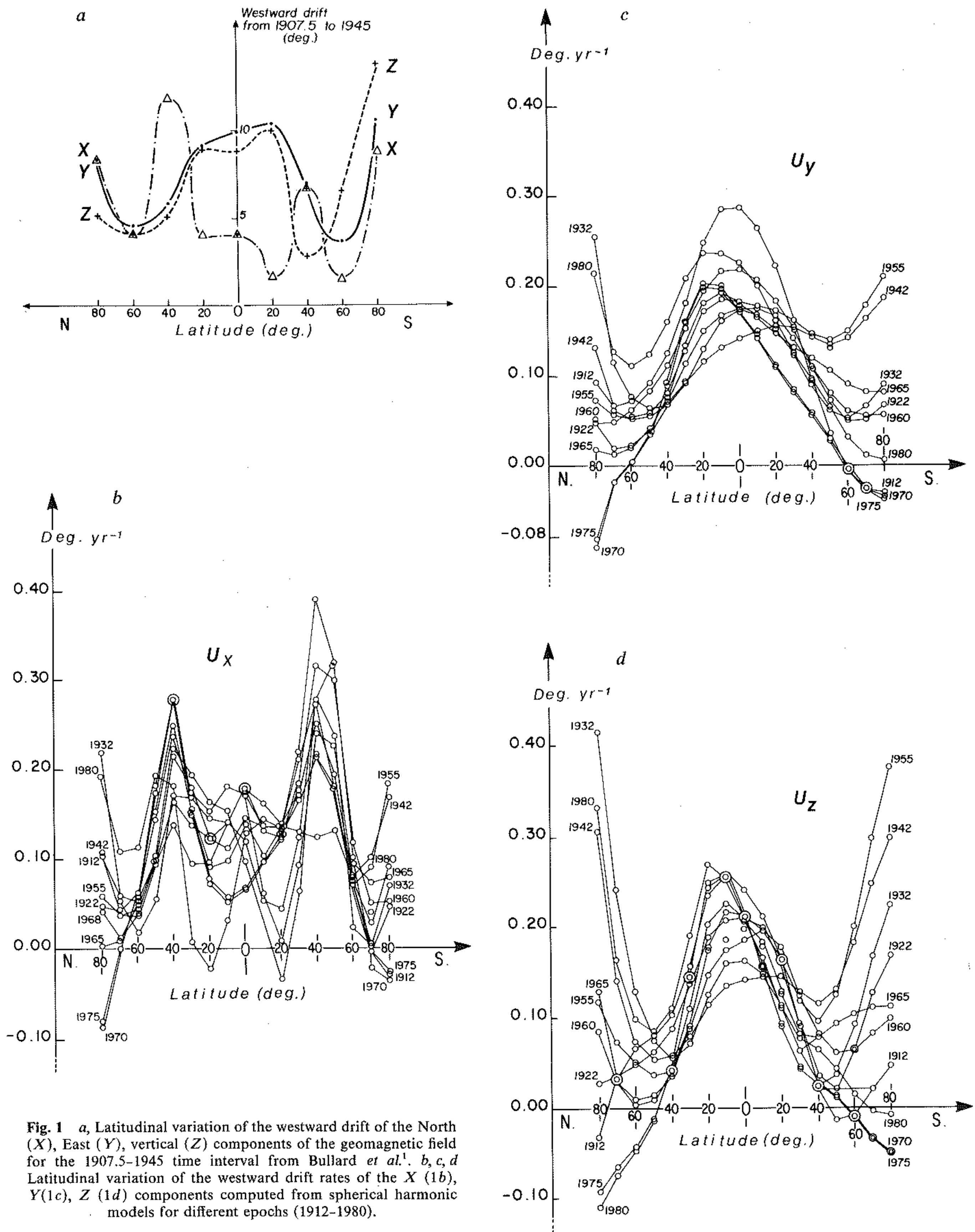

Fig. 1 *a*, Latitudinal variation of the westward drift of the North (X), East (Y), vertical (Z) components of the geomagnetic field for the 1907.5–1945 time interval from Bullard *et al.*[1]. *b, c, d* Latitudinal variation of the westward drift rates of the X (1*b*), Y(1*c*), Z (1*d*) components computed from spherical harmonic models for different epochs (1912–1980).

Table 1 Values of the angular rotations $d1/b^2$, $d3/b^2$, W_e (b being the core's radius) in degrees per year computed from different models, which are referenced

Year	Model	$d1/b^2$	$d3/b^2$	W_e	Present on Fig. 2?	Ref.
1969	SVHO 69	−0.073	0.014	−0.049	y	13
1970	SVAW 70	−0.074	0.015	−0.048	n	6
1970	SVDK 70	−0.064	0.011	−0.045	n	14
1975	SVDK 75	−0.069	0.009	−0.052	y	14
1980	GSFC 80	−0.089	0.018	−0.057	y	16
1980	USGS 80	−0.094	0.015	−0.068	y	15
1982	SVDK 82	−0.098	0.018	−0.067	n	14
1982	USGS 82	−0.117	0.027	−0.072	n	18
1982	SVIZM 82	−0.098	0.018	−0.067	n	19
1982	SVQKB 82	−0.128	0.027	−0.082	n	20
1985	IGRF 85	−0.116	0.030	−0.064	y	17
1987	WC 87	−0.16	0.047	−0.079	n	21
1987	SVBK 87	−0.159	0.048	−0.077	n	22

Symbol y means that the model has been used to draw Fig. 2, symbol n that it has not.

for the westward drift, then the changes in the drift will be quicker in low-latitude zones (because of the reduced inertia of the external cylinders). The surface flow (at the CMB) corresponding to these cylindrical annuli motions is an axisymmetric toroidal flow; we will identify it with the motion $\mathbf{t}_{ax}$ computed above (implying that $\mathbf{t}_{ax}$ must be symmetrical about the Equator). Therefore, knowing the motion $\mathbf{t}_{ax}$ and its time variation at the CMB will allow us to compute the time changes in the cylindrical annuli motions and then the time changes in the angular momentum of the core.

The so-called decade variations in length of day are attributed for the main part to exchanges of angular momentum between the liquid core and the solid mantle[1,28–32]. Nevertheless, atmospheric phenomena, especially South Pacific oscillations[33] and secular decrease of the rotation's rate[34] also contribute to the changes in angular momentum at the periods considered here. To estimate the changes $\Delta\sigma_m$ in the mantle angular momentum that must be balanced by changes $\Delta\sigma_c$ in the core angular momentum, this contribution has been removed. We have therefore used the estimates of the Earth's rotation rate for the 1969–87 period, as proposed by Feissel and Gavoret[35] who have separated the effect of pacific oscillations from the Earth's rotation data. The secular drift has been approximated by a linear trend (2 ms per century). The estimate of the angular momentum of the core depends only on the zonal toroidal component of the flow that, as shown above, is organized in co-axial annular cylinders whose surface expressions are known

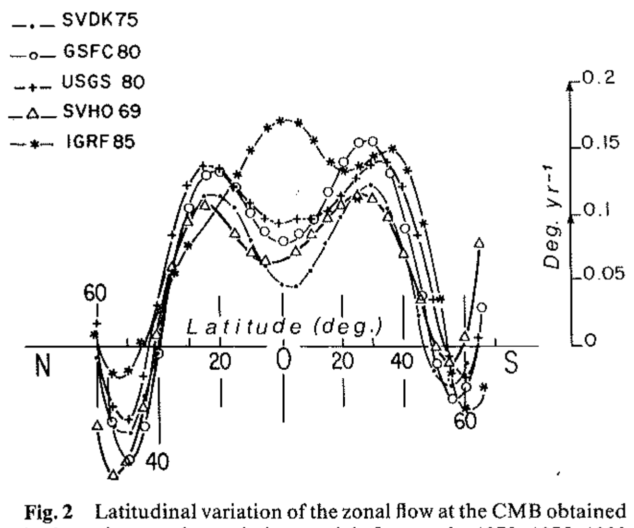

Fig. 2 Latitudinal variation of the zonal flow at the CMB obtained by inverting secular variation models for epochs 1970, 1975, 1980 and 1985 (see text).

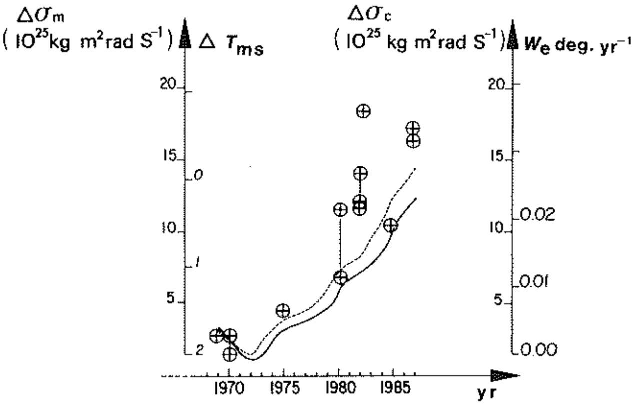

Fig. 3 Changes in the angular momentum of the mantle and of the core for the recent epoch (1969–1987). Solid line, $\Delta\sigma_m$ (see text) or ΔT (in ms); broken line, $\Delta\sigma_m$ (or ΔT) corrected from the secular decrease in length of day T; ⊕, estimates of $-\Delta\sigma_c$.

($\mathbf{t}_{ax}$). Then, if in a first approximation the core density is supposed to be uniform, the change in the core angular momentum depends only on the changes Δd_1, Δd_3 of the two coefficients d_1, d_3 of expansion (2)

$$\Delta\sigma_c = -(8\pi/15)\rho b^3(\Delta d_1 + (12/7)\Delta d_3)$$
$$= -(8\pi/15)\rho b^5 \Delta w_e$$

Δw_e is the change of the 'equivalent rotation' w_e

$$b^2 w_e = d_1 + (12/7) d_3 \qquad (5)$$

To estimate w_e, we have used the estimates of d_1 and d_3 (Table 1) obtained when assuming in addition the symmetry of the zonal toroidal part of the motion. Fig. 3 displays the changes $\Delta\sigma_m$ and $\Delta\sigma_c$ for the time-span 1969–1985. Taking into account the uncertainties on both estimates, the relation $\Delta\sigma_m = -\Delta\sigma_c$ can be considered as reasonably checked. Overestimates of the westward drift rate changes had led to the idea that only a superficial shell, the thickness of which being taken as an adjustable parameter, would be involved in the balance[1,31]. On the other hand, the flow considered here allows us to verify the balance between the angular momentum of the core and the mantle without the need of any variable parameter. We must investigate whether this relation holds for earlier epochs. In the present state of our computations the curves $t_{ax}(\theta)$ present an increasing dissymmetry from 1965 to 1940 forbidding the strict application of formula (3); it could be because of the poor quality of the corresponding SV models (in particular the earlier models are based on very few observatories in the South Hemisphere (see Fig. 2 of Langel *et al.*[17])). Nevertheless, the estimates of $\Delta\sigma_c$ before 1970 remain fairly well correlated with the estimates of $\Delta\sigma_m$; in particular we observe a decreasing trend of ($-\Delta\sigma_c$) before 1970. Decade changes in the mantle angular momentum as observed during the past 20 yr can be balanced by a flow made of cylindrical axial annuli and compatible with the values computed at the CMB from SV data. The phenomenon responsible for the (differential) westward drift of the geomagnetic field would then be a genuine motion of fluid and not a MHD wave propagating westwards. In this paper we have only described the global budget of angular momentum; it is now necessary to study the exchange and coupling mechanisms both inside the core[27] and between the core and the mantle[30].

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